Fundamental Driven Liquidity Traps: A Unified Theory of the Great Depression and the Great Recession

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Abstract

This paper integrates the New Keynesian literature on the liquidity trap to offer a unified theory of the Great Recession and the Great Depression in the United States. We first give an overview of fast moving forces that can lead to a liquidity trap, such as a banking crisis and debt deleveraging shocks and then turn to long acting forces such as the aging of the population and a rise in inequality. These latter forces are often associated with the idea of secular stagnation. We then analyze a number of monetary and fiscal policy interventions within the standard New Keynesian model as well as in a model in which there is a permanent trap. The paper closes by applying the theory to understand the onset and recovery from the U.S. Great Depression and addressing key questions related to the U.S. Great Recession.

— Preliminary, First Draft, Comments Welcome, please address to Gauti_Eggertsson@brown.edu —-

Introduction

It is sometimes argued that the crisis of 2008, often referred to as the Great Recession (GR), caught the economic profession by a surprise, that this suggested some major embarrassment to the profession, and to the field of macroeconomics in particular. While it is true that the events of the 2008 were not expected by many, lost in this narrative is that prior to the crisis there was a relatively mature literature that studied in some detail the type of events that lead to the crisis. Moreover, immediately following it the profession reacted quickly to fill in some gaps in the existing literature – gaps that were in our view relatively well understood – so that we think it is fair to say that now, 11 years later as of writing, the events that unfolded not only in 2008, but also during the Great Depression (GD), can be understood within a common conceptual framework. The objective of this article is to give an overview of this literature, with a few new extensions that are aimed at simplifying and unifying it.

We use the term "liquidity trap" to describe the economic environment faced by the much of the world economy in 2008 and during the Great Depression. To be clear, what we mean by using this term is plainly the observation that during this time period the short-term nominal interest rate was very close to zero. This very fact leads to some fundamental considerations related to the nature of money which are not as apparent during other periods of time. The reason for this is that as long as paper currency is available for people to hold as an asset, the nominal interest rate cannot go (much) below zero: nobody wants to lend away one dollar, unless they get at least one dollar back. Otherwise they would sit on the dollar instead of lending it out. A key aspect of the liquidity trap is that it is often associated with unemployment, subpar growth and inflation below the target of the central bank. This implies that even if the central bank would like to stimulate the economy with interest rate cuts, it cannot do so due to the zero bound.

Several central banks experimented with paying slightly negative interest rate on bank reserves following 2008. This has lead researchers to talk about an effective lower bound on in interest rate, which may be somewhat below zero. Accordingly we will use the term effective lower bound (ELB), but much of the literature instead refers to the zero lower bound (ZLB). Nothing in the paper relies on the exact number which constrains monetary policy, all that matters is that there is some constraint on how much central bank can cut their policy rate. Many of the insights of the literature considered in this paper also apply to more general environments in which governments are for some reason constrained in their monetary policy choices, for example due to the country in question belonging to a currency union or if they have pegged their exchange rate.

Broadly speaking the theory of liquidity trap outlined here relies on two pillars. The first is the *impulse*, or *trigger* of the crisis. The second is a *propagation mechanism* for the trigger to morph into unemployment and/or subpar output growth. By trigger we mean the fundamental shock that gives rise to the drop in interest rate. By propagation mechanism we mean frictions that prevent a natural adjustment to the impulse at full employment. The literature has provided a number of plausible triggers for the crisis. Broadly, the trigger explanations can be categorized into fast and slow acting. The fast acting are mainly stories related to turbulence in the banking sector, over accumulation of private (or public) debt, as well as waves of pessimism or other "demand shocks". In the category of slow acting are forces related to demographics, productivity or income inequality, often discussed under the rubric of *secular stagnation*. All these stories have in common that they would require negative real interest rates for some time for full employment and stable prices. The principal propagation mechanism we review – which we take to be reason why there is not a natural adjustment to full employment – is the ELB and nominal frictions in product or labor markets, i.e. prices and/or wages do not seamlessly adjust as would be required by fully efficient markets. These mechanisms combined imply that the real interest rate, i.e. the nominal interest rate net of expected inflation, cannot fall enough to match the natural rate of interest, a concept we discuss in detail. The result is unemployment.

We start the paper by illustrating a simple parable – a miniature model – that puts this story into a pair of equations. Relative to old style accounts of the Great Depression, such as the standard IS-LM model of undergraduate textbooks, the fundamental difference is the central role played by expectations. This leads to fundamentally different prescription about the effect of policy on economic outcomes and requires a careful description of *policy regimes*, both fiscal and monetary, a central theme in the recent literature and this paper. It also changes in important way how one thinks about the role of monetary policy. Another important difference relative to old vintage models often found in undergraduate textbooks is that the modern theory applier here puts much more structure on the environment, which allows a more careful study of the plausibility of alternative stories for the trigger but also allows one to evaluate the plausibility of the suggested propagation mechanisms, policy options and the way in which they may operate.

The title of this paper is "fundamental driven liquidity traps." The choice of words here is to distinguish the current theory from another well developed literature which treats the zero bound as a consequence of self-fulfilling expectations, originating in the important and influential work of Benhabib et al. (2001). We have chosen to focus on this part of literature to narrow the scope of the paper. It is worth noting that the two theories are not necessarily inconsistent with one another. It is very well possible that a given crisis contains both some elements of self-fulfilling beliefs as well as fundamental factors, even if this kind of integration has not been the focus of the recent literature. [1] It is worth highlighting two other other important limitations of this overview that also limit it scope. We have chosen here to focus on a closed economy setting but an important strand of the literature considers open economy dimensions.[2] Similarly, the models considered assume rational expectations. An important strand of the literature explores the consequences of relaxing this assumption.[3]

The literature this paper builds upon, as most modern macroeconomics since the 1970's, is different from the the old Keynesian literature (from which the study of liquidity traps originated) in one fundamental respect. The behavior of people in the economy is derived by careful description of the choices and constraints the economic agents face and the equations describing the economy are essentially first order conditions of maximization problem of the economics actors along with resource

[1] A recent notable exception is Brendon et al. (2019).

[2] For an early overview of this literature see Svensson (2003).

[3] Examples in this literature include Benhabib et al. (2014) and Evans et al. (2008). For an overview of the literature that deviates from rational expectations, see Woodford (2013)

constraints. A given model, thus, often requires considerable space in describing the economic environment. To be able to summarize the literature in one article, however, we are forced to go somewhat against its spirit and – in the interest of space – relegating complete and careful description of the environment to Appendixes. We want to be upfront about that in doing so, however, we are in no way minimizing the importance of being explicit about the formal environment. Indeed it is the central advantage of the modern framework. If there are questions that arise about the plausibility of particular mechanism and/or the effect of a particular policy, for example, the modern approach allows one a natural way of addressing such questions: simply ask the agents inside the model about why they do what they are doing or how they would react to different alternatives.

We picture a reader of this paper as doing so with one eye on the main text and the other on the Appendix. We have striven to keep the Appendix detailed with formal definitions of the economic environment and careful description of the technical arguments laid out. In conjunction with the article, with several co-authors we have written a "toolkit" designed to solve problems of the same kind as outlined here. All the simulation in the article, as well as the data, are available online for replication or for use for further investigation. The hope is that this will facilitate further work in this area.

1 A Simple Parable to Explain a Liquidity trap

1.1 Fast moving forces

We start of with a simple parable, a variation of Krugman (1998) and Eggertsson and Krugman (2012), to illustrate the two basic foundations of the modern theory of the liquidity trap, i.e., the trigger and propagation of the crisis. The example has the virtue of being simple enough to be boiled down to two equations, and thus may be useful for readers to understand the gist of the issues underlying the modern theory.[4] The first step of the analysis clarifies what we mean when we say that the trigger of the crisis is a force that leads to a reduction in the "natural rate of interest" to a negative level. The second step clarifies why the ELB may make the adjustment difficult when there are nominal frictions in prices and/or wage adjustment.

The trigger of the crisis is a negative natural rate of interest. The natural rate of interest is often defined as the real interest rate if prices and wages are flexible. For pedagogic purposes the natural rate of interest can also be thought of as the real interest rate that would take place in an *endowment economy,* i.e. a world in which output is exogenously given and all the prices are flexible. This definition has the virtue that the details of the firms problem can omitted. As it turns out, the real interest rate derived in this way is often (as will be the case here) the same as in the more general setting where output is endogenous.

[4]Even if we leave blow by blow derivation to Appendix.

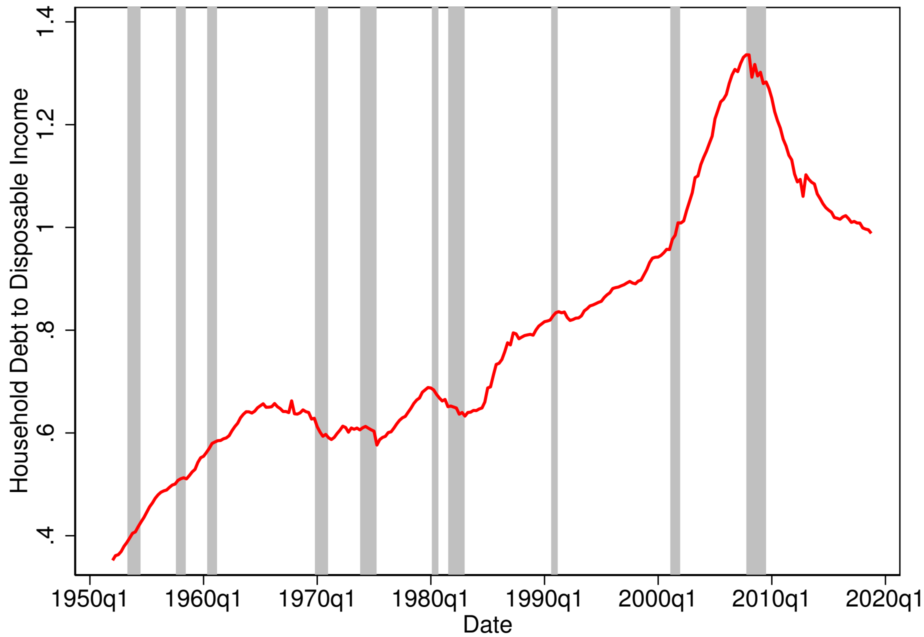


Figure 1: Ratio of Household Debt to Disposable Income

The main idea behind the first theory of the trigger is most easily explained by looking at the data. Figure 1 shows the ratio of household debt to disposable income for the US. As this figure reveals, there was a rapid increase in the household debt from less that 100 percent of disposable income in 2000 to over 130 percent of disposable income at the eve of the crisis. The rapid accumulation of private debt has been carefully documented by several authors, notably in important work by Mian and Sufi (2009). A key aspect of the crisis, and one that we will model, is that it corresponds roughly to the "realization" of a vast number of agents in the economy that the debt accumulation process has gone "too far". The crisis is thus modeled as a rapid "debt deleveraging" of several of the players that fueled their investment and consumption by borrowed money leading up to the crisis. The moment of truth, when indebted players – or alternatively the banks that lent them the money – realize this has gone to far is often referred to as a Minsky moment. We now illustrate this idea.

Imagine a world inhabited by two types of people, savers (s) and borrowers (b), that only differ between one and the other in that the saver is more patient. His discount factor is given by $\beta^s > \beta^b$. Each derives utility via a log utility function

$$E_t \sum_{t=0}^{\infty} \beta^{(i)t} \log C_t(i) \tag{1}$$

s.t.

$$D_t(i) = (1 + r_{t-1}) D_{t-1}(i) - \frac{1}{2} Y + C_t(i)$$

where E_t is an expectation operator, $C_t(i)$ is consumption and $i = \{b, s\}$ denotes borrower and saver respectively. $D_t(i)$ is one period real debt, r_t is the real interest rate, and Y is per period output endowment. In addition each agent faces a debt limit given by

$$(1 + r_t)D_t(i) \leq D^{high} \tag{2}$$

as in Eggertsson and Krugman (2012). Limit is due to some incentive constraint that for simplicity is taken to be exogenous.

It is easy to show that for D^{high} low enough the impatient agent is up against his borrowing constraint in steady state, implying equation (3) which says that the debtor fully consumes his income endowment in steady state net of the interest rate payment. This being an endowment economy, all production is consumed, so the saver similarly consumes his endowment and the interest he gets paid on the debt owed to him by the debtor, as shown in (4)

$$C^b = \frac{1}{2}Y - \frac{r}{1+r}D^{high} \tag{3}$$

$$C^s = \frac{1}{2}Y + \frac{r}{1+r}D^{high} \tag{4}$$

While the borrower is at a corner solution, consuming as much as his borrowing limit permits, the consumption of the saver satisfies a standard Consumption Equation which determines the real interest rate in equilibrium

$$\frac{1}{C_t^s} = (1 + r_t)\,\beta^s E_t \frac{1}{C_{t+1}^s}$$

suggesting a steady state real interest rate of $r = \frac{1-\beta}{\beta}$.

This basic model is useful to think about the crisis as a debt driven phenomenon that is induced by a temporarily negative natural rate of interest. Debt driven theories of crisis, typically, propose that the safe level of debt varies over time, sometimes abruptly. A drop in the safe level of debt is how we model the Minsky moment, i.e. it is an unanticipated reduction in the debt limit so that $D^{high} \to D^{low}$. Agents in the economy suddenly come to realize that the debt level is "too high" to be sustainable and accordingly the debtors need to pay it down, or "deleverage". It is easy to show that the model above adjusts to a new steady state in only one period which looks the same as the old one, but with D^{high} replaced with D^{low}. Let us call the adjustment period "short-run", denoted S, and the new steady state "long-run", denoted L. With a little algebra, the real interest rate in the short run can then be written as

$$1 + r_S = \frac{\frac{1}{2}Y + D^{low}}{\beta\frac{1}{2}Y + \beta D^{high}} \tag{5}$$

which can be negative if D^{low} is low enough. Meanwhile, after this adjustment period, the real interest is once again given by $r_L = \frac{1-\beta}{\beta}$.

What happens here is that Minsky moment triggers the need for borrowers (who had "over-extended themselves" in borrowing) to suddenly reduce their spending. As all production is consumed, the only way to make up for the borrowers cutting their spending is by the saver increasing their spending instead. The way savers are induced to spend more happens via a fall in the real interest rate. The debt deleveraging shock, thus, gives a simple story for a fall in the natural rate of interest during Great Recession and the Great Depression. The crisis forced a large part of the economy to cut back on their spending. To make up for this drop somebody else needed to spend for aggregate spending to remain unchanged, requiring a drop in the interest rate.

Generally, almost any source of a fall in aggregate spending can be cast as a culprit, and as we shall see in the next subsection, any force that may affect the relative supply and demand of savings in the economy.

But is this a problem? It is easy to see, even without introducing any further frictions, why negative real interest rates may pose a challenge, using a simple modeling device introduced by Krugman (1998). Let us introduce money, M_t, and a price level which is the price of the consumption good in terms of money, P_t. Furthermore, as in Krugman (1998), let us assume a simple cash constraint $M_t \geq \chi P_t Y_t$ where $\chi > 0$. Let us suppose that policy in the long-run is given by $M_L = M^*$. In the long-run steady state then the price level is proportional to the money stock so that $P_L = \chi^{-1} M^* Y_L$.

We can introduce a short-term nominal interest rate by assuming that a one period risk-free government bond is traded. The pricing of this bond implies a standard consumption Euler equation:

$$\frac{1}{C_t^s} = (1 + i_t)\beta^s E_t \frac{1}{C_{t+1}^s} \frac{P_t}{P_{t+1}} \tag{6}$$

and since the household can hold money as an asset the household then

$$i_t \geq i^{elb}$$

where we allow for the possibility that effective lower bound may be below zero (for example, due to storage costs). For the purpose of the current discussion, we suppose that $i^{elb} = 0$. The two consumption Euler Equations imply equation (7) and the cash constraint (8).

$$1 + i_S = (1 + r_S)\frac{P_L}{P_S} \tag{7}$$

$$M_S \geq \chi P_S Y_S \tag{8}$$

that are plotted together in Figure 2, where P_L is exogenously pinned down by the policy regime and equal to $\chi^{-1} M^* Y_L$ and r_S is exogenous given by (5). A normal equilibrium is found in A, where the nominal interest rate is given by a the steady state real interest rate $\beta^{-1} - 1$. If the government increases money supply via open market operations in government bonds, then the LM curve shifts from LM_1 to LM_2, reducing the nominal interest rate and increasing the price level. The important case occurs when there is a debt deleveraging shock so that $r_S < 0$. In this case intersection occurs at point C.

Point C already reveals the key proposition highlighted by Krugman (1998). Increasing money sup-

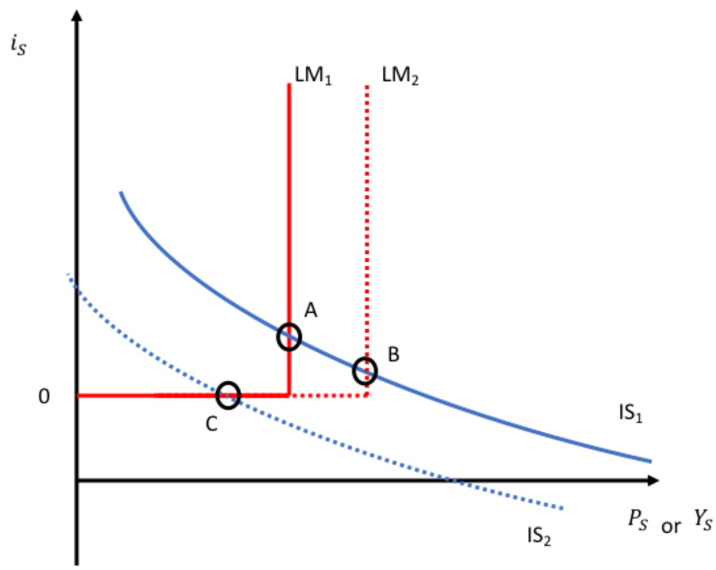


Figure 2: Liquidity Trap

ply via open market operations in government bonds has no effect – no matter how much money is printed. The key behind this proposition is that the future money supply is fixed. Hence, the increase in money supply is expected to be reversed as soon as the debt deleveraging shock is over and the economy normalizes. Accordingly, there is no effect on the long run price level $P_L = \chi^{-1}M^*$. Meanwhile, at point C the the nominal interest rate is zero. Thus, money and government bonds are perfect substitutes so increasing the money supply by buying government debt is akin to substituting blue M&M's with yellow M&M's: one is taking from the agent one piece of paper with zero interest rate (government bond) while giving the other (money). Households are completely indifferent between holding the two. Accordingly, there is no effect on prices. At zero interest rate LM equation is slack, households are no longer holding money exclusively to finance purchases, but also as an asset.

It is important to stress, however, that increasing money supply *permanently* is in fact going to have an effect in this economy. This is because a permanent increase in money supply increases the long-run price level, P_L, which is akin to shifting IS_2 curve towards IS_1 via expected inflation. If this shift is large enough both future *and* current prices will increase.

But why is nominal interest rate stuck at zero? The key reason is that the debt deleveraging shock shifts the IS curve back from IS_1 to IS_2 so that the real interest rate is negative. As we see from equation (7), with future price level fixed by the policy regime the only way real interest rate can be negative without violating the zero bound is via fall in the price level P_S, i.e., the short-run price level needs drop down to generate expected inflation. This fall in the price level happens even if the money supply is fixed in the short-run, or even if it is increased by an arbitrary amount. This is because expected inflation is *needed* to generate negative real interest rate, which in turn is needed to make savers increase their spending making up for the collapse in the spending of debtors.

The fact that the current price level needs to fall in response to the debt deleveraging shock may already give a hint towards why an adjustment to a shock of this kind is problematic in a more general settings where prices or wages do not adjust instantaneously. Consider now an extension in which output is not exogenously given, but instead produced by a profit maximizing representative firm that has a production function L_t^α where $1 > \alpha > 0$ and L_t is labor. Suppose labor is inelastically supplied by the two households. The firm decides how much to produce by hiring workers, taking the price level P_t and the wage rate W_t as given, resulting in the first order condition $\frac{W_t}{P_t} = L_t^{\alpha-1}$. A theory of how debt deleveraging shock translates into an output slump can now be understood by assuming that either prices or wages do not adjust, so that output adjust instead. The simplest extension (we will see more elaborate examples of below) would be to assume that the price-level in the short-run, P_S, is fixed, while it remains flexible in the long-run. In this case output is demand determined in the short-run.[5]

Output is the sum of both consumption of the borrower and the saver, which are given respectively by

$$C_S^b = \frac{1}{2}Y_S + \frac{D^{low}}{1+i_s}\frac{P_L}{P_S} - D^{high}$$

$$C_S^s = \frac{1}{1+i_S}\frac{P_L}{P_S}\beta^{-1}C_L^s$$

With flexible prices these equations together pinned down P_S while output was then exogenously given by the endowment. When the short-run price level is fixed, instead, then it is determined by aggregate spending, which by combining these two equations, taking account of $Y_S = C_S^s + C_S^b$, results in an IS equation that is analogous to the Fisher equation derived before and given by

$$1 + i_S = \frac{\frac{P_L}{P_S}(1 + r_S^n)}{1 + \frac{1+r_S^n}{2D^{low}}(Y_S - Y^n)} \tag{9}$$

where r^n is given by equation (5). The determination of Y_S and i_S can once again be plotted up as in Figure 2 but now with output on the x-axis.

The assumption of price rigidities gives equation (9) a fundamentally different interpretation relative to the Fisher equation which is just a pricing equation – in our example pinning down P_S given expectation about long-run prices. With rigid prices equation (9) instead says how much people spend, given expectations of the future price level and taking the prices today as given. Spending now translates directly into production – output is completely demand determined. In the absence of the zero bound the central bank can also set the interest rate so that the output demanded is equal to the natural rate of output by selecting $i_S = r_S^n$. This solution is infeasible, however, when r_S^n is negative and i_S becomes bounded by zero.

[5]This set of assumptions is an example of the general disequilibrium model proposed in Barro and Grossman (1971). For a review of this literature see Bénassy (1993).

Assuming that the monetary policy is set so that $P_L = P_S$ output is then given by

$$Y_S = Y^n + 2D^{low} - \frac{1}{1+r^n}$$

that is the stronger is the decline in natural interest rate, triggered by a more powerful deleveraging shock, the lower is output.

As in the case with flexible prices, increasing money supply has no effect on output. An important observation, which can be gauged by equation (9), is that commitment to increasing the future price level, P_L, is expansionary. This hints to a more general principle: affecting expectations about future policy can play a critical role in a liquidity trap. And in particular, positive expected inflation can be of great use, a major theme of what is to come in this survey. To analyze this channel, however, it is useful to move beyond the simple parable of the current section. This will be a major the focus of the remainder of the paper.

Overall, the simple example given here provides the most basic theory of 1) how a crisis is triggered via debt deleveraging shock and of 2) how this it gets propagated into an output drop via the ZLB and pricing frictions. A stark assumption in parable was that the borrower faced a hard borrowing constraint and cut down his spending in only one period. Benigno, Eggertsson, and Romei (2019) instead model the borrower as facing a borrowing cost that smoothly increases for debt above the "safe" level. In that model there are two interest rates, a risk free rate faced by the saver and a risky rate faced by the borrower. This model also gives rise to a debt deleveraging cycle. The duration of the debt deleveraging cycle is now endogenously determined. It is characterized by an increase in the spread between the two interest rates, as observed during the crisis. An important point highlighted in Benigno et al. (2019) is that the interpretation of a Minsky moment can either be that it corresponds to a reduction in the debt capacity of the household, or as being due to balance sheet problems of the banking sector. In either case they show that this disturbance shows up as an increase in the spread between interest rates faced by savers and borrowers.

Figure 3 shows one measure of spread, the discrepancy between Baa Moody corporate bonds and 10-year Treasuries. As this figure illustrates, there was a sharp spike in the spread during the crisis. However, shortly after the crisis (and certainly into late-2013 and 2014) the spread has returned to pre-crisis levels. Moreover, the run-up in debt, reported in Figure 1, appeared to have reversed itself. They key prediction of the deleveraging hypothesis was that this should imply that interest rate should normalize or – if a central bank would not increase rate – there would be inflation. During 2013 and 2014, however, interest rate remained at zero in the US, there were very little inflationary pressures in the economy and, in general, there was an alarm about anemic recovery. This led researchers to explore alternative forces that could be driving low interest rates that were not related exclusively to the financial crisis.

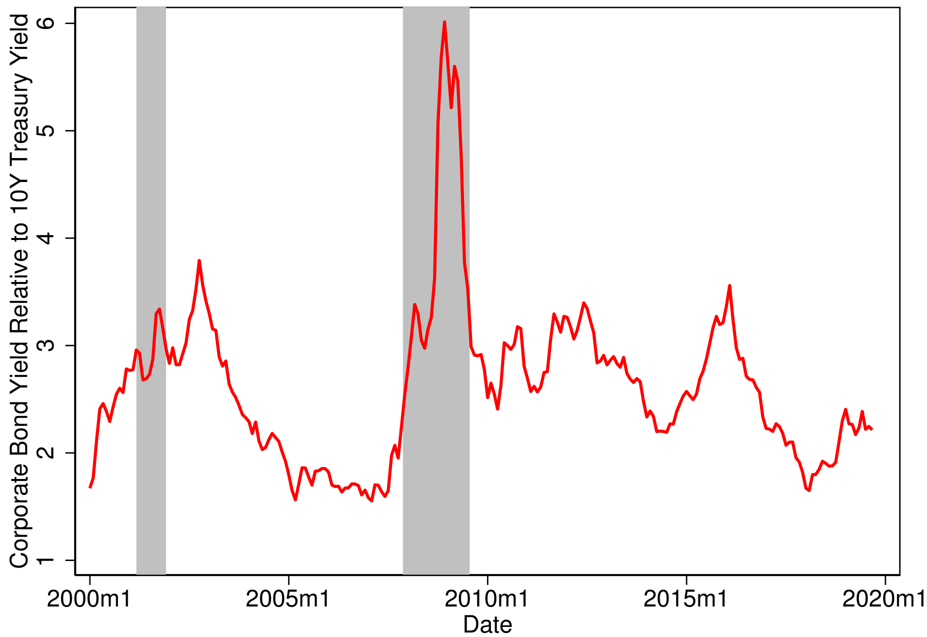


Figure 3: Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity

1.2 Slow moving forces

There is a sense in which the message of the previous section – modeling the trigger for the crisis – is somewhat optimistic. It says that once the debtors have reduced their debt down to a new, sustainable level the debt deleveraging cycle has reached its end and interest rate normalizes to a positive level. One strategy for dealing with a slump associated with a debt deleveraging cycle is then simply to wait it out – a policy that seemed to describe well the reaction of many governments to the crisis of 2008.

Figure 4 hints at why this assessment might be too optimistic for a reason we now formalize. While the short term interest rates collapsed to zero only following the financial crisis of 2008, the decline in interest rates has been continuous over the past several decades. This can be seen by the the evolution of long term interest rates depicted by 10-year Treasuries in the figure. This suggests that while the financial crisis was surely a harbinger for a strong, and perhaps temporary, collapse in the natural rate of interest rate, there may also have been other slower moving forces that also worked towards lower real interest rates, thus accounting for the lack of "normalization" of interest rates immediately following the crisis. It is much less obvious if those fundamental drivers would revert themselves. The possibility that the natural rate of interest is permanently negative is often termed the secular stagnation hypothesis, originally due to Hansen (1939), but recently resurrected by Summers (2014) in response to the absence of inflationary pressures and anemic growth in late-2013.

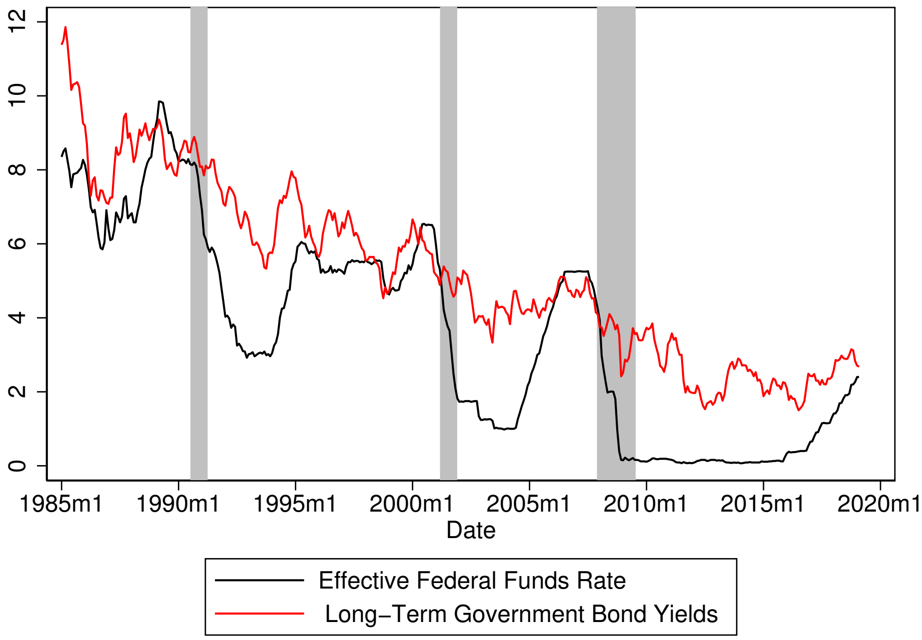


Figure 4: Short and long run interest rates

To highlight this possibility, consider a simple overlapping generations model as in Eggertsson, Mehrotra, and Robbins (2019c). This household has the following utility function:

$$\max_{C_t^y, C_{t+1}^m, C_{t+2}^o} \mathbb{E}_t \left\{ \log\left(C_t^y\right) + \beta \log\left(C_{t+1}^m\right) + \beta^2 \log\left(C_{t+2}^o\right) \right\},$$

where C_t^y is the consumption of the household when young, C_{t+1}^m its consumption when middle aged, and C_{t+2}^o its consumption while old. We assume that borrowing and lending take place via one-period risk-less bonds denoted B_t^i, where $i = y, m, o$ at an interest rate r_t. Given this structure, we can write the budget constraints facing households born at time t in each period as

$$C_t^y = B_t^y \tag{10}$$

$$C_{t+1}^m = Y_{t+1} - (1 + r_t) B_t^y + B_{t+1}^m \tag{11}$$

$$C_{t+2}^o = -(1 + r_{t+1}) B_{t+1}^m \tag{12}$$

$$(1 + r_t) B_t^i \leq D_t, \tag{13}$$

The young household does not work but borrows subject to debt limit as in Eggertsson and Krugman (2012). The middle aged household is the only one who works, so their income is given by Y_t. These households repay the debt they contracted when young and save for retirement when old. The old retire from work so their only income is their savings from working age. For low enough value of the debt limit it is now straightforward to show that the consumption of the young is given by

$$C_t^y = \frac{D_t}{1 + r_t} \tag{14}$$

while the middle-aged satisfy a consumption Euler Equation

$$C_t^m = \beta E_t \frac{1 + r_t}{C_{t+1}^o} \tag{15}$$

and the old consume all their savings so that

$$C_t^o = -(1 + r_{t-1}) B_{t-1}^m \tag{16}$$

We assume that the size of each generation is given by N_t. Let us define the growth rate of the new cohort by $1 + g_t = \frac{N_t}{N_{t-1}}$. Equilibrium in the bond market requires that borrowing of the young equals the savings of the middle aged so that $N_t B_t^y = -N_{t-1} B_t^m$ or

$$(1 + g_t) B_t^y = -B_t^m. \tag{17}$$

To analyze equilibrium determination, let us focus on equilibrium in the market for savings and loans given by equation (17) using the notation L_t^s and L_t^d; the left-hand side of (17) denotes the demand for loans, L_t^d, and the right-hand side its supply, L_t^s. Hence the demand for loans (using (14)) can be written as

$$L_t^d = \frac{1 + g_t}{1 + r_t} D_t \tag{18}$$

while an expression for loan supply, assuming perfect foresight, can be derived by combining the household budget constraints and the middle-generation Euler equation:

$$L_t^s = \frac{\beta}{1 + \beta} (Y_t^m - D_{t-1}) \tag{19}$$

with equilibrium given by

$$r_t = \frac{1 + \beta}{\beta} \frac{(1 + g_t)}{Y_t - D_{t-1}} \tag{20}$$

Simple as they are, we can make several observations from these derivations. First, and perhaps the most important one, is that in contrast to the model with a representative saver we saw before the steady state interest rate is no longer given by β^{-1} and thus is no longer constrained to be positive. Instead, it can be permanently negative. The second is that a debt deleveraging shock, if permanent, no longer has only temporary negative effect on the natural rate of interest. Instead, the effect on the natural rate of interest is permanent. Third, a slowdown in population growth, by reducing the relative size of borrowers (the young) to savers (the middle aged), if strong enough also can lead to the real internet rate being permanently negative. This is important, since most of the industrialized world is experiencing aging population, a phenomenon that is only expected to become more and more pronounced going forward. To the extent that these developments are driving the secular fall in interest rates in Figure 4, such decline is unlikely to reverse itself. Fourth, and more generally, with this being only one illustration, virtually any force that affects the relative supply of savings and demand for loans can affect the natural rate of interest – even in the steady state. Eggertsson, Mehrotra, and Robbins (2019c) and Eggertsson, Mehrotra, Singh, and Summers (2016a) suggest several possible candidates such as the increase in income inequality (increases relative savings), fall in productivity (reduces propensity to invest), fall in the relative price of investment and, specifically for the US, the global glut. Other forces work in the opposite direction (i.e. raising the natural rate of interest) such as the increase in government debt in the US and the pay as you go social security system.

