Optimal mechanism for the sale of a durable good

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A classic

Question

What's the revenue-maximizing mechanism to sell a durable good to a privately informed agent?

- If the seller has commitment: a **posted price**
- For instance, if $v \in \{v_L, v_H\}$, the seller sets a price of
  - v_L if prior belief that $v = v_H$ is below v_L/v_H
  - v_H if prior belief that $v = v_H$ is above v_L/v_H
  - both prices are optimal if prior equals v_L/v_H.

- This result does not depend on binary valuations or the length of the interaction (e.g., Baron and Besanko (1984))

constant posted price

- This result does depend on the seller's **commitment**: the optimal mechanism is **time inconsistent**

A classic revisited

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  2. Skreta (2006): under finite horizon <u>and</u> continuum of types, optimal <u>mechanism</u> is to post a price

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How come? Lack of tractability

- no 'revelation principle'
- Optimal mechanisms in finite horizon:

  Laffont and Tirole (1986,1990), Kumar (1985), Bester and Strausz (2000,2001,2007), Skreta (2006,2015), Deb and Said (2015), Fiocco and Strausz (2015), Beccuti and Möller (2018).
- Contracting in infinite horizon:

  Strulovici (2017), Acharya and Ortner (2017), Gerardi and Maestri (2018).

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- New tool: Revelation principle for limited commitment: Doval and Skreta (2018)

This paper

1. characterize revenue-maximizing mechanism with limited commitment and infinite horizon
    - there is no last period where we know what the optimal mechanism is

   with binary types.
2. revenue-maximizing PBE can be implemented as a sequence of posted prices
    - even when the seller can offer any mechanism
    - echoes the result for the case of commitment.
    - microfoundation for the strategy space in the literature that studies the sale of a durable good.
    - price dynamics are the ones from the price-posting game.
3. methodology for mechanism design w/ limited commitment and transferable utility.

The how matters: a recipe for transferable utility

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Commitment:

1. Revelation principle
2. Optimum: search for binding constraints
3. Use binding constraints to replace transfers: virtual surplus
4. Decision problem: find optimal allocation.
5. Recover transfers from constraints and check global ones.

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3. Use binding constraints to replace transfers: virtual surplus
4. Intrapersonal game: find optimal allocation.
5. Go back to original game: build PBE assessment.to induce line break

Related Literature

Optimal Design in finite horizon
e.g., Laffont & Tirole (1986), Hart & Tirole (1988), Bester & Strausz (2000,2001,2007), Skreta (2006,2015), Deb & Said (2015), Fiocco & Strausz (2015), Beccuti & Möller (2018)

Contracting in infinite horizon:
e.g., Strulovici (2017), Acharya & Ortner (2017), Gerardi & Maestri (2019)

Coasian dynamics and bargaining:
e.g., Stokey (1981), Bulow (1982), Sobel and Takahashi (1983),Fudenberg et al. (1985), Gul et al. (1986), Ausubel and Deneckere (1989), McAfee & Vincent (1997), Caillaud and Mezzetti (2004), McAfee & Wiseman (2008), Board & Pycia (2014), Liu et al. (2018), Nava & Schiraldi (2018)

Intrapersonal games:
e.g., Strotz (1955), Pollak (1968), Peleg & Yaari (1973), Harris & Laibson (2001), Bernheim et al. (2015), Cao & Werning

infinite horizon
+
recipe
+
optimal design

+

microfoundation of strategy space

+

new application

Setup

Primitives:

- A seller and a buyer interact over infinitely many periods.
- The seller owns one unit of a durable good.
- The buyer's valuation for the good is her private information, $v \in \{v_L, v_H\}$. $\mu_0 = P_0(v = v_H)$
- An allocation is $(q, x) \in \{0, 1\} \times \mathbb{R}$.
- Quasilinear flow payoffs: $u_B(q, x; v) = vq - x$, $u_S(q, x; v) = x$
- Common discount factor $\delta \in (0, 1)$

Timing: If in period t, the good has yet to be sold:

t.1 The seller offers the buyer a **mechanism**,

t.2 Observing the mechanism, the buyer accepts or rejects

  t.2.1 If she rejects, no trade and no payments $\rightarrow$ period $t + 1$

  t.2.2 If she accepts, she participates in the mechanism, which determines the rules of trade

  - If the allocation is no trade $\rightarrow$ period $t + 1$

Mechanisms

As in Doval and Skreta (2018), when we say mechanism, we mean:

$$\mathbf{M} = \left(\langle M^{\mathbf{M}}, \beta^{\mathbf{M}}, S^{\mathbf{M}}\rangle, \alpha^{\mathbf{M}}\right)$$

where

$$\underbrace{\beta^{\mathbf{M}} : M^{\mathbf{M}} \mapsto \Delta^*(S^{\mathbf{M}})}_{\text{communication device}} \text{ and } \underbrace{\alpha^{\mathbf{M}} : S^{\mathbf{M}} \mapsto \Delta^*(\{0,1\} \times \mathbb{R})}_{\text{allocation}}$$

- $q^{\mathbf{M}} : S^{\mathbf{M}} \mapsto [0,1]$ is a probability of trade,
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How does this work?

- Buyer inputs privately $m \in M^{\mathbf{M}}$, unobserved by the seller
- An output message $s \in S^{\mathbf{M}}$ is drawn from $\beta^{\mathbf{M}}(\cdot|m)$, public
- The allocation $(q^{\mathbf{M}}(s), x^{\mathbf{M}}(s))$ is determined, public

Extensive form game and equilibrium

- Strategies:
  - For the seller, choose a mechanism for every history Γ.
  - For the buyer, when her type is $v \in \{v_L, v_H\}$, participation, π_v, and reporting, r_v, for each private history.

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- Beliefs: at each history, the seller holds beliefs about the buyer's:
  - payoff relevant type, $v \in \{v_L, v_H\}$,
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equilibrium

A Perfect Bayesian Equilibrium is a tuple $\langle \Gamma, (\pi_v, r_v)_{v \in \{v_L, v_H\}}, \mu \rangle$ such that:

1. Given beliefs μ, strategies are sequentially rational,
2. Beliefs are obtained via Bayes' rule where possible.

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goal

Characterize maximum equilibrium revenue, $u_S^*(\mu_0)$

Main result

Theorem

There is a PBE assessment $\langle \Gamma^*, (\pi_v^*, r_v^*)_{v \in V}, \mu^* \rangle$ that achieves $u_S^*(\mu_0)$ such that each period the seller posts a price.

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What does it mean that the seller posts a price? indirect implementation

- In each period, the mechanism will have two inputs/outputs $\{m_\emptyset, m_B\}$
- $(q(m_\emptyset), x(m_\emptyset)) = (0, 0)$
- $(q(m_B), x(m_B)) = (1, p)$

Step 1: revelation principle Doval & Skreta (2018)

1. Equilibria of simpler game: only offer canonical mechanisms

$$\mathbf{M} = \langle(\underbrace{V}_{\text{input=type}}, \beta^{\mathbf{M}}, \underbrace{\Delta(V)}_{\text{output=belief about type } \mu}), (q^{\mathbf{M}}, x^{\mathbf{M}})\rangle = \textbf{canonical mechanism}$$

2. The seller's equilibrium choice of a mechanism has to satisfy
   - Participation constraints for each type,
   - Incentive compatibility constraints for each type.

   $\}\simeq$ Mechanism design

3. Output messages have a literal meaning: information design
4. Buyer's strategy does not depend on the payoff irrelevant part of the private history Public PBE outcomes=PBE outcomes; can (eventually!) invoke self-generation and check one-step deviations.

