Stock Returns, Market Trends, and Information Theory:

A Statistical Equilibrium Approach

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Introduction

The search for serial correlations in stock returns has been one the main criteria to assess market efficiency (Fama, 1965; Bhowmik and Wang, 2020).

Three main issues associated with serial correlations:

1. Sample size and power of econometric test (Shiller, 2015).
2. Sources of statistical dependence and randomness (Fama, 1965).
3. Informational component of stock prices (Mantegna and Stanley, 1999).

Objectives

- Analyzing the **statistical regularities** of stock returns through an **entropy-constrained statistical equilibrium model**.
- Explaining **randomness in stock prices** as the result of **unintended consequence of investors** seeking higher rates of return.
- Providing an **original assessment of the Efficient Market Hypothesis** by considering the role of **unfulfilled expectations of investors**, and how they impact stock market volatility.

Data Collection

S&P 500 individual companies' daily adjusted stock prices (Yahoo Finance), from which we compute daily logarithmic returns:

$$r_{i,t} = \log[p_{i,t}] - \log[p_{i,t-1}] \tag{1}$$

Sample:

- January 1, 1988 - December 31, 2019.
- Observations: 3,004,150.
- Divided into **bull markets, bear markets, and corrections.**

We consider the **cross-sectional distributions** of individual companies' returns, and then analyze their statistical regularities.

