Rational Bubbles with Competitive Fund Managers

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Abstract

Financial bubbles are often described as the result of behavioral biases and financial constraints. The model presented in this paper shows how a rational expectation equilibrium (REE), in which asset prices can deviate from their fundamental value, can still exist in a world with agents rationally and unbiasedly evaluating an asset's future return. Price deviation is the result of risk-averse fund managers asymmetrically informed who compete to outperform a benchmark. Stronger competition incentives lead to larger price deviations, worse risk-adjusted returns and deteriorates investors' ability to separate different types of managers. However, competition hampers the effect of non-fundamental shocks on the price making them more informational efficient.

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1 Introduction

Do bubbles exist? This question has sparked a long-standing debate in the finance literature. In this paper, I develop a model in which asymmetrically informed fund managers, driven by contractual incentives, compete against a benchmark portfolio. To outperform the benchmark, they trade more aggressively, tilting their portfolios toward a risky asset whose price increases in response to stronger competition incentives. This upward pressure on the asset's valuation can lead to potential losses if fund managers price the asset based on exceptionally favorable signals about future returns.

Investors, in turn, allocate their wealth rationally by updating their beliefs about which fund manager is better informed. However, as the contractual incentives to compete intensify, investors' ability to distinguish between informed and uninformed managers deteriorates. This is because stronger competition incentives cause fund managers to herd on the same asset, resulting in more similar portfolio allocations.

Stronger competition also enhances price informativeness, allowing the uninformed manager to better infer the signal that the informed manager acts upon. This occurs because competition incentives anchor fund managers' allocations more closely to the benchmark portfolio, causing their demand to depend more heavily on the benchmark and less on their private information. As a result, prices reflect this distortion, becoming less sensitive to signals. This reduced sensitivity makes future prices more predictable for the uninformed manager.

The model developed in this paper builds on the framework of Grossman and Stiglitz (1980). Fund managers trade a risky and a risk-free asset and can be either informed or uninformed. The informed type receives a private signal about the future value of the risky asset. The uninformed type recognizes that the price reflects the signal traded upon by the informed manager and updates her expectations about the asset's future payoff accordingly. Investors observe the portfolio allocations of the fund managers, adjust their beliefs about each manager's type, and allocate their wealth between the fund they perceive as managed by the informed type and the risk-free asset.

Fund managers are compensated by investors through a combination of assets under management (AUM) fees and performance fees calculated relative to a benchmark. A fund manager compensated solely through an AUM fee will trade based only on the information she can gather, focusing on attracting as much wealth from investors as possible. In contrast, a fund manager compensated exclusively through a performance fee is not concerned with attracting investors. Her primary goal is to outperform the benchmark, leading her portfolio to closely align with it. The greater the weight of the performance fee in a fund manager's total compensation, the stronger her incentive to compete.

The static version of the model focuses exclusively on the effects of stronger competition incentives on fund managers' portfolio allocations and the pricing of the traded assets. In the dynamic extension, I further examine the informativeness of prices and the influence of competition incentives on investors' decision-making processes and beliefs.

This paper makes a dual contribution to the literature. First, the model provides a simple framework that replicates empirical evidence on the risk-taking behavior of fund managers subject to competition incentives and their tendency to anchor portfolios to a benchmark (Kirchler et al., 2018; Kempf and Ruenzi, 2008; Chevalier and Ellison, 1997; Brown et al., 1996). Second, the paper highlights the role of competition incentives in generating upward pressure on prices, enhancing information efficiency, and reducing investors' ability to differentiate between better-informed fund managers. Importantly, these results are not driven by behavioral biases; fund managers rationally process the information they gather and trade with the objective of outperforming a benchmark and increase their compensation.

The paper is organized as follows: Section 2 reviews the relevant literature, Section 3 presents the static model and analyzes the effect of competition incentive on the pricing of an asset, Section 4 extends the model to a dynamic setting and further investigates the informativeness of the price and investors' beliefs, and Section 5 concludes.

2 Literature Review

The classical argument that excludes the existence of bubbles can be traced back to Fama (1965). Simply stated, if prices do not reflect the fundamentals and assets are traded above (or below) their 'fair' value, a profit opportunity arises. Thus, as long as arbitrageurs are free to trade they will step in, take advantage of such deviations, and force the price back to its fundamental value. Subsequent works have generalized the non-existence argument and confined bubble formations to either peculiar market conditions or non-rational agents (Santos and Woodford, 1997; Tirole, 1982).

This paper focuses on the price distortions that stem from the contractual incentives in asset managers compensation. The empirical evidence suggests that under-performing asset managers suffer career and compensation concerns (Ellul et al., 2020). More generally, past performances appear to be a determinant factor for hiring investment managers or for recommending an actively managed fund product (Jenkinson et al., 2016; Goyal and Wahal, 2012). However, contractual incentives may end up rewarding undesired outcome (Dow and Gorton, 1997). On the other hand, Dass et al. (2008) shows instead the potential inhibiting effect of some contractual incentives, providing empirical evidence that only mutual fund managers concerned about their relative performance were herding into bubble stocks. Li and Tiwari (2009), instead, highlights the importance of choosing an appropriate benchmark to correctly incentivize managers and reduce the moral hazard problem between investors and portfolio managers. In a laboratory experiment, James and Isaac (2000) have even provided evidence of the distorting effect on prices of tournament-like incentives.

The existence of price distortions has been proven to hold true in the presence of various common constraints too. Capital constraints, short sale restrictions, and the absence of perfect substitutes for arbitrage are just some of the potential venues through which price distortions can appear (Ofek and Richardson, 2003; Wurgler and Zhuravskaya, 2002; Shleifer and Vishny, 1997; Miller, 1977). The economic reasoning behind these results is straightforward when compared to the argument elaborated by Fama. All these frictions result in limits to arbitrage, thus prices may persistently not converge to their fundamental level even in the presence of agents that recognize arbitrage opportunities.

In Tirole (1985), it is argued that a bubble on an intrinsically useless asset may emerge in an OLG setting as opposed to an infinitely-lived agent economy. Interestingly, it demonstrates that this distortion does not shrink relative to the size of the economy and it is able to bring the economy on an efficient path too since it slows down excessive capital accumulation. Weil (1987) further elaborates on this. It demonstrates the existence of stochastic bubbles in dynamically inefficient economies if a sufficiently high level of "confidence" allows for intergenerational transfer of wealth.

Arbitrageurs may find it optimal to ride the bubble rather the trade against it. Sentiment predictability or over-optimism by market participants for the bubble to grow can incentivize arbitrageurs to take advantage of price distortions. For instance, empirical analyses by Griffin et al. (2011), and Brunnermeier and Nagel (2004) provide evidence in support of the synchronization risk theory. Specifically, Abreu and Brunnermeier (2003, 2002) show how arbitrageurs may find it convenient to not short-sell an asset whose valuation has inflated due to over-optimistic agents but rather prefer to ride it until a sufficient number of arbitrageurs are able to coordinate their selling strategies. In Sato (2016), fund managers are characterized as agents concerned about their performance ranking. The paper shows how such concerns eventually lead fund managers to ride the bubble in order to outperform each other. Nevertheless, as in Abreu and Brunnermeier (2003), managers only refrain from bursting the bubble rather than causing it.

Sentiment predictability has also been found to foster asset bubbles (Temin and Voth, 2004). The "irrational exuberance" described by Shiller (2000) can be modeled so as to show how sophisticated investors can take advantage of it or be deterred to take riskier allocations (Dumas et al., 2009; De Long et al., 1990a,b). Finally, heterogeneous beliefs and overconfidence are at the foundation of the resale option theory of bubbles (Xiong and Yu, 2011; Scheinkman and Xiong, 2003; Harrison and Kreps, 1978).

DeMarzo et al. (2008) develops a model on financial bubbles around the notorious *Catching up with the Joneses* by Abel (1990). They show that competition over future investment opportunities can induce agents to endogenously generate relative wealth concerns. In turn, this leads them to herd on risky assets and drive up their prices.

3 Static Model

The static model presented in this section is based on the Grossman and Stiglitz (1980) model. Fund managers are asymmetrically informed: the informed type receives a private signal on the future payoff of a risky asset, and the uninformed type does not. Managers' portfolio allocations are then revealed, investors have prior beliefs on managers' type and allocate their wealth between the fund they believe is led by the informed manager and a risk-free asset.

The game is sequential. First I solve the investors' problem and retrieve the optimal amount of fund shares that they want to hold, and then I solve the fund managers' problem where investors' decision is anticipated by the managers. I look for a rational expectation equilibrium, where both fund managers and investors understand that portfolio allocations and the price function convey information on future payoff.

