

# Cyclical and Asymmetry of the User Cost of Labor: Evidence and Theory\*

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## Abstract

The user cost of labor (UCL) plays an allocative role in a wide class of macroeconomic models. Despite this appealing feature, estimating the UCL involves empirical challenges as it requires a sequence of wages from hiring until separation. The cyclical changes in the average quality of new matches weigh on measuring the cyclical of the UCL. In this paper, we overcome these challenges by exploiting unique Japanese data, which tracks wages at each tenure after school graduation. Our empirical findings are twofold. First, the UCL remains highly procyclical after controlling for the cyclical changes in the average job-match quality, whereas the new-hire wage is no longer more cyclical than the incumbent-worker wage. Second, downward adjustments of the UCL are smaller than upward ones. The downward rigidity arises from the combination of that of the new-hire and incumbent-worker wages. We then develop a wage posting model to account for these observations. We demonstrate that under asymmetric information, firms use wages as a screening tool to receive the application from targeted workers and maintain a high value of a posted contract in recessions, leading to downward rigidity and overall high cyclical of the UCL.

**JEL classification:** E24; E32; J31.

**Keywords:** User cost of labor; Wage cyclical; Dwonward wage rigidity; Wage posting.

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\*The views expressed herein are those of the author and should not be attributed to the IMF, its Executive Board, or its management.

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# 1 Introduction

Wage cyclicality is a central question in macroeconomics. The literature has found that wage cyclicality (or rigidity) is one of crucial factors to account for unemployment and other aggregate dynamics. Responding to [Shimer \(2005\)](#)'s puzzle that a standard search and matching model fails to generate the sizable unemployment fluctuations observed in the data, [Hall \(2005\)](#) argues that the incorporation of wage rigidity substantially increases the sensitivity of unemployment to shocks. [Christiano et al. \(2005\)](#) find that wage rigidity is the most important friction to replicate the responses of aggregate variables to a monetary policy shock in a medium-scale model.

The criticality of wage cyclicality hinges on the assumption that wage plays an allocative role in the labor market. In a textbook model, wage, as a price of labor service, lets labor demand and supply match, thereby determining the labor market allocation. However, it is quite difficult to observe such “allocative” wage in the data. Workers and firms may explicitly or implicitly agree to a long-term employment contract. In this situation, the payment that workers receive in each period does not necessarily represent the price of labor service in that period. For example, the total compensation can be allocated evenly across the contract periods between risk-neutral firms and risk-averse workers (e.g., [Beaudry and DiNardo 1991](#)). The cyclicality of current wages does not have meaningful implications to the labor market dynamics under this circumstance (e.g., [Barro 1977](#)).

To address this issue, [Kudlyak \(2014\)](#) proposes a concept of the user cost of labor (UCL). In her view, labor is a long-term asset with costly adjustments. The UCL is defined as the difference between the present values of total labor costs at two points in time. It is verified that the UCL plays an allocative role in a wide class of models, including an implicit contract model ([Basu and House 2016](#)) and search and matching model ([Kudlyak 2014](#)).

Despite this appealing feature, estimating the UCL is no easy task. This is partly because the estimation requires a sequence of wages from hiring until separation. Previous studies utilize the panel structure in the National Longitudinal Surveys of Youth (NLSY) to estimate wages at different lengths of service. However, the length of period and sample size of the NLSY79 leave only 20 to 30 observations of the estimated UCL with little cross-sectional

variation, limiting the range of possible analysis.<sup>1</sup> More importantly, [Gertler et al. \(2020\)](#) point out that the estimated UCL is subject to cyclical changes in the average job-match quality. Workers receive more job offers in expansions, and thus more likely to move to better job matches even if each draw of job-match quality is idiosyncratic (e.g., [Hagedorn and Manovskii 2013](#)). The composition effect renders the average quality of new matches procyclical, overstating the cyclicity of the new-hire wage and therefore that of the UCL.

In this paper, we overcome these empirical challenges by exploiting unique Japanese wage data. Our primary dataset is the Basic Survey on Wage Structure (BSWS) conducted by the Ministry of Health, Labour and Welfare of Japan. The BSWS is a nationally representative survey in which randomly selected firms report their employees' wages and hours worked according to the payroll records. The sample covers 20 million of workers—around one third of the total labor force in the country—with various firm-worker characteristics, including gender, age, educational attainment, and firm size. Notably, the BSWS compiles workers' wages at each length of service, which means that we can directly observe a sequence of wages from hiring until separation.

To control for the aforementioned cyclical change in the average job-match quality, we focus on new graduates from school. As the procyclical upgrading of job-match quality occurs through job changes, new school graduates are supposed to be free from it. The approach is inspired by [Gertler et al. \(2020\)](#), who use the sample of unemployment-to-employment flows in the Survey of Income and Program Participation (SIPP) to control for the composition effects. However, the SIPP does not cover wages of each worker for a long enough period after hiring to estimate the UCL, while the limited sample size of the NLSY poses a difficulty in adopting this approach. In contrast, the design of the BSWS, as well as its ample sample size, allows us to keep track of wages of new school graduates to estimate the UCL.

We use the estimated series of the UCL to investigate its cyclical properties. Our main findings are twofold. First, after controlling for the composition effects of job-match quality, the UCL remains highly procyclical. The estimated semi-elasticity of the UCL with respect

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<sup>1</sup>There are three existing studies that estimate the UCL to our knowledge: [Kudlyak \(2014\)](#); [Basu and House \(2016\)](#); and [Doniger \(2021\)](#). All of them use the NLSY79. [Kudlyak \(2014\)](#) and [Basu and House \(2016\)](#) estimate the aggregate series for the UCL whereas [Doniger \(2021\)](#) constructs the series for each level of educational attainment.

to the unemployment rate is around twice as large as that of the average wage. In contrast, the new-hire wage is no longer more cyclical than the incumbent-worker wage after controlling for the composition effects consistent with [Gertler et al. \(2020\)](#). These results indicate that the high cyclical nature of the UCL arises from the rigidity of the incumbent-worker wage. To see this point, suppose that wages are persistently high (low) for workers hired during expansions (contractions). Then the total cost attributed to their labor service reflects not only the high (low) wage as of hiring but also the wage differences among hiring cohorts in the course of their tenure. The UCL captures the dynamics of such total labor cost.

Second, the UCL is downwardly rigid. Its adjustments are found statistically significant only in an upward phase. While the downward wage rigidity of incumbent workers has been widely reported in the literature, it does not suffice for the downward rigidity of the UCL if the new-hire wage is downwardly flexible. Indeed, our data suggests the downward wage rigidity of both new hires and incumbent workers. To clarify, our measure of new-hire wage is at the job level, i.e., a series of wages for new hires in each period, though previous studies report downward flexibility of the new-hire wage at the worker level, i.e., wage changes of each worker before and after a job change. Our view is that wages at the job level are relevant for a measure of labor cost from the firms' perspectives.

While the high cyclical nature of the UCL is in line with the conventional wisdom of incumbent-worker wage rigidity, relatively less is known about the downward rigidity of the UCL.<sup>2</sup> Thus, we develop a model to reconcile our empirical findings. Our benchmark is [Rudanko \(2009\)](#), who introduced a wage contract of [Beaudry and DiNardo \(1991\)](#) and [Thomas and Worrall \(1998\)](#) into a search and matching framework. A firm-worker match is attained in the form of directed search, i.e., firms post a wage contract in a submarket and workers choose which submarket to apply to. The model embeds incumbent worker's wage rigidity stemming from the insurance that risk-neutral firms offer to risk-averse workers. Our extension involves productivity heterogeneity in both sides of firms and workers. High-productivity firms seek to match with skilled workers, but they have limited ability to distinguish worker type. The setting gives rise to an adverse selection where unskilled workers have an incentive to apply

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<sup>2</sup>An often-used explanation is a fairness constraint that the new-hire wage should be set equal to the incumbent-worker wage (e.g., [Snell and Thomas 2010](#); [Rudanko forthcoming](#)). However, our empirical evidence does suggest the presence of cohort wage differences.

to high-productivity firms. Against this backdrop, firms use wages as a screening tool to receive the application only from the targeted type of workers. Firms keep the value of a posted contract high enough to make their labor market “too competitive” for unskilled workers to apply to. The mechanism accounts for the downward rigidity of the UCL, as well as its overall high cyclical. An implication of the model is that the downward rigidity of the UCL generates asymmetric labor market dynamics consistent with the data.

**Related literature.** There is a long-lasting and unsettled debate on wage cyclical. The primary area of this paper’s contribution is to elaborate on a wage measure in the presence of a long-term employment contract. This issue was originally raised by [Beaudry and DiNardo \(1991\)](#) and has recently been tackled by [Kudlyak \(2014\)](#) and [Basu and House \(2016\)](#) by introducing the concept of the UCL. As explained above, our analysis addresses empirical challenges raised by previous studies (e.g., [Gertler et al. 2020](#)) and confirms the overall high cyclical of the UCL.<sup>3</sup> In addition, our rich dataset enables us to examine asymmetry of cyclical to uncover downward rigidity of the UCL.

At the same time, this paper revisits the cyclical of new-hire and incumbent-worker wages.<sup>4</sup> Various studies report greater wage cyclical for new hires than for incumbent workers (see a survey by [Pissarides 2009](#)). However, recent studies raise a question on this conclusion by controlling for worker productivity and job-match quality (e.g., [Grigsby et al. 2021](#); [Gertler et al. 2020](#)) and focusing on job-level wages, e.g., posted wages for each job (e.g., [Hazell and Taska 2020](#)). Our empirical finding that there is no significant difference in the cyclical of new-hire and incumbent-worker wages is consistent with these studies.

This paper also touches upon the measurement of compensation. In this regard, cyclical may differ across the components of compensation. For example, [Swanson \(2007\)](#)

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<sup>3</sup>[Basu and House \(2016\)](#) construct a proxy for the job-match quality following [Hagedorn and Manovskii \(2013\)](#) to find that the UCL remains highly procyclical. We take a more direct route by limiting the sample to new school graduates.