1.3 Future directions: Towards a more general theory

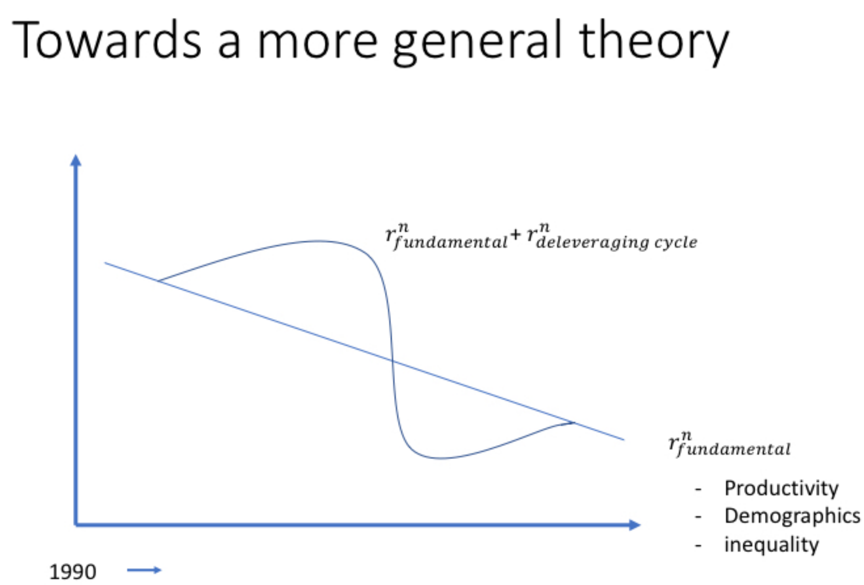


Figure 5: Combining a debt deleveraging and secular stagnation narrative

The literature we have reviewed about the *trigger* of a binding ZLB in the last two subsections has emphasized both fast and temporary reduction in the natural rate of interest (the financial crisis of 2008) as well as slow and more persistent forces (associated with the secular stagnation hypothesis). To date, however, the literature has not integrated these two perspectives. Figure 5 has a graphical illustration of how such an integration may look like. The linear downward sloping line is meant to illustrate forces associated with the secular stagnation that have led to the persistent decline in interest rates observed throughout the industrialized economies over the past 40 years. The dashed line shows temporary forces that may have masked the secular and persistent decline in interest rates. The figure highlights three points.

The first point is that prior to the crisis of 2008 there was relatively compelling evidence of the bubble in the housing market in the US and elsewhere. Such bubble would then have masked the fall in interest rate one would have expected to see according to the slow moving secular forces leading up the crisis. The second point is that this bubble would have made the drop in interest rate more extreme than it otherwise would have been on account of a deleveraging shock. The third point is that once the temporary factors driving the bubble and the crash are removed, if the secular stagnation perspective is correct one would expect a recovery of the real interest rate to a new and lower trend line. This would suggest that even if the Federal Reserve in the US started normalizing interest rates in 2015, the new normal may in fact correspond to a permanently lower natural rate of interest which well may turn permanently negative in the future, thus greatly complicating business cycle stabilization.

1.4 Key Empirical Prediction: The Irrelevance of Increasing Money Supply and Quantitative Easing at the ELB

It is useful to highlight a fundamental empirical prediction of the framework derived so far, which extends to the other models considered. This is an empirical prediction made in 1998 by Krugman (1998) and it is fair to say that it was met with relatively strong degree of skepticism. The key prediction was an irrelevance result: that increases in the money supply via open market operations in government bonds has no effect on either prices and output as long expectations about the future money supply are fixed. Eggertsson and Woodford (2003) extend this irrelevance result, but instead of assuming a fixed future money supply they assume a generic interest rate rule of a similar form often used in applied setting. Moreover, they allow the government to not only increase money supply via open market operations in short term bonds but also allow for money supply to be increased via purchases of long term bonds that may have positive interest rate, or in general any financial asset with arbitrary state contingent payoffs (examples could include foreign exchange or stocks). They establish that once again the increase in money supply has no effect on the equilibrium in the model. Finally Eggertsson (2006) shows that the same type of irrelevance holds even if there are distortions associated with taxation, and even when it is assumed that the government is setting monetary policy optimally under discretion, a topic we will return to later.

Eggertsson and Woodford (2003) is best interpreted as a generalization of the irrelevance result of open market operation by Wallace (1981), extended to include a model with sticky prices, monetary frictions and the zero bound. Moreover, unlike Wallace's results that suggest open market operation are always irrelevant, in Eggertsson and Woodford (2003) model they do matter as long as interest rate is positive but become irrelevant at the zero lower bound, provided they don't affect expectations about future interest rates.

These irrelevance results may seem to be inconsistent with some other results derived in the literature such as Auerbach and Obstfeld (2005), who argue that open market operation in government debt are effective at the zero bound due to the fiscal benefits they imply, or Buiter (2003) who argues that "helicopter drops" of money will work to increase demand, or finally Svensson (2000) that argues that buying foreign exchange by printing money is a "foolproof way" to escape a liquidity trap. The way to reconcile these results is that these latter papers assume that open market operations, helicopter drops or foreign exchange rate interventions are *also* associated with a permanent increase in the money supply – or, equivalently, a change in the interest rate reaction function in the language of Eggertsson and Woodford (2003). Hence these papers do in fact paint a consistent picture: Increasing the money supply is irrelevant at the ELB, regardless of how it is done, *unless* it changes expectations of future money supply/nominal interest rates. Due to these irrelevance result, we will procede in the next section by abstracting from central bank balance sheet, and instead characterize equilibrium only by taking account of the path for the nominal interest rate. We will get back to the central bank balance sheet in section 3 and **??**cred where different assumption underlying the irrelevance results are violated.

1.5 Notes on the literature and alternative modeling strategies for the trigger

The first model in this section is a simplified version Eggertsson and Krugman (2012). Independently, Guerrieri and Lorenzoni (2017) developed the same idea in a model where agent heterogeneity is due to idiosyncratic income risk in the tradition of Bewley (1977). Overall, however, the predictions of the two models are similar.

For simplicity, this section abstracted from capital. Including capital gives rise to other natural candidates for the trigger which typically explain a large drop in investment. Rognlie, Shleifer, and Simsek (2018), for example, analyze the consequence of over-accumulation of capital, e.g. due to a housing boom, or what they term investment hangover. This leads to a period of low investment as the capital stock adjust, which in turn can generate a period of negative natural rate of interest. Del Negro et al. (2017a) model an investment collapse via different mechanism. In their model, entrepreneurs face occasional stochastic investment opportunities, and when they face such opportunities, they sell all their assets to invest. The problem is that private assets are imperfectly liquid, i.e. they may take long time to sell. The trigger is a sudden "freeze" in secondary markets, which make private assets difficult to sell, which in turn leads to a collapse in investment. An important branch of the literature considers different kind of market imperfection where the trigger is a shortage of specific type of asset, safe assets, see for example Caballero and Farhi (2017). An example could be the collapse of the mortgage backed securities market, that while previously viewed as save, became considered to be highly risky at the onset of the Great Recession. Caballero and Farhi (2017) show this type of shock can lead to a reduction in the natural rate of interest that can be arbitrarily persistent, thus providing a complementary way of modeling a secular stagnation.

There is a emerging and growing literature that tries to quantify the gradual reduction in real interest rate documented in section 1.2, both empirically and via model simulations, tracing it to various factors. In addition to the work already cited, Carvalho et al. (2016) and Gagnon et al. (2016) consider demographic factors. Auclert and Rognlie (2018) and Straub (2018) consider inequality. Examples that attempt to account for several forces at once include, Farhi and Gourio (2018), Del Negro et al. (2017b) and Summers and Rachel (2019). Overall, this literature is able to account relatively well for the observed decline in interest rate, suggesting that the there is not necessarily a strong a priori reason to expect a re-normalization in interest rate going forward.

The basic modern formulation of the liquidity trap followed in this section has its origin in Krugman (1998). There are a few important papers, however, that pre-date that paper. This work originates from the conquest of inflation in the early 1990's in the US when there was growing awareness that the zero lower bound might pose a challenge going forward. An early warning of this posssibiliy is an insightful commentary by Summers (1991). Pointing out that real interest rate had been negative by about 1/3 of the time since WWII, Summers argued the possibility of allowing for negative rates was largely lost with a zero inflation target due the ELB. On basis of this observation Summers suggested that the optimal long term inflation rate should be above zero, possibly in the range 2 to 3.

Using Summers observation as motivation, Fuhrer and Madigan (1997) studied the output volatility of a small forward looking model that had a policy reaction function with 0 and 4 percent inflation respectively. They confirmed Summers suggestion, that the economy with 0 percent inflation target would hit the ZLB and experience output contraction.

The influential paper Rotemberg and Woodford (1997) is mostly known for being one of the papers that launched the large literature on estimated dynamic stochastic general equilibrium (DSGE) models for policy analysis. In it, however, the authors indirectly account for the ELB constraint by penalizing variations in the nominal interest rate. An important conclusion of their analysis, was that if the central bank moved the interest rate only slowly in reaction to shocks (or with policy inertia, later studied in more detail in Woodford (1999)), output and inflation could be stabilized relatively close to their target, without violating the ELB or increasing average inflation, which substantially qualified Fuhrer and Madigan (1997) findings. This result was an important precursor to a central result in the literature which will be a focal point of section 3.1: Optimal monetary policy at the ELB is to commit to keeping interest rate low for a considerable period of time after the shock that gave rise to it subside. Moreover, in realistic simulation, the policy can achieve relatively good outcomes for output without requiring substantial increase in inflation. Coibion et al. (2012) corraborates this finding when considering the optimal policy rule, while **??** finds stronger case for higher inflation using a menu cost model, on account of the cost of inflation being lower in that model. cite{billi2011optimal finds that the optimal inflation rate can be as high as 17 percent if one accounts for possible model misspecification and assumes the government re-optimizes in every period, but can commit in advance to a long run inflation target. See also Ball (2013) and Blanchard et al. (2010) for an overview of the issue.

2 A Dynamic Model within a Standard Monetary Policy Regime

2.1 The Textbook New Keynesian Model of Monetary and Fiscal policy

We now move to the textbook New Keynesian model which allows for a better treatment of price dynamics but compromises in focusing on a representative household which implies a more reduced-form driving force for the fall in the natural rate of interest.[6] The derivation of the model is largely standard and is shown in the Appendix, the main assumptions and notation conventions are summarized in the next paragraph.

A representative household maximizes

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} \xi_T \left[\frac{C_T^{1-\sigma_C^{-1}}}{1-\sigma_C^{-1}} + \frac{G_T^{1-\sigma_G^{-1}}}{1-\sigma_G^{-1}} - \int_0^1 \frac{l_T^{1+\omega}}{1+\omega} dj \right] \tag{21}$$

s.t.

$$(1+\tau_t^s) P_t C_t + B_t + (1+i_{t-1}) B_{t-1} + \left(1-\tau_t^I\right) \left[\int_0^1 Z_t(i)\, di + P_t \int_0^1 W_t(j)\, l_t(j)\, dj \right] - P_t T_t \tag{22}$$

where $C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta}{\theta-1}} di \right]^{\frac{\theta-1}{\theta}}$ is a Dixit-Stiglitz aggregate of consumption of variety $c_t(i)$ with a corresponding price index $P_t \equiv \left[\int_0^1 p_t(i)^{\frac{1}{1-\theta}} di \right]^{1-\theta}$ where $p_t(i)$ is the price of each good variety i and $\theta > 1$ is the elasticity of substitution. G_T is the utility of public consumption which is similarly combined of the same consumption varieties. The coefficients σ_C and σ_G are coefficients of intertemporal elasticity of substitution of private and public consumption, $l_t(j)$ is labor of type j, $\omega > 0$ is the inverse of the Frisch elasticity of labor supply, β is a discount factor between 0 and 1 and ξ_T is a shock to preferences. τ_t^s is a proportional tax on all consumption (sales tax) τ_t^I a proportional tax on all income. B_t is a one period risk-free nominal bond and i_t is the one period risk-free interest rate. Z_t is nominal profits while $W_t(j)$ is the nominal wage rate. The household maximizes its objective by a choice of bond holdings, labor and consumption. On the firm side there are monopolistically competitive firms (each producing each variety of the consumption good) with a production function that is linear in labor (we abstract from capital). A firm gets to choose its price with a probability $(1-\alpha)$ in each period. If a firm does not get to choose it indexes the price at the inflation target of the government π^*. The government sets monetary policy via the nominal interest rate. In equilibrium, all bonds are government debt. Total government spending is given by $F_t = G_t + R_t$ where R_t represents tax collection cost associated with lump-sum taxes, T_t. These costs will not play a role until optimal policy under discretion is discussed in section 7.1, and thus we defer discussion until that point. The aggregate resource constraint is $Y_t = C_t + F_t$. Fiscal policy is the choice of total government spending,

[6] As Eggertsson and Krugman (2012) point out, the debt deleveraging shock in the last section does in fact correspond exactly in reduced form to variation in time preference assumed here, and the debt deleveraging parable is one possible microfoundation for the negative natural or "efficient" rate of interest.

F_t, and taxes. It will be convenient to write down the government budget constraint it in terms of the real value of government debt in period t inclusive of interest rate, defined as $w_t \equiv \frac{B_t}{P_t}(1+i_t)$.

Rather than work through the details of the derivation of the household and firm problems, which are readily available elsewhere and reproduced in the Appendix for convenience, we focus on the aggregate relationships which can be derived via log-linear approximation of the equilibrium conditions. For simplicity we assume that the only source of uncertainty is the shock to preferences (inducing people to consume more today relatively to future). One interpretation of this shock is the debt deleveraging story from the last subsection. This model is not suitable for considering permanent demand recession, but this case is considered in section 4 under the rubric of secular stagnation.

The model is summarized by the following three equations, together with a description of monetary and fiscal policy. These are the Investment-Savings (IS), Aggregate Supply (AS) and the government budget constraint (BC) respectively:

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(\hat{\imath}_t - E_t\hat{\pi}_{t+1} - \hat{r}^e_t) + (\hat{F}_t - E_t\hat{F}_{t+1}) + \sigma\chi^s E_t(\hat{\tau}^s_{t+1} - \hat{\tau}^s_t) \tag{23}$$

$$\hat{\pi}_t = \kappa\hat{Y}_t + \beta E_t\pi_{t+1} + \kappa\psi(\chi^I\hat{\tau}^I_t + \chi^s\hat{\tau}^s_t - \sigma^{-1}\hat{F}_t) \tag{24}$$

$$\beta\hat{w}_t = \hat{w}_{t-1} + \beta w_y\hat{\imath}_t - w_y\hat{\pi}_t + (1+\tau^s)\hat{F}_t - (\bar{\tau}^I + \bar{\tau}^s)\hat{Y}_t - \hat{T}_t - c_y\hat{\tau}^s_t - \hat{\tau}^I_t \tag{25}$$

All variables are expressed in log-deviation from their steady-state unless otherwise noted. Here $\hat{Y}_t$ is output, $\hat{\pi}_t$ $\hat{\imath}_t$ is the policy instrument of the central bank. $\hat{F}_t$ is government spending, $\hat{T}_t$ lump-sum taxes, and both expressed in percentage deviation from the steady-state relative to steady-state output. $\hat{w}_t$ is the real value of government debt in percentage from steady-state also expressed relative to steady-state output and steady-state inflation, the sales tax, $\hat{\tau}^s_t$, and income tax, $\hat{\tau}^I_t$, are expressed in deviation from their steady-state rates. $\kappa, \sigma > 0$, $0 < \beta < 1$, $\chi^s > 0$, $\chi^I > 0$ are coefficients.[7] The term $\hat{r}^e_t$ is an exogenous disturbance. It has the interpretation of being equal to the natural rate of interest in deviation from its steady-state if all the fiscal instruments are kept at their steady-state. Under the assumption that that steady-state is efficient it has the interpretation of being the *efficient interest rate*.

We impose the effective lower bound on the nominal interest rate

$$\hat{i}_t \geq i^{elb} \tag{26}$$

The interest rate is expressed in log-deviation from the steady-state, i.e. $\hat{\imath}_t \equiv \log(1+i_t) - \log(1+\bar{\imath})$, where $1+\bar{\imath}$ is the steady-state interest rate which is given by $1+\bar{\imath} \equiv \beta^{-1}(1+\pi^*)$. If the nominal lower bound is zero then $i^{elb} = -\log(1+\bar{\imath})$. The model is approximated around the inflation target of π^*. An important point about the notation convention followed here is that a higher inflation target has the effect of reducing the effective lower bound, i^{elb} directly. The intuition for this is that a higher steady-state inflation leaves the central bank more room to cut rates, a point we will develop in more detail below.

[7] More precisely, let us denote steady-state variable x by $\bar{x}$. We define $\hat{Y}_t \equiv \log Y_t - \log \bar{Y}, \hat{\pi}_t = \log P_t - \log P_{t-1}, F_t \equiv \frac{F_t - F}{\bar{Y}}$, $\hat{\imath}_t \equiv \log(1+i_t) - \log(1+\bar{\imath}), \sigma \equiv \sigma_c \frac{\bar{C}}{\bar{Y}}, \hat{\tau}^s_t = \tau^s_t - \bar{\tau}^s$, $\tau^I_t - \bar{\tau}^I$, $T_t = \frac{T_t - \bar{T}}{\bar{Y}}$, $\chi_s \equiv \frac{1}{1-\bar{\tau}^s}$, $\chi_I \equiv \frac{1}{1+\bar{\tau}^s}, \psi \equiv \frac{1}{\sigma^{-1}+\omega}$, $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\sigma^{-1}+\omega}{1+\omega\theta}$.

In what follows, the exact bound on interest rate can either be positive or negative. It could for example be determined by the level below which a central bank believes further cuts are contractionary, even if technically feasible (see section 6). Or it could arise due to the fact that money can be used to store wealth and there are no storage costs, the traditional motivation for the zero lower bound further discussed in section6.

Equation (23), the IS equation, says that output demanded depends on the real interest rate and expectation of future output. It is instructive to forward it to yield

$$\hat{Y}_t = -\sigma E_t \sum_{j=0}^{\infty} \Big[(\hat{\imath}_{t+j} - \hat{\pi}_{t+1+j} - \hat{r}^e_{t+j}) + (\hat{F}_{t+j} - E_t \hat{F}_{t+1+j}) + \sigma \chi^s E_t (\hat{\tau}^s_{t+1+j} - \hat{\tau}^s_{t+j}) \Big] \tag{27}$$

which reveals that the output depends not only on the real interest rate today but also on the expected future path of interest rates. That expectations of future interest rates influence spending plays a major role in the modern theory of liquidity traps. It implies that the government can affect spending at date t even if the ELB is binding by having an effect on expectations about the future interest rate policy. By the same token, it is not only the variation of fiscal spending and taxes today that matter but rather the entire path of future spending and taxes have an impact on aggregate demand. To close the model we need to be explicit about monetary and fiscal policy, which we do in the context of specific assumptions about stochastic variation in $\hat{r}^e_t$.

2.2 An Analytic Example

A useful assumption about the the natural rate of interest, first applied in Eggertsson and Woodford (2003), makes the New Keynesian model boil down to the solution of only two equations at the ELB. It also allows for graphical illustration of the effect of fiscal policy via basic AS-AD diagram as first shown in Eggertsson (2011).

The assumption is shown in figure 6. The efficient rate of interest drops in period zero to some low level $\hat{r}^e_t = \hat{r}^e_S$ (S denotes "short-run") where we have in mind the stories modeled in the last section. In every period $t > 0$ there is a fixed probability $1 - \mu$ that the shock reverts back to its steady-state, $\hat{r}^e_L = 0$. Thus at time 0 there is a $1 - \mu$ chance the shock is "over" in period 1, while if the the shock remains in period 1 the probability that the shock is over in period 2 is once again $1 - \mu$. We call the stochastic date at which the normalization occurs t_L and assume that once back to the normal the shock stays there. It is easy to confirm, then, that the expected duration of the shock/recession is $1/(1 - \mu)$. The reason this assumption is convenient is that that the model becomes perfectly forward looking and inflation, output and interest rate jump immediately to their steady-state values in the long-run, as long as policy is also forward looking. Moreover, as a result, the short-run looks exactly the same in every period.

The model is closed by specifying monetary and fiscal policy. Monetary policy follows a standard Taylor rule

$$\hat{\imath}_t = \max(i^{elb}, \hat{r}^e_t + \phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t) \tag{28}$$

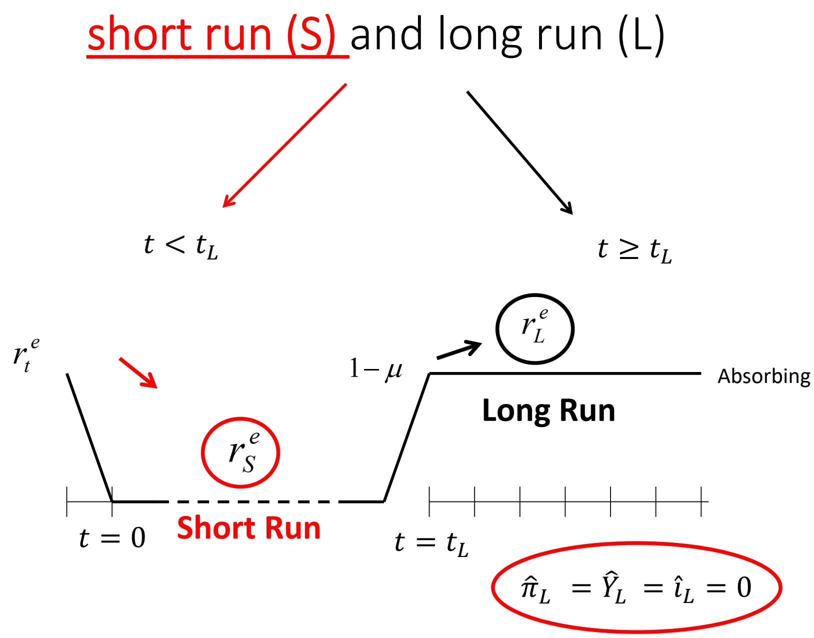


Figure 6: Key Assumptions

All fiscal instruments, with the exception of lump-sum taxes, $\hat{T}_t$, are at steady-state in the long-run so that

$$\hat{\tau}_t^s = \hat{\tau}_t^I = \hat{F}_t = 0 \text{ for } t \geq t_L \tag{29}$$

but may be adjusted in reaction to the shock in the short-run

$$\hat{\tau}_S^s \geq 0, \hat{\tau}_S^I \geq 0, \hat{F}_S \geq 0 \text{ for } t < t_L \tag{30}$$

This assumption implies that the lump-sum taxes, $\hat{T}_t$, are adjusting in the background to clear the government budget constraint. Because taxes are lump-sum, their timing has no effect on interest rate, output or inflation due to Ricardian Equivalence.[8] For now, we will not be specific on how lump-sum taxes adjust. This is the topic we turn to in section 2.9. By having lump-sum taxes adjust in the background we are able to identify the effect of each of the fiscal policy instruments on output and prices abstracting from how these instruments may affect future distortionary taxes.

The assumption of the two-state process with an absorbing long-run simplifies the analytics. The model can be solved backwards starting from the steady-state (call it longer run denoted by subscript L). The IS, AS and the MP rule can be solved together to yield

$$\hat{\pi}_t = \hat{\pi}_L = 0, \hat{Y}_t = \hat{Y}_L = 0, \hat{\imath}_t = \hat{\imath}_L = \hat{r}_L^e = 0 \tag{31}$$

[8]Their timing will matter, however, for welfare if we assume there are tax collection costs because for a fixed F_t higher taxes imply that there is less resources to spend on government spending G_t that contributes to welfare.

which is obtained using the equilibrium selection of unique bounded solution, and by disregarding the possibility that ELB is binding on account of self-fulfilling expectations, a key abstraction in this paper, as mentioned in the introduction. We will later show that the equilibrium we select, corresponds to a Markov Perfect Equilibrium if the central bank maximizes welfare under discretion.

Due to the purely forward looking nature of the model every period $t < t_L$ looks the same since $\hat{r}^e_t = \hat{r}^e_S$ remains constant. Hence, as long as the following condition is satisfied

$$L(\mu) = (1-\mu)(1-\beta\mu) - \kappa\sigma\mu > 0 \tag{32}$$

inflation and output take on constant values $\hat{Y}_S$ and $\hat{\pi}_S$. The equilibrium is then a solution to the pair of equations, the AD curve in (33) and the AS curve in (34):

$$\hat{Y}_S = \begin{cases} -\frac{\sigma(\phi_\pi-\mu)}{1-\sigma+\sigma\phi_y}\hat{\pi}_S + \frac{(1-\mu)}{1-\sigma+\sigma\phi_y}\hat{F}_S - \sigma\chi^s\frac{(1-\mu)}{1-\sigma+\sigma\phi_y}\hat{\tau}^I_S & \text{if } \hat{\imath}_S \geq i^{elb} \\ \\ \frac{\sigma\mu}{1-\mu}\hat{\pi}_S + \frac{\sigma}{1-\mu}(\hat{r}^e_S - i^{elb}) + \hat{F}_S - \sigma\chi^s\hat{\tau}^I_S < 0 & \text{if } \hat{\imath}_S = i^{elb} \end{cases} \tag{33}$$

$$\hat{\pi}_S = \frac{\kappa}{(1-\beta\mu)}\left[\hat{Y}_S + \psi\left(\chi^I\hat{\tau}^I_S + \chi^s\hat{\tau}^s_S - \sigma^{-1}\hat{F}_S\right)\right] \tag{34}$$

which are plotted together in Figure 7. The first row is obtained by substituting the interest rate

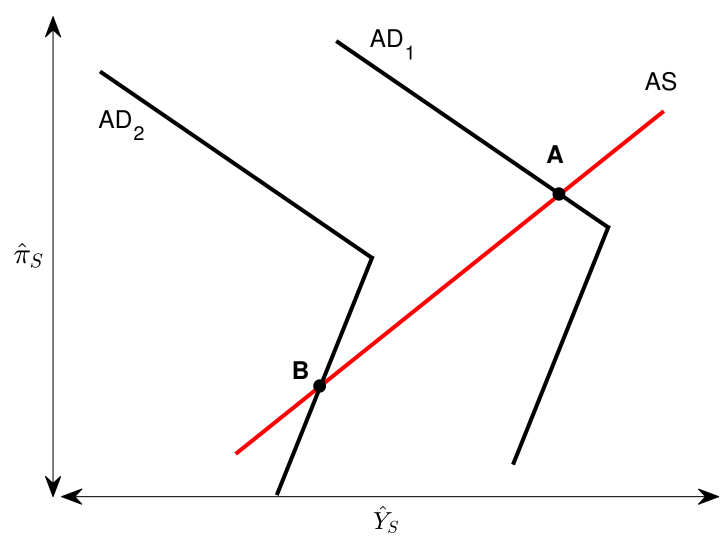


Figure 7: A Normal Equilibrium at positive interest rates (point A) and a Liquidity Trap Equilibrium (point B).

rule at a positive interest rate into (23), taking account of the fact that the solution for the long-run is given by (31) and the stochastic process of the shock implies that $E_t\hat{Y}_{t+1} = \mu\hat{Y}_t = \mu\hat{Y}_S$ and $E_t\hat{\pi}_{t+1} = \mu\hat{\pi}_t = \mu\hat{\pi}_S$. The second row of (23), following the same steps, is derived assuming that the ELB is binding. How fiscal policy affects the equilibrium is discussed in coming section; for now it is kept at the steady-state. It is then easy to confirm that the condition for the ELB to be binding depends on whether $\hat{r}^e_S < i^{elb}$.

The AD curve is downward sloping at positive interest rate while it inverts and becomes upward sloping once the ELB is binding. The AD curve slopes downwards at positive interest rate because with falling inflation the central bank cuts the nominal interest rate more than one-to-one with the drop in inflation, thus reducing the interest rate and stimulating demand. At some point, however, the central bank is prevented from doing so due to the ELB. This occurs if $\hat{\imath}_S < i^{elb}$. At that point lower inflation reduces inflation expectations and this, with the interest rate fixed at a lower bound, increases the real interest rate, thus reducing demand. Meanwhile, the AS curve is always upward sloping. As when output increases the firms face higher marginal costs and choose to increase their prices.

The model has two possible intersections depending on the exogenous shock $\hat{r}^e_S$ and the value of fiscal instruments. Let us assume that $\hat{\tau}^s_S = \hat{\tau}^I_S = \hat{F}_S = 0$. If $\hat{r}^e_S \geq i^{elb}$ there is a regular equilibrium at point A in Figure 7 while if $\hat{r}^e_S < 0$ the shock is large enough for the ELB to be binding:

$$\hat{Y}_S = \begin{cases} 0 & \text{if } \hat{r}^e_S \geq i^{elb} \\ \frac{\sigma(1-\beta\mu)}{(1-\mu)(1-\beta\mu)-\sigma\kappa\mu}\left(\hat{r}^e_S - \hat{i}^{elb}\right) < 0 & \text{if } \hat{r}^e_S < i^{elb} \end{cases} \tag{35}$$

$$\hat{\pi}_S = \begin{cases} 0 & \text{if } \hat{r}^e_S \geq i^{elb} \\ \frac{\sigma\kappa}{(1-\mu)(1-\beta\mu)-\sigma\kappa\mu}\left(\hat{r}^e_S - \hat{i}^{elb}\right) < 0 & \text{if } \hat{r}^e_S < i^{elb} \end{cases} \tag{36}$$

where a requirement for existence of the equilibrium is condition (32).

