Population Dynamics and Family Policies

(work in progress)

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Background

Growing popularity and scale of family policies in developed countries:

- Since 1980s, more countries start adopting formal pronatal policies trend
- OECD countries spend 2% of GDP on family benefits in 2015 scale
- Delivered in the forms of: child-related cash transfers (baby bonus), public spending on services (universal childcare), and financial support through tax system (child tax credit)

Reasons why governments of developed countries use family policies:

1. Mitigate population aging caused by low fertility[1] tfr
2. Immigration alone is not the full solution (e.g. political opposition)

In the long-run, aggregate fertility is crucial for sustainability & growth:

- Jones (2020): policies related to fertility may determine whether we converge to an "empty planet" or to an "expanding cosmos"; they may be much more important than we have appreciated

[1] In principle, the analysis in this paper applies equally to the case where fertility is "too high". Countries rely more on non-fiscal, e.g. planned parenthood, rather than fiscal policies to reduce fertility. See Liao (2013) for an macroeconomic analysis on one-child policy.

Motivation

Existing literature focuses on empirical evaluation of family policies. They find:

- Size of policy effects varies across policies, yet “the directional finding that pronatal benefits boost fertility is nearly uniform" (Mcdonald 2006, Stone 2020)
- Elasticity estimates: increase in *present value* of child benefits equal to 10% of household income lead to 0.5-4.1% increase in fertility (Stone 2020) estimates
- Emphasis on short-run cost-effectiveness given the current pool of (potential) parents

Several important questions remain unanswered:

1. What are the impacts of family policies on future generations?
2. What are the trade-offs in the policy design?
3. With multiple policy instruments to raise fertility, which one(s) should we use?
4. Chu and Koo (1990) argues in favor of policies that restrain fertility among the poor as it improves human capital distribution - Is this a sensible policy recommendation?

This Paper

This paper proposes a tractable micro-founded model with four key elements:

1. Endogenous fertility choices and child human capital investments
2. Heterogeneity in population - income-based policies + heterogeneous response
3. Endogenous human capital dist. (population dynamics) - children are future parents
4. Family benefits and public education expenditures - policy complementarities

We use the calibrated model to:

1. Study the effects of commonly used family policies in transition and in the long-run
2. Find optimal (ex ante Ramsey) policy to achieve replacement fertility (TFR=2.1)
3. Explore the desirability of a different target (TFR=N) via the lens of reproduction possibility frontier (c.f. pandemic possibility frontier in Kaplan et al. 2020) RPF

Preview of Key Results

On the positive side,

1. We propose a flexible framework to embrace economic and ethical considerations on the design family policies
2. The calibrated model generates untargeted elasticities in the range of existing estimates
3. Family policies that are short-run cost-effective could be more costly in the long-run

On the normative side, with assumptions on welfare criteria,[2]:

1. Optimal family policy achieving replacement fertility combines expansion in public education and subsidized childcare
2. Reproduction possibility frontier identifies sizable trade-off between aggregate fertility and output per capita

[2] See *Reasons and Persons* by Parfit (1984) and *"Weighing Lives"* by John Broome (2006) for excellent discussions

Related Literature

“Macro-fertility", population dynamics, inequality and growth

- Doepke (2004), Greenwood et al. (2005), Golosov et al. (2007), Manuelli and Seshadri (2009), Jones et al. (2013), Schroonbroodt and Tertilt (2014), Petit (2019), de Silva and Tenreyro (2017, 2020)
- Chu and Koo (1990), **de la Croix and Doepke (2003), Knowles and Schoonbroodt (mimeo)**, Córdoba and Liu (2013), Liao (2013)
- Study optimal policy design with endogenous human capital formation and heterogeneous agents

“Micro-fertility" and policy evaluation

- **Becker and Tomes (1976, 1979)**, Jones et al. (2010), Bar et al. (2018), Córdoba et al. (2016, 2019)
- Whittington et al. (1990), Zhang et al. (1994), Milligan (2005), Laroque and Salanié (2008), Cohen et al. (2013), Luci-Greulich and Thévenon 2013, González (2013), Raute (2019), Kim (2020)
- Havnes and Mogstad (2011), Adda, Dustmann and Stevens (2017)
- Provide structural model to evaluate policy effects and study optimal policy design

Optimal taxation and Education Policies

- **Benabou (2002)**, Groezen, Leers and Meijdam (2003), Farhi and Werning (2012), Heathcote, Storesletten and Violante (2017), Guner, Kaygusuz and Ventura (2020)
- Domeij and Klein (2013), Ho and Pavoni (2019), Mullins (2019), Kurnaz (2020), **Daruich (2020)**
- Consider both endogeneous fertility and human capital formation in dynamic general equilibrium

Outline

- Simple planner's problem to build intuition
- Quantitative model with calibration
- Policy evaluations:
  1. Baby bonus
  2. Expand public childcare
  3. Expand public education
- Numerical results:
  1. Optimal family policies to reach replacement fertility
  2. Reproduction possibility frontier

Simple Model

Environment

- Generalizes Knowles and Schoonbroodt (mimeo) by including direct utility from fertility and education spendings
- Economy populated by heterogeneous agents with productivity $h_L = 0$, $h_H = 1$
- Each agent's working time $t(n)$ is decreasing in fertility n
- For simplicity of exposition, we make the following assumptions:
  1. Agents utility is given by:
  $$U = \underbrace{c}_{\text{consumption}} + u(\underbrace{n}_{\text{fertility}})$$
  2. Social planner achieves aggregate fertility N
  3. Social planner maximizes steady-state average utility of those who are actually born (c.f. $\mathcal{A}$-efficiency in Golosov, Jones and Tertilt 2007)
- Planner's choices include:
  1. ϕ: fraction of children born by agents with h_H
  2. E: education expenditure per child

Planner's Problem

- Denote steady-state share of agents with h_H as $p(\phi, E)$. It increases in ϕ, E
- Define the number of children *per agent* by productivity type:

$$n_L = \frac{(1-\phi)N}{1-p(\phi,E)}, \quad n_H = \frac{\phi N}{p(\phi,E)} \qquad \text{(accounting identity)}$$