3.1 Assumptions

Consider a simple economy with two tradable assets: a risky asset, which takes the value $\tilde{d}$ in the next period, and a risk-free asset that generates a gross return R with certainty. The risky asset is interpreted as a market portfolio that is too expensive for any single investor to buy. Thus, only fund managers can trade the risky asset alongside the risk-free asset, while investors only trade fund shares and the risk-free asset. The payoff of the risky asset is given by:

$$\tilde{d} = \overline{d}_0 + \delta + u; \tag{1}$$

$$\delta \sim \mathcal{N}(0, \sigma_\delta^2); \quad u \sim \mathcal{N}(0, \sigma_u^2); \quad \delta \perp u.$$

The future value of the risky asset, $\tilde{d}$, is determined by three components. The unconditional expectation, $\overline{d}_0$, is public information. The realization of the random variable δ is known by managers only, while the future realization u is unknown by all agents. As a result, fund managers face lower uncertainty (*i.e.*, lower variance) than investors since δ is known to them. This setup captures the idea that fund managers are more sophisticated or possess more resources to forecast $\tilde{d}$ than investors. Define $\overline{d} = \overline{d}_0 + \delta$. The term $\overline{d} - \overline{d}_0$ also represents the difference in expectations of the future value of the risky asset between fund managers and investors.

Fund managers can either be informed or uninformed. The informed type receives a signal s on the future value $\tilde{d}$ which is unobservable by the uninformed type. The signal is given by the sum of the true realization $\tilde{d}$ and some random noise ϵ.

$$s = \tilde{d} + \epsilon; \tag{2}$$

$$\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2); \quad \epsilon \perp \tilde{d}.$$

The signal is unbiased and does not fully reveal the future value of the risky asset. The conditional moments of $\tilde{d}$ are reported in the Appendix for convenience. In equilibrium, even if investors can observe managers' portfolio allocations, they cannot reverse-engineer the actual realization of both s and δ so that the types of the two managers are not fully revealed.

The risk-free asset is infinitely supplied, while the risky asset has a fixed supply. The current price, p, clears the market for the risky asset. Thus, the price is such that the total supply of the risky asset, Z, must be equal to the total demand $\omega_I + \omega_U$, where ω_I and ω_U stands for the informed and uninformed type demand of the risky asset respectively.

The total supply Z is random:

$$Z = \theta + z; \tag{3}$$

$$z \sim \mathcal{N}(0, \sigma_z^2); \quad z \perp \tilde{d}.$$

The unconditional expectation of the total supply of risky asset, θ, is public information, but a supply shock z (like a fire-sale) randomly affects the total available amount of shares that can be traded in the market.

In equilibrium the price of the risky asset is a linear combination of the signal, the supply shock z, and the difference $\overline{d} - \overline{d}_0$:

$$p = \alpha_0 - \alpha_d(\overline{d} - \overline{d}_0) + \beta\left[(s - \overline{d}) - \gamma z\right]. \tag{4}$$

The constants α_0, α_d, β, and γ are determined by the moments of the distribution $\tilde{d}$ and are known by both fund managers and investors.

The price function (4) highlights the information that fund managers and investors can retrieve from the price since it is publicly observable. The informed manager knows $\overline{d}$ and s, enabling her to reverse-engineer z from the price. In contrast, the uninformed manager cannot deduce the supply shock z, as she does not observe the signal s. Finally, investors allocate after fund managers when portfolio allocations ω_I and ω_U are observable. Thus, they can retrieve the supply shock z from the market-clearing condition, but they cannot reverse-engineer $\overline{d}$ and s.

3.1.1 Fund managers

I assume that fund managers have CARA utility functions. They collect an asset under management fee and a performance fee from investors. The higher the performance fee compared to the assets under management fee, the higher the incentive for managers to compete against their benchmark. Set the subscript $i \in \{I, U\}$ for the informed, I, and the uninformed type, U. The maximization problem of the type i fund manager is:

$$\max_{\omega_i} \mathbb{E}\left[-e^{-\tau\left(F_i^A + F_i^P\right)} \middle| \mathcal{I}_i\right] \quad \text{with } i \in \{I, U\}. \tag{P1}$$

In the maximization problem [P1], τ is the risk-averse parameter, while F_i^A and F_i^P are the total assets under management fee and the total performance fee collected by the fund manager respectively.

F_i^A and F_i^P are given by:

$$F_i^A = c_A W_i; \tag{5}$$

$$F_i^P = c_P \left[\omega_i(\tilde{d} - pR) - \omega^*(\tilde{d} - pR)\right]. \tag{6}$$

The assets under management fee, c_A, for the type i fund manager, is a percentage of the total assets in the next period W_i. The performance fee, c_P, is a percentage of the gain over the benchmark portfolio whose risky asset demand is ω^*. Set $c_T = c_A + c_P$ to be fixed, then the ratio $\frac{c_P}{c_T}$ measures how much of the managers' compensation is affected by their performance. This ratio is always positive and such that $\frac{c_P}{c_T} \in [0, 1]$. The higher the ratio, the more the managers are incentivized to compete against the benchmark portfolio.

The total assets under management, W_i, for the type i fund manager is the sum of the wealth collected from investors and the revenue (or loss) derived from her portfolio allocation. Investors can invest in the fund of the type i fund manager by buying fund shares at a price $q_i(\omega_i)$. The price of fund shares is a function of the fund portfolio allocation and is anticipated by the fund manager. The total wealth invested in the fund is then equal to $nq_i(\omega_i)$, that is the product between the total number of fund shares, n, and the price of each share $q_i(\omega_i)$.

$$W_i = (nq_i(\omega_i) - \omega_i p)R + \omega_i \tilde{d}.$$

Define W_i^* the total assets under management in the next period if the fund portfolio allocation is $\omega_i = \omega^*$. Then the total performance fee (6) is just the proportion c_P of the difference $W_i - W_i^*$ given the same initial wealth collected.

I assume that ω^* is the demand for the risky asset that a fund manager with a CARA utility function would choose unconditionally, without anticipating investors' decisions, and without any competition incentive (*i.e.*, $c_P = 0$). Therefore, the benchmark portfolio is a 'passive' allocation that is only determined by the unconditional moments of $\tilde{d}$. The allocation of the benchmark portfolio ω^* is publicly known and is simply equal to:

$$\omega^* = \frac{\mathbb{E}(\tilde{d}) - pR}{c_A \mathbb{V}(\tilde{d})\tau}$$

The expected utility of the type i fund manager is conditional on the information $\mathcal{I}_i$ she has. The informed fund manager's information set is $\mathcal{I}_I = \{s, \overline{d}\}$ while the uninformed information set is $\mathcal{I}_U = \{p, \overline{d}\}$. Because the price does not provide additional information on the future payoff $\tilde{d}$, it is not beneficial for the informed fund manager to compute the expected utility conditional on the price. Conversely, the uninformed manager does not observe the signal and she knows that the price coarsely conveys information on the future realization $\tilde{d}$ in equilibrium, so her expected utility is conditional on the price.

3.1.2 Investors

Investors also have CARA utility functions, and they receive the result of their investment after fund managers charge their fees. Investors do not know which fund manager is the informed type and they have some prior beliefs on the type of fund managers.

With probability ψ they correctly believe that the fund manager whose true type is informed has the private signal. Alternatively, with probability $1-\psi$ they wrongly believe that she is the uninformed fund manager. For convenience, I call the former the 'informed' type investor, I, and the latter the 'uninformed' type, U. I do so since I assume that investors invest only in the risk-free asset and the fund they believe is led by the informed fund manager. Therefore, the type I investor invests in the fund managed by the type I fund manager, and vice versa for the type U investor. Investor's type is randomly drawn before they allocate their wealth and it is known to them. Assuming there are m investors, then $m\psi$ are informed and $m(1-\psi)$ are uninformed.

The type i investor maximizes her expected utility by choosing the optimal demand of fund share ϕ_i. The investor maximization problem is:

$$\max_{\phi_i} \mathbb{E}\left[-e^{-\tau W_i^v} \middle| \mathcal{I}_i^v\right]; \quad \text{with } i \in \{I, U\}. \tag{P2}$$

Investors allocate their initial wealth W_0 between fund shares of the fund managed by the type i manager and the risk-free asset. Thus, the next period wealth after fees, W_i^v, shown in problem [P2] is equal to:

$$W_i^v = \frac{\phi_i}{n}\left(W_i - F_i^A - F_i^P\right) + (W_0 - \phi_i q_i(\omega_i))R \tag{7}$$

Since the type i investor believes that the type i fund manager is informed, then her expected utility is conditional on ω_i only. Therefore, investors' expected utility is conditional on the information set $\mathcal{I}_i^v = \{\omega_i\}$.