Seller's expected revenue

$$u_S^*(\mu_0) = \sum_{v\in V} \mu_0(v) \sum_{\mu'\in\Delta(V)} \beta^{\mathbf{M}_0^*}(\mu'|v)[x^{\mathbf{M}_0^*}(\mu') + \delta(1 - q^{\mathbf{M}_0^*}(\mu')) \underbrace{U_S^*(h^1)}_{\text{cont. at } h^1}$$

$$h^1 = \mathbf{M}_0^*, 1, \mu', 0, x^{\mathbf{M}_0^*} \text{ a public history}$$

Steps 2 & 3: Binding constraints and virtual surplus

Necessary conditions at seller-optimal PBE–preliminary observations

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1. wlog, if buyer rejects the seller's equilibrium choice of mechanism at h^t, the seller assigns probability 1 to the buyer's valuation being v_H. (non-participation cont. payoff 0)
2. seller-optimal PBE is incentive efficient: given buyer's cont. values, seller obtains best payoff

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Key theme: different μ' evaluate rents differently

Whenever the seller sells to both types, he leaves rents Δv to v_H. The "cost" depends on μ:

$$v_L = \mu(v_H - \Delta v) + (1-\mu)v_L = \mu v_H + (1-\mu)\left(v_L - \frac{\mu}{1-\mu}\Delta v\right)$$
$$= \mu v_H + (1-\mu)\hat{v}_L(\mu)$$

at $\bar{\mu}_1 \equiv \dfrac{v_L}{v_H}$ we have that $\hat{v}_L(\bar{\mu}_1) = 0$

“Getting rid" of the agent $\rightarrow$ towards a recursive formulation

From reporting & communication to info-design

Truth-telling allows us to replace reporting-strategy + communication device by distribution of posteriors. Let $\tau^{\mathbf{M}_0^*}(\mu_0, \mu_1') = \sum_{v \in V} \mu_0(v) \beta^{\mathbf{M}_0^*}(\mu_1'|v)$, $\sum_{\mu_1' \in \Delta(V)} \tau^{\mathbf{M}_0^*}(\mu_0, \mu_1')\mu_1' = \mu_0$

Virtual surplus representation

Actually, for all histories on the path of play, we have

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+ (1 - q^{\mathbf{M}_0^*}(\mu_1'))\delta & \left(U_S^*(h^1) + (\tfrac{\mu_1'}{1-\mu_1'} - \tfrac{\mu_0}{1-\mu_0})(1-\mu_1')U_{H|L}^*(h^1) \right)
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$$u_S^*(\mu_0) = \begin{array}{c} \tau^{\mathbf{M}_0^*}(\mu_0,\mu_1')\Big[q^{\mathbf{M}_0^*}(\mu_1')(\mu_1' v_H + (1-\mu_1')\hat{v}_L(\mu_0)) \\ +(1-q^{\mathbf{M}_0^*}(\mu_1'))\delta\Big(U_S^*(h^1) + (\frac{\mu_1'}{1-\mu_1'} - \frac{\mu_0}{1-\mu_0})(1-\mu_1')U_{H|L}^*(h^1)\Big)\Big] \end{array}$$

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$$U_S^*(\mu^*(h^t)) = \sum_{\mu' \in \Delta(V)} \tau^{\mathbf{M}_t^*}(\mu^*(h^t), \mu') \Big[q^{\mathbf{M}_t^*}(\mu')(\mu' v_H + (1-\mu')\hat{v}_L(\mu^*(h^t))) + (1 - q^{\mathbf{M}_t^*}(\mu'))\times \delta \Big(U_S^*(\mu') + (\tfrac{\mu'}{1-\mu'} - \tfrac{\mu^*(h^t)}{1-\mu^*(h^t)})(1-\mu')U_{H|L}^*(\mu') \Big) \Big]$$

necessary conditions for seller optimal PBE: takeaway the recursion

incentive efficiency implies that given beliefs and buyer's rents $\rightarrow$ history does not matter:

$$R^*(\mu_0, \mu_0) = \sum_{\mu' \in \Delta(V)} \tau^{\mathbf{M}_0^*}(\mu_0, \mu_1') \left[q^{\mathbf{M}_0^*}(\mu_1')(\mu_1' v_H + (1-\mu_1')\hat{v}_L(\mu_0)) + (1-q^{\mathbf{M}_0^*}(\mu_1')) \times \delta R^*(\mu_1', \mu_0) \right],$$

necessary conditions for seller optimal PBE: takeaway the recursion

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$$R^*(\mu_0, \mu_0) = \sum_{\mu' \in \Delta(V)} \tau^{\mathbf{M}_0^*}(\mu_0, \mu_1') \left[q^{\mathbf{M}_0^*}(\mu_1')(\mu_1' v_H + (1-\mu_1')\hat{v}_L(\mu_0)) + (1 - q^{\mathbf{M}_0^*}(\mu_1')) \times \delta R^*(\mu_1', \mu_0) \right],$$

and for all histories on the path of play, we have

$$R^*(\mu^*(h^t), \mu^*(h^t)) = \sum_{\mu' \in \Delta(V)} \tau^{\mathbf{M}_t^*}(\mu^*(h^t), \mu') \left[q^{\mathbf{M}_t^*}(\mu')(\mu' v_H + (1-\mu')\hat{v}_L(\mu^*(h^t))) + (1 - q^{\mathbf{M}_t^*}(\mu')) \times \delta R^*(\mu', \mu^*(h^t)) \right]$$

Step 4: solve recursive problem its solution is Intrapersonal equilibrium

to solve problem: find policy $(\tau, q) : \Delta(V) \mapsto \Delta(\Delta(V)) \times [0,1]^{\Delta(V)}$ and value function $R^{(\tau,q)}$ s.t.

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$$R^{(\tau,q)}(\mu', \mu_0) = \int \left[q(\mu', \widetilde{\mu})(\widetilde{\mu} v_H + (1-\widetilde{\mu})\hat{v}_L(\mu_0)) + \delta(1 - q(\mu', \widetilde{\mu}))R^{(\tau,q)}(\widetilde{\mu}, \mu_0) \right] \tau(\mu', d\widetilde{\mu}),$$

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3. For all $\mu_0 \in \Delta(V)$, policy is optimal given value function

$$(\tau(\mu_0, \cdot), q(\mu_0, \cdot)) \in \arg\max_{\tau', q'} \int \left[q'(\mu')(\mu' v_H + (1 - \mu')\hat{v}_L(\mu_0)) + \delta(1 - q'(\mu'))R^{(\tau,q)}(\mu', \mu_0) \right] \tau'(d\mu')$$

where $\int \mu' \tau'(d\mu') = \mu_0$ and $q'(\cdot) \in [0, 1]$.

