



Identifying Causal Effects under Kink Bunching



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Abstract

This paper develops a generalized framework for causal inference under kink bunching, where marginal rates change at a cutoff and agents can manipulate their choices around it. Diamond and Persson (2017) pioneered causal inference under notch bunching, and we focus on the kink bunching. While existing literature focuses on agents' response elasticities, this paper identifies how kinked policy affect other outcomes of interest, providing a reduced-form approach akin to RDD and RKD. We start with the sharp bunching scenario and then extend to the scenarios with diffuse bunching, misreporting, optimization frictions, and heterogeneity. The estimation method accounts for interior responses above the cutoff, and requires minimal assumptions. Applying the proposed approach, we estimate how kinked medical subsidies affect outpatient behaviors in China.

Introduction

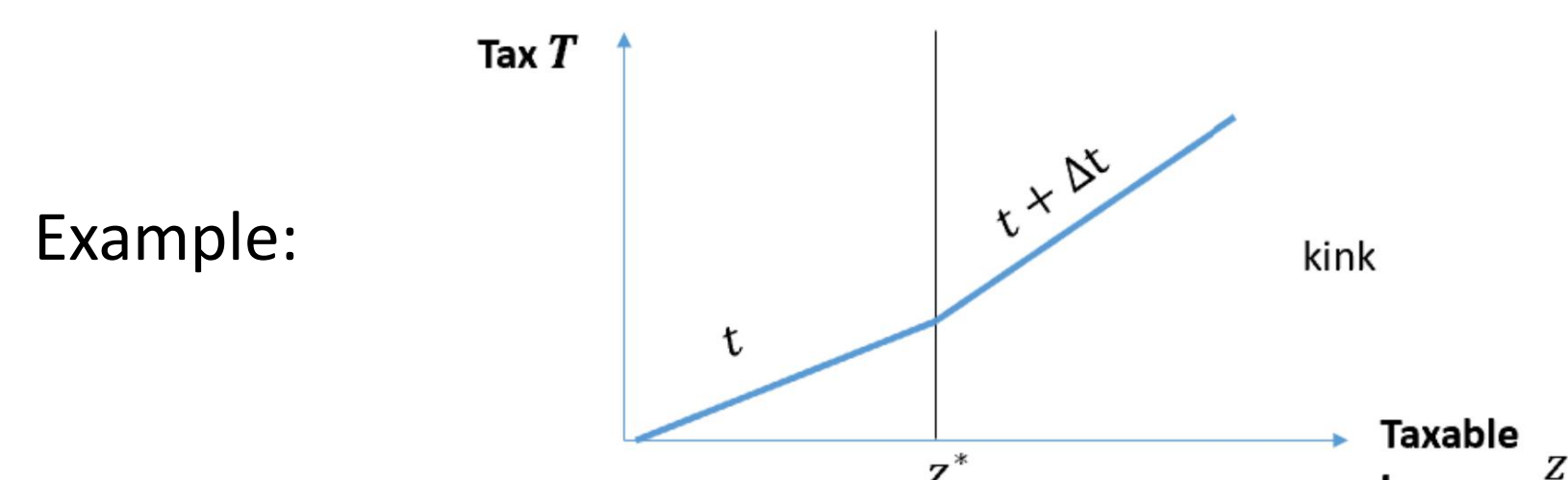
- What do we do in this paper?
 - ✓ **Causal inference** when agents face discrete slope change in choice sets ("**Kink**") and they can manipulate around the cutoff ("**Bunching**").

Nonlinear Policy	Cannot Manipulate	Can Manipulate
	Density is smooth at the cutoff	Density has Bunching at the cutoff
Level change at a cutoff	Regression Discontinuity Design (RDD)	Diamond and Person (2017)
Slope change at a cutoff	Regression Kink Design (RKD)	This Paper

- What is the workflow?
 - ✓ First, review features of kink bunching in the literature
 - ✓ Second, layout causal inference method under kink bunching
 - ✓ Third, extensions and application example.

Kink Bunching Setting

Kink Policy: Agents face a payment rate of t if their z is below the cutoff z^* , but face a higher rate of $t + \Delta t$ if their $z > z^*$.



Counterfactual Policy: Agents face linear payment rate of t .

Due to disutility from work, agents' optimal z depends on marginal tax rate \tilde{t} and their ability n ; that is, $z(\tilde{t}, n)$ (following literature).