Finally, fund share price $q_i(\omega_i)$ is determined by clearing the fund share market. The fund share price $q_i(\omega_i)$ is such that the total demand for type i fund shares must be equal to the total supply of shares n.

3.2 Equilibrium

The investor problem [P2] for the type i investor takes the portfolio allocation ω_i and the price of fund shares q_i as given.

Because the type i investor believes that the type i fund manager is informed, she can inverse the demand function for risky asset ω_i to obtain the conditional expectation $\mathbb{E}(\tilde{d}|\mathcal{I}_I)$ of future payoff $\tilde{d}$. Thus, for the informed investor, it is $\mathbb{E}(\tilde{d}|\mathcal{I}_I) = \mathbb{E}(\tilde{d}|\mathcal{I}_I^v)$, but for the uninformed investor it is $\mathbb{E}(\tilde{d}|\mathcal{I}_I) \neq \mathbb{E}(\tilde{d}|\mathcal{I}_U^v)$ since the uninformed manager does not observe the signal s. I conjecture and later verify that the conditional expectation of future payoff for investors can be expressed as:

$$\begin{cases} \mathbb{E}(\tilde{d}|\mathcal{I}_i^v) = \kappa_0 + \kappa_\omega \omega_i; \\ \mathbb{E}(\tilde{d}|\mathcal{I}_I^v) = \mathbb{E}(\tilde{d}|\mathcal{I}_I) \end{cases} \tag{8}$$

Since both investors believe that they have reversed the informed fund manager demand, then they also both assume they face the same uncertainty, that is $\mathbb{V}(\tilde{d}|\mathcal{I}_I) = \mathbb{V}(\tilde{d}|\mathcal{I}_i^v)$ for $i \in \{I, U\}$.

The expected utility of the investor is maximized when the proportion of fund shares of the type i fund held by the type i investor, $\frac{\phi_i}{n}$, is equal to:

$$\frac{\phi_i}{n} = \frac{\mathbb{E}(\tilde{d}|\mathcal{I}_i^v) - pR}{\mathbb{V}(\tilde{d}|\mathcal{I}_I) f(\omega_i)} - \frac{n c_A q_i(\omega_i) R}{\mathbb{V}(\tilde{d}|\mathcal{I}_I)(f(\omega_i))^2}. \tag{9}$$

Where $f(\omega_i)$ stands for the effective number of shares of the risky asset owned by investors after fees:

$$f(\omega_i) = (1 - c_A - c_P)\omega_i + c_P \omega^*.$$

The optimal proportion of fund shares (9) is increasing in the expected excess return of the fund $\mathbb{E}(\tilde{d}|\mathcal{I}_i^v) - pR$ and is decreasing in the price of the fund share $q_i(\omega_i)$ and the asset under management fee c_A.

Set $\psi_I = \psi$ and $\psi_U = 1 - \psi$. Then, the clearing market condition for the fund share of the type i fund is satisfied when the price of the fund share, $q_i(\omega_i)$, is such that:

$$m \psi_i \phi_i = n.$$

By solving the above condition for $q_i(\omega_i)$, the price of the fund share that clears the market is:

$$q_i(\omega_i) = \frac{f(\omega_i)\left(\mathbb{E}(\tilde{d}|\mathcal{I}_i^v) - pR\right)}{c_A n R} - \frac{\tau \mathbb{V}(\tilde{d}|\mathcal{I}_I) f(\omega_i)^2}{m \psi_i c_A n R}.$$

Similarly to the demand for fund shares (9), the price $q_i(\omega_i)$ is an increasing function of the expected excess return of the fund, and it is decreasing in the fee c_A, the supply of fund shares n, and the portfolio variance. Most importantly, a higher number of investors, m, and a higher likelihood of being recognized as the informed type fund manager, ψ_i, decreases the effect that the portfolio variance has on the price $q_i(\omega_i)$.

Given that the number of investors in a fund is usually very high and that each investor is infinitesimally small compared to the size of the fund, I focus on the limit case $m \to \infty$.

Thus, the price of fund shares $q_i(\omega_i)$ simplifies to:

$$q_i(\omega_i) = \frac{f(\omega_i)\left(\mathbb{E}(\tilde{d}|\mathcal{I}_i^v) - pR\right)}{c_A n R}. \tag{10}$$

An important advantage of (10) is that it is not affected by the probability ψ_i any longer. Thus, when fund managers internalize $q_i(\omega_i)$, the maximization problem is not affected by any discontinuity determined by extreme values of ψ_i.

The price of fund shares can be rewritten as a quadratic function in ω_i once conjecture (8) is plugged in (10). In particular, it is:

$$q_i(\omega_i) = \frac{\lambda_2\omega_i^2 + \lambda_1\omega_i + \lambda_0}{c_A n R}. \tag{11}$$

Where constants λ_2, λ_1, and λ_0 are:

$$\begin{aligned}
\lambda_2 &= \delta_\omega(1 - c_A - c_P); \\
\lambda_1 &= (\delta_0 - pR)(1 - c_A - c_P) + c_P\omega^*\delta_\omega; \\
\lambda_0 &= \omega^* c_P(\delta_0 - pR).
\end{aligned}$$

Fund managers internalize the price function (11), so they maximize their expected utility by taking into account the effect that a change in their optimal demand of risky asset has on the price of their fund shares.

Solving problem [P1] and rearranging terms to satisfy conjecture (8) leads to the following optimal demand schedule ω_i:

$$\omega_i = \frac{\mathbb{E}(\tilde{d}|\mathcal{I}_i) - pR}{(c_A + c_P)\mathbb{V}(\tilde{d}|\mathcal{I}_i)\tau} h_i^\omega + \omega^* h_i^0. \tag{12}$$

The multiplying factors h_i^ω and h_i^0 are:

$$\begin{aligned}
h_I^\omega &= \frac{2}{c_A + c_P} - 1; \\
h_I^0 &= \frac{2c_P}{c_A + c_P}; \\
h_U^\omega &= \frac{(2 - c_A - c_P)\mathbb{V}(\tilde{d}|\mathcal{I}_U)}{(2 - c_A - c_P)\mathbb{V}(\tilde{d}|\mathcal{I}_U) - 2(1 - c_A - c_P)\mathbb{V}(\tilde{d}|\mathcal{I}_I)}; \\
h_U^0 &= \frac{c_P}{c_A + c_P}\left(1 + \frac{(c_A + c_P)\mathbb{V}(\tilde{d}|\mathcal{I}_I)}{(2 - c_A - c_P)\mathbb{V}(\tilde{d}|\mathcal{I}_U) - 2(1 - c_A - c_P)\mathbb{V}(\tilde{d}|\mathcal{I}_I)}\right).
\end{aligned}$$

A detailed derivation of both (12) and (8) is reported in the Appendix.

The optimal demand schedule ω_i is determined by two components. First, fund managers increase their demand for the risky asset if the conditional expectation $\mathbb{E}(\tilde{d}|\mathcal{I}_i)$ increases. Second, a higher exposure to the risky asset of the benchmark portfolio ω^* also increases the demand for the risky asset ω_i. Therefore, incentives to outperform a benchmark leads to a higher demand for the risky asset by fund managers when $\overline{d}_0 - pR > 0$ even if the conditional expectation $\mathbb{E}(\tilde{d}|\mathcal{I}_i) < \overline{d}_0$. This distortion in ω_i disappears when $c_P = 0$. Instead, when c_P increases, both types herd on the risky asset since they invest more as the benchmark portfolio rather than based on their information.

The optimal demand in (12) shows the effect of portfolio delegation too. By decreasing both the assets under management fee, c_A, and the performance fee, c_P, the demand ω_i for the risky asset increases. This effect is due to the higher tolerance to risk that fund managers perceive when they do not manage their own wealth. Indeed, the maximization problem [P1] when $c_A = 1$ and $c_P = 0$ reduces to the model in Grossman and Stiglitz (1980) where agents allocate their wealth and bear all the risk, which is a special case of the model presented in this paper.

Finally, the multiplying factor h_i^ω and h_i^0 are both always positive, since the conditional variance of the uninformed type $\mathbb{V}(\tilde{d}|\mathcal{I}_U) > \mathbb{V}(\tilde{d}|\mathcal{I}_I)$, and $h_i^\omega > 1$ for both types. Moreover, the multiplying factors for the uninformed type converge to the ones of the informed type, if the variance of the supply shock σ_z^2 decreases. Indeed, the less noisy the price is, the more similar the allocations of the two fund managers.

