

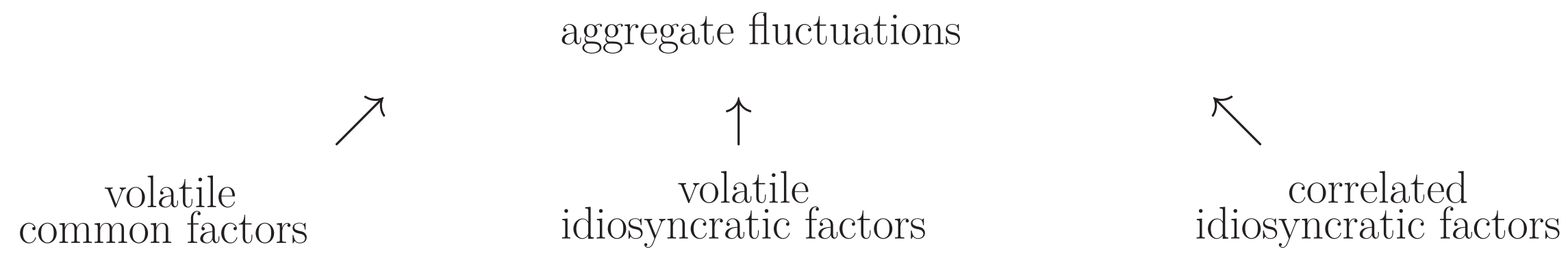
# Aggregate Fluctuations from Clustered Micro Shocks

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## Introduction

When can micro shocks generate macro fluctuations?



Idiosyncratic firm-level shocks can generate macroeconomic fluctuations where they correlate across firms. However, most business cycle research has ignored the cross-firm pairwise correlations. This paper investigates how correlated idiosyncratic movements across firms within a cluster lead to macroeconomic fluctuations.

## Framework

Firm  $i$ 's fluctuations come from two uncorrelated random variables (common and idiosyncratic parts) with zero mean:

$$\underbrace{\hat{y}_{it}}_{\text{firm fluctuation}} = \underbrace{\varepsilon_{A,t}}_{\text{common true factor}} + \underbrace{\varepsilon_{F,it}}_{\text{idiosyncratic true factor}} \quad \text{and} \quad \underbrace{\hat{Y}_t}_{\text{aggregate fluctuation}} = \sum_i \underbrace{w_{it}}_{\text{share}} \times \hat{y}_{it}$$

$$= \underbrace{e_{A,t}}_{\text{pseudo common factor}} + \underbrace{e_{F,it}}_{\text{pseudo idiosyncratic factor}} \quad \text{where} \quad e_{A,t} = N_t^{-1} \sum_{i'} \hat{y}_{i't} \quad \text{and} \quad e_{F,it} = \hat{y}_{it} - e_{A,t}$$

Many studies use the pseudo factors; the sample mean and the deviation from it because it is hard to identify true common and idiosyncratic factors from firm fluctuations. That is because  $\hat{y}_{it}$ ,  $e_{F,it}$ , and  $e_{A,t}$  are directly observable, however,  $\varepsilon_{F,it}$ , and  $\varepsilon_{A,t}$  are not.

## Identical variance and covariance

Consider a cluster where firms have identical standard deviation and pairwise correlation of idiosyncratic shocks;  $\text{sd}(\varepsilon_{A,t}) = \sigma_{F,t} > 0$  and  $\text{corr}(\varepsilon_{A,t}, \varepsilon_{F,it}) = \rho_{F,t} \in (-1, 1)$ .

The aggregate fluctuations come from three sources.

$$\underbrace{\sigma_{Y,t}^2}_{\text{aggregate volatility}} = \underbrace{\sigma_{A,t}^2}_{\text{common factor volatility}} + \underbrace{h_t^2 \sigma_{F,t}^2}_{\text{idiosyncratic factor volatility}} + \underbrace{(1 - h_t^2) \rho_{F,t} \sigma_{F,t}^2}_{\text{idiosyncratic factor dependency}}$$

where  $h_t = [\sum_{i'} w_{i't}^2]^{1/2}$  is Herfindahl Hirschman Index.

