Aggregate Fluctuations from Clustered Micro Shocks

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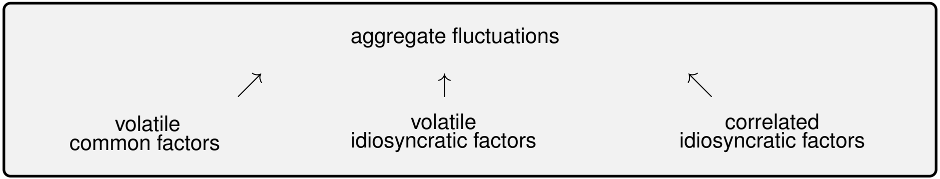
Where do business cycle fluctuations come from?

When can micro shocks generate macro fluctuations?



- granular origins: fat tail distribution leads non-diversification
- clustered origins: cross-firm correlated idiosyncratic factors

Even if most business cycle research does ignored the cross-firm pairwise correlations, idiosyncratic co-movements potentially lead to macro fluctuations.

Correlated idiosyncratic factors and macro fluctuations

A simple example with identical variance-covariance

$$\underbrace{\hat{y}_{it}}_{\text{firm fluctuation}} = \underbrace{\varepsilon_{\mathrm{A},t}}_{\substack{\textbf{true}\\ \text{common factor}}} + \underbrace{\varepsilon_{\mathrm{F},it}}_{\substack{\textbf{true}\\ \text{idiosyncratic factor}}} \quad \text{and} \quad \underbrace{\hat{Y}_t}_{\text{agg. fluctuation}} = \sum_i \underbrace{w_{it}}_{\text{share}} \times \hat{y}_{it}$$

Aggregate fluctuations with identical variance and covariance

$$\underbrace{\mathsf{var}(\hat{Y}_t)}_{\substack{\text{aggregate}\\ \text{volatility}}} = \underbrace{\sigma^2_{\mathrm{A},t}}_{\substack{\text{common factor}\\ \text{volatility}}} + \underbrace{h_t^2\sigma^2_{\mathrm{F},t}}_{\substack{\text{idiosyncratic factor}\\ \text{volatility: granularity}}} + \underbrace{\big(1-h_t^2\big)\rho_{\mathrm{F},t}\sigma^2_{\mathrm{F},t}}_{\substack{\text{idiosyncratic factor}\\ \text{dependency}}}$$

Notation, notes, and remarks:

h_t	Herfindahl Hirschman Index,	$[\sum_{i'} w^2_{i't}]^{1/2} \in [N_t^{-1/2}, 1]$
$\sigma^2_{\mathrm{A},t}$ and $\sigma^2_{\mathrm{F},t}$	firm i's variance of true common and idiosyncratic factor,	$\mathsf{var}(\varepsilon_{\mathrm{A},t})$ and $\mathsf{var}(\varepsilon_{\mathrm{F},it})$
$\rho_{\mathrm{F},t}$	correlation b/w firms i and i''s true idiosyncratic factors,	$\mathsf{corr}(\varepsilon_{\mathrm{F},it}, \varepsilon_{\mathrm{F},i't})$

Why and when can we ignore pairwise correlation?

Fluctuations: true vs pseudo factors

$$\underbrace{\hat{y}_{it}}_{\text{firm fluctuation}} = \underbrace{\varepsilon_{A,t}}_{\substack{\textbf{true}\\ \text{common factor}}} + \underbrace{\varepsilon_{F,it}}_{\substack{\textbf{true}\\ \text{idiosyncratic factor}}} = e_{A,t} + e_{F,it} = \underbrace{\overbrace{\frac{1}{N_t}\sum_{i'}\hat{y}_{i't}}^{\text{mean}}}_{\substack{\textbf{pseudo}\\ \text{common factor}}} + \underbrace{\overbrace{\hat{y}_{it} - \frac{1}{N_t}\sum_{i'}\hat{y}_{i't}}^{\text{deviation from mean}}}_{\substack{\textbf{pseudo}\\ \text{idiosyncratic factor}}}$$

The identical variance and covariance across firms imply

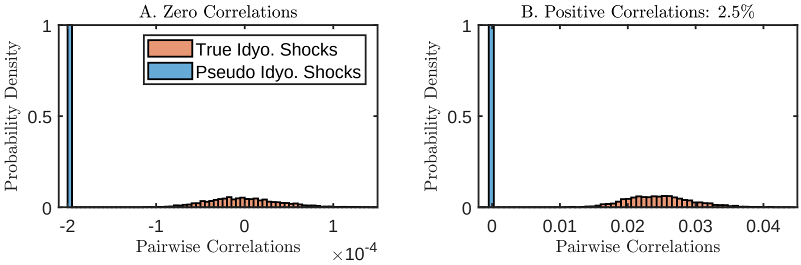
- corr$(e_{A,t}, e_{F,it}) = 0$ and corr$(e_{F,it}, e_{F,i't}) \approx 0$ for $i \neq i'$.
- business cycle studies with pseudo variables are OK (well-defined) where dependency does not matter.

The heterogeneous variance and covariance across firms imply

- corr$(e_{A,t}, e_{F,it}) \neq 0$ and corr$(e_{F,it}, e_{F,i't}) \neq 0$ is disconnected to corr$(\varepsilon_{F,it}, \varepsilon_{F,i't})$.
- business cycle studies with pseudo variables are spurious and not well-defined.

Simulations (1/4)

Sample pairwise correlations: true vs pseudo idiosyncratic factors



- pseudo idiosyncratic factors ignore true factors' pairwise correlations

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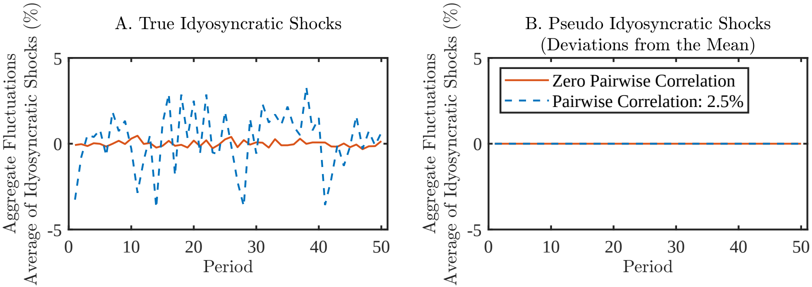
Notation, notes, and remarks:

- 3,000 simulations, 50 periods, 5,000 firms, S.D. of $\varepsilon_{\mathrm{F},it}$ is 12%.

Simulations (2/4)

Aggregate fluctuations: $N_t^{-1} \sum \varepsilon_{\mathrm{F},it}$ and $N_t^{-1} \sum e_{\mathrm{F},it}$



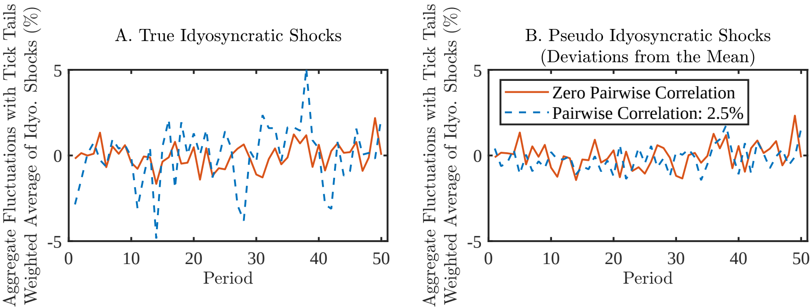
- 2.5% pairwisely correlation ⇒ notable aggregate fluctuations

Notation, notes, and remarks:

- Here, we ignored the common factor. 50 periods, 5,000 firms, S.D. of $\varepsilon_{\mathrm{A},t}$ is 12%.