Limited commitment as an intrapersonal game

Key conceptual innovation

Optimal design with limited commitment as an Intrapersonal Game

- A solution to the recursive problem is an intrapersonal equilibrium
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- Hence, we construct the equilibrium by hand:

Intrapersonal equilibrium: an informed guess and verify

- Fix some policy (τ, q) and construct the continuation values $R^{(\tau,q)}$.
- Fix the seller's prior, μ_0. We want to find

$$\max_{\tau',q'} \int \left[q'(\mu')(\mu' v_H + (1-\mu')\hat{v}_L(\mu_0)) + (1-q'(\mu'))\delta R^{(\tau,q)}(\mu', \mu_0) \right] \tau'(d\mu')$$

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- At most two posteriors
- Without information about $R^{(\tau,q)}$ difficult to draw conclusions; we guess and verify
  1. For all μ_0, there exist two beliefs in the support of $\tau(\mu_0, \cdot)$, $\mu_D(\mu_0) \leq \mu_0 \leq \mu^S(\mu_0)$,
  2. for all μ_0, $\mu^S(\mu_0) = 1$ and $q(\mu_0, \mu^S(\mu_0)) = 1$,
  3. if $\frac{v_L}{v_H} \leq \mu_0$, $q(\mu_0, \mu_D(\mu_0)) = 0$ (if $\mu_0 < \frac{v_L}{v_H}$, then $q(\mu_0, \mu_D(\mu_0)) = 1$–identical to "commitment")

Intrapersonal equilibrium: existence

Theorem

There exists a unique intrapersonal equilibrium $\langle(\tau^*, q^*), R^{(\tau^*,q^*)}\rangle$. It is characterized by a sequence of optimal delay beliefs: $\overline{\mu}_0 < \overline{\mu}_1 < \cdots < \overline{\mu}_n < \ldots$ with $\overline{\mu}_0 = 0, \overline{\mu}_1 = v_L/v_H$, such that if $\mu_0 \in [\overline{\mu}_i, \overline{\mu}_{i+1})$,

1. if $i \geq 1$, then $\mu^{D^*}(\mu_0) = \overline{\mu}_{i-1}$ while
2. and $i = 0$, then $\tau^*(\mu_0, \mu_0) = 1, q^*(\mu_0, \mu_0) = 1$ (zero delay for low priors)

- Indifferences are resolved in favor of maximizing delay
- Conflict in tie-breaking: if $\mu_0 \in [\overline{\mu}_i, \overline{\mu}_{i+1})$, then
  - Prefers maximum delay for $\mu' \leq \overline{\mu}_i$
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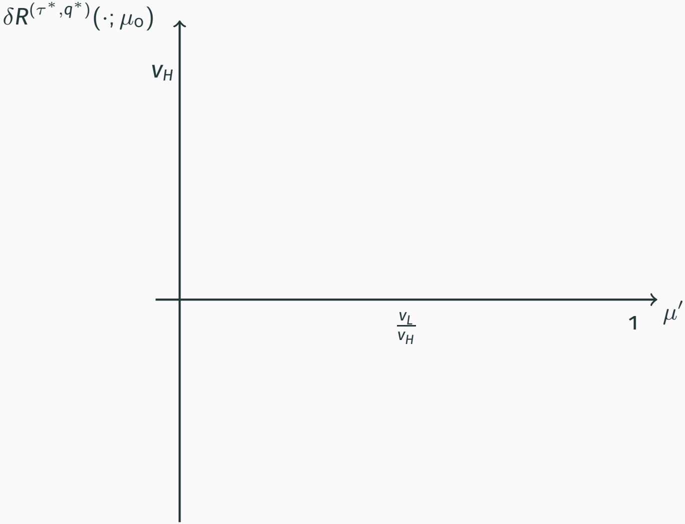
- Uniqueness: the original game does not have a unique equilibrium.

Intrapersonal equilibrium: an informed guess and verify

If $\langle(\tau^*, q^*), R^{(\tau^*,q^*)}\rangle$ **is an intrapersonal equilibrium:**



(in the picture, $\mu_0 > v_L/v_H$*)*

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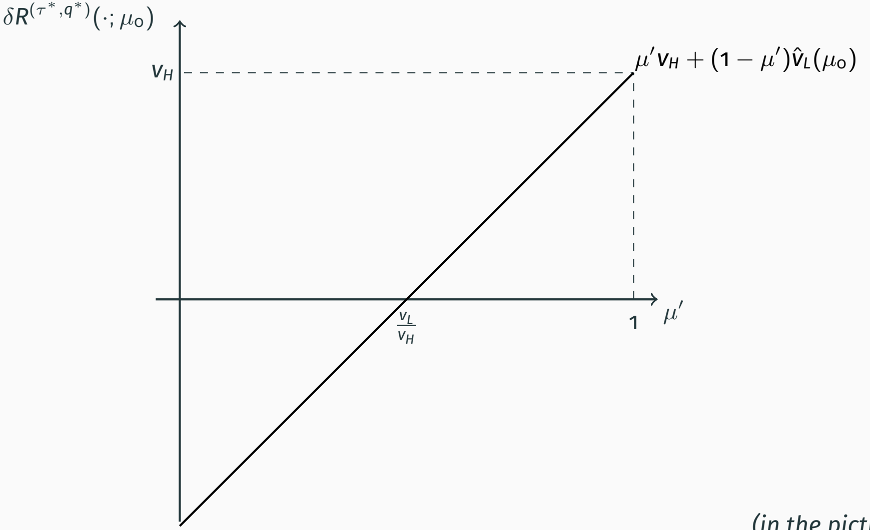

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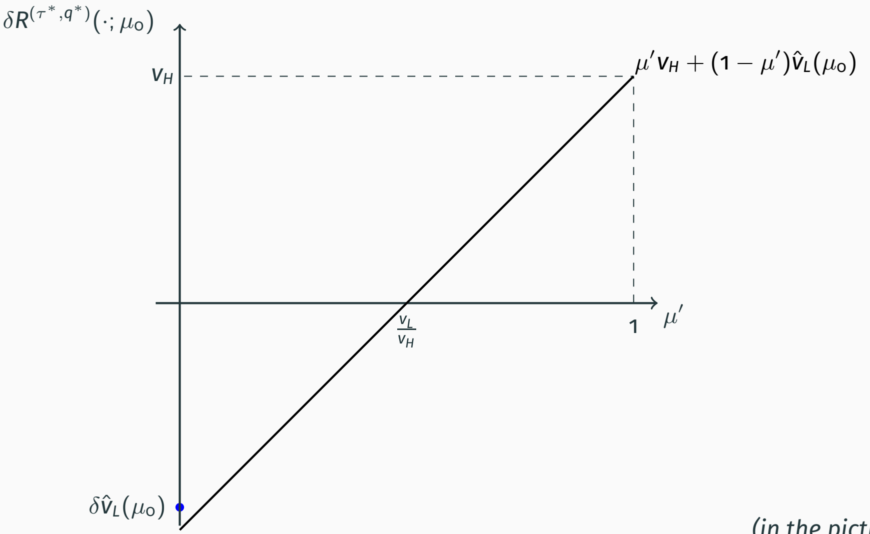

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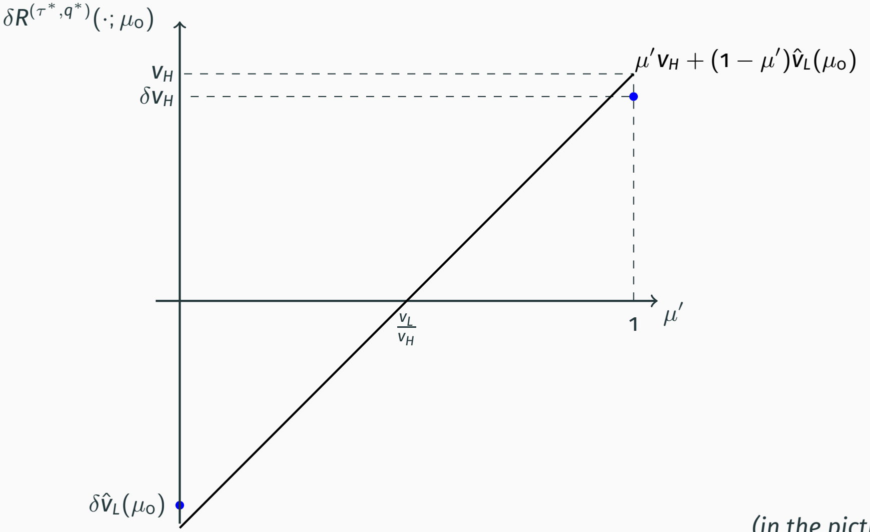