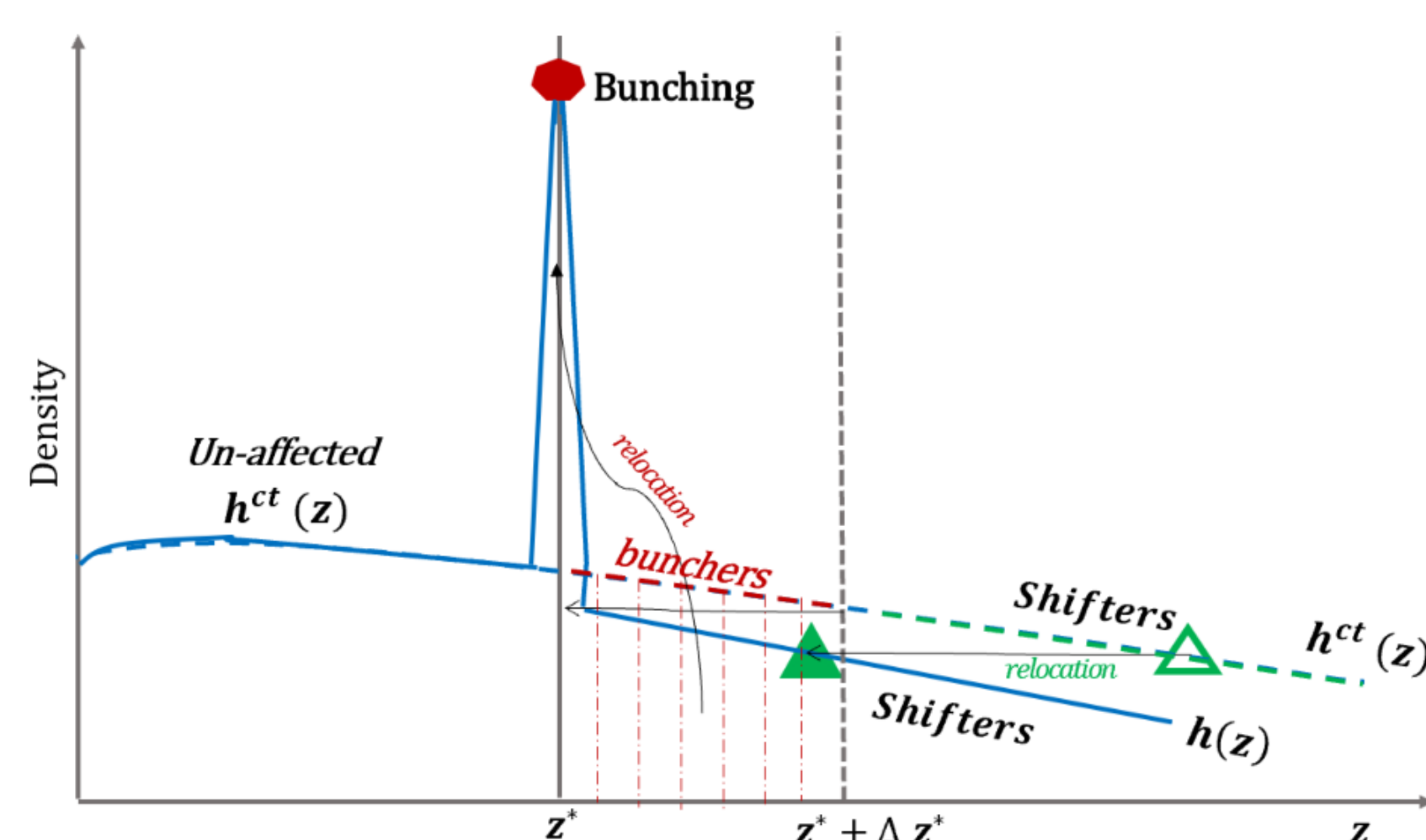
Assumption 1 Assume $z(\tilde{t}, n) = f(\tilde{t}; e)g(n; e)$, where e is a parameter (elasticity). Under kinked policy, we have

$$z_n^{kp} = \begin{cases} z(t, n) = z_n^{ct} & \text{if } z_n^{ct} \leq z^* \text{ Unaffected} \\ z^* & \text{if } z_n^{ct} \in (z^*, z^* + \Delta z^*] \text{ Bunchers,} \\ z(t + \Delta t, n) = z_n^{ct} \frac{f(t + \Delta t; e)}{f(t; e)} & \text{if } z_n^{ct} > z^* + \Delta z^* \text{ Shifters} \end{cases}$$

Amount of excess bunching: $B = \int_{z^*}^{z^* + \Delta z^*} h^{ct}(z) dz$
Marginal bunching agent is also a shifting agent: $\frac{z_n^{kp}}{z_n^{ct}} = \frac{f(t + \Delta t; e)}{f(t; e)} = \frac{z^*}{z^* + \Delta z^*}$

- **Shifting agents adjust their z by the same ratio.**

Figure 1: Change in the Density Distribution



Estimate counterfactual density $h^{ct}(z)$, marginal response Δz^* and elasticity e : (Chetty et al. 2011; Blomquist et al. 2021; Bertanha et al. 2023; and our method).

