Belief Polarization, Information Bias, and Financial Markets

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Abstract

This paper studies how belief polarization affects financial markets. I develop an equilibrium model with two groups of investors whose polarized views are driven by biased private signals. Investors trade competitively in the market based on public information revealed by the equilibrium asset price and private information accumulated through word-of-mouth communication. Investors' unconscious biases lead to belief divergence and generate excess volatility and trading volume. The information-sharing process amplifies these effects. The public asset price does not fully eliminate investors' unconscious biases.

Motivation

Why are people's views polarized? One possible explanation: **unconscious bias** (Hirshleifer, 2020; Akcay and Hirshleifer, 2021) in information possibly from

- Biased news to capture particular audience (CNN vs. FOX)
- Social media creates **echo chambers** (Cookson et al., 2021)

Research Questions: i) How is the financial market affected by the unconscious bias? ii) How is this process affected by the social transmission of information?

Theoretical Model

Standard CARA-normal competitive market with dynamic NREE.

- **OLG framework**: i from generation $t-1$ trades at date t, consumes at date $t+1$.
- **Two groups**: $g \in \{\mathcal{A}, \mathcal{B}\}$ has a continuum of investors with risk-aversion $\frac{1}{\gamma}$.
- **K risky assets**: pay dividends $\mathbf{D}_t = \boldsymbol{\eta} U_t + \mathbf{e}_t$, $U_t \sim \mathcal{N}(0, \tau_u^{-1})$ is a common factor.
- **Private signal**: At date t, $i \in g$ receives **biased** noisy signals about $\mathbf{D}_{t+1}$:

$$\mathbf{S}_t^i = \boldsymbol{\eta}(U_{t+1} + \delta_g) + \mathbf{e}_{t+1} + \boldsymbol{\varepsilon}_t^i, \quad \mathbf{e}_t \sim \mathcal{N}_K(\mathbf{0}, \tau_e^{-1}\mathbf{I}), \quad \boldsymbol{\varepsilon}_t^i \sim \mathcal{N}_K(\mathbf{0}, \tau_s^{-1}\mathbf{I})$$

  Unconscious bias δ_g is unobservable, and investors are unaware of it.
- **Public signal**: At date t, i observes equilibrium market prices $\mathbf{P}_t$.
- **i's distorted beliefs** about i) private information:

i — view of myself: $\mathbf{S}_t^{ii} = \mathbf{D}_{t+1} + \boldsymbol{\varepsilon}_t^i$ — views of group members: $\mathbf{S}_t^{ij} = \mathbf{D}_{t+1} + \boldsymbol{\varepsilon}_t^j$ — views of the other group: $\mathbf{S}_t^{il} = \mathbf{D}_{t+1} + \boldsymbol{\varepsilon}_t^l$

ii) public market prices:

$$\mathbf{P}_t^i = \mathbf{a}\underbrace{\left(\int_{j\in\mathcal{A}} \mathbf{S}_t^{ij}dj + \int_{l\in\mathcal{B}} \mathbf{S}_t^{il}dl\right)}_{\text{perceived aggregate private information}} + \mathbf{b}\mathbf{X}_t = \mathbf{a}\mathbf{D}_t + \mathbf{b}\mathbf{X}_t$$

- **Trading**: i maximizes terminal wealth by making portfolio choice $\mathbf{x}_t^i$ at date t facing noisy aggregate supply $\mathbf{X}_t \sim \mathcal{N}_K(\mathbf{0}, \tau_x^{-1}\mathbf{I})$.

An Endogenous Interpretation for Unconscious Biases

Unconscious bias δ_g is the endogenous result of information percolation process (Duffie et al., 2009) with echo chambers.