Simulation (3/4): with unequal size distributions

Aggregate fluctuations: $\sum w_{it}\varepsilon_{\mathrm{F},it}$ and $\sum w_{it}e_{\mathrm{F},it}$



- 2.5% pairwisely correlation + fat-tailed size distribution $\Rightarrow$ aggregate fluctuations

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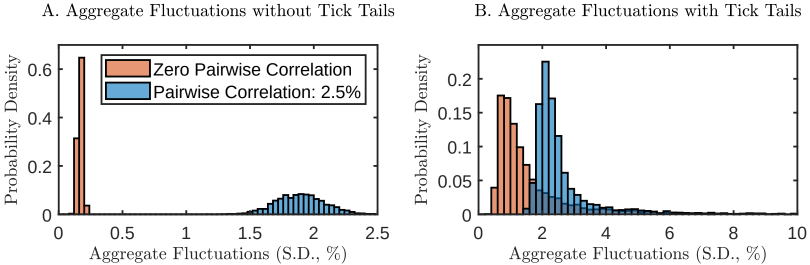
Notation, notes, and remarks:

- Here, we ignored the common factor. 50 periods, 5,000 firms, S.D. of $\varepsilon_{\mathrm{A},t}$ is 12%.

Simulations (4/4)

Aggregate volatility: S.D. of $N_t^{-1} \sum \varepsilon_{\mathrm{F},it}$ and $\sum w_{it}\varepsilon_{\mathrm{F},it}$



- 2.5% pairwisely correlation + fat-tailed size distribution $\Rightarrow$ aggregate fluctuations

Notation, notes, and remarks:

- Here, we ignored the common factor. 3,000 simulations, 50 periods, 5,000 firms, S.D. of $\varepsilon_{\mathrm{A},t}$ is 12%.

This paper does

This paper provides the micro-foundations for (aggregate) business cycle fluctuations.

- cluster origins (dependency within an industry)
  - idiosyncratic shocks are correlated across firms
  - variance and pairwise covariance differ across firms

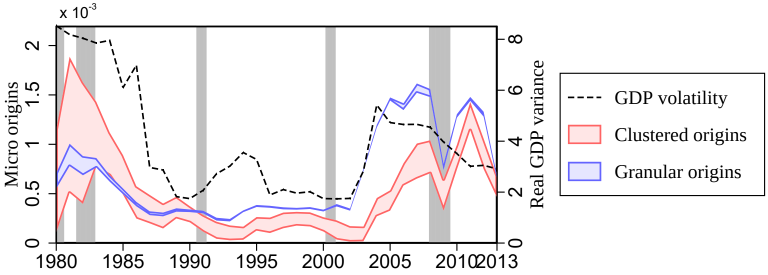
I need to identify true factors ($\varepsilon_{\mathrm{A},t}$ and $\varepsilon_{\mathrm{F},it}$) from observation ($\hat{y}_{it}$)... maybe challenging...

I compute the upper- and lower-bounds of granular and clustered origins instead of estimating point values. This approach

- relies on some statistical facts rather than additional assumptions and/or information.
- avoids misspecification issues.

This paper finds

Clustered and granular origins:



- The clustered origins explain 1) the great moderation and 2) the recent increase in the US business cycle volatility.

Notation, notes, and remarks:

- Compustat Annual Fundamentals North America database 1976–2018 details.
- Aggregate and industrial GDPs and deflators are from Bureau of Economic Analysis.

Related literature

GDP volatility – related to origins:

- Stock and Watson (2002); Comin and Philippon (2005); Comin and Mulani (2006); Davis, Haltiwanger, Jarmin, Miranda, Foote and Nagypal (2006); Carvalho and Gabaix (2013)

Granularity:

- Jovanovic (1987); Gabaix (2011); di Givonanni and Levchenko (2012); Carvalho and Gabaix (2013); Bremus, Buch, Russ and Schnitzer (2018); Gaubert and Itoskhoki (2018)
- Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012); Carvalho (2014); Oberfield (2018); Herskovic, Kelly, Lustig and Van Nieuwerburgh (2020)

Dependency:

- Long and Plosser (1983); Horvath (1998); Dupor (1999); Foerster, Sarte and Watson (2011); Atalay (2017)
- Oberfield (2018); Schaal and Taschereau-Dumouchel (2018); Mullen (2020); Fiori and Scoccianti (2021)

Heterogeneous firm volatility:

- Stanley, Amaral, Buldyrev, Havlin, Leschhorn, Maass, Salinger and Stanley (1996); Xu and Malkiel (2003); Comin and Philippon (2005); Comin and Mulani (2006); Chun, Kim, Morck and Yeung (2008); Castro, Clementi and Lee (2015); Tweedle (2018)

Theoretical motivation and key concepts

- **Origins of business cycle fluctuations**

Origins of business cycle fluctuations (1/2)

Aggregate fluctuations

$$\underbrace{\hat{Y}_t}_{\text{aggregate fluctuations}} = \underbrace{\textstyle\sum_i w_{it}\hat{y}_{it}}_{\text{weighted sum of firm fluctuations}} \quad \text{where} \quad \underbrace{\hat{y}_{it}}_{\text{firm's fluctuations}} = \underbrace{\varepsilon_{\mathrm{A},t}}_{\text{common factor}} + \underbrace{\varepsilon_{\mathrm{F},it}}_{\text{idiosyncratic factor}} \quad \text{and} \quad \varepsilon_{\mathrm{A},t} \perp \varepsilon_{\mathrm{F},it}$$

$$\underbrace{\sigma^2_{\hat{Y},t}}_{\text{aggregate volatility}} = \underbrace{\sigma^2_{\mathrm{A},t}}_{\text{common factor volatility}} + \underbrace{\textstyle\sum_i w^2_{it}\sigma^2_{\mathrm{F},it}}_{\text{idiosyncratic factor volatility}} + \underbrace{\textstyle\sum_i w_{it}\sum_{i'\neq i} w_{i't}\rho_{\mathrm{F},ii't}\sigma_{\mathrm{F},it}\sigma_{\mathrm{F},i't}}_{\text{idiosyncratic factor dependency}}$$

Notation, notes, and remarks:

w_{it}	firm i's share,	size weight
$\sigma^2_{\hat{Y},t}$	variance of aggregate business cycles,	var$(\hat{Y}_t)$
$\sigma^2_{\mathrm{A},t}$ and $\sigma^2_{\mathrm{F},it}$	firm i's variance of true common and idiosyncratic factor,	var$(\varepsilon_{\mathrm{A},t})$ and var$(\varepsilon_{\mathrm{F},it})$
$\rho_{\mathrm{F},ii't}$	correlation of true idiosyncratic factor b/w firms i and i',	corr$(\varepsilon_{\mathrm{F},it}, \varepsilon_{\mathrm{F},i't})$