It is useful to study in more detail how the equilibrium for inflation and output at the ELB depends upon μ as the existence requires that $L(\mu) > 0$ (recall that the deficit has no direct effect on either output and inflation under current assumption).

Figure 8 shows the part of the AD curve which crosses the AS curve (corresponding to point B in Figure 7). Consider first the special case in which $\mu = 0$, i.e. when the shock $\hat{r}^e_S$ reverts back to the steady-state in period 1 with probability 1. This case is shown in Figure 8 with solid line. It applies only to the equilibrium determination in period 0. The equilibrium is shown where the two solid lines intersect at the point A. At this point output is completely demand-determined by the vertical AD curve and pinned down by the shock $\hat{r}^e_S$. For a given level of output, then, inflation is determined by intersection of AD curve with the AS curve. It is worth emphasizing again: *Output is completely demand-determined, i.e., it is completely determined by the AD equation.*

Consider now the effect of increasing $\mu > 0$. In this case the contraction is expected to last longer than one period. Because of the simple structure of the two-state Markov process for the shock the equilibrium displayed in the figure corresponds to all periods $0 \leq t < t_L$. The expectation of a possible future contraction results in movements in both the AD and the AS curves, and the equilibrium is determined at the intersection of the two dashed curves, at point B. Observe that the AD equation

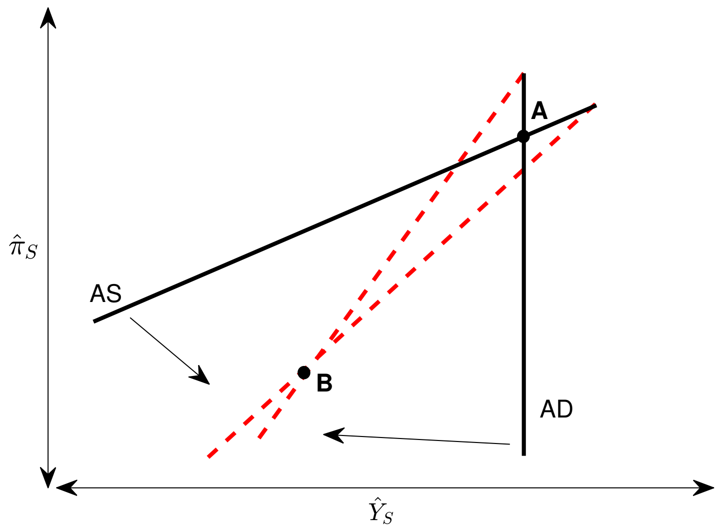


Figure 8: AD and AS at the ZLB for different values of μ

is no longer vertical but upward sloping in inflation, i.e., higher inflation *expectations* $\mu\hat{\pi}_S$ increase output. The reason is that for a given nominal interest rate ($\hat{i}_S = i^{elb}$ in this equilibrium) any increase in expected inflation reduces the real interest rate making current spending relatively cheaper and increasing demand. Conversely, expected deflation, a negative $\mu\hat{\pi}_S$, causes current consumption to be relatively more expensive than future consumption, thus suppressing spending. Observe furthermore the presence of the expectation of future contraction, $\mu\hat{Y}_S$, on the right-hand side of the AD equation. The expectation of future contraction makes the effect of both the shock and the expected deflation even stronger.

Let us now turn to the AS equation (34). Its slope is now steeper than before because the expectation of future deflation leads the firms to cut prices by more for a given demand slack, as shown by the dashed line. The net effect of the shift in both curves is a more severe contraction and deflation shown by the intersection of the two dashed curves at point B in Figure 8. As μ increases the drop in output becomes more pronounced, and if the two curves become parallel the model explodes and no equilibrium exists. The $L(\mu) > 0$ is the requirement that the AD curve is steeper than the AS curve and is needed for an equilibrium to exist, an issue developed in more detail in Eggertsson and Singh (2019). As this discussion should make clear, the model is thus capable to generate a substantial drop in output by having sufficiently persistent shock. This helps to clarify how the model can account for the Great Depression as is illustrated next.

2.3 A Quantitative Example

The model is parameterized to generate two numerical examples that are meant to illustrate the drop in output and inflation observed in the US during the Great Depression (GD) and the Great Recession (GR). Figures 9 and 10 show the evolution of output, inflation and interest rates in both episodes. Both

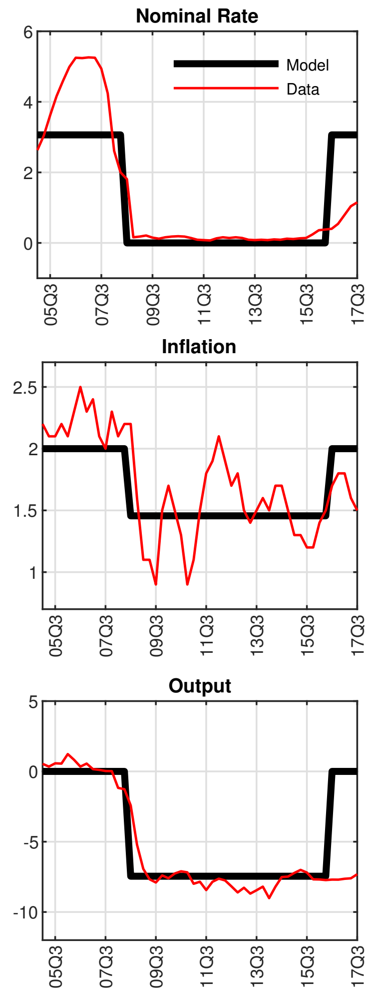


Figure 9: Great Recession: In the Model and in the Data

events feature a fall in interest rates, output and inflation. The key difference is that while deflation was on the order of 10% during the contraction phase of the GD 1929-33, in the case of GR there was no deflation. Instead, inflation fell modestly and was about about 1.5% on average from 2008-2015 once the Federal Reserve started raising rates, or 0.5 p.p. below the official 2% percent target. Output contracted significantly during the GD, and was about 30% lower in 1933 relative to 1929. The output contraction was smaller during the Great Repression, the figure shows that output was about 7.5% below pre-recession trend during this period.[9]

The numerical examples are constructed as follows: For the GD example it is assumed that the inflation target of the Federal Reserve is 0% and parameters and shocks are chosen to match a 30% drop in output and 10% deflation. The output of the model is plotted against the data from 1929-1937 under the assumption that the shock remains in the low state throughout that period. There is a vertical

[9]For the Great Recession the figure reports output in deviation from a linear trend estimated from 2000 while in the case of Great Depression we report actual output. Measured in deviation from the trend GD contraction was even larger.

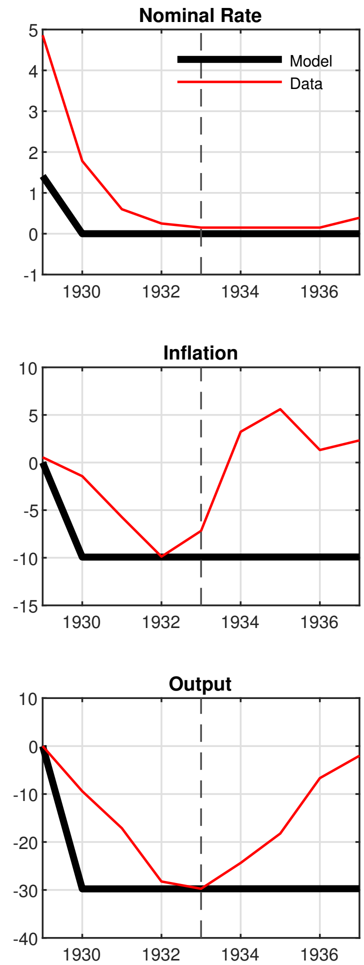


Figure 10: Great Depression: In the Model and in the Data

line denoting when Hoover leaves office in 1933 and section 7 accounts for the recovery in output via regime change that occurred once FDR took the office. For the GR it is assumed that the inflation target of the Federal Reserve is 2%, and parameters and shocks are chosen to match a deviation of output from trend of 7.5% and a drop in inflation from 2% to 1.5%. The output of the model is plotted in 9 against the data from 2005-2017. It is assumed that the shock reverts back to the steady-state in Q4 of 2015 once the Federal Reserve started increasing rates [10].

Table 1 shows the parameters used to generate the two numerical examples. The procedure used to choose the parameters is described in more detail in Denes and Eggertsson (2009). It consists of two steps. The first step is to construct priors about the structural parameters and the shock. The priors capture reasonable values for the variables based upon existing literature as shown in Table 2[11]. The

[10]While this assumption is debatable, the return date plays no role for the analysis. Instead, what matters is the expected duration of the shock at the time the ELB is binding.

[11]For example, we know that the relative risk-aversion parameter σ^{-1} is greater than 0. A typical value in the literature is 2, but one often sees values below or above 2. To capture the first observation the prior is specified as a gamma distribution, which has the property that the variable in question can only be positive. To capture the second observation the mean of the

Parameter	Great Recession	Great Depression
β	0.997	0.997
σ^{-1}	0.909	1.373
$1-\mu$	0.151	0.085
κ	0.003	0.007
r^e_S	-0.017	-0.015

Table 1: Mode of posterior distribution

Parameter	Distribution	Mean	Std. deviation
α	Beta	0.66	0.05
$1-\mu$	Beta	0.083	0.05
σ^{-1}	Gamma	2	0.5
r^e_S	Gamma	-0.0102	0.0025
κ	Model	0.15	0.108

Table 2: Priors: mean and standard deviation

second step is to choose parameters and shocks to match the empirical moments of interest i.e. the drop in output and inflation, while keeping these variables as consistent with the priors as possible.

Comparing the parameterization for the GR and the GD example in Table 1 reveals that the main difference is that the shock process is more persistent in the GD example (μ is larger) and prices are more rigid. This explains the larger drop in inflation in the GD example.

An important difference across the two calibrations is that it is assumed that the central bank has a 2 percent inflation target during in the GR while it is 0 in the GD example [12]. A higher inflation target translates into higher steady-state inflation. This means that the central bank has more room to cut the interest rate in the GR example than in the GD. Because the model is written in terms of deviations from the steady-state, a higher inflation target directly reduces i^{elb} which measures the percentage reduction in the interest rate possible relative to it steady-state. In the GR example, the central bank has room to cut rates by about 3%, while in the GD this room is only 1.4%. Thus, as emphasized by Summers (1991) and Krugman (1998), the most straightforward way of circumventing the bound on nominal interest rate bound is to increase the inflation target of the central bank. This translates directly into a lower i^{elb} and gives the central bank more room to cut rates. Consider for example a central bank that targets 7 percent inflation. The size of the shock, r^e_S, which has the interpretation of being the efficient rate of interest is -6.4 for GD and 6 for the GR, expressed in annual terms. With expected inflation at 7 percent, the central bank would thus have been able to offset the shock in both numerical examples by cutting the nominal interest rate to 0.6 in the GD example and to 1 percent for GR and thereby fully offset the shock. An important element for this strategy to work is that inflation expectations remain stable at 7%.

gamma distribution is chosen to be 2 and the standard deviation 0.5

[12] See Eggertsson (2008) for a discussion of the policy regime in the US during the Great Depression and further discussion in section 7

	Great Recession	Great Depression
$\bar{r}$	1.1	1.4
π^*	2.0	0.0
i^{el}	-3.0	-1.4

Table 3: Steady-state real rate, inflation target and effective lower bound

Parameter	5-th percentile	Median	95-th percentile	Mode
α	0.798	0.845	0.883	0.84
β	0.996	0.997	0.998	0.997
$1-\mu$	0.078	0.135	0.204	0.151
σ^{-1}	0.687	1.092	1.654	0.909
ω	1.129	2.36	4.958	1.843
θ	11.418	17.489	25.945	15.989
r^e_S	-0.022	-0.017	-0.014	-0.017

Table 4: Posterior distribution under Great Recession scenario

Parameter	5-th percentile	Median	95-th percentile	Mode
α	0.828	0.838	0.847	0.835
β	0.995	0.997	0.998	0.997
$1-\mu$	0.077	0.084	0.093	0.085
σ^{-1}	1.244	1.37	1.496	1.373
ω	2.656	2.75	2.845	2.777
θ	5.64	6.198	6.79	6.346
r^e_S	-0.019	-0.015	-0.011	-0.015

Table 5: Posterior distribution under Great Depression scenario

2.4 Is the Economy Self-Equilibriating? The Paradox of Price Flexibility

The two main foundations of the theory of this paper is the *trigger* of the Great Recession and the Great Depression – which generates a temporary reduction in the natural rate of output – and a propagation mechanism – rigidities in prices or wages – which makes the required reduction in the equilibrium interest rate difficult. A natural conjecture, then, is to suppose that contraction gets mitigated as prices become more flexible. One might even suspect that over time, as prices and wages naturally adjust, a self-sustaining recovery emerges in the model. This conjecture, however, is incorrect. Paradoxically, the opposite happens at the ELB. As prices and/or wages become more flexible, the drop in output is exacerbated. This is coined the paradox of price flexibility by (Eggertsson and Krugman, 2012). While the degree of price flexibility is reduced-form in the model, this comparative static cast doubt on the degree to which the market price mechanisms self-equilibriate, i.e. that there is a natural pull to full employment absent policy interventions, or a reversion in the exogenous shock.

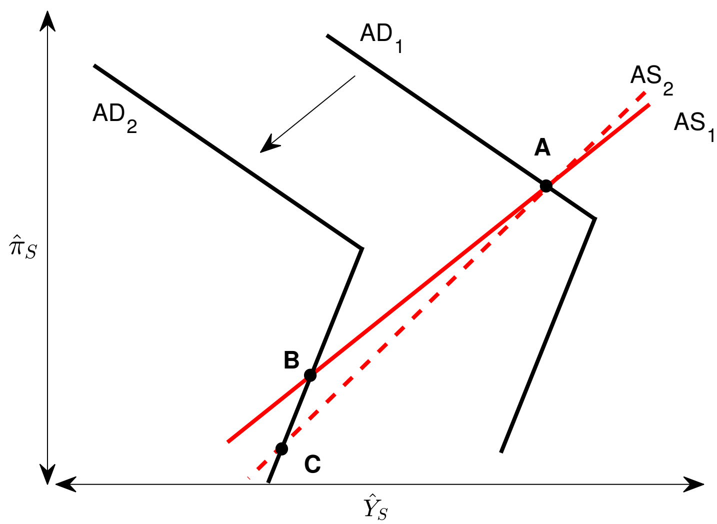


Figure 11: The effect of increasing price flexibility at positive interest rate (A) and at the ZLB (B to C)

The paradox is illustrated in figure 11. As prices become more flexible, i.e. κ increases, the AS curve becomes more steep. The AS curve rotates at point A, i.e. at full output when inflation is at target, thus leaving the equilibrium unchanged at positive interest rates. At the ELB equilibrium, point B, then making prices more flexible generates a worse outcome, however. The reason for this is that higher price flexibility not only reduces prices today, cutting inflation π_S, but also generates a drop in *expected inflation* $\mu\pi_S$ upon which demand directly depends. This reduces demand and moves the economy to a new equilibrium with lower output at the intersection of the dashed "high flexibility" AS curve and AD curve (point C).

The possibility that price and wage flexibility could be destabilizing was noted by Fisher as early as 1923 (Fisher (1923), Fisher (1925)) who saw the business cycle as Ïargely a dance of the dollar:̈ According to Fisher, expected deflation leads to high anticipated real interest rates which suppresses investment and output. This is the same mechanism as in the current model. The increase in flexibility leads to lower inflation and higher real interest rate that is not offset by monetary policy.

De Long and Summers (1986) document that this and similar arguments can be found in Keynes' earlier writing, as well as in the writing of later authors such as Tobin (1975). De Long and Summers (1986) further illustrate conditions under which this result obtains in the dynamic IS-LM model. Bhattarai, Eggertsson, and Schoenle (2018) establish general conditions under which price flexibility is destabilizing in the same framework as considered in this paper, taking into account wide range of structural shocks and policy regimes. While price flexibility is destabilizing in the current model because of its effect on expected inflation and the real interest rate, Eggertsson and Krugman (2012) show that a similar paradox occurs in the saver-borrower model if debt is nominal due a Fisherian debt-deflation channel. There it happens because unexpected deflation generates redistribution from borrowers to savers. This in turn increases the need of debtor to deleverage, meaning that the natural rate of interest becomes even more negative, thus intensifying the contraction.

The paradox of price flexibility is that as prices become more flexible output drops by more at the ELB. Yet, once prices become fully flexible – and here lies the paradox – output is by definition at the flexible price level which is constant. How can one reconcile this comparative static with the fact that one knows that in the limit output is stable? The way to reconcile these two observations is somewhat subtle. Consider the comparative static in figure 11 as prices are made more and more flexible. The AS curve becomes steeper, output drops by more. At a certain point, in the log-linearized model, the drop is unbounded, and the two lines are parallel and do not intersect at all, i.e. there is *no equilibrium*. If one studies the fully non-linear model, as done is Eggertsson and Singh (2019), non-existence can be established formally, i.e. that if prices are "too flexible" there is no equilibrium. Importantly, the same applies even under perfectly flexible prices. If the central bank targets too low inflation at positive interest rates at flexible prices, then there is *no equilibrium* in the model at the ELB. The best way to see this is via simple example, which is shown in Eggertsson and Giannoni (2019) and illustrated in the footnote.[13] The key assumption that generates non-existence of equilibrium is the assumed policy regime given by the rule (28). Under a different policy rule, as for example shown in Bhattarai, Eggertsson, and Gafarov (2019), the paradox need not apply. An important example of a policy regime in which there is no price flexibility paradox is one in which a government commits to the optimal policy, as we show in section 3.1.

The paradox of price flexibility gives an interesting perspective on one reasons for why the Great Recession was so much less severe than the Great Depression, but table 1 suggests prices, as measured

[13]**Non-existence example**. Consider a flexible price economy, given by a Fisher Equation and ZLB

$$r_t = i_t - E_t \pi_{t+1}$$

$$i_t \geq 0$$

Because this economy has flexible prices, r_t is exogenous. It is the real interest rate while and i_t is the nominal interest rate in levels controlled by the central bank and π_t is inflation in levels. This economy is for example derived in Woodford (2003) and corresponds to a classic endowment economy. Consider now a shock to the real interest rate so that it is negative in period 0, $r_0 < 0$, and $r_t = \beta^{-1} - 1 > 0$ for all $t > 0$. We can see that in period 1 the ELB is no longer binding, so that the central bank can set any inflation rate as the model is perfectly forward looking. Suppose, for example, that it sets $\pi_t = 0$ for all $t > 0$. But the implication is that there is no equilibrium in period 0 because $r_0 = i_0 - \pi_1 = i_0 > 0$ and we just stated that r_0 was negative.

by κ are two times more flexible in the GD numerical example. Consider the GR numerical example but replace the value κ in Table 1 for the GR with the value from the GD. In this case, using expression (35) and (36), the fall in output would have been 9.4% percent rather than 7.5% and the US economy would have experienced 0.2% inflation instead of 1.5%.

2.5 What can Policy do in the absence of a Monetary Policy Regime Change?

What can policy do? A key element for determining the equilibrium is the monetary policy regime (28). One way in which a monetary policy can be effective is via a change in this policy regime, e.g., via increase in inflation target or a commitment to keeping nominal rates low beyond what is implied by this policy rule. This is a major theme in the literature and the topic of section 3. Before getting there, however, it is worth considering what can be done without a monetary policy regime change, i.e. what can be done by increasing or reducing various fiscal policy instruments in the short-run without a change in (28).

There are at least two rationales for considering policy interventions that are short of a monetary policy regime change. First, a monetary policy regime change may be difficult to accomplish (at least to its fullest extent, and hence there should be a plan B). It will typically involve monetary commitment to actions that should be undertaken once the ELB stops being binding. Thus, there is an inherent credibility problem with regime changes, as discussed in more detail in section 3. Second, a policy maker may want to understand costs and benefits of alternative policies absent a regime change if a regime change involves considerable costs (e.g. loss of credibility in fighting inflation), something that weighted heavily on policy makers at the Federal Reserve during the crisis of 2008.

It is worth pointing out that the old-style Keynesian literature implicitly ruled out monetary policy regime changes. This is because the standard IS-LM model assumes that expectations are exogenous, and it is via expectation of future policy that regime changes obtain their power. The analysis inside the conventional policy regime, however, is instead quite similar to what would have been found in the standard IS-LM model. This is because in the absence of a regime change, just as in the old IS-LM model, the expectations remain exogenous in certain respects, a point we make precise below.

2.6 Fiscal multipliers within a conventional monetary policy regime

Fiscal policy is the oldest suggestion solution to the problem created by the ELB and was a major theme of early Keynesian economics. Within the standard New Keynesian model fiscal policy can have large effects at the ELB assuming the monetary policy regime (28). This impact is usefully summarized by fiscal *multipliers*. The government spending multiplier, for example, measures by how many dollars output increases for a dollar increase in government spending. Recall that the assumption about fiscal policy in (29)-(30) is that in the long-run all fiscal variables, with the exception

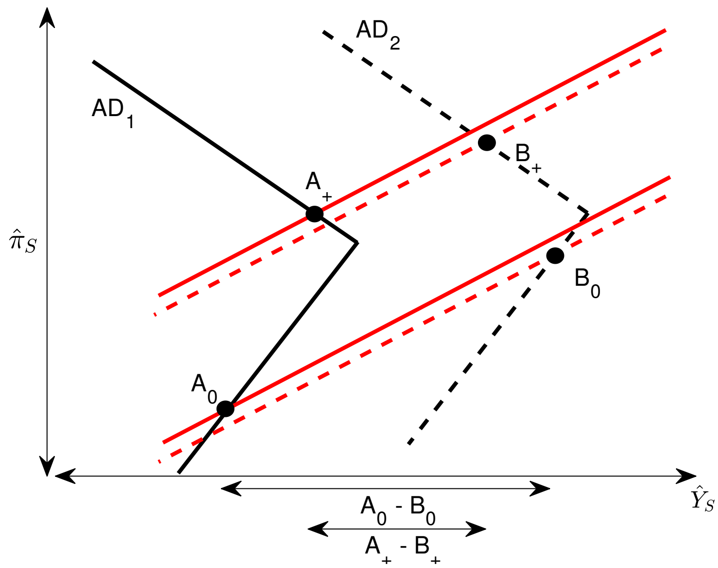


Figure 12: The effect of government spending at positive interest rate (A_+ to B_+) and at the ZLB (A_0 to B_0))

of lump-sum taxes, are back to their steady-state values. The focus here is on the effect of changing these fiscal variables in the short-run holding the long-run fixed.

While government spending, $\hat{F}_S$, and sales taxes, $\hat{\tau}^s_S$, directly change aggregate demand in the equation (23), aggregate demand is independent of the income tax $\hat{\tau}^I_S$. Income tax only affects aggregate supply. If the government cuts taxes on income, for example, people are willing to work more, and this shifts out the aggregate supply while leaving aggregate spending unchanged due to Ricardian equivalence. In a more general setting income tax cuts may have an effect on spending, for example, due to some agents being liquidity constrained. Here, however, it is useful to use the income tax instead as a general stand in for supply side policies, while $\hat{F}_S$ is a general stand-in for demand side policy (observe that τ^s_S has exactly the same effect as $\hat{F}_S$ only scaled by σ).

Figure 12 depicts two initial equilibria A_+ when the AD and AS curve intersect at a positive interest rate and A_0 when the intersection is the ELB. Consider the impact of higher government spending, $\hat{F}_S$, which triggers a rightward shift of both the AD and the AS curve, resulting in a new equilibrium in B_+ and B_0 respectively. The important point this figure illustrates is that the expansionary effect of fiscal spending at the ELB is in general larger than at a positive interest rate, i.e the line segment $B_0 - A_0 > B_+ - A_+$, which is an analytic result illustrated below. The key reason is that at the ELB the central bank does not raise the interest rate in response to a government spending shock (unless it is very large) while at positive rates the central bank will increase interest rate more than one to one in response to the inflationary effect of government spending.

That government spending is more expansionary at the ELB can be formally shown by computing multiplier of government spending. The *dynamic multiplier of government spending* measures much output is expected to increase in net present values as a result of a given fiscal stimulus, also measured in net present value. Each sum is discounted using the discount factor of the representative household, i.e.,

$$M_F = \frac{E_0 \sum\limits_{t=0} \beta^t \Delta \hat{Y}_t}{E_0 \sum\limits_{t=0} \beta^t \Delta \hat{F}_t} = \frac{\frac{1}{1-\mu\beta}\Delta \hat{Y}_S}{\frac{1}{1-\mu\beta}\Delta \hat{F}_S} = \frac{\Delta \hat{Y}_S}{\Delta \hat{F}_S} \tag{37}$$

The multiplier takes a simple form for the two-state Markov Process, along with the assumption that spending only increases in the short-run. This is shown in the right hand side of equation (37) which yields $\frac{\Delta \hat{Y}_S}{\Delta \hat{F}_S}$ which is the *impact multiplier of government spending*. For the two-state Markov Process the impact multiplier and the dynamic multiplier are one and the same. They can be expressed in closed form as:

$$\frac{\Delta \hat{Y}_S}{\Delta \hat{F}_S} = \begin{cases} \frac{(1-\mu\beta)(1-\mu)+\kappa\psi(\phi_\pi-\mu)}{(1-\mu+\sigma\phi_\pi)(1-\mu\beta)+\sigma\kappa(\phi_\pi-\mu)} < 1 & \text{if } \hat{\imath}_S \geq i^{elb} \\ \\ \frac{(1-\mu\beta)(1-\mu)-\psi\mu\kappa}{(1-\mu\beta)(1-\mu)-\sigma\mu\kappa} > 1 & \text{if } \hat{\imath}_S = i^{elb} \end{cases}$$

suggesting the multiplier is always smaller than 1 at positive interest rate and greater one at the ELB. Using the numerical examples from the last section Table 6 shows the value for the government spending multiplier. At positive interest rate the multiplier is around 0.2. It is above 1 in value for the Great Recession calibration and is even larger for the Great Depression numerical example. The two numerical examples illustrate a more general principle. As the recession becomes worse, the the multiplier increases, a "divine co-incidence" emphasized by Christiano et al. (2011). Formally it can be shown that the severity of recession increases with the duration of the shock, i.e. as μ becomes larger. A sales tax cut has the same effect as the increase in government spending and is also reported in Table 6, which has the interpretation of measuring the response to a percentage *rate* change in taxes that are proportional to output.

As seen in figure 12 both the the increase in government spending and a sales tax cut do not only increase aggregate demand. They also shift out aggregate supply. This is because an increase in government spending takes away resources from private consumption: marginal utility of consumption increases and labor supply shifts out. This reduces wages and shifts the AS curve out as firms are willing to produce more at a given level of prices. The effect of the increase in aggregate supply is subtle at the ELB as it is contractionary even if this force is not strong enough to offset the demand effect of government spending. This somewhat paradoxical result has been termed the paradox of toil (Eggertsson (2010)), which we now turn to.

2.7 The Paradox of Toil and Temporary Supply Shocks at the ELB

Suppose everyone wakes up one day with a desire to work more. Presumably, some would find more work to do, and there would be more work in the aggregate. Instead, the paradox of toil is that at the ELB this would lead to less work in equilibrium. Thus less work, if everyone tires to work more!

Multiplier	Great Recession $\hat{i}_t = \hat{i}^{elb}$	$\hat{i}_t > \hat{i}^{elb}$	Great Depression $\hat{i}_t = \hat{i}^{elb}$	$\hat{i}_t > \hat{i}^{elb}$
$\frac{\Delta \hat{Y}_S}{\Delta \hat{F}_S}$	1.1	0.2	2.3	0.2
	$[1, 1.2]$	$[0.2, 0.3]$	$[1.9, 2.9]$	$[0.2, 0.2]$
$\frac{\Delta \hat{Y}_S}{\Delta \hat{\tau}_S^s}$	-1.1	-0.2	-1.5	-0.1
	$[-1.5, -0.6]$	$[-0.3, -0.1]$	$[-2.1, -1.2]$	$[-0.1, -0.1]$
$\frac{\Delta \hat{Y}_S}{\Delta \hat{\tau}_S^I}$	0.07	-0.01	0.7	-0.03
	$[0.02, 0.1]$	$[-0.01, 0]$	$[0.5, 1]$	$[-0.03, -0.02]$

Table 6: Short-run multipliers of government spending, sales and income tax. 95% set in parentheses.

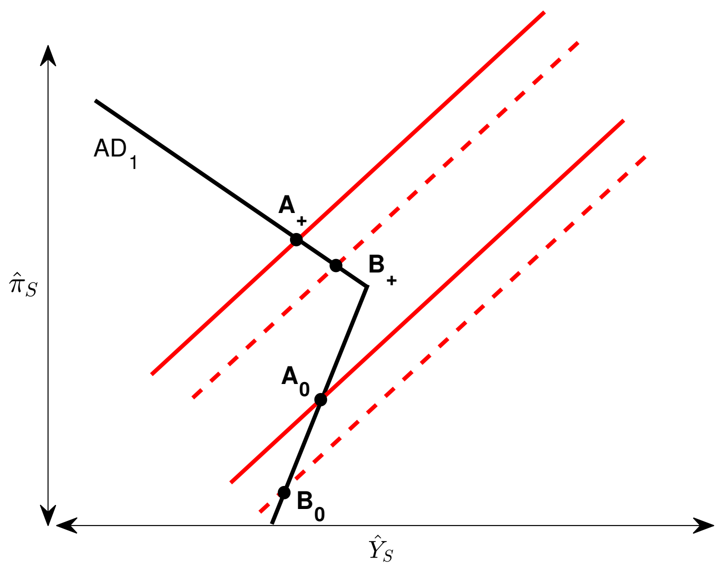


Figure 13: The effect of a supply shock at positive interest rate (A_+ to B_+) and at the ZLB (A_0 to B_0)

The paradox is shown in figure 13. Consider first the effect of an increase in aggregate supply at positive interest rate (for example due to an increase willingness of people for work due to an income tax cut $\hat{\tau}_S^I$). The initial equilibrium is at point A_+. An increase in aggregate supply leads to a shift rightward, denoted by the dashed line, with a new intersection at point B_+ at higher output and lower inflation. Increase in willingness to work leads to a reduction in wages so that firms lower their prices. The reduction in inflation leads the central bank to cut the interest rate more than one to one, so that the real interest rate goes down. This triggers an increase in spending and leads to a movement along the AD curve to a new equilibrium at point B_+. There is more spending and production in this equilibrium.