Causal Inference in Kink Bunching

Example: a reduction in income z due to the kink tax policy could affect people's health expenses y .

- **Unaffected Agents** ($z_n^{ct} \leq z^*$): Unaffected
- **Bunching Agents** ($z^* < z_n^{ct} \leq z^* + \Delta z^*$): Affected
- **Shifting Agents** ($z_n^{ct} > z^* + \Delta z^*$): Affected

For Shifting Agents ($z_n^{ct} > z^* + \Delta z^*$):

- ✓ Observe (z_n^{kp}, y_n^{kp}) under kink policy, need to **recover** y_n^{ct} .
- ✓ **Have Inferred** z_n^{ct} , given estimated Δz^* in density part and $z_n^{ct} = \frac{z_n^{kp} z^* + \Delta z^*}{z^*}$
- ✓ What drives change in y ?
 - 1st, changes in z .

Define parameter $\mu \equiv \frac{\partial y}{\partial \Delta z / z}$. Given $\frac{z_n^{kp}}{z_n^{ct}} = \frac{z^* + \Delta z^*}{z^*}$, we have $y_n^{kp} - y_n^{ct}$ due to direct changes in $z =$ **unknown constant**

2nd, changes in payment scheme T .

Define parameter $\lambda \equiv \frac{\partial Y}{\partial T}$. As payment scheme changes from linear to kink,

$(y_n^{kp} - y_n^{ct})$ due to a change in $T =$ **unknown constant + linear function of z_n^{ct}** .

Assumption 2 Effects from changes in z and T on outcome y are additive.

$$\begin{aligned} \Rightarrow y_n^{kp} - y_n^{ct} &= c_1 + c_2 z_n^{ct} \\ \Rightarrow E[y_n^{kp} | z_n^{ct}] - E[y_n^{ct} | z_n^{ct}] &= c_1 + c_2 z_n^{ct}. \end{aligned}$$

Level + Slope Change

Estimate conditional mean $E[y_n^{ct} | z_n^{ct}]$

- (1) Relocate Shifters from z_n^{kp} to z_n^{ct}
- (2) Use data of **unaffected** (z_n^{ct}, y_n^{ct}) & of **relocated shifters** (z_n^{ct}, y_n^{kp}) to fit a regression, with **level and slope change above cutoff**.

$$y_j^{reg} = f(z_j^{ct}; \beta) + a_0 I[z_j^{ct} > z^*] + a_1 I[z_j^{ct} > z^*] z_j^{ct} + \varepsilon_j$$

if $z_j^{ct} < z^*$ or $z_j^{ct} > z^* + \Delta z^*$,

$$(3) E[y_j^{ct} | z_j^{ct}] = f(z_j^{ct}; \beta)$$

➔ average effect on shifters

- (4) Level & slope change coeffs: a_0, a_1
- ➔ parameters μ, λ ➔ simulations

For Bunching Agents ($z^* < z_n^{ct} \leq z^* + \Delta z^*$):

- ✓ **Average Treated Outcome:** separate from unaffected agents at cutoff
- ✓ **Average Counterfactual Outcome:** infer from $E[y_n^{ct} | z_n^{ct}]$ in $(z^*, z^* + \Delta z^*)$

Discussion & Extensions

- ✓ **Functional Form for the conditional outcome mean** $E[y_n^{ct} | z_n^{ct}]$: exploit policy variations (over-time, or across-group) as robustness check.
- ✓ **Multiple determinants of agents' z :** ability, gender, experience, etc.
 - Shifting agents might still adjust by constant ratio (testable assumption).
 - Shifting agents might adjust by constant ratio, after adding controls. It is testable and the method will work with small modification.
- ✓ **Diffuse Bunching:** include diffusion region around cutoff
- ✓ **Rounding in z :** control for rounding.
- ✓ **Misreporting in z :** method has no bias, as shifters misreport by the same degree.
- ✓ **Heterogeneity in Parameters:**
 - heterogeneity in μ, λ : does not bias the estimates
 - heterogeneity in e : take the $\ln(z)$ (due to constant change), apply Taylor expansion, use density change as additional moment. (good approx.)