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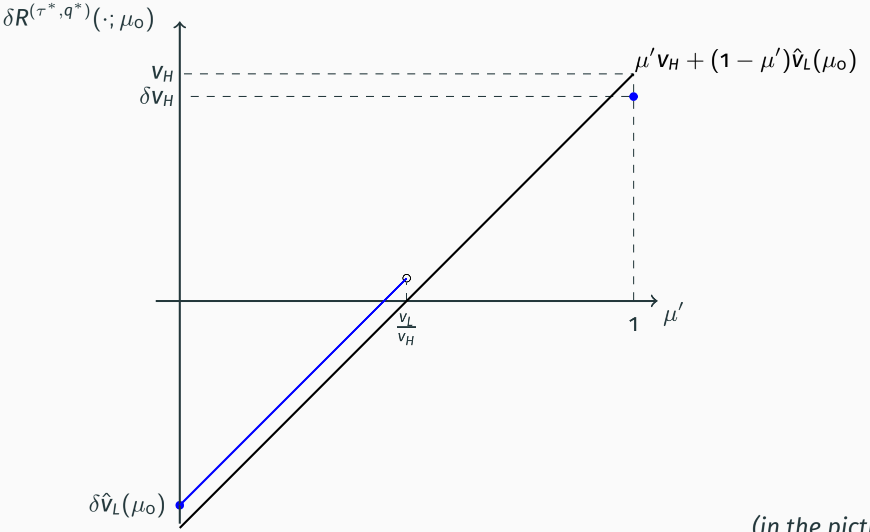

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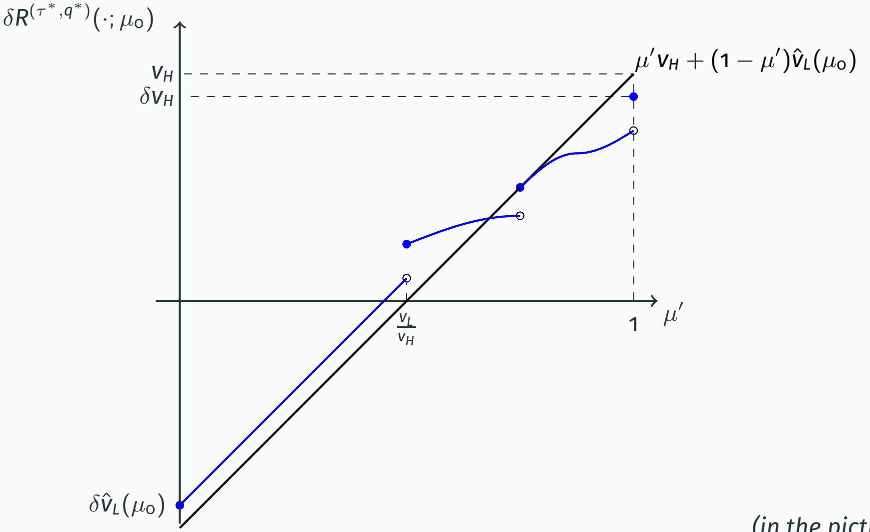

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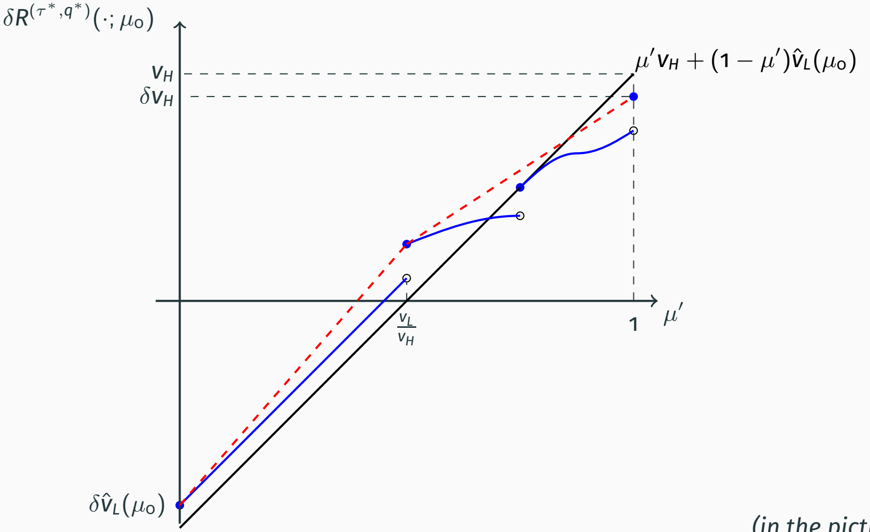

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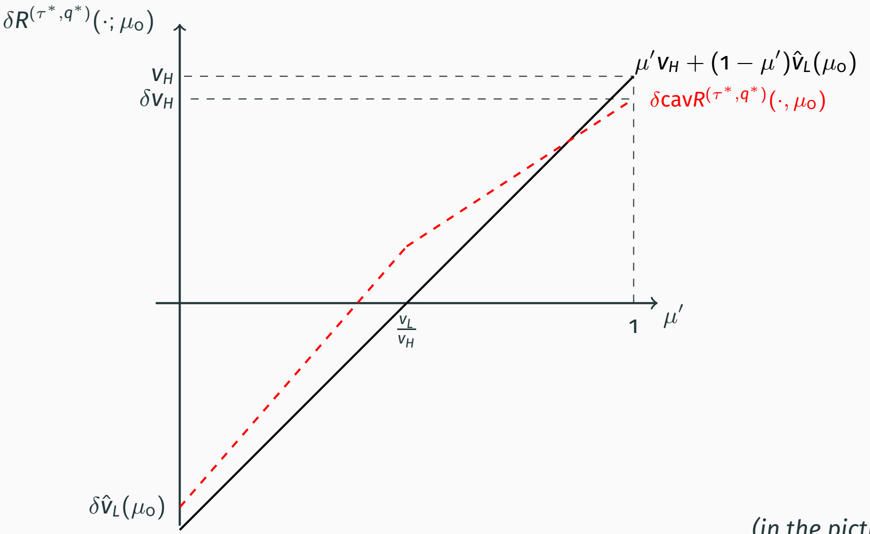

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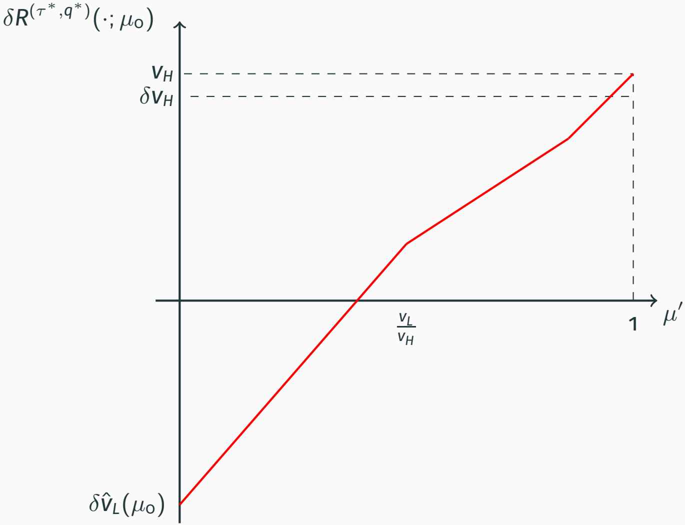

