Dilutive Financing

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Motivation: why do firms hold cash reserves at all?

Fundamentally, an issue of financial market imperfections.

- Suppose expenses could be frictionlessly financed on demand and incrementally.
  1. Promptly spend any earnings in either investment or dividend payout.
  2. Exhaust cheaper internal funding[1] sources before tapping into costlier financing.[2]

'Financial slack': departure from 'pecking order.'

- Graham (2022): corporate managers across firm sizes cite financial flexibility as the primary factor in capital structure.
  - And its weakening as the main driver of underinvestment.

Important implications on corporate investment and stock returns.

[1] Most saliently, cash and cash equivalents; also, short-term debt and lines of credit.
[2] Long-term debt, equity issuance.

Why does financial slack arise?

At its core, classic problem of return dominance.

- Canonical explanation: Baumol (1952) and Tobin (1956)
  - Fixed transaction cost: high-return illiquid $\rightarrow$ low-return liquid.
  - Standard modeling tool to generate lumpiness.
- Needs 'stochastic' fixed cost to explain violation of pecking order.
  - Exogenous state dependence of exogenous model input...

This paper: alternative explanation with bargaining in financing.

- Bargaining $\implies$ rent extraction $\implies$ financing cost (or 'dilution')
- Lumpy financing to bargain infrequently.
- Early financing to strengthen bargaining position.
  - Financial flexibility reduces financing cost.

Financial slack is firms' costly bargaining tool against financiers.

Core mechanism

Two periods and a terminal date, no time discounting.

- A *farmer* ('she') owns a *crop*.
  - Each period, the crop needs a unit of *fertilizer* to survive.
  - In the terminal date, the farmer sells the crop for $\overline{v} > 2$.

- There are two *chemists* ('he'), each visiting her at each period, who can produce fertilizer.
  - Unit marginal cost of fertilizer production.
  - In bargaining, the farmer only retains $\theta \in (0, 1)$ fraction of surplus.

- The farmer has imperfect technology to store fertilizer.
  - Stored fertilizer decays down by a factor of $\beta \in (0, 1)$ in each period.

Core mechanism (I): lumpy purchase

Will the farmer choose to...

1. purchase a unit fertilizer from both chemists? Or...

	Period 1	Period 2	Terminal Date
Social	-1	-1	$\overline{v}$
Equity	$\theta\left(v_0^2 - 1\right)$	$v_0^2 := \theta\left(\overline{v} - 1\right)$	$\overline{v}$

2. purchase from the first chemist enough to sustain both periods?

	Period 1	Period 2	Terminal Date
Social	$-\left(1 + 1/\beta\right)$	0	$\overline{v}$
Equity	$\theta\left(\overline{v} - (1+1/\beta)\right)$	$\overline{v}$	$\overline{v}$

She chooses lumpy purchase (2) if second chemist's rent is greater than storage cost.

$$(1-\theta)(\overline{v} - 1) \geq \frac{1}{\beta} - 1$$

Core mechanism (II): early purchase

Now the farmer starts with a unit of fertilizer. When will she bargain?

1. Spend the inventory upfront and purchase from the second chemist?

$$v_0^2 = 0 + \theta\Big(\overline{v} - 0 - 1\Big).$$

   - Her outside option against the second chemist is loss of her crop.

2. Buy $1/\beta$ from the first chemist to skip second-period purchase?

$$v_0^2 + \theta\left(\overline{v} - v_0^2 - \frac{1}{\beta}\right).$$

   - Her outside option is bargaining with the second chemist.

She purchases early (2) if gain from better outside option outweighs her share of storage cost.

$$(1-\theta)\Big(v_0^2 - 0\Big) \geq \theta\left(\frac{1}{\beta} - 1\right)$$

General

Key predictions

1. Financial slack increases in 'price-to-earnings.'
   - Dilution as a fraction of surplus from averting damage to firm value.
   - Effect stronger with greater investment.

2. Early financing compresses the size of financing cost.
   - Distance to termination & backstop strategies improve firms' outside option at bargaining.
   - Arises even without precautionary motive against liquidity crisis.

Additional predictions

3. Robust access to financing $\implies$ 'excessive' financial slack, investment internally funded.
   - Reliance on concentrated financiers $\implies$ may finance investment despite sufficient funds, forgo investment with even more funds.
4. Business fundamentals matter critically in amplification of dilution when financing/capital market environments drastically deteriorate.
   - If firms can't find other financiers or sell off capital, early financing cannot boost outside option sufficiently. . .
   - unless both revenue *and* internal investment remain robust.

Main contribution

Conventional view on cash-holdings: Baumol (1952)-Tobin (1956)

- Fixed transaction cost of withdrawing from higher-yield sources.
- In application to corporate cash-holdings and equity financing,
  - Décamps, Mariotti, Rochet and Villeneuve (2011)
  - Bolton, Chen and Wang (2011, 2013)

This paper: bargaining induces cash-holdings and lumpiness.

- Tractable 'microfoundation' for fixed transaction costs.
- Early financing for non-precautionary purposes.
  - It may reduce financing cost, a novel direction of causality.

Empirics of corporate cash-holdings

- Opler, Pinkowitz, Stulz and Williamson (1999)
  - Firms with higher growth prospects hold more cash.
- Bates, Kahle and Stulz (2009)
  - Cash-holdings substantially increased 1980 through 2006 as firms became R&D intensive.
  - Agency frictions à la Jensen (1986) fail to explain the trend.
- Graham and Leary (2018)
  - Increased cross-sectional divergence in cash-holdings since 1980s.
  - Smaller firms and tech/health firms exhibit higher cash ratios.
- Graham (2022)
  - CFOs across firm sizes consider financial flexibility as the primary factor in capital structure.
  - Low current profitability & small cash-holdings drive decisions to reduce investment.

Other related literature

- Debt maturity management and early refinancing: Froot, Scharfstein and Stein (1993), Rampini and Viswanathan (2010), Mian and Santos (2018)
- Debt renegotiation: Hart and Moore (1998), Bolton and Scharfstein (1996)
- Capital structure/investment under dynamic contracting: DeMarzo and Fishman (2007a, 2007b), DeMarzo, Fishman, He and Wang (2012)
- Search friction in financing: Hugonnier, Malamud and Morellec (2014)
- Dynamic bargaining: McClellan (2024)
- Strategic conflicts between different classes of stakeholders: Myers (1977), Rajan (1992), Admati, DeMarzo, Hellwig and Pfleiderer (2018), DeMarzo and He (2021), Donaldson, Gromb and Piacentino (2020), Dangl and Zechner (2021)
- Investment irreversibility, financing friction, and productivity: Caggese (2007), Kurlat (2013), Lanteri (2018), Cui (2022)
- Bargaining in OTC markets and durability of a match: Duffie, Gârleanu and Pedersen (2005), Farboodi, Jarosch, Menzio and Wiriadinata (2019), Hendershott, Li, Livdan and Schürhoff (2020)

Overview

- Section 2: Model
  - Exogenous cash flow.
- Section 3: Investment Extensions
  - Endogenizes cash flow with investment.
  - Both stochastic/lumpy investment and smooth adjustment cost.

Model

Business

Continuous time, every agent risk-neutral, common discount rate $\rho > 0$.