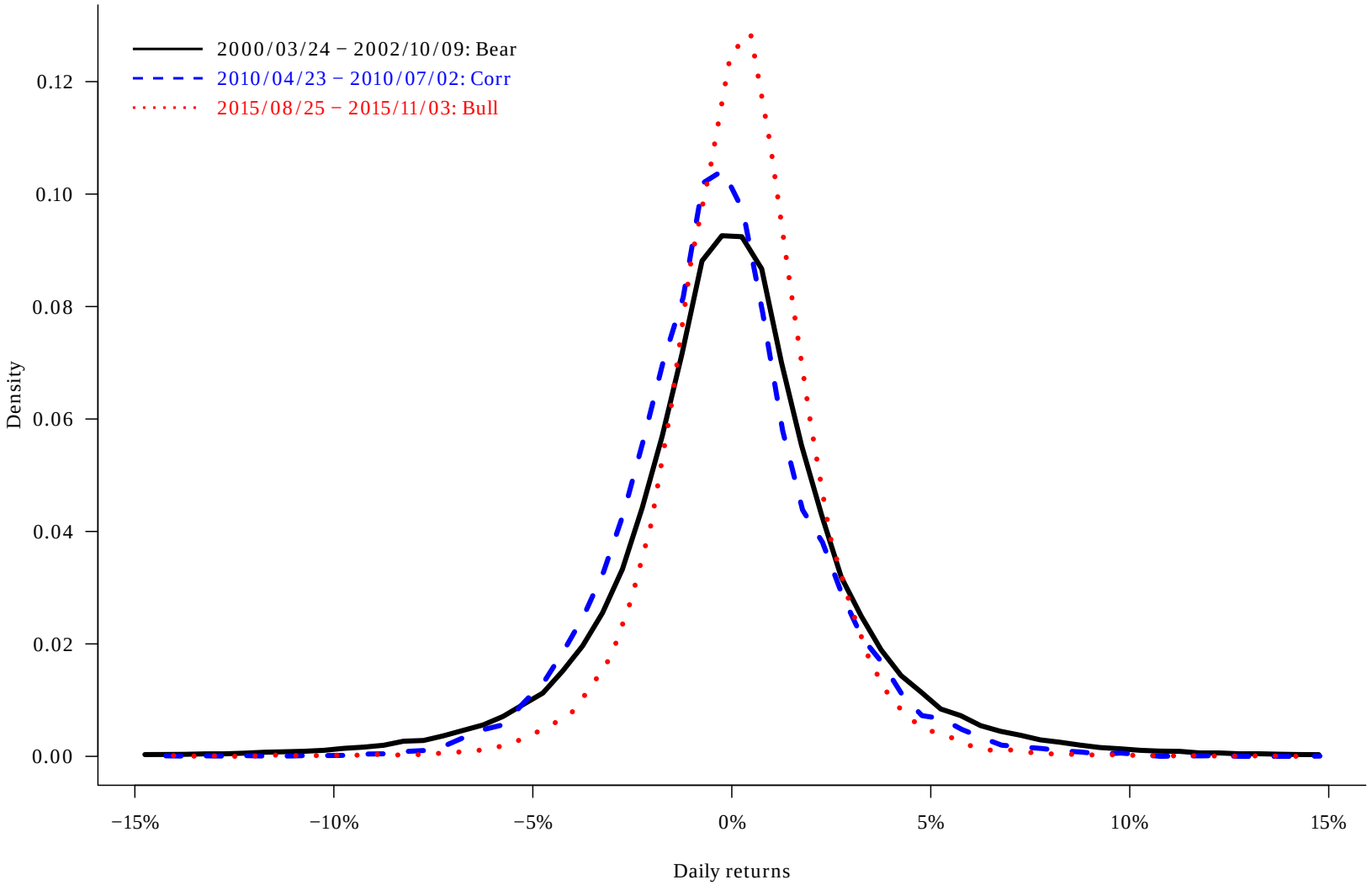


Figure 1: Cross-sectional distributions over bull, bear markets, and corrections.