Now, consider widely used pseudo common and idiosyncratic factors. They are orthogonal. Thus, it can be well-defined macro and micro shocks. In this case, the clustered origins with pseudo terms are negligible where there exist many firms.

**[Proposition 1]** The cross-sectional sample mean and the deviations from it have the following correlations. For  $\forall i \neq i'$ ,

$$\text{corr}(e_{A,t}, e_{F,it}) = 0 \quad \text{and} \quad \text{corr}(e_{F,it}, e_{F,i't}) = -\frac{1}{N_t - 1}$$

**[Corollary 1]** The variance of aggregate fluctuations can be decomposed into the pseudo common and idiosyncratic shocks' variances asymptotically.

$$\sigma_{Y,t}^2 = \text{var}(e_{A,t}) + h_t^2 \text{var}(e_{F,it}) - \left( \frac{1 - h_t^2}{N_t - 1} \right) \text{var}(e_{F,it})$$

## Heterogeneous variance and covariance

Consider a cluster where firms have different standard deviation and pairwise correlation of idiosyncratic shocks;  $\text{sd}(\varepsilon_{A,t}) = \sigma_{F,it} > 0$  and  $\text{corr}(\varepsilon_{A,t}, \varepsilon_{F,it}) = \rho_{F,it} \in (-1, 1)$ .

Now, consider widely used pseudo common and idiosyncratic factors. They are not orthogonal. Also, their correlations are not zero and not systemically linked with correlations of true idiosyncratic factors. Thus, they lead to an incomplete understanding of business cycles. Therefore, the business cycle research should identify true factors instead of using the pseudo factors.

**[Proposition 2]** Consider a cluster where idiosyncratic shocks' standard deviation and pairwise correlation are different across firms. Then, the covariance between the cross-sectional sample mean and firm  $i$ 's deviation from it is non-zero in general.

$$\text{cov}(e_{A,t}, e_{F,it}) = N_t^{-1} [\sigma_{F,it}^2 - N_t^{-1} \sum_{i'} \sigma_{F,i't}^2] + [N_t^{-1} \sigma_{F,it} \sum_{i' \neq i} \rho_{F,i't} \sigma_{F,i't} - N_t^{-1} \sum_{i'} N_t^{-1} \sigma_{F,i't} \sum_{i'' \neq i'} \rho_{F,i''t} \sigma_{F,i''t}]$$

## Clustered Origins

This paper defines clustered origins by contribution of pairwise correlations within cluster. An individual firm  $i \in I_{st} \subset I_t = \cup_{s \in S} I_{st}$  is established in one cluster  $s \in S$ .  $S$  and  $I_{st}$  denote the set of clusters and the set of firms in cluster  $s$ .  $w_{st}$  and  $w_{sit}$  are the share of cluster  $s$  in the whole economy and the firm share within cluster  $s$ . ( $w_{it} = w_{st} w_{sit}$ )

The clustered and granular origins of cluster  $s$  can be rewritten as follows.

$$\chi_{st} = \sum_{i \in I_{st}} w_{sit} \sum_{i' \in I_{st}, i' \neq i} w_{si't} \rho_{F,i't} \sigma_{F,it} \sigma_{F,i't} \quad \text{and} \quad \Gamma_{st} = \sum_{i \in I_{st}} w_{sit}^2 \sigma_{F,it}^2$$

The business cycle component of GDP is  $\widehat{\text{GDP}}_t = d_t \sum_{i \in I_t} w_{it} \hat{y}_{it} = d_t \sum_{s \in S} w_{st} \hat{Y}_{st}$  where  $\hat{Y}_{st} = \sum_{i \in I_{st}} w_{sit} \hat{y}_{it}$ .  $d_t$  is the Domar weight adjustment. The GDP volatility is

$$\text{var}(\widehat{\text{GDP}}_t) = d_t^2 \sum_{s \in S} w_{st}^2 [\sigma_{A,st}^2 + \chi_{st} + \Gamma_{st}] + \text{BIO}_t,$$

where  $\text{BIO}_t$  represents GDP fluctuations from between-industry comovements. Finally, the overall clustered and granular origins are  $\chi_t = d_t^2 \sum_{s \in S} w_{st}^2 \chi_{st}$  and  $\Gamma_t = d_t^2 \sum_{s \in S} w_{st}^2 \Gamma_{st}$ .