- A *business* with underlying cash flow is owned by *shareholders*.
  - Cash flow has mean $\mu \in \mathbb{R}$ and volatility $\sigma \geq 0$

$$\mu \; dt + \sigma \; dB_t.$$

  - At $\lambda \geq 0$ Poisson rate, 'succeeds' with terminal payoff $\Pi \in \mathbb{R}$.
  - Exogenous cash flow i.e. 'Lucas tree': no investment choice for now.

Stylized examples

Cash flow examples:

- 'Startups': constant loss $-\kappa \; dt$ $(\kappa > 0,\ \sigma = 0)$.
  - Success, arriving at Poisson rate $\lambda > 0$, gives a terminal payoff $\Pi > \kappa/\lambda$.
- 'Operating firms': $\pi \; dt + \sigma \; dB_t$ $(\pi,\ \sigma > 0)$, and $\lambda = 0$.

Π is future upside potential, π is current cash flow profitability.

Internal funds and financing

- Business holds internal funds $h_t \geq 0$, from which *dividend* is paid.
  - h_t earns internal yield $r \in [0, \rho)$. $\rho - r > 0$ is the carry cost.
  - No friction for positive dividend. Negative dividend not allowed.
  - Zero funds without prompt financing: terminates with zero payoff.
- Shareholders are penniless, so they can finance only from deep-pocketed *financiers*.
  - Shareholders can choose the timing of financing.
- But financiers are not competitive 'price-takers.'
  - So they engage in Nash bargaining for financing.

Outside option

If shareholders can walk out from bargaining and immediately find other financiers, they have a *take-it-or-leave-it* offer, i.e. full bargaining power.

- Shareholders must wait a nonzero interval to find alternative financiers after walking out: call it *exclusion*.
  - Discrete time: bargain in the next period.
- Excluded shareholders are *re-included* into the financial market at Poisson rate $\gamma \in [0, \infty)$, i.e. stochastic duration of exclusion.
  - Tractability: keep track of just one more value function V, V_o.
  - γ parametrizes accessibility of alternative financing.

“Essentially a search friction, but only for off path. . . ?”

This is actually quite plausible. . .

1. CFOs forecast short-term cash flows and approach financial institutions in advance.
2. No double engagement to induce Bertrand competition.

Search friction is overcome, but its latency affects bargaining on path.

Interpretations

- Direct access to only a handful of specialized financiers (e.g. VCs).
- Concentrated investment banks syndicate dispersed investors.
  - Time lag of financing because of due diligence process.

Conservatism in modeling

- The $n+1^{\text{th}}$ alternative financiers are found at the same time lag γ and have the same funding cost ρ as the n^{th} alternative financiers.

Dividend payout and HJB equation

By risk-neutrality, optimal dividend policy is a payout threshold $\overline{h} \geq 0$.

- That is, pay $h_t - \overline{h} \geq 0$ only when $h_t \geq \overline{h}$.

When shareholders are inactive (i.e. neither financing nor paying dividend), their value function V satisfies the HJB equation

$$\rho V(h) - rhV'(h) = \lambda\Big(\Pi + h - V(h)\Big) + \mu V'(h) + \frac{1}{2}\sigma^2 V''(h).$$

Nash bargaining

Shareholders' *Nash bargaining weight* $\theta \in (0,1)$.

Let $V_o(h)$ their reservation value with funds h ($\implies V_o(0) = 0 < V(0)$).

Bargaining at h solves

$$\max_{\substack{\overline{h}\geq 0,\\ x\in[0,1]}} \left(xV(\overline{h}) - V_o(h)\right)^{\theta} \left((1-x)V(\overline{h}) - \left(\overline{h} - h\right)\right)^{1-\theta}$$

$$\implies \qquad \overline{h} \in \arg\max_{h\geq 0} V(h) - h, \quad \text{i.e. } V'(\overline{h}) = 1,$$

$$x(h)V(\overline{h}) = V_o(h) + \theta\Big(V(\overline{h}) - V_o(h) - (\overline{h} - h)\Big)$$

$$= \theta\Big(V(\overline{h}) - (\overline{h} - h)\Big) + (1-\theta)V_o(h).$$

- Funding target = dividend payout threshold.
- If optimal to finance at h, then $V(h) = x(h)V(\overline{h})$.
    - Recall: no search friction on path.

Outline

Illustration (I): stylized startup financing

Burn cash $\kappa\ dt$ until success arrives at rate λ with terminal payoff $\Pi > 0$.

- $\rho = 0.05,\ r = 0,\ \theta = 0.5,\ \gamma = 1,\ \kappa = 2,\ \lambda = 0.1,\ \Pi = 50.$

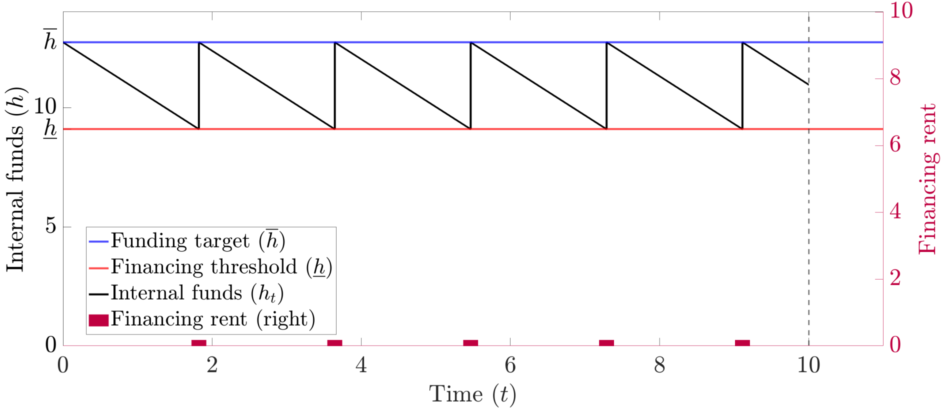
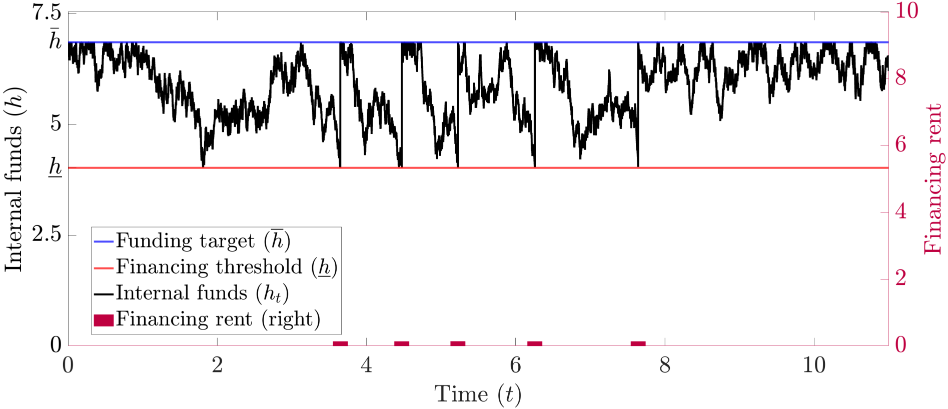


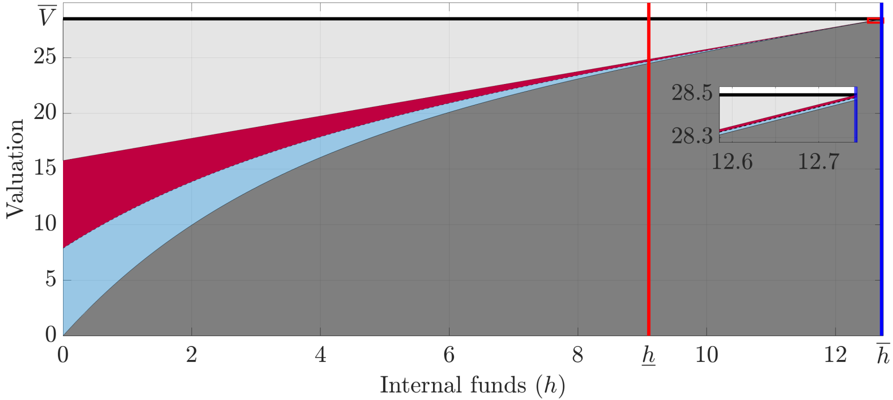
Illustration (II): stylized operating firm financing

Constant average profit with volatility $\pi\ dt + \sigma\ dB_t$.

