# Counterpoint Theory and the Novel Function Able to Classify All Continuous Solutions to the Nash Bargaining Problem*

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## Abstract

This paper is the first to solve the Nash bargaining problem of how two firms optimally share a surplus they jointly create in repeated bargaining rounds[1] by defining the single mathematical function able to classify all continuous solutions to the Nash bargaining problem consistent with Nash's axioms of: Pareto optimality, symmetry, invariance with respect to affine transformations of utility, and independence of irrelevant alternatives,[2] *and* the alternative axiom of monotonicity.[4] Here, a unique equilibrium point is determined, representative of an infinite number of points in $R^3$ that cannot be reached by any two vector combinations.[1]

## Introduction

This paper solves the long-standing Nash bargaining enigma in economic and mathematical game theory. It defines a single mathematical function able to classify all continuous solutions to the Nash bargaining problem while satisfying both Nash's axioms of: Pareto optimality, symmetry, invariance with respect to affine transformations of utility, independence of irrelevant alternatives,[2] *and* the alternative axiom of monotonicity[4] without relaxing any of their constraints. It is well known that if C is the set of all possible bargaining problems, Nash's function $f$: C→ γ in $R^2$, cannot classify all continuous solutions to the bargaining problem such that given a solution ω under Nash's axioms and one under the alternate axiom of monotonicity φ, are the same point as $f: C \rightarrow \gamma: \ \omega \notin f(\varphi)$ and $\varphi \notin f(\omega)$.[4] Although many tried they failed to define a single function able to define all continuous solutions to the bargaining problem without relaxing Nash's axioms.

## Methodology

Counterpoint theory uses the unique mathematical properties of the *open* Möbius band, to model a *pair* of points at point P on the band that *corresponds to a pair of paired equilibrium points on the simple closed curve* γ *in* $R^2$ *such that at point* $P = f(\{l,m\}) = f(\{l',m'\})$ as $f: C \rightarrow R^3$ .

## A Unique Solution

Counterpoint theory defines the unique solution to the Nash bargaining problem where two players' *independent* pure strategies *i,j*, played together form a superior interdependent strategy *k* that provides each player a higher payoff by simply playing their best strategy. The unique solution is defined by paired point theory[1] where the unique equilibrium point solution represents a pair of equilibrium points in $R^3$ that *corresponds* to a pair of paired equilibrium points in $R^2$. This unique equilibrium solution is modeled using the function $f: C \rightarrow R^3$ of C, an *open* Möbius band on which each point represents a pair of equilibrium points and corresponds to a pair of paired equilibrium points on a simple closed curve γ in $R^2$,[1] which for each player represents the set of their "good" strategies in the "convex polyhedral subset" of their strategy space and holds for each player's equilibrium strategy set in all solvable games.[3]

## Applications

Counterpoint Theory's unique equilibrium solution is useful in explaining the unique equilibrium solution of balanced vertically integrated firms which integrate both forward and backward to avoid the economic hold-up problem of incomplete markets, in addition to the applications described by Nash in *The Bargaining Problem*.[2]

## References

1. Knebel, E., Counterpoint Theory and the Novel Function Able to Classify All Continuous Solutions to the Nash Bargaining Problem ( 2021-2015) , University of Oxford. Copyright Protected 2015-2021.
2. Nash, John. 1950. "The Bargaining Problem." Econometrica 18: 155–62.
3. ------------ "Non Cooperative Games" Princeton University pg. 12
4. Kalai, Ehud, and Meir Smorodinsky. 1975. "Other Solutions to Nash's Bargaining Problem." Econometrica 43 (3): 513–18.

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