## Identification Strategy

The goal is to quantify the clustered origins of aggregate fluctuations. This is challenged by the notion that I observe a firm's business cycle component ( $\hat{y}_{it}$ ), but its true common and idiosyncratic parts ( $\varepsilon_{A,t}$  and  $\varepsilon_{F,it}$ ) are not observed. Without the help of additional information, it is impossible to decompose one into two parts.

Instead of estimating their point values, I provide an individual firm's upper and lower bound of the common shock's variance from the variance and covariance of the observed  $\{\hat{y}_{it}\}_{i \in I_{st}}$  using some statistical properties.

**[Proposition 3]** In cluster  $s$ , the common shock's variance is not larger than  $\sigma_{A,st}^{*2}$ .

$$0 \leq \sigma_{A,st}^2 \leq \sigma_{A,st}^{*2} = \min_{i, i' \in I_{st}} \{ \text{var}(\hat{y}_{it}), [1 + \text{corr}(\hat{y}_{it}, \hat{y}_{i't})] \text{sd}(\hat{y}_{it}) \text{sd}(\hat{y}_{i't}) \}$$

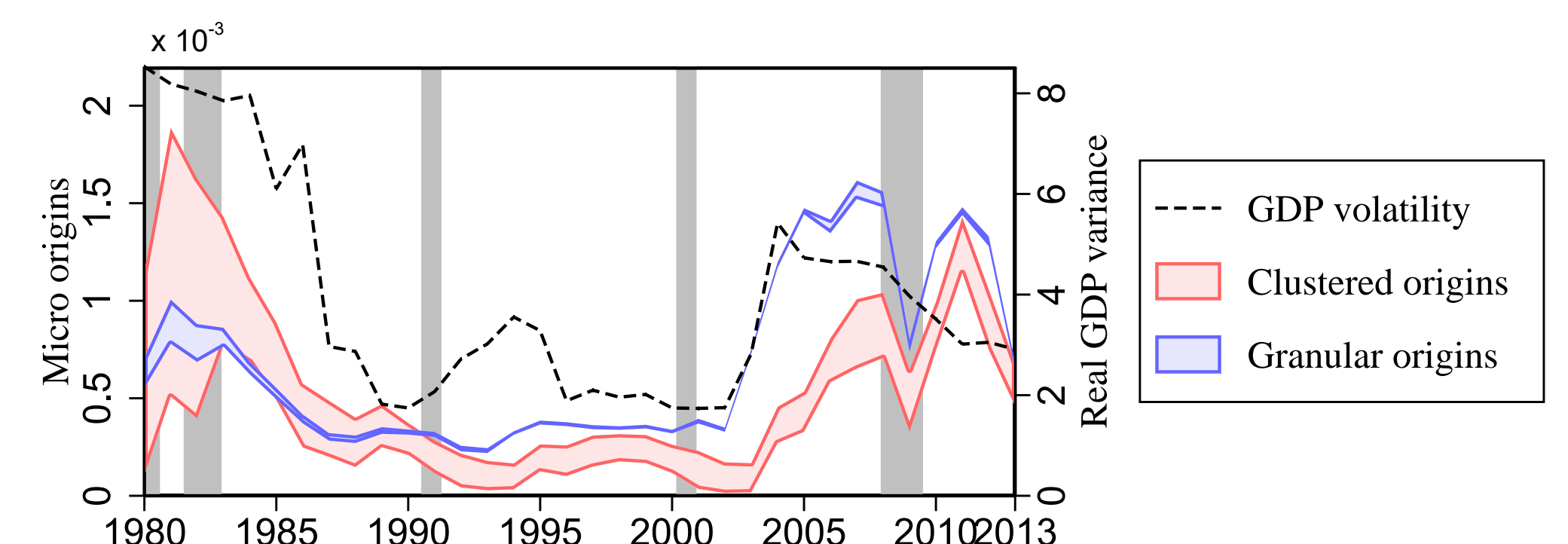
**[Corollary 2]** The clustered and granular origins ( $\chi_{st}$  and  $\Gamma_{st}$ ) are bounded as follows.