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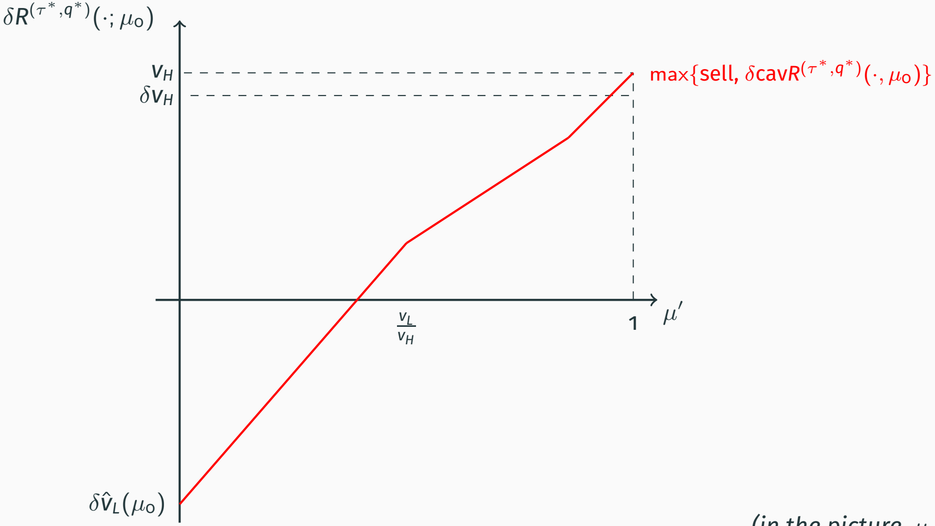

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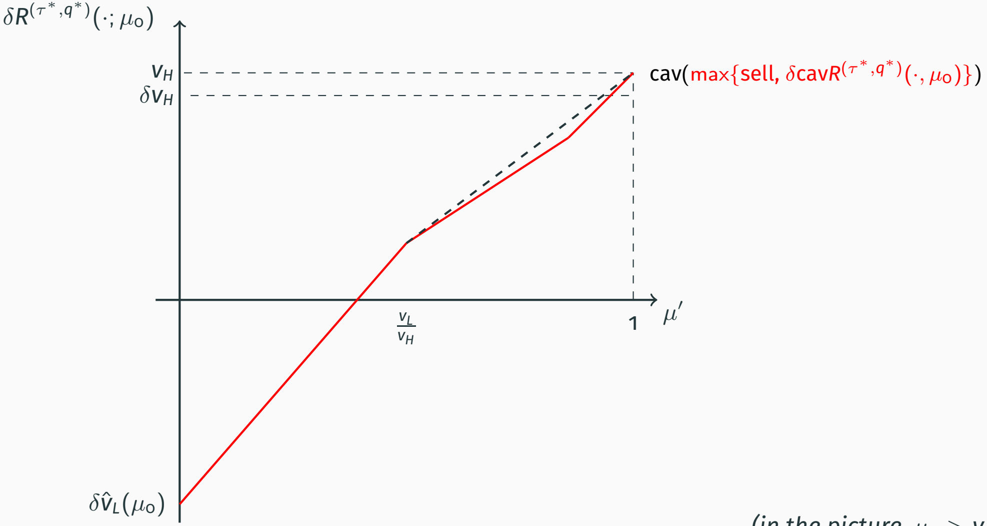

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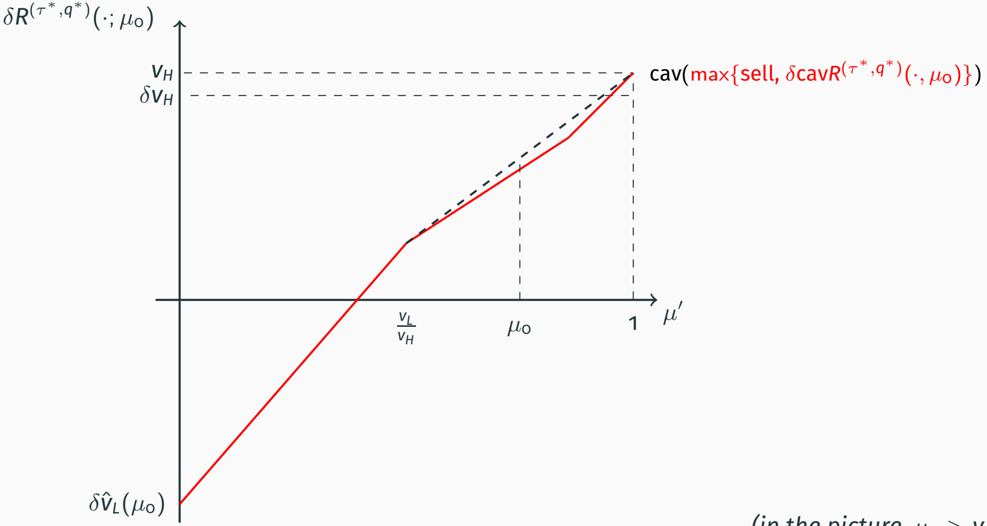

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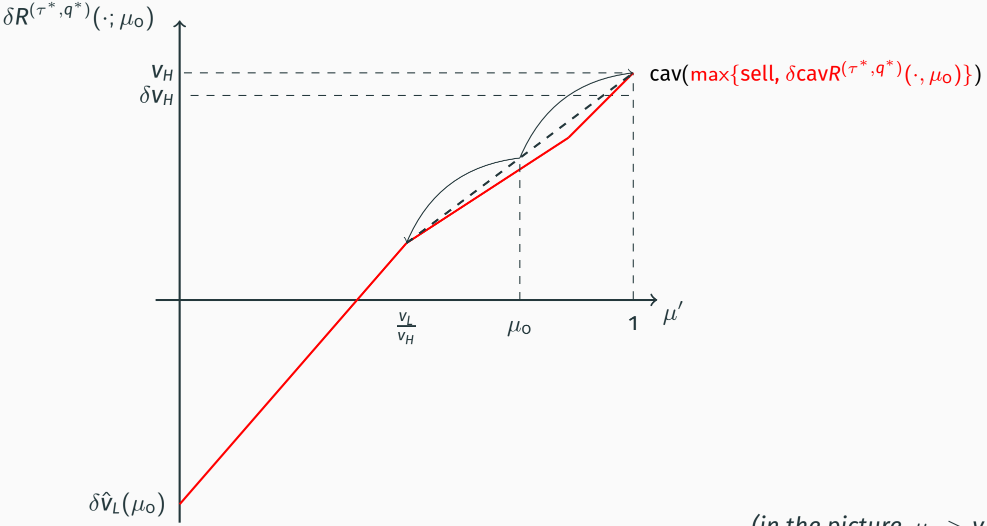

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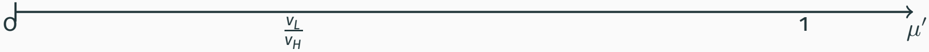
Intrapersonal equilibrium: simple policies



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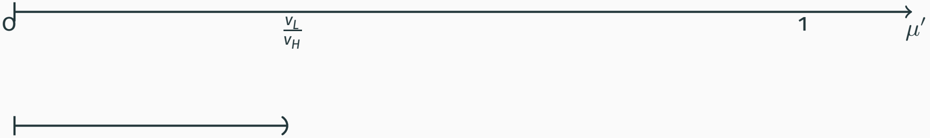

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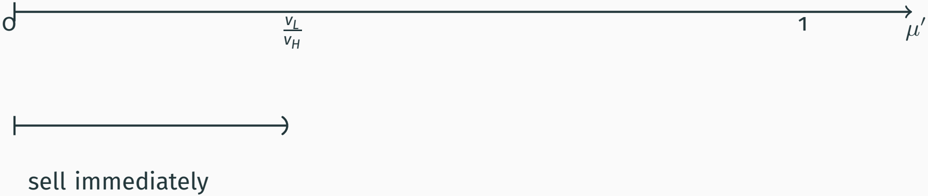