This result is overturned at the ELB. The initial intersection of the AD and AS curve at the ELB is shown by A_0. Consider a shift of the AS curve to the right as before. Again this leads to a reduction in wages which again leads the firms to reduce their prices, thus reducing inflation. At the ELB, however, the central bank is unable to cut interest rates in response to the reduction in inflation. The reduction in inflation, and expectation of lower future inflation, then leads to an increase in the real interest rate – which leads to a movement down along the upward sloping segment of the demand curve to the point B_0. Hence at the ELB, as everybody attempts to work more, there is less work in equilibrium. This is the paradox of toil.

The Paradox of Toil echoes the old Keynesian paradox of thrift. The paradox of thrift is that if everybody tries to save more, there is less aggregate savings in equilibrium. This occurs because the increase in savings reduces demand, which in turns reduces aggregate income, thus curtailing peoples *ability* of to save. The paradox of thrift also applies here, as shown in Eggertsson (2010), if the model is extended to incorporate capital accumulation. Both paradoxes correspond to the classical fallacy of composition.

The analytics of the paradox, which we've explained graphically above, are shown below where the supply shock is captured by $\hat{\tau}_s^I$:

$$\frac{\Delta \hat{Y}_S}{\Delta \hat{\tau}_s^I} = \begin{cases} -\chi^I \frac{\sigma\kappa\psi(\phi_\pi-\mu)}{\left(1-\mu+\sigma\phi_y\right)(1-\beta\mu)+\sigma\kappa(\phi_\pi-\mu)} < 0 & \text{if } \hat{\imath}_S > i^{elb} \\ \\ \chi^I \frac{\sigma\mu\kappa\psi}{(1-\mu)(1-\beta\mu)-\sigma\mu\kappa} > 0 & \text{if } \hat{\imath}_S = i^{elb} \end{cases} \tag{38}$$

The income tax, $\hat{\tau}_s^I$, is a stand-in for a generic supply shock. A tax cut corresponds to a positive supply shock, for it induces people to work more. Yet, as the model is Ricardian, it implies no aggregate demand effect, an abstraction that does not apply in more general setting such as the spender saver model of Eggertsson and Krugman (2012) or the OLG model of Eggertsson, Mehrotra, and Robbins (2019c). More generally, the shock can be interpreted as anything that shifts aggregate supply, such as oil shock, positive technology shock, or policies that reduce inefficiencies often associated with structural reform.

The main point of the paradox of toil is not that the labor tax cuts are contractionary, since generally one would expect income tax cuts also to have demand effect. Instead, it is that the main problem at the ELB is insufficient demand, and thus the key aim of a policy intervention should be on policies that increase aggregate spending. Policies that are aimed at increasing aggregate supply are much less likely to be successful, and indeed can backfire because they are typically associated with deflationary pressures. Since monetary policy is unable to react to deflationary pressures at the ELB, they will increase the real interest rate which in turn will contract aggregate demand.

2.8 Long-run Fiscal Expansion and Long-run Supply Shocks

Last section considered demand and supply policies that were temporary, that is, they were executed in direct response to the shock that gave rise to the ELB. It is of considerable interest to understand, however, the effect of permanent policy changes. In this case the effect of a fiscal expansion becomes smaller, and positive supply shocks are no longer be contractionary because they increase expectation

Multiplier	Great Recession $\hat{i}_t \geq \hat{i}^{elb}$	Great Depression $\hat{i}_t \geq \hat{i}^{elb}$
$\frac{\Delta \hat{Y}_S}{\Delta \hat{F}_L}$	-0.8	-1.9
	$[-0.9, -0.6]$	$[-2.6, -1.6]$
$\frac{\Delta \hat{Y}_S}{\Delta \hat{\tau}_L^S}$	0.8	1.3
	$[0.4, 1.2]$	$[1, 1.8]$
$\frac{\Delta \hat{Y}_S}{\Delta \hat{\tau}_L^I}$	-0.6	-1
	$[-0.8, -0.3]$	$[-1.4, -0.8]$

Table 7: Long-run multipliers of government spending, sales and income tax. 95% set in parentheses.

of output once the ELB no longer applies, a point emphasized by Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2014).

Consider a policy intervention that only affect the long-run. In this case both $\hat{Y}_L$ and $\hat{\pi}_L$ are affected by the policy. The AD in the short-run is

$$\begin{aligned} \hat{Y}_t &= E_t\hat{Y}_{t+1} - \sigma(\hat{\imath}_t - E_t\hat{\pi}_{t+1} - \hat{r}_t^e) + (\hat{F}_t - E_t\hat{F}_{t+1}) \\ \hat{Y}_S &= \mu\hat{Y}_S + \underbrace{(1-\mu)\hat{Y}_L + \sigma(1-\mu)\hat{\pi}_L}_{\text{long-run Demand Effect}} - \sigma i_S + \sigma\mu\hat{\pi}_S + \sigma\hat{r}_S^e + \underbrace{(1-\mu)\hat{F}_S - (1-\mu)\hat{F}_L}_{\text{Direct Demand Effect}} \end{aligned} \tag{39}$$

where we denote by "long-run demand effect" the effect of demand via the effect of the policy through $\hat{\pi}_L$ and $\hat{Y}_{L,}$ and denote as "direct effect" the last term in (39) which was the focus of last section (even if we kept $\hat{F}_L = 0$).

To understand the effect of policy on long-run output and inflation let us assume that monetary policy is set so that inflation is always at target in the long-run, i.e. $\hat{\pi}_L = 0$. It follows directly from this assumption from the AS equation that

$$\hat{Y}_L = -\psi\chi^I\hat{\tau}_L^I + \psi\sigma^{-1}\hat{F}_L$$

i.e. long-run output changes with long-run income tax and fiscal spending. We can now analyze the effect of an increase in either long-run fiscal spending, $\hat{F}_L$, or increase in long-run taxes, $\hat{\tau}_L^I$, on the short-run variables.

Consider first the effect of $\hat{F}_L$. The first part of its effect is the "direct" demand effect, the last term on the right hand side of equation (39). This term is negative. The reason is that for a given output, an increase in long-run government spending takes away resources from private consumption. This induces households to save more which reduces demand. The second part of the effect arises due to expectations of higher output in future: higher permanent government spending increases $\hat{Y}_L$. By taking away resources from private spending, the marginal utility of consumption of agents increases,

Multiplier	Great Recession $\hat{i}_t = \hat{i}^{elb}$	Great Depression $\hat{i}_t = \hat{i}^{elb}$
$\frac{\Delta \hat{Y}_S}{\Delta \hat{F}_{perm}}$	0.3	0.3
	$[0.2, 0.5]$	$[0.32, 0.35]$
$\frac{\Delta \hat{Y}_S}{\Delta \hat{\tau}^s_{perm}}$	-0.3	-0.2
	$[-0.4, -0.1]$	$[-0.23, -0.21]$
$\frac{\Delta \hat{Y}_S}{\Delta \hat{\tau}^I_{perm}}$	-0.5	-0.3
	$[-0.7, -0.2]$	$[-0.4, -0.3]$

Table 8: Permanent multipliers of government spending, sales and income tax. 95% set in parentheses.

thus increasing labor supply and output. Higher future output, in turn, increases spending today via higher permanent income.

For simplicity, let us assume that in the short-run, monetary policy will set $\hat{\pi}_S = 0$ if it can. Then a little bit of algebra suggests that

$$\frac{\Delta \hat{Y}_S}{\Delta \hat{F}_L} = \begin{cases} 0 & \hat{\imath}_S > i^{elb} \\ -\frac{(1-\mu)(1-\beta\mu)\psi\omega}{(1-\mu)(1-\beta\mu)-\mu\sigma\kappa} < 0 & \hat{\imath}_S = i^{elb} \end{cases} \tag{40}$$

Expectations of higher government spending have no effect at positive interest rate in the short-run because in that case the central bank can offset any effect it has on demand. It is always negative at the ELB, however, due the forces we have described above.

We can now consider the multiplier of a permanent increase in government spending. It takes into account both the expansionary effect of the short-run increase in spending and the contractionary effect arising form the increase in $\hat{F}_L$. Let us define $\hat{F}_{perm}$ as a permanent fiscal expansion, i.e. $\hat{F}_{perm} = \hat{F}_S = \hat{F}_L$. A little bit of algebra suggest that in this case, the multiplier is given by

$$\frac{\Delta \hat{Y}_S}{\Delta \hat{F}_{perm}} = \psi\sigma^{-1} \tag{41}$$

which is the same regardless of whether or not the ELB is binding.

Consider next the effect of an aggregate supply shock such as a reduction in long-run income tax, τ^I_L. This policy increases long-run output $\hat{Y}_L = -\chi^I \psi \hat{\tau}^I_L$. As seen by (39), the expectation of higher future output will then, for a given interest rate, increase demand. If again monetary policy will set $\hat{\pi}_S = 0$ if it can in the short turn, then

$$\frac{\Delta \hat{Y}_S}{\Delta \hat{\tau}^I_L} = \begin{cases} 0 & \text{if } \hat{\imath}_S \geq i^{elb} \\ -\frac{(1-\mu)(1-\mu\beta)\psi\chi^I}{(1-\mu)(1-\mu\beta)-\sigma\mu\kappa} < 0 & \text{if } \hat{\imath}_S = i^{elb} \end{cases} \tag{42}$$

so that while expectation of a supply shock is offset in the short-run if the central bank can adjust the interest rate, then at the ZLB expectation of a negative supply shocks (high τ_L^I) is contractionary, while an positive supply shock is expansionary because it increases expectation about future output. The paradox of toil no longer applies if the supply shock are sufficiently persistent, if $\hat{\tau}^I_{perm} = \hat{\tau}^I_S = \hat{\tau}^I_L$ the paradox disappears,i.e.,

$$\frac{\Delta \hat{Y}_S}{\Delta \hat{\tau}^I_{perm}} = -\psi\chi^I \tag{43}$$

which is the same regardless of whether or not the ELB is binding.

2.9 Self-defeating Austerity Budget Measures

Following the Great Recession public debt increased considerably in several advanced economies. This led several countries to adapt "austerity measures" (such as spending cuts) to balance the budget. One implication of large multipliers of government spending at the ELB is that these type of measures may instead increase the deficit rather than decrease it due to the negative effect they have on tax base.

So far, however, we have assumed that lump-sum taxes are adjusting in response to fiscal adjustment. Because of Ricardian equivalence their timing was irrelevant, and thus the deficit was indetermined. We now make plausible assumptions under which the deficit is well defined. But why would budget deficits matter? Since a deficit must ultimately be financed via future taxes or spending cuts a straightforward way in which they can affect the short-run is via expectation of $\hat{F}_L$ or $\hat{\tau}^I_L$, which we have already seen have big impact on short term demand as shown in table 2.8. This is an issue we get to in the next subsection, but before getting there, let us consider the evolution of the deficit and debt in isolation.

Suppose that lump-sum taxes in the short-run are unchanged at the steady-state so that [14]

$$\hat{T}_t = 0 \text{ for } t < t_L \tag{44}$$

but in the long-run adjust to stabilize public debt, i.e.

$$\hat{T}_t \text{ adjust so that } \hat{w}_t = \hat{w}_{t_L-1} = \hat{w}_L \;\text{ for } t \geq t_L \tag{45}$$

As before, because only lump-sum taxes adjust, the paths for inflation and output are determined independently from fiscal policy. Given (31) and (45) the government budget constraint implies that

[14] For technical reasons we assume that lump-sum taxes are set so that public debt cannot exceed a debt limit $\bar{w} > 0$ that can be arbitrarily high and is never reached in the simulation considered. By imposing this limit we guarantee that lump-sum taxes are set so that the transversality condition of the representative household is always satisfied, and hence deficits do not have any effect on inflation. The simplest way of ensuring this is to make the assumption that the stochastic process governing the shock has some terminal date T$\geq t_L$ at which the shock reverts back to steady state with probability 1.

$$\hat{T}_L = (1-\beta)\hat{w}_L = (1-\beta)\hat{w}_{t_L-1}$$

The deficit, defined as $\hat{D}_t = \beta\hat{w}_t - \hat{w}_{t-1}$, is then in the short-run

$$\hat{D}_S = \beta\hat{w}_t - \hat{w}_{t-1} = \beta w_y \hat{\imath}_S - w_y \hat{\pi}_S - (\bar{\tau}^I + \bar{\tau}^s)\hat{Y}_S + (1+\bar{\tau}^S)\hat{F}_S - c_y \hat{\tau}_S^I - \hat{\tau}_S^I \tag{46}$$

yielding the following expression for deficit (given output and inflation, as determined by (35) and (36) under the assumption that other fiscal variables are kept at steady-state:

$$\hat{D}_S = \begin{cases} 0 & \text{if } \hat{r}_S^e \geq i^{elb} \\ \beta w_y i^{elb} - w_y \hat{\pi}_S - (\bar{\tau}^I + \bar{\tau}^s)\hat{Y}_S & \text{if } \hat{r}_S^e > i^{elb} \end{cases} \tag{47}$$

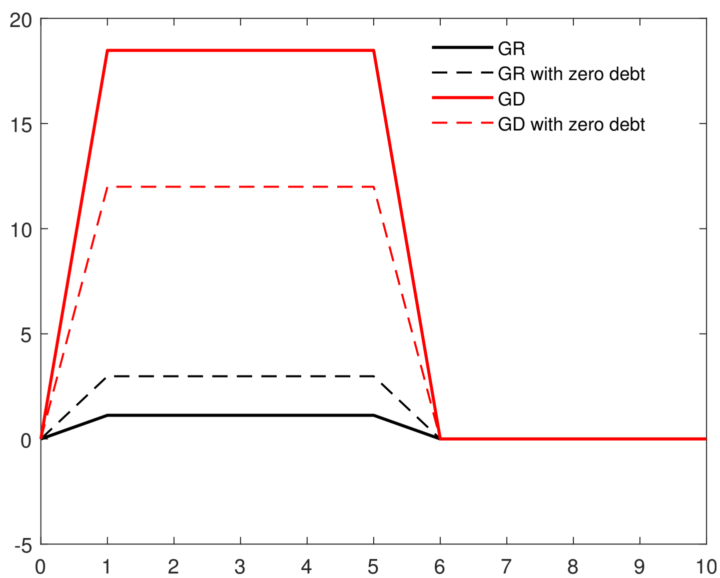


Figure 14: Deficits under Taylor Rule

The implications for the deficit in the GD and GR numerical examples can be computed once values are also chosen for w_y,$\bar{F}$, $\bar{\tau}^I$ and $\bar{\tau}^s$. Assuming $\bar{\tau}^I$=0.3, $\bar{\tau}^s$=0.1 and $\bar{F}$=0.2, figure 14 shows the increase in the deficit for the GD and GR numerical examples assuming either w_y=0 or w_y=0.75/4 (corresponding to 75 percent of annual output). If the government has no outstanding debt, the increase in the budget deficit is proportional to the drop in output. For example if output drops by 7.5 percent then the budget deficit is 3 percent of GDP in the GR example. A positive level of debt gives rise to two additional considerations. First, lower interest rate reduces the interest rate cost of the debt, as seen by the first term in equation (47). Second, the fall in inflation increases the real value of debt, that is the second term. In the GR calibration these considerations reduce the deficit due to the fall in interest costs, as seen in figure 14 via the dashed line, while in the GD example outstanding debt would tend to increase the deficit due to the debt-deflation effect. We do not attempt to match the deficits seen in the data in either the GD or the GR numerical examples. In the GD, President Hoover explicitly tried (but failed) to balance the budget with tax increases, while FDR explicitly had a strategy of deficit spending, as further discussed in section 7. In the GR, the government also had a fiscal expansion that had implication of increasing the deficit.

Despite the fact that we do not explicitly attempt to match the deficit, it is still of interest to compute comparative statics for the effect on the deficit of various fiscal interventions. This comparative static answers the question of what a particular policy change does to the deficit, assuming other fiscal variables are unchanged, a reasonable assumption for a policy maker that is considering the cost and benefits of a policy change.

It is useful to decompose the deficits into two components

$$\hat{D}_S = \underbrace{\beta w_y \hat{\imath}_S - w_y \hat{\pi}_S - (\bar{\tau}^I + \bar{\tau}^s)\hat{Y}_S}_{\text{Endogenous Component}} + \underbrace{(1+\bar{\tau}^S)\hat{F}_S - c_y \hat{\tau}_S^I - \hat{\tau}_S^I}_{\text{Fiscal Policy Component}} \tag{48}$$

The policy component of the deficit reflects the direct effect of a tax increase or spending cut on the deficit, for example a 1 dollar decrease in government spending will reduce the deficit by one dollar, holding everything else constant. The endogenous component reflects that the cut in government spending may also reduce aggregate output, and thus the tax base, which in turn may increase the deficit. Similarly, if the cut in spending reduces output, then inflation will decrease as well, and since debt is nominal, this will lead to an increase in the real value of government debt, which similarly has negative effect on the deficit. Finally, at positive interest rate, then there may be an effect on interest costs, which depends on the endogenous reaction of the central bank.

It then becomes interesting to understand what it takes to balance the budget. At positive interest rate cutting government spending always reduces the deficit in numerical experiments, however, this need not be the case at the ELB:

$$\frac{\Delta \hat{D}_s}{\Delta \hat{F}_S} = \begin{cases} 1 + \bar{\tau}^s + \beta w_y \psi \omega \sigma^{-1}(1-\mu) - \left(\bar{\tau}^s + \bar{\tau}^I\right)\sigma^{-1}\psi & \hat{r}_S^e > i^{elb} \\ 1 + \bar{\tau}^s - w_y \frac{\kappa}{(1-\beta\mu)}\left(\frac{\Delta \hat{Y}_S}{\Delta \hat{F}_S} - \psi\sigma^{-1}\right) - \left(\bar{\tau}^s + \bar{\tau}^I\right)\frac{\Delta \hat{Y}_S}{\Delta \hat{F}_S} & \hat{r}_S^e \le i^{elb} \end{cases} \tag{49}$$

Here the second line shows that cutting government spending to reduce the deficit can easily be self-defeating at the ELB, provided the spending multiplier is large enough. A little bit of algebra given the condition

$$\frac{\Delta \hat{D}_s}{\Delta \hat{F}_S} < 0 \text{ if } \frac{\Delta \hat{Y}_S}{\Delta \hat{F}_S} > \Gamma = \frac{1 + \bar{\tau}^s + w_y \frac{\kappa}{(1-\beta\mu)}\psi\sigma^{-1}}{\bar{\tau}^s + \bar{\tau}^I + w_y \frac{\kappa}{(1-\beta\mu)}}$$

Table 9 suggest that this condition is satisfied in the GD example, that is, cutting government spending increases the deficit rather than reducing it, while it is not in the GR example. The converses of

Multiplier	Great Recession $\hat{i}_t = \hat{i}^{elb}$	Great Recession $\hat{i}_t > \hat{i}^{elb}$	Great Depression $\hat{i}_t = \hat{i}^{elb}$	Great Depression $\hat{i}_t > \hat{i}^{elb}$
$\frac{\Delta \hat{D}_S}{\Delta \hat{F}_S}$	0.6	1.2	-0.3	1.2
	$[0.6, 0.7]$	$[1.1, 1.4]$	$[-0.7, -0.08]$	$[1.2, 1.2]$
$\frac{\Delta \hat{D}_s}{\Delta \hat{\tau}^s_S}$	-0.3	-0.9	0.1	-0.9
	$[-0.5, -0.1]$	$[-1.1, -0.8]$	$[-0.06, 0.5]$	$[-0.9, -0.9]$
$\frac{\Delta \hat{D}_S}{\Delta \hat{\tau}^I_S}$	-1.1	-0.6	-1.5	-0.7
	$[-1.1, -1]$	$[-0.8, -0.4]$	$[-1.8, -1.4]$	$[-0.8, -0.7]$

Table 9: Short-run deficit multipliers of government spending, sales and income tax. 95% set in parentheses.

this result is that in the GD example, then a fiscal expansion is self-financing, a "free lunch", a result that is analyzed in Erceg and Lindé (2014) in a model that closely resembles the current one. DeLong et al. (2012) show that once the economy experiences hysterisis, the likelihood that fiscal expansion is self-financing is enhanced even further.

The effect of a sales tax increase is very similar to that of a cut in government spending so we omit it here. It can be shown that increases in sales taxes can be self-defeating due to the negative effect on the tax base. Increase in the income tax, however, works better in this setting:

$$\frac{\Delta \hat{D}_s}{\Delta \hat{\tau}^I_t} = \begin{cases} -1 + \left(\beta w_y (1-\mu)\, \sigma^{-1} + \left(\bar{\tau}^s + \bar{\tau}^I\right)\right) \psi \chi^I & \hat{r}^e_S > i^{elb} \\ -1 - w_y \frac{\kappa}{(1-\beta\mu)} \left(\frac{\Delta \hat{Y}_S}{\Delta \hat{\tau}^I_S} + \psi \chi^I\right) - \left(\bar{\tau}^s + \bar{\tau}^I\right) \frac{\Delta \hat{Y}_S}{\Delta \hat{\tau}^I_t} & \hat{r}^e_S \leq i^{elb} \end{cases} \tag{50}$$

with numerical values shown in Table 9. The reason why increases in income taxes work better than cuts in spending is due to the Paradox of Toil, i.e., increases in income taxes increase output. However, as we have stressed above, the model does not incorporate possible negative demand effects of this policy that arise, for example, if some agents are liquidity constrained. Nevertheless, it is somewhat ironic that many European countries tried to balance the budget with exactly the wrong mix of instruments with respect to the model, i.e. they tended to cut spending, raise sales taxes and cut income taxes.

2.10 Deficits and Confidence

One rationale for austerity in Europe was that it was necessary to restore "confidence". Discussion of confidence is typically tied to the fiscal solvency of the government or the long-run policy framework. If the central bank successfully targets inflation, there is no room for confidence in the New Keynesian model. At the ELB, however, it can play an important role.

Consider a central bank that successfully keeps inflation on target. In this case the AS equation directly pins down output. It is given by

$$\hat{Y}_S = -\psi\chi^I\hat{\tau}_S^I + \psi\sigma^{-1}\hat{F}_S$$

This formulation clarifies that away from the ELB, output is independent of any measures of confidence or fiscal solvency. Consider now, however, the situation in which the ELB is binding. Then, using (39), AD is

$$\hat{Y}_S = \hat{Y}_L + \sigma\hat{\pi}_L + \hat{F}_S - \hat{F}_L + \frac{\sigma\mu}{1-\mu}\hat{\pi}_S + \frac{\sigma}{1-\mu}r_S^e$$

Critically, output in the short-run is now highly dependent on all three variables, $\hat{Y}_L, \hat{\pi}_L$ and $\hat{F}_{L.}$ Meanwhile, in the long-run

$$\hat{Y}_L = -\psi\chi^I\hat{\tau}_L^I + \psi\sigma^{-1}\hat{F}_L$$

How is this related to confidence? An increase in the budget deficit, will ultimately need to be stabilized one way or another. How this stabilization is achieved then depends on the specification of the *policy regime*.

In general, there is no simple answer to how deficit change expectations about future taxes and spending, this depends ultimately on the fiscal policy regime. Here we show a few examples to clarify this point.

Consider the following fiscal policy regime:

i) $\hat{D}_t = \hat{D}_S$ for $t < t_L$.

ii) $\hat{\tau}_t^s = \hat{\tau}_t^I = \hat{F}_t = 0$ for $t < t_L$

iii) $\hat{w}_t = \delta\hat{w}_{t-1}$ for $t \geq t_L$ where $0 < \delta < 1$

iv) $\hat{T}_t = \beta w_y \hat{\imath}_t$ for $t \geq t_L$

The key assumption is that, while distortionary taxes (or government spending) are not the direct source of the short-run deficit they need to adjust in the long-run to bring down debt to its pre-crisis level (note that $\hat{w}_t$ is the deviation of debt from the steady state that we linearize around). The last section that if all taxes rates are held at their steady state, and lump sum taxes fixed, then a recession at the ELB generates a deficit. We can also consider variation in short term lump sum taxes that generates further increase in the deficit.

If short-run variations in $\hat{w}_t$ are met by increases in long-run lump-sum taxes, then that would be the end of the story since then the model would satisfy Ricardian equivalence. Instead, we now suppose that long-run lump-sum taxes stay close to their steady state at time $t \geq t_L$ and thus other taxes or spending need to adjust to bring public debt back to steady state.[15] This is made explicit in iii) where we assume that long-run fiscal policy adjusts to stabilize the debt level at its original level and that this adjustment takes place over some period of time at a rate δ. This means that while the government may run up deficits in the short-run, in the long-run debt is stabilized at whatever level it was prior to the crisis.

Using the budget constraint in equation (25), we can see that by assumptions (iii) and (iv), it follows that

$$\begin{aligned}\hat{D}_t = -(1-\beta\delta)\delta^{t+1-t_L}\hat{w}_{t_L-1} &= \left[1+\bar{\tau}^s-\sigma^{-1}\psi(\bar{\tau}^I+\bar{\tau}^s)\right]\hat{F}_t - \left[1-(\bar{\tau}^I+\bar{\tau}^s)\psi\chi^I\right]\hat{\tau}_t^I \\ &\quad -\left[\frac{\bar{C}}{\bar{Y}}-(\bar{\tau}^I+\bar{\tau}^s)\psi\chi^s\right]\hat{\tau}_t^s \text{ for } t\geq t_L.\end{aligned}$$

A negative $\hat{D}_t$ means the government is running a budget surplus. This equation says that, in the long-run, the short run deficit needs to be financed by either a cut in government spending or an increase in labor or sales taxes. [16] How is this surplus financed? By tax increases and/or spending cuts. Suppose each fiscal instrument is adjusted in fixed proportions. Let γ_I be the share of $\hat{D}_{t+j}$ financed by labor tax, γ_s be the share financed with sales tax, and $\gamma_F = 1-\gamma_I-\gamma_s$ be the share financed by a reduction in government spending. Under this assumption, a full specification of long-run fiscal policy is now given by the choice of the weights γ_F, γ_I and γ_s. [17]

Given this policy specification we can now do the following thought experiment: What is the effect of an increase in the deficit on aggregate demand? Recall that the deficit is driven by either lump-sum taxes or shortfall in revenues due to the drop in output, hence, it has no "direct effect" on demand. Its effect, then, only comes about via expectations about *long-run taxes and the long-run size of the government*. This effect will critically depend upon how the deficit is financed, i.e. with tax or spending cuts.

In the short-run, however, output and inflation are now longer constant. They are functions of the debt, the value of the deficit, $\hat{D}_S$, and the shock, $\hat{r}_S^e$. Accordingly, we look for a solution of the form

$$\hat{Y}_{S,t} = Y^w\hat{w}_{S,t-1} + Y^D\hat{D}_S + Y^r\hat{r}_S^e \tag{51}$$

[15] More specifically, assumption iv) states that lump-sum taxes pay for the excess interest rate cost of debt in the transition phase. This has quantitatively small effects, but simplifies the algebra somewhat.

[16] The coefficient in front of each fiscal variable is in general positive.

[17] This assumption implies that the deviation of taxes and government spending from the steady-state will decline at the same rate as the debt, i.e. at the rate δ. We can define these variables as follows. The sequence of $\{D_t\}$ at $t\geq t_L$ is financed by $\hat{\tau}_t^I$, $\hat{\tau}_t^s$ and $\hat{F}_t$ in the fixed proportions

$$\begin{aligned}\gamma_{F,t} &\equiv \frac{\left[1+\bar{\tau}^s-\sigma^{-1}\psi(\bar{\tau}^I+\bar{\tau}^s)\right]\hat{F}_t}{\hat{D}_t} = \gamma_F \\ \gamma_{I,t} &\equiv -\frac{\left[1-(\bar{\tau}^I+\bar{\tau}^s)\psi\chi^I\right]\hat{\tau}_t^I}{\hat{D}_t} = \gamma_I \\ \gamma_{s,t} &\equiv -\frac{\left[\frac{\bar{C}}{\bar{Y}}-(\bar{\tau}^I+\bar{\tau}^s)\psi\chi^s\right]\hat{\tau}_t^s}{\hat{D}_t} = \gamma_s.\end{aligned}$$

Multiplier	Great Recession $\hat{i}_t = \hat{i}^{elb}$	$\hat{i}_t > \hat{i}^{elb}$	Great Depression $\hat{i}_t = \hat{i}^{elb}$	$\hat{i}_t > \hat{i}^{elb}$
$\frac{\Delta \hat{Y}_{S,t}}{\Delta \hat{D}_S} \vert \gamma_F = 1$	0.2	0	2.1	0
$\frac{\Delta \hat{Y}_{S,t}}{\Delta \hat{D}_S} \vert \gamma_s = 1$	0.3	0	2.1	0
$\frac{\Delta \hat{Y}_{S,t}}{\Delta \hat{D}_S} \vert \gamma_I = 1$	-0.2	0	-1.2	0

Table 10: Deficit multipliers of government spending, sales and income tax. 95% set in parentheses.

for output and an analogous solution for the other endogenous variables.[18] The model can then be solved numerically using the method of undermined coefficients. Table 2.10 reports the numerical value of the coefficient Y^D in (51) under three different assumptions for how the deficit is paid of in the long-run. If the short-term deficit is paid off with a reduction in the long-run size of the government (so $\gamma_F = 1$), if it is paid off with increases in long-term sales taxes (so $\gamma_s = 1$) and if it is paid off with long-term income taxes (so $\gamma_I = 1$). This coefficient has the following interpretation: If the deficit increases by one dollar in every period t in the short-run, by how many dollars will output go up or down in that period as a consequence? Hence, this number is comparable with the multipliers of spending and taxes than we explored in earlier sections.

As shown in the table, the effect of the deficit depends on how it will be paid off in the future. Consider the GR calibration. A one dollar increase in the deficit will increase output by 20 cents if the government pays it down by cutting long-run government spending. In contrast, if a one dollar deficit is financed by an increase in long-term income taxes, then output will decrease by 20 cents. The numbers are much larger for the GD example. Observe that at positive interest rate the deficit plays no role, for in that case any effect of expected tax or spending changes will be offset by changes in the interest rate by the central bank.

We are now in a position to study the effect of various types of fiscal policy, accounting for debt dynamics. The effect of government spending can approximately be split up into three pieces. First, a direct effect comes about even if taxes are lump-sum. Second, there is an indirect effect on the deficit. Finally, a third effect is how those deficits influence expectations about future fiscal variables and thereby current output. By now, we have already explored each of these effects separately and now we combine the individual pieces. Consider the following policy regime at a zero interest rate:

Fiscal Regime 1 In the short-run ($t < t_L$), $\hat{F}_t = \hat{F}_S$, $\hat{\tau}_t^I = \hat{\tau}_t^s = \hat{T}_t = 0$ and $\hat{D}_t = \hat{D}_S$. In the long run, the government behaves according to (i)-(iv) with $\gamma_F = 1$.

