Stock Returns, Market Trends, and Information Theory: A Statistical Equilibrium Approach

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INTRODUCTION

Since the advent of the Efficient Market Hypothesis (EMH), the search for **serial correlations** in stock returns has been one the main criteria to assess martet efficiency (Fama, 1965; Bhowmik and Wang, 2020).

Three main theoretical and empirical issues are associated to serial correlations:

1. Sample size and power of econometric tests (Shiller, 2015).
2. Sources of statistical dependence and randomness (Fama, 1965).
3. Informational component of stock prices (Mantegna and Stanley, 1999).

Given the centrality of the **informational content** of stock prices raised by the EMH, how could we deploy such information to better understand the behavior of stock returns?

METHODOLOGY

To explain the observed statistical regularities in the distributions of stock returns, we adopt the **Quantal Response Statistical Equilibrium** (QRSE) model (Scharfenaker and Foley, 2017).

The logic of the model is based on a process of Smithian competition. Investors, seeking above-average rates of return from their transactions, generate "tendential gravitation" around an average rate of return as an unintentional result of their interactions with other actors.

Based on an entropy-constrained framework, the model derives equilibrium as an information theoretic probability distribution representing all possible states of the system.

OBJECTIVES

1. Analyzing **statistical regularities** of stock returns over different market periods, by a developing an **entropy-constrained statistical equilibrium model**.

2. Explaining **randomness in stock prices** as the **unintended consequence** of investors seeking higher rates of return.

3. Providing an **original assessment of the EMH** by considering the role of **unfulfilled expectations** of investors, and how they impact stock market volatility.

THE QRSE MODEL

1. **Quantal response behavior** of market participants:
   - μ : Fair value (expected average payoff).
   - T : Agent responsiveness to variations in returns.

2. **Negative feedback** of individual actions on market outcomes:
   - α : "Conventional" (market) rate of return.
   - S : Market responsiveness to variations in returns.

3. **The role of expectations**, as captured by the skewness:
   - $\zeta = \mu - \alpha$ ($= 0, > 0, < 0$): Degree of expectation fulfilment.

DATA COLLECTION

We compute **logarithmic daily returns** of the **individual companies** listed in the **S&P 500**, over the period **01/01/1988 – 12/31/2019**.

We divide our sample into **bull**, **bear markets** (declines of 20% or more over at least a two-month period), and **corrections** (declines of 10% from the most recent peak).

We consider the **cross-sectional distributions** of individual companies' returns, and then analyze their **statistical regularities** (Figures 1 and 2).

CONCLUSIONS

We find evidence of **punctuated statistical equilibrium** over multiple market periods, disrupted by structural changes affecting the stock market (Figure 3).

We find evidence of **significant deviations of individual expectations from market outcomes** over extended time periods, even though they remain quire consistent over the long-run (average $\zeta = 0.06$ %\day).

We show how the **stochastic nature of stock prices** can be explained as the **spontaneous convergence** of the system towards a **market convention**.

Figure 1. Cross-sectional distributions over bull, bear markets, and corrections.

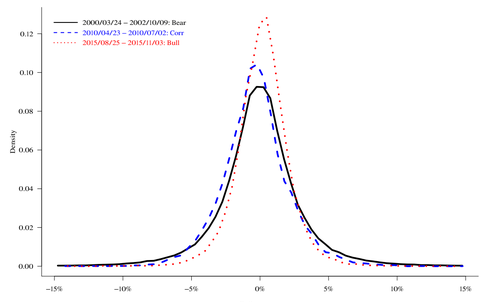


Figure 2. Empirical moments over bull, bear markets (red bars), and corrections (gray bars).

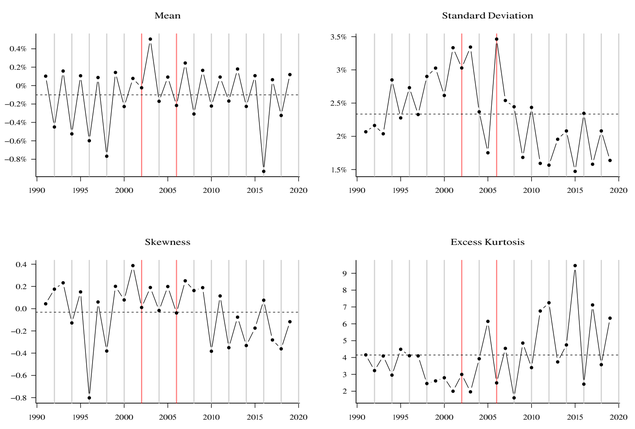
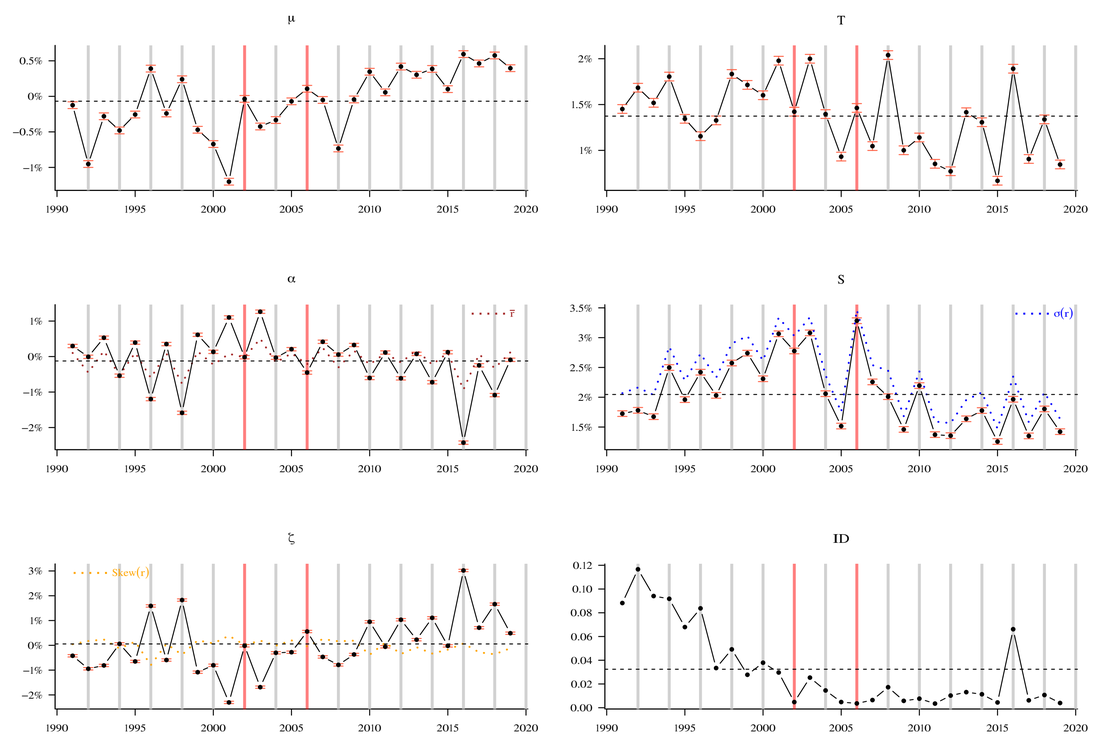


Figure 3. Time series of parameter estimates (%/day).



Note: The grey bars denote corrections, whereas the red bars bear markets. The red segments shows the 95% credibility interval, whereas the dashed lines show the average value of each parameter over the whole sample.

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