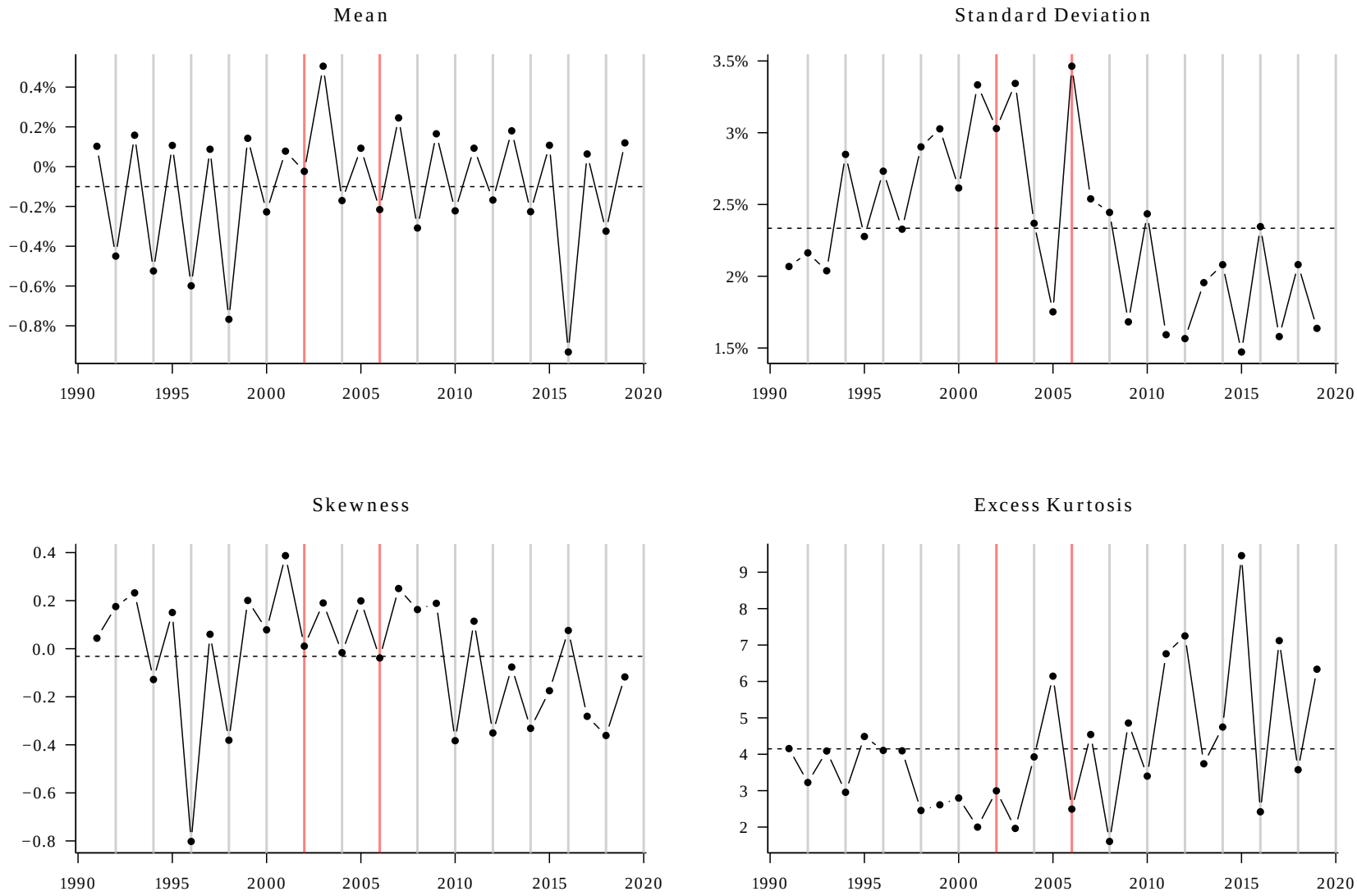


Figure 2: Empirical moments over bull, bear markets (red bars), and corrections (grey bars).

Quantal Response Statistical Equilibrium

The Quantal Response Statistical Equilibrium model (Scharfenaker and Foley, 2017) provides a theoretical framework which explaines observed statistical regularities through a process of Smithian competition.

Entropy-constrained model, which derives equilibrium as an information