<sup>4</sup>The distinction is important because different workers’ wages can play considerably different allocative roles. On the one hand, [Pissarides \(2009\)](#) argues that the new-hire wage, which is proportional to the UCL in his setting, is relevant for the labor market dynamics in a canonical search and matching model. On the other hand, [Fukui \(2020\)](#) demonstrates that in the presence of on-the-job search, the incumbent-worker wage is a critical determinant of the unemployment fluctuations. Though our model does not take into account on-the-job search, our proposed mechanism can be compatible with it.

finds that non-base components, such as overtime pay, are more procyclical than base pay. [Grigsby et al. \(2021\)](#) report that bonuses are mainly driven by idiosyncratic factors and do not necessarily respond to aggregate conditions. Moreover, non-cash benefits, including employee contributions to social security and health insurance, may display distinctive patterns depending on their nature (e.g., [Lebow et al. 2003](#); [Gu et al. 2020](#)). Built upon these studies, we focus on total cash earnings, including bonuses, overtime pay, and other cash benefits, to accommodate adjustments by using various compensation components. Though our data does not cover non-cash benefits, there is limited room for the arbitrage use of non-cash benefits in the legal framework of Japan, as discussed later.

On the theoretical front, this paper contributes to a model selection for wage setting. A seminal work by [Beaudry and DiNardo \(1991\)](#) finds that the current wage depends on the history of the labor market conditions at the time of and after hiring. [Thomas and Worrall \(1998\)](#) formally show that this pattern is consistent with an implicit wage contract with limited commitment under which a risk-neutral firm offers a fixed wage contract to risk-averse workers as insurance against future shocks as far as the match entails surplus. [Hagedorn and Manovskii \(2013\)](#) challenge this view by arguing that on-the-job search in the spot market leads to a cyclical upgrading of job-match quality, replicating the observed history dependence of the current wage. To reconcile the debate, [Bellou and Kaymak \(2021\)](#) introduce a measure of job-match quality that accounts for the dissolution of poor matching due to quits in expansions and lay-off in contractions, and find evidence in favor of an implicit contract. This paper takes a more direct approach by focusing on workers who remain at the same job after graduation from school to exclude the cyclical upgrading of job-match quality through job changes. We still find the dependence of incumbent worker wage on the past labor market conditions, supporting the presence of an implicit contract.

In addition, our model proposes a novel explanation of the downward rigidity of the UCL. Although the literature has proposed various sources of the downward wage rigidity for incumbent workers, including sciological factors (e.g., [Bewley 1999](#)), sharking (e.g., [Shapiro and Stiglitz 1984](#)), and collective bargaining (e.g., [Holden 1994](#)), that is not warranted for the UCL or the new-hire wage. The mechanism we propose is closely related to the efficiency wage (e.g., [Solow 1979](#); [Yellen 1984](#)), which hypothesizes a positive association between work-

ers' productivity and wage. Among others, [Stiglitz \(1976\)](#) analyzes various wage policies and the allocation of workers with different productivity in the spot labor market. Our setting is also related to [Guerrieri et al. \(2010\)](#), who study adverse selection in directed search models. A key distinction from these studies is that in our model, a single wage policy can attain a separation equilibrium by resorting to the different trade-off between wage and job-finding rate that each type of workers face.

**Layout.** The remainder of the paper is organized as follows. Section 2 describes the definition and measurement of the UCL. Section 3 explains the BSW data. Section 4 presents our main empirical results, and Section 5 is devoted to robustness check and discussion. Section 6 develops a model to account for the empirical results. Section 7 concludes.

## 2 Methodology

**Definition.** Let  $PDV_t$  be the expected present discounted value (PDV) of future allocative wages, or the user cost of labor,  $UCL_{t+\tau}$ :

$$PDV_t := \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} \beta^\tau (1-s)^\tau UCL_{t+\tau} \right], \quad (1)$$

where  $\beta$  is the discount factor and  $s$  is the separation rate. Rearranging this equation, the UCL can be written as the differential of the present values of the current and future labor:

$$UCL_t = PDV_t - \mathbb{E}_t [\beta(1-s)PDV_{t+1}], \quad (2)$$

Notice that  $PDV_t$  should be equal to the PDV of future remitted wages,  $w_{t,t+\tau}$ :

$$PDV_t = \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} \beta^\tau (1-s)^\tau w_{t,t+\tau} \right], \quad (3)$$

where  $w_{t,t+\tau}$  denotes the wage paid at  $t + \tau$  to the worker hired at  $t$ . Equations (2) and (3) map the remitted wages to the UCL.

**What does the UCL capture?** Before turning to an empirical counterpart, it is helpful to rewrite the UCL as follows.

$$UCL_t = w_{t,t} + \mathbb{E}_t \left[ \sum_{\tau=1}^{\infty} \beta^\tau (1-s)^\tau (w_{t,t+\tau} - w_{t+1,t+\tau}) \right]. \quad (4)$$

Equation (4) indicates that the UCL consists of the wage at hiring, or the new-hire wage,  $w_{t,t}$ , in the first term and the discounted sum of the wage differences between hiring cohorts in the subsequent periods in the second term.

Two special cases would help gain intuition on the cyclicity of the UCL. The first case is where all workers are treated equally regardless of the timing of their hiring. In the absence of cohort differences, the UCL coincides with the new-hire wage, i.e.,  $UCL_t = w_{t,t}$  if  $w_{t,t+\tau} = w_{t+1,t+\tau}$  for all  $\tau \geq 1$ . The other extreme case is that each cohort of workers is treated differently throughout their tenure. Suppose that wages are fixed permanently after hiring, i.e.,  $w_{t,t+\tau} = w_{t,t}$ . It is immediate to show  $UCL_t = 1/(1-\beta(1-s))w_{t,t} - \beta(1-s)/(1-\beta(1-s))\mathbb{E}_t[w_{t+1,t+1}]$ , implying that the impact of  $w_{t,t}$  is scaled up by  $1/(1-\beta(1-s)) > 1$ . These examples show that the cyclicity of UCL is closely linked to that of the new-hire wage, but also depends on the extent to which the cohort differences persist after hiring with the rigidity of the incumbent worker wage amplifying the cyclicity of UCL.

**Estimation strategy.** Our objective is to construct a measure of UCL from the data. This is isomorphic to obtaining a sequence of wages from hiring until separation,  $\{w_{t,t+\tau}\}_{\tau=0}^{\infty}$ . We construct the real hourly wage  $w_{t,t+\tau}^c$  and the number of workers  $N_{t,t+\tau}^c$ , where  $\tau$ ,  $c$ , and  $t$  represent the length of each service, the characteristics of each employer-employee, and time of hiring, respectively. These observations allow us to compute the UCL accordingly:

$$UCL_t^c = w_{t,t}^c + \sum_{\tau=1}^{T-1} \beta^\tau (1-s)^\tau (w_{t,t+\tau}^c - w_{t+1,t+\tau}^c), \quad (5)$$

$$\text{where } s^c = 1 - \frac{1}{t_{end}} \sum_{t=1}^{t_{end}} \left( \frac{N_{t,t+T-1}^c}{N_{t,t}^c} \right)^{\frac{1}{T-1}}, \quad (6)$$



with  $t_{end}$  being the end period of sample.  $T$  is the length of period used for the calculation of the PDV, as described shortly. The discount factor  $\beta$  is set to 0.97, following [Basu and House \(2016\)](#).

**Assumption and discussion.** Assumptions of our measure of UCL (5)–(6) in comparison to the definition (2)–(3) are summarized below.

*Assumption 1 (Perfect foresight).* The expectation operator in equations (2)–(3) is replaced with the realized values, as the expected wages are not observed in the data. In other words, our UCL measure is an ex-post one. The assumption follows previous studies ([Kudlyak 2014](#); [Basu and House 2016](#)).

*Assumption 2 (Years of truncation).* The calculation of PDV is truncated at some year  $T$ .  $T$  should be sufficiently long so as not to affect the cyclical properties of the UCL. We set  $T = 10$  years because we find that the cohort differences in the remitted wage remain statistically significant up to ten years after hiring in our sample, as shown in Section 4.3.<sup>5</sup> We verify that a further increase in  $T$  has little impact on our results.

*Assumption 3 (Time-invariant separation rate).* The separation rates are assumed constant over time in each category. Indeed, they are only insignificantly responsive to business cycle fluctuations in our sample, as shown in Section 5.2.<sup>6</sup> Note that our assumption is more general than previous studies, which assume that the separation rate is identical across all workers.

*Assumption 4 (Time-invariant mean of unobserved heterogeneity).* We use semi-aggregate wage series for each employer-employee characteristics compiled in the BSWS. Thus, the wage dynamics are subject to workers' composition changes within each category. In other words, our implicit assumption is that unobserved heterogeneity in workers' productivity and match quality within each category is independent of business cycle fluctuations. This assumption is considered less critical in our analysis for the following reasons. First, as de-

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<sup>5</sup>[Kudlyak \(2014\)](#) and [Basu and House \(2016\)](#) use  $T = 7$  years for U.S. data.

<sup>6</sup>This is not inconsistent with an implicit contract. Even under the limited commitment of workers' participation, firms may adjust the incumbent-worker wage enough to avoid their quits. We obtain empirical evidence consistent with this hypothesis, presented in Section 5.2.

scribed in Section 3, various observed employer-employment characteristics are controlled. These include worker age, years of service, educational attainment, gender, and firm size, with the total data points in each year amounting to 2,500. Second, since we focus on new school graduates, the job-match quality is not affected by the cyclical upgrading through on-the-job search. Moreover, the productivity of new school graduates is supposed to depend mainly on their educational background, and thus their heterogeneity is considered to remain stable over business cycles. Third, the composition changes through separation are less concerning due to the low and stable separation rates in the Japanese labor market. We do not find significant changes in the separation rate over business cycles. Fourth, another concern is that larger (smaller) numbers of new hires in booms (recessions) imply that lower (higher) productivity workers become employed if sorting takes place in the labor market. However, this channel will create counter-cyclicality in the average productivity of new hires, thereby undermining the cyclicity of the UCL. Thus, our result of high cyclicity of the UCL would be further reinforced if the channel were controlled.

**Alternative wage measures.** For comparison purposes, we construct the average wage  $w_t^{ave,c}$ :

$$w_t^{ave,c} = \frac{\sum_{\tau=0}^{\tau_{max}} N_{t-\tau,t}^c w_{t-\tau,t}^c}{\sum_{\tau=0}^{\tau_{max}} N_{t-\tau,t}^c}, \quad (7)$$

where  $\tau_{max}$  is the maximum tenure, and the new-hire wage  $w_t^{new,c}$ :

$$w_t^{new,c} = w_{t,t}^c. \quad (8)$$

### 3 Data

**BSWS.** Our primary dataset is the Basic Survey on Wage Structure (BSWS) of Japan. The BSWS is a nationally representative survey conducted by the Ministry of Health, Labour and Welfare (MHLW). As this survey is classified as a fundamental statistic under the Statistics Act of Japan, survey subjects are obliged to report with a penalty if violating it.