Application & Conclusion

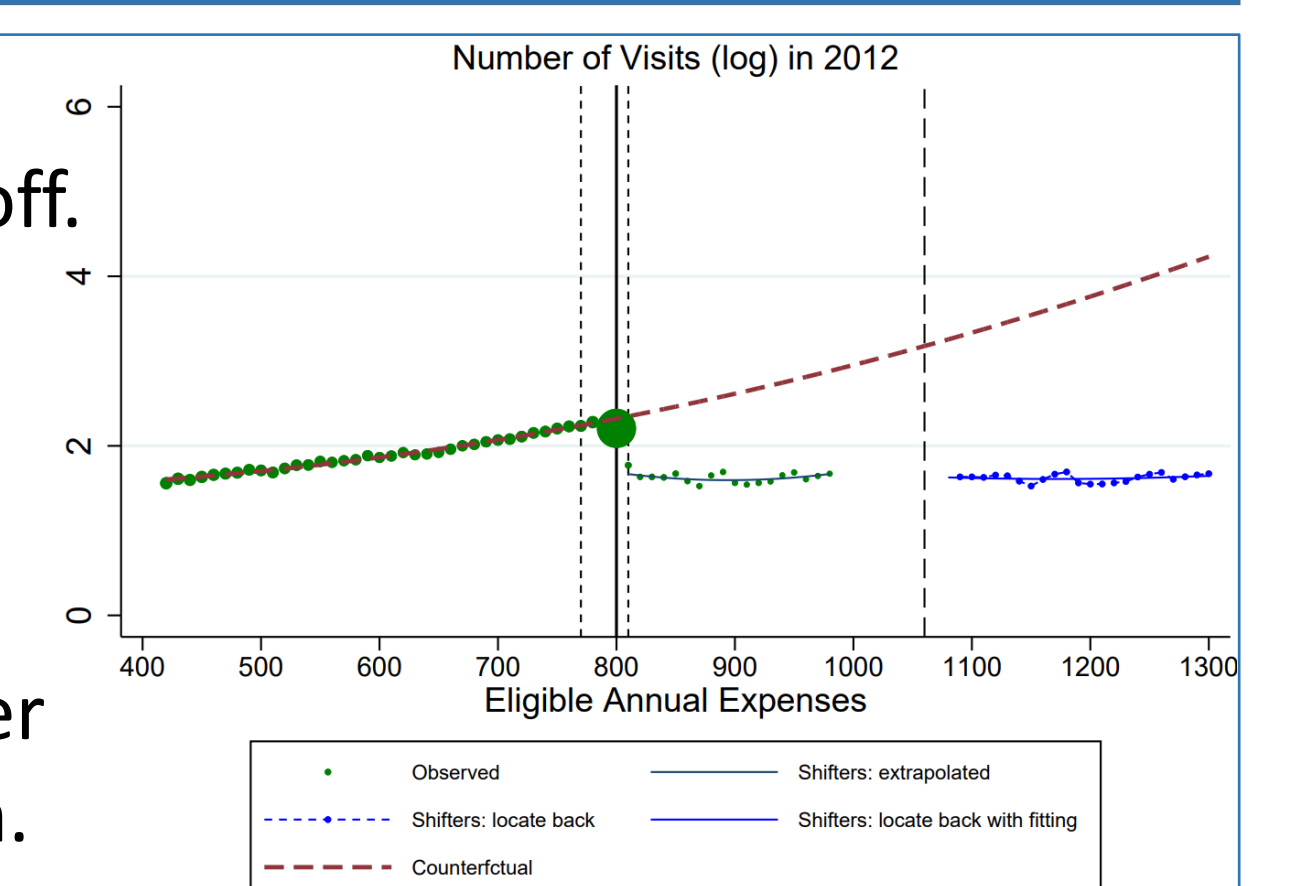
Medical Insurance Example

- Copayment rate rose from 50% to 100% at cutoff.
- Patients reduce hospital visits in response.

Conclusion

Provide causal estimator under kink bunching.

- ✓ Exploit shifters' constant ratio change in z .
- ✓ Link it with level & slope changes in $E[y|z]$ after relocating shifters, which enables identification.



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