- **From t-1 to t**: Investors randomly meet and share endowed signals about U_{t+1}:

$$Z_{t-1}^i = U_{t+1} + \epsilon_{t-1}^i, \quad \epsilon_{t-1}^i \sim \mathcal{N}(0, \tau_z^{-1})$$

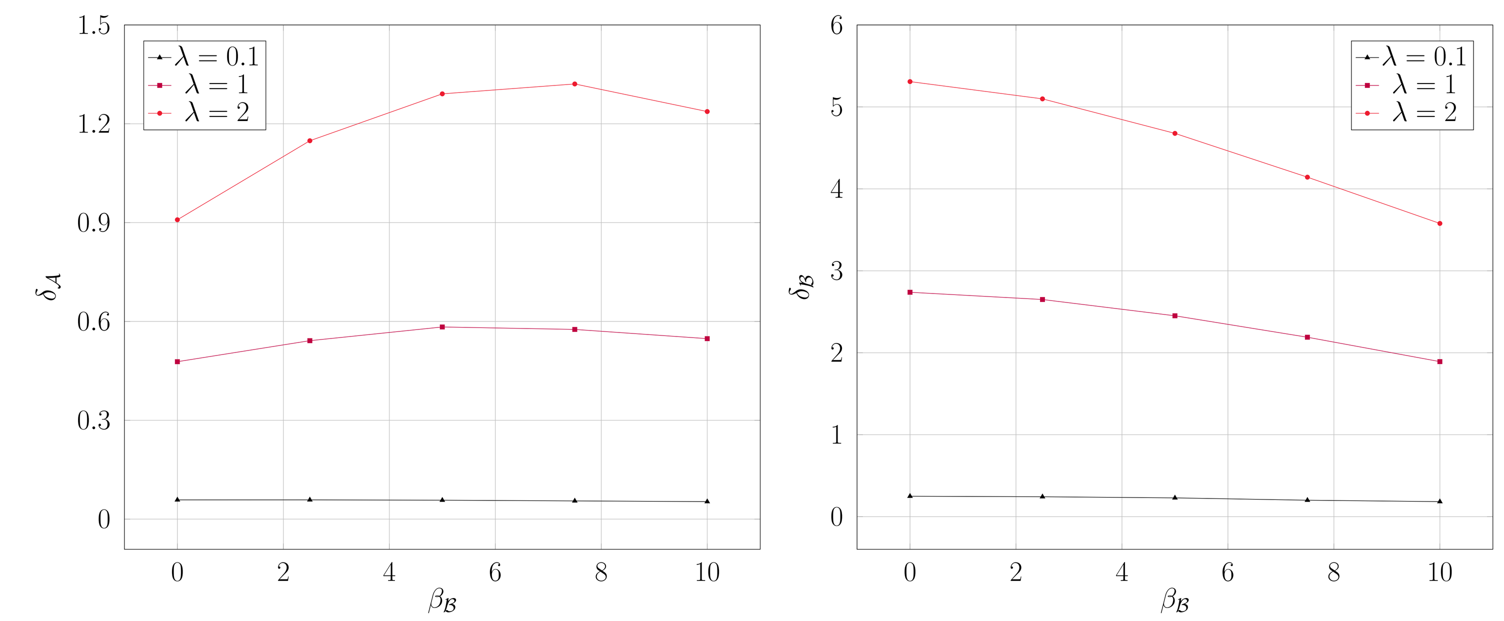
- **Meetings**: take place continuously at Poisson arrival times with intensity λ.
- **Echo Chambers**: governed by "*tolerance-to-listen*" parameter $\beta_g > 0$.

i: $\in \mathcal{A}$ if $\epsilon_{t-1}^i < 0$ — accept signals in the interval $(-\infty, \bar{Z}_s^i + \beta_\mathcal{A}]$

i: $\in \mathcal{B}$ if $\epsilon_{t-1}^i > 0$ — accept signals in the interval $[\bar{Z}_s^i - \beta_\mathcal{B}, +\infty)$

If $\beta_g \to +\infty$, $i \in g$ has open mind and is not restricted by echo chambers.
If $\beta_g \to 0$, $i \in g$ has a "*silo*" mentality and do not accept less extreme views.