Subjects of the survey are firms employing more than five full-time employees. Sampling

takes two steps. In the first step, firms are randomly selected to represent the whole economy in terms of geography, industry, and firm size. In the second step, employees are chosen in each selected firm to represent various categories. The number of firms sampled is around 80 thousand as of 2016 from the population of about 1,400 thousand. The survey is conducted annually in July, and asks the information as of June.

The BSWS has been conducted annually since 1954. We use the 1980 to 2019 surveys to examine the low and stable inflation periods in recent decades. Though the survey separately compiles the data for regular and non-regular workers, we focus on regular workers.<sup>7</sup> The choice is consistent with the purpose of this paper, as regular workers are supposed to be subject to long-term employment. With the years of truncation  $T$  set to 10 years, the UCL series are constructed annually from 1980 through 2010.

**Employer-employee characteristics.** The employer-employee characteristics in the BSWS include educational attainment (junior-high school; high school; junior college or technical school; college),<sup>8</sup> gender (male and female), and firm size (large firms with 1,000 or more full-time employees; medium firms with 100 to 999; and small firms with 99 or fewer). We omit junior-high school graduates due to their small sample size. The number of categories available is 18 ( $= 3 \times 2 \times 3$ ).

Panel (A) of Table 1 displays the number of employees in each category. Workers with high school degree and male workers account for the largest fraction in the sample, whereas workers are distributed somewhat evenly across firm size.

**Remitted wage measure.** Our measure of remitted wages,  $w_{t,t+j}$ , is the real hourly earnings. The hourly earnings are computed by dividing the total cash earnings by the total hours worked. The total cash earnings are the sum of base pay, overtime pay, bonuses, and other cash payments such as commuting and family allowances. These are before taxes and

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<sup>7</sup>These worker types are labeled as “full-time” and “part-time” workers, respectively, in the BSWS. We prefer to call them regular and non-regular workers as a guidance by the MHLW asks surveyed firms to differentiate the two types of workers according to their scope of responsibility and the type of wage and employment contract.

<sup>8</sup>Years of schooling to complete each educational attainment is nine years for junior-high school, 12 years for high school, 14 years for junior college or technical school, 16 years for college.

Table 1: Descriptive statistics

## (A) Number of workers by category

	Junior high school	High school	Junior college or technical school	College	Total
Education	604 (2.7)	9,518 (42.9)	4,097 (18.5)	7,960 (35.9)	22,179 (100)
Gender	Male 14,448 (65.1)	Female 7,731 (34.9)			Total 22,179 (100)
Firm size	Large 8,210 (37.0)	Medium 8,118 (36.6)	Small 5,851 (26.4)		Total 22,179 (100)

*Notes:* Thousands of workers. Portion to total employees is in parentheses. As of 2019.

## (B) Average hourly wage by category

	Junior high school	High school	Junior college or technical school	College	Total
Education	1,740 (71.6)	2,024 (83.3)	2,239 (92.1)	3,068 (126.2)	2,431 (100)
Gender	Male 2,676 (110.1)	Female 1,973 (81.2)			Total 2,431 (100)
Firm size	Large 2,923 (120.2)	Medium 2,308 (94.9)	Small 1,911 (78.6)		Total 2,431 (100)

*Notes:* JPY per hour. Wage levels relative to the average wage are in parentheses. As of 2019.

## (C) Average separation rate by category

	Junior high school	High school	Junior college or technical school	College	Total
Education	24.7%	8.0%	6.3%	6.2%	7.2%
Gender	Male 5.7%	Female 11.0%			Total 7.2%
Firm size	Large 5.5%	Medium 7.8%	Small 10.7%		Total 7.2%

*Notes:* Annual rates within ten years from hiring. Average from 1980 to 2019.

employee's contribution of social securities. The inclusion of non-base components allows us to accommodate wage adjustments using various components of earnings. Firms surveyed report their employees' earnings and hours worked according to their payroll records. Note that the Japanese law requires firms to record their employees' hours worked as well as earnings in their payroll records. Therefore, the BSWS data does not suffer measurement errors, unlike workers' self-reported data. Panel (B) of Table 1 shows the average nominal

hourly earnings in each category. Workers with college degree, male workers, and those in large firms receive higher earnings on average.

The wage series are deflated by the GDP implicit price deflator. The choice follows [Basu and House \(2016\)](#) in the view that the UCL measures labor cost from the firm’s perspectives rather than workers, and thus deflator should correspond to the price at which firms produce.

Non-cash benefits are not covered by the BSWS. To overview the non-cash benefits in the Japanese labor market, [Table 2](#) shows the average share in the General Survey on Working Conditions, which has been conducted periodically since 2001 by the MHLW to supplement the BSWS. The average share of non-cash benefits is a little below 20 percent of the total compensation and stable over time.<sup>9</sup> Legal welfare expenses, i.e., employers’ contributions to social securities, pursue a slight upward trend, while a decline in non-legal welfare expenses and retirement expenses offsets the trend. It is worth noting that the Japanese legal framework leaves little room for discretionary changes by employers in legal welfare expenses, which compose more than a half of total non-cash benefits. Social securities, including health and long-term care insurance, welfare pension insurance, and labor (employment and accident) insurance, are mandatory for all full-time workers. The contributions are legally determined based on employees’ earnings and family conditions, and must be split evenly between employers and employees. Therefore, these components are loosely proportional to cash earnings.

**New graduates.** [Hagedorn and Manovskii \(2013\)](#) claim that the larger number of job offers during expansions provides a higher chance for workers to move to a better job than the current one, leading to procyclicality in the average job-match quality. [Gertler et al. \(2020\)](#) point out that the UCL estimated in previous studies is subject to the cyclical upgrades of job-match quality, thereby overstating its cyclicity.

A key approach of this paper is to restrict the sample to new graduates from school, who

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<sup>9</sup>The share is somewhat lower than in the U.S. data mainly due to the lower public health insurance premium in Japan. [Gu et al. \(2020\)](#) report that, for the U.S. economy, the average share of benefit expenditures is 27 percent in 1982 (25 percent if nonproduction bonus, premium pay, and shift differential, which are likely paid by cash, are excluded) and 32 percent in 2018 (29 percent) in a sample of the employer cost surveys conducted by the Bureau of Labor Statistic (BLS). They find that health insurance premium accounts for 9.5 percent in 2018 with a rising trend.

Table 2: Average share of non-cash benefits

	2001	2005	2010	2015
<b>Total compensation</b>	<b>100.0</b>	<b>100.0</b>	<b>100.0</b>	<b>100.0</b>
<b>Cash earnings</b>	<b>81.7</b>	<b>81.0</b>	<b>81.5</b>	<b>80.9</b>
Scheduled hour earnings	59.6	60.2	62.1	60.3
Non-scheduled hour earnings	5.4	5.7	5.6	5.8
Bonus and other earnings	16.7	15.0	13.8	14.8
<b>Non-cash benefits</b>	<b>18.3</b>	<b>19.0</b>	<b>18.5</b>	<b>19.1</b>
Legal welfare expenses	9.3	10.0	10.8	11.4
Health and long-term care insurance fee	3.0	3.4	3.6	4.0
Welfare pension insurance fee	5.1	5.2	5.8	6.2
Labor insurance fee	1.2	1.4	1.3	1.0
Other legal welfare expenses	0.1	0.1	0.1	0.2
Non-legal welfare expenses	2.3	2.1	2.0	1.6
House expenses	1.1	1.0	1.0	0.8
Food expenses	0.2	0.2	0.2	0.1
Other welfare expenses	0.9	0.8	0.8	0.7
Retirement expenses	5.8	6.0	5.0	4.5
In-kind benefits	0.3	0.2	0.1	0.1
Training	0.3	0.3	0.1	0.2
Others	0.4	0.4	0.4	1.2

*Notes:* Figures are the average shares for all full-time workers. Legal welfare expenses are the employers' contributions to each social security item. Non-legal items are non-cash benefits, such as the expenses associated with company housing and canteens. The General Survey on Working Conditions is available since 2001.

*Source:* BSWS, General Survey on Working Conditions.

are free from the cyclical changes in job-match quality through job changes. New school graduates are identified by their age and educational attainment recorded in the BSWS. The standard age of new graduates is 18 years old for high school graduates, 20 years old for junior college or technical school graduates, and 22 years old for college graduates. The BSWS records workers' age categories as follows: 17 years old or below, 18–19 years old, 20–21 years old, 22–24 years old, and 25 years old and above in five-year range. We assume that new hires aged 18–19 years old with high school degree, 20–21 years old with junior college or technical school degree, and 22–24 years old with college degree are new graduates from school.

Strictly speaking, it is not impossible that each age category in the BSWS includes non-new graduates or excludes new graduates. However, these risks are considered minimal for the following reasons. First, it is not allowed to graduate from high school under the age of

18 as the Japanese education system adopts the age principle, as opposed to the program principle in other countries including the U.S., and schooling years up to high school are strictly linked to students' age. In addition, it is extremely rare to graduate from higher degree school earlier than expected years. Only those with outstanding qualities in certain fields, such as sports, can be admitted to university under the age of 18. Cumulatively, there have been less than 100 cases in history out of hundreds of millions of students. Second, separation rates are pretty low and stable in the Japanese labor market. In our sample, the annual separation rates are lower than 10 percent even in the initial few years of employment. Thus, there are few job changes within the same age category (e.g., a high school graduate starting to work at 18 years old, quitting and moving to another job before 19 years old). Moreover, workers with less than three years of work experiences are called "second new graduates." They go through a application and selection process and are employed under a condition similar to new graduates in the Japanese labor market practice.

New graduates are tracked after hiring according to their age. For example, college graduates with a tenure of ten years are supposed to be 32 years old. The categories for length of service in the BSWs are as follows: 0 years (new hires), 1–2 years, 3–4 years, and 5 years and above in five-year range. We interpolate wages for each year of service within a category by using the fourth-order polynomial.

**Separation rate.** Panel (C) of Table 1 shows the average annual separation rates within ten years after hiring. The rates tend to be higher for lower wage workers such as those with non-college degree, female workers, and small firms. These features are similar to those reported by Farber (1999) on the U.S. economy. The overall average separation rate is 7.2 percent per year. The rate is substantially lower than that in the NLSY sample, in which Kudlyak (2014) reports the average separation rate is around 30 percent per year, while it should be noted that our sample focuses on regular workers whose tenure tends to be long.