$$\sum_{i, i' \in I_{st}, i' \neq i} w_{sit} w_{si't} \text{cov}(\hat{y}_{it}, \hat{y}_{i't}) - (1 - h_{st}^2) \sigma_{A,st}^{*2} \leq \chi_{st} \leq \sum_{i, i' \in I_{st}, i' \neq i} w_{sit} w_{si't} \text{cov}(\hat{y}_{it}, \hat{y}_{i't})$$

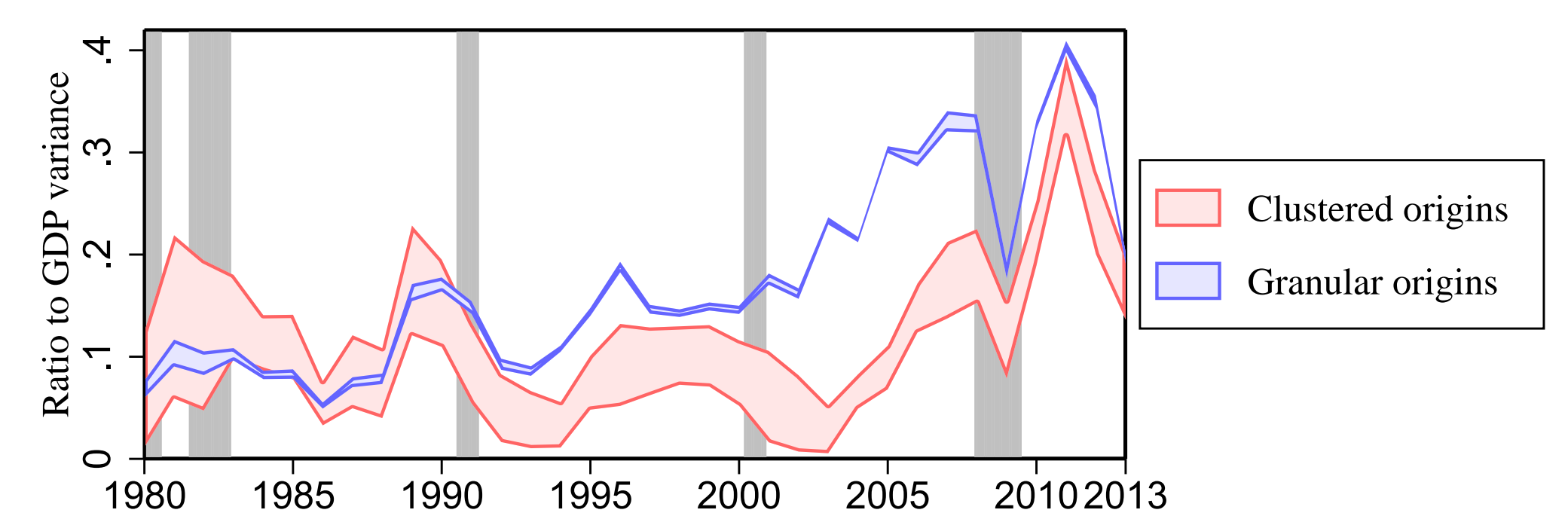
$$\sum_{i \in I_{st}} w_{sit}^2 \text{var}(\hat{y}_{it}) - h_{st}^2 \sigma_{A,st}^{*2} \leq \Gamma_{st} \leq \sum_{i \in I_{st}} w_{sit}^2 \text{var}(\hat{y}_{it})$$

## The Evolution of Cluster Origins of Macro Fluctuations

Clustered and granular origins



Ratio of clustered and granular origins to GDP volatility



Data Source: Compustat Annual Fundamentals North America database 1976–2018

## Conclusion

Demeaned (pseudo) fluctuations misrepresent their pairwise correlations of micro shocks and contributions to macro fluctuations when their variance-covariance differs across firms.

This paper highlights the importance of pairwise correlations (clustered origins) in US.

- Clustered origins well explain the evolution of the US GDP's volatility, especially the great moderation.
- Clustered origins rise from around 10% to 25% in the recent two decades.



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