This policy regime suggests that, in the short-run, the government will increase spending and that this spending will be associated with constant short-run deficits (or surpluses) $\hat{D}_S$. Those deficits will

[18] Here $\hat{w}_{S,t-1}$ denotes the value of debt conditional on the shock in its low state, and similarly $\hat{Y}_{S,t}$ is output conditional on the shock being in the low state.

be paid off at time $t \geq t_L$. The net effect of increasing government spending in this policy regime, in the short-run, can be approximated by

$$\begin{array}{lcccccl} & \underbrace{\frac{\Delta\hat{Y}_s}{\Delta\hat{F}_s}_{\,(from\ Table\ 6)}} & + & \underbrace{\frac{\triangle\hat{D}_S}{\triangle\hat{F}_S}_{\,(from\ Table\ 9)}} & * & \underbrace{\frac{\Delta\hat{Y}_{S,t}}{\Delta\hat{D}_S/\gamma_F=1}_{\,(from\ Table\ 10)}} & \\ GR\text{ (mode)} & \mathbf{1.1} & & \mathbf{0.6} & & \mathbf{0.2} & = \mathbf{1.3} \\ GD\text{ (mode)} & \mathbf{2.3} & & \mathbf{-0.3} & & \mathbf{2.1} & = \mathbf{1.7}. \end{array}$$

For the GR calibration, we see that the effect of increasing government spending is higher in policy Regime 1 than when spending is financed just by current or future lump-sum taxes as in Table 6. Meanwhile, we see that the multiplier is smaller in the GD calibration. What is the intuition? Let us start with the GR case. The fact that government spending creates deficits triggers expectations of a future reduction in the size of the government. These long-run expectations also increase demand as shown by equation (40) and shown in Table 2.8. The reason is that lower long-run government spending leaves more room for private consumption in the long-run and thus, by the permanent income hypothesis, this means that the consumers want to consume more today (since they are trying to smooth consumption). In the GD calibration, however, the spending multiplier is reduced. The reason is that now government spending in the short-run decreases the budget deficit and, thus triggers expectation of higher long-run government spending, which is contractionary in the short-run.

Consider the following policy regime with the assumption that deficits are paid off using future income taxes:

Fiscal Regime 2 In the short-run, $t < t_L$, $\hat{F}_t = \hat{F}_S$, $\hat{\tau}_t^I = \hat{\tau}_t^s = \hat{T}_t = 0$ and $\hat{D}_t = \hat{D}_S$. In the long run, the government behaves according to (i)-(iv) with $\gamma_I = 1$.

We can now again compute the marginal effect of government spending as before by:

$$\begin{array}{lcccccl} & \underbrace{\frac{\Delta\hat{Y}_s}{\Delta\hat{F}_s}_{\,(from\ Table\ 6)}} & + & \underbrace{\frac{\triangle\hat{D}_S}{\triangle\hat{F}_S}_{\,(from\ Table\ 9)}} & * & \underbrace{\frac{\Delta\hat{Y}_{S,t}}{\Delta\hat{D}_S/\hat{\tau}_{L,t}^I>0}_{\,(from\ Table\ 10)}} & \\ GR\text{ (mode)} & \mathbf{1.1} & & \mathbf{0.6} & & \mathbf{-0.2} & = \mathbf{0.9} \\ GD\text{ (mode)} & \mathbf{2.3} & & \mathbf{-0.3} & & \mathbf{-1.2} & = \mathbf{2.7} \end{array}$$

Here we see that the marginal effect of government spending in Regime 2 is now lower than in the case when government spending is financed by lump-sum taxation in the GR scenario. The intuition is straightforward: Since government spending generates deficits, this implies that long-run income taxes need to be higher, which reduces output in the long-run. This reduces demand in the short-run. In the GD scenario, we see that the marginal effect of government spending is even higher. This is because government spending generates a budget surplus rather than a deficit, and thus implies lower labor taxes in the future.

Overall, the message of these examples is that how a fiscal expansion is financed, through adjustment of future taxes and spending, can have an important effect on demand in the short-run. Thus, a given government stimulus at a zero interest rate should be complemented with a plan about how short-run budget deficits or surpluses will be met in the future. The bottom-line, therefore, is that the effects of a fiscal intervention are highly dependent on the fiscal policy regime, and in particular, peoples expectations about the future fiscal policy regime.

2.11 Can Fiscal Policy Eliminate the Problem of the Zero Bound?

In 2001 Martin Feldstein suggested to the Japanese government in and op-ed in the Wall Street Journal that it should suspend the their equivalence the sales tax, τ_t^s., and only gradually lift it to it previous level (see Feldstein (2001)). He furthermore suggested, that the any revenue loss could be made up by raising taxes on income, $\tau_{L.}^I$.

Eggertsson and Woodford (2004) propose to model Feldstein's idea as follows:

$$\hat{\tau}_S^s = \frac{(\chi^S)^{-1}}{1-\mu}\hat{r}_S^e < 0$$

$$\hat{\tau}_S^I = -\frac{\chi^s}{\chi^I}\hat{\tau}_S^s > 0$$

It is easy to verify in the current model, that if the two tax instruments are adjusted in this way, the shock is completely offset, and there is no output contraction. Correia et al. (2013) extend this result to a model that incorporates capital. In this more general case a capital subsidy is required in addition to these two tax variations. The general point is that with a rich enough tax structure, shock to the efficient rate of interest can typically be offset.

There are several important caveats to this result if translated into pracical policy advise. First, the use of sales taxes to stimulate spending critically depend upon that the prices set by the firms (at staggered intervals) do not incorporate the tax when setting their prices. If prices are set inclusive of the tax, then $\hat{\tau}_t^s$ does not appear in the IS equation, only in the AS, and the policy would have no effect. Eggertsson and Woodford (2004) argue that this is likely to be the case in countries that have VAT tax, as is the case for most European countries, even if it does not appear to apply to sales taxes in the US. Second, sales taxes are often levied at local level, e.g. in the US, and thus not a tool of national fiscal policy. Third, in the numerical example above, another problem, is that the implies reduction in sales taxes imply that that $\hat{\tau}_t^s = -0.101$, i.e., the policy would require a sales subsidy. Hence, if one supposes that the tax rates must be non-negative, Eggertsson and Woodford (2004) policy might not fully offset the shock.

Correia et al. (2013) show that rather than requiring a reduction in the sales taxes that is then returned back to normal level, a policy of committing to continuously increasing the sales taxes to levels above steady-state yields the same result. In this case, however, another complication is that this might ultimately require negative income taxes.

While one might question if the tax system is flexible enough to enable the government to overcome the ELB, a more fundamental criticism of Feldstein's proposal is found in Lancaster (2019). Lancaster (2019) shows that the policy prescription is highly model depended and sensitive to the assumption that no agents are liquidity constraint. Considering the OLG model in section 2 with nominal frictions, he shows that the Feldstein's proposal has the opposite effect in that model, i.e. it makes the ELB problem worse. We revisit this result in in section 4.

Taken together, these consideration imply that it is worth considering alternatives such as regime changes for monetary policy that we now turn to. The broader take-away, however, is that there are potentially very large gains in reforming the tax system to adjust to shocks that give rise to the ELB. If large enough, these reforms can be sufficiently strong to eliminate the problem.

2.12 Notes on the Literature

The textbook New Keynesian model used in this section is covered in detail in Woodford (2003) and Galí (2015). The first paper to analyze fiscal policy in this model at the ELB is Eggertsson (2001a) that finds significant role for real government spending, provided the central bank cannot commit to future policy. Independently, Christiano (2004) similarly found large role for fiscal policy the ELB. Large parts of this section are adapted from Eggertsson (2011) and Denes et al. (2013). Relative to these papers, the model has been calibrated to account for the Great Recession, and the pricing assumption of Benigno et al. (2019) is used to explain absence of deflation. Christiano, Eichenbaum, and Rebelo (2011) analyze the size of the fiscal multiplier in a simple New Keynesian model but also extend the analysis to a medium scale quantitative model and find that that the same main conclusion apply, see also Woodford (2011). Farhi and Werning (2016a) analyze fiscal multipliers in both closed and open economies and extend the analysis to account for non-Ricardian agents that consume all their income.

A difficulty in assessing the empirical impact of fiscal expansion is that the model predicts significant different response at the ELB relative to away from it, while most post WWII data is generated in a period of positive interest rates. Thus the empirical literature on fiscal policy rarely contains direct evidence. Two important exceptions are Ramey and Zubairy (2018) and Miyamoto, Nguyen, and Sergeyev (2018) using historical data from the US and data from Japan respectively. Both estimate multipliers at positive interest rates below 1, and at the ELB above 1, at about 1.5. These numbers are well consistent with the GR and GD numerical examples. Other literature, such as Nakamura and Steinsson (2014) has exploited regional variation in government spending to draw inference about the impact of government spending at the ELB. For a broader overview of the literature about the effect of fiscal policy on output, see Nakamura and Steinsson (2018) and Ramey (2019).

Wieland (2019) proposes using earth quakes and variation in oil prices as empirical tests of the paradox of toil using Japanese data from recent decades. He argues that his findings are inconsistent with the paradox of toil. One interpretation of his finding is that the oil shocks are persistent enough so that the result of section 2.8 apply instead, in which case the paradox disappears. Eggertsson et al. (2014)

extend the paradox of toil to an open economy, and analyze condition under which structural reforms may be contractionary, but these type of reforms were high on the agenda in Europe in response to the Great Recession.

Boneva, Braun, and Waki (2016) argue that several of results shown in this section are an artifact of log-linearization. They establish this result by analyzing a non-linear version of Rotemberg (1982). Eggertsson and Singh (2019) show that the non-linear version of the model studied in this section can be solved in closed form under the two state Markov process and confirm that the key results are unchanged, even the quantitative result in the GD example are largely unchanged. They show that Boneva et al. (2016) result are driven by extreme resource costs of price changes so that a large fraction of economic resources are devoted to changing prices. They argue that this is implausible, and suggest an modified variation of the non-linear Rotemberg (1982) model, that they suggest is better consistent with the original model, which in turn closely matches the results reported in this section.

Kiley (2016) suggests that the large spending multiplier at the ELB, the paradox of toil, as well as the power of forward guidance which is the focus of next section, are an artifact of the assumption of sticky prices. He suggest that once the assumption of sticky prices is replaced with sticky information, the multipliers are instead small and the paradox of toil disappears. Eggertsson and Garga (2019) show that Kiley's result are due to that he is considering different thought experiment than considered in this section. In Kiley's thought experiment the ELB bound is not binding due to an exogenous shock, as in this section, but instead due to an exogenous peg. Eggertsson and Garga (2019) show that if the same thought experiment is considered as in this section, where the ELB is binding due to a fundamental trigger, the results are even stronger under sticky information than prices (that is, the government spending multiplier is larger, and the paradox of toil more extreme.)

Cochrane (2017) suggests that the paradox of toil, high government spending multiplier, as well as the price flexibility paradox depend on equilibrium selection. In section 3.4 we corroborate this finding by showing that if monetary policy is conducted under optimal commitment, then spending multipliers are low, there is no paradox of toil, and the price flexibility paradox disappears. Our interpretation of this finding is suggesting that the results are highly dependent on the *monetary policy regime*. We will show in section 3.4 that the policy regime analyzed in this section is equivalent to the Markov Perfect Equilibrium, i.e. policy when the Federal Reserve is unable to commit to future inflation and output boom once the ELB is no longer a constraint. We argue that this is a more realistic description of actual policy in the US during the GR, relative for example to what Cochrane (2017) calls the "local to frictionless equilibria" which is close to the optimal policy under commitment, as discussed in more detail in Eggertsson and Egiev (2019).

3 Monetary Policy Regime Changes

The discussion so far has been focused on policy responses taking as given that monetary policy is determined according to a Taylor rule or, alternatively, a strict inflation target. We now consider what can be done if the systematic component of monetary policy is changed, i.e., if there is a *regime change*.

There are several examples of monetary regime changes in economic history, and we will document some in sections 7 and 8. Examples of regime changes include countries leaving gold standard during the Great Depression. One can also argue that introduction of "forward guidance" by the Federal Open Market Committee (FOMC) was an example of a regime change (in that case the FOMC deviated from the reaction function which the market had assumed it had been following).

We start with fully optimal monetary policy. We then consider simple variations of the policy regime given by equation (28) that can approximate the optimal policy. We then move to the difficult issue of policy credibility.

3.1 The Optimal Monetary Policy

To analyze the optimal policy we assume that the government maximizes utility of a representative household. The utility of the household, as shown by Woodford (2003), can be expressed to a second order as

$$U_t \approx -E_t \sum_{t=0}^{\infty} \beta^t \{\hat{\pi}_t^2 + \lambda_y \hat{Y}_t^2\} \tag{52}$$

where $\lambda_y \equiv \kappa/\theta$. The optimal policy is the set of state contingent paths for $\{\hat{\pi}_t, \hat{Y}_t, \hat{\imath}_t\}$ that maximizes (52) subject to (23) and (24). This problem is solved using Lagrangian methods. Denote the Lagrange multiplier for the IS, AS and ELB constraints by ϕ_{1t}, ϕ_{2t} and ϕ_{3t} respectively. The first order conditions of the government's problem are

$$\hat{\pi}_t + \sigma\beta^{-1}\phi_{1t-1} + \phi_{2t} - \phi_{2t-1} = 0 \tag{53}$$

$$\lambda_Y \hat{Y}_t + \phi_{1t} - \beta^{-1}\phi_{1t-1} - \kappa\phi_{2t} = 0 \tag{54}$$

$$-\sigma\phi_{1t} + \phi_{3t} = 0 \tag{55}$$

$$\hat{\imath}_t \geq i^{elb}, \phi_{3t} \geq 0, \phi_{3t}(\hat{\imath}_t - i^{elb}) = 0 \tag{56}$$

where the last condition is a Kuhn-Tucker complementary slackness condition. The equilibrium is a set of stochastic processes for $\{\hat{\pi}_t, \hat{Y}_t, \hat{\imath}_t\}$ that solve these conditions, together with the IS and the AS equations given the stochastic process for $\{\hat{r}_t^e\}$. The difficulty in solving for an equilibrium stems from the inequality in the complementary slackness condition that renders standard solution techniques inapplicable.

The stochastic process assumed here makes the solution tractable: one can first solve the system for the period when the shock is over and the ELB is not binding, and then solve the model backwards using this solution as input. The main complication is that it not known for how long the ELB binds after the shock reverts to steady state. Eggertsson and Woodford (2003) apply a guess and verify algorithm to solve this problem which is generalized and automated in Eggertsson et al. (2019a).

The optimal policy is shown in figure 15 for the GR numerical example. The figure shows the contingency when the shock is over in period 5, but the figure is representative of other contingencies as shown in Eggertsson and Woodford (2003). The key element of optimal commitment is that it involves a policy of keeping the nominal interest rate at the ELB beyond the time at which the shock is over. This contrasts sharply with the standard Taylor rule where the central bank re-normalizes the interest rate as soon as the shock is over.

As can be seen in the figure, this policy greatly mitigates the drop in output and inflation relative to the Taylor rule. Several channels are at work here. First, lower future nominal interest rate at a time when, according to previous expectations, the bank was supposed to raise it directly stimulates spending. This happens via IS equation that depends upon the entire path of interest rate as seen in equation (27). Second, the commitment is associated with expectations of higher inflation which lowers the real interest rate. Finally, commitment generates expectation of higher output which similarly increases spending via the permanent income hypothesis (i.e. via expectation of higher future output on the right hand side of the IS equation (23)). Observe that due to the small reduction inflation according to the GR calibration, the expectation of future inflation plays a relatively small role according to the optimal commitment plan.

3.2 Implementing the Optimal Monetary Policy When the Government Can Commit

The interest rate commitment shown in figure 15 reflects the path for interest rates for one realization of the shock (when it subsides in period 5). In this particular case the central bank holds the nominal rate at the lower bound for 5 additional periods. Figure 16 shows the time paths for interest rate for several different possibilities, i.e. when the shock subsides in period 1, 5, 10 and 15. As the figure makes clear, the duration of the ELB is state contingent: the longer the shock – the longer it binds *after the shock* is over, in contrast to the Taylor rule when the ELB stops being binding as soon as the shock is over. The fact that the interest rate commitment is contingent on the shock makes the communication of the optimal commitment policy challenging. The central bank needs to communicate to the market not only that it will keep the interest rate low beyond the duration of the shock, but also that that such time extension cannot be approximated by a simple calendar time. For example, if the shock subsides in period 5, then the duration of the ELB is an extra 5 quarters (or a total of 10), while if it last for 10 quarters the extra is 7 quarters instead (total of 17). Thus the duration of the optimal commitment depends on how economic conditions evolve over time, i.e. the underlying time path for the shocks (which in realistic applications would presumably depend on host of factors that may be difficult to articulate explicitly.)

Eggertsson and Woodford (2003) show the somewhat surprising result that it is possible to communicate the optimal commitment policy in a way that is independent of the stochastic process for the underlying shock (and thus the expected duration or the severity of the economic recession). The key

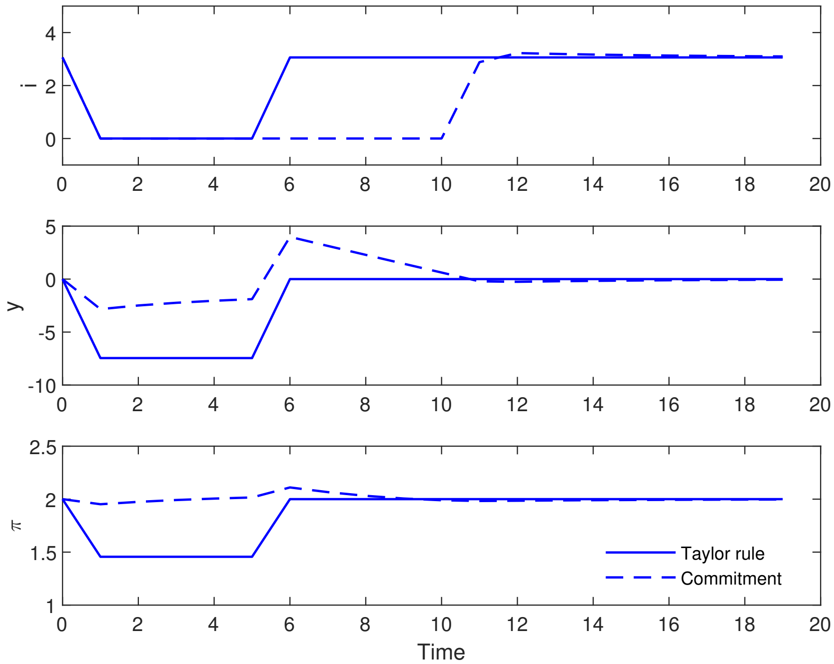


Figure 15: Taylor rule and commitment monetary policy regime when the shock lasts 5 periods

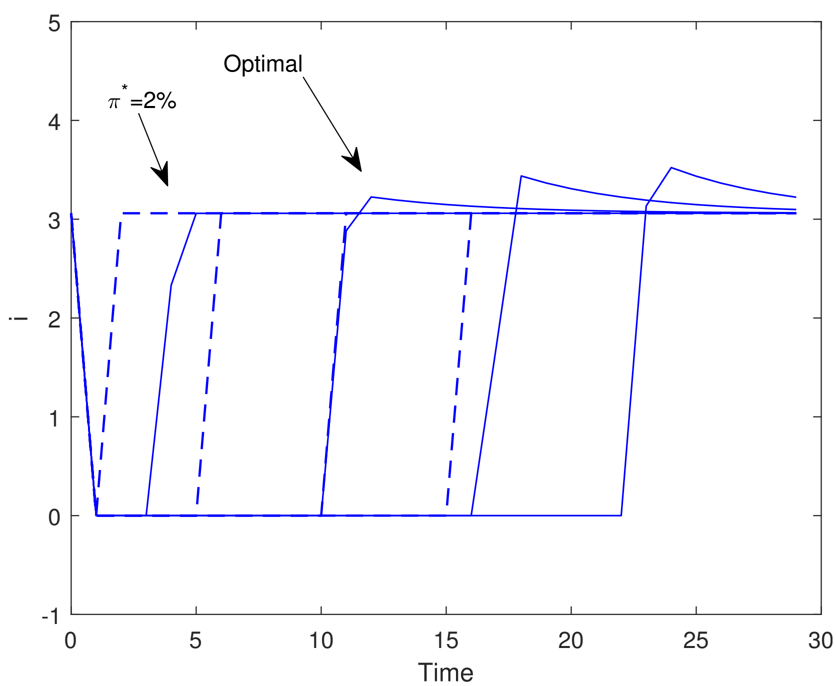


Figure 16: Interest rate under under commitment policy regime and Taylor rule

idea is that the central bank announces a "threshold" which captures for how long it will keep the interest rate at the ELB. This threshold is formulated in terms of economic fundamentals, i.e. the output gap and the price level rather than on the unobserved shock. They propose an output gap-adjusted price level target to characterize such a threshold. It takes the following form. First, in each and every period there is a predetermined value p_t^*. The central bank commits to not increase the increase rates unless

$$\tilde{p}_t = p_t^* \tag{57}$$

where $\tilde{p}_t$ is an output adjusted price index defined by

$$\tilde{p}_t \equiv p_t + \frac{\kappa}{\lambda}\hat{Y}_t. \tag{58}$$

The key to this commitment is how the threshold is formulated. Eggertsson and Woodford (2003) show that the optimal monetary policy commitment is replicated in the case p_t^* is computed according to the following formula

$$p_{t+1}^* = p_t^* + \beta^{-1}(1+\kappa\sigma)\Delta_t - \beta^{-1}\Delta_{t-1} \tag{59}$$

where Δ_t is a variable that measures by how much the central bank misses its target in period t due the ELB:

$$\Delta_t \equiv p_t - p_t^*. \tag{60}$$

Figure 17 illustrates this by showing target and gap-adjusted price levels for a single contingency in which shock lasts for 5 periods. This corresponds to the optimal commitment we showed in figure 15. A key feature is that if the price level target is not reached in a particular period, then the response of the central bank is to increase the price level target even further.

In the crisis of 2008 several central banks attempted forward guidance. Initially, however, most central bank attempted guidance based on fixed calendar dates. For example the Bank of Canada announced in 2009 that it would keep the interest rate rate at zero until at least mid-2010, while the Federal Reserve announced that it would keep rates low for an "extended" period of time. The policy that came closest to the type of a threshold suggested by Eggertsson and Woodford (2003) was the Federal Reserve's announcement in 2012. Then the Fed announced it would keep interest rate at zero until employment was at least below 6.5% and inflation was not projected two years ahead to be more than 50 basis point above the inflation target of the bank. While this policy did not go as far as the proposal by Eggertsson and Woodford (2003), it did have the essential feature that it specified only the condition under which rates would increase, *a threshold*, rather than some deterministic time horizon for low interest rates. It had a desirable property: it was up to the market to estimate the duration of the ELB and thus, were economic conditions to deteriorate, the market would've automatically updated its expectations about a longer duration of ELB.

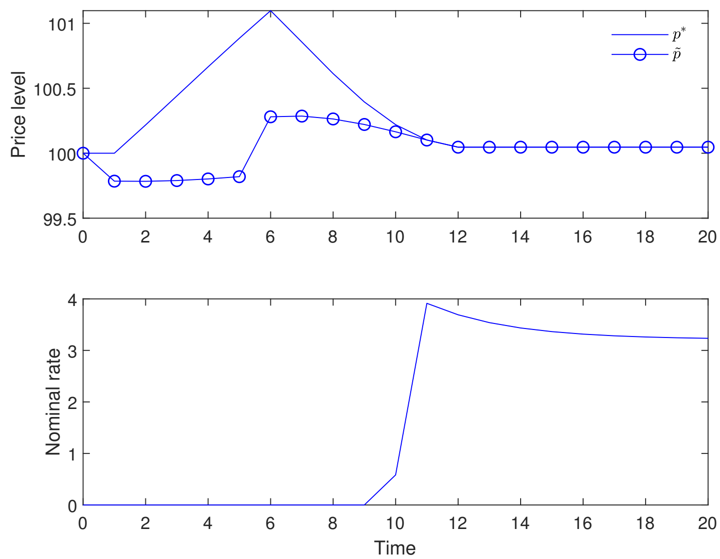


Figure 17: Price level, actual and target, under optimal monetary policy

3.3 Alternative policy proposals

There have been several monetary polices suggested to mitigate the problem of the zero bound. An early proposal, and perhaps the simplest one, is Krugman (1998) suggestion for the Bank of Japan. He suggested a policy of 4% inflation for 15 years. As we have already noted in section 2.3, this effectively gives the central bank more space to cut rates when faced with an adverse shock, i.e. in terms of the notation of the model it amounts to reducing i^{elb}. With a higher inflation target of 4 percent then the nominal steady state interest rate is in the GR example is 5.2 percent. The central bank can then cut the interest rate by 530 basis point in response to a shock, and thus accommodate shock that implies negative real interest rate of - 4 percent. This turns out to be too little in the GR example, but the shock there correspond to -6.8 percent, suggesting that 7 percent inflation target would be required. A higher inflation target has the obvious disadvantage of requiring permanently higher inflation rates – even at times when they are no longer desirable. As seen in Figure 15, the optimal policy also prescribes a trivial overshooting of inflation, less than 50 basis points. Hence eliminating the ELB via higher average inflation target would be severely sub-optimal.

In an early and important paper on the ELB Reifschneider and Williams (2000) showed, by simulating the FRB/US model, that a simple modification of the Taylor rule could greatly reduce the cost of the lower bound without increasing average inflation. According to Reifschneider and Williams (2000) rule the Fed should keep track of when the Taylor rule could not be satisfies due to the ZLB. Let us denote these "misses" by $d_t = i_t - i_t^{Taylor}$. Such misses are then accumulated into Z_t and policy follows

$$\hat{i}_t = \max(\hat{i}_t^{Taylor} - \alpha Z_t, 0) \tag{61}$$

which has the implication in the context of the current model that once the shock is over, the central bank still keeps the interest rate low for some time, depending on the value of α. Figure 18 shows

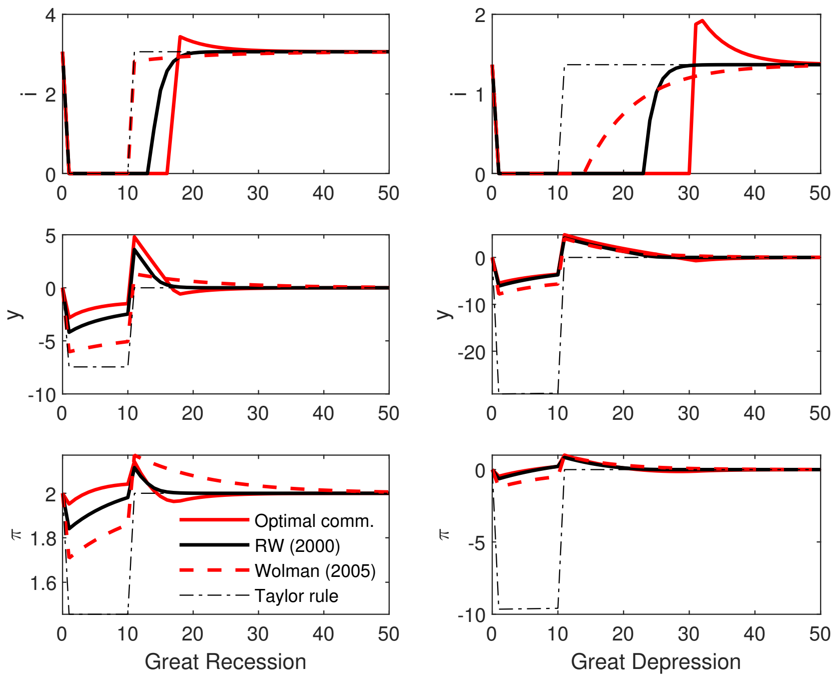


Figure 18: Optimal commitment policy regime, Reifschneider and Williams (2000) and Wollman (2005) in a contingency when shock lasts for 10 periods under Great Recession (left column) and Great Depression (right column) parameterization.

that this policy rule replicates the optimal commitment relatively closely for both the GR and the GD numerical examples.

Wolman (1999) is another early example (eventually published in Wolman (2005)). He suggested that the cost of the ELB could be substantially reduced if the inflation rate in the Taylor rule of the central bank was replaced with the price level. The intuition for this result is that in a recession, as the price level falls, people expect it to revert to its original level under price level targeting. This automatically increases inflationary expectation at the ZLB and thus increases demand. Thus, price level targeting has an important automatic stabilizing effect. Figure 18 compares Wolman's proposal to the optimal commitment. While this rule replicates the optimal policy relatively closely for the GD example, it does less well for the GR example. The reason for this is the small reduction in inflation in the GR example, thus this policy rule prescribes insufficient expansion once the ELB stops being binding relative to the optimal commitment.

3.4 Fiscal Multipliers and Paradoxes Revisited and the Optimal Fiscal Commitment

A key conclusion from our previous analysis is that government spending multiplier is above 1 at the ELB, giving strong motivation for fiscal stimulus, while supply policies are much less effective due

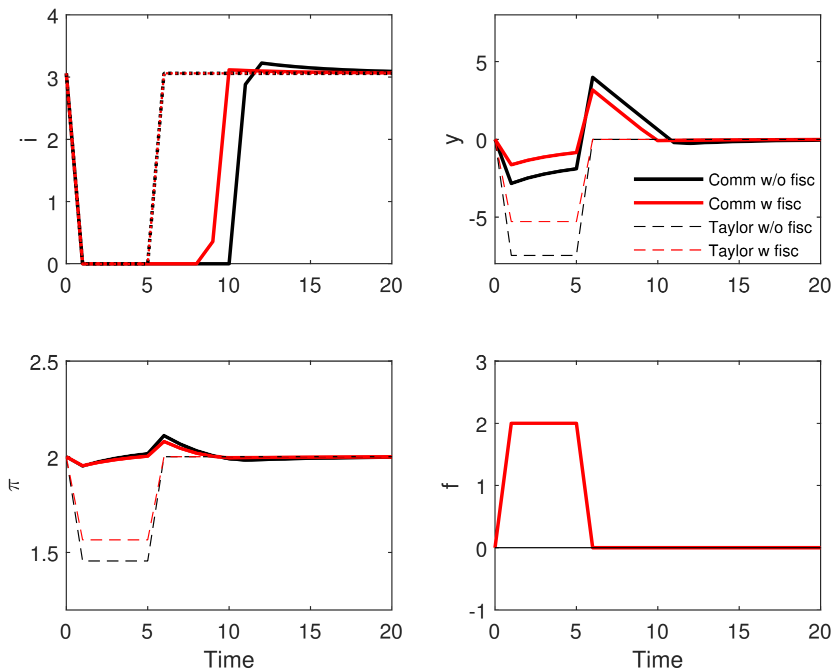


Figure 19: 2% Fiscal shock under monetary commitment and assuming Taylor Rule

to the deflationary effect they create. Another interesting result is that increasing price flexibility is destabilizing.

These conclusions are closely related. The main driving force is that monetary policy, given by the Taylor rule, does not respond to the increase in government spending or the deflationary pressures triggered by more price flexibility or positive supply shocks. Under the optimal monetary policy under commitment, however, the monetary authority responds to all shocks or structural changes. Consider, for example, if there is a positive supply shock that triggers deflationary pressures. Then the central bank responds by keeping the nominal interest rate low for a longer period of time than previously. Similarly, if there is an increase in government spending, then the central bank provides smaller policy accommodation than previously, since it assumes (correctly) that less stimulus is needed to achieve its objectives.