theoretic probability distribution representing all possible states of the system.

Three main components of the model:

1. Quantal response behavior of market participants (μ, T).
2. Negative feedback of individual actions on outcomes (α, S).
3. Role of expectations $(\zeta = \mu - \alpha)$.

Quantal Response

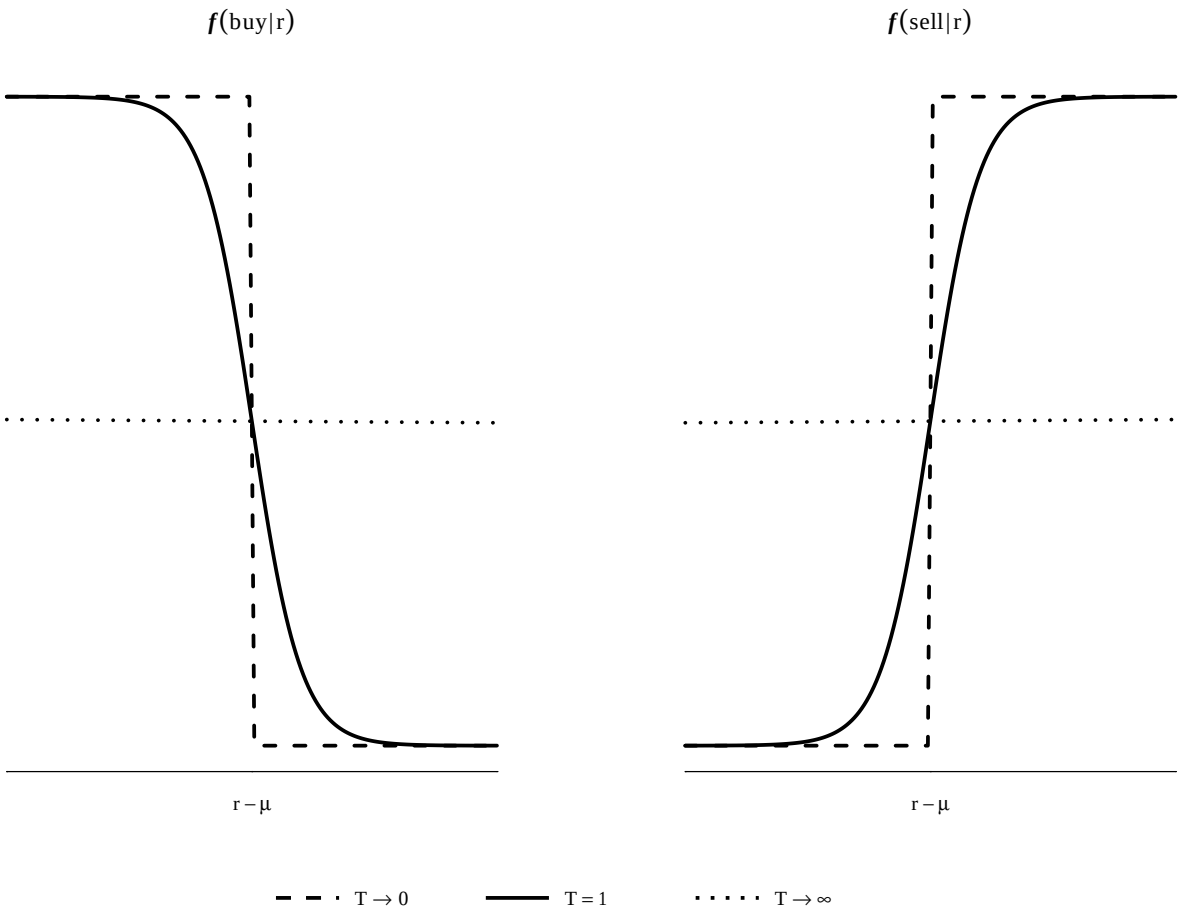


Figure 3: Logit quantal response conditional probabilities for various values of T and $\mu = 0$.