## 4 Main results

### 4.1 Cyclical regression

To examine the cyclical of the UCL, we run the following regression:

$$y_t^c = \gamma x_{t-1} + \alpha^{edu} + \alpha^{gen} + \alpha^{size} + \epsilon_t^c, \quad (9)$$

where  $\alpha^{edu}$ ,  $\alpha^{gen}$ , and  $\alpha^{size}$  are the fixed effects for educational attainment, gender, and firm size, respectively.  $y_t^c$  and  $x_{t-1}$  are wage and cyclical measures described below. Regarding the timing of  $x_{t-1}$ , the BSWS records wages as of June in each year, and there should be a time lag for reflecting business cycle conditions on wages. In particular, as the academic year ends in March in Japan, most new graduates start working in April with a predetermined working contract presumably reflecting the economic conditions until the previous year. For these reasons, we take one period lag for  $x_t$ . Own lag is not included because the series display little auto-correlation at the annual frequency.

The coefficient of our interest is  $\gamma$ . A positive value of  $\gamma$  indicates procyclicality of  $y_t^c$ . The weighted least square is used with the weight equal to the number of new graduates in each category,  $N^{new,c} = 1/t_{end} \sum_{t=0}^{t_{end}} N_{t,t}^c$ .

Our wage measures,  $y_t^c$ , are the UCL, new-hire wage, and average wage. We take the average of wage measures for three years forward, e.g.,  $y_t^c = (UCL_t^c + UCL_{t+1}^c + UCL_{t+2}^c)/3$  for the UCL, to accommodate a potential time lag of correlation. As for a cyclical measure,  $x_t$ , we follow the convention in the literature to use the unemployment rate as the baseline case. To extract the cyclical components, we use the HP-filter for both  $y_t^c$  and  $x_t$  with a standard smoothing parameter of  $\lambda = 100$  for annual data (e.g., [Backus and Kehoe 1992](#)). Robustness in terms of cyclical measure and detrending method is assessed in [Section 5](#).

### 4.2 Baseline result

[Table 3](#) shows the results of the cyclical regression (9). The table reports the coefficient of the unemployment rate,  $\gamma$ . As the sign of the unemployment rate is reversed and the series is standardized,  $\gamma$  represents the percent response of wage measures to a one standard deviation



Table 3: Baseline result: Cyclicalty of the UCL, average wage, and new-hire wage

	(1)	(2)	(3)	(4)	(5)
	UCL		New-hire wage		Ave. wage
	New grad.	All workers	New grad.	All workers	All workers
$x_{t-1}$ : Unemp. rate (sign reversed)	0.877*** (0.114)	1.076*** (0.084)	0.525*** (0.066)	0.923*** (0.079)	0.475*** (0.060)
R-squared	0.12	0.25	0.11	0.23	0.10
N of categories	18	18	18	18	18
N of observations	540	540	540	540	540

Robust standard errors are in parentheses.

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

*Notes:* The table shows the estimated values of  $\gamma$  in (9). The unemployment rate is standardized, and its sign is reversed. The estimated coefficients represent the percent response of wage measures to a one standard deviation improvement in the unemployment rate.

change in the unemployment rate. Note that one standard deviation of the unemployment rate is around 0.3 percent in our sample.

Column (1) is our baseline specification of the UCL for new graduates, while column (2) shows that for all workers for comparison purposes. The estimated coefficient is 0.877 in column (1). The lower cyclicalty of the UCL for new graduates than for all workers is consistent with the procyclical composition effects of job-match quality. However, even after correcting the composition effect, the UCL is almost twice as procyclical as the average wage,  $w_t^{ave,c}$ , shown in column (5). Compared to previous studies on the U.S. data, [Kudlyak \(2014\)](#) reports the estimated coefficient of the UCL is around three times larger than that of the average wage without controlling for the composition effect, while [Basu and House \(2016\)](#) find it is around three to six times larger across specifications.

The new-hire wage,  $w_t^{new,c}$ , is also subject to the composition effects of job-match quality. Interestingly, once the composition effects are controlled in column (3), its cyclicalty becomes close to the average wage. Our result is in line with the recent studies that control for workers' productivity and job-match quality ([Gertler et al. 2020](#); [Grigsby et al. 2021](#)), while earlier studies report that new-hire wages are highly cyclical without such correction.

It is also notable that the UCL is more cyclical than the new-hire wage. The difference becomes clearer once controlling for the composition effects of job-match quality. The result implies that the high cyclicalty of the UCL arises not only from that of the new-hire wage, but also from the wage dynamics of incumbent workers. As discussed in Section 2, the

incumbent-worker wage’s rigidity increases the cyclicalness of the UCL. In that sense, our result is also consistent with the previous studies, which claim incumbent worker wage’s rigidity (e.g., [Yamamoto 2007](#) for the Japanese labor market).

While previous studies on the U.S. economy typically report insignificant cyclicalness of the average wage, the estimated coefficient in our sample is relatively high. This is partly because the series is free from the composition changes of employee-employer characteristic over business cycles, which could dampen the cyclicalness (e.g., low-wage worker group tends to be more cyclical). This could also reflect institutional features of the Japanese labor market, including the substantial fraction of variable components in earnings, such as overtime payment and bonuses, as shown in [Table 2](#).

### 4.3 History dependence of the incumbent-worker wage

To scrutinize the factors behind the high cyclicalness of the UCL, we regress wages at each tenure on the unemployment rate in the year of hiring in [Table 4](#). A positive coefficient indicates that the labor market condition as of hiring persistently affects wages, implying the incumbent workers’ wage rigidity. A seminal work by [Beaudry and DiNardo \(1991\)](#) argues that the pattern is consistent with an implicit contract under which a risk-neutral firm offers insurance against business cycle fluctuations to risk-averse workers. As our sample of new graduates is not subject to the cyclical changes of the average job-matching quality, it is suited to test for this hypothesis. [Panel \(A\)](#) of [Table 4](#) shows that the labor market condition as of hiring has significantly positive effects. The effects gradually decay as the length of service increases, with significant impacts remaining up to nine years after hiring.

[Panel \(B\)](#) adds as an independent variable the lowest unemployment rate, i.e., most favorable labor market condition for workers, in the course of tenure. If workers’ commitment to the continuation of a wage contract is limited, firms have an incentive to raise wages in expansions to prevent workers quits for another job as long as the match creates surplus. The regression result is consistent with the model’s implication. The longer the tenure is, the more likely workers receive a large enough shock to trigger a wage change and thus less dependent on the labor market condition as of hiring their wages become.

This history dependence of the incumbent-worker wage results in the higher cyclicalness

Table 4: History dependence of the incumbent-worker wage

(A) Effect of labor market condition as of hiring				
	(1)	(2)	(3)	(4)
	Wages with the length of service			
	1-2y	3-4y	5-9y	10-14y
$x_t^{lag}$ : Lagged unemp. rate (sign reversed)	0.782*** (0.137)	0.712*** (0.161)	0.382*** (0.131)	0.249 (0.238)
R-squared	0.06	0.11	0.24	0.03
N of categories	18	18	18	18
N of observations	540	540	540	540

Robust standard errors are in parentheses.  
\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

(B) Effect of labor market condition in the course of tenure				
	(1)	(2)	(3)	(4)
	Wages with the length of service			
	1-2y	3-4y	5-9y	10-14y
$x_t^{lag}$ : Lagged unemp. rate (sign reversed)	0.782*** (0.137)	0.824*** (0.194)	0.271** (0.133)	0.276 (0.237)
$x_t^{min}$ : Lowest unemp. rate (sign reversed)	-0.120 (0.143)	-0.308 (0.269)	0.647*** (0.171)	0.822** (0.322)
R-squared	0.06	0.12	0.26	0.04
N of categories	18	18	18	18
N of observations	540	540	540	540

Robust standard errors are in parentheses.  
\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

*Notes:* Dependent variables are wages of new graduates with each length of service.  $x_t^{lag}$  is the unemployment rate in the year of hiring. The current unemployment rate (one period lagged) and fixed effects for educational attainment, gender, and firm size are included as independent variables. The sample is restricted to new graduates.

of the UCL compared to the new-hire wage. In other words, the cost of labor is not just the remitted wage at each point of time. Rather, it entails the persistent wage differences across hiring cohorts in their entire tenure.

## 4.4 Asymmetry

Another area of potential interest is the asymmetry in cyclicalities, given the substantial debate in the literature on downward rigidity of wage adjustments (e.g., [Tobin 1972](#); many others afterward) and asymmetry in the labor market dynamics (e.g., [Friedman 1969](#); more recently [Dupraz et al. 2021](#)). Table 5 shows the cyclicalities of each wage measure in ex-

Table 5: Asymmetry in cyclicality

	(1)	(2)	(3)	(4)	(5)
	UCL		New-hire wage		Ave. wage
	New grad.	All workers	New grad.	All workers	All workers
$x_{t-1} \times 1_{x_{t-1} \geq 0}$	1.359*** (0.232)	1.857*** (0.175)	0.890*** (0.139)	1.789*** (0.172)	1.173*** (0.135)
$x_{t-1} \times 1_{x_{t-1} < 0}$	0.363 (0.235)	0.245 (0.161)	0.137 (0.122)	0.000 (0.139)	-0.268** (0.124)
R-squared	0.13	0.29	0.12	0.29	0.17
N of categories	18	18	18	18	18
N of observations	540	540	540	540	540

Robust standard errors are in parentheses.

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

*Notes:* The same notes as Table 3 apply. The independent variable is split into positive and negative values.

pansions and contractions. We split the HP-filtered unemployment rate into positive and negative values. The table indicates that the cyclicality is significantly positive only in expansions. A similar pattern is obtained for the new-hire and average wages.<sup>10</sup> The result is surprising in the sense that the incumbent worker wage’s downward rigidity itself could allow for downward adjustment of the UCL as long as the new-hire wage were to downward flexible. The combination of downward rigidity of the new-hire and incumbent-worker wages can explain that of the UCL.

In the literature, numerous studies point to the downward wage rigidity of incumbent workers, including in the Japanese labor market (e.g., [Yamamoto 2007](#)). Regarding the new-hire wage, various studies find that wages frequently changes when a worker moves to another job (e.g, [Barattieri et al. 2014](#)). However, at the job level, a recent work by [Hazell and Taska \(2020\)](#) shows that wages posted by firms are downwardly rigid in U.S. data. [Fukui et al. \(2020\)](#) report a similar pattern in the Japanese labor market in the data during the COVID-19 pandemic. Our result is in line with these studies.