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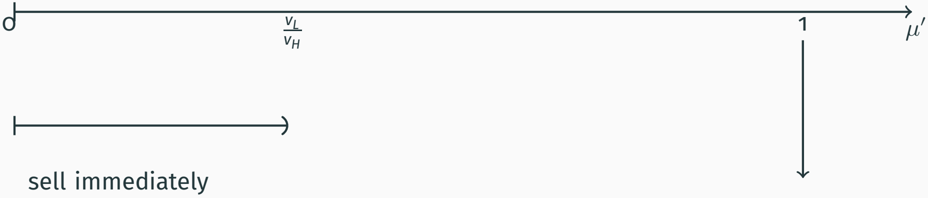

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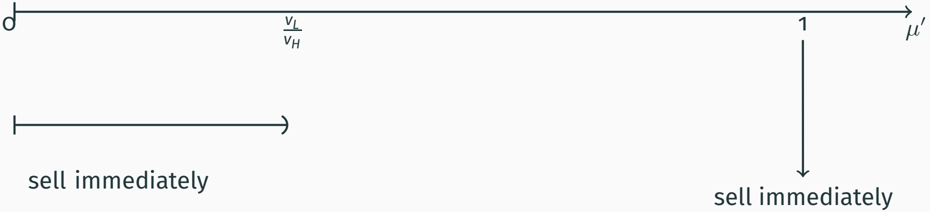

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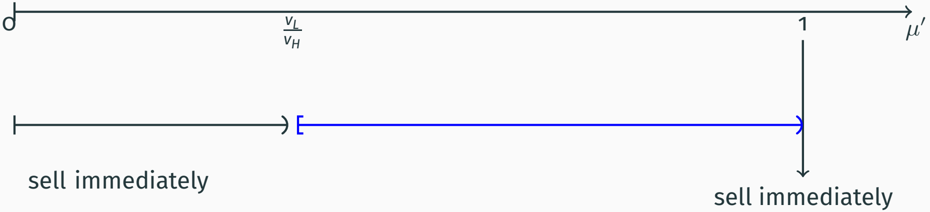

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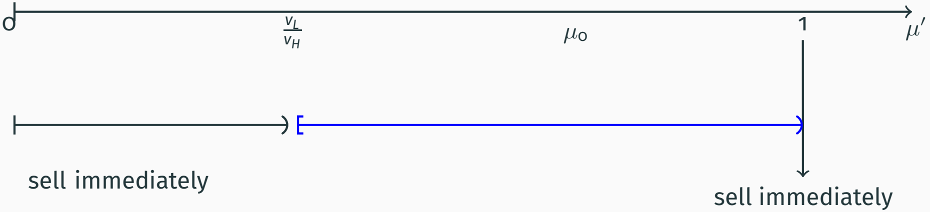

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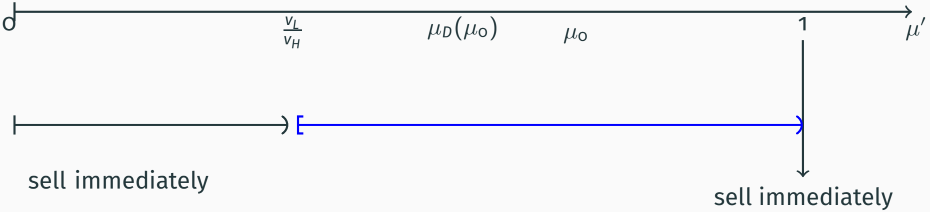

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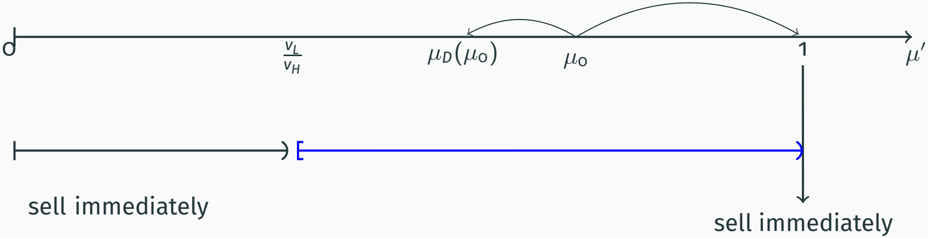

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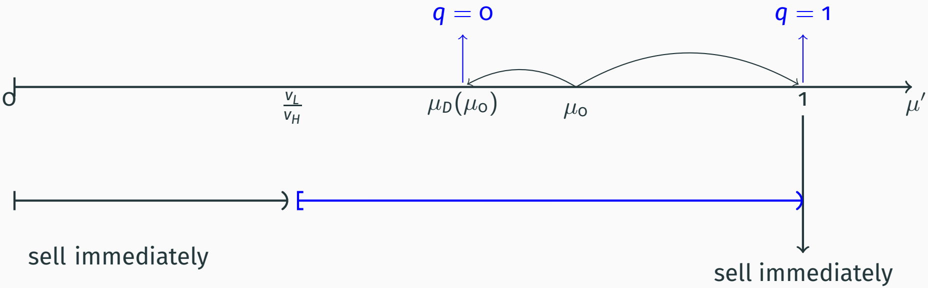

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Updating below v_L/v_H
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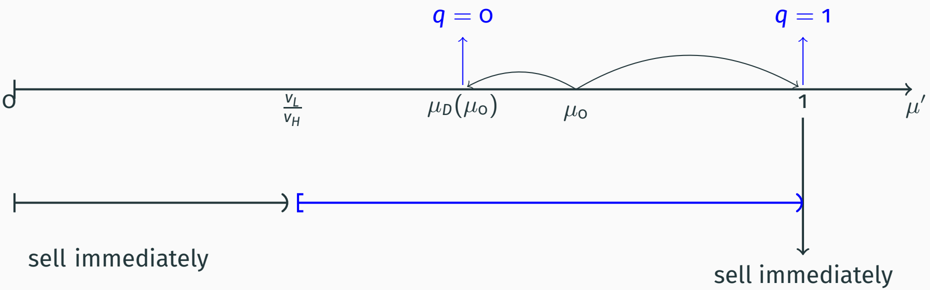

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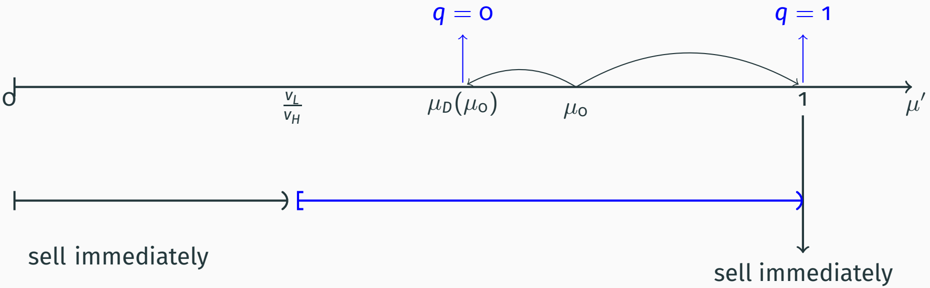
- Fudenberg, Levine, Tirole (1985): this is optimal for a seller who faces a myopic buyer
- Logic here is slightly different: today's seller is happy with infinite delay.