- Assume n_H increases in ϕ, hence $t(n_H)$ decreases in ϕ
- The planner's problem is given by

$$\max_{\phi,E} \underbrace{\left(\underbrace{Y}_{\text{aggregate output}} - \underbrace{NE}_{\text{costs of education}} \right)}_{\text{aggregate consumption}} + \underbrace{\Pi(\phi,E)}_{\text{average utility from fertility}}$$

$$Y = \underbrace{1}_{h_H} \underbrace{p(\phi,E)}_{\text{share of } h_H} \cdot \underbrace{t(n_H)}_{\text{working time of } h_H}$$

$$\Pi(\phi,E) = p(\phi,E)u(n_H) + (1-p(\phi,E))u(n_L)$$

Optimal Fertility Profile ϕ

- First-order condition of ϕ:

$$\underbrace{\frac{\partial p(\phi, E)}{\partial \phi} \cdot t(n_H)}_{\text{intergenerational transmission}} = \underbrace{p(\phi, E) \cdot -\frac{dt(n_H)}{d\phi}}_{\text{cost of childbearing}} + \underbrace{-\frac{\partial \Pi(\phi, E)}{\partial \phi}}_{\text{distributive justice of fertility}}$$

- Policy recommendation in Chu and Koo (1990) is incomplete even in the social planner's problem as it ignores:
  1. Raising children reduces market time for parents
  2. Utility changes with ϕ - Lucas' Critique (c.f. Córdoba and Liu 2013)
- Human capital distribution is alertnot the policy objective in itself
- Optimal fertility profile ϕ will depend on:
  - Aggregation of individuals' preferences on fertility
  - Relative magnitude of intergenerational transmission and cost of childbearing

Optimal Education Expenditure E

- First-order condition of E:

$$\underbrace{\frac{\partial(Y+\Pi)}{\partial p}}_{\text{effects of composition on welfare}} \cdot \underbrace{\frac{\partial p(\phi, E)}{\partial E}}_{\text{effects of education on composition}} = \underbrace{N}_{\text{marginal cost}}$$

- The term $\frac{\partial p(\phi,E)}{\partial E}$ capture both:
  1. direct effects on children's productivity, and
  2. effects on future generations through intergenerational transmission
- Equating direct benefits to costs leads to under-investment (Daruich 2020)
- Education and family policies are closely related as E^* and ϕ^* are interdependent

Quantitative Model

Key Trade-offs in Quantitative Model

- Government expenditures, funded by distortionary labor taxes, should be distributed efficiently subject to achieving replacement fertility
- Potential uses of tax revenues:
  1. Family benefits targeting low-income parents
     - Low opportunity cost of child-raising in terms of market production ✓
     - More responsive to per dollar incentives
     - Economies of scale in child-raising
     - Overcome borrowing constraints in child investment
  2. Family benefits targeting high-income parents
     - Utilize intergenerational spillover of human capital ✓
  3. Increase public education expenditure uniformly
     - Raises human capital level for all children - hence future parents ✓
     - Affects fertility indirectly - direct and composition effects

Model Overview

- Overall: extend De la Croix and Doepke (2003) with family policies
- Household
  - Two-period overlapping generations model: child and adult
  - Unitary households that are heterogeneous in human capital level h
  - Choose fertility, labor supply, consumption and investment in children
- Representative firm takes labor as the only input
- Government
  - Imposes labor taxes that depend on income and fertility
  - Uses tax revenues to finance education, family benefits, and other spendings
- Population externalities in the form of idea creation and pollution
- General equilibrium with endogenous human capital distribution

Household Problem

- Households solve:

$$u(h) = \max_{c,n,l,e} \log(c) + \nu \log(\underbrace{n}_{\text{fertility}} \cdot (\mathbb{E}_\epsilon \underbrace{h'}_{\text{child h.c.}})) + \zeta \log(\underbrace{l}_{\text{leisure}}) - \underbrace{\mathcal{C}(N)}_{\text{congestion}} \tag{1}$$

where $\mathcal{C}(N)$ captures congestion externalities (e.g. pollution, scarce resources)[3]

- Household budget constraint:

$$\underbrace{c}_{\text{consumption}} + n \times \underbrace{e}_{\text{inv. per child}} = \underbrace{y}_{\text{total income}} - \underbrace{\mathcal{T}(y, n)}_{\text{net taxes}} \tag{2}$$

$$y = \underbrace{w}_{\text{wage}} \underbrace{h}_{\text{parents' h.c.}} \underbrace{(1 - n^\rho \cdot \chi - l)}_{\text{workinghours}} + \underbrace{\mathcal{F}(h, n)}_{\text{family benefits}} \tag{3}$$

- Child human capital production function with idiosyncratic shock $\log(\epsilon) \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma^2)$:

$$\underbrace{h'}_{\text{child h.c.}} = Z \cdot \underbrace{\epsilon}_{\text{shock}} \cdot \underbrace{h^\theta}_{\text{ige}} \cdot (\underbrace{E}_{\text{public edu.}} + \underbrace{e}_{\text{private edu.}})^\gamma \qquad e \geq 0 \tag{4}$$

where h^θ includes nature, interactions within family, and progressitivity in education

[3] We assume that $\mathcal{C}(N)$ is increasing and $\lim_{N \to \infty} \mathcal{C}(N) = +\infty$

First-order Conditions

Fertility choice

$$\underbrace{\frac{\nu}{n}}_{\text{mu of fertility}} = \underbrace{\lambda_{bc}\left(\underbrace{-\frac{dy}{dn}}_{\text{direct cost}} + \underbrace{\frac{\partial \mathcal{T}(y,n)}{\partial y}\cdot\frac{dy}{dn} + \frac{\partial \mathcal{T}(y,n)}{\partial n}}_{\text{effects via taxes}} + \underbrace{\frac{\partial \mathcal{F}(h,n)}{\partial n}}_{\text{effects via family benefits}} + \underbrace{e}_{\text{education expense}}\right)}_{\text{mc of an additional child}}$$

Fixed cost $\chi +$ quality "endowment" $E \Rightarrow$ quality-quantity tradeoff

Education investment

$$\underbrace{\frac{\nu\gamma}{E+e}}_{\text{mu from child quality}} = \underbrace{\lambda_{bc}\cdot n}_{\text{direct cost}} + \underbrace{\lambda_e}_{\text{non-negativity of investment}}$$

$$\underbrace{\lambda_e \cdot e = 0}_{\text{complementary slackness}}$$

With $E > 0$, there exists a threshold h^* such that $e^*(h) = 0$ when $h \leq h^*$, and $e^*(h)$ monotonically increases with h as $h > h^*$

Firms and the Stationary Equilibrium

extensions

- Representative firm takes labor as the only input

$$Y = e^{\mathcal{A}(N)} \cdot H \tag{5}$$

where $e^{\mathcal{A}(N)}$captures externalities in ideas production[4]

- Denote government policies as $\mathcal{P}$. Stationary distribution $F_{\mathcal{P}}(h)$ solves:

$$F_{\mathcal{P}}(k) = \frac{1}{N} \int_{\Omega_h} \int_{\Omega\epsilon} n^*(h) \mathbb{1}_{h'<k} \, dG(\epsilon) \, dF_{\mathcal{P}}(h) \tag{6}$$

$$N = \int_{\Omega_h} n^*(h) \, dF_{\mathcal{P}}(h) \tag{7}$$

- Multi-type branching (Galton-Watson) process. Existence, uniqueness, and convergence of stationary distribution $F_{\mathcal{P}}(h)$ are shown in Mode (1971)
- Fertility choices lead to externalities from $\mathcal{C}(N), \mathcal{A}(N)$ and $F_{\mathcal{P}}(h)$ - reasons why laissez faire N could be too high or too low

[4] We assume that $\lim_{N\to 0} e^{\mathcal{A}(N)} = 0$ and $\lim_{N\to\infty} e^{\mathcal{A}(N)} = 0$ is bounded.