Origins of business cycle fluctuations (2/2)

Aggregate fluctuations with identical variance and covariance

$$\underbrace{\sigma^2_{\hat{Y},t}}_{\substack{\text{aggregate}\\ \text{volatility}}} = \overbrace{\underbrace{\sigma^2_{\mathrm{A},t}}_{\substack{\text{common factor}\\ \text{volatility}}}}^{\text{aggregate origins}} + \overbrace{\underbrace{h_t^2 \sigma^2_{\mathrm{F},t}}_{\substack{\text{idiosyncratic factor}\\ \text{volatility}}}}^{\text{granular origins: }\Gamma_t} + \overbrace{\underbrace{\left(1-h_t^2\right)\rho_{\mathrm{F},t}\sigma^2_{\mathrm{F},t}}_{\substack{\text{idiosyncratic factor}\\ \text{dependency}}}}^{\text{clustered origins:}\chi_t}$$

- Lucas (1977)'s diversification argument: idiosyncratic shocks average out : it only holds when i) $h_t \to 0$ as $N_t \to \infty$ and ii) $\rho_{\mathrm{F},t} = 0$

Notation, notes, and remarks:

h_t	Herfindahl Hirschman Index,	$[\sum_{i'} w^2_{i't}]^{1/2} \in [N_t^{-1/2}, 1]$
$\sigma^2_{\hat{Y},t}$	variance of aggregate business cycles,	var$(\hat{Y}_t)$
$\sigma^2_{\mathrm{A},t}$ and $\sigma^2_{\mathrm{F},t}$	firm i's variance of true common and idiosyncratic factor,	var$(\varepsilon_{\mathrm{A},t})$ and var$(\varepsilon_{\mathrm{F},it})$
$\rho_{\mathrm{F},t}$	correlation b/w firms i and i''s true idiosyncratic factors,	corr$(\varepsilon_{\mathrm{F},it}, \varepsilon_{\mathrm{F},i't})$

Are clustered origins non-negligible?: A simple example

Size of clustered origins relative to granular origins (with identical variance-covariance)

$$\frac{\chi_t}{\Gamma_t} = \frac{(1-h_t^2)\rho_{\mathrm{F},t}\sigma_{\mathrm{F},t}^2}{h_t^2\sigma_{\mathrm{F},t}^2} = \left(\frac{1}{h_t^2} - 1\right)\rho_{\mathrm{F},t}$$

- With $h_t = 0.12$ as in Gabaix (2011)'s example
- $\rho_{\mathrm{F},t} \in [0.01,\ 0.05]$ implies $\chi_t \in \Gamma_t \times [0.68,\ 3.42]$

Why has the predominant research long ignored pairwise correlation across firms?

Notation, notes, and remarks:

h_t	Herfindahl Hirschman Index,	$[\sum_{i'} w_{i't}^2]^{1/2} \in [N_t^{-1/2}, 1]$
$\sigma_{\mathrm{F},t}^2$	variance of true firm i's idiosyncratic factor,	var$(\varepsilon_{\mathrm{F},it})$
$\rho_{\mathrm{F},t}$	correlation b/w firms i and i''s true idiosyncratic factors,	corr$(\varepsilon_{\mathrm{F},it}, \varepsilon_{\mathrm{F},i't})$

The Framework with pseudo variables

- **Homogeneous variance-covariance**
- Heterogeneous variance-covariance
- Evidence from the US public firms

Pseudo factors and spurious relations

True vs Pseudo common and idiosyncratic factors

$$\underbrace{\hat{y}_{it}}_{\text{firm fluctuations}} = \underbrace{\varepsilon_{\mathrm{A},t}}_{\text{true common factor}} + \underbrace{\varepsilon_{\mathrm{F},it}}_{\text{true idiosyncratic factor}} \quad \Rightarrow \quad \hat{y}_{it} = \underbrace{e_{\mathrm{A},t}}_{\text{pseudo common factor}} + \underbrace{e_{\mathrm{F},it}}_{\text{pseudo idiosyncratic factor}}$$

- True of common and idiosyncratic factors are not directly observable
- Many studies use the pseudo factors; the sample mean and the deviation from it.
- Spurious relations

$$\text{var}(e_{\mathrm{A},t}) \approx \sigma^2_{\mathrm{A},t} + \rho_{\mathrm{F},t}\sigma^2_{\mathrm{F},t} \quad \text{and} \quad \text{var}(e_{\mathrm{F},it}) \approx \sigma^2_{\mathrm{F},t} - \rho_{\mathrm{F},t}\sigma^2_{\mathrm{F},t}$$

  - Systemically over- or under-estimated volatility of factors

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Notation, notes, and remarks:

$e_{\mathrm{A},t}$ pseudo common factor, $e_{\mathrm{A},t} = N_t^{-1}\sum_{i'}\hat{y}_{i't} = \varepsilon_{\mathrm{A},t} + N_t^{-1}\sum_{i'}\varepsilon_{\mathrm{F},i't}$

$e_{\mathrm{F},it}$ pseudo idiosyncratic factor, $e_{\mathrm{F},it} = \hat{y}_{it} - e_{\mathrm{A},t} = \varepsilon_{\mathrm{F},it} - N_t^{-1}\sum_{i'}\varepsilon_{\mathrm{F},i't}$

Properties of homogeneous variance and covariance

Proposition 1

Consider a cluster where firms have identical standard deviation and pairwise correlation of idiosyncratic shocks; $\sigma_{\mathrm{F},t} > 0$ and $\rho_{\mathrm{F},t} \in (-1, 1)$. Then, the cross-sectional sample mean and the deviations from it have the following correlations. For $\forall i \neq i'$,

$$\text{corr}(e_{\mathrm{A},t}, e_{\mathrm{F},it}) = 0$$
$$\text{corr}(e_{\mathrm{F},it}, e_{\mathrm{F},i't}) = -(N_t - 1)^{-1}.$$

- Spurious but well-defined!
  - Pseudo common and idiosyncratic factors are orthogonal
  - Pseudo idiosyncratic factors are asymptotically orthogonal to each other

  : true dependency does not matter for the pseudo dependency

Notation, notes, and remarks:

- Note that these results do not hold when I use the weighted mean.

details

Irrelevance of correlated pseudo idiosyncratic factors

Corollary 1

The variance of aggregate fluctuations can be decomposed into the pseudo common and idiosyncratic shocks' variances asymptotically.

$$\sigma^2_{\hat{Y},t} = \mathsf{var}(e_{\mathrm{A},t}) + h_t^2 \mathsf{var}(e_{\mathrm{F},it}) - \left(\frac{1-h_t^2}{N_t-1}\right)\mathsf{var}(e_{\mathrm{F},it})$$

- we can use the pseudo factors where clustered origins (dependency) do not matter asymptotically.