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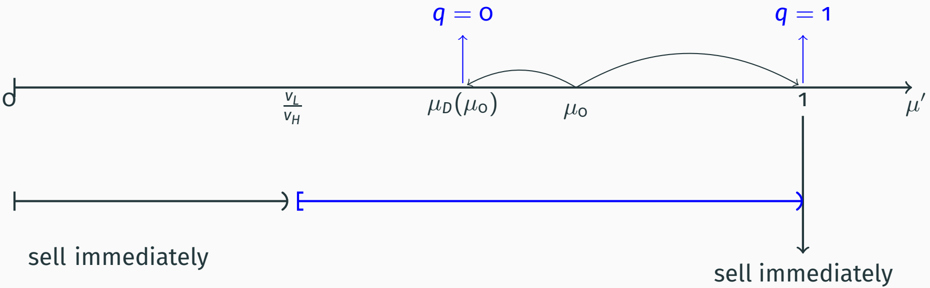
Classify seller priors

$$D_n = \{\mu_0 : \mu_D^{(n)}(\mu_0) = 0\}$$



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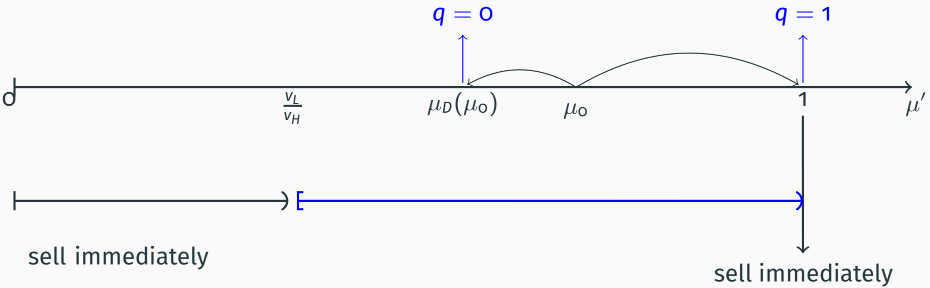

Intrapersonal equilibrium: simple policies

Monotonicity
Fix $\mu_0, \mu_0' \geq v_L/v_H$. If $\mu_0 < \mu_0'$, then $\mu_D(\mu_0) \leq \mu_D(\mu_0')$.

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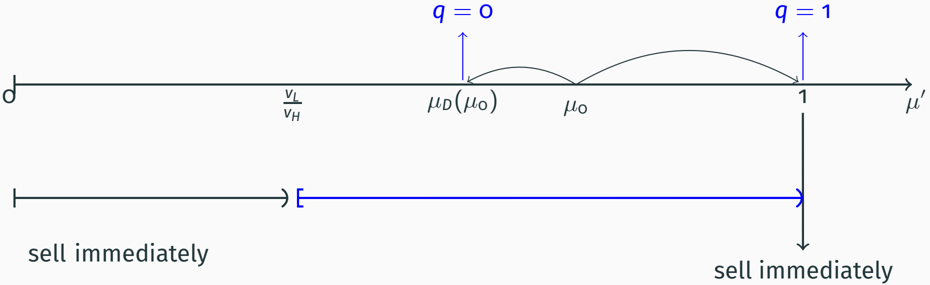

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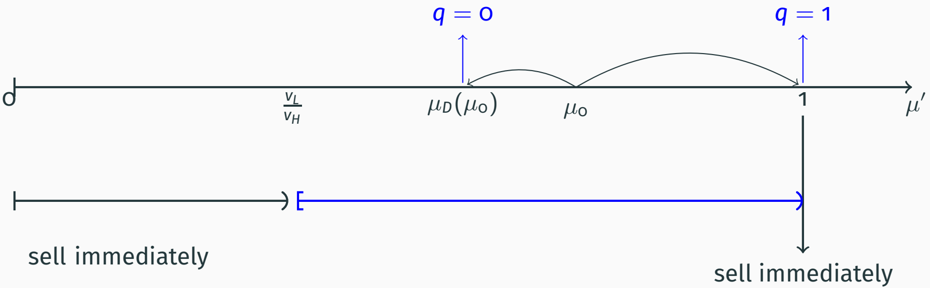

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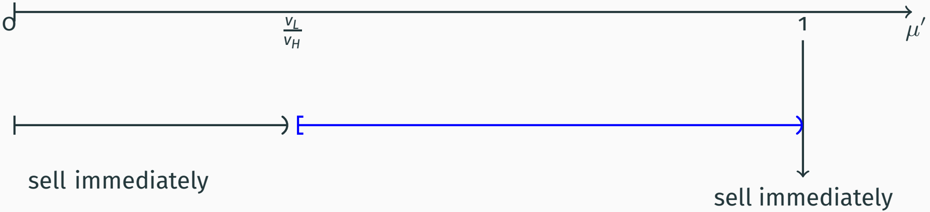

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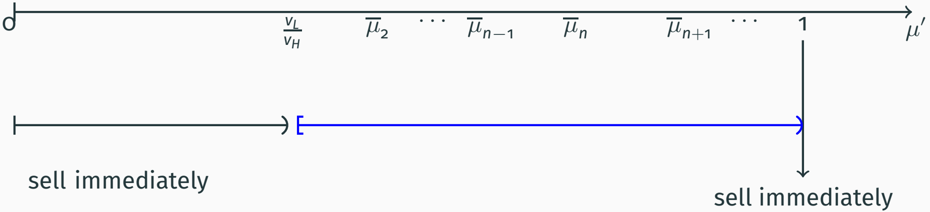

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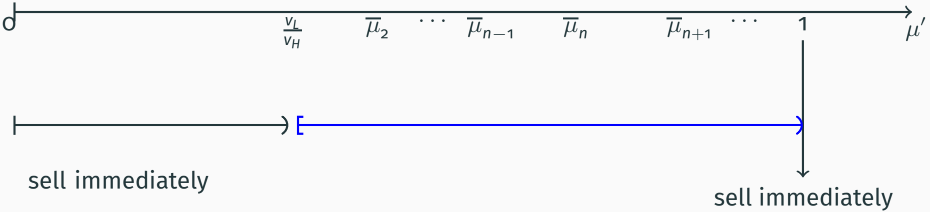

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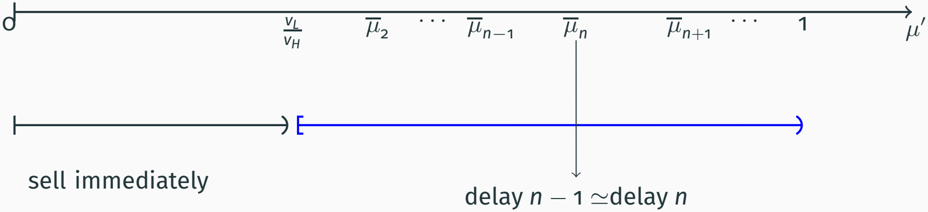

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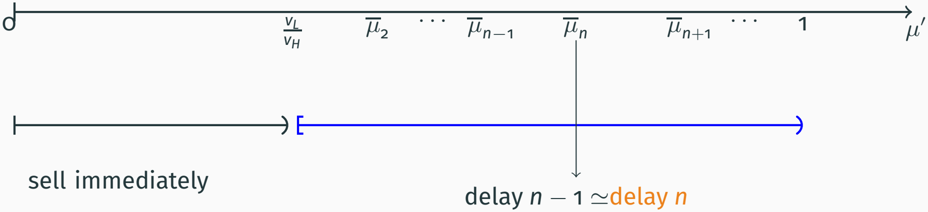