Government

- The government raises revenues from income taxes $\mathcal{T}(y, n)$
- Government expenditures include exogenous spending X, education expenditure per child E, and family benefits $\mathcal{F}(\cdot)$ capturing two widely used pronatal policies:[5]

$$\mathcal{F}(h, n) = \underbrace{\alpha_1 \cdot n}_{\text{baby bonus}} + \underbrace{\alpha_2 \cdot h \cdot n^{\rho}}_{\text{universal child care}} \quad (8)$$

where $\alpha_2 \cdot h \cdot n^{\rho}$ is equivalent to reducing time costs χ by $\frac{\alpha_2}{w}$ per child for all h

- Enriching the universe of policy tools would allow for more targeting and better policies
- Fiscal budget constraint:

$$\underbrace{\int \mathcal{T}(y^*(h), n^*(h))\, dF_{\mathcal{P}}(h)}_{\text{net taxes revenue}} = \underbrace{\int \mathcal{F}(h, n^*(h))\, dF_{\mathcal{P}}(h)}_{\text{family benefits}} + \underbrace{N \cdot E}_{\text{public education}} + \underbrace{X}_{\text{others}} \quad (9)$$

[5] Another policy instrument not studied here is parental leave. Incorporating it in the analysis requires model extensions including gender roles and employment risk (Wang 2020).

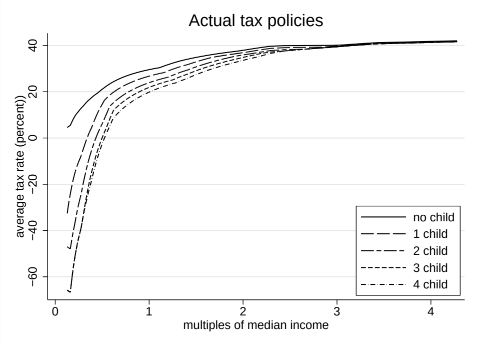
Calibration and Positive Analyses

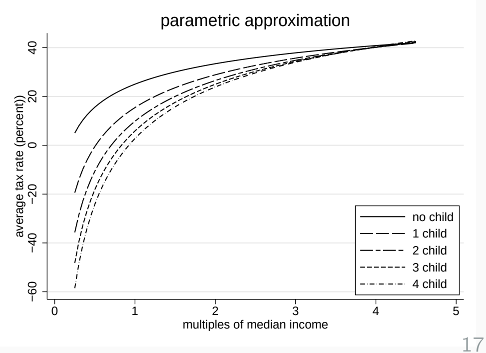
Current Tax System

- Actual policies in the U.S. simulated using TAXSIM
- Parametric specification following Heathcote, Storesletten and Violante (2017): $T(y) = y \cdot \left(1 - \tau y^{-\lambda}\right)$. Adding # of children ($n$):

$$T(y, n) = y \cdot \left[1 - (\tau_1 + \tau_2 \log(n+1)) y^{-(\tau_3 + \tau_4 \log(n+1))}\right] \tag{10}$$

$$\tau_1 = 0.699, \quad \tau_2 = 0.088, \quad \tau_3 = 0.151, \quad \tau_4 = 0.096$$





Model Parameters

Table 1: Calibrated parameters - matched to the United States in 2010

	Interpretation	Value	Source/Target
χ	fixed cost per child	0.15	Folbre (2008)
ρ	economies of scale in child-raising	0.80	Folbre (2008)
ν	utility from fertility	0.269	total fertility rate (World Bank)
ζ	utility from leisure	0.447	average working hours (CPS)
E	government spending on education	0.078	OECD Education Statistics
Z	normalizing scalar	3.968	median income equals one
θ	intergenerational spillover	0.176	IGE = 0.34 (Chetty et al. 2014)
γ	productivity of goods investment in h'	0.092	fertility differential (CPS supplement)
σ	dispersion of idiosyncratic shock	0.696	income dispersion (Census)

Policy Effects

- Consider an unexpected, permanent policy change from $\mathcal{P}$ to $\mathcal{P}'$ at $t = 0$
- Total effects between steady-states:

$$\underbrace{\Delta X_{\mathcal{P}\to\mathcal{P}'}}_{\text{total effects between two s.s.}} \approx \underbrace{\int \Delta x_{\mathcal{P}\to\mathcal{P}'}(h)\, dF_{\mathcal{P}}(h)}_{\text{short-run effects}} + \underbrace{\int x_{\mathcal{P}}(h)\, d\Delta F_{\mathcal{P}\to\mathcal{P}'}(h)}_{\text{dynamic composition effects}} \quad (11)$$

- Short-run effects evaluated under $F_{\mathcal{P}}(h)$ - compare untargeted model elasticities with empirical estimates
- The calibrated model provides estimates of:
  1. Dynamic composition effects, hence long-run policy effects under $F_{\mathcal{P}'}(h)$
  2. Transition path of the economy (population dynamics):

$$F_{t+1}(k) = \frac{1}{N_t} \int_{\Omega_h} \int_{\Omega\epsilon} n^*(h) \mathbb{1}_{h'<k}\, dG(\epsilon)\, dF_t(h)$$

where household choices are under $\mathcal{P}'$, and initial condition is $F_0(h) = F_{\mathcal{P}}(h)$

Baby Bonus in the Short-Run

Policy counterfactual # 1

- Consider a baby bonus of \$5,000 per new-born child independent of birth order (0.13% of annual GDP with current level of fertility) with stationary $F_{\mathcal{P}}(h)$

Short-run effects:

- Aggregate fertility increases from 1.92 to 1.962 on impact (24% towards 2.1)
- Magnitude: pv 6.4% of median household income leading to 2.2% increase in fertility (c.f. Stone 2020: pv 6.4% $\rightarrow$ 0.32-2.62% increase in fertility)
- Low- to middle-h.c. families have larger responses to uniform cash transfers (c.f. Bonner and Sarkar 2020 on Australian baby bonus)
- Total hours decrease by 0.90% - raising children is time-costly
- Per capita output decreases by 0.49% - further "costs" beyond baby bonus