Notation, notes, and remarks:

- $\mathsf{cov}(e_{\mathrm{F},it}, e_{\mathrm{A},t}) = 0$
- $\mathsf{cov}(e_{\mathrm{F},it}, e_{\mathrm{F},i't}) = -(N_t-1)^{-1}\mathsf{var}(e_{\mathrm{F},it})$

The Framework with pseudo variables

- Homogeneous variance-covariance
- **Heterogeneous variance-covariance**

Properties of homogeneous variance and covariance

Proposition 2

Consider a cluster where idiosyncratic shocks' standard deviation and pairwise correlation are different across firms. Then, the covariance between the cross-sectional sample mean and firm i's deviation from it is non-zero in general.

$$\text{cov}(e_{\mathrm{A},t}, e_{\mathrm{F},it}) = \frac{1}{N_t}\left[\sigma^2_{\mathrm{F},it} - \frac{1}{N_t}\sum_{i'}\sigma^2_{\mathrm{F},i't}\right] + \left[\frac{1}{N_t}\sum_{i'\neq i}\rho_{\mathrm{F},ii't}\sigma_{\mathrm{F},it}\sigma_{\mathrm{F},i't} - \frac{1}{N_t}\sum_{i'}\frac{1}{N_t}\sum_{i''\neq i'}\rho_{\mathrm{F},i'i''t}\sigma_{\mathrm{F},i't}\sigma_{\mathrm{F},i''t}\right]$$

- not well-defined
  - pseudo common and idiosyncratic factors are correlated
  - pseudo idiosyncratic factors are correlated to each other
- We need to recover true idiosyncratic factors' volatility and dependency.

Are variance and covariance heterogeneous? (1/2)

Evidence on heterogeneous variance and covariance of true idiosyncratic factor

- homogeneous variance: identical $\text{var}(\hat{y}_{it})$ across firms

$$\text{var}(\hat{y}_{it}) = \sigma^2_{\mathrm{A},t} + \sigma^2_{\mathrm{F},it}$$

- homogeneous covariance: identical $\text{cov}(\hat{y}_{it}, \hat{y}_{i't})$ across firms

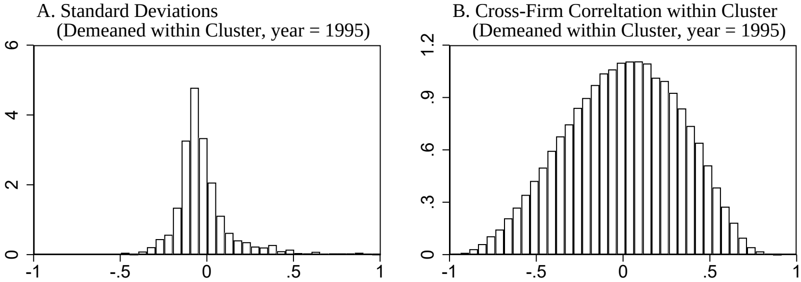
$$\text{cov}(\hat{y}_{it}, \hat{y}_{i't}) = \sigma^2_{\mathrm{A},t} + \rho_{\mathrm{F},ii't}\sigma_{\mathrm{F},it}\sigma_{\mathrm{F},i't}$$

Notation, notes, and remarks:

$\hat{y}_{it}$	firm's fluctuations,	$= \varepsilon_{\mathrm{F},it} + \varepsilon_{\mathrm{A},t} = e_{\mathrm{F},it} + e_{\mathrm{A},t}$
$\sigma^2_{\mathrm{A},t}$	variance of true common factor,	$\text{var}(\varepsilon_{\mathrm{A},t})$
$\sigma^2_{\mathrm{F},it}$	firm i's variance of true idiosyncratic factor,	$\text{var}(\varepsilon_{\mathrm{F},it})$
$\rho_{\mathrm{F},ii't}$	correlation of true idiosyncratic factor b/w firms i and i',	$\text{corr}(\varepsilon_{\mathrm{F},it}, \varepsilon_{\mathrm{F},i't})$

Are variance and covariance heterogeneous? Yes! (2/2)

Evidence on heterogeneous variance and covariance of true idiosyncratic factor



Notation, notes, and remarks:

- Source: Compustat Annual Fundamentals North America database 1976–2018 ▸ details
- In each t, I calculate a firm's standard deviation and correlations of labor productivity in $[t-4, t+5]$. I report the statistics after demeaning within industry in each year.

Origins of aggregate fluctuations

- **Empirical Strategy**
- The evolution of micro origins in the US

Origins of industrial fluctuations

Industry (cluster) s fluctuations: $\hat{Y}_{st} = \sum_{i \in I_{st}} w_{sit} \hat{y}_{it}$ where $\hat{y}_{it} = \varepsilon_{\mathrm{A},st} + \varepsilon_{\mathrm{F},it}$

$$\begin{aligned}\sigma^2_{\hat{Y},st} &= \sigma^2_{\mathrm{A},st} + \sum_{i \in I_{st}} w^2_{sit} \sigma^2_{\mathrm{F},it} + \sum_{\substack{i,i' \in I_{st} \\ i' \neq i}} w_{sit} w_{si't} \rho_{\mathrm{F},ii't} \sigma_{\mathrm{F},it} \sigma_{\mathrm{F},i't} \\ &= \underbrace{\sigma^2_{\mathrm{A},st}}_{\text{aggregate}} + \underbrace{\sum_{i \in I_{st}} w^2_{sit} \mathsf{var}(\hat{y}_{it}) - h^2_{st} \sigma^2_{\mathrm{A},st}}_{\text{granular origins : } \Gamma_{st}} + \underbrace{\sum_{\substack{i,i' \in I_{st} \\ i' \neq i}} w_{sit} w_{si't} \mathsf{cov}(\hat{y}_{it}, \hat{y}_{i't}) - (1 - h^2_{st}) \sigma^2_{\mathrm{A},st}}_{\text{cluster origins : } \chi_{st}}\end{aligned}$$

Notation, notes, and remarks:

h_{st} Herfindahl Hirschman Index in industry s, $[\sum_{i \in I_{st}} w^2_{st}]^{1/2} \in [N_{st}^{-1/2}, 1]$

w_{sit} share of firm i in industry s, size weight

Origins of industrial fluctuations

Industry (cluster) s **fluctuations:** $\hat{Y}_{st} = \sum_{i \in I_{st}} w_{sit} \hat{y}_{it}$ where $\hat{y}_{it} = \varepsilon_{A,st} + \varepsilon_{F,it}$

$$\sigma^2_{\hat{Y},st} = \sigma^2_{A,st} + \sum_{i \in I_{st}} w^2_{sit} \sigma^2_{F,it} + \sum_{\substack{i,i' \in I_{st} \\ i' \neq i}} w_{sit} w_{si't} \rho_{F,ii't} \sigma_{F,it} \sigma_{F,i't}$$

$$= \underbrace{\sigma^2_{A,st}}_{\text{aggregate}} + \underbrace{\sum_{i \in I_{st}} w^2_{sit} \text{var}(\hat{y}_{it}) - h^2_{st} \sigma^2_{A,st}}_{\text{granular origins : } \Gamma_{st}} + \underbrace{\sum_{\substack{i,i' \in I_{st} \\ i' \neq i}} w_{sit} w_{si't} \text{cov}(\hat{y}_{it}, \hat{y}_{i't}) - (1 - h^2_{st}) \sigma^2_{A,st}}_{\text{cluster origins : } \chi_{st}}$$