<sup>10</sup>The estimated response of average wage in contractions is negative, though the significance level is a little low. This is not inconsistent with the previous studies that report counter-cyclicalities of real wages in recessions (e.g., [Gu et al. 2020](#)). In this regard, fixed components of earnings can be more expensive in recessions when converted to an hourly rate.

## 5 Robustness check

### 5.1 Specification

Table 6 assesses the robustness of our baseline result. Panel (A) explores different cyclical measures. We first use the job-openings to job-applicants ratio in the Job/Placement Services Statistics (JEPSS). The JEPSS compiles vacancies posted and job-seekers registered in the public employment security offices, covering around 20 percent of the total job creation. The job-openings to job-applicants ratio is considered as an indicator of labor market tightness, which is often defined in a search and matching model. We also use the technology shock identified by the long-run restriction of [Gali \(1999\)](#).<sup>11</sup> The panel confirms high cyclicalities of the UCL. The relative magnitude of cyclicalities remains unchanged from the baseline case when the job-opening to job-applicants ratio is used as a cyclical measure, reported in columns (1)–(3). In the case of the technology shock of [Gali \(1999\)](#) in columns (4)–(6), the estimated coefficient is statistically significant with an intended sign only for the UCL.

Panel (B) examines different methods of detrending. In columns (1)–(3), we employ the Hamilton filter proposed by [Hamilton \(2018\)](#). The filtered values are obtained as residuals by regressing the current value on the two to five periods lagged values. We use the year-on-year growth rate in columns (4)–(6). These alternative detrending methods lead to a similar pattern of the relative cyclicalities among the UCL, average wage, and new-hire wage. The difference in the estimated coefficient of the UCL and average wage is in the range of 2.5 to 4 times—somewhat higher than the baseline result with the HP-filter.

### 5.2 Separation rate

[Nekarda and Ramey \(2020\)](#) criticize the assumption of exogenous separation rates in calculating the UCL. They argue that on-the-job search along with incumbent workers’ wage rigidity leads to a lower separation rate of workers hired in expansions, as these workers can enjoy high wages of the current job in the course of their tenure. Those hired in contractions

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<sup>11</sup>The details are presented in Appendix A.

Table 6: Robustness check

(A) Alternative cyclical measures

	(1)	(2)	(3)	(4)	(5)	(6)
	Job-opening to job-applicants ratio			Technology shock of Gali (1999)		
	UCL	New-hire wage	Ave. wage	UCL	New-hire wage	Ave. wage
$x_{t-1}$ : Job opening ratio	1.229*** (0.157)	0.870*** (0.091)	0.707*** (0.080)			
$x_{t-1}$ : Technology shock				0.840** (0.422)	0.313 (0.211)	-0.374* (0.210)
R-squared	0.14	0.18	0.13	0.14	0.12	0.09
N of categories	18	18	18	18	18	18
N of observations	540	540	540	540	540	540

Robust standard errors are in parentheses.

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

(B) Alternative methods of detrending

	(1)	(2)	(3)	(4)	(5)	(6)
	Hamilton filter			Year-on-year changes		
	UCL	New-hire wage	Ave. wage	UCL	New-hire wage	Ave. wage
$x_{t-1}$ : Unemp. rate (sign reversed)	1.731*** (0.255)	1.118*** (0.143)	0.462*** (0.106)	0.675*** (0.135)	0.384*** (0.078)	0.260*** (0.068)
R-squared	0.13	0.13	0.13	0.05	0.03	0.04
N of categories	18	18	18	18	18	18
N of observations	450	450	450	522	522	522

Robust standard errors are in parentheses.

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

*Notes:* The same notes as Table 3 apply. The UCL and new-hire wage are those for new school graduates, whereas the average wage is that for all workers. For the technology shock of Gali (1999) reported in columns (4)–(6) of panel (A), two-period lagged unemployment rate is included in the independent variables to control for the past labor market conditions.

and thus stuck in a low-wage contract are more willing to move to another job.

In our view, the validity of the exogenous separation rates is an empirical question since there can be mitigating and offsetting forces to the channel proposed by Nekarda and Ramey (2020). For example, firms may compensate workers to prevent their separation, as Beaudry and DiNardo (1991) argue. In addition, Fujita and Ramey (2009) report that the employment-to-unemployment flows are counter-cyclical in U.S. data, rather than procyclical implied above, due to job losses in contractions.

Table 7 investigates the history dependence of separation rates. We calculate the separation rates of each hiring cohort during ten years after hiring, and regress them on the

Table 7: History dependence of separation rates

	(1)	(2)	(3)
$x_{t-1}$	-0.005 (0.005)		
$x_t^{fore,ave}$		-0.011 (0.008)	
$x_t^{fore,min}$			0.010 (0.011)
R-squared	0.12	0.12	0.12
N of categories	18	18	18
N of observations	531	531	531

Robust standard errors are in parentheses.

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

*Notes:* Dependent variables are the average annual separation rates within ten years after hiring. For independent variables,  $x_{t-1}$  is the one period lagged unemployment rate, which represents the labor market condition as of hiring.  $x_t^{fore,ave}$  and  $x_t^{fore,min}$  are the average and minimum unemployment rates within ten years from hiring, respectively. Fixed effects for educational attainment, gender, and firm size are included in the regression.

unemployment rate in the year of hiring in column (1). Though the estimated coefficient is negative consistent with [Nekarda and Ramey \(2020\)](#)'s argument, it is insignificant. In columns (2) and (3), moreover, neither the average or lowest unemployment rates in the course of tenure affects the separation rate. The result is in line with the story that firms compensate workers so that they remain in the current job in expansions. It is also consistent with the relatively high cyclical of the average wage reported in [Table 3](#). The stable separation rates in contractions may also reflect strict restrictions on firing in the Japanese legal framework (e.g., [Auer and Cazes 2003](#)). In a nutshell, the evidence suggests that our sample is suited for assuming exogenous separation rates.

## 6 Model

This section develops a model to reconcile our empirical findings. As we have obtained ample empirical support to an implicit wage contract of [Beaudry and DiNardo \(1991\)](#), our starting point is [Rudanko \(2009\)](#), who introduced an implicit wage contract into a search and matching framework. The resulting incumbent worker's wage rigidity would generate overall high cyclical of the UCL. Our model is also intended to capture the downward

rigidity of the UCL. To this end, the extension involves productivity heterogeneity in both sides of firms and workers. High-productivity firms seek to match with skilled workers by using wages as a screening tool under imperfect information regarding the type of workers. We demonstrate that firms maintain a high value of a posted contract in contractions to keep their labor market competitive enough to exclude the application from unskilled workers. Consequently, the extended model replicates our empirical findings, namely, the downward rigidity of the UCL, as well as its overall high cyclicalities.

## 6.1 Environment

**Matching framework.** Model environment follows that of [Rudanko \(2009\)](#) except for heterogeneity of firms and workers. Matching of firms and workers is attained in the form of directed search. In the beginning of each period, firms post vacancies in a submarket  $i$  with a wage contract  $\sigma_i$ , which specifies state-contingent period wages:

$$\sigma_i(z_t) = \{w_{it+\tau}(z^{t:t+\tau}) \in [\underline{w}, \bar{w}] \text{ for all } z^{t:t+\tau} = (z_t, z_{t+1}, \dots, z_{t+\tau})\}_{\tau=0}^{\infty}, \quad (10)$$

where  $z_t$  denotes the aggregate productivity.  $z_t$  takes one of values in a set,  $Z = \{z_1, z_2, \dots, z_K\}$ , with  $z_k < z_{k+1}$  for all  $k = 1, 2, \dots, K$ , and follows a stationary first-order Markov process with transition probabilities  $\pi(z_{t+1}|z_t)$ . Notice that the UCL can be defined as in [Section 2](#).

Within each submarket, a worker-firm match is formed according to a matching function  $m_i(u_{it}, v_{it})$ , where  $u_t$  is the measure of unemployed workers searching for a measure  $v_t$  of vacancies. Following standard assumptions,  $m_i(u_{it}, v_{it})$  is a constant return-to-scale Cobb-Douglas function:

$$m_i(u_{it}, v_{it}) = \kappa_i u_{it}^{\alpha_i} v_{it}^{1-\alpha_i}, \quad (11)$$

where the function is submarket-specific with parameters  $\kappa_i$  and  $\alpha_i$ . Workers' job-finding rate is given by  $m_i(u_{it}, v_{it})/u_{it} = m_i(1, \theta_{it}) = \mu_i(\theta_{it})$  with  $\theta_{it} = v_{it}/u_{it}$  being a measure of labor market tightness. Similarly, the arrival rate of workers for a vacancy is given by  $m_i(u_{it}, v_{it})/v_{it} = q_i(\theta_{it}) (= \mu_i(\theta_{it})/\theta_{it})$ .



**Workers.** There is a continuum of workers on a unit interval. Workers provide one unit of labor service and receive labor income if employed, whereas they receive benefits if unemployed.<sup>12</sup> They consume their income each period under preferences  $\mathbb{E}_t[\sum_{\tau=0}^{\infty} u(c_{t+\tau})]$  with  $u(\cdot)$  being a CRRA utility function. A wage contract terminates with an exogenous separation rate. Once it terminates, workers become unemployed and search for another job. Job search is costless for workers.

There are two types of workers, skilled ( $S$ ) and unskilled ( $N$ ). Their skill difference materializes when matched with a specific type of firms, which is described shortly. Their wages differ depending on the type of firms they match with.

Let  $U_{ji}$  and  $E_{ji}^{\sigma}$  denotes type- $j$  worker's value of unemployment and employment with a wage contract  $\sigma$  in submarket  $i$ . They can be written as

$$U_{ji}(z_t) = u(b) + \beta \mathbb{E}_t [\mu_i(\theta_i(z_{t+1})) E_{ji}^{\sigma}(z_{t+1}) + (1 - \mu_i(\theta_i(z_{t+1}))) U_{ji}(z_{t+1})], \quad (12)$$

$$E_{ji}^{\sigma}(z_t) = u(w_i^{\sigma}(z_t)) + \mathbb{E}_t \left[ \sum_{\tau=1}^{\infty} \beta^{\tau} (1 - s_i)^{\tau-1} \{ (1 - s_i) u(w_i^{\sigma}(z^{t:t+\tau})) + s_i U_{ji}(z_{t+\tau}) \} \right], \quad (13)$$

for  $j = S, N$ , where  $\beta$  is the subjective discount factor,  $b$  is the unemployment benefit, and  $s_i$  is the separation rate in each submarket  $i$ . Workers choose the submarket that maximizes the expected surplus of employment  $V_{ji}^{\sigma}$ :

$$V_{ji}^{\sigma}(z_t) = \mu_i(\theta_i(z_t))(E_{ji}^{\sigma}(z_t) - U_{ji}(z_t)). \quad (14)$$

**Firms.** There are two types of firms with different productivity, high ( $H$ ) and low ( $L$ ). Type- $H$  firms offer in non-routine jobs, thereby achieving higher productivity than type- $L$  firms, but production only occurs when matched with skilled workers. In contrast, type- $L$  firms offer routine jobs, which are doable by both types of workers. Productivity of skilled and unskilled workers is identical when matched with type- $L$  firms.