Baby Bonus in Transition and the Long-Run

Long-run effects:

- Evaluate the \$5,000 baby bonus as distribution transits to $F_{\mathcal{P}'}(h)$
- Aggregate fertility rises to 1.963 - almost all effects are realized at $t = 0$ plot
- Per capita output decreases by 0.72% (c.f. short-run drop of 0.49%) plot
- Key intuition: heterogeneous fertility responses + intergenerational transmission of human capital = changing equilibrium human capital distribution plot

Expand Public Childcare Provision

Policy counterfactual # 2

- Consider redirecting the same amount of baby bonus (0.13% of GDP) to expanding public childcare (reduce child fixed costs χ by 1%)

	aggregate fertility	total hours	total output
short-run	+0.025	-0.55%	-0.47%
long-run	+0.025	-0.55%	-0.49%

- Effects on aggregate fertility is positive but only 60% of baby bonus
- Magnitude is again within the range of estimates summarized in Stone (2020)
- Loss in hours is smaller - public childcare encourages the combination of employment and motherhood (Rindfuss et al. 2010, Bauernschuster et al. 2013)
- Human capital distribution is unaffected in the long-run

plots

Expand Public Education

Policy counterfactual # 3

- Consider redirecting the same amount of baby bonus (0.13% of GDP) to expanding public education expenditure (increase E by 4.3%)

	aggregate fertility	total hours	total output
short-run	same	same	same%
long-run	-0.001	+0.03%	+0.50%

- Fertility is unchanged in the short-run, and even decreases in the long-run
- Increased birth intention is balanced by changing human capital distribution (DeCicca and Krashinsky 2016) plots
- Education raises output in the long-run with same hours worked

Take-Aways

1. Besides matching aggregate data, in policy counterfactuals the model generates fertility elasticities that are in the range of existing estimates
2. Policy that achieves short-run cost-effectiveness could be more costly in the long-run when human capital distribution changes
3. Each “naive" policy tool has its strengths and weaknesses - policy maker needs to consider them jointly (echoes Ufuk et al. 2020 - coupling education and innovation policies) plots
4. If the mechanism of fertility growth is reliant on families with low educational attainment, “the incentives need to be supplemented by human-capital-augmenting programs to enhance the productivity of their children" (Bonner and Sarkar 2020)

Normative Policy Analyses

Ramsey Problem

- We assume that the government is maximizing steady-state ex ante welfare of those who are actually born (c.f. $\mathcal{A}$-efficiency in Golosov, Jones and Tertilt 2007):

$$\text{SWF}_{\mathcal{P}} = \underline{u} + \left[\int \left(u(h) - \underline{u}\right)^{\frac{\psi-1}{\psi}} \, dF_{\mathcal{P}}(h) \right]^{\frac{\psi}{\psi-1}}$$

where $1/\psi$ governs inequality aversion in the society[6], and $\underline{u} \equiv \min_{h\in\Omega_h} u(h)$

- Magnitude of $\mathcal{A}(N)$ and $\mathcal{C}(N)$ is uncertain (e.g. Jones 2020, Bohn and Stuard 2015)
- The key idea to make further progress is to decompose the maximization of $\text{SWF}_{\mathcal{P}}$ into two problems (c.f. two-stage budgeting):

$$\max_{\mathcal{P}} \text{SWF}_{\mathcal{P}} \equiv \max_{N} \left[\left(\max_{\mathcal{P}} \widetilde{\text{SWF}}_{\mathcal{P}}(N) \right) + \left(\mathcal{A}(N) - \mathcal{C}(N)\right) \right]$$

where $\max_{\mathcal{P}} \widetilde{\text{SWF}}_{\mathcal{P}}(N)$ is a constrained optimization problem with $\mathcal{A}(N) = \mathcal{C}(N) = 0$ subject to fiscal budget constraint and “aggregate fertility constraint"

$$\int n^*(h) \, dF_{\mathcal{P}}(h) = N \tag{12}$$

[6] As $\psi \to +\infty$, we are in the case of utilitarianism; as $\psi \to 0$, we are in the case of maxmin.

Optimal Policy to Reach Replacement Fertility

- We solve the constrained optimization problem with some additional assumptions:
  1. $N = 2.1$ - commonly accepted long-run fertility target
  2. $\psi = 0.1$ - conservative and close to Rawlsian maxmin principle
  3. $\alpha_1, \alpha_2 \geq 0$ - not allowing for explicitly taxing childbearing
  4. Majority support for policy reform: $\int \mathbb{1}_{\mathcal{P} \succsim_h \mathcal{P}_0} \, dF_{\mathcal{P}_0}(h) > 0.5$
- Optimal policy $\mathcal{P}^*(2.1) = \{E^*, \alpha_1^*, \alpha_2^*\}$ leads to upward shift of fertility profile: plot

	magnitude
subsidized childcare (α_2)	reduce fixed costs χ by 6.0%
increased education (E)	increase E by 15.4%
baby bonus (α_1)	not used $\alpha_1^* = 0$
ex ante c.e.	+2.16%
output per capita	-2.14%

Table 2: Optimal family policy reaching replacement fertility

Optimal Policy to Reach Replacement Fertility - Discussions

- Baby bonus is cost-effective in the short-run and is more progressive. Yet it is not used in the optimal policy combination due to its adverse effects on $F_{\mathcal{P}}(h)$
- Baby bonus would be used if the policy maker makes education system more progressive by reducing θ - more measurements/decomposition needed
- Moral judgments and policy assumptions matter for optimal policy results:
  1. When we relax the restriction on $\alpha_1, \alpha_2 > 0$, optimal policy would include $\alpha_1 < 0$ (uniform child tax) and $\alpha_2 \uparrow$. The resulting fertility profile is hump-shaped
  2. As inequality-aversion $(1/\psi)$ decreases, α_1 decreases while α_2 increases - overall policy becomes less progressive

Transition

- Policy recommendations based on steady-state comparisons should consider transition path (Conesa and Krueger 2006)
- In our context, different $\mathcal{P}$ will induce different transition path along which population will not be the same in general
- $\mathcal{P}^*(2.1)$ will be closer to being “dynamically optimal" when:
  - The government is more patient
  - The transition takes fewer periods to complete
- Transition to new steady-state is accomplished fairly quickly in two generations

Optimal Aggregate Fertility Rate

- Optimal N solves the second-step problem:

$$\max_{N} \widetilde{\text{SWF}}_{\mathcal{P}^*}(N) + (\mathcal{A}(N) - \mathcal{C}(N))$$

with first-order condition:

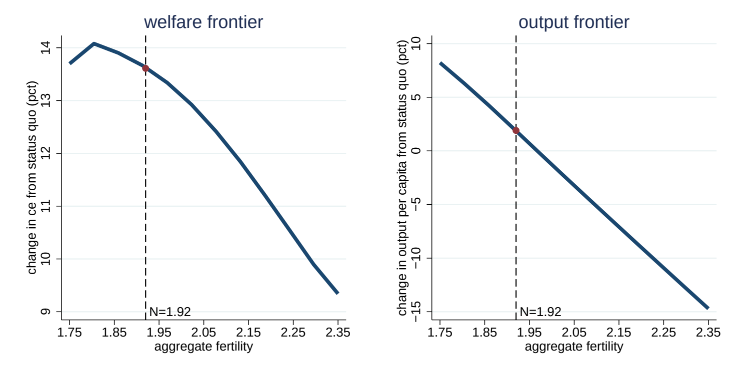
$$\underbrace{\frac{d\widetilde{\text{SWF}}_{\mathcal{P}^*}(N)}{dN}}_{\text{marginal benefits of higher fertility}} = \underbrace{\frac{d(\mathcal{C}(N) - \mathcal{A}(N))}{dN}}_{\text{marginal ``cost" from externalities}}$$

- Theoretically, optimal N^* should have an interior solution in the model as:

1. Assumptions on utility function: $\lim_{N\to 0} \widetilde{\text{SWF}}_{\mathcal{P}^*}(N) = \lim_{N\to\infty} \widetilde{\text{SWF}}_{\mathcal{P}^*}(N) = -\infty$
2. Assumptions on externalities: $\lim_{N\to 0} e^{\mathcal{A}(N)} = 0$, $\lim_{N\to\infty} e^{\mathcal{A}(N)} < \infty$, $\lim_{N\to\infty} \mathcal{C}(N) = +\infty$

Reproduction Possibility Frontier

- We trace out $\widetilde{\text{SWF}}_{\mathcal{P}^*}(N)$ and $Y_{\mathcal{P}^*}(N)$ to illustrate aggregate tradeoff while further research on measuring $\mathcal{A}(N)$ and $\mathcal{C}(N)$ are needed (e.g. Bohn and Stuard 2015)



Conclusion

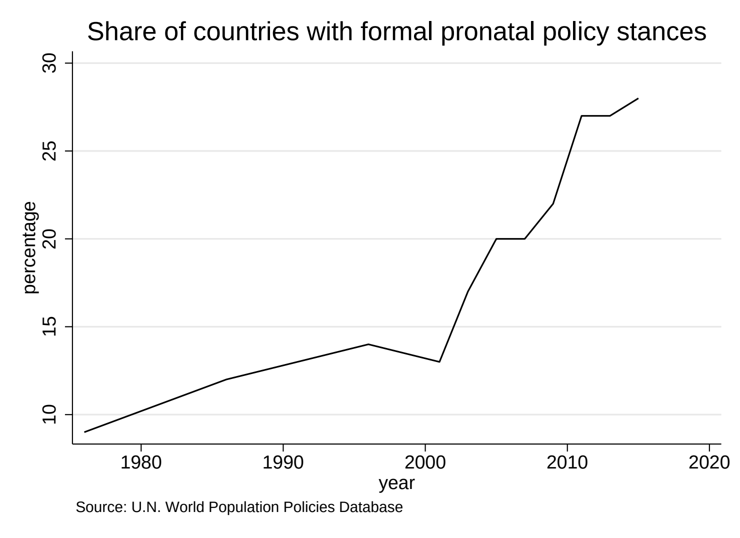
Conclusion

- Build a tractable GE-OLG model with heterogeneous agents, endogenous fertility, and human capital formation to study family policies
- The model generates untargeted elasticities in the range of existing estimates
- We find the following results:
  1. Intergenerational transmission of h.c., costs of childrearing, and productivity of education are the key determinants of the aggregate trade-off
  2. Family policies that are short-run cost-effective could be more costly in the long-run
  3. Various “naive" policy tools need to be considered jointly
  4. Under preferred welfare criteria, optimal family policy achieving replacement fertility combines expansion in public education and subsidized childcare

Comments are greatly appreciated ✉ anson.zhou@wisc.edu

Appendix

Trend in Pronatal Policies Around the World



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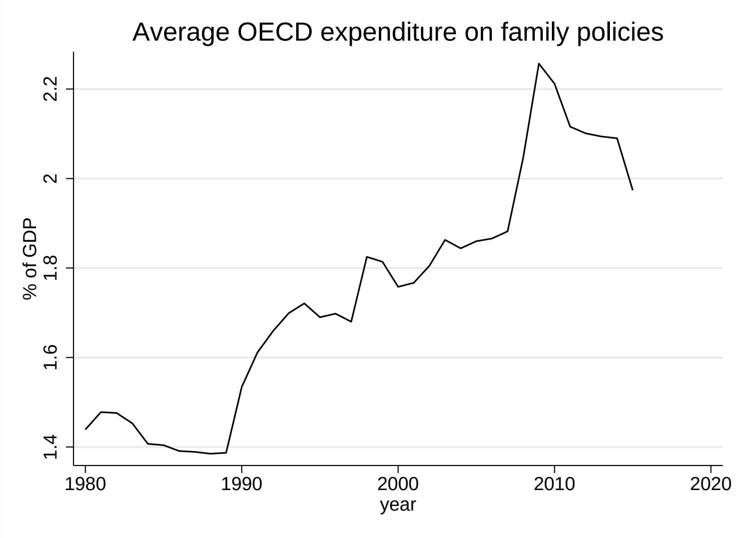
Trend in Family Benefits Expenditures, OECD



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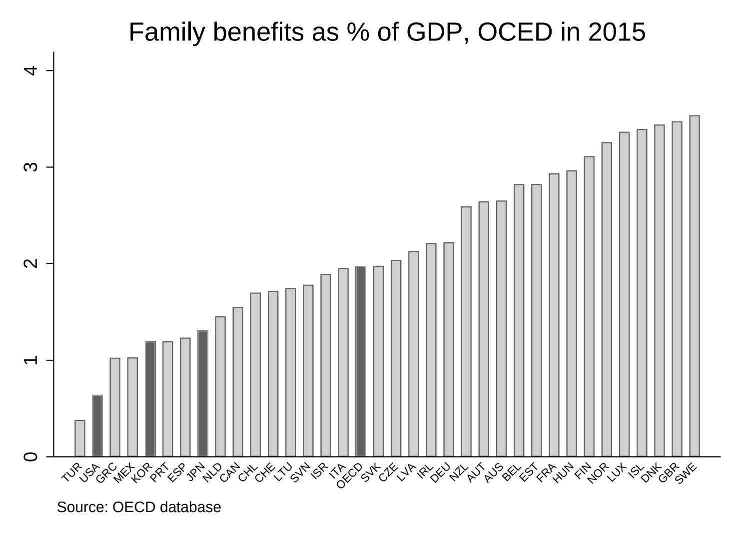
Family Benefits as % of GDP, OECD 2015