This is illustrated in figure 19 for the GR numerical example which shows the effect of a 2 percent increase in government spending through the duration of the ELB. The solid line black line shows the evolution of the economy for the GR example if the central bank commits to optimal monetary policy and there is no fiscal stimulus. The solid red line shows the evolution of the economy under the optimal monetary commitment if there is a 2% fiscal stimulus. There is now less monetary stimulus required, hence monetary policy raises interest rate faster than in the absence of the stimulus. Thus monetary policy is tightened in response to a fiscal expansion. As a consequence of this, the effect of fiscal policy is smaller than if we assume a Taylor rule, since in this case the nominal interest rate remain unchanged in response to the fiscal policy expansion. The dashed line shows the evolution of the economy if monetary policy follows a Taylor rule. In this case, there is no change in interest rates

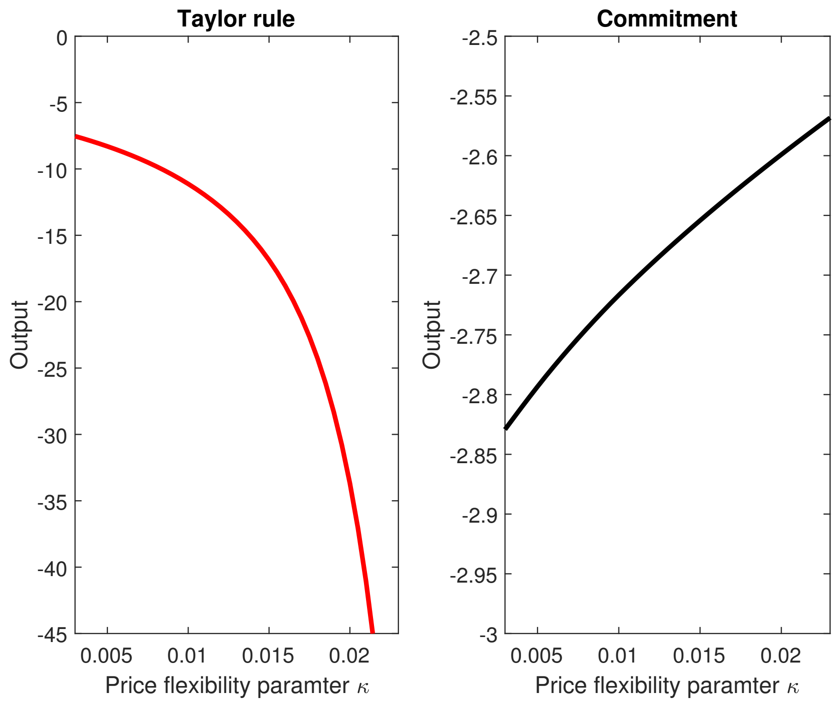


Figure 20: Taylor rule and commitment monetary policy regimes under different values of price flexibility κ

as a response to the fiscal expansion. The result of this is that fiscal policy is more expansionary.

This basic insight is confirmed in Table 11. Under optimal commitment, the fiscal spending multiplier goes down, relative to when a Taylor rule is assumed, resembling closely the value of the multiplier at positive interest rate when the government is unconstrained by the ELB. Moreover, it does not increase when comparing the Great Recession scenario to the Great Depression scenario.

Exactly the same logic applies when one considers supply shocks. If the monetary policy is able to commit to optimal policy, it is effectively replicating the flexible price allocation. Thus the economy the reaction of this economy to shocks is also similar to that of the flexible price allocation. In effect, there is no paradox of toil in the model under the optimal monetary policy commitment.

The paradox of price flexibility similarly disappears under the optimal monetary policy commitment. Figure 20 shows for the Great Recession parameterization that increasing price flexibility parameter κ leads, as expected, to unbounded output loss under Taylor Rule. This result disappears if the central bank commits to the optimal policy. Under optimal commitment increasing price flexibility leads to gradual improvement in output as shown in 20.

These result help explaining the findings in Cochrane (2017). Cochrane (2017) argues that the large government multipliers and paradoxes documented assuming the Taylor rule are the result of a selection of equilibrium which he argues is implausible. The equilibria he suggest is more plausible, however, is instead closer to the optimal commitment equilibrium, a point developed in more detail

Multiplier	Great Recession Commitment	Great Recession Taylor Rule	Great Depression Commitment	Great Depression Taylor Rule
$\frac{E_0 \Delta \hat{Y}_S}{E_0 \Delta \hat{F}_S}$	0.3	1.1	0.3	2.3
	$[0.2, 0.4]$	$[1, 1.2]$	$[0.2, 0.4]$	$[1.9, 2.9]$
$\frac{E_0 \Delta \hat{Y}_S}{E_0 \Delta \hat{\tau}^I_S}$	-0.3	0.07	-0.3	0.7
	$[-0.4, -0.2]$	$[0.02, 0.1]$	$[-0.4, -0.3]$	$[0.5, 1]$

Table 11: Short-run multipliers of government spending and income tax under commitment and under discretion monetary regimes

in Eggertsson and Egiev (2019). Eggertsson and Egiev (2019), however, argue that the optimal discretion equilibrium, analyzed in the next section, provides a better description of US policy maker' behavior during the Great Recession. In the next section we will see that the optimal discretion equilibrium is in fact equivalent to the Taylor rule we have already assumed. Eggertsson and Egiev (2019) main argument in favor of the discretionary equilibrium/Taylor rule as a description of the US experience during the Great Recession is that policy makers never made an explicit commitment to allow the inflation to overshoot the 2% inflation target of the Federal Reserve nor did it commit to output to beeing substantially above potential for an extended period of time. Inflation and output overshooting is the fundamental characteristic of the optimal commitment (and, equivalently, of the local to frictionless equilibria analyzed in Cochrane (2017)).

3.5 The Deflation Bias and the Difficult Problem of Credibility

A key practical problem with optimal policy is that it may not be credible. More precisely, it is not time consistent. Macroeconomists have known about this problem at least since Kydland and Prescott (1977). This problem is particularly severe at the ELB, since the optimal monetary policy involves *only* making promises about future interest rate policy rather than interest rate policy today. Moreover, the government needs to make promises about behavior that is fundamentally different from past behavior. A key problem with a regime change involving only announcements, then, is that it may not be "credible."

Krugman (1998), when suggesting that the Bank of Japan should commit to inflation, recognized that at the ELB the problem of the central bank was essentially the opposite of the regular credibility problem of the central bank. Hence Krugman suggested that the central bank needed to "credibly commit to being irresponsible," although he does not provide any explicit guidance of how this can be achieved beyond making announcements about a higher inflation target.

The credibility problem of the central bank at the ELB was first formalized in Eggertsson (2001b) in the standard New Keynesian model assuming that government spending was fixed and taxes were lump sum. Eggertsson (2001b) analyzes a Markov Perfect Equilibrium (MPE) where it assumes that, unlike in the commitment equilibrium, the central bank takes the expectations of private sector as *given*, assuming the same stochastic process for the efficient rate of interest as we have assumed here. Adam and Billi (2007), using numerical methods, generalize the analysis by considering a generic AR(1) process for the efficient rate of interest.

Consider first the MPE abstracting from fiscal policy. The model is perfectly forward looking: there are no state variables in the AS and IS equations of the model, and BC is irrelevant as we abstract from fiscal policy. Thus, the expectation variables in the IS and AS equation are given by some functions $E_t\hat{\pi}_{t+1} = \pi_r \hat{r}^e_S$ and $E_t Y_{t+1} = Y_r \hat{r}^e_S$ (where π_r and Y_r are constants) that the government takes as given when solving the maximization problem. The problem of the government, once again denoting the Lagrange multiplier for the IS curve, AS curve and the ELB by ϕ_{1t}, ϕ_{2t} and ϕ_{3t} respectively, gives rise the first order conditions:

$$\hat{\pi}_t + \phi_{2t} = 0 \tag{62}$$

$$\lambda_Y \hat{Y}_t + \phi_{1t} - \kappa\phi_{2t} = 0 \tag{63}$$

$$-\sigma\phi_{1t} + \phi_{3t} = 0 \tag{64}$$

$$\hat{i}_t \geq \hat{i}^{el}, \phi_{3t} \geq 0, \phi_{3t}(\hat{i}_t - \hat{i}^{el}) = 0 \tag{65}$$

and an equilibrium can once again be defined as a set of stochastic processes for $\{\hat{\pi}_t, \hat{Y}_t, \hat{i}_t\}$ that satisfy these conditions and the IS and AS equations given the stochastic process $\{\hat{r}^e_t\}$. Relative to the commitment solution, the key difference is the absence of any lagged variables in these conditions. Thus at a given point in time t, the model is perfectly forward looking.

This has fundamental implications which are easy to see for the simple stochastic process we have assumed. Consider the time period once the shock has reversed to steady state i.e. $t \geq t_L$. It is easy to verify that the first order conditions are satisfied when $\hat{\pi}_t = \hat{Y}_t = 0$ which is exactly the same solution as when the central bank follows the Taylor rule we have already analyzed. The solution for $t < t_T$ is then once again given by 33 and 34 for the case when $\hat{r}^e_S < 0$ as given by (35) and (36) and shown in figure 15 where the solution under discretion coincides with the solution under the Taylor rule.

Figure 15 illustrates the credibility problem associated with optimal policy under commitment. Recall that the central bank achieves policy stimulus by committing to keeping the nominal interest rate at the ELB even as the natural rate of interest has already reverted back to steady state in period 6. The optimal policy under discretion formalizes the credibility problem associated with this strategy. As the figure illustrates, the optimal commitment policy reduces the drop in output and inflation in period 1 through 5 relative to discretion. In period 6 on-wards, however, the optimal discretion yields the best possible outcome, i.e. output is at its steady state and inflation on target. Meanwhile the optimal commitment policy deviates from the first best in period 6 onwards by having output above steady state and inflation above it target. Hence, ideally, the government would like people to believe it was doing the optimal commitment in period 1 through 5, and then come period 6 to renege on the policy and achieve the new first best. If the government maximizes under discretion, and the public is rational, the public would understand these incentives, and thus the optimal policy commitment is not credible. This is the deflation bias of discretionary policy.

This problem has been widely recognized by policymakers and is one of the main reasons why forward guidance was not pursued as vigorously as theory suggested might be ideal. John Williams, for example, then the President of the Federal Reserve Bank of San Francisco, noted in a transcript of a FOMC meeting in 2011 when discussing the possibility of adopting more aggressive forward guidance language: "In the jargon of academics, our commitment technology is very limited. It is simply impossible for us to set a predetermined course of policy that will bind future Committees." Similarly, the current Chair of the Federal Reserve, Jerome Powell, noted in the context of policies that involved committing to a future expansion at the ELB: "Part of the problem is that when the time comes to deliver the inflationary stimulus, that policy is likely to be unpopular, what is known as the time consistency problem in economics." Powell (2019). In the language of the model, Powell was recognizing the incentive of the government in period 6 in figure 15 to renege on the "inflationary stimulus" implied by the optimal commitment, as it is not "popular" ex-post since at that time both the inflation and output target can be reached without any complication.

3.6 A Credible Regime Change using Fiscal Policy Instruments

We next address how optimal policy can be made credible. At a practical level a first step, and perhaps the most important and natural one, is that the central bank simply states clearly its intentions about future development of policy in a way that is consistent with the optimal commitment, similar to Krugman (1998) suggestion of committing to being irresponsible by announcing a 4 percent inflation target for 15 years. To the extent that the central bank has much credibility in the market, and if there are large costs perceived from going back on its words, this may be enough. The policies discussed in sections 3.2 and 3.3 were examples of ways in which a central bank could go beyond a simple inflation target and communicate the optimal commitment.

Those policy approaches, however, do not address the objections raised by Powell and Williams in the last section: that the optimal commitment is dynamically inconsistent (at least if the costs of reneging on verbal promises is expected to be low). What options are available to enhance the credibility of the optimal monetary policy commitment?

How should one approach that question? One approach, followed in Eggertsson (2006) and Bhattarai et al. (2019), is to assume that the government cannot make any credible verbal announcement or commitment about future policy because it assumes that all the government cares about is the how its policy, constrained by the physical economic constraints, affects its current and future objectives – regardless of past promises. This was indeed the approach of the last subsection in the analysis of the MPE, although there we made the additional assumption that government spending was fixed and lump sum taxes could be adjusted. We now consider how fiscal policy can be used to enhance the credibility of an inflationary monetary policy regime. An important element of this policy is that it assumes monetary and fiscal policy are coordinated to jointly optimize social welfare, an issue discussed in section 3.8.

Let us assume that lump sum taxes are costly as in Barro (1979), that is that the "production" of government revenues, T_t, requires spending some resources corresponding to $f(T_t)$, where $f(.)$ is increasing and convex (for example, due to tax collection costs). Let us assume this is the only source of government revenues. Total government spending is thus given by

$$F_t = G_t + f(T_t) \tag{66}$$

where G_t is government spending that generates utility for the households while the collection cost is a dead-weight loss. The advantage of this specification of taxes is that it captures the idea that taxation is "costly" for society, while it abstracts from any effect it has on relative prices, i.e. below we abstract from changes in sales and labor taxes but both have an effect on relative prices. If we assume that the steady state is efficient, it can now be shown that the social welfare, as captured by a second order approximation of the representative household utility function, is

$$E_t \sum_{t=0}^{\infty} \beta^t \{\pi_t^2 + \lambda_y (\hat{Y}_t - \delta \hat{F}_t)^2 + \lambda_F (\hat{F}_t)^2 + \lambda_T \hat{T}_t^2\} \tag{67}$$

where $\delta \equiv \sigma^{-1}\psi$.

The introduction of taxation costs gives a straightforward way for the government to credibly commit to an "inflationary stimulus" in the words of Chairman Powell. Consider the incentive effect of the government having a one trillion dollar debt. Suppose the government announces Krugman (1998) strategy of four percent inflation for 15 years. This inflation strategy, if successful, would erode about 45 percent of the real value of the 1 trillion dollar debt if interest rates are fixed [19]. Suppose, now, that instead of delivering 4 percent inflation for 15 years, the government delivers no inflation at all. In this case, the real value of the debt would be correspondingly higher, i.e. the real value of debt would be higher corresponding to 450 billions dollars which would thus have to be met by current or future tax collection. Higher nominal debt is thus the most straight forward resolution to the deflationary bias.

Increasing government spending is another way of generating a economic stimulus in a way that is not subject to a credibility problem. The logic for it is straightforward: while monetary policy involves a commitment to a future stimulus (when the ELB is no longer binding) increasing fiscal spending involves instead actions today as shown below.

The insights are formalized in Eggertsson (2001a) considering optimal policy under discretion, or a MPE taking into account the objective (67). Assuming the lump sum tax is the only source of revenues, the government budget constraint is given by

[19] The plausibility of different interest rate assumption, and price rigidities, is discussed in detail in Bhattarai et al. (2019) as well as how it interact with the duration of the public debt

$$\hat{w}_t = \beta^{-1}\hat{w}_{t-1} + w_y\hat{\imath}_t - \beta^{-1}w_y\hat{\pi}_t + \beta^{-1}\hat{F}_t - \beta^{-1}\hat{T}_t \tag{68}$$

The government maximizes (67) subject to IS, AS, ELB along with the budget constraint (68). The analysis below is a bit simpler than Eggertsson (2006) in that we characterize the game via linear quadratic approximation which simplifies the exposition. Relative to the maximization problem in the last subsection, however, the difference is that the model is no longer purely forward looking because it has a state variable that is given by $\hat{w}_{t-1}$.. The assumption of MPE mandates that the expectations of the agents only depend upon the state variables of game. Hence

$$E_t\hat{Y}_{t+1} = Y_r r_t^e + Y_w\hat{w}_t \tag{69}$$

$$E_t\pi_{t+1} = \pi_r r_t^e + \pi_w\hat{w}_t \tag{70}$$

$$E_t\hat{F}_{t+1} = F_r r_t^e + F_w\hat{w}_t \tag{71}$$

where $Y_r, Y_w, F_r, F_w, \pi_r, \pi_w$ are constants to be solved for.

The problem of the government under discretion can be characterized by the Bellman Equation

$$V(\hat{w}_{t-1}, \hat{r}_t^e) = \max_{\hat{T}_t, \hat{F}_t, \hat{\imath}_t} \{\pi_t^2 + \lambda_y(\hat{Y}_t - \delta\hat{F}_t)^2 + \lambda_F(\hat{F}_t)^2 + \lambda_T\hat{T}_t^2 + \beta E_t V(\hat{w}_t, \hat{r}_{t+1}^e)\} \tag{72}$$

subject to the ELB, IS and AS equation, where the expectations have been replaced with (69) - (71), and (68). The first order conditions for this problem, as well as the envelope condition, are shown in the Appendix.

The key aspect of the solution is that now the model incorporates the state variable $\hat{w}_{t-1}$, the real value of the debt, which allows the government to commit to an inflationary policy. The main complication of the solution is that the expectation functions (70) - (71) are unknown. This problem can be solved using the method of undetermined coefficients.

Figure 21 illustrates how the government can use fiscal policy to stimulate output under discretion. We compare 4 solutions. The first solution, in black, shows the optimal discretion holding the fiscal instruments fixed. The second, in dashed red, show the case when the government can use real government spending to stimulate spending. The third, in magenta, shows if the government can use debt to commit to inflation. Finally, in blue is the solution when the government can use both instruments to achieve its objective. Critically, it is nominal debt which renders the inflation policy credible, as seen by the blue and pink lines.

As stressed by Bhattarai et al. (2019), there are two main forces that make the nominal debt commit the future government to inflation, the balance sheet incentive and the rollover incentive. The balance

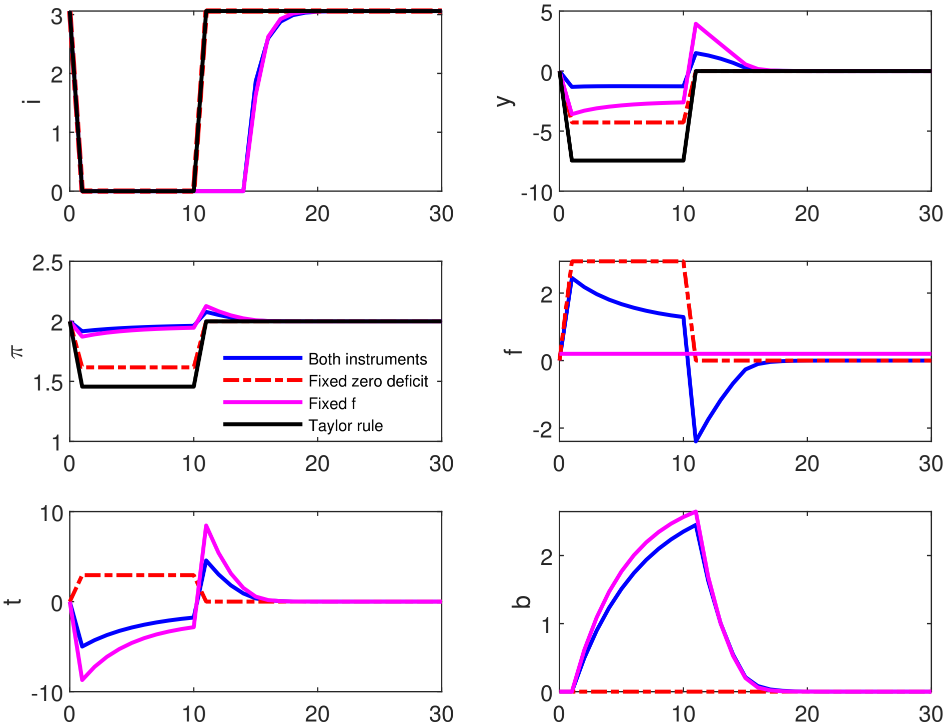


Figure 21: Government policy under MPE using different sets of instruments

sheet incentive what we explained with the example we gave of by how much inflation erodes the real value of 1 trillion dollar debt at fixed interest rate. The rollover incentive arises because the debt the government holds needs to be rolled over at a certain interest rate which directly affects the financing cost of the government. For example, if the government to hold 1 trillion dollar debt, then it matters a great deal for it rolls it over at 3 percent interest rate or 4 percent interest rate. The higher the debt, the more incentive the government has to keep rates low to minimize the interest rate cost. And the government controls this rate via the central bank. Taken together, these two forces explain why the higher nominal debt in Figure 19 allows the government to credibly commit to low future nominal interest rates in the MPE.

3.7 Quantitative Easing and Foreign Exchange Rate Intervention to Enhance the Credibility of Inflationary Policy

The last subsection showed how increasing government debt renders inflationary policy policy credible. The thought experiment was that the government cuts taxes to increase nominal debt – and the debt gave the government the incentive to inflate in future periods. Since this is the policy needed at the ELB, it gave a straightforward solution to the deflationary bias. Cutting taxes to increase government liabilities is only one way of increasing government debt, however.

The government can also increase nominal debt by instead purchasing any privately owned assets such as stocks or foreign exchange. An important point is that it does not matter if the government purchases these assets by issuing debt or by printing money (or more precisely, in practice, bank reserves). Both are a government liabilities with zero rate of return. In the early phases of the Great Recession, for example, the Federal Reserve expanded its balance sheet by about $1.2 trillions, and did so by expanding its credit in exchange for various private assets. This was largely done by issuing reserves thus increasing total government nominal liabilities, thus directly affecting the inflation incentives of the government.

The game considered in the last section can be extended to incorporate the possibility that government debt is increased not only by cutting taxes but also by acquiring other types of assets. This is done explicitly in Eggertsson et al. (2016b). One key finding of that paper, however, is that the required purchases of assets may need to be implausibly large to operate as an effective commitment device. The reason for this is that if the government buys real assets at the ELB, the cost of reversing these transactions is relatively small once the ELB is no longer binding. Thus, this commitment mechanism may not have sufficient bite. This property could explain why, for example, the Central Bank of Switzerland had limited success in reversing deflationary pressures despite increasing the monetary base by very large amounts when buying foreign exchange during the Great Recession.

Bhattarai et al. (2019) consider an alternative strategy by which the government can commit to an expansionary policy which does not require an increase in government debt. They consider instead a policy of purchasing long term government debt in exchange for reserves. While this means that the total government debt is unchanged, it reduces the maturity of the government debt in the hands of the public because reserves and short term debt are equivalent. This policy was in fact pursued in the US during the Financial Crisis under the rubric of Quantitative Easing, which involved the Federal Reserve purchasing large quantities of long term treasuries by issuing reserves.

The reason this policy works in Bhattarai et al. (2019) is that if the government holds more short-term debt it has an incentive to keep the nominal interest rate low in the future. They offer the following simple intuition for their results: consider your own incentives if you have a mortgage and are evaluating two loan contracts: A 30-year loan with interest rates that are already fixed or a loan contract with floating interest rates that are determined monthly. Consider now your incentives if you get the opportunity to set the federal funds rate. If you have a 30-year loan, then your own interest rate is unaffected by an increase in the federal funds rate, leaving the interest costs unchanged. Meanwhile, if you have a flexible loan, you have much to lose on account of a rise in interest rates for it will directly increase your interest payments. It is as if you are rolling over your debt from one period to the next with variable rates. Accordingly, the more short-term debt you hold, the less willing you are to raise the federal funds rate.

3.8 Fiscal Incentives and Central Bank Independence

The discussion in the previous two sections has centered around how fiscal policy can be used to render the optimal monetary commitment credible. The key assumption is that monetary and fiscal policies are coordinated to maximize social welfare. This does not need to be the case under all circumstances. Consider the most simple example illustrated in Eggertsson (2001b): imagine that the central bank is on the gold standard. In this case $M = k \times \text{gold}$, where k is some constant. As long as the deficit spending does not influence gold (which one has no reason to expect in the model) it has no effect on either output or prices. Deficit spending only increases demand if it influences the central bank's choice of money supply *in the future*. A central bank on a gold-standard is only one example of a "goal independent" central bank. It can, in principle, have any objectives different from social welfare. One example of a goal-independent bank that is of particular interest is a bank that maximizes the utility of a representative household but ignores the welfare consequences of government spending. The loss criterion of a bank with this objective is to a second order equal to the one we assumed when analyzing optimal monetary policy in isolation. In this case, deficit spending has no implication for future inflation.

What are the implications of purchases in real assets (such as foreign exchange) if the central bank is goal independent? Suppose, for example, that it prints money and buys foreign exchange or real assets. Will this increase inflation expectations? As stressed by Eggertsson (2001a), costly taxation can be thought of as a parable for having the government, or a goal-independent central bank, being concerned about its balance sheet e.g. caring about capital gains and losses. The central bank, therefore, has to care only about its own capital gains or losses for open market operations in foreign exchange or real assets to be effective. Do goal-independent central banks care about capital losses? If a central bank incurs capital losses (for example, by buying real assets that lose value) it can compensate for them by one of two ways: printing money or obtaining tax revenues collected by the treasury. The first may imply excessive inflation, the other a bailout from the treasury associated with a loss of independence. For most central banks, this choice is like choosing between death by fire or by drowning. There is, therefore, every reason to expect a goal-independent central bank to care about capital losses, and this concern enables the bank to use foreign exchange intervention or buying real assets as a commitment device, a point that is formally established in Jeanne and Svensson (2007).

As stressed by Eggertsson (2003), however, if the central bank is risk averse there is a catch which is not incorporated in Jeanne and Svensson (2007) analysis for they consider a model with perfect foresight. The central bank may be reluctant to buy any assets that have uncertain returns; and, apart from short-term government bonds, any asset has an uncertain return. If the bank prints money and buys foreign exchange, to take one example, there is always a positive probability that at some future date the bank will have to choose between high inflation or high capital losses. Risk aversion, therefore, may limit the bank from taking any actions that carry risk when the zero bound is binding. This remains true even if these actions enable the bank to achieve some of its goals, such as output and price stabilization. One implication is that even though a central bank has several policy instruments with which to escape a liquidity trap, in its arsenal the bank's concern about its independence may limit its use of them. By most account, this was indeed the reason for why the central bank of Switzerland stopped intervening in foreign exchange markets in 2015. While this problem was anticipated by Eggertsson (2003), an even earlier identification of this problem is due to Paul Samuelson. He suggested that the Federal Reserve did not intervene in markets in a more robust way during the

Great Depression because it was a "prisoner of its own independence"[20]. Samuelson's idea was that the Fed did not act due to fears of balance sheet losses. In general, a cooperation between the treasury and the central bank may thus be useful as a goal-independent central bank may be "too risk-averse" to effectively commit to end deflation.

3.9 Notes on the Literature

The analysis of optimal policy in this section follows closely Eggertsson and Woodford (2003). That paper in turn build on Rotemberg and Woodford (1997) and Woodford (1999) but in the latter paper the ZLB is indirectly taken into account for the optimal policy problem by introducing a quadratic penalty in the loss function of the government, so that standard solution methods applied. The contribution of Eggertsson and Woodford (2003) was to instead explicitly account for the ZLB. Independently, Jung et al. (2005) also derived the optimal monetary policy commitment but assumed instead a deterministic AR(1) process. Adam and Billi (2006) generalized these results by considering stochastic AR(1) process and solving for the optimal policy numerically.

Preceding this work, however, were several important papers written mostly at the Federal Reserve where there was growing recognition that with low inflation, the ZLB could pose a challenge. These papers were typically written following the research agenda laid out by John Taylor, that involved searching for optimal rules. The best known of these papers is Reifschneider et al. (1999) discussed in section 3.2 that largely anticipated the form of optimal policy that was later derived analytically.In a well-known Board of Governors staff study that was presented to the FOMC on June 3, 1998 (later published as Clouse et al. (2003)) much of this early work is reviewed and references.

Krugman (1998) was the first to recognize the inherent credibility problem facing a central bank at the ELB. This insight was first formalized in Eggertsson (2001b) which analyzed a MPE at the ZLB, later derived in the standard New Keynesian model, assuming the same kind of stochastic process as in this paper, in Eggertsson (2006) which coined it the problem the "deflation bias." Independently, Jung et al. (2005) also characterized the optimal policy under discretion, assuming deterministic AR(1) process. Adam and Billi (2007) extended the analysis to consider an general AR(1) process as the driving force. Nakata (2018) shows how the optimal policy can be made credible by considering sustainable plans, provided the frequency of ZLB episodes is high enough.

4 Secular Stagnation

In section 1.2 we discussed the possibility that the natural rate of interest could be permanently negative, an idea often associated with secular stagnation hypothesis that dates back to Hansen (1939). The review so far, however, has contemplated the possibility that the natural rate of interest is only

[20] See Mayer (1993).

temporarily negative, as in each period there is a constant probability of it reverting back to normal. This section considers how the results change once we consider the natural rate that is permanently negative. Summers (2014) suggested this as a way of formalizing Hansen's idea.

It might seem at first that studying the possibility of a permanent trap would be a straightforward extension of the discussion so far, that we only need to contemplate the special case in which the probability of the natural rate of interest reverting back to steady-state is zero. There are two reasons for why things are not that simple. The first is related to aggregate demand, the second to aggregate supply. Both considerations require us to move beyond the linearized equations we have relied upon so far.

Consider first aggregate demand in the standard New Keynesian model which is characterized by the IS equation linearized around a steady-state. Abstracting from growth the real interest rate in steady-state is

$$r = 1/\beta - 1 > 0$$

and thus the long-run interest rate has to be greater than zero since $0 < \beta < 1$. The assumption of a representative agent drives this result: it implies that the real interest rate is given by the rate of time preference. One might think that assuming $\beta > 1$ helps. The problem in this case, however, is that the intertemporal budget constraints of the household "blows up" and an optimal consumption plan is not well defined.

Moving to the saver-borrower model of Eggertsson and Krugman (2012) is of little help either, as there the real interest rate in steady-state is pinned down by the discount factor of the representative saver. One possibility, without abandoning a representative agent, is to assume that the economy is constantly shrinking, or that there is such a high level of uncertainty that people are willing to pay for storage of money. Either assumption can imply negative interest rate.

An alternative solution is proposed by Eggertsson, Mehrotra, and Robbins (2019c). In that model, the representative agent is replaced with overlapping generations: young, middle aged and old as we reviewed in section 1.2. In this case, as seen in equation 20, the real interest rate can be negative in steady-state.

Let us now extend the OLG model in section 20 to include nominal price level determination and production so show how a secular stagnation equilibrium with sub-par employment and output may arise. We follow the same steps as in the simple savers and borrowers model in section 1.1. To introduce nominal price determination we assume there is a one period risk-free nominal bond traded, which gives rise the following pricing equation for the middle aged generation

$$\frac{1}{C_t^m} = \beta E_t \frac{1}{C_{t+1}^o}(1 + i_t)\frac{P_t}{P_{t+1}}, \tag{73}$$

Monetary policy is set according to a standard Taylor rule in its non-linear form

$$1 + i_t = \max\left(1, (1 + i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right) \tag{74}$$

If we combine equations (15), (20), (73) and (74) in steady-state, we arrive at the following aggregate demand relationship:

$$Y = \begin{cases} D + \frac{(1+\beta)(1+g)D\Gamma^*}{\beta}\frac{1}{\Pi^{\phi_\pi - 1}} & \text{for } i > 0, \\ D + \frac{(1+\beta)(1+g)D}{\beta}\Pi & \text{for } i = 0. \end{cases} \tag{75}$$

where $\Gamma^* \equiv (1 + i^*)^{-1}(\Pi^*)^{\phi_\pi}$ is the composite policy parameter in the monetary policy reaction function.