- $\rho = 0.05,\ r = 0,\ \theta = 0.5,\ \gamma = 1,\ \pi = 1,\ \sigma = 2.$



Optimizing financing rent with slack

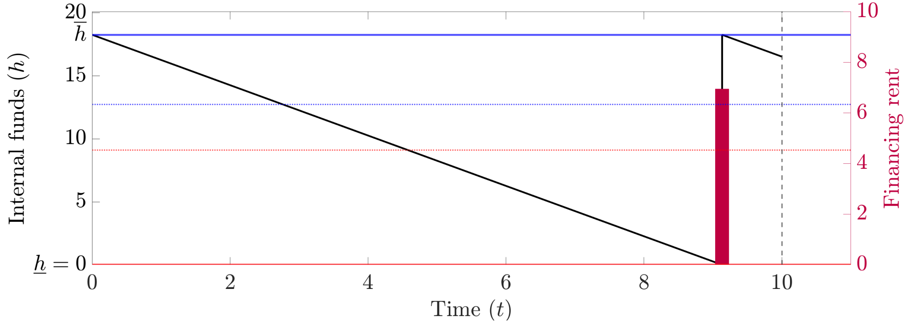


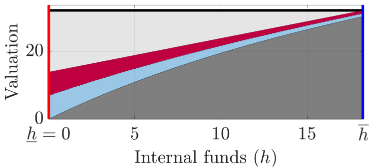
- Outside option rises steeply for low h. Financing amount falls one-to-one.
- Financing surplus $\left(= \text{Rent } 1-\theta + \text{Retention } \theta\right)$ shrinks steeply.

Analysis

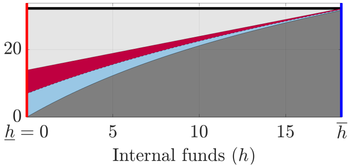
When NOT to finance early

Ineffective backstop strategy: $\gamma \leq \underline{\gamma}$ for some $\underline{\gamma} \in (0, \infty) \implies \underline{h} = 0$.





No alternative financing $\gamma = 0$



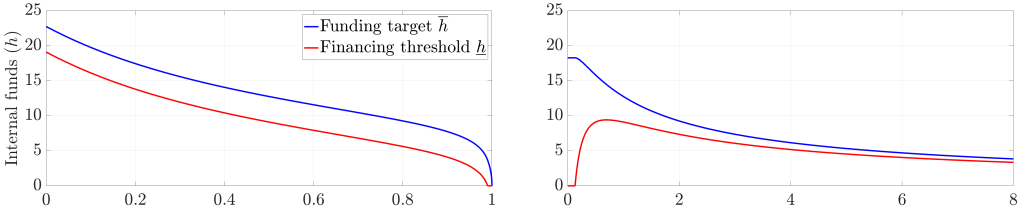
Breakeven accessibility $\gamma = \underline{\gamma}$

Analysis

Outline

Comparative statics in strategic parameters (θ, γ)

Proposition 3



(a) Nash bargaining weight (θ) **(b)** Alternative financing access (γ)

- $\gamma \to \infty$ equivalent to $\theta = 1$, where there is no financial slack.
- Sizable lumpiness even for little bargaining power by financiers:

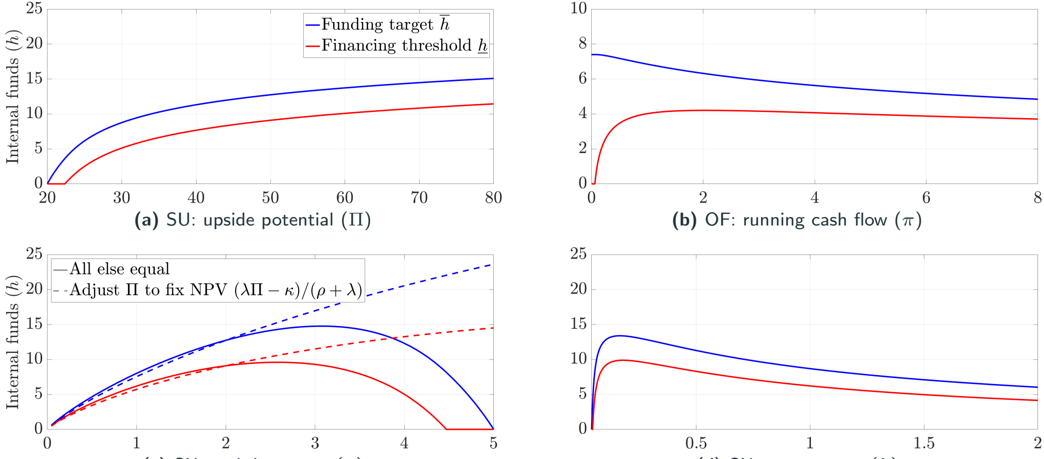
$$\lim_{\theta \to 1^-} \frac{\partial \overline{h}}{\partial \theta} = -\infty.$$

- No early financing with $\theta < 1$ high enough or γ low enough.
  - With high θ, dilution is already small enough.
  - With low γ, early financing does not improve outside option enough.

Comparative statics in business parameters

Compare two examples again: $\rho = 0.05,\ r = 0,\ \theta = 0.5,\ \gamma = 1.$

- Startup (SU): $\lambda = 0.1,\ \Pi = 50,\ \kappa = 2$ / Operating firm (OF): $\pi = 1,\ \sigma = 2$



(a) SU: upside potential (Π)

(b) OF: running cash flow (π)

(c) SU: cash burn rate (κ)

(d) SU: success rate (λ)

Current profitability vs future value

Π (startups): future value, π (operating firms): current profitability.

- $\Pi \uparrow$ raises financial slack despite no change in running cash flow.
  - Higher value $\implies$ dilution more painful.
- $\pi \uparrow$ additionally makes cash rundowns less likely.

Segue into investment—i.e. reducing π but raising Π.

Investment Extensions

Outline

Additional setup

Business has running cash flow: $\pi\ dt + \sigma\ dB_t$.