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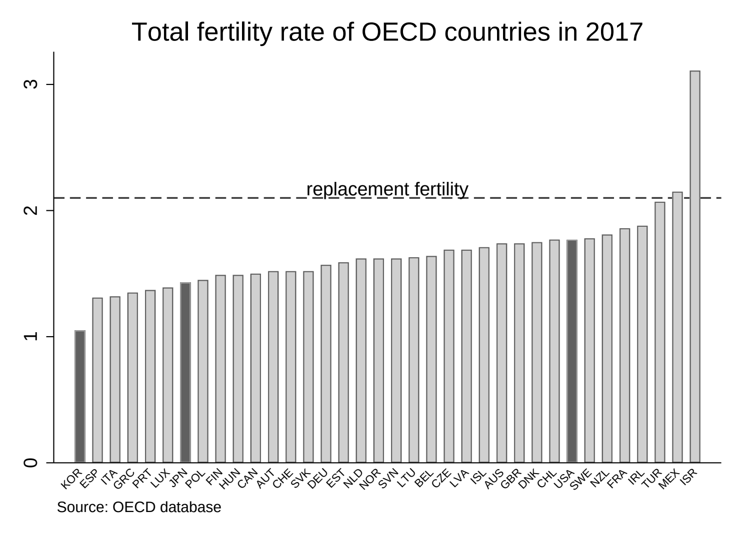
Total Fertility Rate of OECD Countries in 2017



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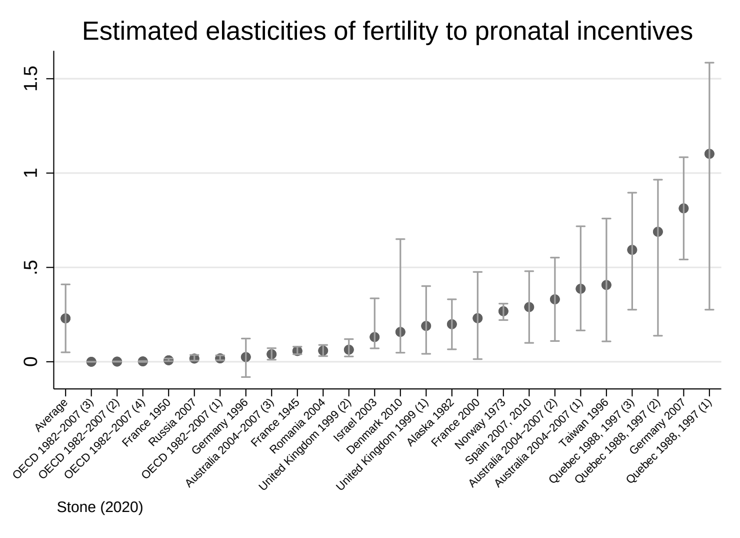
Estimated Elasticities of Fertility to Pronatal Incentives



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Achieving Replacement Fertility

‘When asked what would be a desirable fertility level, most politicians, journalists, and even demographers would answer slightly above two children per woman; many would mention the precise level of the total fertility rate (TFR): 2.1."

– *Lutz (2014)*

“The National Population Policy 2000 — released on Feb.15th — aims to bring the total fertility rate (TFR) to replacement level by 2010 and to achieve a stable population by 2045, at a level consistent with sustainable economic growth, social development, and environmental protection."

– *Ministry of Health, India*

back

Reproduction Possibility Frontier

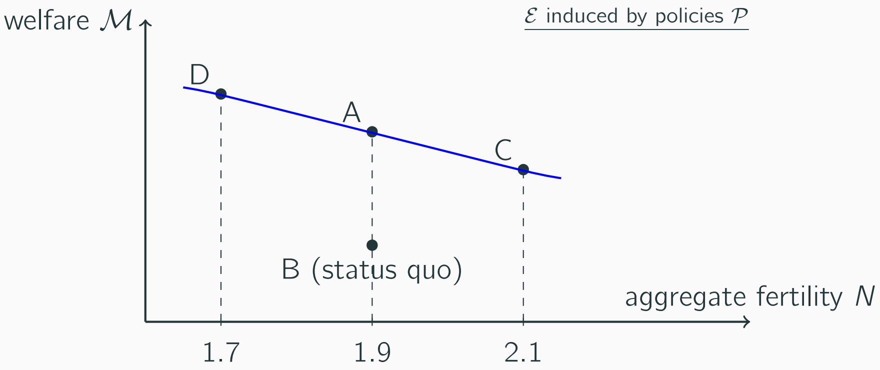


Figure 1: Reproduction Possibility Frontier (RPF)

- RPF shows the highest achievable objective $\mathcal{M}$ for every level of aggregate fertility in a stationary environment

back

Potential Extensions

- Marketable childcare
- Life-cycle with more periods allowing for:
  1. Idiosyncratic productivity shocks and wealth accumulation
  2. Retirement, pension system, inter-vivos transfers and bequests
  3. Human capital accumulation with dynamic complementarity
- Human capital production function permitting:
  1. Imperfect substitution between public and private expenditures
  2. Endogenous time investment in child human capital formation
- Behavioral component in fertility determination
- Production function allowing for:
  1. Productivity growth
  2. Physical capital in the production function
  3. Heterogeneous human capital