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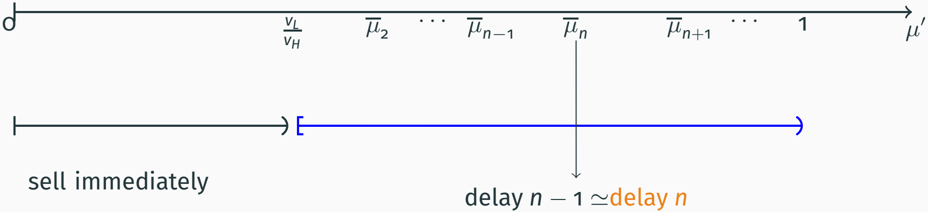

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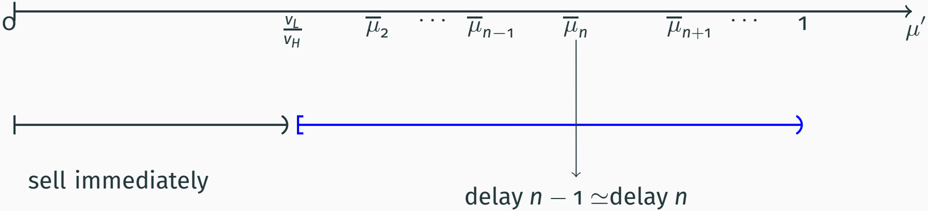

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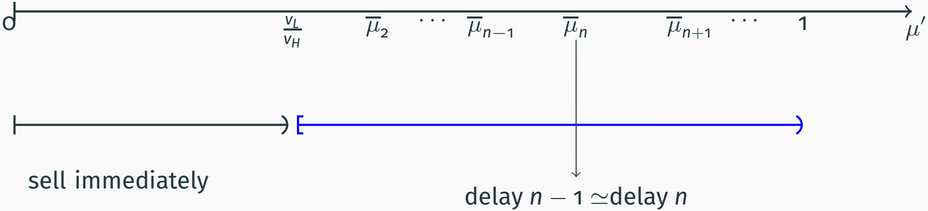

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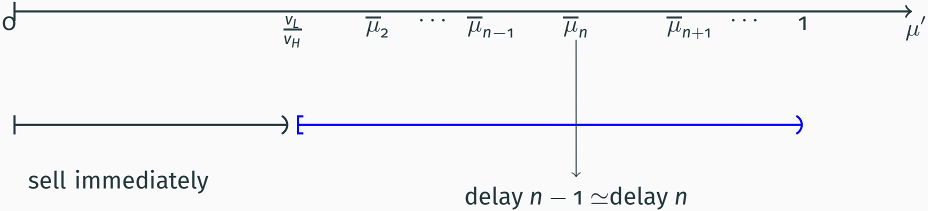

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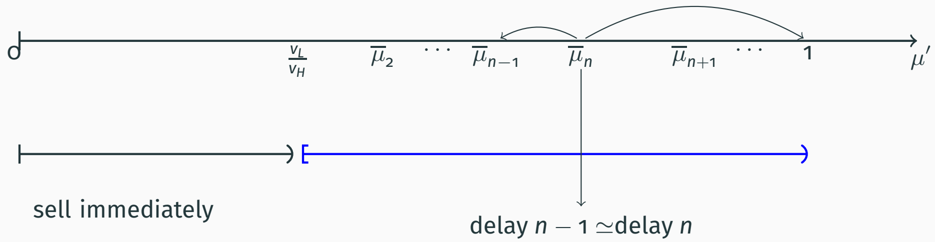

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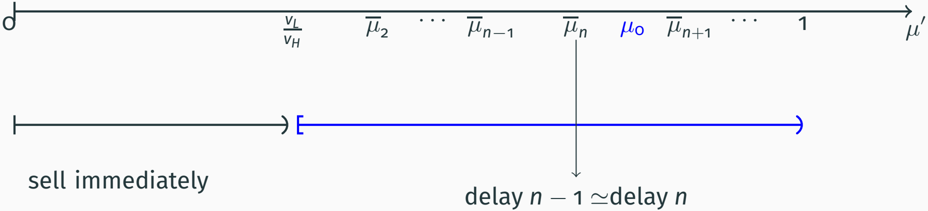

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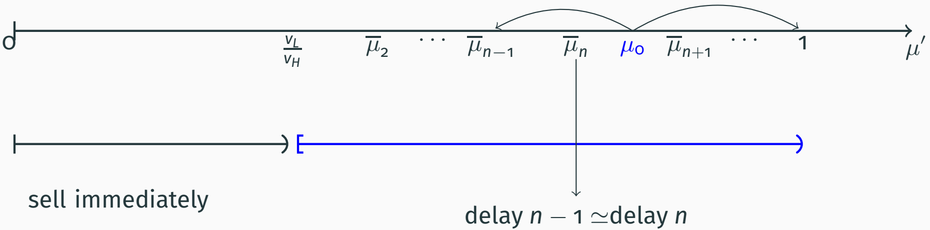

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- Self-generation.

Conclusions: a recipe for mechanism design w/ limited commitment

Commitment:

1. Revelation principle
2. Optimum: search for binding constraints
3. Use binding constraints to replace transfers: virtual surplus
4. Decision problem: find optimal allocation.
5. Recover transfers from constraints and check global ones.

Limited Commitment:

✓ Revelation principle (Doval & Skreta (2018))

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Thank you!! ☺

Perfect Bayesian Equilibrium

Bayes' rule where possible

- Fix a strategy profile $(\Gamma, (\pi_v, r_v)_{v \in V})$, and two nodes v and v' such that v precedes v'.
- We can use the strategy profile to define a probability, $P^{(\Gamma, (\pi_v, r_v)_{v \in V})}(v' | v)$, of reaching node v' conditional on being at node v.
- Extend this probability to all nodes by making it 0 for nodes v' that do not succeed v.

Consecutive information sets

Say that information set h^t precedes information set h^{t+1} if there exists a mechanism, $\mathbf{M}$, such that either of the following hold:

1. there is a posterior, μ', such that $\sum_{v \in V} \beta^{\mathbf{M}}(\mu' | v) > 0$ and $h^{t+1} = (h^t, \mathbf{M}, 1, \mu', (0, x^{\mathbf{M}}(\mu')), \omega_{t+1})$, or
2. $h^{t+1} = (h^t, \mathbf{M}, 0, \emptyset, (0, 0), \omega_{t+1})$.

Bayes' rule where possible

Fix an assessment, $\langle \Gamma, (\pi_v, r_v)_{v \in V}, \mu \rangle$, and two consecutive information sets h^t, h^{t+1}.

- h^{t+1} is reached with positive probability from h^t under $\langle \Gamma, (\pi_v, r_v)_{v \in V}, \mu \rangle$, if

$$P^{\langle \Gamma, (\pi_v, r_v)_{v \in V}, \mu \rangle}(h^{t+1}|h^t) \equiv \sum_{v \in h^t, v' \in h^{t+1}} \mu^*(v|h^t) P^{(\Gamma, (\pi_v, r_v)_{v \in V})}(v'|v) > 0$$

- h^{t+1} can be reached from h^t through a deviation by the seller if there exists Γ' such that $P^{\langle \Gamma', (\pi_v, r_v)_{v \in V}, \mu \rangle}(h^{t+1}|h^t) > 0$.

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An assessment $\langle \Gamma, (\pi_v, r_v)_{v \in V}, \mu \rangle$ satisfies Bayes' rule where possible if for all $t \geq 0$ and for all consecutive h^t, h^{t+1}, $\mu(v'|h^{t+1})$ is obtained via Bayes' rule from $\mu(\cdot|h^t)$ if either

1. $P^{\langle \Gamma, (\pi_v, r_v)_{v \in V}, \mu \rangle}(h^{t+1}|h^t) > 0$, or
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Back