Notation, notes, and remarks:

h_{st}	Herfindahl Hirschman Index in cluster s,	$[\sum_{i \in I_{st}} w^2_{st}]^{1/2} \in [N^{-1/2}_{st}, 1]$
w_{sit}	share of firm i in cluster s,	size weight

How to identify a range of common factor

Proposition 3

In a cluster, the common shocks' variance should not be larger than $\sigma^{*2}_{\mathrm{A},st}$.

$$0 \le \sigma^2_{\mathrm{A},st} \le \sigma^{*2}_{\mathrm{A},st} = \min_{i,i' \in I_{st}} \left\{ \mathsf{var}(\hat{y}_{it}),\ \left[1 + \mathsf{corr}(\hat{y}_{it}, \hat{y}_{i't})\right] \mathsf{sd}(\hat{y}_{it}) \mathsf{sd}(\hat{y}_{i't}) \right\}$$

- $\mathsf{var}\big(\hat{y}_{it}\big) = \sigma^2_{\mathrm{A},st} + \sigma^2_{\mathrm{F},it}$: since variance is non-negative,

$$\mathsf{var}\big(\hat{y}_{it}\big) \ge \sigma^2_{\mathrm{A},st} \quad \text{and} \quad \mathsf{var}(\hat{y}_{it}) \ge \sigma^2_{\mathrm{F},it}$$

- $\mathsf{cov}(\hat{y}_{it}, \hat{y}_{i't}) = \sigma^2_{\mathrm{A},st} + \rho_{\mathrm{F},ii't}\sigma_{\mathrm{F},it}\sigma_{\mathrm{F},i't}$: since correlation is b/w –1 and 1,

$$\mathsf{cov}(\hat{y}_{it}, \hat{y}_{i't}) \ge \sigma^2_{\mathrm{A},st} - \sigma_{\mathrm{F},it}\sigma_{\mathrm{F},i't} \ge \sigma^2_{\mathrm{A},st} - \mathsf{sd}\big(\hat{y}_{it}\big)\mathsf{sd}\big(\hat{y}_{i't}\big)$$

Notation, notes, and remarks:

$\sigma^2_{\mathrm{A},st}$	variance of true common factor in cluster s,	$\mathsf{var}(\varepsilon_{\mathrm{A},t})$
$\sigma^2_{\mathrm{F},it}$	firm i's variance of true idiosyncratic factor,	$\mathsf{var}(\varepsilon_{\mathrm{F},it})$
$\rho_{\mathrm{F},ii't}$	correlation of true idiosyncratic factor b/w firms i and i',	$\mathsf{corr}(\varepsilon_{\mathrm{F},it}, \varepsilon_{\mathrm{F},i't})$

How to identify granular and cluster origins

Corollary 2

The clustered and granular origins are bounded as follows.

$$\sum_{\substack{i,i'\in I_{st}\\ i'\neq i}} w_{sit}w_{si't}\mathsf{cov}(\hat{y}_{it},\hat{y}_{i't}) - \left(1-h_{st}^2\right)\sigma_{\mathrm{A},st}^{*2} \leq \chi_{st} \leq \sum_{\substack{i,i'\in I_{st}\\ i'\neq i}} w_{sit}w_{si't}\mathsf{cov}(\hat{y}_{it},\hat{y}_{i't})$$

$$\sum_{i\in I_{st}} w_{sit}^2\mathsf{var}(\hat{y}_{it}) - h_{st}^2\sigma_{\mathrm{A},st}^{*2} \leq \Gamma_{st} \leq \sum_{i\in I_{st}} w_{sit}^2\mathsf{var}(\hat{y}_{it})$$

Notation, notes, and remarks:

χ_{st}	cluster origins,	$\sum_{i\in I_{st}} w_{sit}\sum_{i'\in I_{st}\setminus\{i\}} w_{si't}\rho_{\mathrm{F},ii't}\sigma_{\mathrm{F},it}\sigma_{\mathrm{F},i't}$
Γ_{st}	granular origins,	$\sigma_{\mathrm{A},st}^2 + \sum_{i\in I_{st}} w_{sit}^2\sigma_{\mathrm{F},it}^2$
h_{st}	Herfindahl Hirschman Index in cluster s,	$[\sum_{i\in I_{st}} w_{st}^2]^{1/2} \in [N_{st}^{-1/2}, 1]$
w_{sit}	share of firm i in cluster s,	size weight
$\sigma_{\mathrm{A},st}^{*2}$	upper bound of variance of true common factor in cluster s	

Origins of macroeconomic fluctuations

Macro fluctuations: $\widehat{\text{GDP}}_t = d_t \sum_{i \in I_t} w_{it} \hat{y}_{it} = d_t \sum_{s \in S} w_{st} \hat{Y}_{st}$

$$\mathsf{var}(\widehat{\text{GDP}}_t) = d_t^2 \left[\sum_{s \in S} w_{st}^2 \sigma_{\hat{Y},st}^2 + \sum_{\substack{s,s' \in S \\ s \neq s'}} w_{st} w_{s't} \mathsf{cov}(\hat{Y}_{st}, \hat{Y}_{s't}) \right] = d_t^2 \sum_{s \in S} w_{st}^2 \left[\underbrace{\sigma_{\text{A},st}^2}_{\text{macro}} + \underbrace{\Gamma_{st}}_{\text{granular}} + \underbrace{\chi_{st}}_{\text{cluster}} \right] + \text{BIO}_t$$

- Domar weights — Domar (1961); Hulten (1978)

$$\widehat{\text{GDP}}_t = \sum_{i \in I_t} \underbrace{\frac{\text{sales}_{it-1}}{\text{GDP}_{t-1}}}_{\text{Domar weight}} \hat{y}_{it} = \underbrace{\left(\frac{\sum_{i' \in I_t} \text{sales}_{it-1}}{\text{GDP}_{t-1}} \right)}_{\text{Domar adjustment: } d_t} \sum_{s \in S} w_{st} \sum_{i \in I_{st}} w_{it} \hat{y}_{it}$$

Notation, notes, and remarks:

w_{it} and w_{sit} share of firm i in total and in cluster s, — size weight

w_{st} share of cluster s in total, — $w_{it} = w_{st} w_{sit}$

BIO_t between-industry origins,
$\text{BIO}_t = d_t^2 \sum_{\substack{s,s' \in S \\ s \neq s'}} w_{st} w_{s't} \left[\mathsf{cov}(\varepsilon_{\text{A},st}, \varepsilon_{\text{A},s't}) + \sum_{\substack{i \in I_{st} \\ i' \in I_{s't}}} w_{sit} w_{si't} \mathsf{cov}(\varepsilon_{\text{F},it}, \varepsilon_{\text{F},i't}) \right]$

Origins of aggregate fluctuations

- Empirical Strategy
- **The evolution of micro origins in the US**

Data and measurements

details & summary statistics

Sales and employments, $\texttt{sale}_{it}$ and , $\texttt{emp}_{it}$

- from Compustat North America: Fundamental Annuals (1975–2018)

Industry-level deflators, $\texttt{p}_{st}$

- from the US Bureau of Economic Analysis
- Chain-Type Price Indexes for Gross Output by Industry [2012=100]

Logged labor productivity and its business cycle components: y_{it} and $\hat{y}_{it}$