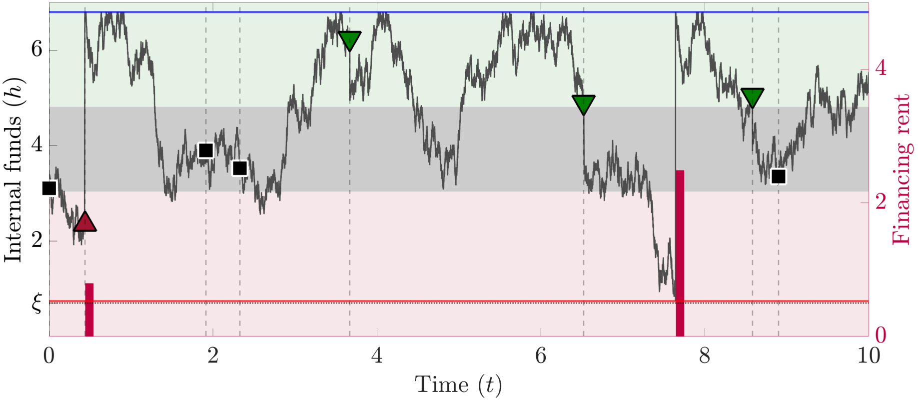
- Chance to scale up by $\eta > 1$ arrives at Poisson rate $\lambda > 0$.
  - Must pay $\xi > 0$ in upfront investment expense to scale up.
- Upon λ arrival, shareholders with h may
  1. **Fund investment** internally: $\eta V\left(\frac{h-\xi}{\eta}\right)$.
  2. **Forgo investment**: $V(h)$.
  3. **Finance investment** externally:

$$V_o(h) + \theta\left(\underbrace{\eta V(\overline{h}) - V_o(h) - \left(\eta\overline{h} + \xi - h\right)}_{\text{Financing \& investment surplus}}\right).$$

Outside option: exclusion & missing investment opportunity.

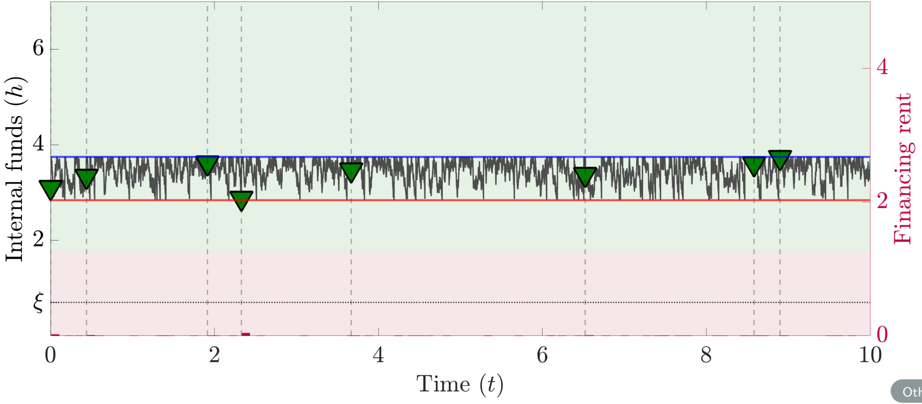
Three ways to handle opportunities

- Deep parameter: $\rho = 0.07,\ r = 0,\ \theta = 0.5,\ \gamma = 0.3.$
- Business parameters: $\pi = 1,\ \sigma = 2,\ \lambda = 0.5,\ \xi = 0.7,\ \eta = 1.1.$



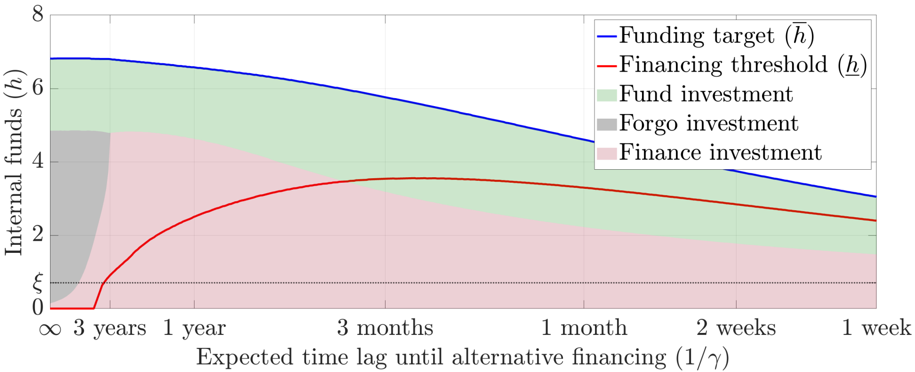
‘Mature’ firm $\gamma = 26$

- Can find alternative financiers just in two weeks.
- Always funds investment internally, high financing threshold.



Other examples

Comparative statics in ‘access to financing’ γ



- **Fixed-cost**: $\underline{h} \in \{0, \xi\}$, and $\underline{h} = \xi$ only if investment must be paid out of pocket first.
  - If not, $\underline{h} = 0$ and finance investment only when $h \leq \xi$.
- **Bargaining framework**: rationalizes $\underline{h} \gg \xi$ with $\gamma \gg 0$.
  - Robust financing access may induce ‘excessive’ financial slack.

Outline

'Off-the-shelf' firm technology

'AK' cash production technology with investment adjustment cost.

- **Production:** K_t produces volatile cash inflow of

$$(A\,dt + \sigma\,dB_t)\,K_t.$$

- **Investment:** $i_t K_t\,dt$ of flow investment incurs a convex adjustment cost $\Psi(i_t)K_t\,dt$. Capital stock evolves as $\frac{dK_t}{K_t} = \left(i_t - \delta\right)\,dt$. Let

$$\Psi(i) := \psi\frac{i^2}{2}, \quad \psi > 0.$$

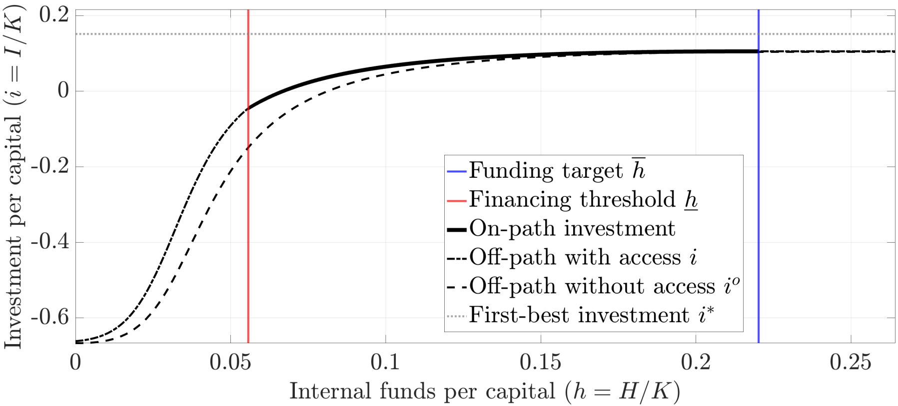
  - No explicit capital trades: needs a sufficiently frictional model.

- **Cash flow:** $dH_t = \Big(A - i_t - \Psi(i_t)\Big)K_t\,dt + \sigma K_t\,dB_t$.

Homogeneity in (K, H): $h := H/K$, $V(h)$ value per capital.