back1
back2 back3

Model Fit - Fertility

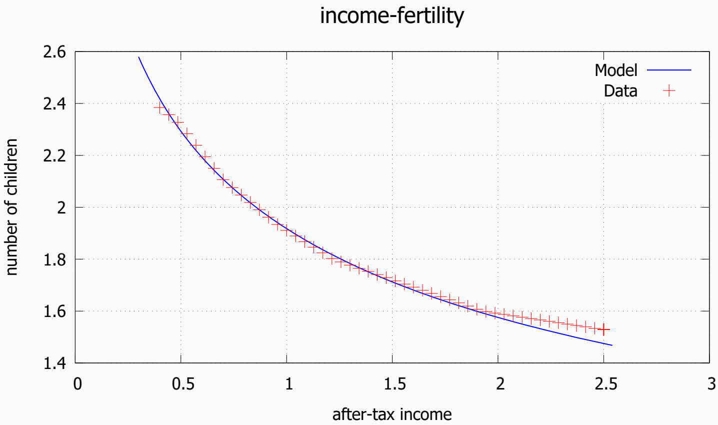


Figure 2: Income-Fertility Profile

Model Fit - Mobility

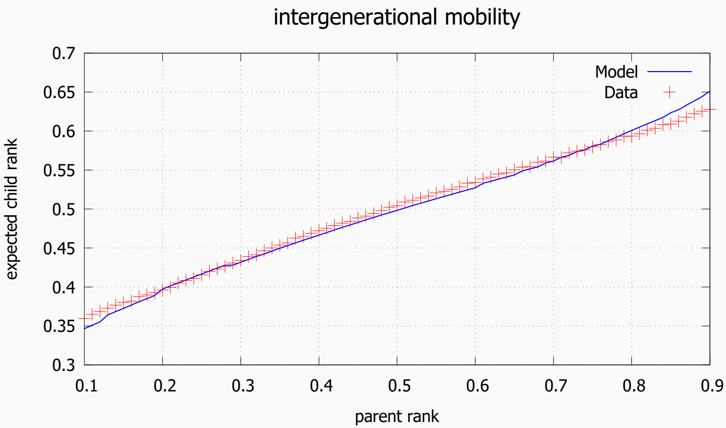


Figure 3: Intergenerational Mobility

back

Transition of Aggregate Fertility under Baby Bonus

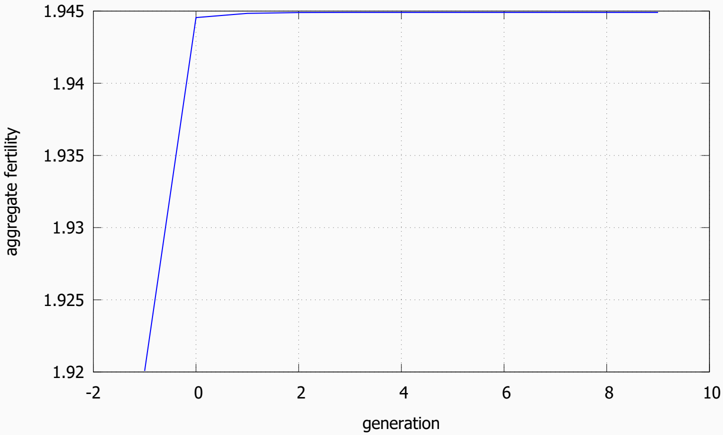


Figure 4: Transition of Aggregate Fertility under Baby Bonus

Transition of Per Capita Output under Baby Bonus

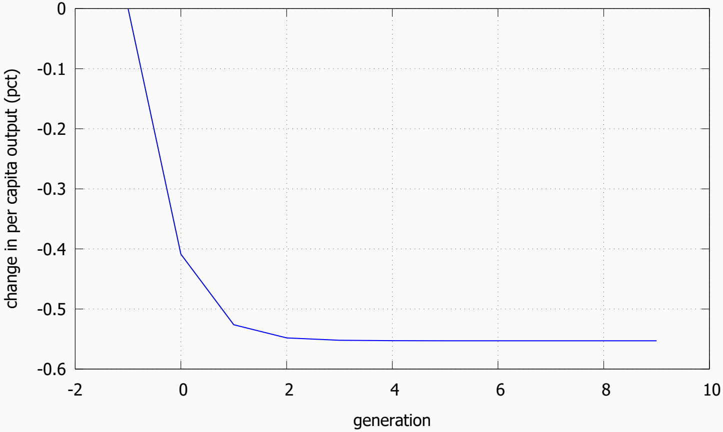


Figure 5: Transition of Per Capita Output under Baby Bonus

Transition of Average Human Capital under Baby Bonus

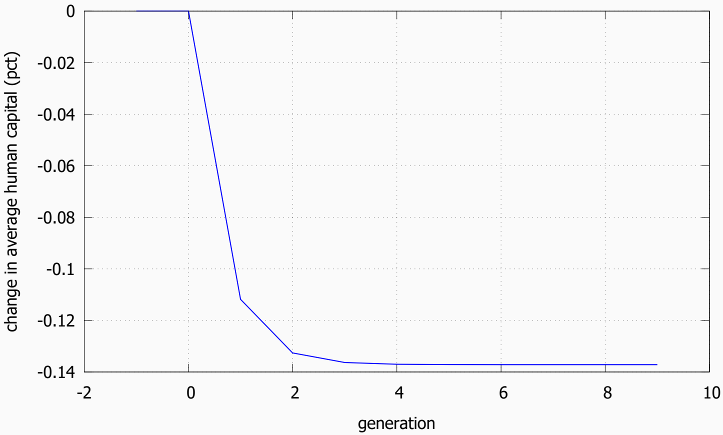


Figure 6: Transition of Average Human Capital under Baby Bonus

back

Transition of Aggregate Fertility under Expanded Public Childcare

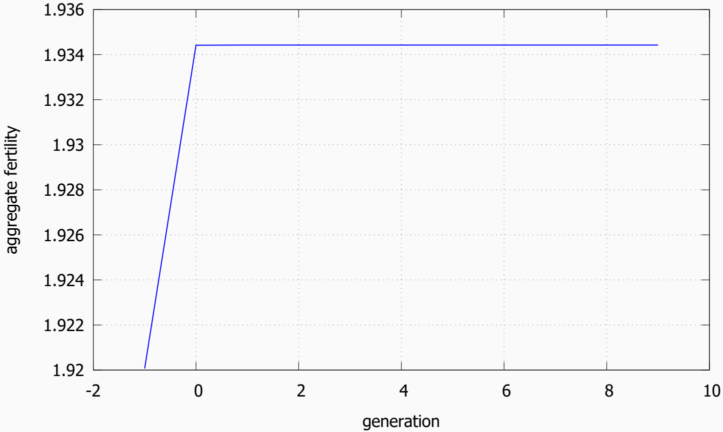


Figure 7: Transition of Aggregate Fertility under Expanded Public Childcare

Transition of Per Capita Output under Expanded Public Childcare

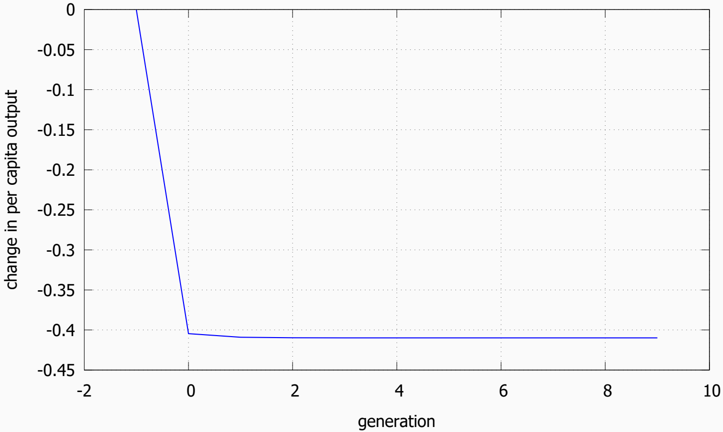