3.2.1 Price and comparative statics

Given the optimal demand for the risky asset, ω_i, the price p can be immediately derived from its clearing market condition. Formally, the following must hold:

$$\omega_I + \omega_U = Z.$$

Solving the above equation for p, and rearranging terms, returns the linear combination in (4):

$$p = \alpha_0 - \alpha_d(\overline{d} - \overline{d}_0) + \beta\left[(s - \overline{d}) - \gamma z\right].$$

Set $\sigma_d^2 = \mathbb{V}(\tilde{d})$, $\sigma_{d,I}^2 = \mathbb{V}(\tilde{d}|\mathcal{I}_I)$, and $\sigma_{d,U}^2 = \mathbb{V}(\tilde{d}|\mathcal{I}_U)$. Then, the constants α_0, α_d, β, and γ are:

$$\begin{aligned}
\alpha_0 &= \frac{\overline{d}}{R} - \frac{c_A(c_A + c_P)\sigma_{d,I}^2\sigma_{d,U}^2\theta}{R\left[c_A\sigma_d^2\left(h_U^\omega\sigma_{d,I}^2 + h_I^\omega\sigma_{d,U}^2\right) + (c_A + c_P)(h_I^0 + h_U^0)\sigma_{d,I}^2\sigma_{d,U}^2\right]}\tau\sigma_d^2;\\
\alpha_d &= \frac{(c_A + c_P)(h_I^0 + h_U^0)\sigma_{d,I}^2\sigma_{d,U}^2}{R\left[c_A\sigma_d^2\left(h_U^\omega\sigma_{d,I}^2 + h_I^\omega\sigma_{d,U}^2\right) + (c_A + c_P)(h_I^0 + h_U^0)\sigma_{d,I}^2\sigma_{d,U}^2\right]};\\
\beta &= \frac{c_A\sigma_d^2\left(b_P h_U^\omega\sigma_{d,I}^2 + b_s h_I^\omega\sigma_{d,U}^2\right)}{R\left[c_A\sigma_d^2\left(h_U^\omega\sigma_{d,I}^2 + h_I^\omega\sigma_{d,U}^2\right) + (c_A + c_P)(h_I^0 + h_U^0)\sigma_{d,I}^2\sigma_{d,U}^2\right]};\\
\gamma &= \frac{(c_A + c_P)\sigma_{d,I}^2\tau}{b_s h_I^\omega}.
\end{aligned}$$

Parameters b_s and b_P are equal to:

$$b_s = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\epsilon^2}; \qquad b_P = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\epsilon^2 + \gamma^2\sigma_z^2}.$$

The effect of risk-aversion is observable through the parameter α_0. The unconditional expectation of the price is not equal to the discounted future value of the risky asset, $\frac{\overline{d}_0}{R}$, but it is further discounted due to risk-averse managers trading the asset. Incentivizing competition against a benchmark by introducing a performance fee, c_P, positively affects α_0 since managers are willing to load more risk to outperform the benchmark portfolio and increase the demand schedule ω_i.

The parameter α_d captures the effect that the different expectations on future payoff between fund managers and the benchmark portfolio have on the price. If $\overline{d} - \overline{d}_0$ is positive, managers would like to invest more than the benchmark portfolio imposes but, because of the performance incentive, they anchor their allocation to the benchmark. This distortion brings the price down. Vice versa, if $\overline{d} - \overline{d}_0$ is negative, the price surges.

Finally, the parameter β measures how sensitive the price is to changes to the private signal s, while γ captures the propagation of the supply shock z in the price. If the performance fee c_P increases, the parameter β decreases while γ increases, since managers have a greater incentive to allocate as the benchmark rather than based on their information. Conversely, if the precision of the signal b_s increases (or the variance σ_ϵ^2 decreases), the price becomes more sensitive to the signal and is less affected by supply shocks.

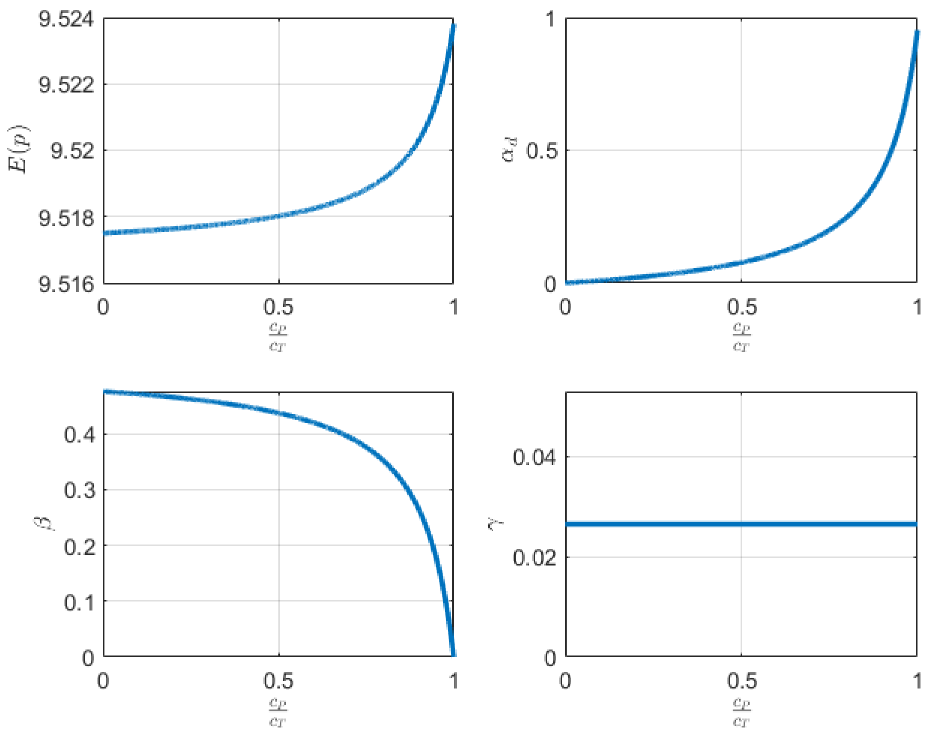

Figure 1: The figure plots the value of the unconditional expectation of the price, $\mathbb{E}(p)$, and price parameters α_d, β, and γ to different values of c_P such that $c_A = c_T - c_P$. The model parameters used are $\sigma_\delta = \sigma_u = \sigma_z = \sigma_\epsilon = 0.5$, $d_0 = 10$, $\theta = 1$, $R = 1.05$, $c_T = 0.3$, and $\tau = 2$.

Figure 1 shows the effect of a change in $\frac{c_P}{c_T}$ to the unconditional expectation of the price, $\mathbb{E}(p)$, and the parameters α_d, β, and γ. In order not to alter the risk-bearing capacity of fund managers determined by changes to c_T, any change in c_P is such that $c_A = c_T - c_P$ so that c_T is held constant.

An increase in the incentive to compete against a benchmark tilt fund managers' portfolio allocation towards the risky asset. The higher demand is not determined by a more optimistic signal, and it is symmetric between the two types of fund managers. Therefore, the price surges in expectation but becomes less sensitive to the signal s. Since both fund managers increase their demand for the risky asset due to performance concerns only, then the distortion determined by the difference in expectations $\overline{d} - \overline{d}_0$ is amplified too.

In the static version of the model, the parameter γ is unaffected by any change in c_P offset by an opposite change in c_A. As Section 4 shows, this is not anymore the case in a dynamic setting, since future prices become more predictable when managers' demand is anchored to a benchmark due to stronger incentives to outperform it.

4 Dynamic Model in T periods

The model presented in Section 3 can be extended to a T-period framework to better capture the impact of signals and competition incentives on price dynamics and investors' beliefs. For the most part, the assumptions listed in the static version of the model are the same and only extended to a dynamic setting.

In each period, a new generation of investors enters the market, while the previous generation exits and consumes all the wealth obtained from their investments into the funds. Fund managers do not retain their fees in the fund but consume them in every period. A sequential game between fund managers and investors, similar to the one of the static model, takes place at every time t. The type of fund manager is determined at time 0 and is never revealed. Investors' beliefs are updated once managers' allocations become observable.

4.1 Assumptions

Consider an economy where fund managers can trade a risk-free asset and a risky asset. The risk-free asset yields a constant gross return R each period, whereas the risky asset's dividend is revealed and paid only at time T, accruing over time.

$$\tilde{d}_T = \sum_{t=1}^{T-1} \tilde{d}_t; \tag{13}$$

$$\tilde{d}_{t+1} = \overline{d}_0 + \delta_t + u_{t+1}. \tag{14}$$

The random variables δ_t and u_t follow a normal distribution. While δ_t is revealed to fund managers at time t, u_t never becomes publicly observable.

$$\delta_t \sim \mathcal{N}(0, \sigma_\delta^2) \;\; i.i.d. \qquad u_t \sim \mathcal{N}(0, \sigma_u^2) \;\; i.i.d.$$

The terminal dividend, $\tilde{d}_T$, represents the cumulative earnings revealed at period-end and is derived from the accruals of earnings over previous periods. During the periods leading up to T, the price of the risky asset is determined endogenously by fund managers' demand, allowing for potential profits even before the final dividend is paid.