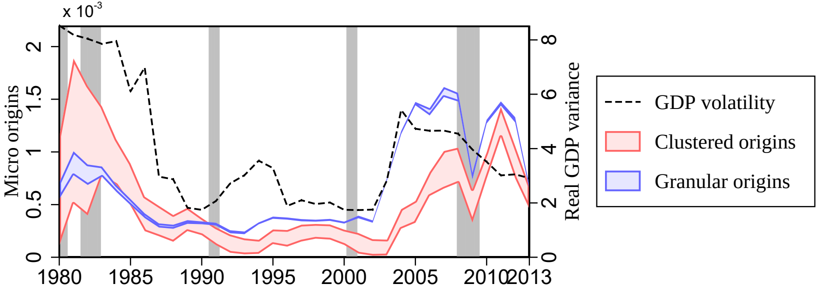
- $y_{it} = \ln \texttt{sale}_{it} - \ln \texttt{p}_{st} - \ln \texttt{emp}_{it}$
  alternatively, logged real sales $y_{it} = \ln \texttt{sale}_{it} - \ln \texttt{p}_{st}$
- its business cycle components are from

$$\hat{y}_{it} = y_{it} - \beta_s y_{it-1} - \psi_s^{\text{age}} \times \ln \text{age}_t - \psi_s^{\text{emp}} \times \ln \text{emp}_t - \psi_s^{\text{time}} \times t - \delta_i$$

alternatively, the growth rates (log-difference)

Results: Origins of macroeconomic fluctuations (1/3)

Clustered and granular origins:

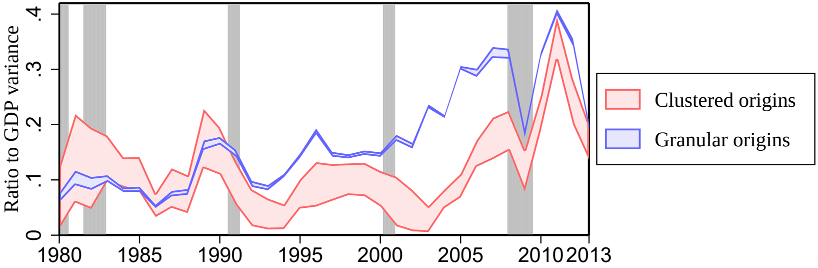


Notation, notes, and remarks:

- Compustat Annual Fundamentals North America database 1976–2018 details.
- Aggregate and industrial GDP and deflators are from Bureau of Economic Analysis.
- 53 clusters (industries).

Results: Origins of macroeconomic fluctuations (2/3)

Ratio of clustered and granular origins to GDP volatility:



Notation, notes, and remarks:

- Compustat Annual Fundamentals North America database 1976–2018 ▸ details.
- Aggregate and industrial GDP and deflators are from Bureau of Economic Analysis.
- 53 clusters (industries).

Results: Origins of macroeconomic fluctuations (3/3)

Robustness check

- with vs. without Domar adjustment
- business cycle component vs. growth rate of labor productivity
- labor productivity vs. firm (real) sales

Conclusion

I introduce cross-firm idiosyncratic shocks

- Demeaned (pseudo) productivities misrepresent cross-firm dependency when their productivities' variance-covariance is heterogeneous

Revisit micro origins of aggregate fluctuations

- Clustered micro shocks are important.
- Granularity is still important.
- Recently, I observed the rise of micro origins.

Thank you!

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Overview

A.1. Mathematical appendix

A.2. Data appendix

Pseudo common and idiosyncratic factors and their relations : homogeneous variance and covariance

back

Spurious relations with homogeneous variance and covariance

$$\text{var}(\hat{y}_{it}) = \sigma_{\Xi}^2 + \sigma_{\mathrm{F},it}^2$$

$$\text{var}(e_{\mathrm{F},it}) = \left(1 - \frac{1}{N_t}\right)(1-\rho_{\mathrm{F},t})\sigma_{\mathrm{F},t}^2 \quad \to \quad \sigma_{\mathrm{F},t}^2 - \rho_{\mathrm{F},t}\sigma_{\mathrm{F},t}^2$$

$$\text{var}(e_{\mathrm{A},t}) = \sigma_{\mathrm{A},t}^2 + \frac{\sigma_{\mathrm{F},t}^2}{N_t} + \left(1 - \frac{1}{N_t}\right)\rho_{\mathrm{F},t}\sigma_{\xi}^2 \quad \to \quad \sigma_{\mathrm{A},t}^2 + \rho_{\mathrm{F},t}\sigma_{\mathrm{F},t}^2$$

$$\text{cov}(e_{\mathrm{F},it}, e_{\mathrm{A},t}) = \text{corr}(e_{\mathrm{F},it}, e_{\mathrm{A},t}) = 0$$

$$\text{cov}(e_{\mathrm{F},it}, e_{\mathrm{F},i't}) = -\frac{1}{N_t}(1-\rho_{\mathrm{F},t})\sigma_{\mathrm{F},t}^2 \quad \to \quad 0$$

$$\text{corr}(e_{\mathrm{F},it}, e_{\mathrm{F},i't}) = -\frac{1}{N_t - 1} \quad \to \quad 0$$

Nice properties of pseudo common and idiosyncratic factors : homogeneous variance and covariance (1/2)

back

Pseudo common and idiosyncratic factors with weight

$$e^{\mathrm{w}}_{\mathrm{A},t} = \sum_{i'} \mathrm{w}_{i't}\hat{y}_{i't} = \varepsilon_{\mathrm{A},t} + \sum_{i'} \mathrm{w}_{i't}\varepsilon_{\mathrm{F},i't} \quad \text{and} \quad e^{\mathrm{w}}_{\mathrm{F},it} = \hat{y}_{it} - e_{\mathrm{A},t} = \varepsilon_{\mathrm{F},it} - \sum_{i'} \mathrm{w}_{i't}\varepsilon_{\mathrm{F},i't}$$

- small idiosyncratic variance of firms with large weights

$$\mathrm{var}(e^{\mathrm{w}}_{\mathrm{F},it}) = (1 - 2\mathrm{w}_{it} + \mathrm{m}^{\mathrm{w}}_2)(1 - \rho_{\mathrm{F},t})\sigma^2_{\mathrm{F},t}$$

- true dependency, ρ_ξ, does not matter for correlation b/w idiosyncratic shocks

$$\mathrm{corr}(e_{\mathrm{F},it}, e_{\mathrm{F},i't}) = -\frac{\mathrm{w}_{it} + \mathrm{w}_{i't} - \mathrm{m}^{\mathrm{w}}_2}{\sqrt{1 - 2\mathrm{w}_{it} + \mathrm{m}^{\mathrm{w}}_2}\sqrt{1 - 2\mathrm{w}_{i't} + \mathrm{m}^{\mathrm{w}}_2}}$$

  - more unequal weight tends to generate positive dependency
  - less (more) weighted firms tends to be positively (negatively) correlated

---

Notation, notes, and remarks:

w_{it} arbitrary weight, $\sum_{i'} \mathrm{w}_{i't} = 1$ and $\mathrm{w}_{it} > 0$

$\mathrm{m}^{\mathrm{w}}_2$ measurements how much equally weighted, $\mathrm{m}^{\mathrm{w}}_2 = \sum_{i''} \mathrm{w}^2_{i''t} \in [N_t^{-1}, 1]$

