Collateral, Haircuts and Rates: A Theory of Repo

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What is repo?

- Repo is a form of lending collateralized by a portfolio of securities.
- Repo market is systemically important (Gorton and Metrick, 2012), with a daily turnover of € 3 trillion globally (ICMA, 2019).
- A repo deal has not only a price condition (interest rate, r), but also a degree of collateralization (haircut, h).

Two questions

Q1: How does collateral quality affect repo parameters?

Finding 1: Value-at-Risk (VaR) and Expected Shortfall (ES) arise endogenously as sufficient statistics of the quality of collateral, i.e. its return distribution.
Finding 2: ES↑⇒ h↑, r↑
Finding 3: VaR↑⇒ h↑, r↓

Q2: How do borrower's properties affect repo parameters?

Finding 4: While riskier borrowers face higher haircuts, they do not necessarily pay higher rates.
Finding 5: Borrowers that possess more profitable investment opportunities borrow with a smaller haircut at a cost of paying a higher rate.

In brief, this paper...

1. Endogenizes the effect of collateral quality on haircuts and rates (Adrian and Shin (2013), Dang et al. (2013)).
2. Suggests a solution to the VaR vs ES debate (Artzner (1999), Acerbi and Tasche (2002), BIS (2016)).
3. Suggests a framework to resolve some puzzling empirical patterns (Benmelech and Bergman (2009), Auh and Landoni (2016)).

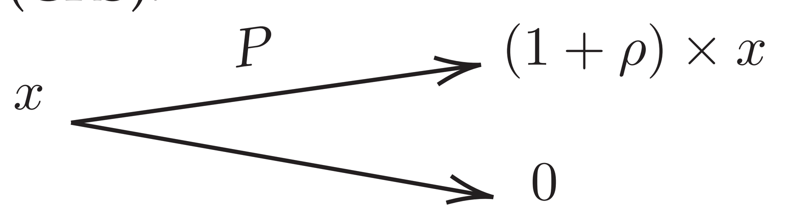
Model

A two-period model with two risk-neutral agents, borrower (b) and lender (l).

Borrower: penniless, has a private investment opportunity, possesses one unit of pledgeable financial asset worth $1.

Lender: competitive, deep-pocketed, can invest in a riskless asset with return $(1+r_f)$ or lend the borrower some amount (M).

Investment opportunity: binomial, scalable (CRS).



Pledgeable financial asset: return R distributed with a cdf $F(R) \in C^1$, independent of the borrower's investment opportunity.

Assumption 1: difference in beliefs. Agent i believes $P = P_i$, $i \in \{b, l\}$, so that
$NPV_b \triangleq (1+\rho)\times(1-P_b) - (1+r_f) > 0$,
$NPV_l \triangleq (1+\rho)\times(1-P_l) - (1+r_f) < 0$.

Assumption 2: borrower prefers to keep the financial asset rather than selling it (i.e., due to immediate selling costs).

Borrower's expected utility:

$$W(r,M) = \overbrace{M\times(\rho-r)}^{\text{inv. opp. successful}}\times(1-P_B) + \underbrace{\mathbb{E}[max(R-(1+r)M,0)]}_{\text{inv. opp. fails}}\times P_B.$$

Lender's expected utility:

$$U(r,M) = \overbrace{(1+r)M}^{\text{inv. opp. successful}}\times(1-P_L) + \underbrace{\mathbb{E}[min(R,(1+r)M)]}_{\text{inv. opp. fails}}\times P_L - \underbrace{(1+r_f)M}_{\text{opport. costs}}.$$

Equilibrium

Definition (haircut): $(1+h) \triangleq \frac{1}{M}$.

Definition (equilibrium): The repo market equilibrium is a contract (r_{eq}, h_{eq}) such that the borrower's utility W is maximized subject to the lender's break-even condition $U(r,M)=0$.

$$1+r_{eq} = (1+r_f)\times\left(1-\overbrace{P_L\times\alpha}^{\text{PD}}\times\underbrace{\left[\frac{ES(\alpha)-VaR(\alpha)}{1-VaR(\alpha)}\right]}_{\text{LGD}}\right)^{-1},$$

$$1+h_{eq} = [1-VaR(\alpha)]^{-1}\times(1+r_{eq})$$

where α – const., $\alpha\in[0,1]$.

