Peer Effects in Random Consideration Sets

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In many environments choices of peers affect one's decisions: school or restaurant choice, reposts in social media

(i) Usually, peer effect models assume that peers' choices affect one's preferences.

(ii) In many situations, peers can only shape the sets of alternatives one pays attention to – consideration sets.

We study:

(i) A continuous time dynamic model of peer effects

(ii) Choices of friends affect consideration sets but not preferences

(iii) Fully heterogeneous agents

(iv) Discrete- or continuous-time data on choices of people in the network

Results:

(i) We identify:
   (a) Network structure – sets of friends of every agent
   (b) Preferences
   (c) Consideration set formation mechanism

(ii) Relation between the structure of the network and peoples' mistakes

(iii) Maximum likelihood estimator of all aspects of the model

(iv) Analysis of the visual focus of attention using experimental data. **Main finding – robust left-to-right bias.**

Baseline Model

Network, Preferences, Choice Configuration

- $\Gamma = (\mathcal{A}, e)$ – social network
- $\mathcal{A} = \{1, 2, \dots, A\}$ – agents; e – connections
- $\mathcal{N}_a$ – set of friends of agent a
- $\overline{\mathcal{Y}} = \mathcal{Y} \cup \{o\}$, where $\mathcal{Y} = \{1, 2 \dots, Y\}$ – set of options, o – default option
- $\succ_a$ – strict preference order of agent a over $\overline{\mathcal{Y}}$
  1. o is the worst – can be relaxed (see the paper)
  2. Preferences do not need to be deterministic (see the paper)
- $\mathbf{y} = (y_a)_{a \in \mathcal{A}}$ – choice configurations

Choice Revision

- Every agent is endowed with independent Poisson alarm clocks with rates $\lambda = (\lambda_a)_{a \in \mathcal{A}}$.
- When agent a's alarm clock goes off
  1. She looks at current choices of her friends,
  2. She forms a consideration set,
  3. She picks the best alternative from the consideration set according to $\succ_a$.

Consideration Sets

- $Q_a(v \mid \mathbf{y})$ – the probability that agent a pays attention to alternative v given a choice configuration $\mathbf{y}$
- Probability of facing $\mathcal{C}$ is

$$\prod_{v \in \mathcal{C}} Q_a(v \mid \mathbf{y}) \prod_{v \notin \mathcal{C}} (1 - Q_a(v \mid \mathbf{y})) .$$

- Probability that agent a selects (at the moment of choosing) alternative v is given by

$$P_a(v \mid \mathbf{y}) = Q_a(v \mid \mathbf{y}) \prod_{v' \in \mathcal{Y}, v' \succ_a v} (1 - Q_a(v' \mid \mathbf{y})) .$$

1. o is always considered – can be relaxed (see the paper)
2. Probability of selecting o is $\prod_{v \in \mathcal{Y}} (1 - Q_a(v \mid \mathbf{y}))$.
3. Manski (1977) and Manzini & Mariotti (2014)

Main Assumptions

- $N_a^v(\mathbf{y})$ – the number of friends of agent a who select option v in choice configuration $\mathbf{y}$

(A1) For each $a \in \mathcal{A}$, $v \in \mathcal{Y}$, and $\mathbf{y} \in \overline{\mathcal{Y}}^A$, $1 > Q_a(v \mid \mathbf{y}) > 0$.

(A2) For each $a \in \mathcal{A}$, $|\mathcal{N}_a| > 0$.

(A3) For each $a \in \mathcal{A}$, $v \in \mathcal{Y}$, and $\mathbf{y} \in \overline{\mathcal{Y}}^A$,

$$Q_a(v \mid \mathbf{y}) \equiv Q_a\left(v \mid y_a, N_a^v(\mathbf{y})\right) \text{ is strictly increasing in } N_a^v(\mathbf{y}) .$$

- A1 states that every option is considered and not considered with positive probability.
- A2 requires every agent to have at least one friend since our identification strategy is based on the variation of friends' choices.
- A3 means the probability paying attention depends on the current choice and the number (but not the identity) of friends that currently selected it. A3 states that each person pays more attention to a particular option if more of her friends are adopting it.