- $\mathrm{cov}(e_{\mathrm{F},it}, e_{\mathrm{F},i't}) = -(\mathrm{w}_{it} + \mathrm{w}_{i't} - \mathrm{m}^{\mathrm{w}}_2)(1 - \rho_{\mathrm{F},t})\sigma^2_{\mathrm{F},t}$

Nice properties of pseudo common and idiosyncratic factors : homogeneous variance and covariance (2/2)

back

Pseudo common and idiosyncratic factors

$$\text{var}(e_{\mathrm{A},t}) = \sigma^2_{\mathrm{A},t} + \mathsf{m}_2^{\mathsf{w}}\sigma^2_{\mathrm{F},t} + (1 - \mathsf{m}_2^{\mathrm{w}})\rho_{\mathrm{F},t}\sigma^2_{\mathrm{F},t}$$

$$\text{corr}(e_{\mathrm{A},t}, e_{\mathrm{F},it}) = -\frac{\mathrm{w}_{it} - \mathsf{m}_2^{\mathsf{w}}}{\sqrt{\frac{\sigma^2_{\mathrm{A},t}/\sigma^2_{\mathrm{F},t} + \rho_{\mathrm{F},t}}{1 - \rho_{\mathrm{F},t}} + \mathsf{m}_2^{\mathsf{w}}}\sqrt{1 - 2\mathrm{w}_{it} + \mathsf{m}_2^{\mathrm{w}}}}$$

- pseudo common and idiosyncratic shocks are correlated...it is not ideal...
  - idiosyncratic factor of firm with a small (large) weight tends to be positively (negatively) correlated to the common factor

- we need $\mathrm{w}_{it} = \mathsf{m}_2^{\mathrm{w}}$ to get uncorrelated pseudo common and idiosyncratic shocks.
  - how? set $\mathrm{w}_{it} = 1/N_t$ for all i!

Notation, useful for the next few slides, I promise...

Heterogeneous variance and covariance

numbers: 1
covariance matrix
$i = 1$

firm	1	2	3	$\cdots$	$N_t - 1$	N_t
1	✓					
2						
3						
$\vdots$						
$N_t - 1$						
N_t						

Notation:

$\sigma^2_{\mathrm{F},it}$	firm i's variance of true idiosyncratic factor, var$(\varepsilon_{\mathrm{F},it})$	
$\overline{\sigma}^2_{\mathrm{F}}$	average of true idiosyncratic factor,	$N_t^{-1} \sum_i \sigma^2_{\mathrm{F},it}$
$\mathrm{C}_{\mathrm{F},ii'}$	covariance of true idiosyncratic factor b/w firms i and i',	cov$(\varepsilon_{\mathrm{F},it}, \varepsilon_{\mathrm{F},i't})$
$\overline{\mathrm{C}}_{\mathrm{F},i}$	average of firms i's covariance of true idiosyncratic factor,	$(N_t - 1)^{-1} \sum_{i' \neq i} \mathrm{C}_{\mathrm{F},ii'}$

Notation, useful for the next few slides, I promise...

Heterogeneous variance and covariance

numbers: N_t

covariance matrix

firm	1	2	3	$\cdots$	$N_t - 1$	N_t
1	✓					
2		✓				
3			✓			
$\vdots$				$\ddots$		
$N_t - 1$					✓	
N_t						✓

Notation:

$\sigma^2_{\mathrm{F},it}$	firm i's variance of true idiosyncratic factor,	var$(\varepsilon_{\mathrm{F},it})$
$\overline{\sigma}^2_{\mathrm{F}}$	average of true idiosyncratic factor, $N_t^{-1} \sum_i \sigma^2_{\mathrm{F},it}$	
$\mathrm{C}_{\mathrm{F},ii'}$	covariance of true idiosyncratic factor b/w firms i and i',	cov$(\varepsilon_{\mathrm{F},it}, \varepsilon_{\mathrm{F},i't})$
$\overline{\mathrm{C}}_{\mathrm{F},i}$	average of firms i's covariance of true idiosyncratic factor,	$(N_t - 1)^{-1} \sum_{i' \neq i} \mathrm{C}_{\mathrm{F},ii'}$
$\overline{\overline{\mathrm{C}}}_{\mathrm{F}}$	average covariance of true idiosyncratic factor,	$N_t^{-1} \sum_i \overline{\mathrm{C}}_{\mathrm{F},i}$

Notation, useful for the next few slides, I promise...

Heterogeneous variance and covariance

numbers: 1

covariance matrix

$i = 1$

$i' = 2$

firm	1	2	3	$\cdots$	$N_t - 1$	N_t
1						
2	✓					
3						
$\vdots$						
$N_t - 1$						
N_t						

Notation:

$\sigma^2_{\mathrm{F},it}$	firm i's variance of true idiosyncratic factor,	var$(\varepsilon_{\mathrm{F},it})$
$\overline{\sigma}^2_{\mathrm{F}}$	average of true idiosyncratic factor,	$N_t^{-1} \sum_i \sigma^2_{\mathrm{F},it}$
$\mathrm{C}_{\mathrm{F},ii'}$	covariance of true idiosyncratic factor b/w firms i and i', cov$(\varepsilon_{\mathrm{F},it}, \varepsilon_{\mathrm{F},i't})$	
$\overline{\mathrm{C}}_{\mathrm{F},i}$	average of firms i's covariance of true idiosyncratic factor,	$(N_t - 1)^{-1} \sum_{i' \neq i} \mathrm{C}_{\mathrm{F},ii'}$
$\overline{\overline{\mathrm{C}}}_{\mathrm{F}}$	average covariance of true idiosyncratic factor,	$N_t^{-1} \sum_i \overline{\mathrm{C}}_{\mathrm{F},i}$

Notation, useful for the next few slides, I promise...

Heterogeneous variance and covariance

numbers: $N_t - 1$
covariance matrix
$i = 1$

firm	1	2	3	$\cdots$	$N_t - 1$	N_t
1						
2	✓					
3	✓					
$\vdots$	$\vdots$					
$N_t - 1$	✓					
N_t	✓					

Notation:

$\sigma^2_{\mathrm{F},it}$ firm i's variance of true idiosyncratic factor, var$(\varepsilon_{\mathrm{F},it})$

$\overline{\sigma}^2_{\mathrm{F}}$ average of true idiosyncratic factor, $N_t^{-1} \sum_i \sigma^2_{\mathrm{F},it}$

$\mathrm{C}_{\mathrm{F},ii'}$ covariance of true idiosyncratic factor b/w firms i and i', cov$(\varepsilon_{\mathrm{F},it}, \varepsilon_{\mathrm{F},i't})$

$\overline{\mathrm{C}}_{\mathrm{F},i}$ average of firms i's covariance of true idiosyncratic factor, $(N_t - 1)^{-1} \sum_{i' \neq i} \mathrm{C}_{\mathrm{F},ii'}$

$\overline{\overline{\mathrm{C}}}_{\mathrm{F}}$ average covariance of true idiosyncratic factor, $N_t^{-1} \sum_i \overline{\mathrm{C}}_{\mathrm{F},i}$

Notation, useful for the next few slides, I promise...