The information sets $\mathcal{I}_{i,t}$, for $i \in \{I, U\}$, contain all public information disclosed up to time t and the current dividend mean $\overline{d}_t = \overline{d}_0 + \delta_t$. In addition, the informed manager observes a private signal s_t defined by:

$$\begin{aligned} s_t &= \tilde{d}_{t+1} + \epsilon_t; \\ \epsilon_t &\sim \mathcal{N}(0, \sigma_\epsilon^2) \;\; i.i.d. \end{aligned} \tag{15}$$

Since fund managers' portfolio allocations are observable after the price has been set and trades have cleared, the uninformed manager can infer the private signal at time t and incorporate it into their information set from time $t+1$ onward. Thus, the following relationships hold:

$$\begin{aligned} \mathcal{I}_{I,t} &= \mathcal{I}_{U,t} \cup \{s_t\} \\ \mathcal{I}_{U,t} &= \left\{\{\overline{d}_j\}_{j=0}^{t}, \{s_j\}_{j=0}^{t-1}\right\} \end{aligned}$$

Since all the realizations of u_t, for $t \in [1, T-1]$, are not publicly observable from time $t+1$ onward, both types of fund managers (and investors) never resolve their uncertainty on $\tilde{d}_t$ and their information sets do not include it.

In each period t, the total supply of the risky asset is random. A supply shock z_t affects the total supply, such that:

$$\begin{aligned} Z_t &= \theta + z_t; \\ z_t &\sim \mathcal{N}(0, \sigma_z^2); \;\; i.i.d. \end{aligned} \tag{16}$$

In equilibrium the price of the risky asset is a linear combination of all the signals realized up to time t, the current supply shock z_t, and the current difference $\overline{d}_t - \overline{d}_0$:

$$p_t = \frac{1}{R^{T-t}} \left[\alpha_{0,t} - \alpha_{d,t} \left(\sum_{j=1}^{t} \overline{d}_j - t\overline{d}_0 \right) + \beta_{t,t}\left((s_t - \overline{d}_t) - \gamma_t z_t \right) + \sum_{j=1}^{t-1} \beta_{j,t}(s_j - \overline{d}_t) \right]. \tag{17}$$

The price function in the dynamic model mirrors the price of the static model in Section 3. The main difference is due to the accruals of the signals from the first period, which is caused by the final payoff $\tilde{d}_T$ paid only in the last period. Because past signals enter future price function, an overly-optimistic (or overly-pessimistic) signal can permanently alter prices from time t onward.

4.1.1 Fund managers

The wealth dynamic of investors and fund managers now accounts for the dividend payment occurring at time T. Similarly to Section 3, define $W_{i,t}$ the wealth before fees at time t that a type $i \in \{I, U\}$ fund manager realizes through her portfolio allocation at time $t-1$. That is:

$$W_{i,t} = (n_i q_{i,t-1} - \omega_{i,t-1} p_{t-1}) R + \omega_{i,t-1} p_t$$

Then, the asset under management fee collected at time t, $F^A_{i,t}$, and the performance fee, $F^P_{i,t}$, are defined as:

$$F^A_{i,t} = c_A W_{i,t}; \tag{18}$$

$$F^P_{i,t} = c_P \left[\omega_{i,t-1}(p_t - p_{t-1}R^f) - \omega^*_{t-1}(p_t - p_{t-1}R^f)\right]; \tag{19}$$

For both the wealth $W_{i,t}$ and the fees $F^A_{i,t}$, $F^P_{i,t}$, it is $p_T = \tilde{d}_T$ in the last period T.

Because fund managers consume all the fees they collect in every period, their utility maximization problem at time t is simply:

$$\max_{\{\omega_{i,j}\}_{j=t}^{T-1}} \mathbb{E}\left[\sum_{s=t}^{T-1} -\beta^{j-t+1} e^{-\tau\left(F^A_{i,j+1}+F^P_{i,j+1}\right)} \middle| \mathcal{I}^i_t\right]; \quad \text{with } i \in \{I, U\}. \tag{P3}$$

Due to short-lived investors who liquidate their investments to consume all their wealth, and to fund managers not retaining their fees into the fund, the dynamic problem [P3] reduces to a simple static problem at time t that is solvable in closed-form.

The price set by fund managers' allocation at time t naturally has a dual role in the dynamic setting. A high price at time t increases the profitability (and the relative fees) of time $t-1$ portfolio allocation, but reduces the excess return that is expected in period $t+1$. In equilibrium, I show that the closer fund managers are to time T, the faster the price of the risky asset grows if managers are subject to higher competition incentives.

4.1.2 Investors

Investors can allocate their wealth in a risk-free asset or in the fund they believe is managed by the informed type manager. Their wealth at time t, $W^v_{i,t}$, after fees are paid is:

$$W^v_{i,t} = \frac{\phi_{i,t-1}}{n}\left(W_{i,t} - F^A_{i,t} - F^P_{i,t}\right) + (W_{0,t-1} - \phi_{i,t-1} q_{i,t-1}) R^f \tag{20}$$

Because investors are short-lived, their utility maximizing problem is identical to the one of Section 3 but for the time index:

$$\max_{\phi_{i,t}} \mathbb{E}\left[-e^{-\tau W^v_{i,t+1}} \middle| \mathcal{I}^i_{v,t}\right]. \tag{P4}$$

The investor information set, $\mathcal{I}^i_{v,t}$, is affected by whether the individual investor is informed or not. If the investor is informed, she can retrieve the conditional mean of the price distribution under the current signal s_t from the informed fund manager demand, so that $\mathbb{E}(p_{t+1}|\mathcal{I}^v_{I,t}) = \mathbb{E}(p_{t+1}|\mathcal{I}_{I,t})$. Conversely, if she is uninformed, $\mathbb{E}(p_{t+1}|\mathcal{I}^v_{U,t}) \neq \mathbb{E}(p_{t+1}|\mathcal{I}_{I,t})$.

Contrary to the static version of the investor problem, in the dynamic setting the investor can update their probability of dealing with the informed fund manager, ψ_t, in every period. Define the indicator variable $\mathbb{1} = 1$ if the investor type is informed, and $\mathbb{1} = 0$ if she is uninformed. Set the difference between the portfolio allocations of the two fund managers $\xi_t = \omega_{I,t} - \omega_{U,t}$. If the investor is informed, then she will correctly assign the informed demand $\omega_{I,t}$ to the informed type and retrieve ξ_t. If she is uninformed, she will wrongly assume that $\omega_{U,t}$ is the informed demand and obtain $-\xi_t$.

Thus, set $\xi_{i,t}$ for $i \in \{I, U\}$ such that $\xi_{I,t} = \xi_t$ and $\xi_{U,t} = -\xi_t$. Then the probability ψ_t is defined:

$$\begin{aligned} \psi_t &= Pr(\mathbb{1} = 1|\mathcal{I}^v_{i,t}); \\ \mathcal{I}^v_{i,t} &= \{\xi_{i,t} \lesseqgtr 0\} \cup \mathcal{I}^v_{i,t-1}. \end{aligned} \tag{21}$$

The probability in (21) can be easily computed at time t in a recursive way. Given the time t portfolio allocations, by the Bayesian theorem the probability ψ_t is equal to:

$$\psi_t = \frac{Pr(\xi_{i,t} \lesseqgtr 0|\mathbb{1} = 1)\psi_{t-1}}{Pr(\xi_{i,t} \lesseqgtr 0|\mathbb{1} = 1)\psi_{t-1} + Pr(\xi_{i,t} \lesseqgtr 0|\mathbb{1} = 0)(1 - \psi_{t-1})}.$$

If the indicator variable is $\mathbb{1} = 1$, then $\xi_{I,t} = \xi_t$. Otherwise, it is $\xi_{U,t} = -\xi_t$.

Probability ψ_t is affected by ψ_{t-1}, thus previous portfolio allocations are all going to influence the probability at time t.

4.2 Equilibrium

The model is solved through backward-induction from time $T - 1$ back to time 1, fund managers internalize investors' wealth allocations, and investors update their beliefs before maximizing their expected utility.

Since both maximization problem [P3] and [P4] reduce to a static problem solved at time t, the optimal fund managers' demand of risky asset and investors' demand of fund shares are the same but for the time indexation and the conditional moments of p_{t+1} reported in the Appendix.

For the above reasons I now discuss only the effect of competition incentives on the price dynamic and investors' beliefs.