Type- $H$  firms offer a state-contingent wage contract under limited commitment on the

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<sup>12</sup>We abstract the variations of hours worked for simplicity. [Thomas and Worrall \(2007\)](#) show that the main implications of an implicit contract model are preserved under variable hours.

worker side, which is the focus of our analysis. Type- $L$  firms are assumed to offer a wage contract under full commitment for simplicity.

Let  $F_i^\sigma$  denote type- $i$  firm's value of a matched wage contract  $\sigma$ . It can be written as

$$F_i^\sigma(z_t) = a_i z_t - w_i^\sigma(z_t) + \mathbb{E}_t \left[ \sum_{\tau=1}^{\infty} \beta^\tau (1 - s_i)^\tau (a_i z_{t+\tau} - w_i^\sigma(z^{t:t+\tau})) \right], \quad (15)$$

for  $i = H, L$ . Firms produce output  $a_i z_t$  using one unit of labor and pay period wages determined under a contract  $\sigma$ . Once a contract terminates, firm's expected value is zero under the free entry assumption, as explained below. The firm-specific productivity  $a_i$  is given by

$$a_H = \begin{cases} \bar{a} > 1 & \text{if matched with a type-}H \text{ worker} \\ 0 & \text{if matched with a type-}L \text{ worker} \end{cases}, \quad (16)$$

$$a_L = 1. \quad (17)$$

Firms have to pay vacancy posting cost  $k_i$  in each period. Free entry entails that the expected profit of job posting  $J_i$  becomes zero:

$$J_i(z_t) = -k_i + q_i(\theta_i(z_t)) F_i^\sigma(z_t) = 0. \quad (18)$$

## 6.2 Equilibrium

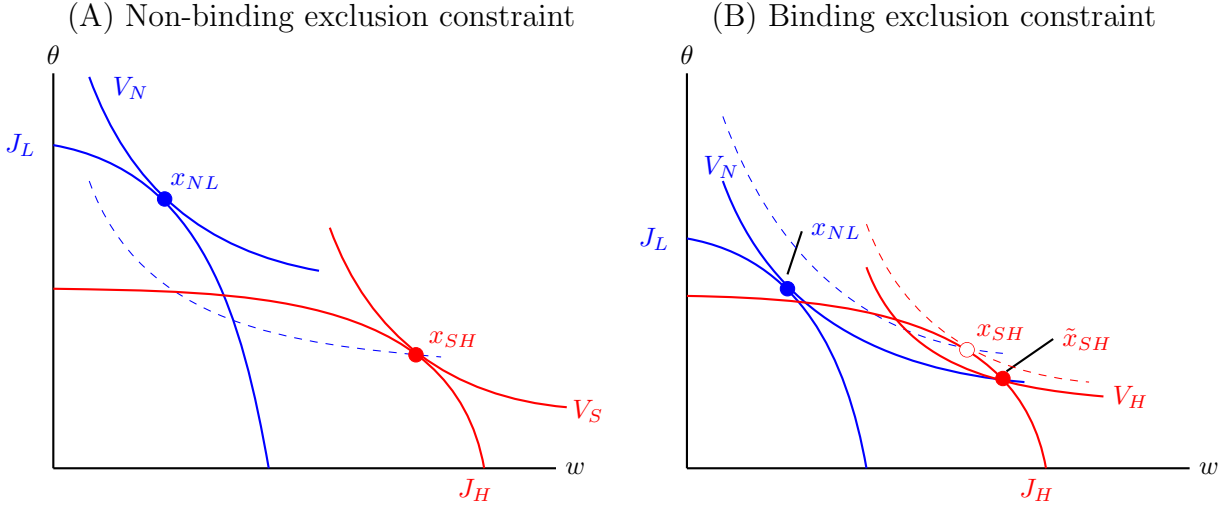
**Separation of submarkets.** Type- $H$  and  $L$  firms offer different wage contracts in each submarket. As Moen (1997) discusses, two types of workers are separated into different submarkets if firms could announce skill requirements and screen applications accordingly. In our case, type- $H$  firms seek to match with skilled workers whereas type- $L$  firms are indifferent in terms of worker type. Thus, a natural separation is the type- $H$  firm–type- $S$  worker pair and type- $L$  firm–type- $N$  worker pair, as described in panel (A) of Figure 1.<sup>13</sup>

We consider an alternative circumstance in which the screening by type- $H$  firms is im-

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<sup>13</sup>As we will formalize shortly, a directed search equilibrium is located on a Pareto frontier on the  $(w, \theta)$  plane as a result of workers' choice of submarket and firms' profit maximization.

Figure 1: Illustration of equilibrium



perfect. Specifically, if unskilled (type- $N$ ) workers apply to type- $H$  firms, type- $H$  firms can detect the worker type and decline the application with probability  $p \in (0, 1)$ . There remains a positive probability  $(1 - p)$  with which type- $H$  firms may hire the unintended type of workers (type- $N$ ). In such a case, production cannot occur due to the lack of required skill of the type- $N$  worker, but the type- $H$  firm has to pay wages as a contract is already signed.

To avoid this undesirable consequence, type- $H$  firms use their posted wage contract as a screening tool. Specifically, a high value of a wage contract can make the submarket “too competitive” for type- $N$  workers, while maintaining the application from type- $S$  workers. This differentiation is possible with a single posted contract because workers’ objective function, i.e., the expected surplus, takes into account job-finding rate, and therefore a change in the value of wage contract can have different impacts on each type of workers. As type- $N$  workers face a lower job-finding rate than type- $S$  workers in type- $H$  firms’ submarket due to the screening probability  $p$ , the marginal benefit of higher value contract is smaller. As a result, type- $N$  workers may not tolerate a low labor market tightness (low job-finding rate) associated with a high value of the wage contract posted in the submarket.

The situation is illustrated in panel (B) of Figure 1. The contact point of type- $H$  firm’s profit frontier and type- $S$  worker’s indifferent curve ( $x_{SH}$ ) makes type- $N$  workers better off

when they apply to type- $H$  firm's submarket. On the other hand, a lower job-finding rate of type- $L$  workers implies a flatter indifference curve on the  $(w, \theta)$  plane, leaving them in type- $L$  firm's submarket when type- $H$  firms post a wage contract with a high enough value ( $\tilde{x}_{SH}$ ). This can be described as an exclusion constraint of type- $N$  workers in type- $H$  firms' problem:<sup>14</sup>

$$V_{NL}^\sigma(z_t) \geq V_{NH}^\sigma(z_t). \quad (19)$$

We assume that type- $L$  firms take the type- $H$  firms' screening as given and do not change their own behavior.

**Equilibrium.** The definition of a directed search equilibrium is in line with [Moen \(1997\)](#) and [Rudanko \(2009\)](#) and is presented in Appendix B. We confirm that a risk-neutral firms post a fixed wage contract to risk-averse workers while offering wage increases in expansions to ensure workers' participation under their limited commitment, as shown in the previous studies. These account for incumbent workers' wage rigidity.

The novelty of our model is the addition of the exclusion constraint (19). While we leave the investigation of its impact for quantitative analysis in the following subsection, our intuition suggests that the constraint becomes more binding for lower  $z$ . Lower productivity implies a smaller surplus of a match for workers since the value of employment declines while that of unemployment is bounded by the unemployment benefit. Hence, the marginal benefit of ensuring higher labor market tightness diminishes. Workers lean toward a high wage contract with low market tightness offered by type- $H$  firms, rather than a low wage contract of type- $L$  firms, which let type- $H$  firms post a high value contract to satisfy the exclusion constraint.

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<sup>14</sup>Type- $S$  workers' participation in type- $H$  firm's submarket,  $V_{SH}(z_t) \geq V_{SL}(z_t)$ , is also necessary to ensure the separation of workers in each submarket. However, this condition is satisfied under  $\bar{a} > 1$  and reasonable values of other parameters for an intuitive reason.

### 6.3 Numerical analysis

**Calibration.** Calibrated parameters are listed in Panel (A) of Table 8. The time-frequency of the model is quarterly. As the numerical analysis aims to compare the qualitative implications of the model with empirical observations, the calibration is conducted in a parsimonious manner to capture key features of the Japanese labor market. Parameters in the matching function are set according to the JEPSS. The number of vacancies and job-seekers recorded in the JEPSS enables us to construct empirical counterparts of the vacancy filling rate,  $q$ , job finding rate,  $\mu$ , and labor market tightness,  $\theta$ . We use these series of regular workers for calibrating parameters of type- $H$  firms and those of non-regular workers for type- $L$  firms. Specifically, we regress the HP-filtered values of  $\ln q_{it}$  on those of  $\ln \theta_{it}$  in the sample of 1972–2019 to obtain  $\alpha_H = 0.57$  and  $\alpha_L = 0.78$ . These estimates are broadly in line with those in U.S. and European countries’ data surveyed by [Petrongolo and Pissarides \(2001\)](#).  $\kappa_i$  is set consistent with the time-average, i.e.,  $\kappa_i = \bar{q}_i \bar{\theta}_i^{\alpha_i}$  with  $\bar{x}$  denoting the average value in the sample, yielding  $\kappa_H = 0.26$  and  $\kappa_L = 0.38$ . The separation rate for type- $H$  firms,  $s_H$ , is the one in the BSWS data reported in Table 1, whereas that of type- $L$  firms  $s_L$  is obtained from the Survey on Employment Trends (SET) conducted by the MHLW. Though somewhat smaller than the BSWS, the SET surveys around 15 thousand firms following a sampling procedure similar to the BSWS. Job-posting costs  $k_i$  are calibrated to match  $\mu_i$  in the model to the data using the simulated method of moments. Productivity of type- $H$  firms  $\bar{a}$  is targeted to the wage gap between type- $H$  and  $L$  firms. These parameter values capture a sharp contrast between regular (type- $H$ ) and non-regular (type- $L$ ) labor markets. Regular workers enjoy high-productivity (high  $\bar{a}$ ) under a long-term contract (low  $s$ ) while requiring a costly recruiting process on both worker (low  $\mu$ ) and firm sides (high  $k$ ). The unemployment benefit  $b$  is set to the average replacement rate 0.55 reported in SET. Regarding workers’ preference, the subjective discount factor  $\beta$  is set to  $0.97^{1/4}$ . The period utility function is assumed to take the logarithm. The evolution of aggregate productivity is parameterized as a discretized AR(1) process with the persistence parameter  $\rho_z = 0.90$  and the standard deviation of shocks  $\sigma_z = 0.015$ . The screening probability of type- $H$  firms is fixed at  $p = 0.85$ .