HJB

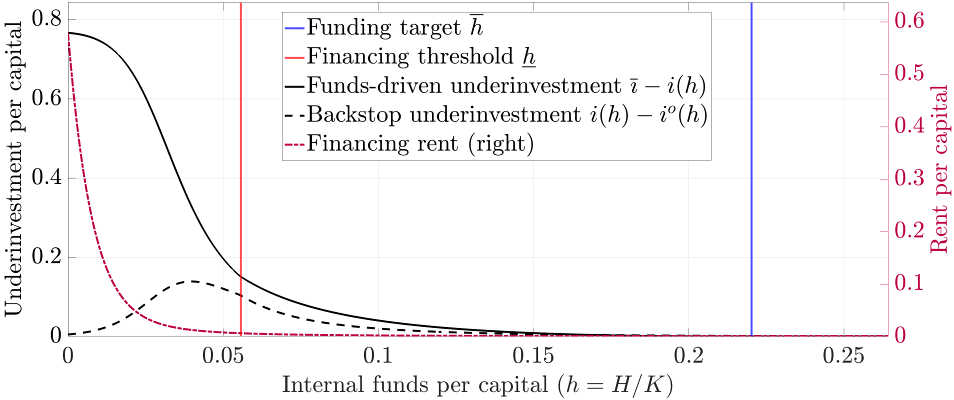
Strategic investment and early financing



Early financing $\underline{h} > 0$ even without alternative financing $\gamma = 0$. Why...?

- Efficiency motive (prevent extremely low investment at low h)?
- But it fails to deliver $\underline{h} > 0$ under fixed cost (see BCW 2011).

Strategic underinvestment and dilution

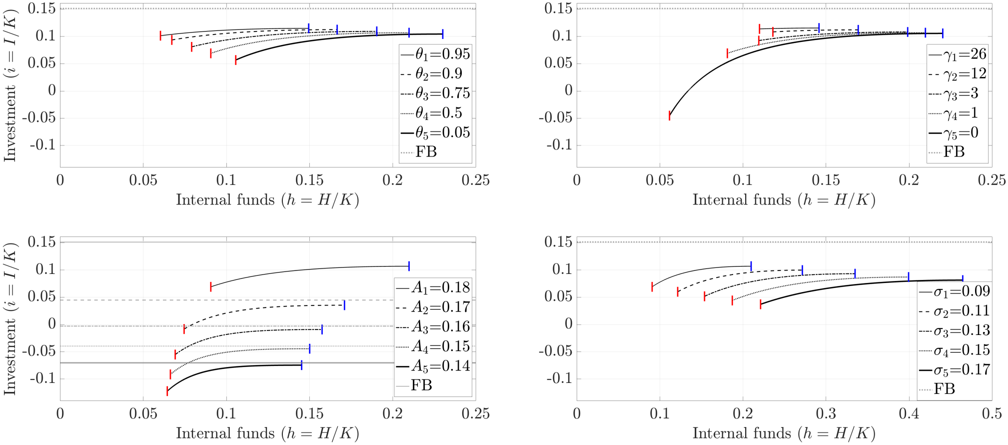


- On-path underinvestment increases financing surplus, hence dilution.
  - To reduce on-path underinvestment, reduce dilution. But how...?
- Backstop underinvestment reduces dilution.

Analysis Lumpy divestment

Comparative statics $(\theta, \gamma, A, \sigma)$

Baseline parameters: $\rho = 0.06,\ r = 0.05,\ \theta = 0.5,\ \gamma = 1,$
$A = 0.18,\ \delta = 0.1007,\ \sigma = 0.09,\ \psi = 1.5$. BCW 2011 except (θ, γ)



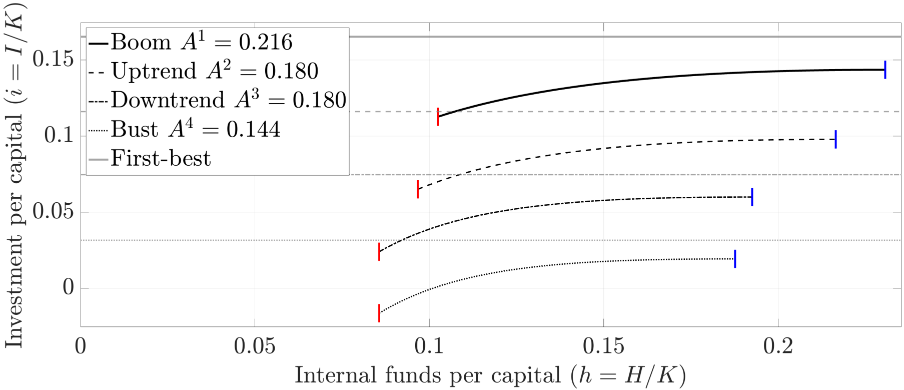
Fluctuating returns to investment

TFP fluctuates, as described by Markov transition in Poisson rates:

From\To	A^1	A^2	A^3	A^4	Distribution
$A^1 = 0.216$	$\cdot$	0.3	0	0	25%
$A^2 = 0.180$	0.3	$\cdot$	0.3	0	25%
$A^3 = 0.180$	0	0.3	$\cdot$	0.3	25%
$A^4 = 0.144$	0	0	0.3	$\cdot$	25%

- $A^2 = A^3$ has the same revenue, but A^2 merits increased investment.
  - $A^3 \to A^2$ raises upside potential with better investment returns. . .
  - but reduces net cash inflow due to greater investment.

Investment returns increase financial slack



Slack varies the most between $A^2 \leftrightarrow A^3$, the same current TPF. Compare

- Improved investment returns expedite financing/delay dividend...
  - even though firms are not cash-constrained.

Investment irreversibility and early financing

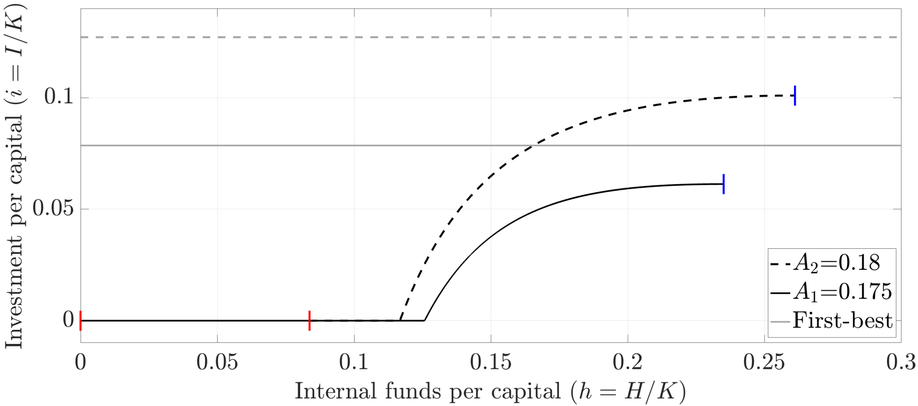
- Alternative financing and underinvestment boost outside option.
  - Underinvestment encompasses divestment.
- Divestment can be more difficult than investment—'irreversibility.'
  - Modify adjustment cost as follows:

$$\Psi_\phi(i) := \begin{cases} \psi \frac{i^2}{2}, & i \geq 0, \\ \frac{\psi}{\phi} \frac{i^2}{2}, & i < 0. \end{cases}$$

$\phi = 0$: divestment is prohibited, i.e. perfect irreversibility.

Small variation in A, a vast difference in $\underline{h}$

Suppose $\gamma = \phi = 0$, i.e. no alternative financing or divestment.



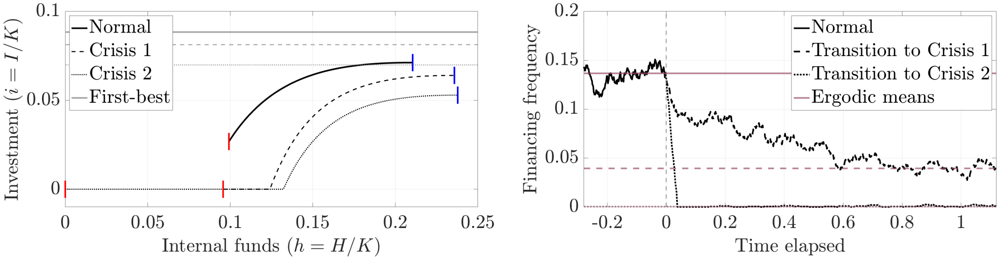
Underinvestment possible at high h, useful only with strong cash drift.