4.2.1 Price dynamic

At any time t, the clearing market condition for the risky asset is satisfied when the price p_t is equal to:

$$p_t = \frac{1}{R^{T-t}} \left[\alpha_{0,t} - \alpha_{d,t} \left(\sum_{j=1}^{t} \bar{d}_j - t\bar{d}_0 \right) + \beta_{t,t} \left((s_t - \bar{d}_t) - \gamma_t z_t \right) + \sum_{j=1}^{t-1} \beta_{j,t}(s_j - \bar{d}_t) \right].$$

Set $\sigma^2_{p,t} = \mathbb{V}(p_{t+1})$, $\sigma^2_{p,I,t} = \mathbb{V}(p_{t+1}|\mathcal{I}_{I,t})$, and $\sigma^2_{p,U,t} = \mathbb{V}(p_{t+1}|\mathcal{I}_{U,t})$.

The constant $\alpha_{0,t}$ can be split in two components: the discounted expected future payoff revealed at time T, and the discount that a risk-averse agent requires to trade:

$$\alpha_{0,t} = (T - t - 1)\bar{d}_0 + \sum_{j=1}^{t} \bar{d} + \alpha^*_{0,t};$$

$$\alpha^*_{0,t} = \alpha^*_{0,t+1} - \frac{R^{-1} c_A (c_A + c_P) \sigma^2_{p,I,t} \sigma^2_{p,U,t}}{c_A \sigma^2_{p,t} \left(h^\omega_{U,t} \sigma^2_{p,I,t} + h^\omega_I \sigma^2_{p,U,t} \right) + (c_A + c_P)(h^0_I + h^0_{U,t}) \sigma^2_{p,I,t} \sigma^2_{p,U,t}} \sigma^2_{p,t} \theta \tau.$$

The price parameter $\alpha^*_{0,t}$ is inversely related to the performance fee c_P since fund managers' demand increases with an increase in c_P. Moreover, $\alpha^*_{0,t}$ includes all future parameters $\alpha^*_{0,j}$ for $j > t$, so the closer the revelation period T is, the lower is the discount that fund managers need to bear the risk associated to holding the risky asset. When $t = T - 1$, $\alpha^*_{0,t} = 0$ since at time $t = T$ the price is exogenously set by $\tilde{d}_T$.

The multiplying factor $\alpha_{d,t}$ associated to the difference in expectations $\sum_{j=1}^{t} \bar{d}_j - t\bar{d}_0$ is:

$$\alpha_{d,t} = \frac{(c_A + c_P)(h^0_I + h^0_{U,t}) \sigma^2_{p,U,t} \sigma^2_{p,I,t} - c_A \left(h^\omega_{U,t} \sigma^2_{p,I,t} + h^\omega_I \sigma^2_{p,U,t} \right) \alpha_{d,t+1}}{c_A \sigma^2_{p,t} \left(h^\omega_{U,t} \sigma^2_{p,I,t} + h^\omega_I \sigma^2_{p,U,t} \right) + (c_A + c_P)(h^0_I + h^0_{U,t}) \sigma^2_{p,I,t} \sigma^2_{p,U,t}}.$$

The parameter $\alpha_{d,t}$, just as the static version of the model, disappears if $c_P = 0$ and is positively related to the performance fee c_P. It is trivial to see that when the period t approaches time T, the distortion effect of the difference in expectations shrinks. When $t = T - 1$, then $\alpha_{d,t+1} = 0$.

The sensitivity to the signal $\beta_{j,t}$ depends on whether the signal s_j is for the contemporaneous period $j = t$ or for previous periods. The difference is due to the different information sets of the fund managers. Previous signals are known to both types of fund manager, but the time t signal is not. Therefore:

$$\beta_{t,t} = \frac{c_A \sigma^2_{p,t} \left(b_{P,t} h^\omega_{U,t} \sigma^2_{p,I,t} + \beta_{t,t+1} h^\omega_I \sigma^2_{p,U,t} \right)}{c_A \sigma^2_{p,t} \left(h^\omega_{U,t} \sigma^2_{p,I,t} + h^\omega_I \sigma^2_{p,U,t} \right) + (c_A + c_P)(h^0_I + h^0_{U,t}) \sigma^2_{p,I,t} \sigma^2_{p,U,t}};$$

$$\beta_{j,t} = \frac{c_A \sigma^2_{p,t} \left(h^\omega_{U,t} \sigma^2_{p,I,t} + h^\omega_I \sigma^2_{p,U,t} \right) \beta_{j,t+1}}{c_A \sigma^2_{p,t} \left(h^\omega_{U,t} \sigma^2_{p,I,t} + h^\omega_I \sigma^2_{p,U,t} \right) + (c_A + c_P)(h^0_I + h^0_{U,t}) \sigma^2_{p,I,t} \sigma^2_{p,U,t}}; \quad \forall j < t.$$

Notice that the main difference between $\beta_{t,t}$ and $\beta_{j,t}$ for all $j < t$ is due to the parameter $b_{P,t}$ that summarizes the precision of current price p_t to predict the next period price. If $t = T - 1$, the sensitivity factor $\beta_{j,t+1} = b_s$.

Parameters b_s and $b_{P,t}$ are equal to:

$$b_s = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\epsilon^2}; \qquad b_{P,t} = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\epsilon^2 + \gamma_t^2 \sigma_z^2}.$$

Finally, the supply shock parameter γ_t that in Section 3 was not affected by changes to c_P is now inversely related to the performance fee:

$$\gamma_t = \frac{\tau(c_A + c_P) R^{T-t-1} \sigma_{p,I,t}^2}{h_I^\omega \beta_{t,t+1}}.$$

The inverse relationship is determined by the effect that a higher c_P has on the sensitivity $\beta_{j,t}$ for all $j \in [1, t]$. Indeed, by increasing the performance fee, the sensitivity to signal decreases as it was for the static model too. Because future prices becomes less sensitive to signals, then the conditional variance $\sigma_{p,I,t}^2$ also decreases for all t since prices become more predictable. The combined effect of an increase in c_P to both the sensitivity factor and the conditional variance is an overall decrease of γ_t and an increase in the informational efficiency of the price.

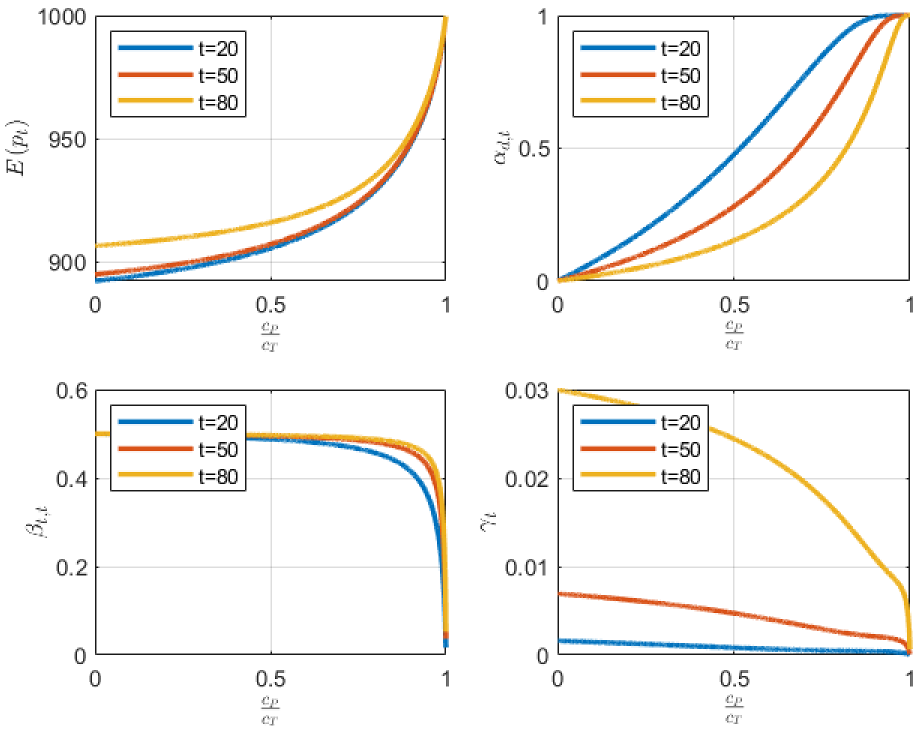

Figure 2: The figure plots the value of the unconditional expectation of the price, $\mathbb{E}(p_t)$, and price parameters $\alpha_{d,t}$, $\beta_{t,t}$, and γ_t to different values of c_P, such that $c_A = c_T - c_P$, and at different time periods. The model parameters used are $\sigma_\delta = \sigma_u = \sigma_z = \sigma_\epsilon = 0.5$, $d_0 = 10$, $\theta = 100$, $R = 1.05$, $c_T = 0.3$, $\tau = 2$, and $T = 101$.