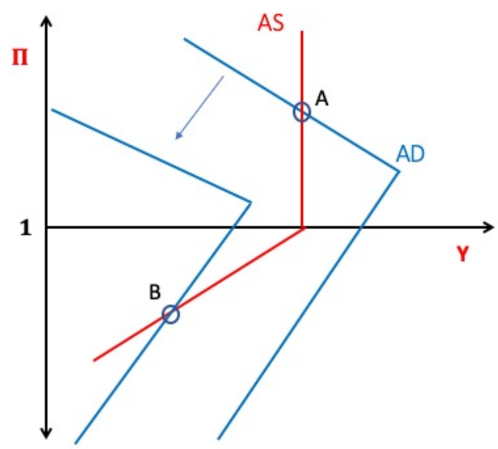


Figure 22: A normal steady state (point A) and secular stagnation (point B).

The upper portion of the AD curve shown in Figure 22 depicts this relationship. It is similar to our earlier figures, except now it refers to steady-state output. As inflation increases, the central bank raises the nominal interest rate by more than one for one (since $\phi_\pi > 1$), which in turn increases the real interest rate and reduces demand.[21]. At the ELB (for simplicity here assumed to be zero) the

[21]The mechanism of why aggregate demand depends upon the real interest rate, however, is fundamentally different. Here lower real interest rate increases demand because lower interest rate loosens the borrowing constraint of the young generation

AD curve is upward sloping. As inflation increases, the nominal interest rate remains constant, thus reducing the real interest rate. This reduction in the real rate raises consumption demand.

It is not only the aggregate demand side of the framework that needs to be revisited in order to contemplate the permanent demand recession envisioned by Hansen (1939) and Summers (2014). Many theories of price rigidities imply that in the long-run there is monetary neutrality. Thus, by assumption, these models preclude the possibility of a permanent demand recession. Somewhat subtly, the way this shows up in general equilibrium models is that if the central bank is targeting low enough inflation, there is no equilibrium (a point elaborated in Eggertsson and Giannoni (2013)). While the New Keynesian model we have applied in this survey does not have full long-run neutrality, it is relatively close to that (as can be seen in equation 24 in its linear form since β is close to one), which implies that secular stagnation is either associated with implausibly large hyper-deflation, or that no equilibrium exist.

The presumption of long-run neutrality is a fairly strongly held prior in the economic profession. This presumption is probably largely based upon the experience of the 1970s, at which time there were significant increases in inflation without corresponding improvement in employment. The general consensus is that this happened because inflation expectations adjusted with higher inflation. As pointed out by Eggertsson, Mehrotra, and Robbins (2019c), however, there has never been as strong of a consensus about wages readily adjusting downwards in face of high unemployment, as recently documented for example by Schmitt-Grohé and Uribe (2016). Thus, a simple way to generate long-run non-neutralities is to assume that wages are rigid downwards, an assumption that has a long history in economics dating at least back to Keynes (1937).

Following Eggertsson et al. (2019c) suppose that output is produced by a representative firm that has a production function

$$Y_t = L_t^{\alpha} \tag{76}$$

where $1 > \alpha > 0$ and L_t is labor. Labor is in-elastically supplied by the middle-aged generation. The firm decides how much to produce by hiring workers taking the price level P_t and the wage rate W_t as given, resulting in the first order condition

$$\frac{W_t}{P_t} = L_t^{\alpha-1} \tag{77}$$

Nominal wages, however, are downward rigid and given by

$$W_t = max\{\tilde{W}_t, W_t^{flex}\} \tag{78}$$

where $\tilde{W}_t$ is a wage norm, and W_t^{flex} is the wage rate if wages are flexible. This assumption says that households will not supply labor unless the wages are at least equal to a social norm given by $\tilde{W}_t$. However, they are perfectly willing to work for higher wages, and hence if the market clearing wage is higher than the wage norm, then firm will bid up the wages to the market clearing level.

Keynes (1937) original idea was that wages are rigid downwards, that is, workers will not accept a nominal wage rate lower than what it was last year. This is one example of a wage norm. Here we generalize this idea slightly by considering a wage norm of the form [22]

$$W_t = W_{t-1}^{\gamma} W_t^{flex\,1-\gamma}. \tag{79}$$

Imposing steady state, combining (76)-(79) yields the aggregate supply relationship:

$$Y = \begin{cases} \bar{Y} & \text{for } \Pi \geq 1 \\ \\ \bar{Y}\Pi^{\frac{1-\gamma}{\gamma}\frac{\alpha}{1-\alpha}} & \text{for } \Pi < 1. \end{cases} \tag{80}$$

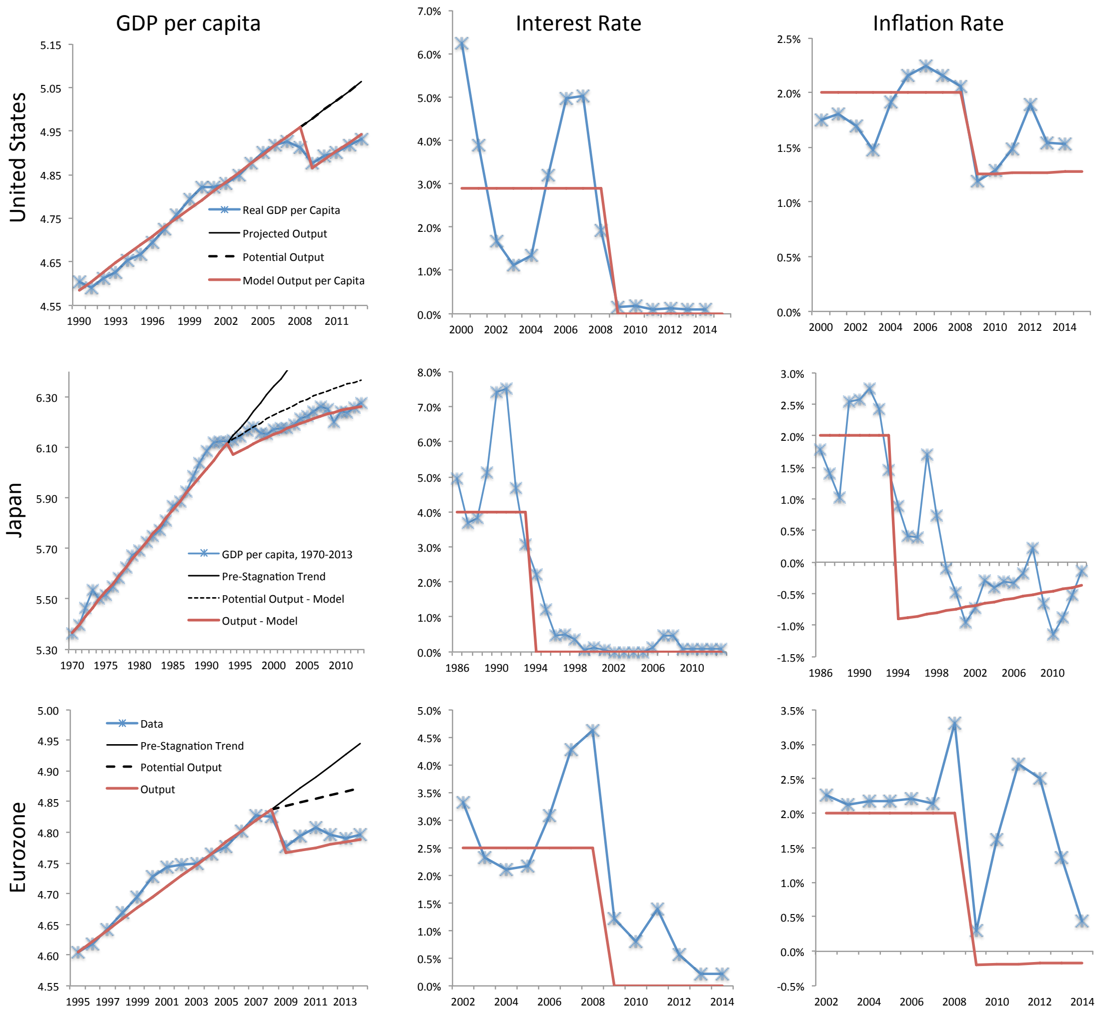
which is plotted up in figure 22 where gross inflation and output refer to their steady-state values. The AS curve is vertical at positive inflation, that is when $\Pi > 1$. In this case the wage norm is not binding and wages are at their flexible level so the entire labor endowment is employed. Once there is deflation, however, then real wages are above the market clearing level, and firms are not hiring the entire labor endowment. Point A shows the intersection of the AS and AD curve when the natural rate of interest is positive (this intersection is shown if the central bank has a positive inflation target). If the natural rate of interest is sufficiently negative, however, then the intersection is at point B where the interest rate is zero. In this case there is deflation and unemployment.

Point B is interesting for several reasons as a description of the economy. First, it represents a permanent recession without any force pulling the economy to full employment. One might imagine, since the downward rigidity of wages is one of the main culprits, that reducing wage rigidities would be of help – this is equivalent to reducing γ in the wage norm. This, however, would only make the upward sloping segment of the AS curve steeper, which makes the recession worse rather than better, a result reminiscent of the price flexibility paradox in the New Keynesian model. Note that the New Keynesian model "exploded" as the persistence of the shock became bigger. Here, however, the natural rate of interest is permanently negative, and yet the recession is bounded and the equilibrium well behaved (and determinate).

The model can generate output and inflation dynamics of the kind observed in the Euro zone, Japan and the US, as shown in figure 23, but the calibration underlying this figure is taken from Eggertsson et al. (2019c). According to this simulation there is no tendency for output and interest rates to revert back to normal. Relative to the model just outlined, the model in the simulation has been extended so that the wage norm in increasing at the same rate as the inflation target of the central bank, so that no deflation need not be observed in the recession. Moreover there is an exogenous path for total factor productivity that is increasing over time. The simulation for Japan and the Euro zone also imposes a reduced form type representation of hysteresis, i.e. the suppressed state of demand feeds into lower total factor productivity, so that the recession affects the growth rate potential of the economy. There is a growing literature that explicitly models hysteresis effect in endogenous growth models, examples include Benigno and Fornaro (2017) and Garga and Singh (2018) who also explore

[22]relative to Eggertsson et al. (2019c) we have written the wage norm in exponential form which makes the algebra slightly cleaner

Figure 23: Data versus model transition paths: US, Japan, and the Eurozone



implication for monetary policy.

There are important implications for monetary and fiscal policy once one contemplates the possibility of a permanent recession. The most obvious one, perhaps, is that a policy of waiting for the shock to subside (because stimulative policies are too risky or costly) is not a good strategy. Another immediate observation is that a policy of forward guidance, that is, promises of lower future nominal interest rate is ineffective. If the ELB is expected to be of infinite duration, it obviously has no effect to promise lower interest rate for even longer. A more subtle point, however, is that even if the monetary authority has full credibility, announcing a higher inflation target may have little or no effect.

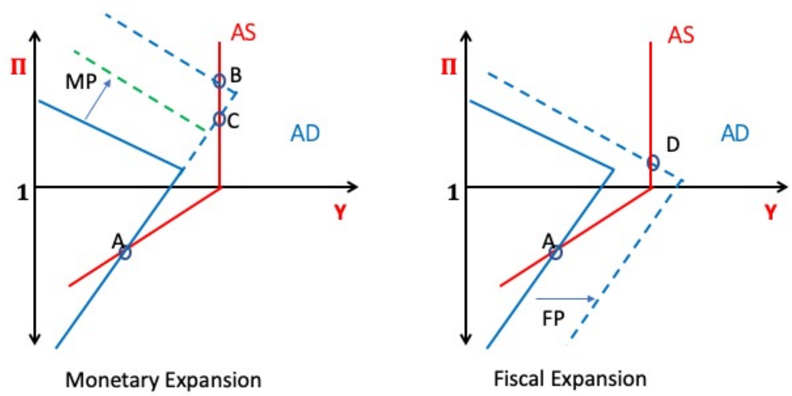


Figure 24: The effect of monetary and fiscal expansion in a secular stagnation

There are two new difficulties associated with announcing a higher inflation target, even if the public believes the central bank is fully committed to it. The first problem, or what Krugman (1999) coined the "law of the excluded middle" is that if the inflation target is not high enough, then the central bank will never be able to reach it. A higher inflation target shifts the upper section of AD curve directly upwards, as shown in figure 24. If the inflation target is not higher enough, however, the then there is no intersection between AD and AS at full employment. The law of the excluded middle also applies in the New Keynesian model, but only in the short-run. Once the natural rate of interest recovers the central bank will ultimately reach the new inflation target. This expectation, in itself, can have large effect. In a secular stagnation, however, the inflation target, if too low, will never be reached, and hence announcing a too low target has no effect, even if the public believes the central banks intentions.

The problem of the excluded middle is quite likely to be a serious problem in practice. Consider the policy of the Bank of Japan in 2013 when it decided to announce a new inflation target of 2 percent. This result suggest that if the natural rate of interest was below -2 percent, then this policy would have no effect. Since announcing its target, the BoJ has never managed to reached it. This result suggests that the problem with the policy was not the target itself, but the fact that it was too low. For a practical policy makers this implies a bit of a dilemma, for it implies that the right course of action, if an inflation target at the ELB is not reached, would be to announce an even higher one. The conundrum is that the public might the ask, "you have not even reached the last one, and now you expect us to believe a new and even higher target?".

A second problem is that even if a sufficiently higher inflation target can be reached, as suggested by point B in Figure 24, this fact, in itself, does not eliminate the possibility of the original secular stagnation equilibrium (point A). A credible inflation target, therefore, only allows for the possibility of a better equilibrium. It does not exclude the original secular stagnation equilibrium. This is fundamentally different from the analysis in the New Keynesian model where a sufficiently high inflation target guaranteed a unique determinate equilibrium at full employment.There typically exists, however, an equilibrium with inflation below the target at the ELB that is indeterminate. This type of equilibria, and how they can be eliminated, is the focus on Benhabib et al. (2001). Note that point C in figure 24 correspond to this kind of equilibria. The reason for this is that in the New Keynesian model we assumed the there was a state of the world in which the natural rate of interest had re-normalized and the inflation target could be reached. This state of the world worked as an anchor, which pinned down the desirable equilibria. In a secular stagnation, however, no such anchor exists.

Relative to monetary policy, fiscal policy retains much of its power in secular stagnation, even if its effects are somewhat more subtle. In particular a successful fiscal policy directly increases the natural rate of interest and shifts the AD curve to the right in figure 24, generating a unique equilibria at full employment at point D. Thus if aggressive enough, fiscal policy eliminates the possibility of secular stagnation altogether. The way fiscal policy works, however, is different from the standard New Keynesian model. The reason is that the model has different generation of agents, and thus the distributional consequences of fiscal policy become crucial.

Consider, for example, the effect of increasing government spending via directly taxing the young generation. Since the young generation was borrowing constrained, then every dollar in spending will be met by a corresponding decrease in spending by the young, and thus, the spending multiplier is zero. Consider next the effect of increasing government spending by a tax on the old who will spend all their income. Again every dollar of government spending is now met with a reduction of spending but now due to the old generation. In addition, however, the middle aged – anticipating higher taxes when old, will cut down on their own spending to save, which will reduce demand even further. The government spending multiplier, under this specification, is thus negative! If we consider the more plausible scenario, that it is the working age population that finances the spending, or simply debt financing, the spending multiplier is higher. Eggertsson et al. (2019c), for example show that it is greater than 1 when financed by middle aged agents and greater than 2 when financed via debt issuance.

5 Financial Imperfections and Credit Interventions

(very preliminary) So far we have not discussed the possibility of credit interventions at the ELB, policies that played a prominent role in the reaction of Federal Reserve to the Great Recession. There is a relatively rich and growing literature that has emerged following the crisis incorporates friction so that the Eggertsson and Woodford (2003) irrelevance result does not apply. Examples include, among others, Gertler and Karadi (2011), Del Negro et al. (2017a). Gertler and Kiyotaki (2010) offer an excellent survey of this literature, and thus we only comment briefly on how we think this literature fits into the current framework.

One way of thinking about much of this literature is that it models in more detail how savings get transmitted into the spending of borrowers, for example along the lines of the Eggertsson and Krugman (2012) example at the beginning of this survey. A sudden reduction in the borrowing limit triggers a drop in the natural rate of interest. Some of this work can be interpreted as providing a more detailed model of how and why there is a the reduction in the debt limit of the borrower. Del Negro et al. (2017a), for example, assume that the crisis brought about a reduction in the liquidity of various private assets which in turn led to drop in investment spending. The credit intervention was helpful, for it replaced some of these assets with perfectly liquid paper thus stimulating investment and directly increasing the natural rate of interest. Importantly, work along these lines violates the Wallace Irrelevance result and Eggertsson and Woodford's extension of it, thus providing a possible role for quantitative easing that is independent of the expectation channel that has been the focus of this survey.

By focusing on the textbook New Keynesian model, rather than for example the model with hetorogenous agents as in section 1.1, we have limited the scope of policy intervention that can be considered. We already noted that Farhi and Werning (2016a) consider the consequence of liquidity constrained agents, in which case lump-sum tax cut directly increase demand. The model of Eggertsson and Krugman (2012) and related ones, can also be used to understand the effect of debt forgiveness. Korinek and Simsek (2016) and Farhi and Werning (2016b) consider the implications for macro-prudential polices, an important topic which we have now touched upon.

6 What is the Effective Lower Bound?

(very preliminary) The traditional motivation for the zero lower bound is because people can hold paper currency as an asset. You would not want to lend anybody one dollar, unless you get paid back at least one dollar. Otherwise it would be preferable for you to hold onto the dollar instead as it will not depreciate in value.

Following the crisis of 2008, so central banks in Switzerland, Japan, the Eurozone, Denmark and Sweden started charging commercial bank negative nominal interest rate for cash balances they hold at the central bank which has been used as indicator for the policy stance. The reason these rates can go negative, is that commercial banks are willing to pay for the transaction services the central bank reserve accounts provides, but in the aggregate, the alternative to holding reserves would be for banks to hold cash which in itself implies storage costs. The experience so far, however, suggest that despite negative interest on reserves, commercial bank increase of cash balances has been relatively modest (see Eggertsson et al. (2019b)).

Eggertsson et al. (2019b) document that the interest paid on banks deposit and policy rate move closely together at positive interest rate. They document, however, that at once the policy rate goes negative commerical banks do not transmit the negative rates to their depositors [23] Moreover, once the policy rates turn negative, the transmission to actual bank lending rates is similarly muted.

Eggertsson et al. (2019b) propose a model in which banks deposits are the main funding source for commercial banks, but there is a zero bound on deposits rates due the the presence of money as a store of value. Yet, banks are willing to pay negative interest on reserves held at the central bank, due to the transaction service provided by a central bank which reduces their intermediation cost. The prediction of the model is that the transmission mechanism of policy to lending rates breaks down at zero, due to the bound on deposit, and that negative policy rates are contractionary, provided bank intermedition costs depend on net worth. Hence according to Eggertsson et al. (2019b) then the effective lower bound is 0 even if policy rates can technically be set at negative level.

Brunnermeier and Koby (2016) also analyze the possibility that interest rate cuts can become contractionary, what they call the reversal rate, due how interest rate cuts affect bank's balance sheet. They find that the reversal rate is -1 percent.

7 The Great Depression

7.1 Great Expectations and the End of the Depression

The framework studied in section 2 and 3 can be used to explain the recovery from the Great Depression. As shown in figure 25 the drop in output during the Great Depression ended abruptly in 1933. The recovery coincided with the inauguration of Franklin Delano Roosevelt (FDR) at a time when the short term interest rate was zero. This section shows how the recovery can be modeled as a regime change.

The key element of the regime change in 1933 was the commitment to increase price level to the pre-depression level. Since the price level had fallen by about 30 percent, this implied a commitment to substantial inflation. Thus, the policy corresponds closely to the optimal monetary commitment considered in section 3.1. Recall, that a key result of Eggertsson and Woodford (2003) is that the optimal commitment closely resembles a price level target.

Roosevelt made several announcements in the early months of his administration to suggest that the overriding objective of economic policy was reflation, i.e., to increase the price level, to pre-depression levels. At a press conference on April 19, 1933, for example, Roosevelt stated the âdefinitive objectiveâ of raising commodity prices. This press conference was called after Congress had passed the Thomas Amendment, a bill that gave Roosevelt broad powers to inflate and effectively eliminated

[23] Even if there is some evidence that negative rates have been transmitted to large depositors.

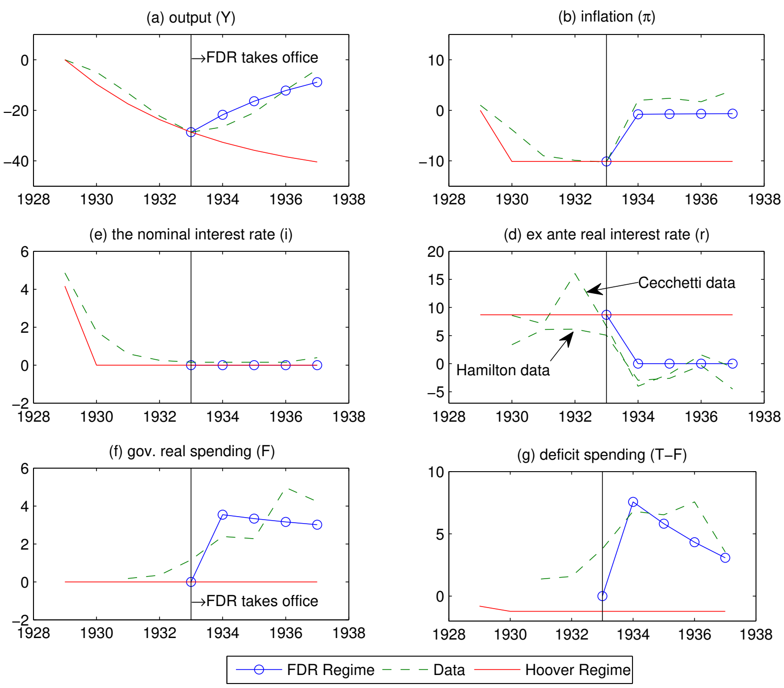


Figure 25: The FDR regime change implies a collapse in real interest rates and a robust recovery in prices and output. The regime change can account for 67 percent of the recovery in prices and 79 percent of the recovery in output in the period 1933â1937.

the independence of the Federal Reserve. Similarly, Roosevelt was quoted in the Wall Street Journal May 1, 1933: âWe are agreed in that our primary need is to insure an increase in the general level of commodity prices. To this end simultaneous actions must be taken both in the economic and the monetary fields.â Roosevelt reiterated this view in a radio address to the nation in one of his âfireside chatsâ on May 7. By late spring in 1933, there could be no doubt in the minds of market participants that the administration was aiming to inflate.

But even if one could argue that FDR committed to increasing he price level, why was it credible? After all, he predecessor in office, Herbert Hoover, also made several announcements that recovery was just around the corner, with an associated improvement in output and prices. Eggertsson (2008) argues that FDR's *policy actions* made the inflation policy credible as in the MPE equilibrium studied in section 7.1. Below we show how this can formally be modeled in a infinitely repeated game, according to which FDR made the regime change credible by violating certain policy dogmas that arguably constrained policy under Herbert Hoover. In the next subsection we also suggest that certain New Deal policies can be viewed trough the same prism.

Temin and Wigmore (1990) first proposed that the end of the Great Depression was triggered by a regime change. This idea is formally modeled in Eggertsson (2008) where the period 1929-33 is assumed to be governed by a Hoover regime, while the period 1933-1937 is governed by the FDR regime. The model is the same as in section 7.1. Under both policy regimes the government is maximizing objective (67). The difference between the two is that the Hoover regime is constrained by three policy dogmas while the FDR regime is not. The regime change is thus modeled as the elimination of the policy dogmas, and this endogenously leads to a credible increase in inflation expectations triggered by policy actions. Below we formalize the policy dogmas in the context of the model of section 25.

First, there is a "small government dogma" such that real government spending is constant at all times, or in the language of the model of section 7.1:

$$F_t = \bar{F} = \bar{G} + s(T_t) + A_t. \tag{81}$$

where A_t is residual spending that does not contribute to utility. [24] This dogma captures Hoover's views on fiscal policy: that the government should be kept "small", at its current level. In an address to the American Legion on September 21, 1931, for example, he stated: "Every additional expenditure placed upon our government in this emergency magnifies itself out of all proportion into intolerable pressures, whether it is by taxation or by loans. Either loans or taxes [...] will increase unemployment.[...] We can carry our present expenditures without jeopardy to national stability. We can carry no more without grave risks."

Second, there is a "balanced budget dogma" such that the government never spends beyond its means. This is modeled so that the government collects taxes to keep the real value of the debt constant:

$$w_t = w_{t-1} = \bar{w} \tag{82}$$

This dogma also represents President Hoover's views at the time. In a press statement at the early stages of the Depression on July 18, 1930, for example, he stated:[25] "For the Government to finance by bond issues deprives industry and agriculture of just that much capital for its own use and for employment. Prosperity cannot be restored by raids on the public Treasury."

Hoover's views on deficits remained unchanged throughout the Depression although he was unable to prevent them during parts of his presidency. For simplicity, the third dogma, the "gold standard" dogma is excluded in the analysis here, but Hoover was a strong defender of the gold standard. Eggertsson (2008) shows that this dogma can be added without changing the results because the U.S. government held gold in excess of the monetary base at the time, so this constraint was not binding. Nevertheless, abolishing it was important for FDR's regime change for it implied that the gold would not constrain the growth of the money supply in the later years.

These three dogmas did not only describe Hoovers views at the time, but more generally the conventional wisdom at the time as argued by Eggertsson (2008).

[24] In writing the dogmas in this way, we abstract from any welfare effects of variations in taxes under the Hoover regime by setting $G_t = \bar{G}$ and assuming that the residual spending A_t in (81) adjusts to counteract any changes in tax collection costs. This simplifies the characterization of the Hoover regime considerably.

[25] Hoover (1934).

If the two policy dogmas (81) and (82) constrain the government, it is relatively straightforward to show that the government problem is *identical* to optimal monetary under discretion in section 3.5 where we have abstracted from fiscal policy. In other words, the Hoover regime implies that as soon as the shock that gave rise to the ELB subsides, the central bank stabilizes inflation at zero. This, in turn, leads to the type of output contraction considered in the GD numerical example.

The consequence of the Hoover regime is shown in figure 25, which compares the output implied by the model to the decline in the data. Had the Hoover regime remained in place, the economic contraction would have continued from 1933 onwards as shown by the continuation of the red line. Instead, the data registers a recovery. To simulate the model we have deviated modestly from the model in section 3.5 to match the gradual drop in output. [26]

The FDR policy regime is defined by optimal discretion once the three policy dogmas have been abolished. The FDR regime, the optimization of (67) subject to the constraints of the model, is thus identical to the problem we studied in section . Figure 25 shows the effect of the regime change in the model assuming that the regime change was unexpected. As the figure reveals, the model can account for a substantial (about 80%) part of the recovery in output and inflation. We will have more to say about what could explain the unexplained portion in the next section.

There is ample historical evidence for the abrupt regime change that took place once FDR took office. As if mobilizing the nation for war, the government went on an aggressive spending campaign, nearly doubling government consumption and investment in one year as shown in the Figure 25. This clearly violated the first policy dogma that constrained the Hoover regime. The spending spree was not financed by tax increases but instead by some of the largest budget deficits in US history outside of wartime, thus violating the second policy dogma. On the monetary side Roosevelt announced that the value of the dollar was no longer tied to the price of gold, effectively giving the administration unlimited power to print money. The overarching goal of these policies was to inflate the price level, and Roosevelt announced that this would be achieved through all possible means, stating: "If we cannot do this one way, we will do it another. Do it, we will."

Interestingly, the end of the gold standard and the monetary and fiscal expansion were largely unexpected, since all these policies violated the Democratic presidential platform. It is hard to overstate how radical the regime change was. "This is the end of Western civilization" said Director of the Budget Lewis Douglas, and resigned from office. During Roosevelt's first year in office, several other senior government officials resigned in protest, as the regime change went against the prevailing conventional wisdom of what was considered as responsible policy.

[26] Relative to our earlier model, we have introduced *habit persistence* in consumption and labor as in Eggertsson (2008). This modest extension means that the formulas for output we derived in section 2 and 2 are unchanged if we replace $\hat{Y}_t$, with the quasi growth rate of output $\tilde{Y}_t$. All other formulas remain the same. The two are related by

$$\tilde{Y}_t = \hat{Y}_t - \rho \hat{Y}_{t-1} \tag{83}$$

where ρ measures the degree of habit persistence. The model is re-parameterized using the same methods as before, and the parameter values used are shown in the Appendix.

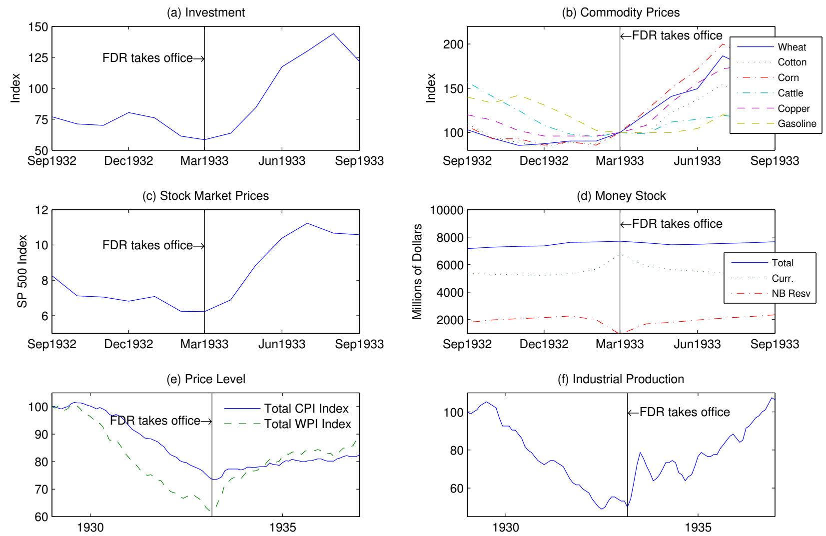


Figure 26: Investment, commodity prices, and the stock market rebounded once FDR took office. The large change in these forward-looking variables cannot be explained by contemporaneous changes in the money supply, which did not change around the turning point. Similarly, prices and industrial production reversed their three-year downward trend when FDR took office.

The data are highly suggestive of a regime change, beyond the aggregate annual statistics reported in the model simulation in figure 25. Figure 26 shows several measures of prices and industrial production with a vertical line denoting the month of Roosevelt's inauguration. Panels (a)-(c) show a one-year window for commodity prices, the stock market, and a monthly investment index, all of which are highly volatile and should respond strongly to a shift in expectations. All indicators rebounded strongly once Roosevelt took office. The stock market, for example, increased by 66 percent in Roosevelt's first 100 days and commodity prices skyrocketed. Similarly, investment nearly doubled in 1933 with the turnaround in March that year. Panels (e) and (f) take a broader view and show that Roosevelt's inauguration turned the persistent deflation in the wholesale and the consumer price indexes from 1929 to March 1933 into modest inflation from March 1933 to 1937. Roosevelt's inauguration also marked a turning point in monthly industrial production, which bottomed out in March 1933 after falling for three consecutive years. Overall, the comparison between Roosevelt's first term in office (1933-37) and Herbert Hoover's last (1929-33) is striking. Hoover's last term resulted in 26 percent deflation, while Roosevelt's first registered 13 percent inflation. Similarly, output declined 30 percent from 1929-33. This was the worst depression in U.S. history. In contrast, 1933-1937 registered the strongest output growth (39 percent) of any four year period in U.S. history outside of wartime. This dramatic turning point, the defining moment of the recovery, requires a careful description, and the model the model offers one such account.