Negative Feedback

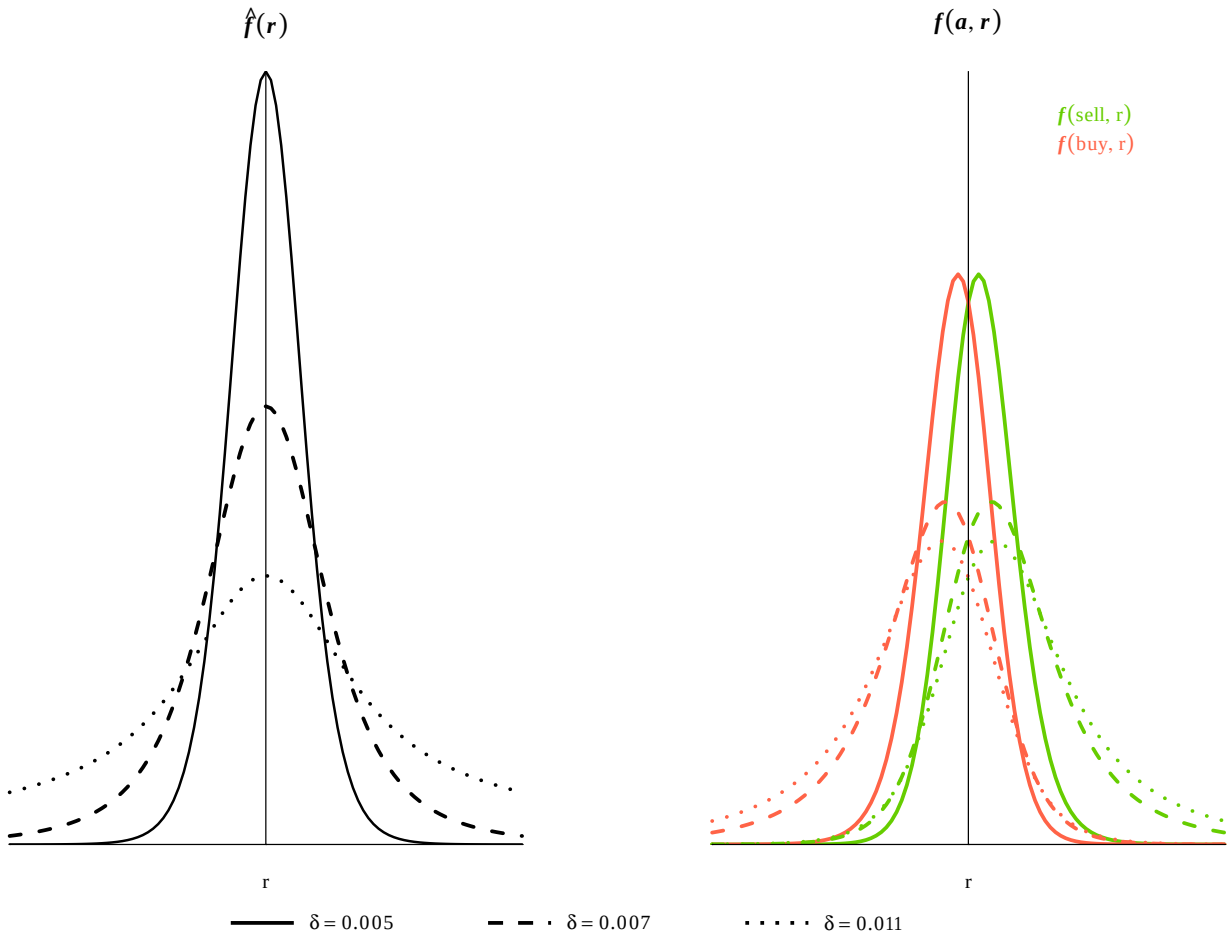
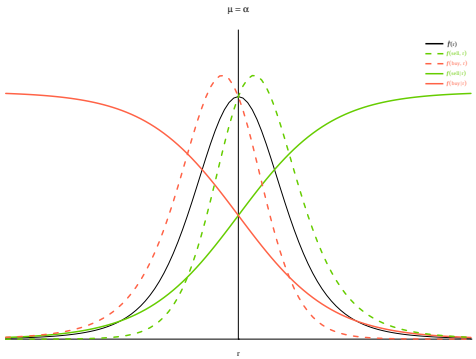
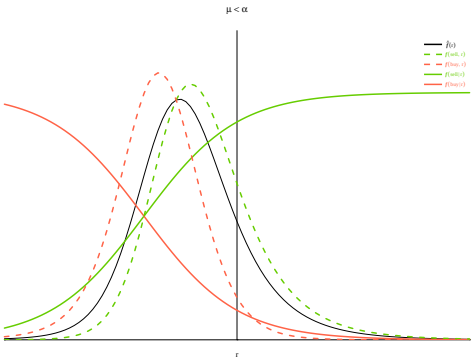