- **Polarization case**: information percolation + echo chambers.
- **Benchmark case**: no echo chamber channel by letting $\beta_\mathcal{A}, \beta_\mathcal{B} \to \infty$.
- $\boldsymbol{\delta_g}$ is defined as the group distortion comparing the two cases.
- **An simulated example**: $\beta_\mathcal{A} = 10,000$, number of investors $= 100,000$, $\tau_z = 0.01$.



$\Rightarrow$ δ_g is endogenously generated by echo chamber effect in communications and is amplified with the information percolation process speeding up.

Result 1: Distorted Learning and Equilibrium

At trading date t, i learns about $\mathbf{D}_{t+1}$ under Gaussian updating. The misinterpretation of information distorts investor i's learning process.

- Conditional variance is not affected by the unconscious bias:

$$\text{Var}^{-1}[\mathbf{D}_{t+1}|\mathcal{F}_t^i] = (\boldsymbol{\eta}\boldsymbol{\eta}'\tau_u^{-1} + \mathbf{I}\tau_e^{-1})^{-1} + \tau_s\mathbf{I} + (\mathbf{b}_t^{-1}\mathbf{a}_t)^2\tau_x\mathbf{I}$$

- Conditional expectation is affected by the unconscious bias:

$$\mathbb{E}[\mathbf{D}_{t+1}|\mathcal{F}_t^i] = \underbrace{\text{Var}[\mathbf{D}_{t+1}|\mathcal{F}_t^i]\tau_x(\mathbf{b}^{-1}\mathbf{a})^2\mathbf{a}^{-1}\mathbf{P}_t}_{\text{learning from price}} + \underbrace{\text{Var}[\mathbf{D}_{t+1}|\mathcal{F}_t^i]\tau_s\mathbf{S}_t^i}_{\text{learning from private signal}}$$

The true equilibrium asset pricing function is given by

$$\left[\mathbf{I} + \frac{(\tau_u^{-1}\boldsymbol{\eta}\boldsymbol{\eta}' + \tau_e^{-1}\mathbf{I})^{-1}}{\tau_s + \gamma^2\tau_s^2\tau_x}\right]\mathbf{P}_t = \left[\mathbf{D}_{t+1} + \frac{1}{2}(\delta_\mathcal{A} + \delta_\mathcal{B})\boldsymbol{\eta}\right] - \frac{1}{\gamma\tau_s}\mathbf{X}_t$$

Main finding: Unconscious biases have an equilibrium aggregate effect!

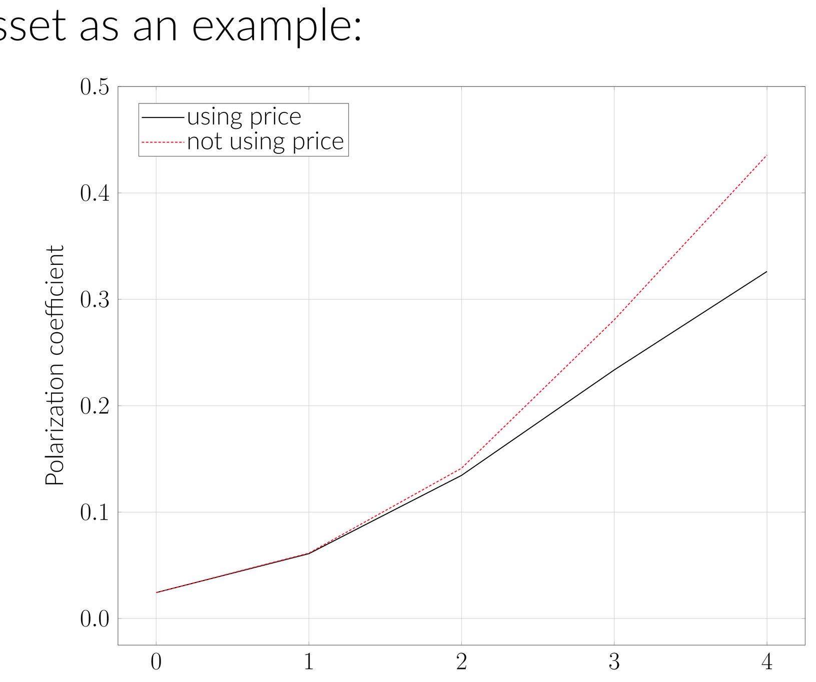
Result 2: Belief Polarization

Definition: Belief polarization $\mathcal{P}_t$ is measured as the distance between the average beliefs of the two groups about the future dividends $\mathbf{D}_{t+1}$:

