Identification of Random Coefficient Latent Utility Models

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ASSA 2021

This Paper

- Identifies distribution of random coefficients for large class of models.
- Covers discrete choice, bundles models, consideration set models, among others.
- Key feature: need to identify average demand function.
- Get traction by exploiting envelope theorem.

Example: Discrete Choice

Discrete choice with linear random coefficients

- $$v_k = \beta_k' x_k + \varepsilon_k.$$
- β and ε are random
- Special case is random coefficients logit, in which ε has a *known* distribution up to location.
  - Studied in Fox, il Kim, Ryan, Bajari (2012, JoE).
  - We differ by letting both β and ε have nonparametric distribution.
- Identify all moments of $\beta = (\beta_1, \ldots, \beta_K)$.

General Model

These models (and others) can be written as perturbed utility models of the form

$$Y(X, \beta, \varepsilon) \in \operatorname*{argmax}_{y \in B} \sum_{k=1}^{K} (\beta_k' X_k) y_k + D(y, \varepsilon).$$

- Y is quantity vector for K goods.
- X collects regressors.
- ε can be infinite dimensional.
- B is nonrandom however $D(y, \varepsilon)$ can be $-\infty$ for certain combinations.
  - Allows "consideration sets."

Identified Primitive

This paper starts with average structural function

$$\overline{Y}(x) = \int Y(x, \beta, \varepsilon) d\tau(\beta, \varepsilon)$$

and asks what we can learn about distribution of β.

- When X and (β, ε) are independent,

$$\overline{Y}(x) = \mathbb{E}[Y \mid X = x].$$

- Also identifiable with endogeneity.
  - We complement Berry and Haile (2014, ECTA), who identify $\overline{Y}(x)$ in a demand setting with instruments.

Envelope Theorem

Lemma

Let

$$V(\beta_1' x_1, \ldots, \beta_K' x_K) = \int \left(\max_{y \in B} \sum_{k=1}^{K} y_k (\beta_k' x_k) + D(y, \varepsilon) \right) d\mu(\varepsilon).$$

Then

$$\int Y(x, \beta, \varepsilon) d\mu(\varepsilon) = \nabla V(\beta_1' x_1, \ldots, \beta_K' x_K)$$

at any point of differentiability.

- Related to Williams-Daly-Zachary theorem of discrete choice.

Slope-Intercept Independence

Assumption

β and ε are independent so we can write

$$\overline{Y}(x) = \int \int Y(x, \beta, \varepsilon) d\mu(\varepsilon) d\nu(\beta).$$

Notation:

$$\overline{Y}(x, \beta) = \int Y(x, \beta, \varepsilon) d\mu(\varepsilon)$$

$$\overline{Y}(x) = \int \overline{Y}(x, \beta) d\nu(\beta).$$

Technique

Assume each x_k is scalar for simplicity.

Write envelope theorem as

$$\overline{Y}_k(x, \beta) = \partial_k V(\beta_1 x_1, \ldots, \beta_K x_K).$$

Differentiate envelope theorem further to get

$$\partial_{x_j} \overline{Y}_k(x, \beta) = \partial_{j,k} V(\beta_1 x_1, \ldots, \beta_K x_K)\beta_j$$

and

$$\partial_{x_\ell} \partial_{x_j} \overline{Y}_k(x, \beta) = \partial_{\ell,j,k} V(\beta_1 x_1, \ldots, \beta_K x_K)\beta_\ell \beta_j.$$

Technique

Take expectations over β and evaluate at $x = 0$ to get:

$$\partial_{x_\ell}\partial_{x_j}\overline{Y}_k(0) = \partial_{\ell,j,k}V(0)\int \beta_\ell\beta_j d\nu(\beta).$$

- Key feature: $\partial_{\ell,j,k}V(0)$ does not depend on β.

Example: Identification of Second Moments

$$\partial_{x_1}\partial_{x_1}\overline{Y}_2(0) = \partial_{1,1,2}V(0)\int \beta_1^2 d\nu(\beta)$$

$$\partial_{x_1}\partial_{x_2}\overline{Y}_2(0) = \partial_{1,2,2}V(0)\int \beta_2\beta_1 d\nu(\beta)$$

$$\partial_{x_2}\partial_{x_1}\overline{Y}_1(0) = \partial_{2,1,1}V(0)\int \beta_1\beta_2 d\nu(\beta)$$

$$\partial_{x_2}\partial_{x_2}\overline{Y}_1(0) = \partial_{2,2,1}V(0)\int \beta_2^2 d\nu(\beta)$$

$$\Longrightarrow$$

Example: Identification of Second Moments

$$\frac{\partial_{x_1}\partial_{x_1}\overline{Y}_2(0)}{\partial_{x_2}\partial_{x_1}\overline{Y}_1(0)} = \frac{\partial_{1,1,2}V(0)\int \beta_1^2 d\nu(\beta)}{\partial_{2,1,1}V(0)\int \beta_1\beta_2 d\nu(\beta)} = \frac{\int \beta_1^2 d\nu(\beta)}{\int \beta_1\beta_2 d\nu(\beta)}$$

- Uses symmetry $\partial_{1,1,2}V(0) = \partial_{2,1,1}V(0)$.
- Symmetry has been used without random coefficients in Allen and Rehbeck (2019, ECTA).

Example: Identification of Second Moments

Combining other equations identifies the ratio of any two second moments.

- Identification given a scale assumption $\int \beta_1^2 d\nu(\beta) = 1$.

Main Result

Theorem

Assume $\int \beta_{1,1}^M d\nu(\beta)$ is known. Under regularity conditions, each M-th order moment of the form

$$\int \beta_{k_1,\ell_1} \cdots \beta_{k_M,\ell_M} d\nu(\beta)$$

is identified. In addition, for each $\gamma \in \{1, \ldots, K\}^{M+1}$,

$$\partial_\gamma V(0)$$

is identified.

Counterfactuals and Welfare

Can identify counterfactual/welfare objects under additional assumptions.

- Integrated indirect utility

$$V(\beta_1' x_1, \ldots, \beta_K' x_K) = \int \left(\max_{y \in B} \sum_{k=1}^{K} y_k(\beta_k' x_k) + D(y, \varepsilon) \right) d\mu(\varepsilon).$$

Identification of V (Welfare)

How to identify V?

- Main result identified partial derivatives of V at 0.
- If V is real analytic we can identify the function globally from these derivatives.
- (Paper presents two other techniques.)

Counterfactuals

Once V is identified, counterfactuals can be identified from envelope theorem.

$$\overline{Y}_k(x, \beta) = \partial_k V(\beta_1' x_1, \ldots, \beta_K' x_K).$$

Conclusion

- Identification of moments of linear random coefficient distribution in class of perturbed utility models.
- Covers several examples in a single framework.
- Requires only the average structural function.
- Exploits the envelope theorem.