In-depth

Business fundamentals matter in amplification of dilution

Suppose that backstop strategies may become temporarily unavailable.

From\To	(γ^s, ϕ^s, A^s)	$s=0$	$s=1$	$s=2$
Normal: $s=0$	$(1, 0.5, 0.18)$	$\cdot$	0.1	0.1
Crisis 1: $s=1$	$(0, 0, 0.17)$	0.5	$\cdot$	0
Crisis 2: $s=2$	$(0, 0, 0.16)$	0.5	0	$\cdot$



Financing rent is 0.11%/0.13%/42.01% of $\overline{V}^s$ in $s = 0/1/2$.

Conclusion

Financial slack arises from bargaining in financing

Key predictions

- Continuation value amplifies dilution and increases financial slack.
  - The effect is stronger for firms that invest intensively.
  - Rationalizes why ‘growth’ firms hold more cash.
- Early financing compresses dilution endogenously.
  - Strengthens outside option with backstop strategy.
  - Non-precautionary: even without any risk of a spike in financing cost.

Additional predictions

- Robust financing access $\implies$ investment always internally funded.
  - Weak access $\implies$ may finance/forgo investment opportunities.
- No alternative financing & investment irreversibility may amplify dilution.
  - But strong revenue stream and high internal investment may prevent such amplification.

Bargaining at the heart of financial slack and financing dynamics.

Thank you!

Supplements

Lumpy financing

Let $B \subset [0, \overline{h}]$ the set of funds h where shareholders optimally finance.

Proposition 1 (Lumpy financing)

Financing is lumpy and intermittent, i.e. $\sup B < \overline{h}$.

Proof.

Suppose not. Then, from Nash bargaining

$$V(\overline{h}) = x(\overline{h})V(\overline{h}) = \theta V(\overline{h}) + (1-\theta)V_o(\overline{h}).$$

Since $\theta < 1$, $V(\overline{h}) = V_o(\overline{h})$, which contradicts $\gamma < \infty$ & $\theta > 0$. □

Back

Lumpiness arises from bargaining

Financiers' rent when shareholders finance at h is

$$(1-\theta)\Bigg[\underbrace{\Big(V(\bar{h})-V(h)-(\bar{h}-h)\Big)}_{Y}+\underbrace{\Big(V(h)-V_o(h)\Big)}_{Z}\Bigg].$$

Y: social surplus from financing, Z: cost of exclusion (i.e. no 'TIOLI')

- Financiers receive $(1-\theta)>0$ of not just Y but also Z.
  - As $h\to\bar{h}$, $(1-\theta)Z$ is bounded away from zero...
  - while the frequency of rent blows up to infinity. Basic
- Nontrivial bargaining: statically $\theta<1$ and dynamically $\gamma<\infty$.
  - If $\theta=1$, then no rent.
  - If $\gamma\to\infty$, then $Z\to 0$ for $h>0$ and $Y\to 0$, implying no rent.

$$\underbrace{V(h)=x(h)V(\bar{h})}_{\text{Optimally-timed financing}}=\underbrace{V_o(h)+\theta\Big(Y+Z\Big)}_{\text{Nash bargaining}}\to V(h)+\theta Y.$$

Back

Funding cushion and off-path backstop strategy

Lemma 2 (Monotone financing strategy)

$h \in B \implies [0, h] \subset B.$

- Equilibrium fully characterized by $(\bar{h}, \underline{h})$, where $\underline{h} := \sup B$.

Corollary 1

Given other parameters, there exists $\underline{\gamma} \in (0, \infty)$ such that $\underline{h} = 0 \iff \gamma \leq \underline{\gamma}$. In particular, $\gamma = 0$ always implies $\underline{h} = 0$.

- Weak backstop strategy $\gamma \leq \underline{\gamma} \implies$ no reason to finance early.
  - With investment choice, underinvestment is also a backstop strategy, so $\gamma = 0 \not\Longrightarrow \underline{h} = 0$.

Back

Idea behind Proof sketch: Bargaining-relevant comparison

Imagine shareholders at $h_t \in B \setminus \{0\}$ comparing immediate financing against a one-shot deviation of delaying financing by a dt instant.

- No risk of fund depletion due to delay because $h_t > 0$.
- Running cash inflow identical during the instant regardless of delay.
  - Essentially a parallel shift of the set of feasible payoffs.
- Three nontrivial changes relative to instantaneous delay:
  1. Variation in carry cost due to running cash inflow.
  2. Extra carry cost from earlier financing, $(\rho - r)(\overline{h} - h_t)\, dt$.
  3. Chance of instantaneous alternative financing, $\gamma\big(V(h_t) - V_o(h_t)\big)\, dt$.
- (1) vanishes $(dt + dB_t)\, dt$. So, consider (2) and (3) only.
  - Financing at h_t raises reservation value by (3) and lowers bargaining surplus by (2)+(3); shareholders only bear θ of the surplus reduction.
  - Therefore, finance immediately at $h_t = h > 0$ if

$$(1-\theta)\gamma\big(V(h) - V_o(h)\big) \geq \theta(\rho - r)(\overline{h} - h).$$

Basic

Back

Costs and benefits of financial slack

Net equity value: $\boxed{V(\overline{h}) - \overline{h} = \frac{\text{NPV} - \mathcal{C}}{1 + \mathcal{D}}}$ where $\text{NPV} := \frac{\mu + \lambda \Pi}{\rho + \lambda}$

Carry cost $\mathcal{C} := \underline{\mathcal{C}} + \mathcal{C}_\Delta$ with $\underline{\mathcal{C}} := (\rho - r)\frac{\underline{h}}{\rho + \lambda}$

and $\mathcal{C}_\Delta := (\rho - r)\mathbb{E}_0\left[\int_0^\tau e^{-\rho t}(h_t - \underline{h})\, dt\right]$

Dilution $\mathcal{D} := (1 - \underline{x})\mathbb{E}_0\left[\sum_{m=1}^{n_\tau} e^{-\rho \tau_m}\right]$

τ: time of terminal 'success,' at rate λ.

n_τ: total number of financing, τ_m: m^{th} financing time.

- $\underline{h}$ reduces size $1 - \underline{x}$ & $\Delta h := \overline{h} - \underline{h}$ reduces frequency $\mathbb{E}\left[\sum e^{-\rho \tau_m}\right]$.
- $\underline{h}$ incurs fixed carry cost $\underline{\mathcal{C}}$ & Δh incurs variable carry cost $\mathcal{C}_\Delta$. $\frac{\partial \underline{\mathcal{C}}}{\partial \underline{h}} = \frac{\rho - r}{\rho + \lambda} > \frac{\partial \mathcal{C}_\Delta}{\partial \Delta h}$.