Figure 4: Marginal and joint frequency distributions for different values of δ.

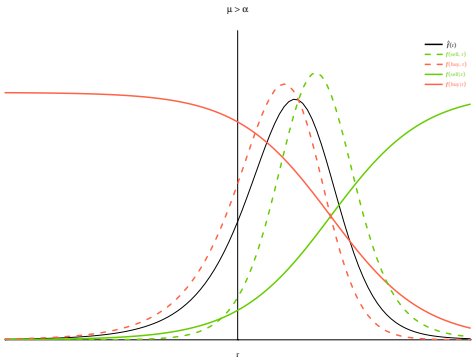
Expectations



(a) Symmetry ($\zeta = 0$).



(b) Positive skewness ($\zeta < 0$).



(c) Negative skewness ($\zeta > 0$).

Figure 5: Marginal, conditional, and joint frequency distributions for fulfilled and unfulfilled expectations.

Results

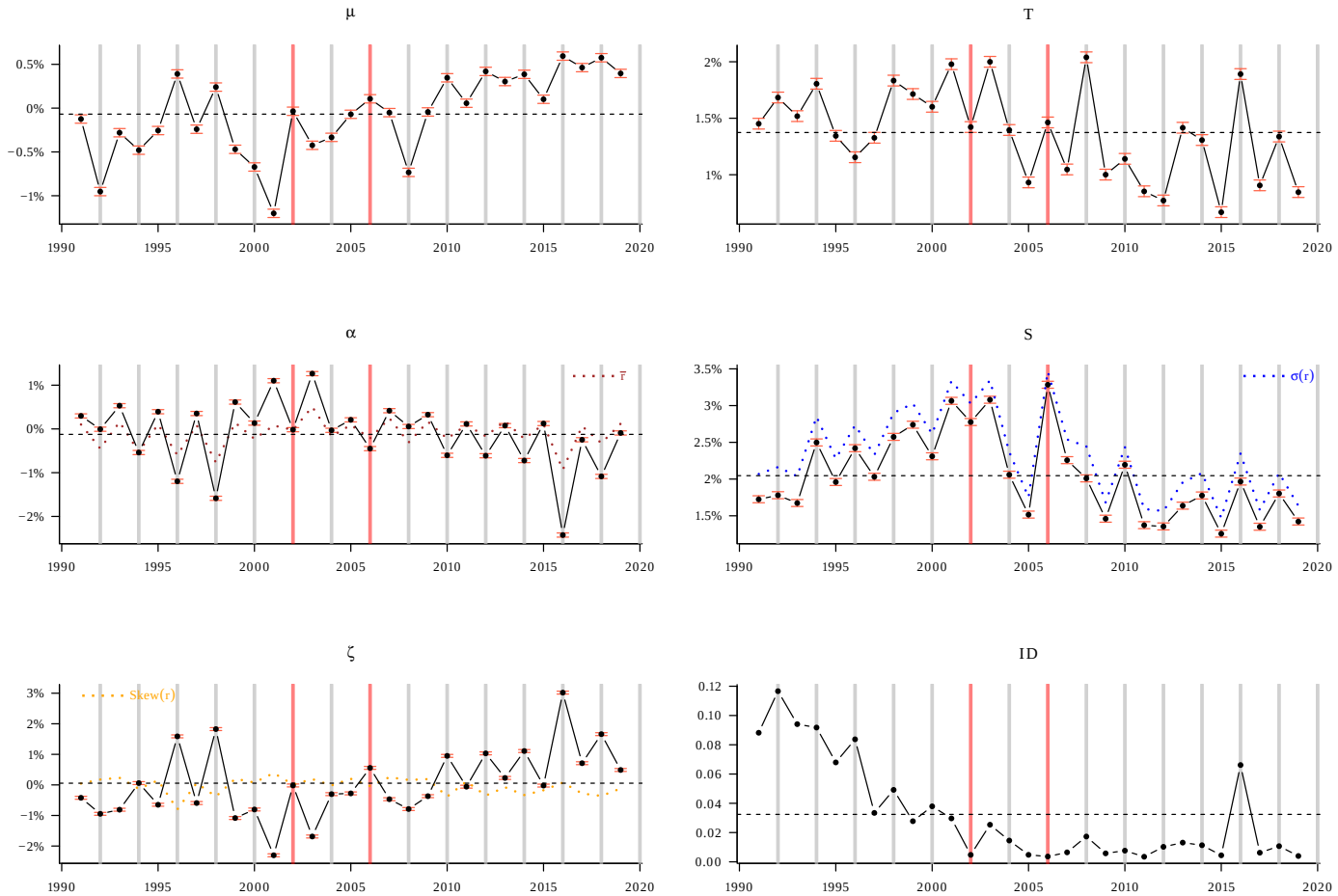


Figure 6: Parameter estimates (%/day).

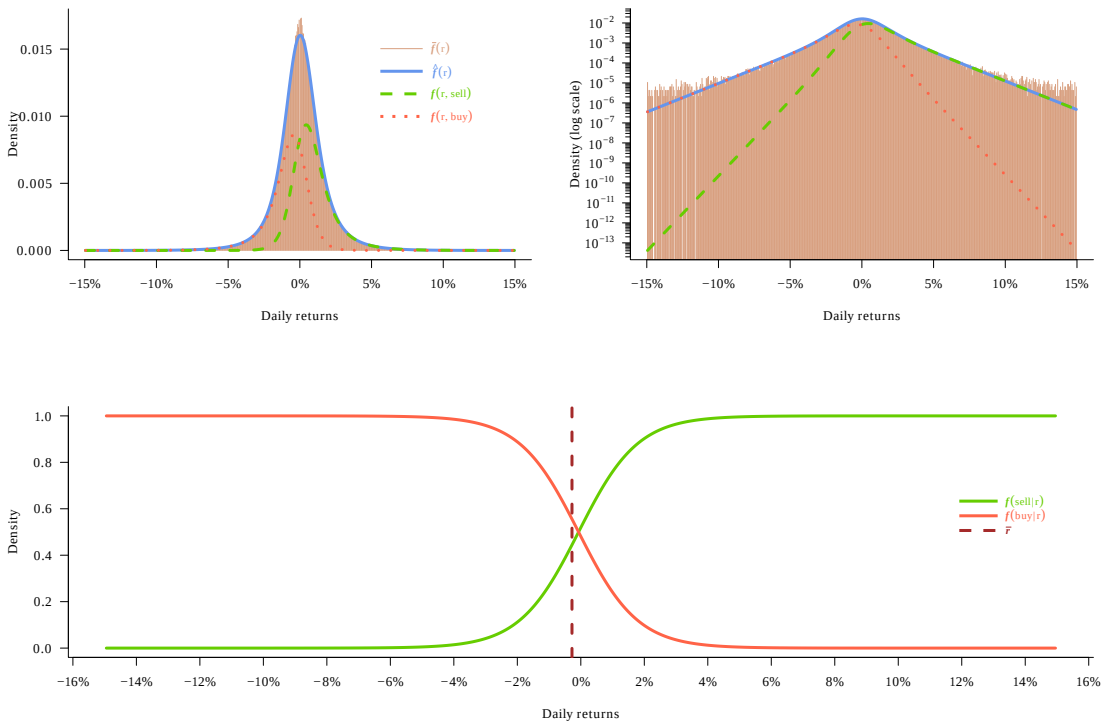


Figure 7: Bull market: 2003/03/11 – 2007/10/09.