$$\mathcal{P}_t \equiv \int_{i\in\mathcal{B}} \mathbb{E}\left[\mathbf{D}_{t+1}|\mathcal{F}_t^i\right]di - \int_{i\in\mathcal{A}} \mathbb{E}\left[\mathbf{D}_{t+1}|\mathcal{F}_t^i\right]di$$

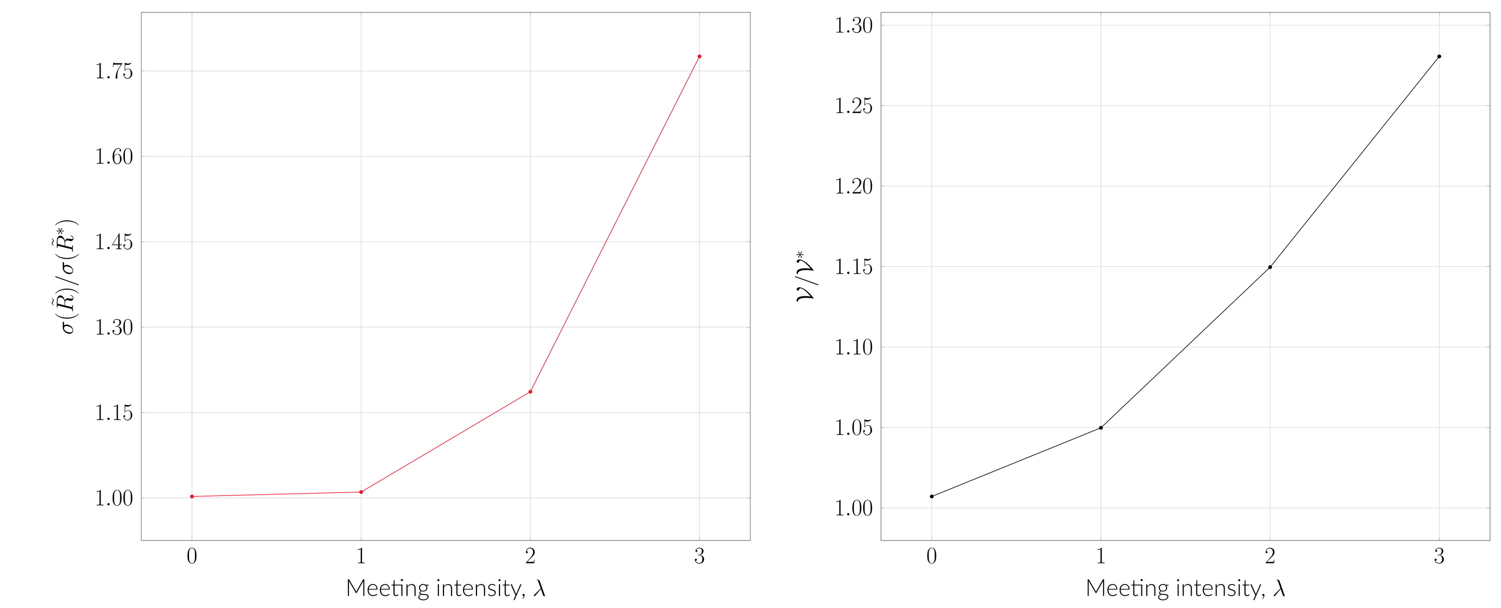
$$= (\delta_\mathcal{B} - \delta_\mathcal{A})\underbrace{\frac{\tau_s}{\tau_e+\tau_s+\gamma^2\tau_s^2\tau_x}\left[1 + \frac{\tau_e^2\sum_{k=1}^K\eta_k^2}{\tau_e\tau_u + (\tau_u+\tau_e\sum_{k=1}^K\eta_k^2)(\tau_s+\gamma^2\tau_s^2\tau_x)}\right]\boldsymbol{\eta}}_{\text{polarization coefficient}}$$