The turning point cannot be explained by contemporaneous changes in the money supply, an hypothesis often associated with monetarism. As shown in panel (d) of Figure 26, the money supply did not

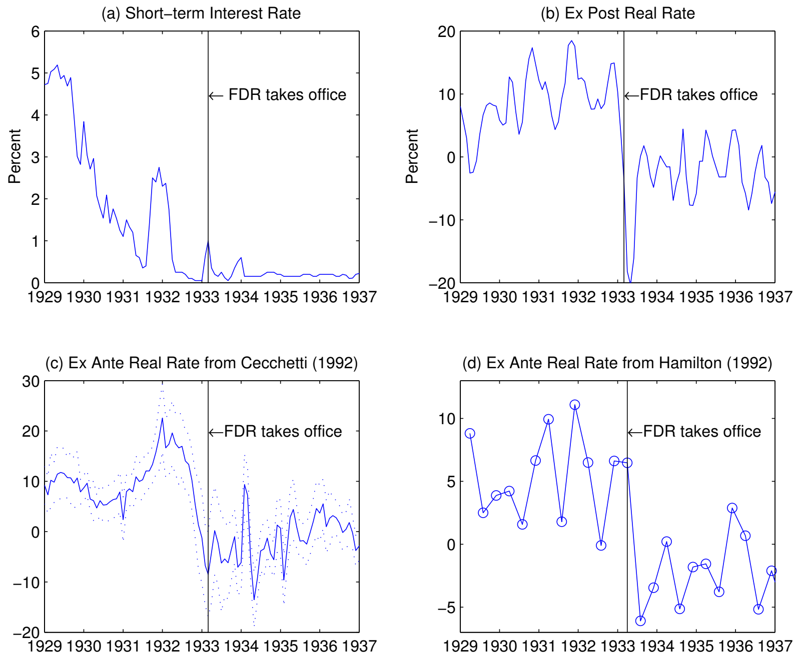


Figure 27: Real interest rates collapsed around the turning point, even if the short-term nominal interest rate did not change much. This suggests a large change in inflation expectations.

change around the turning point. Similarly, the turning point cannot be explained by interest rate cuts. The short-term interest rate was already close to zero in the beginning of 1933, as can be seen in panel (a) in Figure 27. The yield on three-month Treasuries, for example, was only 0.05 percent in January 1933 and could clearly not go much lower due to the zero bound on the short-term nominal interest rate. Yet, despite the fact that neither the nominal interest rate nor the money supply changed much at the turning point, the elimination of the policy dogmas drastically changed the systematic part of monetary policy in the model, i.e., the framework that governed the policy setting going forward. Thus according to the model, what changed was expectations about how the interest rate and the money supply would be set in the future, leading to a dramatic change in inflation expectations. One way of seeing this in the data is to observe that the short-term real interest rate, the difference between the short-term nominal interest rate and expected inflation, collapsed around the turning point in 1933, dropping from high levels during 1929-33 to modestly negative in 1933-37. Figure 27 shows several measures of real interest rates that document this pattern.

Another hypothesis for the recovery, is that whatever shocks drove the Great Depression, reverted themselves in the spring of 1933. One such candidate, for example, might be the banking crisis abated at that time. The argument would then be that the recovery was driven by recovery in banking sector and that this was unrelated to the regime change we just illustrated. We have already documented that a banking shock can indeed generate a drop in the efficient interest rate, and thus account for the drop in output and inflation. As shown in figure 28 a reversal of this shock, illustrated by the circled

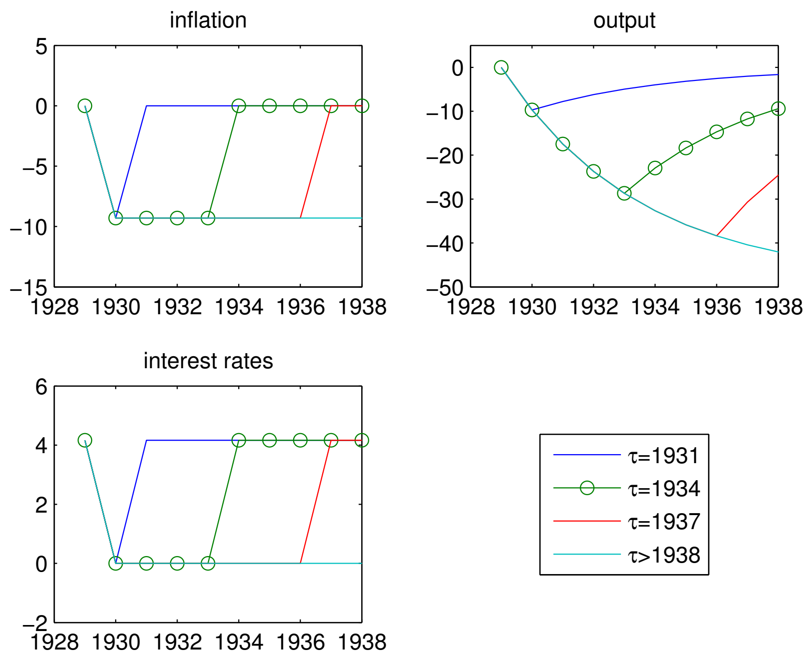


Figure 28: A reversal of the shock that gave rise to the Great Depression cannot account for the recovery, for this would have implied a renormalization of interest rates.

green line, could in principle account for the recovery in inflation and output. The problem with this hypothesis from the perspective of the model is shown in the panel for interest rate. If the shock had reverted, the model implies that this would imply a re-normalization of interest rate as well. Instead, the data suggest that the short-term interest rate remained at zero for several more years to come, without runaway inflation.

It is worth commenting briefly on the role of deficit spending in the FDR policy regime according to the model shown in figure 25. If FDR verbal commitment to increasing the price level was fully believed by the public, then the deficit and nominal debt plays nor role. In figure 25, however, we assume a MPE, so that verbal commitment play no independent role. It is thus the deficit spending that makes the commitment to increase the price level credible in the model. This suggest that an alternative way (and largely equivalent in term of outcome for inflation and output) of modeling the regime change in 1933 is to simply assume that FDR credibly committed to a price level target, while abstracting away from how he did so, an approach we will take when analyzing the "Mistake of 1937".

7.2 The National Industrial Recovery Act and the Paradox of Toil

We have already seen that the regime change in monetary and fiscal policy can account for a good part of the recovery in 1933-37. What could explain the rest? In addition to the drastic regime change in monetary and fiscal policy, FDR implemented a set of policies that were even more controversial. These policies were aimed at increasing prices in a more direct way through the National Industrial Recovery Act (NIRA).

Eggertsson (2012) proposes to model NIRA as an exogenous increase in the collusion wedge that can be expressed as an exogenous increase in firms markups, and similarly that higher unionization can be expressed as an exogenous increase in labor markup. Under these assumptions the AS curve can be written as

$$\hat{\pi}_t = \kappa \hat{Y}_t + \beta E_t \pi_{t+1} + \hat{\omega}_t \tag{84}$$

where $\hat{\omega}_t$ corresponds to variations in NIRA policies. Let us assume that interest rate policy is again given by (28) (or equivalently a Hoover regime) but that the government implements a New Deal according to

$$\hat{\omega}_S = \phi_\omega r_S^e > 0 \text{ when } 0 < t < t_T \tag{85}$$

with $\phi_\omega < 0$ and

$$\hat{\omega}_t = 0 \text{ when } t \geq t_T. \tag{86}$$

There are two reasons for considering this policy rule. The first is theoretical. Eggertsson (2012) shows that the optimal policy takes this form either assuming it is the optimal forward looking policy, or the MPE, but he abstract from fiscal policy. The second reason is empirical. NIRA was an "emergency " legislation that was installed to re-inflate the price level. The NIRA stated:

> A national emergency productive of widespread unemployment and disorganization of industry [...] is hereby declared to exist.

It then went on to specify that, when the emergency would cease to exist

> This title shall cease to be in effect and any agencies established here under shall cease to exist at the expiration of two years after the date of enactment of this Act, or sooner if the President shall by proclamation or the Congress shall by joint resolution declare that the emergency recognized by section 1 has ended.

Hence, a reasonable assumption is that the NIRA was expected to be temporary as an emergency measure and to last only as long as the shock (which creates the deflationary "emergency " in the model).

The characterization of this New Deal policy is exactly the same as when we considered the effect of a negative supply shock in section 2.7 where the higher income tax $t\hat{a}u^I$ was a stand-in for a negative supply shock. Here, instead, it is the facilitation of monopoly collusion by firms and/or increase in bargaining power of workers that play this role. Following the same steps as before, replacing $t\hat{a}u^{I}$ with $\hat{\omega}_t$, we can once again show that a negative supply shock is expansionary and the formula 38 is unchanged.

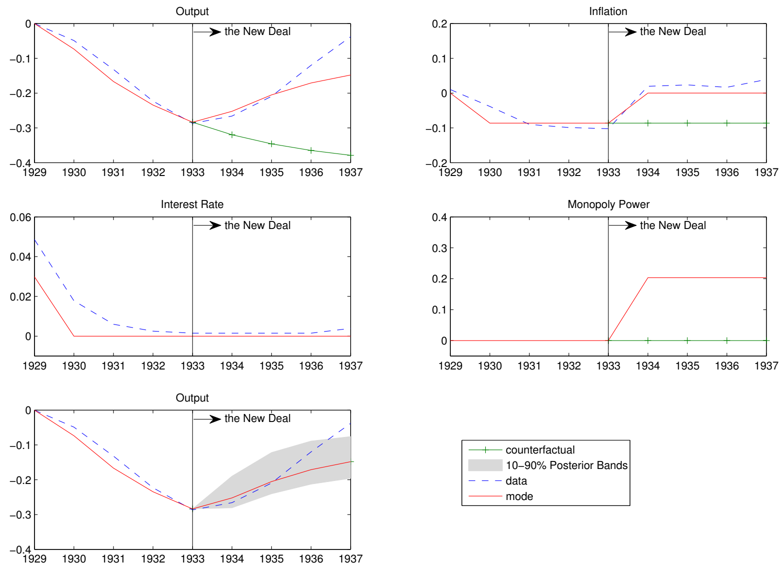


Figure 29: A reversal of the shock that gave rise to the Great Depression cannot account for the recovery, for this would have implied a renormalization of interest rates.

Figure 29 shows the effect of the New Deal, where the coefficient $\phi_\omega < 0$ has been chosen optimally, under the assumption that no other change has been made to policy. The calibration suggests that the New Deal can explain about 55% of the recovery in output and 70% of the recovery in inflation comparing 1937 to 1933. In the absence of any policy intervention, deflation would have continued, and output would have continued on a downward trajectory, reaching close to 40 percent away from its 1929 level in 1937, instead of registering the robust recovery seen in the data. This counterfactual history is shown by the line labeled "counterfactual" in the figures.

While the New Deal policy policy is expansionary according to the New Keynesian model, it has been universally derided by economists ranging from John M. Keynes (1933) to Milton Friedman and Anna Schwartz (1963) and all the way to the modern literature. Keynesâs argument was that demand policies, not supply restrictions, were the key to recovery and that to think otherwise was âa technical fallacyâ related to âthe part played in the recovery by rising prices.â Keynesâs logic will be recognized by a modern reader as a basic IS-LM argument: A demand stimulus shifts the âaggregate demand curveâ and thus increases both output and prices, but restricting aggregate supply shifts the âaggregate supply curve,â and, while this increases prices as well, it contracts output at the same time. Keynesâs argument against the NIRA was later echoed in Friedman and Schwartzâs (1963) classic account of the Great Depression and by countless other authors. Another set of arguments against NIRA are microeconomic. If firms gain monopoly power, they increase prices to increase their profits. The higher prices lead to lower demand. Encouraging workersâ collusion has the same effect. The workers conspire to prop up their wages, thus reducing hours demanded by firms. These results can be derived in a wide variety of models and have been applied by several authors in the context of the US Great Depression. An elegant and well known example is Cole and Ohanian (2004).

It is perhaps not surprising that one of the authors of the NIRA, Regford Guy Tugwell, said of the legislation that "for the economic philosophy which it represents there are no defenders at all." It was only with the development of New Keynesian models, over half a century later, that a co-herent logic could be formalized in mathematical models.

The logic of the argument inside the New Keynesian model, however, is not new. Essentially, it is that these policies are expansionary because they changed expectations from being deflationary to being inflationary, thus eliminating the deflationary spiral of 1929-33. This made lending cheaper and thus stimulated demand. This was also the reasoning of the architects of the NIRA. The New York Times, for example, reported the following on April 29th 1933, when discussing the preparation of the NIRA.

> A higher price level which will be sanctioned by the act, it was said, will encourage banks to pour into industry the credit now frozen in their vaults because of the continuing downward spiral of commodity prices.

The old Keynesian models miss this channel because expectations play little or no role. The other literature cited above misses it because it assumes one or all of the following (i) flexible prices, (ii) no shocks, and/or (iii) abstract from the zero bound. For NIRA to be expansionary all three assumptions have to be abandoned.

In the last section we showed how the monetary and fiscal policy regime change could explain a large part of the recovery. The results in figure 29 suggest the New Deal's NIRA may be the missing link.

While 1933-37 registers the strongest growth in U.S. economic history outside of wartime, there is a common perception among economists that the recovery from the Great Depression was very slow. One way to reconcile these two observations is to note that the economy was recovering from an extremely low level of output. Even if output grew very rapidly in 1933-37, some may argue it should have grown even faster and registered more than 9 percent per year average growth in that period.

Another explanation is that there was a serious recession in 1937-38 as shown in figure 30. If the economy had maintained the momentum of the recovery and avoided the recession of 1937-38, GDP would have reached trend in 1938.[27] To large extent, therefore, explaining the slow recovery is the same as explaining the recession of 1937-38, an issue we now turn to.

[27] This conclusion is drawn by using the data from Romer (1988), which covers 1909â1982, and estimating a linear trend. This trend differs from the one assumed by Cole and Ohanian (2004) because it suggests that the economy was 10 percent above trend in 1929, while they assume it was at trend at that time.

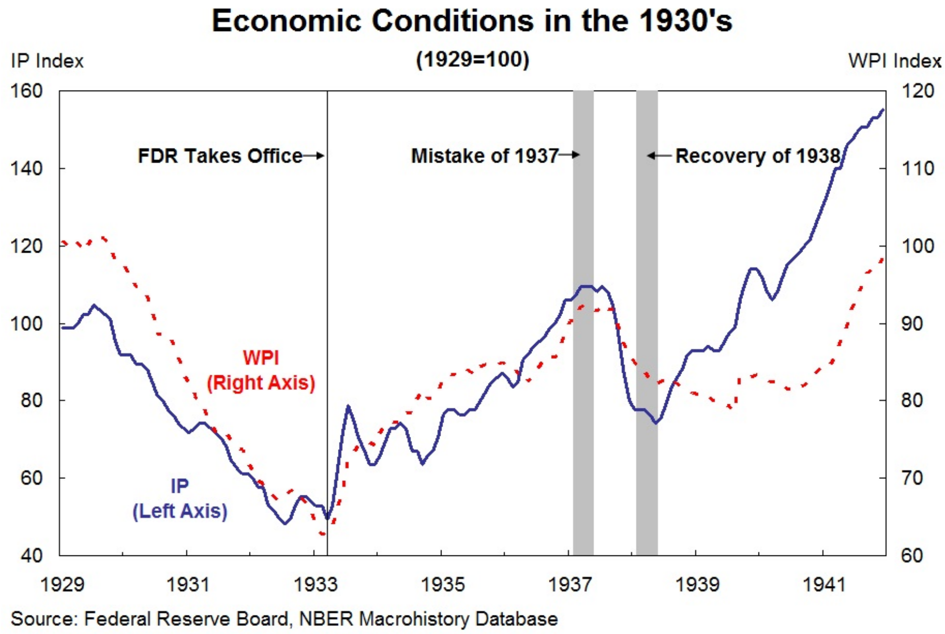


Figure 30: Both wholesale prices (WPI) and industrial production (IP) collapsed in 1929-1933 but abruptly started to recover in March 1933, when FDR took power and implemented a regime change along with New Deal Policies. The second phase of the Great Depression occurs after the Mistake of 1937, after the Supreme Court had striken down the New Deal and the Administration appeared to back away from its commitment to reflate the price level to pre-Depression levels.

7.3 The Mistake of 1937

The FDR regime change corresponds to a single data point in 1933. History has been kind to us in this regard, however, because the historical record documents what are arguable two other regime changes, denoted by the shaded region in figure 30, identified as "The Mistake of 1937" and the "Reversal of 1938" by two gray bars.

Eggertsson and Pugsley (2006) suggest that these two additional turning points correspond to (1) that the public believed that the administration was abandoning the FDR regime change in a return to a Hoover style deflationary regime, resulting in the severe recession in 1937, and then (2) the administration once again re-committed to an inflationary regime in 1938, explaining the resumption in growth at that time shown in figure 30.

We have already shown the contractionary nature of the Hoover regime, and similarly argued that it was the commitment of FDR to increase the price level that can explain a robust recovery. Rather than analyzing a MPE Eggertsson and Pugsley (2006) model the regime change as change in time varying probability of being in i) Hoover style deflationary regime or ii) FDR style inflationary regime characterized as a credibly commitment to inflation. As we have already observed, moving from one policy regime to other can explain large movements in output and prices once the efficient rate of interest is negative.

Below we focus on Eggertsson and Pugsley (2006) narrative of regime change in 1937 and 1938. Why is is it reasonable to think that a regime change occured? We then discuss what might have triggered the turning points.

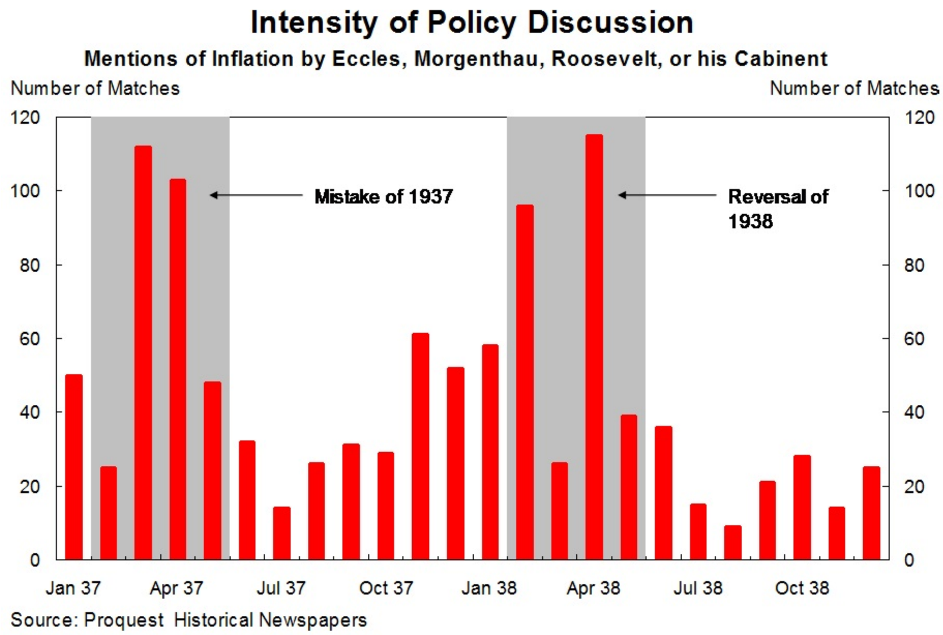


Figure 31: Intensity of Policy Discussion: Mentions of Inflation by Eccles, Morgenthau, FDR, or His Cabinet

Eggertsson and Pugsley (2006) propose a measure of narrative evidence from newspapers . Figure 31 shows the results from searches in Proquest Historical Newspapers database for front page or inner articles in this period that mention inflation, reflation, deflation, or price level and include the name of at least one key government official. Figure 7.3 and 7.3 show some illustrative cases of key announcements the search delivered. The period in which these communication peak (illustrated by the grey bars in 31) are those also highlighted in figure 30 as turning points.

The regime shift is exemplified in FDR's press conference on April 2, 1937: "I am concerned – we are all concerned – over the price rise in certain materials." On the day of this announcement the stock-market fell by 6 percent. The next day the Wall Street Journal reported as follows:

> There was a feeling among some bankers that the President's remarks bore a relation to the recent statement of M. Ecclees, Chairman of the Board of Governors of the Federal Reserve System, advocating prompt balancing of the budget as the only means of averting monetary inflation and the other recent statements of government officials warning of the threat of inflation. All of these remarks, it was said, indicated a change in the trend of the government's recovery measures away from the emphasis which has been placed upon stimulation of industrial activity and the recovery of prices.

Table 3: The Mistake of 1937: Anti-inflationary Communcation

1. July 14, 1936	The Federal Reserve announces the first reserve requirement increase which will become effective on the 15th of August.
2. January 30, 1937	The Federal Reserve announces the second and third reserve requirement increases which will become effective the 1st of March and 1st of May.
3. February 18, 1937	Marriner Eccles, Chairman of the Board of Governors, in Senate hearings: "The short term rates are excessively low and there may be a tendency for rates near the vanishing point to increase." --- Wall Street Journal, February 19, 1937, pg. 1.
4. March 15, 1937	Marriner Eccles, Chairman of the Board of Governors, gives a statement: "The upward spiral of wages and prices into inflationary levels can be as disastrous as the downwards spiral of deflation." --- Chicago Daily Tribune, March 16, pg. 1.
5. March 17, 1937	Commerce Secretary Daniel C. Roper and Secretary of Agriculture Henry A. Wallace hold press conferences: Both Secretaries warn against excessive inflation. --- Wall Street Journal, March 18, 1937, pg. 8.
6. March 24, 1937	Marriner Eccles, Charimain of the Board of Governors, on inflation: "Chairmain Eccles outlines five steps to avert 'dangerous inflation' in Forbes Magazine which are (i) reserve requirement increases "to eliminate excess reserves", (ii) fiscal policy that balances the budget, (iii) reduction in the gold price of the dollar, (iv) increase in the labor share of national income, and (v) antitrust legislation." --- The Christian Science Monitor, March 25, 1937.
7. April 2, 1937	Franklin Delano Roosevelt holds a press conference: "I am concerned -- we are all concerned -- over the price rise in certain materials."
8. August 3, 1937	Franklin Delano Roosevelt's views on price level targeting revealed: Senator Elmer Thomas published a letter from Franklin Delano Rosevelt to him rejecting his proposal that the Federal Reserve should formally target the 1926 price level. --- Wall Street Journal, August 4, 1937, pg. 6.

These announcements were in opposition to FDR's previous commitment to restore prices to their pre-depression levels. At the time. prices as measured by both WPI and CPI were still well below their previous levels. WPI was 13% below its 1926 average and CPI was 20%. With prices below their previously announced targets, the administration's very public alarm over increasing prices suggested that the administration was abandoning its previous goals, and these fears are reflected in the subsequent movements of the price level. Eggertsson and Pugsley (2006) document that both commodity prices, as well as long term interest rate moved during the period associated with "the Mistake of 1937" which are highly consistent with this shift in beliefs.

It is worth noting that a large part of the NIRA, which we argued was important in increasing inflation expectations, had been struck down as unconstitutional in the years leading up to the Mistake of 1937. Moreover, FDR was unsuccessful in "re-stacking" the Supreme Court in early 1937, a move FDR made as he believed the Supreme Court was undoing the New Deal. These development could also have added to the public perception that FDR had lost control over the economic policy and thus reduced the credibility of the reflation.

While the contraction in 1937-1938 is the sharpest on record, its end was just as abrupt. It ended in the period identified in figure 31. What happened? Again, FDR's own statements seem to suggest this period is also a natural candidate for a regime change and that he was reverting back to the inflationary regime installed in 1933. For example, on February 1st the Chicago Tribune reported.

Table 4. The Reversal of 1938: Pro-inflation Communcation

1. February 15, 1938	President Roosevelt holds a press conference: "At his press conference today, the President said that he believes now, as he did in 1933, that achivement of permanent prosperity depends on raising general price levels to those prevailing in 1926." --- Chicago Daily Tribune, February 16, 1938, pg. 1, "Hope Inflation Will Halt Depression."
2. February 18, 1938	President Roosevelt releases a written statement at a press conference that was prepared by Henry Morgenthau, Jr., Secretary of the Treasury; Henry A. Wallace, Secretary of Agriculture; Frances Perkins, Secretary of Labor; Marriner Eccles, Chairman of the Board of Governors of the Federal Reserve System, and economists of various executive departments: It is clear that in the present situation a moderate rise in the general price level is desirable [....] Our program seeks a balanced system of prices such as will promote a balanced expansion in production. Our goal is a constantly increasing national income through increasing production and employment. This is the way to increase the real income of consumers.
3. April 14, 1938	Franklin Delano Roosevelt, gives addresses Congress announcing that the reserve requirement increases will be abandoned: "The measures for expanding excess reserves which were announced on Thursday by President Roosevel will re-create the bases for a great credit inflation [...] Monetary management, after having been directed for some time towards guarding against a possible inflationary boom, has turned, under the pressure of the business depression, toward the other extreme." --- The New York Times, April 17, 1937.

> At his press conference today, the President said that he believes now, as he did in 1933, that achievement of permanent prosperity depends on raising general price levels to those prevailing in 1926.

Shortly thereafter, FDR held another conferences, flanked by the Secretary of the Treasury and the Chairman of the Federal Reserve to announce that a rise in the general price level was desired. Later that spring the administration took several steps to support an inflationary program, such as lowering the reserve requirement back to its 1936 level, increasing deficit spending and desterilizing government gold stocks.

Eggertsson and Pugsley (2006) document evidence from commodity markets and yields on long term bonds that seem once again consistent with shift in believes about future inflation.

It is often argued that it was wartime spending that finally lifted the US economy out of the Great Depression. This "conventional wisdom" is probably colored by the Keynesian view that monetary policy was impotent during this period. While there is no doubt that wartime spending helped stimulate demand, the turnaround from 1937-38 is more appropriately traced back to Roosevelt's recommitment to inflation in the early months of 1938.

To what extent was the Mistake of 1937 are deliberate change in strategy or confusing signals about future policy, or perhaps reflecting a tug of war between the Federal Reserve and the Treasury? These issue remain unresolved, but Eggertsson and Pugsley (2006) offers various hypothesis aimed at addressing them, see also Eggertsson (2016) for discussion of issues related to the independence of the Fed.

8 The Great Recession

8.1 Forward Guidance

While the Great Depression provided what is arguably a relatively clear example of a regime changes, there is a weaker case to be made that the a similar regime change occurred in the US following the Great Recession in 2008.

Perhaps the strongest case for that such a regime change occurred is that the Federal Reserve started using increasingly what has been termed "forward guidance". With the Federal Reserve rate close to zero in 2008, the Fed increasingly started discussing how it would change interest rate in the future. This is precisely the type of commitment we have seen is required under optimal policy. In the beginning, these announcements largely referred to explicit calendar time. An example of this type of policy is when the Federal Reserve said that rates would stay low for "some time" (December 2008), for "extended period" (March 2009) and "at least to mid-2013" (August 2013). **??** reviews these policies, and argues that they were not as effective as they could have been for they did not explicitly tie the hands of the Federal Reserve. Campell, Evans, Fisher, and Justiniano (2012) further stress that this type of guidance can have two effects: An "Odyssian" guidance would serve to commit the policy maker to low future rates, and thus be expansionary, while a "Delphic" announcement would instead simply serve to signal weaker future fundamentals and thus be contractionary. Thus the literature has yet to converge on how effective these policies were.

An announcement made by the FOMC in December 2012, however, was closer to the type of optimal commitment policy considered here. In that the committee announced

> In particular, the Committee decided to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that this exceptionally low range for the federal funds rate will be appropriate at least as long as the unemployment rate remains above 6-1/2 percent, inflation between one and two years ahead is projected to be no more than a half percentage point above the Committeeâs 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored.

but this policy has been termed the "Evans rule" after Charles Evans, President of the Federal Reserve Bank of Chicago who was the main proponent of this formulation. This policy does have the feature of the "trigger strategy" suggested by Eggertsson and Woodford (2003) that stipulates only under what conditions interest rate are raised, rather than a fixed time horizon. Thus, it is left to the market to back out the expected duration of the ELB. It does not explicitly, however, encompass the feature of optimal policy of making up for the fall in inflation below the target of the central bank in the same way as the optimal commitment. Interestingly, however, by only talking about "projected inflation between one and two years" to be no more than 0.5 percent higher than the inflation target, the FOMC was allowing for the possibility that current inflation could substantially above the target – as long as the projected inflation was not too high. Thus the announcement could plausibly be interpreted as representing somewhat of a regime change

8.2 The Curious Case of Missing Deflation

An interesting aspect of the numerical examples reported in the last section is that it captured a substantial fall in output, calibrated to 7.5% without any drop in inflation. Several commentators have suggested that the lack of outright deflation during the GR, in contrast to the GD, calls for fundamental reevaluation of models of price rigidities (**??**) is on example).

The text book Phillips curve has the following form

$$\Delta \pi_t = \kappa \hat{Y}_t$$

where $\Delta \pi_t$ measure the rate of change in inflation. Clearly a Phillips curve of this form predicts a fall in inflation in response to an output contraction.

The New Keynesian Phillips curve, however, has fundamentally different predictions as stressed by Del Negro, Giannoni, and Schorfheide (2015). The AS curve can be forwarded to yield

$$\hat{\pi}_t = \kappa E_t \sum_{t=0}^{\infty} \beta^t \{\hat{Y}_{t+j} + \psi \chi^I \hat{\tau}_t^I + \chi^s \hat{\tau}_t^s - \sigma^{-1} \hat{F}_t\} \tag{87}$$

i.e. inflation depends not mainly on current economic activity, but expectation about future activity,$\hat{Y}_t$. Del Negro, Giannoni, and Schorfheide (2015) argue that expectation of marginal cost (which is proportional to $\hat{Y}_t$) in the model, together with higher degree of price rigidities, were and important reason for why there was not a stronger fall in inflation. This also plays a role in GR example, as κ is assumed to be much smaller than in the GD example. An important element for this story to be plausible is that inflation expectation are well anchored.

The model in the survey incorporates an additional element, as in Benigno, Eggertsson, and Romei (2019) that helps to match the inflation data. It is assumed, that once firms re-optimize their price, they follow a pricing plan in which their prices are indexed to the inflation target of the central bank. The term $\hat{\pi}_t$ represents inflation in deviation from the steady state inflation which is given by the inflation target of the central bank. Thus the model only predicts that inflation is below the inflation target rather than requiring outright deflation.

Another way to rationalize the absence of deflation is that the fall in output generated a simultaneously negative "supply shocks" that increased marginal costs. In the model here, τ_t^I, can be interpreted as stand-in for a generic supply shock. Christiano, Eichenbaum, and Trabandt (2015) model one such mechanism via working capital, where higher spreads in financial market feed directly into higher marginal costs. Coibion and Gorodnichenko (2015) replace the inflation expectation term in the AS equation with household survey expectation which rose sharply after 2009 in response to higher oil prices. They argue that this rise in inflation expectations explains the "missing" deflation.

9 Conclusion

In this survey we have attempted to summarize recent literature on the ELB, with an eye towards offering a unified perspective of the Great Recession and the Great Depression.

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