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Early financing and rent

Proposition 2

$\underline{h} > 0$ if and only if

$$(1-\theta)\gamma > \frac{(\rho - r)\overline{h}}{V(\overline{h}) - \overline{h}},$$

in which case

$$\gamma\big(V(\overline{h}) - V(\underline{h}) - \Delta h\big) = (\rho - r)\Delta h.$$

- Finance early only when bargaining-adjusted effectiveness of backstop strategy $(1-\theta)\gamma$ is higher than the carry cost burden.
  - More concretely, $\underline{h} > 0$ if and only if

$$\frac{(1-\theta)\gamma}{\rho + \lambda + (1-\theta)\gamma} > \frac{(\rho - r)\overline{h}}{\mu + \lambda\Pi}.$$

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Early financing and rent (cont'd)

- When financing early, shareholders pay $\frac{\rho-r}{\gamma}\Delta h$ as optimized rent.

$$\frac{\rho-r}{\gamma}\Delta h = V(\bar{h}) - V(\underline{h}) - \Delta h \; = \underbrace{(1-\underline{x})V(\bar{h})}_{\text{Gross compensation}} \; - \underbrace{\Delta h}_{\text{Fund cost}}$$

  - Immediate financing reduces financing rent by a factor of $\theta\gamma \; dt$:

$$V(\underline{h}) = \theta\big(V(\bar{h}) - \Delta h\big) + (1-\theta)V_o(\underline{h})$$
$$\Longrightarrow (1-\theta)\gamma \underbrace{\big(V(\underline{h}) - V_o(\underline{h})\big)}_{\text{Cost of exclusion}} \; dt = \theta\gamma \underbrace{\big(V(\bar{h}) - V(\underline{h}) - \Delta h\big)}_{\text{Financing rent}} \; dt.$$

  - Shareholders' carry cost burden from immediate financing is $\theta(\rho - r)\Delta h \; dt$.

Optimal interior financing threshold $\underline{h} > 0$ equalizes these margins.

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Comparative statics in strategic parameters (θ, γ)

Proposition 3

1. *$\overline{h}$ decreases in θ and $\gamma \geq \underline{\gamma}$.*
2. *$\underline{h}$ decreases in θ when $\underline{h} > 0$. $\underline{h} = 0$ for θ sufficiently high.*
3. *If $r = 0$, Δh is constant in θ when $\underline{h} > 0$ and decreasing in $\gamma \geq \underline{\gamma}$.*
   If $r \in (0, \rho)$, Δh is increasing in θ when $\underline{h} > 0$.
4. *$\overline{h} \to 0$ as either $\theta \to 1$ or $\gamma \to \infty$.*

- Total slack $\overline{h} = \underline{h} + \Delta h$: decreasing in (θ, γ).
- Funding cushion $\underline{h}$: decreasing in θ, non-monotonic in γ.
- Buffer stock Δh: constant in θ (when $\underline{h} > 0$), decreasing in $\gamma \geq \underline{\gamma}$.
    - Some subtlety when $r > 0$.

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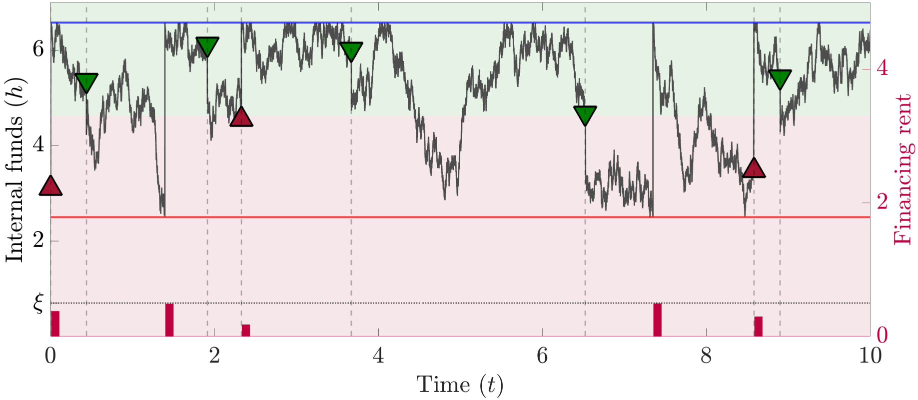
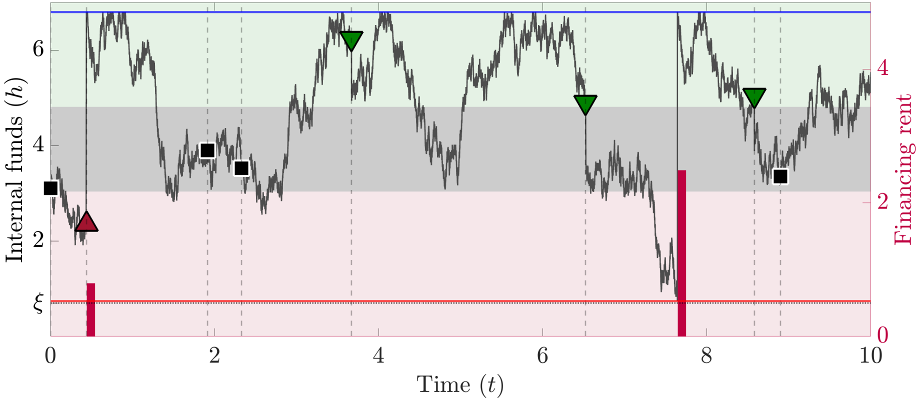
‘Regular’ firm $\gamma = 1$

- Can find alternative financiers in one year.
- Often finances investment, dilution no longer negligible.



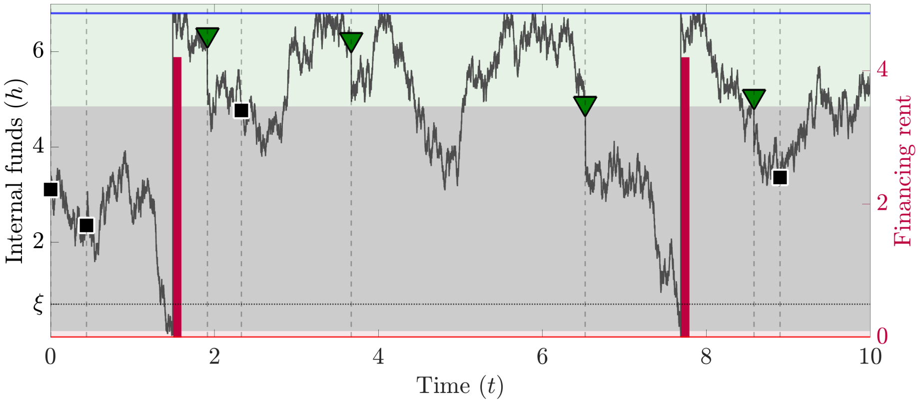
‘Small’ firm $\gamma = 0.3$

- Can find alternative financiers in three years.
- Forgoes investment often.



‘Distressed’ firm $\gamma = 0$

- There are no alternative financiers.
- Forgoes a lot of investments to avoid large dilution.



Back

Per-capital shareholder HJB

Equity value W homogeneous in (K, H). Define $h := H/K$ and $V(A, h) := W(A, K, H)/K$. Under financial inactivity, V solves

$$
\begin{aligned}
&\rho V - rhV_h \\
&= \left(A + \frac{1}{2\psi}\right) V_h + \frac{1}{2}\sigma^2 V_{hh} - \left(\delta + \frac{1}{2\psi}\right)\left(V - hV_h\right) \\
&\qquad + \underbrace{\frac{1}{2\psi}\left(\frac{V}{V_h} - h - 1\right)\left(V - hV_h\right)}_{=:\mathcal{K}(V)} + \mathcal{A}(V).
\end{aligned}
$$

- $\mathcal{K}(V) = \frac{1}{2} i W_K$ is non-linear, reflecting optimized investment.
  - $1/2$ adjusts for the quadratic cost given optimal $i = i(A, h)$.