Figure 2 shows the effect of a change in $\frac{c_P}{c_T}$ to the unconditional expectation of the price, $\mathbb{E}(p_t)$, and the parameters $\alpha_{d,t}$, $\beta_{t,t}$, and γ_t at different time periods. In order not to alter the risk-bearing capacity of fund managers determined by changes to c_T, any change in c_P is such that $c_A = c_T - c_P$ so that c_T is held constant.

The unconditional expectation of the price, $\mathbb{E}(p_t)$, naturally increases with time due to a decreasing discount required by risk-averse fund managers the closer they get to the payoff period T. A similar mechanism affects $\alpha_{d,t}$ which also becomes less sensitive to variation in c_P approaching time T. Instead, while the sensitivity factor $\beta_{t,t}$ shows a similar pattern as other parameters with respect to time, it also appears to be relatively stable for values of $\frac{c_P}{c_T}$ that are small. Finally, γ_t is now negatively related to c_P. Thus, the difference $\sigma^2_{p,U,t} - \sigma^2_{p,I,t}$ shrinks when c_P increases improving the efficiency of the price.

The combination of higher expected prices and inelastic sensitivity factors is a source of potential overvaluation of assets if a positive signal realizes. Suppose that at any time t, the difference $s_t - \overline{d}_t$ is positive and arbitrarily high due to a positive realization of ϵ_t. The price from time t onward reflects this optimistic signal because fund managers trade on that and rationally believe that $\tilde{d}_t$ will be high. If the performance fee is $c_P = 0$, the price is more likely to not be higher than $\tilde{d}_T$ in previous periods. If the performance fee $c_P > 0$, then $\alpha_{0,t}$ will also be higher and a potential loss could realize for managers that loaded on more risk to outperform their benchmark given the signal they observe. Conversely, if $c_P > 0$ and the difference $s_t - \overline{d}_t$ is negative, the higher incentives will support the price even if it should decrease.

4.2.2 Investors' beliefs

The dynamic setting of this section allows to investigate whether investors benefit from fund managers competing against a benchmark. In particular, I analyze the ability of investors to separate the two types of fund managers when the ratio $\frac{c_P}{c_T}$ increases.

The probability function defined in (21) is conditional on the difference between fund managers' portfolio allocation, $\xi_t = \omega_{I,t} - \omega_{U,t}$. Thus, if investors observe a large difference ξ_t, this is revealing of the types unless the precision of ξ_t is low. Ideally, a small $\sigma_{\xi,t}$ and a positive $\mathbb{E}(\xi_t)$ would that investors should expect the informed manager to invest more in the risky asset and a small difference from the mean is enough to reveal managers' type. Figure 3 shows the relationship between $\frac{c_P}{c_T}$ and the unconditional expectation of ξ_t, $\mathbb{E}(\xi_t)$, and the standard deviation $\sigma_{\xi,t}$.

Since fund managers anchor their portfolio to the benchmark when they are subject to higher competition incentives, the difference ξ_t shrinks and becomes more volatile. As expected, ξ_t is expected to become larger when time t approaches the payoff period T since the informed manager demand increases more than the uninformed when there exists a positive excess return for the risky asset. For the same reason, when $T - t$ is large, investors need to observe larger differences to tilt their beliefs in favor of one of the two managers.

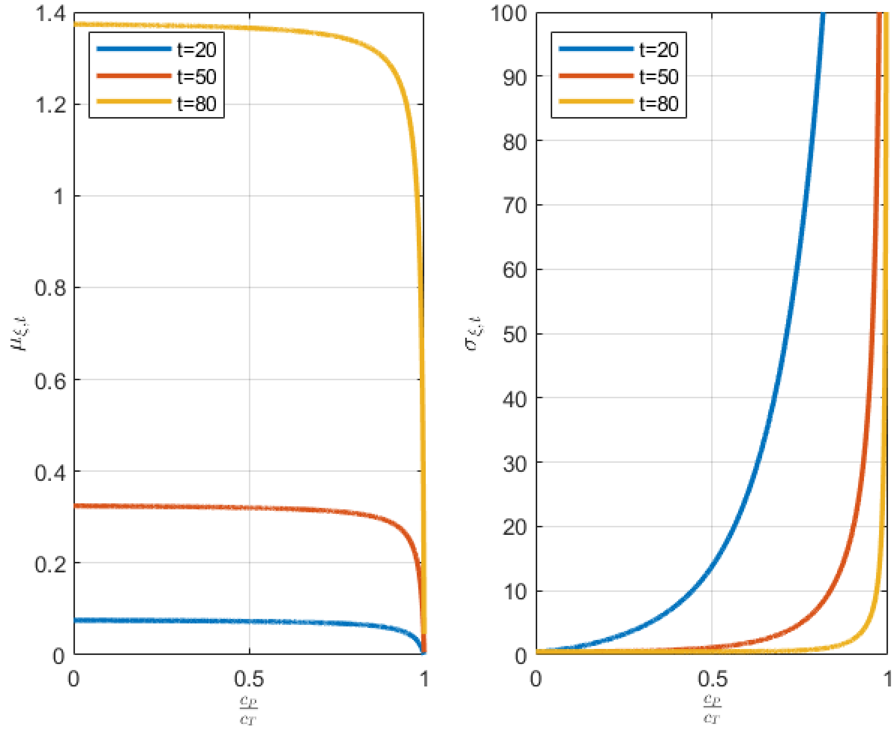

Figure 3: The figure plots the value of the unconditional expectation of the difference between fund managers' portfolio allocation, $\mathbb{E}(\xi_t)$, and its standard deviation, $\sigma_{\xi,t}$ to different values of c_P, such that $c_A = c_T - c_P$, and at different time periods. The model parameters used are $\sigma_\delta = \sigma_u = \sigma_z = \sigma_\epsilon = 0.5$, $d_0 = 10$, $\theta = 100$, $R = 1.05$, $c_T = 0.3$, $\tau = 2$, and $T = 101$.

Incentivizing fund managers to compete against a benchmark through higher performance fees hampers the ability of investors to separate the two types since managers tend to herd on the same asset with similar demand schedules.

5 Conclusion

This work contributes to the bubble literature in two key ways. First, it introduces a comprehensive framework that theorizes common empirical findings linking competition to asset managers' allocation choices. Second, it examines the role of competition incentives in price formation, their impact on informational efficiency, and the ability of investors to distinguish between better-informed fund managers. Crucially, these results are not driven by behavioral biases; fund managers rationally process the information they collect, trading with the objective of outperforming a benchmark and maximizing their compensation.

The findings challenge the conventional view supported by the Arrow-Debreu general equilibrium model that increased competition is always beneficial. Instead, the analysis shows that stronger competition incentives lead fund managers to trade more aggressively and anchor their portfolios to a benchmark. This behavior drives herding on the same risky asset, resulting in greater risk exposure and upward pressure on prices. Consequently, prices become less sensitive to signals, and investors struggle to differentiate between informed and uninformed fund managers.

However, this model does not aim to establish a "golden rule" for competition. It refrains from taking a position on the trade-offs between curbing excessive asset price growth, mitigating the impact of non-fundamental price shocks, and promoting informed trading. While this paper analyzes the prevalent contractual incentives—assets under management fees and performance fees—it does not propose a normative framework for socially optimal contractual incentives. Further research is needed to explore these aspects in greater depth.

Appendix

Projection theorem and conditional moments

In the static model of Section 3 and the dynamic model of Section 4 both the conditional expectation and the conditional variance of future payoffs can be computed by means of the projection theorem, since all random variables in the model are normally distributed.

In its simplest form, the projection theorem states that the expected value and the variance of a random variable $\tilde{x}$ conditional on $\tilde{y}$ is equal to:

$$\mathbb{E}(\tilde{x}|\tilde{y}) = \mathbb{E}(\tilde{x}) + \frac{\mathbb{C}ov(\tilde{x},\tilde{y})}{\mathbb{V}(\tilde{y})}\left[\tilde{y} - \mathbb{E}(\tilde{y})\right]; \quad \mathbb{V}(\tilde{x}|\tilde{y}) = \mathbb{V}(\tilde{x}) - \frac{\mathbb{C}ov^2(\tilde{x},\tilde{y})}{\mathbb{V}(\tilde{y})}.$$

For a more detailed explanation of the projection theorem and its application I refer to Mele (2022).