Table 8: Calibration

## (A) Calibrated parameters

Description	Symbol	Value	Source
Elasticity of $q$ to $\theta$ in type- $H$ firms	$\alpha_H$	0.57	JEPSS
Elasticity of $q$ to $\theta$ in type- $L$ firms	$\alpha_L$	0.78	JEPSS
Constant in matching function for type- $H$ firms	$\kappa_H$	0.26	JEPSS
Constant in matching function for type- $L$ firms	$\kappa_L$	0.38	JEPSS
Separation rate for type- $H$ firms	$s_H$	0.018	BSWS
Separation rate for type- $L$ firms	$s_L$	0.064	SET
Unemployment benefit	$b$	0.55	SET
Job-posting cost for type- $H$ firms	$k_H$	1.192	Internally calibrated
Job-posting cost for type- $L$ firms	$k_L$	0.161	Internally calibrated
Productivity of type- $H$ firms	$\bar{a}$	1.55	Internally calibrated
Subjective discount factor	$\beta$	$0.97^{1/4}$	Externally fixed
Persistence of aggregate productivity	$\rho_z$	0.90	Externally fixed
Size of aggregate productivity shock	$\sigma_z$	0.015	Externally fixed
Probability of screening in type- $H$ firms	$p$	0.85	Externally fixed

*Notes:* The abbreviations are as follows; JEPSS: Job/Employment Placement Services Statistics and SET: Survey on Employment Trends.

## (B) Targeted moments

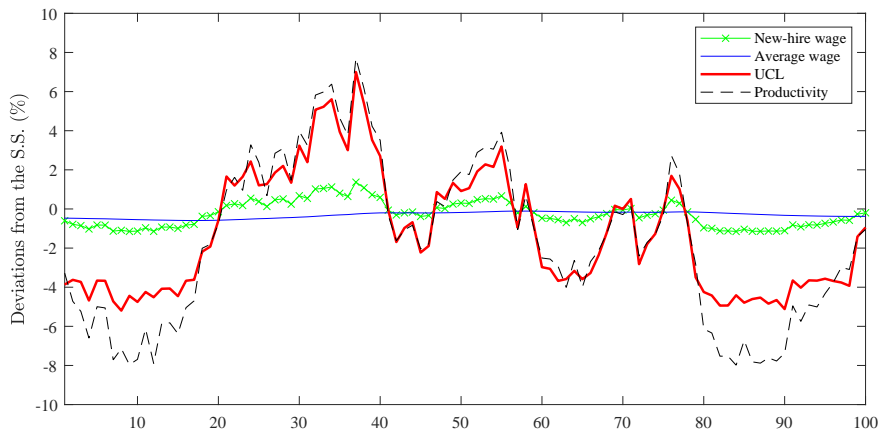
Description	Symbol	Model	Data	Source
Vacancy to job-seeker ratio of type- $H$ firms	$\theta_H$	0.80	0.80	JEPSS
Vacancy to job-seeker ratio of type- $L$ firms	$\theta_L$	1.51	1.51	JEPSS
Arrival rate of workers of type- $H$ firms	$\bar{q}_H$	0.29	0.29	JEPSS
Arrival rate of workers of type- $L$ firms	$\bar{q}_L$	0.28	0.28	JEPSS
Job-finding rate of type- $H$ firms	$\bar{\mu}_H$	0.21	0.21	JEPSS
Job-finding rate of type- $L$ firms	$\bar{\mu}_L$	0.48	0.38	JEPSS
Wage gap between type- $H$ and $L$ firms	$\bar{w}_H/\bar{w}_L$	1.51	1.51	BSWS

**Simulation results.** Figure 2 shows the dynamics of the UCL, new-hire wage, and average wage of type- $H$  firms, as well as underlying productivity process, in simulated data of the model. It is immediate to see that the UCL is much more procyclical than the new-hire and average wages in line with the empirical observation. Note that the average wage in the model is extremely rigid, presumably because it abstracts intensive margin and wage adjustments of incumbent workers through overtime premium and lump-sum bonuses.

It is also notable that the UCL does not fully track productivity in a downward phase. To interpret, the value of a posted wage contract remains high enough even in a recession to exclude type- $N$  workers' participation in type- $H$  firms' market.

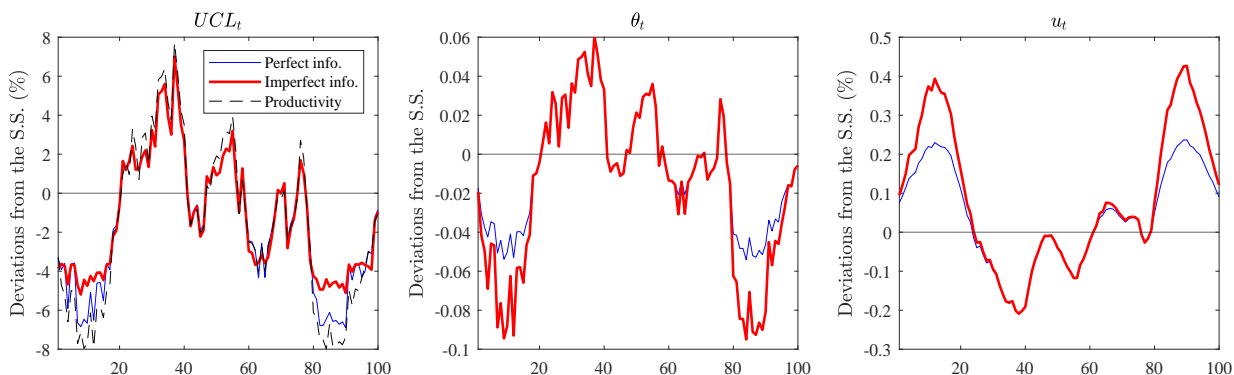
The downward rigidity of the UCL is clearer in Figure 3, which compares the baseline case under imperfect information and an alternative case of perfect information. Under the

Figure 2: Model simulation



*Notes:* The policy function of each variable is obtained through the value function iteration on discretized grids. In the model simulation, the aggregate productivity is generated as a continuous AR(1) series, and endogenous variables are simulated using the linear interpolation of the policy function.

Figure 3: Model simulation with and without imperfect information



perfect information, type- $H$  firms are able to detect worker type and decline unskilled workers' applications if any, as a consequence of which skilled and unskilled workers are separated into the two submarkets. Thus, the flexibility of the UCL is entailed both upwardly and downwardly. In the presence of imperfect information, in contrast, the room for downward adjustments of the UCL is limited. As the UCL serves as the price of labor in the model, the downward rigidity leads to enlarged fluctuations in the labor market tightness,  $\theta_t$ , and unemployment rate,  $u_t$ , in a downward phase. The asymmetry of labor market dynamics is consistent with the plucking model of business cycles (e.g, [Friedman 1969](#); more recently [Dupraz et al. 2021](#)). While previous studies assume an exogenous downward rigidity con-

straint to obtain the asymmetry (e.g., [Dupraz et al. 2021](#)), the novelty of this paper’s model is that the downward rigidity emerges endogenously as a consequence of screening of different types of workers through wage posting.

## 7 Conclusion

In this paper, we have explored the cyclical properties of the user cost of labor (UCL). While the literature sees the advantage of the UCL as a measure of allocative wage, the estimation of the UCL involves empirical difficulties. These include collecting a sequence of wages from hiring to separation and controlling for the cyclical changes in the average quality of new job matches. We address these challenges by tracking wages of new school graduates in large-scale Japanese wage data. Our approach confirms non-negligible effects of the cyclical changes in the average job-match quality, but the UCL remains highly procyclical after the correction. The estimated UCL is more procyclical than the new-hire wage because the rigidity of the incumbent-worker wage amplifies the wage differences among hiring cohorts.

The high cyclical of the UCL sides with earlier studies on the topic, including [Kudlyak \(2014\)](#) and [Basu and House \(2016\)](#), which call for a question on the convention wisdom of rigid real wage. That is to say, the high cyclical of the UCL makes it challenging for a standard macroeconomic model to account for unemployment and other business cycle fluctuations.

One possible resolution from the scope of this paper is the asymmetry in the UCL’s cyclical. We find that the UCL is downwardly rigid. As numerous studies have pointed out, business cycles are characterized by a sharp and rapid contraction followed by a gradual recovery. These asymmetric business cycle fluctuations, often referred to as a plucking model, are consistent with the downward rigidity of the UCL. Indeed, we have developed a wage posting model in which firms use wages to screen a certain type of applicants, resulting in the downward rigidity of the UCL, and have shown that the model replicates the asymmetry in the labor market dynamics.

We conclude this paper with potential applications of our elaborated UCL measure. First, the UCL would be useful for exploring the cyclical of price markup. A key issue in



estimating price markup is the measurement of marginal factor price (e.g., [Bils et al. 2018](#)). The UCL can elaborate on the conventional labor share based on the average wage to measure (inverse of) markup. Second, a medium- and long-run trend of the UCL would be worth investigating. Potential implications include those for cross-sectional and inter-generational wage inequality that may not be fully captured by the remitted wage.<sup>15</sup> Consequence for a secular trend of the labor share could be another area of exploration.

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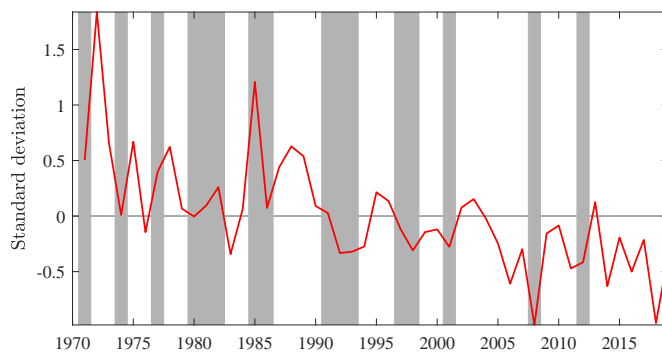
<sup>15</sup>For example, see [Goldin and Katz \(2009\)](#) for cross-sectional wage inequality, and [Glover et al. \(2020\)](#) and [Kiyotaki and Zhang \(2018\)](#) for inter-generational one.