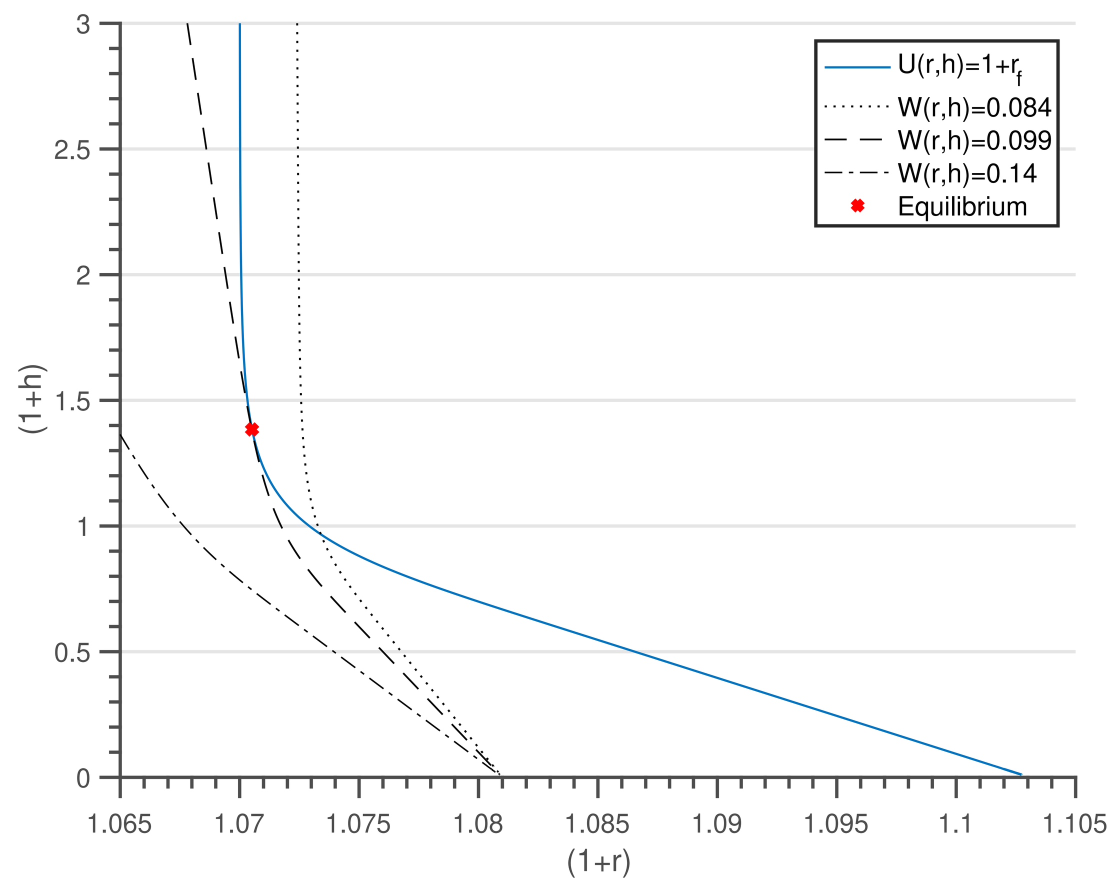


Figure 1: Equilibrium in the repo market is given by the tangency point of the lender's break-even condition and the borrower's utility curve.

Comparative statics -1 (collateral)

VaR and ES are tightly related, but represent different aspects of market risk. One needs to first orthogonalize them.

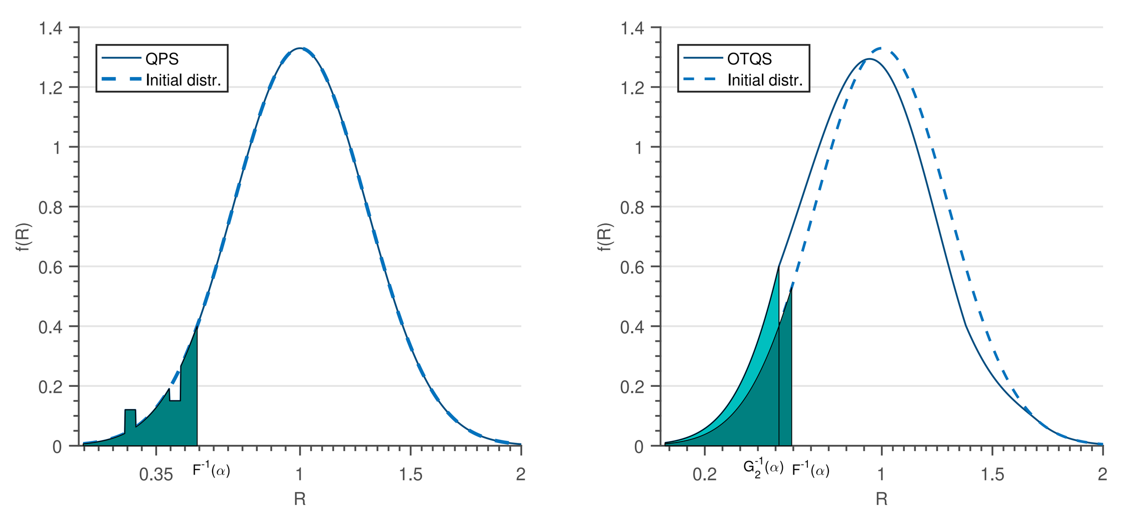


Figure 2: Quantile-preserving spread (QPS) and Over-the-quantile spread (OTQS).

$$\left.\frac{dh}{dES(\alpha)}\right|_{VaR(\alpha)=const} > 0,$$

$$\left.\frac{dr}{dES(\alpha)}\right|_{VaR(\alpha)=const} > 0,$$

$$\left.\frac{dh}{dVaR(\alpha)}\right|_{ES(\alpha)=const} > 0,$$

$$\left.\frac{dr}{dVaR(\alpha)}\right|_{ES(\alpha)=const} < 0,$$

where $dES(\alpha)$ and $dVaR(\alpha)$ are defined in terms of an α-QPS and α-OTQS respectively.

Comparative statics -2 (borrower)

The main parameters of the borrower are
- the probability of failure P_l,
- the return on the borrower's project ρ.

$$\frac{dh_{eq}}{d\rho} < 0, \quad \frac{dr_{eq}}{d\rho} > 0, \quad \frac{dh_{eq}}{dP_L} > 0,$$

$$\frac{dr_{eq}}{dP_L}\begin{cases} > 0 & \text{if } \frac{\kappa\times(1-ES(\alpha))}{ES(\alpha)-VaR(\alpha)} < \epsilon_K^F \\ < 0 & \text{if } \frac{\kappa\times(1-ES(\alpha))}{ES(\alpha)-VaR(\alpha)} > \epsilon_K^F \end{cases},$$

where ϵ_K^F is the elasticity of the CDF $F(\cdot)$ at $K_{eq} = \frac{(1+r_{eq})}{(1+h_{eq})}$, and $\kappa > 0$ - const.