Heterogeneous variance and covariance

numbers: $N_t(N_t - 1)$

covariance matrix

firm	1	2	3	$\cdots$	$N_t - 1$	N_t
1		✓	✓	$\cdots$	✓	✓
2	✓		✓	$\cdots$	✓	✓
3	✓	✓		$\cdots$	✓	✓
$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
$N_t - 1$	✓	✓	✓	$\cdots$		✓
N_t	✓	✓	✓	$\cdots$	✓	

Notation:

$\sigma^2_{\mathrm{F},it}$	firm i's variance of true idiosyncratic factor,	var$(\varepsilon_{\mathrm{F},it})$
$\overline{\sigma}^2_{\mathrm{F}}$	average of true idiosyncratic factor,	$N_t^{-1} \sum_i \sigma^2_{\mathrm{F},it}$
$\mathrm{C}_{\mathrm{F},ii'}$	covariance of true idiosyncratic factor b/w firms i and i',	cov$(\varepsilon_{\mathrm{F},it}, \varepsilon_{\mathrm{F},i't})$
$\overline{\mathrm{C}}_{\mathrm{F},i}$	average of firms i's covariance of true idiosyncratic factor,	$(N_t - 1)^{-1} \sum_{i' \neq i} \mathrm{C}_{\mathrm{F},ii'}$
$\overline{\overline{\mathrm{C}}}_{\mathrm{F}}$	average covariance of true idiosyncratic factor, $N_t^{-1} \sum_i \overline{\mathrm{C}}_{\mathrm{F},i}$	

Pseudo common and idiosyncratic factors and their relations : heterogeneous variance and covariance

back

Spurious relations with homogeneous variance and covariance

$$
\begin{aligned}
\text{var}\left(\hat{y}_{it}\right) &= \sigma^2_{\Xi} + \sigma^2_{\mathrm{F},it} & & \\
\text{var}\left(e_{\mathrm{A},t}\right) &= \sigma^2_{\Xi} + \overline{\Psi} & \rightarrow\ & \sigma^2_{\Xi} + \overline{\overline{\mathrm{C}}}_{\mathrm{F}} \\
\text{var}\left(e_{\mathrm{F},it}\right) &= \left(\sigma^2_{\mathrm{F},it} - \Psi_i\right) - \left(\Psi_i - \overline{\Psi}\right) & \rightarrow\ & \left(\sigma^2_{\mathrm{F},it} - \overline{\mathrm{C}}_{\mathrm{F},i}\right) - \left(\overline{\mathrm{C}}_{\mathrm{F},i} - \overline{\overline{\mathrm{C}}}_{\mathrm{F}}\right) \\
\text{cov}\left(e_{\mathrm{F},it}, e_{\mathrm{A},t}\right) &= \Psi_i - \overline{\Psi} & \rightarrow\ & \overline{\mathrm{C}}_{\mathrm{F},i} - \overline{\overline{\mathrm{C}}}_{\mathrm{F}} \\
\text{cov}\left(e_{\mathrm{F},it}, e_{\mathrm{F},i't}\right) &= \left(\mathrm{C}_{\mathrm{F},ii'} - .5\Psi_i - .5\Psi'_i\right) & \rightarrow\ & \left(\mathrm{C}_{\mathrm{F},ii'} - .5\overline{\mathrm{C}}_{\mathrm{F},i} - .5\overline{\mathrm{C}}_{\mathrm{F},i'}\right) \\
&\quad - .5\left(\Psi_i - \overline{\Psi}\right) - .5\left(\Psi'_i - \overline{\Psi}\right) & & \quad - .5\left(\overline{\mathrm{C}}_{\mathrm{F},i} - \overline{\overline{\mathrm{C}}}_{\mathrm{F}}\right) - .5\left(\overline{\mathrm{C}}_{\mathrm{F},i'} - \overline{\overline{\mathrm{C}}}_{\mathrm{F}}\right)
\end{aligned}
$$

Notation, notes, and remarks:

$$
\begin{aligned}
\Psi_i &= N_t^{-1}\sigma^2_{\mathrm{F},it} + (1 - N_t^{-1})\overline{\mathrm{C}}_{\mathrm{F},i} \quad \rightarrow \overline{\mathrm{C}}_{\mathrm{F},i} \\
\overline{\Psi} &= N_t^{-1}\textstyle\sum_{i''}\Psi''_i = N_t^{-1}\overline{\sigma}^2_{\mathrm{F}} + (1 - N_t^{-1})\overline{\overline{\mathrm{C}}}_{\mathrm{F}} \quad \rightarrow \overline{\overline{\mathrm{C}}}_{\mathrm{F}}
\end{aligned}
$$

Overview

A.1. Mathematical appendix

A.2. Data appendix

Summary statistics

back

Variable		Full sample	1980–1985	1986–2000	2001–2013
Within-firm standard deviation of labor productivity: $\text{var}(\hat{y}_{it})$					
Mean		0.199	0.174	0.205	0.203
Standard deviation		0.226	0.171	0.232	0.238
Quantile	10%	0.058	0.056	0.058	0.059
	50%	0.133	0.126	0.137	0.132
	90%	0.378	0.324	0.396	0.385
Observations (firms)		82,670	13,480	35,750	33,440
Pairwise within-cluster correlation of labor productivity: $\text{corr}(\hat{y}_{it}, \hat{y}_{i't})$					
Mean		0.106	0.086	0.060	0.150
Standard deviation		0.340	0.341	0.328	0.344
Quantile	10%	−0.353	−0.366	−0.380	−0.321
	50%	0.112	0.084	0.064	0.164
	90%	0.559	0.544	0.496	0.602
Observations (pairs)		9,424,466	1,203,324	3,759,910	4,461,232

Notes: I calculate the firm i's standard deviation and pair of i and i''s correlation at time t with a rolling window of 10 years, $[t-4, t+5]$. The correlations are only for the pairs in the same cluster. There are 53 clusters.

Data construction (1/2)

back

[Step 1]
Bureau of Economic Analysis (BEA) database.

- Industry-level deflators ($\mathtt{p}_{st}$): Chain-Type Price Indexes for Gross Output by Industry [2012=100]

Compustat North America: Fundamental Annuals (1975–2018) databases.

- Sales ($\mathtt{sale}_{it}$) and employments ($\mathtt{emp}_{it}$)

[Step 2]
First, I keep the following observations in the Compustat database.

- No major mergers flag: Comparability status ($\mathtt{compst}_{it}$) does not equal to AB.
- Country ISO 3 digit code ($\mathtt{loc}_{it}$): USA
- Currency ISO 3 digit code ($\mathtt{curcd}_{it}$): USD

Data construction (2/2)

back

Then, I exclude firms with the following criteria.

- Non-positive sales
- Non-positive employments
- Utilities sector (NAICS 22)
- Public administration sector (NAICS 91–92)

[Step 3]
I merge the Compustat sample and the industry-level BEA deflator.
I calculate the logged labor productivity as real sales divided by employments ($\ln \texttt{sale}_{it} - \ln \texttt{p}_{st} - \ln \texttt{emp}_{it}$) for firm i in industry s at t.

[Step 4]
Since some clusters have low observations, I merge them.