## Appendix A Technology shock of Gali (1999)

**Construction of shock.** We follow Gali (1999) to run a bivariate VAR of labor productivity and total hours worked. Each series is the quarter-on-quarter growth rate, and lag length is set to 2 according to the AIC. The total hours worked is constructed by multiplying the number of employees in the Labour Force Survey and the hours worked per employee in the Monthly Labour Survey together and is adjusted to the per capita basis. The labor productivity is obtained by dividing the real GDP by the total hours worked. The technology shock is identified as an innovation that has a permanent effect on labor productivity, which is the so-called long-run restriction. The annual series is obtained by summing up the identified shocks in each year. The sample spans from 1970 to 2019.

**Time-series of shock.** Figure A-1 plots the time-series of identified technology shock. The series captures salient business cycle episodes as well as the secular trend of productivity slowdown during the period. In recent years, for example, negative innovations are observed for the burst of the dot-com bubble (2001), the global financial crisis (2008), the great earthquake of east Japan (2011), the European sovereign debt crisis and Japanese yen appreciation (2012). Substantial increases in labor inputs in the last few years, largely accounted for by unskilled female and elderly workers, accompanied by the stagnated GDP growth are also identified as negative productivity shocks.

Figure A-1: Time-series of identified shock



*Notes:* The series is annualized. Shaded areas indicate recessions according to the business cycle dating by the Cabinet Office of Japan.

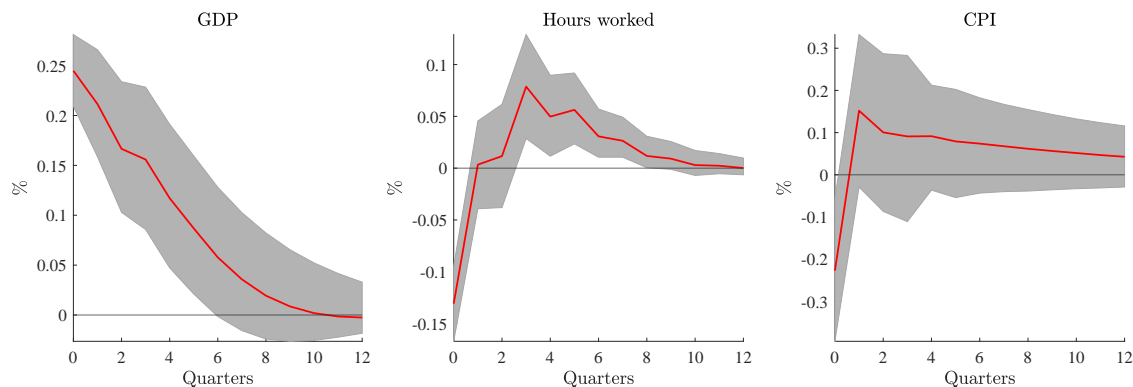
**Quasi-impulse response analysis.** To examine the effects of the identified shocks on aggregate variables, we estimate the quasi-impulse responses. The responses of a target variable  $y_t$  to an exogenous shock  $z_t$  are obtained by estimating the following equation:

$$y_t = \rho_1 y_{t-1} + \dots + \rho_p y_{t-p} + \phi_0 z_t + \phi_1 z_{t-1} + \dots + \phi_p z_{t-p} + \epsilon_t, \quad (\text{A.1})$$

where the lag length is set to  $p = 3$ . Note that the specification is equivalent to a bivariate VAR in which a variable  $z_t$  is treated as exogenous.

Figure A-2 shows the quasi-impulse responses of the GDP, total hours worked, and CPI to each shock. The GDP and total hours worked are the HP-filtered series on a per capita basis where the construction the total hours worked is described above. The CPI is less fresh food and in the quarter-on-quarter growth rate. A positive technology shock leads to an increase in the GDP and an decrease in the CPI on impact. The response of hours worked is negative on impact consistent with [Gali \(1999\)](#)'s result on U.S. data. Note that, although [Gali \(1999\)](#) does not obtain clear results on Japanese data until 1994, our result is based on the extended sample up to 2019.

Figure A-2: Quasi-impulse responses



*Notes:* The percent responses to a 1 standard deviation shock are shown. Dashed lines indicate the 90% confidence interval.

## Appendix B Directed Search Equilibrium

**Posted wage contract.** Firms post a wage contract to maximize its value while ensuring worker's value at the Pareto frontier. It is convenient to start from type- $L$  firm's problem. It can be represented by dynamic programming below.

$$f_L^\sigma(z, E_L^\sigma, U_L(z)) = \max_{w_L, \{E_L^\sigma(z')\}_{z' \in Z}} z - w_L + \beta \mathbb{E}_z [(1 - s_L) f_L^\sigma(z', E_L^\sigma(z'), U_L(z'))], \quad (\text{A.2})$$

$$\text{s.t. } E_L^\sigma = u(w_L) + \beta \mathbb{E}_z [(1 - s_L) E_L^\sigma(z') + s_L U_L(z')]. \quad (\text{A.3})$$

The notation for worker type is dropped as workers' values do not depend on worker type in type- $L$  firms. The first-order conditions (FOCs) along with the envelop condition lead to  $1/u'(w) = 1/u'(w(z'))$  for all  $z' \in Z$ , implying a fixed wage contract. This is a well-known result that a risk-neutral firm offers perfect insurance against aggregate fluctuations to risk-averse workers.

Type- $H$  firm's problem is boiled down to dynamic programming with additional constraints:

$$f_H^\sigma(z, E_{SH}^\sigma, U_{SH}(z)) = \max_{w_H, \{E_{SH}^\sigma(z')\}_{z' \in Z}} \bar{a}z - w_H + \beta \mathbb{E}_z [(1 - s_H) f_H^\sigma(z', E_{SH}^\sigma(z'), U_{SH}(z'))], \quad (\text{A.4})$$

$$\text{s.t. } E_{SH}^\sigma = u(w_H) + \beta \mathbb{E}_z [(1 - s_H) E_{SH}^\sigma(z') + s_H U_{SH}(z')], \quad (\text{A.5})$$

$$E_{SH}^\sigma(z') \geq U_{SH}(z') \quad \text{for all } z' \in Z, \quad (\text{A.6})$$

$$V_{NL}^\sigma(z) \geq V_{NH}^\sigma(z). \quad (\text{A.7})$$

Equation (A.6) denotes worker's participation constraint in the life of the contract due to the limited participation assumption. Equation (A.7) is type- $N$  workers' exclusion constraint, which appears only in the initial period of the contract.  $V_{NL}^\sigma(z)$  is obtained by solving type- $L$

firm's dynamic programming above, whereas  $V_{NH}^\sigma(z)$  is given by

$$V_{NH}^\sigma(z) = p\mu_H(\theta_H(z))(E_{NH}^\sigma(z) - U_{NH}(z)), \quad (\text{A.8})$$

$$E_{NH}^\sigma(z) = u(w_H) + \beta\mathbb{E}_z [(1 - s_H)E_{NH}^\sigma(z') + s_H U_{NH}(z')], \quad (\text{A.9})$$

$$U_{NH}(z) = u(b) + \beta\mathbb{E}_z [p\mu_H(\theta_H(z'))E_{NH}^\sigma(z') + (1 - p\mu_H(\theta_H(z'))U_{NH}(z'))]. \quad (\text{A.10})$$

Notice that the job-finding probability is discounted by the screening probability  $p$ .

As is shown by [Thomas and Worrall \(1998\)](#) and verified in a search and matching framework by [Rudanko \(2009\)](#), period wages take a max function under worker's limited commitment:  $w' = \max\{w, w^*(z')\}$ , where  $w^*(z')$  is the wage that lets the participation constraint [\(A.6\)](#) hold with equality, i.e.,  $E_{SH}^\sigma(z') = U_{SH}(z')$ . This can be seen in the optimality condition  $1/u'(w) = 1/u'(w(z')) - \eta(z')$ , where  $\beta(1 - s_H)\pi(z'|z)\eta(z')$  is the Lagrangian of [\(A.6\)](#). A fixed wage continues if [\(A.6\)](#) holds with inequality ( $\eta(z') = 0$  due to the complementary slackness condition), while a firm raises wage to ensure worker's participation if the outside option would exceed the value of current match under a fixed wage ( $\eta(z') > 0$ ).

**Definition of equilibrium.** A directed search equilibrium is defined in line with [Moen \(1997\)](#) and [Rudanko \(2009\)](#). An equilibrium consists of a worker's employment value,  $E_{ji}^\sigma(z_t)$ , a worker's unemployment value,  $U_{ji}(z_t)$ , a firm's value function,  $f_i^\sigma(z_t)$ , a market tightness,  $\theta_i(z_t)$ , and a wage contract,  $\sigma_i(z_t)$ , for  $j = S, N$ ,  $i = H, L$ , and all  $z_t \in Z$  such that

1. (Free entry) Firms enter a labor market and post vacancies with the associated contract  $\sigma_i(z_t)$  until the value of posting a vacancy becomes zero, i.e., equation [\(18\)](#) in the main text is satisfied.
2. (Firms' optimization) Given  $U$ , the firm's value function  $f_i^\sigma(z_t)$  solves the dynamic programming [\(A.2\)](#) and [\(A.4\)](#).
3. (Workers' unemployment value) The worker's unemployment value,  $U_{ji}(z_t)$ , evolves consistent with other elements of the equilibrium, i.e., equation [\(12\)](#) in the main text is satisfied.

4. (Pareto efficiency) There is no alternative pair  $(\hat{\theta}(z_t), \hat{\sigma}(z_t))$  with which the net surpluses of workers and firms are at least as much as those with  $(\theta(z_t), \sigma(z_t))$  and it is strictly more for one party, i.e.,

$$\mu_i(\hat{\theta}_i(z_t))(E_{ji}^{\hat{\sigma}}(z_t) - U_{ji}(z_t)) > \mu_i(\theta_i(z_t))(E_{ji}^{\sigma}(z_t) - U_{ji}(z_t)), \quad (\text{A.11})$$

and

$$-k_i + q_i(\hat{\theta}_i(z_t))f_i^{\hat{\sigma}}(z_t) > 0. \quad (\text{A.12})$$

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