Equilibrium

- Our model leads to a sequence of joint choices $\mathbf{y}$ that evolve through time according to a Markov process.
- The transition rate from $\mathbf{y}$ to any different one $\mathbf{y}'$ is

$$m(\mathbf{y}' \mid \mathbf{y}) = \begin{cases} 0 & \text{if } \sum_{a \in \mathcal{A}} \mathbb{1}(y'_a \neq y_a) > 1 \\ \sum_{a \in \mathcal{A}} \lambda_a P_a(y'_a \mid \mathbf{y}) \mathbb{1}(y'_a \neq y_a) & \text{if } \sum_{a \in \mathcal{A}} \mathbb{1}(y'_a \neq y_a) = 1 \end{cases} .$$

- We can define the transition rate matrix as

$$\mathcal{M}_{\iota(\mathbf{y})\iota(\mathbf{y}')} = m\left(\mathbf{y}' \mid \mathbf{y}\right) .$$

- An equilibrium is an invariant distribution $\mu : \overline{\mathcal{Y}}^A \to [0, 1]$, with $\sum_{\mathbf{y} \in \overline{\mathcal{Y}}^A} \mu(\mathbf{y}) = 1$, of the process with transition rate matrix $\mathcal{M}$.
- This equilibrium behavior relates to the transition rate matrix as

$$\mu \mathcal{M} = \mathbf{0}.$$

Proposition 1. *If A1 is satisfied, then there exists a unique equilibrium μ.*

Data and Identification

- We consider two possible settings:
  1. Continuous-time data – a long sequence of every choice and timing of every choice is observed,
  2. Discrete-time data – the choices of agents are only observed at prespecified times (e.g, daily or weekly).
- First, we identify the conditional choice probabilities $P = (P_a)_{a \in \mathcal{A}}$.
  1. Continuous-time data – P is directly observed
  2. Discrete-time data – P can be generically identified under mild restriction
- Knowing P, we can identify all the model parameters.

Proposition 2. *Under A1-A3, the set of connections Γ, the profile of strict preferences $\succ$, and the attention mechanism Q are point identified from P.*

- We extend the model to allow for random preferences, absence of the default option, more general consideration set formation processes.

Estimation

- Let $\theta = (\Gamma, \succ, Q)$
- The matrix of transition probabilities implied by the transition rate matrix $\mathcal{M}(\theta)$ is

$$\mathcal{P}(\theta, \Delta) = e^{\Delta \mathcal{M}(\theta)},$$

where e^A is the matrix exponential of A.

- Given sample $\{\mathbf{y}_t\}_{t=0}^T$, we can build the log-likelihood function

$$L_T(\theta) = \Sigma_{t=0}^{T-1} \ln \mathcal{P}_{\iota(\mathbf{y}_t), \iota(\mathbf{y}_{t+1})}(\theta, \Delta)$$

- The ML estimator

$$\widehat{\theta}_T = \arg\max_\theta L_T(\theta) .$$

Simulations

- 5 agents, 3 choices and the default
- Consideration probabilities are functions of the number of friends only: $Q_a(v \mid y_a, N) = Q(N)$
- Specification:

$$2 \succ_1 1, \quad 1 \succ_2 2, \quad 2 \succ_3 1, \quad 1 \succ_4 2, \quad 1 \succ_5 2,$$

$$Q(v \mid 0) = \frac{1}{4}, \quad Q(v \mid 1) = \frac{3}{4}, \quad Q(v \mid 2) = \frac{7}{8},$$

$$\mathcal{N}_1 = \{2, 3\}, \mathcal{N}_2 = \{1, 3\}, \mathcal{N}_3 = \{1, 2\}, \mathcal{N}_4 = \{5\}, \mathcal{N}_5 = \{4\} .$$

- Estimator of Q performs well in finite samples (Table 1).

		Sample Size					
Attention Probabilities		10	50	100	500	1000	5000
$Q(v \mid 0)$	Bias	213.6	164.9	133.2	65.1	42.3	9.4.
	RMSE	259.1	172.9	137.4	66.6	43.3	10.2
$Q(v \mid 1)$	Bias	66.4	53	46.9	27.2	18	2.7
	RMSE	103.9	66.9	55.6	31	20.9	5.6
$Q(v \mid 2)$	Bias	53.8	49.8	42.9	23.6	15.1	0.3
	RMSE	94.3	64.6	52.8	27.2	17.8	4.4

Table 1: Bias and Root Mean Squared Error (RMSE) ($\times 10^{-3}$)

- The network and preferences are estimated correctly in more than 97% simulations just with 100 observations (Table 2).