- Belief polarization exists **only when** there are biases.
- The larger absolute values of $(\delta_\mathcal{A} - \delta_\mathcal{B})$, the larger the belief polarization.
- Take one risky asset as an example:



Main finding: Communication of investors **amplifies** belief polarization while market price helps to **reduce** belief polarization

Result 3: Volatility and Trading Volume



Main finding: Unconscious bias generates excess volatility and trading volume, which increase with the speed of information percolation.

An Application to Political Economy

Motivation: political affiliation affects people's economic expectations (Kempf and Tsoutsoura, 2021; Mian et al., 2021).

- **Two trading dates** $t = 0, 1$, between which a **presidential election** happens.
- One risky asset with final payoff U at $t = 2$ and one risk-free bond with $R_f = 1$.
- Unconscious bias is identified as **partisanship bias** about the fundamental U.
- **Political affiliated groups**: $\{\mathcal{D}, \mathcal{R}\}$ have opposite bias $\delta(-\delta)$ in private signals S_t^i, which depends on the political status of the economy.
- Before the election, Democratic-affiliated (Republican-affiliated) investors are aligned (misaligned) investors.

$$S_0^i = U + \delta + \varepsilon_0^i, \quad i \in \mathcal{D}$$
$$S_0^i = U - \delta + \varepsilon_0^i, \quad i \in \mathcal{R}$$

- After the presidential election, Democratic-affiliated (Republican-affiliated) investors become misaligned (aligned) investors.

$$S_1^i = U - \delta + \varepsilon_1^i, \quad i \in \mathcal{D}$$
$$S_1^i = U + \delta + \varepsilon_1^i, \quad i \in \mathcal{R}$$

- Investors communicate about the private information S_0^i, S_1^i.
- **Polarization case**: strict echo chambers, only communicate within the group.
- **benchmark case**: no echo chambers, communication has no restriction.
- The trading strategy of investor i: $\nabla x_1^i \equiv x_1^i - x_0^i$.

Main finding: When information percolates with echo chambers, after the election, the Republican-affiliated group $\mathcal{R}$ will take more equity shares than the Democratic-affiliated group $\mathcal{D}$ at $t = 1$.

$$\int_{i\in\mathcal{R}} \nabla x_1^i di > \int_{i\in\mathcal{D}} \nabla x_1^i di$$

If the partisanship bias δ is significant or investors meet at high intensity, the Democratic-affiliated group $\mathcal{D}$ **rebalances** into the safe asset while the Republican-affiliated group $\mathcal{R}$ increases the equity holding (Meeuwis et al., 2020). These effects are attenuated if information percolates without echo chambers.

References

Akcay, E. and Hirshleifer, D. (2021). New evolutionary finance: social transmission bias and cultural evolution in financial markets. *Proceedings of the National Academy of Sciences*, 118(26).

Cookson, J. A., Engelberg, J. E., and Mullins, W. (2021). Echo Chambers. *working paper*.

Duffie, D., Malamud, S., and Manso, G. (2009). Information percolation with equilibrium search dynamics. *Econometrica*, 77:1513–1574.

Hirshleifer, D. A. (2020). Presidential Address: Social Transmission Bias in Economics and Finance. *The Journal of Finance*, 75(4):1779–1831.

Kempf, E. and Tsoutsoura, M. (2021). Partisan professionals: Evidence from credit rating analysts, forthcoming. *The Journal of Finance*.

Meeuwis, M., Parker, J. A., Schoar, A., and Simester, D. I. (2020). Belief disagreement and portfolio choice. *NBER working paper No. 25108*.

Mian, A., Sufi, A., and Khoshkhou, N. (2021). Partisan bias, economic expectations, and household spending, forthcoming. *The Review of Economics and Statistics*.