Figure 8: Transition of Per Capita Output under Expanded Public Childcare

Transition of Aggregate Fertility under Expanded Public Education

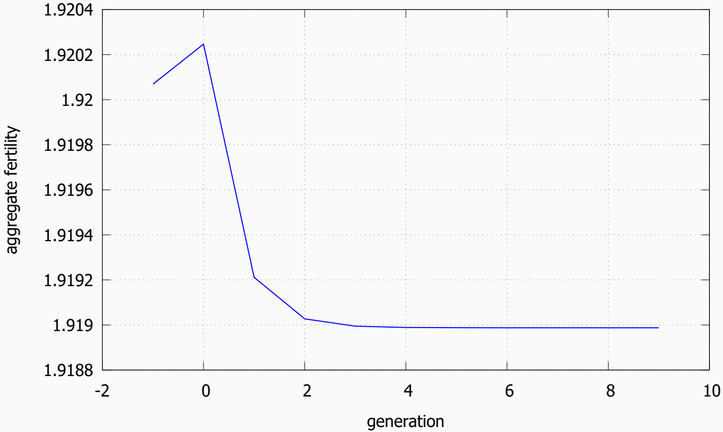


Figure 9: Transition of Aggregate Fertility under Expanded Public Education

Transition of Per Capita Output under Expanded Public Education

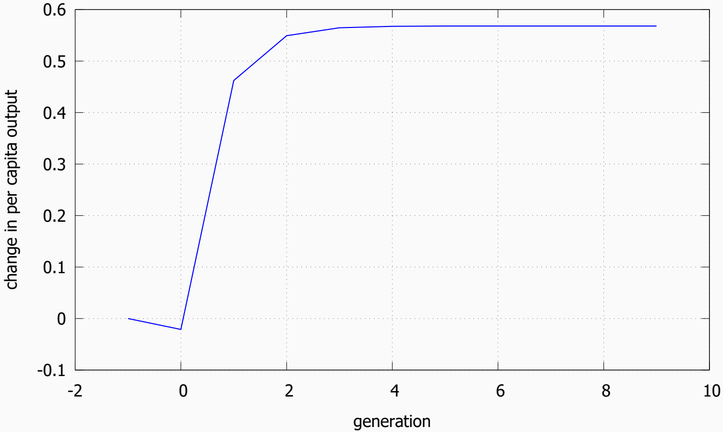


Figure 10: Transition of Per Capita Output under Expanded Public Education

Transition of Average Human Capital under Expanded Public Education

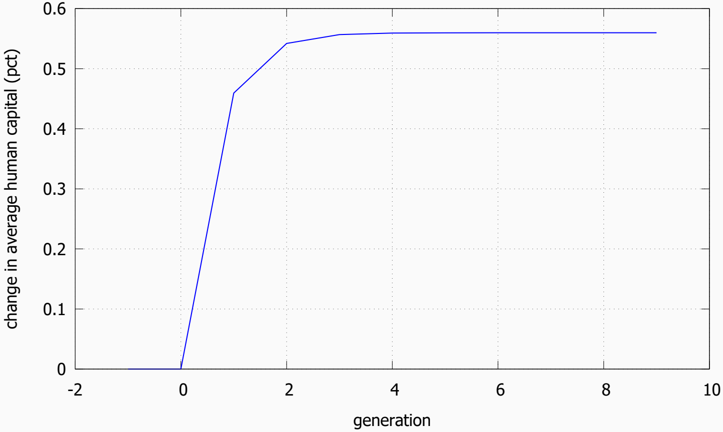


Figure 11: Transition of Average Human Capital under Expanded Public Education

Policy Expansion Paths

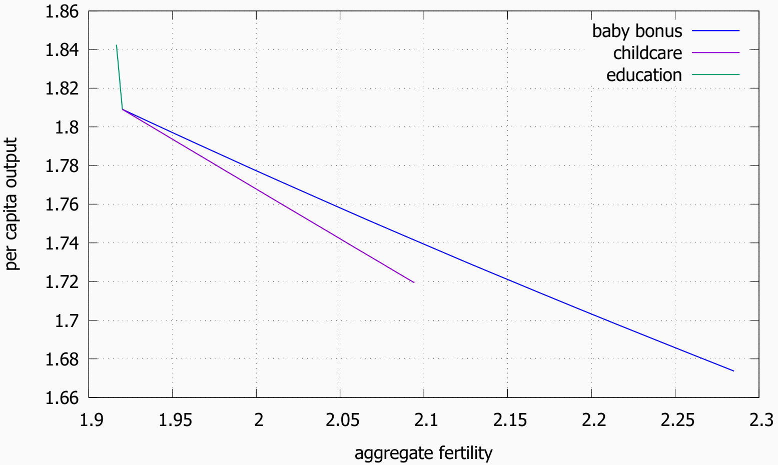
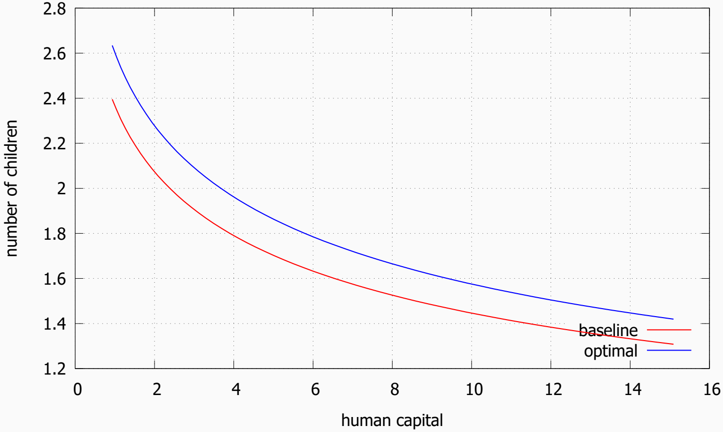


Figure 12: Policy Expansion Paths

Note: For baby bonus and childcare, expenditure ranges from 0% to 1.5% of GDP. For education, the increase of E from baseline ranges from 0% to 10%. Not balancing government budget constraint in this exercise.

back

Optimal Fertility Profile



back