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Does Unobservable Heterogeneity Matter for Portfolio-Based Asset Pricing Tests?

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Overview

Portfolio Sorts

- Theoretical result: Portfolio sorts tend to misattribute parts of the "alpha" to the firm characteristic underlying the sort
- Specification test: Hausman (1978) type test for assessing the reliability of alpha estimates from portfolio sorts

GPS-Model

GPS = Generalized Portfolio Sorts

- Firm-level regression approach for analyzing stock returns
- GPS-Model can be estimated with firm fixed effects, which so far are largely ignored in (portfolio-based) asset pricing tests
- Method nests standard portfolio sorts and solves their weaknesses

Popular method for analyzing stock returns

Portfolio Sorts Approach

Step 1

- Sort stocks into 10 decile portfolios on firm characteristic X
- Compute the high-minus-low decile portfolio return

$r_{\Delta,t} = r_{High,t} - r_{Low,t}$ where $r_{pt} = \sum_i w_{it} r_{it}$ $(p = High, Low)$

Step 2

Estimate k-factor model (time-series regression)

$$r_{\Delta,t} = \alpha_\Delta + \beta_\Delta \, RMRF_t + \ldots + \varepsilon_{\Delta t}$$

➔ Characteristic X predicts cross-section of stock returns if $\hat{\alpha}_\Delta \neq 0$

Question: Is $\hat{\alpha}_\Delta \neq 0$ a *reliable* criterion for identifying characteristics-based factors?

Our approach for analyzing stock returns

Generalized Portfolio Sorts (GPS) Model

Model Setup

Firm-level panel regression: $r_{it} = (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + c_i + v_{it}$

r_{it}	(excess) return of firm i in period t
$\boldsymbol{z}_{it}$	vector of firm characteristics $z_{m,it}$
$\boldsymbol{x}_t$	vector of market-level factor variables $x_{k,t}$
c_i	firm fixed effect ($E(c_i) = 0$)

Econometric Properties

- GPS model handles multivariate & continuous firm characteristics
- In case of $Cov(\boldsymbol{z}_{it}, c_i) \neq \boldsymbol{0}$, coefficient estimates $\widehat{\boldsymbol{\theta}}$ are
  - **biased**, if GPS model is estimated with pooled OLS (i.e. estimation without firm fixed effects)
  - **consistent**, if GPS model is estimated with firm fixed effects

Key result

GPS-Model vs. Portfolio Sorts

Proposition

When estimated with **pooled OLS**, the GPS-Model can perfectly reproduce the

- alpha and factor exposure estimates
- standard errors and t-statistics

of conventional portfolio sorts.

Implications

1. The **GPS model nests portfolio sorts** as a special case
2. If firm fixed effects are present and correlated with the firm characteristic underlying the sort, the portfolio sorts approach
   - yields alpha and factor exposure estimates that are biased
   - does NOT reliably identify characteristics-based factors if $\hat{\alpha}_{\Delta} \neq 0$

Proposition: Formal illustration

Portfolio Sorts Approach

1. Estimate $r_{Low,t} = \alpha_{Low} + \beta_{Low}\, RMRF_t + \varepsilon_{\Delta t}$
2. Estimate $r_{\Delta,t} = \alpha_{\Delta} + \beta_{\Delta} RMRF_t + \varepsilon_{\Delta t}$

GPS Model

(pooled OLS Estimation)

$$r_{it} = (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + \nu_{it} = \left(\begin{bmatrix} 1 & D_{it}^{(High)} \end{bmatrix} \otimes \begin{bmatrix} 1 & RMRF_t \end{bmatrix}\right)\boldsymbol{\theta} + \nu_{it}$$

$$= \theta_{Low,0} + \theta_{Low,1} RMRF_t + \theta_{\Delta,0} D_{it}^{(High)} + \theta_{\Delta,1} D_{it}^{(High)} RMRF_t + \nu_{it}$$

Result

$$\hat{\alpha}_{Low} \equiv \hat{\theta}_{Low,0} \qquad \hat{\beta}_{Low} \equiv \hat{\theta}_{Low,1} \qquad \hat{\alpha}_{\Delta} \equiv \hat{\theta}_{\Delta,0} \qquad \hat{\beta}_{\Delta} \equiv \hat{\theta}_{\Delta,1}$$