ID	μ	T	α	S	$\bar{r}$	ζ	δ	$f[buy]$	$f[sell]$
0.0048	-0.0702	0.9314	0.2077	1.5154	-0.2780	0.0924	0.8641	0.4726	0.5274

Table 1: Parameter estimates (%/day).

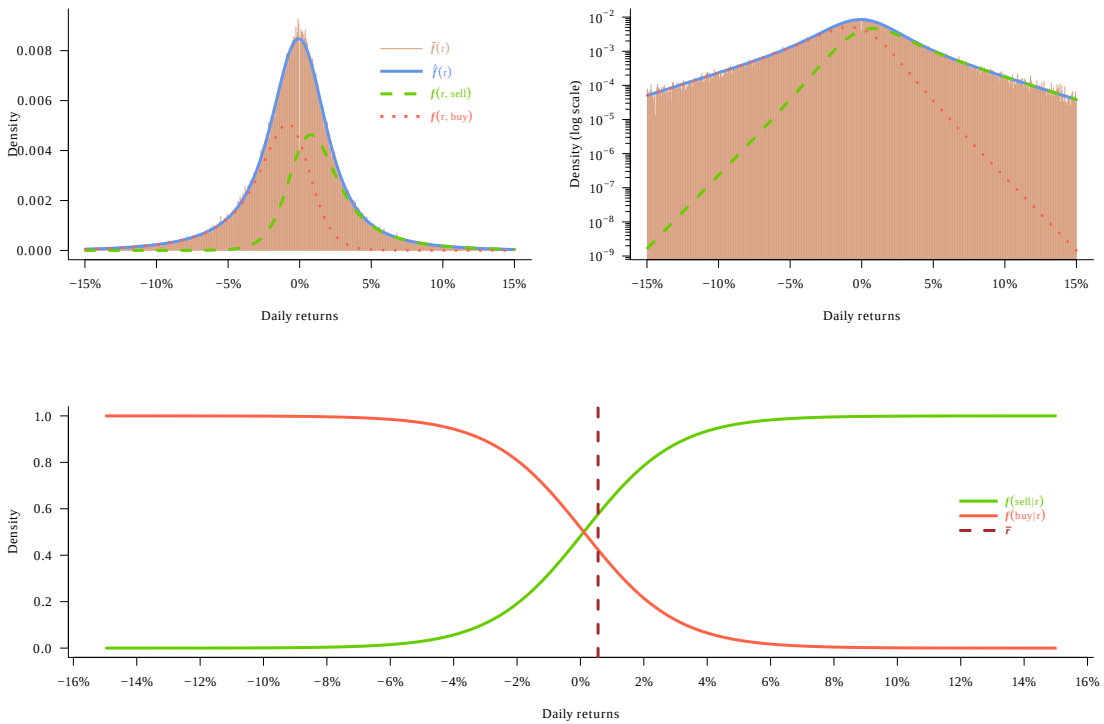


Figure 8: Bear market: 2007/10/09 – 2009/03/09.

ID	μ	T	α	S	$\bar{r}$	ζ	δ	$f[buy]$	$f[sell]$
0.0036	0.1064	1.4626	-0.4501	3.2834	0.5566	-0.2159	1.9037	0.5315	0.4685

Table 2: Parameter estimates (%/day).

Conclusions

- We find evidence of punctuated statistical equilibrium over multiple market periods, disrupted by structural changes affecting the stock market.
- We find evidence of significant deviations of individual expectations from market outcomes over extended time periods, even though they remain consistent in the long-run.
- We show how the stochastic nature of stock prices can be explained as the spontaneous convergence of the system towards a market convention.

Thank You!

References

Bhowmik, R. and S. Wang (2020). Stock market volatility and return analysis: A systematic literature review. *Entropy 22*(5), 522.

Fama, E. F. (1965). The behavior of stock-market prices. *The Journal of Business 38*(1), 34–105.

Mantegna, R. N. and H. E. Stanley (1999). *Introduction to Econophysics: Correlations and Complexity in Finance*. Cambridge University Press.

Scharfenaker, E. and D. K. Foley (2017). Quantal response statistical equilibrium in economic interactions: Theory and estimation. *Entropy 19*(9), 444.

Shiller, R. J. (2015). *Irrational Exuberance: Revised and Expanded Third Edition*. Princeton University Press.