Sample Size	10	50	100	500
Network	32.4%	94.4%	99.8%	100%
Preferences	34.6%	85.6%	97.6%	100%
Network & Preferences	13.4%	83%	97.4%	100%

Table 2: Correctly Estimated Network & Preferences

Application

- Visual attention is a key determinant of decision making in many environments – e.g. picking a newspaper article to read or a chocolate bar in a vending machine to eat.
- Gaze to an alternative may substantially increase the likelihood of picking that alternative (Smith & Krajbich, 2019)
- There is evidence that choices of peers affect visual attention of individuals (Gallup et al., 2012).
- We can separate and quantify the effects visual attention (determined by choices of peers) and subjective individual preferences have on choice.
- People in North America and Western Europe display the left-to-right bias found in experimental studies (Spalek & Hammad,2004,2005 and Reutskaja et al., 2011)

Data

- 5 people, seated around a circle, were asked to play a party game.
- A tablet in front of each player recorded the direction of the player's sight every $\Delta = 1/3$ second.
- We aim to separate the preferences of each player for the direction of sight from peer effects on visual focus of attention on individual behavior.
- Choice sets : look to the left, to the right, or to the tablet.
- Network: everyone is connected to everyone
- Consideration probabilities only depend on the number of friends: $Q_a(v \mid y_a, N) = Q_a(N)$
- No restrictions on preferences. If nothing is considered then agent sticks to the current choice

Some Results

- All agents prefer looking to the **left**, then to tablet, and then to the right – the left-to-right bias.
- Consideration probabilities seem to be monotone in the number of friends.
- Two specifications:
  1. Consideration probabilities are the same across options and agents (Figure 1)
  2. Consideration probabilities are the same across options only (Figure 2)

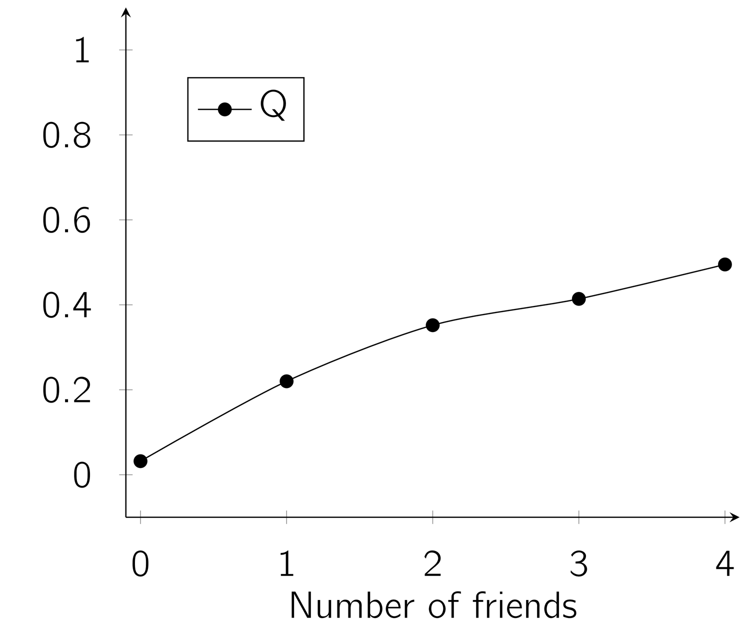

Figure 1: Consideration probability as a function of the number of friends. $Q_a(v \mid y_a, N) = Q(N)$

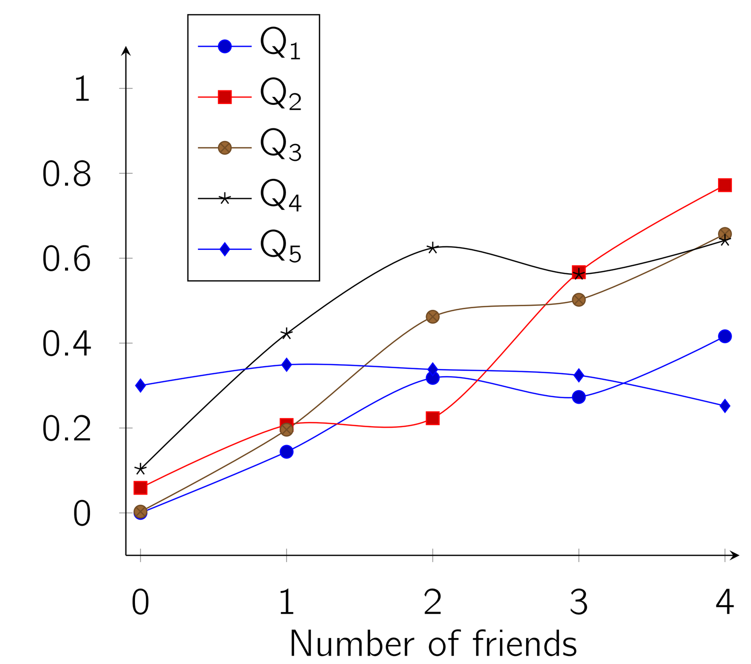

Figure 2: Consideration probability as a function of the number of friends. $Q_a(v \mid y_a, N) = Q_a(N)$