Proposition: Empirical Validation

Table 4: Performance of high vs. low quintile portfolios sorted on operating profitability (OA_{it})

Panel A: Conventional portfolio sorts

	Q1 (low)	Q5 - Q1
a	-0.318***	0.542***
	(-4.26)	(5.48)
b_{RMRF}	1.092***	-0.143***
	(48.10)	(-4.99)
b_{SMB}	0.212***	-0.287***
	(7.65)	(-8.19)
b_{HML}	0.175***	-0.491***
	(4.24)	(-10.35)
R-squared	0.897	0.335
N Obs.	642	642

Panel B: GPS-model (Pooled OLS estimation)

Vector $\boldsymbol{z}_{it}$ →	1	$OA_{it}^{(5)}$
← Vector $\boldsymbol{x}_t$		
1 (Intercept)	-0.318***	0.542***
	(-4.26)	(5.48)
$RMRF_t$	1.092***	-0.143***
	(48.10)	(-4.99)
SMB_t	0.212***	-0.287***
	(7.65)	(-8.19)
HML_t	0.175***	-0.491***
	(4.24)	(-10.35)
R-squared	0.224	
N Obs.	1,025,809	
N Stocks	14,705	

GPS-Model Applications beyond Portfolio Sorts

Multivariate Analysis

- Conduct competing hypotheses tests and robustness checks
- Perform asset pricing tests based on the full sample data
  ➔ no need to focus on top and bottom groups

TS vs. XS Predictability

- Test if TS predictability equals XS predictability
  ➔ Characteristics only predicting the cross-section of returns are susceptible to alpha misattribution

Do Firm Fixed Effects Matter?

- GPS-model allows to perform Hausman (1978) type tests
  ➔ GPS-model specification test
  ➔ Portfolio sorts specification test

Do Firm Fixed Effects Matter?

GPS-model: $r_{it} = ([1 \quad OA_{it} \quad GA_{it} \quad Vola_{it} \quad Beta_{it}] \otimes [\,1 \quad RMRF_t \quad SMB_t \quad HML_t]) \, \boldsymbol{\theta} + (c_i) + e_{it}$

	GPS-models estimated with weighted pooled OLS					GPS-models including firm fixed effects (weighted FE est.)				
Constant	-0.317***	-0.248***	0.275**	0.343***	-0.008	-0.082	-0.335***	0.044	0.422***	-0.062
	(-4.22)	(-3.46)	(2.54)	(3.02)	(-0.06)	(-0.82)	(-3.32)	(0.33)	(3.05)	(-0.37)
Operating Profitability	1.743***				1.105***	0.614				-1.583***
	(4.97)				(2.61)	(1.31)				(-2.63)
Gross Profitability		0.778***			0.338		1.003***			1.744***
		(4.47)			(1.58)		(3.99)			(5.40)
Volatility			-5.423*		-0.142			0.269		4.672
			(-1.94)		(-0.05)			(0.08)		(1.60)
Beta				-0.290**	-0.303**				-0.370***	-0.420***
				(-2.57)	(-2.52)				(-2.67)	(-2.95)
R-squared	0.236	0.235	0.264	0.281	0.295	0.237	0.236	0.265	0.282	0.296
N Obs.	2,115,518	2,115,518	2,289,867	2,275,370	2,059,734	2,115,518	2,115,518	2,289,867	2,275,370	2,059,734
N Stocks	17,008	17,008	19,109	19,124	16,908	17,008	17,008	19,109	19,124	16,908

Conclusion

GPS-Model

GPS-model does everything portfolio sorts can do **+ more**

- Competing hypotheses tests and robustness checks
- Specification testing

➔ Estimation with firm fixed effects ensures coefficient estimates are consistent even in case of $Cov(\boldsymbol{z}_{it}, c_i) \neq \mathbf{0}$

Empirical Insight

Conventional portfolio sorts tend to misattribute parts of the "alpha" to the characteristic underlying the sort

➔ Operating profitability and volatility suffer from such alpha misattribution

➔ Gross profitability and Beta predict the cross-section of stock returns even when accounting for $Cov(\boldsymbol{z}_{it}, c_i) \neq \mathbf{0}$