Stable Market Segmentation against Price Discrimination

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Motivation

Online Price Discrimination: Online sellers.

- They can generate a sequence of tags by data inference to denote the features of consumers.
  - Preference, taste, etc.
- They claim that these tags can be used to improve their services, such as accurately recommending the goods.

Regulations on Data Protection:

- *General Data Protection Regulation* (GDPR), EU, 2018
  - Grants consumers the right to rectify their data. (Art.16)
  - Grants consumers the right to erase their data. (Art.17)
    - Also known as **Right to Be Forgotten**.
- *Internet Information Service Algorithmic Recommendation Management Provisions*, China, 2022
  - Allows consumers to delete their tags after registration.
  - Once allows consumers to change their tags after registration.

Research Questions:

- A monopolist producer can charge different prices in different markets that are divided by tags. (**3rd Price Discrimination**)
- **Strategic** consumers can manipulate their tags in an online environment.
  - Non-cooperative GT + Collusion $\Longrightarrow$ **Stable Market Segmentation**
- The producer sets an optimal price in each market.
- Can consumers fight against a monopolist producer?
  - Is it necessary for consumers to stand united?
- What are those market equilibria look like? (Preliminary)
- What are the possible welfare consequences? (**Core**)
  - The limits of price discrimination (BBM2015@AER) + Strategic and mobile consumers
  - Social-optimal? Buyer-optimal?
  - Anyone is worse off compared with uniform monopoly?

Model

Basic model:

- The producer sells homogeneous products to a continuum of consumers.
  - Homogeneous products.
  - Unit demand.
- Consumers' reservation price can take values from a finite set $\{v_1, v_2, \cdots, v_K\}$ with $0 < v_1 < v_2 < \cdots < v_K$.
- Constant marginal cost, which can be normalized to zero.
- The producer and all consumers can learn value distributions in each market.

Market Segmentation:

- A market (segment) can be represented by a vector $\mathbf{x} = (x_1, \cdots, x_k, \cdots, x_K)$, where $x_k \geq 0$ is the proportion of consumers with reservation price v_k.
- There is an **aggregate market**:

$$\mathbf{x}^* = (x_1^*, \cdots, x_k^*, \cdots, x_K^*)$$

  where $\|\mathbf{x}^*\|_1$ is normalized to 1.
- A segmentation of the aggregate market, denoted by $\sigma(\mathbf{x}^*)$, is a collection (possibly not a set) of segments $\{\mathbf{x}_1, \cdots, \mathbf{x}_t\}$ such that $\sum_{i=1}^{t} \mathbf{x}_i = \mathbf{x}^*$.

Pricing & Surplus (Baseline):

- **Third-degree Price Discrimination**. (i) In each market segment, the producer offers a take-it-or-leave-it price. (ii) Each consumer will buy the product if the price is NO LARGER THAN his reservation price.
- The price v_i is optimal for a given market $\mathbf{x}$ *iff*

$$v_i \sum_{j \geq i} x_j \geq v_k \sum_{j \geq k} x_j, \quad \forall k.$$

Model

- If there are multiple optimal prices in one segment, the producer will take the minimum optimal one, which favors consumers the most. $\phi^{\min}(\mathbf{x})$ denotes the minimum optimal price for market segment $\mathbf{x}$.
- The surplus of producer is defined as

$$\sum_{\mathbf{x} \in \sigma(\mathbf{x}^*)} \underbrace{\phi(\mathbf{x}) \sum_{j: v_j \geq \phi(\mathbf{x})} x_j}_{\text{Profit in } \mathbf{x}}$$

- The surplus of consumers is defined as

$$\sum_{\mathbf{x} \in \sigma(\mathbf{x}^*)} \underbrace{\sum_{j: v_j \geq \phi(\mathbf{x})} (v_j - \phi(\mathbf{x})) x_j}_{\text{Surplus in } \mathbf{x}}$$

- The social welfare is defined as,

$$\sum_{\mathbf{x} \in \sigma(\mathbf{x}^*)} \underbrace{\sum_{j: v_j \geq \phi(\mathbf{x})} v_j x_j}_{\text{Welfare in } \mathbf{x}}$$

- **Uniform Monopoly** A. The producer sets a uniform price to all consumers. (NO market segmentation and NO price discrimination)
  - Uniform monopoly price is $\phi^{\min}(\mathbf{x}^*)$.
  - Producer surplus is denoted by $\pi^* = \phi^{\min}(\mathbf{x}^*) \sum_{j: v_j \geq \phi(\mathbf{x}^*)} x_j$.
  - Consumer surplus is denoted by $u^* = \sum_{j: v_j \geq \phi^{\min}(\mathbf{x}^*)} (v_j - \phi^{\min}(\mathbf{x}^*)) x_j$.

Tag-editable Framework

Timeline:

1. **Strategic consumers choose their tags simultaneously.** Alternatively, a producer/mediator designs a market segmentation, which should be robust to strategic consumers.
2. The producer chooses the **minimum** optimal price. (Robust)

Stable Segmentation

A segmentation $\sigma(\mathbf{x}^*) = \{\mathbf{x}_1, \cdots, \mathbf{x}_t\}$ is **stable**, if for any group of consumers $\mathbf{y}$, there is no decomposition $\mathbf{y} = \sum_{i=1}^{t} \mathbf{y}_i'$ such that all consumers in $\mathbf{y}$ have **strictly** higher utility in the segmentation $\{\mathbf{x}_1 - \mathbf{y}_1 + \mathbf{y}_1', \cdots, \mathbf{x}_t - \mathbf{y}_t + \mathbf{y}_t'\}$ than in $\sigma(\mathbf{x}^*)$.