In Section 3, the variable of interest is $\tilde{d}$ and its moments are computed conditional on δ and the signal s, for the informed type, or the current price p, for the uninformed type. The informed fund manager computes:

$$\mathbb{E}(\tilde{d}|\mathcal{I}_I) = \overline{d} + \frac{\sigma_u^2}{\sigma_u^2+\sigma_\epsilon^2}(s-\overline{d}); \quad \mathbb{V}(\tilde{d}|\mathcal{I}_I) = \sigma_u^2 - \frac{\sigma_u^4}{\sigma_u^2+\sigma_\epsilon^2}; \tag{A.1}$$

While the uninformed fund manager:

$$\mathbb{E}(\tilde{d}|\mathcal{I}_U) = \overline{d} + \frac{\sigma_u^2}{\beta\left(\sigma_u^2+\sigma_\epsilon^2+\gamma^2\sigma_z^2\right)}(p-\alpha); \quad \mathbb{V}(\tilde{d}|\mathcal{I}_U) = \sigma_u^2 - \frac{\sigma_u^4}{\sigma_u^2+\sigma_\epsilon^2+\gamma^2\sigma_z^2}. \tag{A.2}$$

In Section 4, the variables of interest are $\tilde{d}_T$ and p_{t+1} for all $t \in [1, T-2]$. At any time t, the moments of the distribution are computed conditional on all previous realizations of $\{\delta_j\}_{j=1}^t$ and signals $\{s_j\}_{j=1}^t$, for the informed type, or the current price p_t and previous signals $\{s_j\}_{j=1}^{t-1}$, for the uninformed type.

Notice that because $\mathbb{C}ov(\tilde{d}_t, s_j) = 0$ for all $j < t$, the computation of the conditional moments is considerably easier. At time $T-1$, it is:

$$\mathbb{E}(\tilde{d}_T|\mathcal{I}_{I,T-1}) = \sum_{j=1}^{T-1}\left(\overline{d}_j + b_s(s_j - \overline{d}_j)\right); \tag{A.3}$$

$$\mathbb{V}(\tilde{d}_T|\mathcal{I}_{I,T-1}) = (T-1)\left(\sigma_u^2 - \frac{\sigma_u^4}{\sigma_u^2+\sigma_\epsilon^2+\gamma_{T-1}^2\sigma_z^2}\right); \tag{A.4}$$

$$\mathbb{E}(\tilde{d}_T|\mathcal{I}_{U,T-1}) = \sum_{j=1}^{T-1}\overline{d}_j + \frac{b_{P,T-1}R}{\beta_{T-1,T-1}}\left(p_{T-1} - \mathbb{E}(p_{T-1}|\mathcal{I}_{U,T-1})\right) + \sum_{j=1}^{T-2} b_s(s_j - \overline{d}_j); \tag{A.5}$$

$$\mathbb{V}(\tilde{d}_T|\mathcal{I}_{U,T-1}) = (T-1)\sigma_u^2 - (T-2)\frac{\sigma_u^4}{\sigma_u^2+\sigma_\epsilon^2} - \frac{\sigma_u^4}{\sigma_u^2+\sigma_\epsilon^2+\gamma_{T-1}^2\sigma_z^2}. \tag{A.6}$$

Where $\mathbb{E}(p_{T-1}|\mathcal{I}_{U,T-1})$ is:

$$\mathbb{E}(p_{T-1}|\mathcal{I}_{U,T-1}) = \frac{1}{R}\left[\alpha_{0,T-1} + \alpha_{d,t}\left(\sum_{j=1}^{T-1}\overline{d}_j - (T-1)\overline{d}_0\right) - \sum_{j=1}^{T-2}\beta_{j,T-1}(s_j - \overline{d_j})\right].$$

Instead, for time $t < T-1$ it is:

$$\mathbb{E}(p_{t+1}|\mathcal{I}_{I,t}) = R^{-(T-t-1)}\left[\alpha_{t+1} + \alpha_{d,t+1}\left(\sum_{j=1}^{t}\overline{d}_j - t\overline{d}_0\right) + \sum_{j=1}^{t}\beta_{j,t+1}(s_j - \overline{d}_j)\right]; \quad \text{(A.7)}$$

$$\mathbb{V}(p_{t+1}|\mathcal{I}_{I,t}) = R^{-2(T-t-1)}\left[(1+\alpha_{d,t+1})^2\sigma_\delta^2 + \beta_{t+1,t+1}^2(\sigma_u^2 + \sigma_\epsilon^2 + \gamma_t^2\sigma_z^2)\right]; \quad \text{(A.8)}$$

$$\begin{aligned}\mathbb{E}(p_{t+1}|\mathcal{I}_{U,t}) = R^{-(T-t-1)} &\left[\alpha_{t+1} + \alpha_{d,t+1}\left(\sum_{j=1}^{t}\overline{d}_j - t\overline{d}_0\right) + \right. \\ &\left. + \frac{b_{P,t}R}{\beta_{t,t}}(p_t - \mathbb{E}(p_t|\mathcal{I}_{U,t})) + \sum_{j=1}^{t-1}\beta_{j,t+1}(s_j - \overline{d}_j)\right]; \quad \text{(A.9)}\end{aligned}$$

$$\begin{aligned}\mathbb{V}(p_{t+1}|\mathcal{I}_{U,t}) = R^{-2(T-t-1)} &\left[(1+\alpha_{d,t+1})^2\sigma_\delta^2 + \beta_{t+1,t+1}^2(\sigma_u^2 + \sigma_\epsilon^2 + \gamma_t^2\sigma_z^2) + \right. \\ &\left. + \beta_{t,t+1}^2\left(\sigma_u^2 + \sigma_\epsilon^2 - \frac{(\sigma_u^2+\sigma_\epsilon^2)^2}{\sigma_u^2 + \sigma_\epsilon^2 + \gamma_t^2\sigma_z^2}\right)\right]. \quad \text{(A.10)}\end{aligned}$$

Where $\mathbb{E}(p_{T-1}|\mathcal{I}_{U,T-1})$ is:

$$\mathbb{E}(p_t|\mathcal{I}_{U,t}) = \frac{1}{R^{T-t}}\left[\alpha_{0,t} + \alpha_{d,t}\left(\sum_{j=1}^{t}\overline{d}_j - t\overline{d}_0\right) - \sum_{j=1}^{t-1}\beta_{j,t}(s_j - \overline{d_j})\right].$$

Derivation of the optimal demand of fund managers

Solving the maximization problem [P1] of the informed fund manager by conjecturing (8) leads to the following optimal demand:

$$\omega_I = \frac{\left(\mathbb{E}(\tilde{d}|\mathcal{I}_I) - pR + c_P\sigma_{d,I}^2\tau\omega^*\right)(c_A + c_P) - \lambda_1}{(c_A + c_P)^2\sigma_{d,I}^2\tau + 2\lambda_2}$$

The functional form of the optimal demand ω_I is known by investors, and the only value they do not observe is the conditional expectation $\mathbb{E}(\tilde{d}|\mathcal{I}_I)$. Since fund managers and investors maximize their expected utility sequentially, and investors can observe fund managers' allocation after they trade, they can retrieve $\mathbb{E}(\tilde{d}|\mathcal{I}_I)$ by inverting ω_I. Specifically, it is:

$$\mathbb{E}(\tilde{d}|\mathcal{I}_I) = \lambda_1 - (c_A + c_P)(c_P\sigma_{d,I}^2\tau\omega^* - pR) + \omega_I\left[(c_A + c_P)^2\sigma_{d,I}^2\tau + 2\lambda_2\right].$$

The above expression shows that if investors correctly plug in ω_I the portfolio allocation of the informed fund manager, they are able to reverse-engineer the conditional expectation of $\tilde{d}$ that the informed manager observes. Indeed, as shown in (A.1)-(A.2), the conditional variance are only a function of constant parameters that are known by investors. Therefore, for the informed investor it is $\mathbb{E}(\tilde{d}|\mathcal{I}_I) = \mathbb{E}(\tilde{d}|\mathcal{I}_I^v)$, while for the uninformed investor $\mathbb{E}(\tilde{d}|\mathcal{I}_I) \neq \mathbb{E}(\tilde{d}|\mathcal{I}_U^v)$ since she believes that the informed fund manager's demand is ω_U.

To verify conjecture (8) it is sufficient to solve for κ_0 and κ_ω the following system of two equations that is solvable in closed-form:

$$\begin{cases} \lambda_1 - (c_A + c_P)(c_P \sigma_{d,I}^2 \tau \omega^* - pR) - \kappa_0 = 0; \\ (c_A + c_P)^2 \sigma_{d,I}^2 \tau + 2\lambda_2 - \kappa_\omega = 0. \end{cases}$$

Once parameters κ_0 and κ_ω are computed, the optimal demand ω_I and ω_U can be derived by rearranging and collecting terms to match (12).

The same procedure to verify the above conjecture and derive the optimal demand for fund managers applies for the dynamic model of Section 4 too.

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