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Financial slack and underinvestment

Strategic link between underinvestment and early financing.

Proposition 5

Pointwise for every A, and suppressing notation for its dependence,

$$\underline{h} > 0 \iff (1-\theta)\gamma + \frac{1}{2}\underbrace{\left(\bar{\imath} - i(0)\right)}_{(a)} > \frac{(\rho - r)\overline{h}}{\overline{V} - \overline{h}}$$

$$\iff \theta\gamma\left(\overline{V} - \underline{V} - \Delta h\right) + \frac{1}{2}\theta\underbrace{\left(\bar{\imath} - \underline{i}\right)}_{(b)}\underbrace{\left(\overline{V} - \overline{h}\right)}_{=\overline{W}_K}$$

$$= \theta(\rho - r)\Delta h + \frac{1}{2}(1-\theta)\underbrace{\left(\underline{i} - \underline{i}^o\right)}_{(c)}\underbrace{\left(\underline{V}^o - \underline{hV}^o_h\right)}_{=\underline{W}^o_K}.$$

- $(a), (b)$: funds-driven underinvestment
- (c): backstop underinvestment

Optimal early financing threshold $\underline{h} > 0$ with investment

Compare immediate financing at t against instantaneous delay by dt.

Marginal benefits

- $\theta\gamma\left(\overline{V} - \underline{V} - \Delta h\right)$
  - Instantaneous alternative access reduces rent by a factor of $\theta\gamma\ dt$.
- $+\frac{1}{2}\theta\left(\bar{\imath} - \underline{i}\right)\overline{W}_K$
  - Shareholders' θ portion of higher instantaneous investment returns.

$=$ **Marginal costs**

- $\theta(\rho - r)\Delta h$
  - Shareholders' θ portion of instantaneous carry cost.
- $+\frac{1}{2}(1-\theta)\left(\underline{i} - \underline{i}^o\right)\underline{W}^o_K$
  - Less capital at bargaining reduces reservation value, causing shareholders loss by $1-\theta$ factor.

$\underline{h} > 0$ equalizes marginal benefits with marginal costs.

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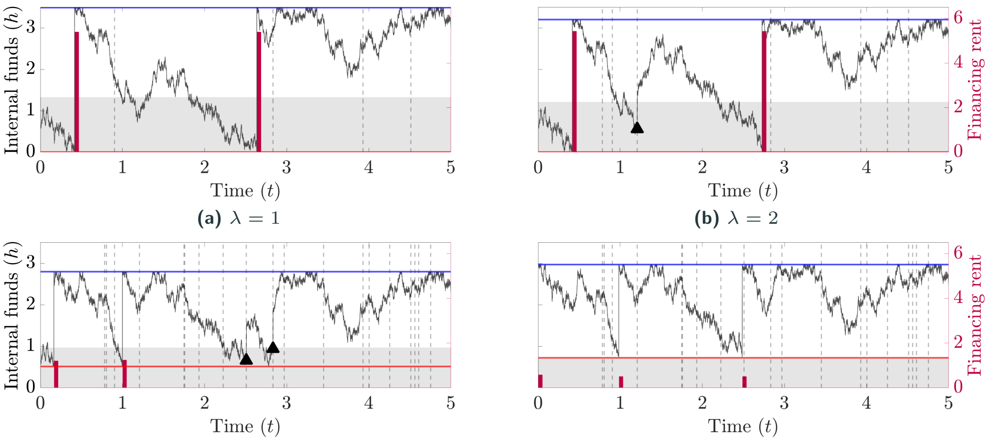
Lumpy divestment without alternative financing access

Consider a stochastic opportunity for lumpy (inefficient) divestment.

$\rho = 0.07,\ r = 0,\ \theta = 0.5,\ \gamma = 0,\ \pi = 1,\ \sigma = 1.2,\ \xi = -0.7,\ \eta = 0.9$



(a) $\lambda = 1$ **(b)** $\lambda = 2$ **(c)** $\lambda = 6$ **(d)** $\lambda = 12$

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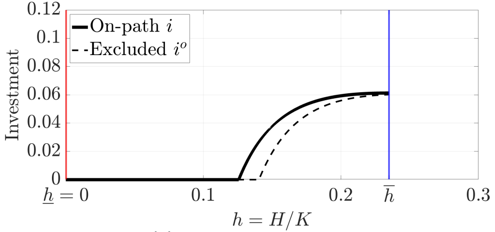
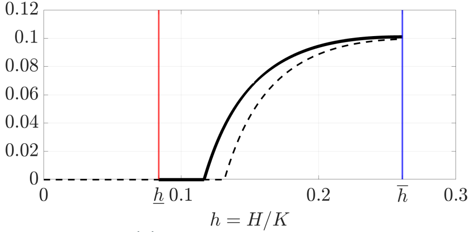
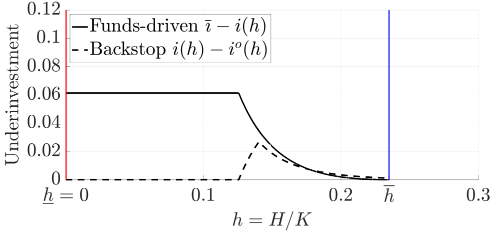
Productivity and financing threshold given $\gamma = \phi = 0$



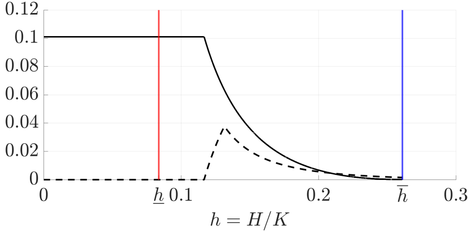
(a) Investment, $A = 0.175$



(b) Investment, $A = 0.18$



(c) Underinvestment, $A = 0.175$



(d) Underinvestment, $A = 0.18$

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Proof sketch for Lemma 2 and Corollary 1

Suppose $B \neq \{0\}$. Absence of immediate search friction implies: $\forall h \in B$,

$$V(h) = \theta\Big(V(\overline{h}) - \overline{h} + h\Big) + (1-\theta)V_o(h). \tag{1}$$

Since immediate financing is better than instantaneous delay on B,

$$\rho V(h) - rhV'(h) \geq \mathcal{H}(V)(h) \quad \forall h \in B, \tag{2}$$

where $\mathcal{H}(V)(h) := \lambda\big(\Pi + h - V(h)\big) + \mu V'(h) + \frac{1}{2}\sigma^2 V''(h)$. Note that

$$\rho V_o(h) - rhV_o'(h) = \mathcal{H}(V_o)(h) + \gamma\Big(V(h) - V_o(h)\Big), \tag{3}$$

$$\rho V(\overline{h}) - r\overline{h} = \mathcal{H}(V)(\overline{h}). \tag{4}$$

(1) and linearity of $\mathcal{H}$ give $\mathcal{H}(V)(h) = \theta\mathcal{H}(V)(\overline{h}) + (1-\theta)\mathcal{H}(V_o)(h)$. Substituting (1), (3), (4) into (2) cancels out $\mathcal{H}(V)(h)$, giving: $\forall h \in B$,

$$(1-\theta)\gamma\Big(V(h) - V_o(h)\Big) \geq \theta(\rho - r)\Big(\overline{h} - h\Big).$$

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