- We require each group to have a positive measure.
- We assume any group of consumers cannot build a new market. This assumption is not a loss of generality since the consumer with the lowest valuation within the group must have zero utility by establishing a new market.

Weak-Stable Segmentation: A segmentation $\sigma(\mathbf{x}^*) = \{\mathbf{x}_1, \cdots, \mathbf{x}_t\}$ is **weak-stable**, if for a **small** group of consumers **with the same valuation** in market $\mathbf{x}_i$, it is not profitable for them to deviate to any other market.

- "Small" indicates the measure of them is positive but arbitrarily close to zero.
- A relaxed concept that facilitates our analysis.
- Manifest individual deviation scenario. Since individual in real world has small but non-negligible market share.

Verification Condition

Weak-stable verification: If $\phi^{\min}(\mathbf{x}_i) < \phi^{\min}(\mathbf{x}_j)$,
$v \in \mathbf{supp}\{\mathbf{x}_j\} \cap \left(\phi^{\min}(\mathbf{x}_i), \phi^{\min}(\mathbf{x}_j)\right]$,
v should be optimal in market $\mathbf{x}_i$.

Stable: no-inflow condition: The segmentation $\sigma(\mathbf{x}^*)$ is stable **iff** the following **no-inflow** condition holds: For any market $\mathbf{x}_i \in \sigma(\mathbf{x}^*)$, there is not a group of consumers $\mathbf{y} \neq \mathbf{0}$ from other markets such that all consumers in $\mathbf{y}$ have strictly higher utility in market $\mathbf{x}_i + \mathbf{y}$ than before.

Welfare Consequences

Main Theorem

The surplus of the producer and consumers (π, u) can be achieved by a **stable segmentation iff** $\pi = \pi^*$ and $u \in [u^*, w^* - \pi^*]$. No consumer is worse off compared with uniform monopoly.

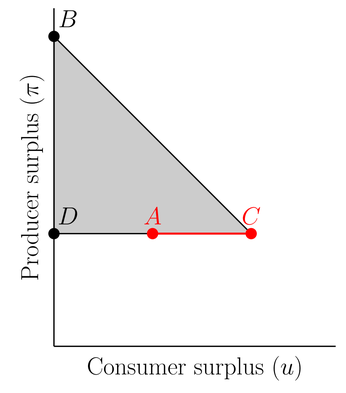

Figure 1:Surplus Triangle

Further Analysis

Stable at the buyer-optimal outcome:

- Stable and weak-stable are equivalent definitions.
- Full characterization of **all** stable, social-optimal, and direct (SSD) segmentations with geometrical characterization.
- For a SSD segmentation, $\sigma(\mathbf{x}^*) = \{\mathbf{x}_1, \cdots, \mathbf{x}_t\}$.
  - Price profile: $\{\phi^{\min}(\mathbf{x}_1), \cdots, \phi^{\min}(\mathbf{x}_t)\}$
  - Revenue profile: $\{\pi_1, \cdots, \pi_t\}$, π_i is the revenue of market $\mathbf{x}_i$.
- For SSD segmentation, revenue profile and price profile are identical which are characterized as following figures. We define revenue function:

$$\hat{\pi}(v_i) = v_i \sum_{j=i}^{K} x_j^*$$

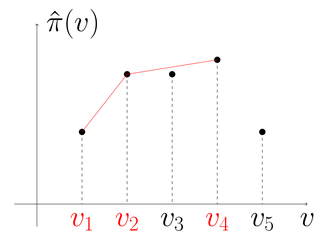

Figure 2:Price Profile

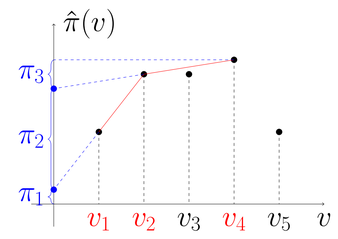

Figure 3:Revenue Profile

Robustness:

- Relaxing minimum pricing rule, i.e. any **ex-post rational** pricing rule.
  - Producer surplus is fixed at the uniform monopoly level.
  - Consumer surplus is at least at the uniform monopoly level (point A) but the upper bound may shrink.
  - No consumer is worse off compared with uniform monopoly.

Policy Implications

1. Release prohibitions on price discrimination.
   - Enabling price discrimination is **Pareto-improving**.
2. Empower consumers with more freedom to edit their tags.
   - Free circulation is desirable.
3. The Right to be **partially** Forgotten should be mandated.
   - First-degree price discrimination outcome is stable if only the Right to be **entirely** Forgotten is enforced.
4. Promoting frictionless second-hand markets may be harmful.
   - Prevent ex-post arbitrage, which originally protects consumers.
5. Data brokers help solve the equilibrium selection problem.
   - A mediator is helpful in selecting the best equilibrium.