Investor sentiment, behavioral heterogeneity and stock market dynamics

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Outline

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Introduction

'I define a speculative bubble as a situation in which **news of price increases spurs investor enthusiasm, which spreads by psychological contagion from person to person**, *in the process amplifying stories that might justify the price increases and bringing in a larger and larger class of investors, who, despite doubts about the real value of an investment, are drawn to it partly through envy of others' successes and partly through a gambler's excitement.'*

- Shiller, Irrational Exuberance, 2015

Motivation

- Pioneering works in heterogeneous agent model (HAM)
  - Day and Huang (1990); Lux (1995); Brock and Hommes (1998); Chiarella and He (2003); He and Westerhoff (2005)
- Only a handful of HAM studies have taken into account investor sentiment
  - Lux (2012); Chiarella et al. (2017)

Objective

To analyze interaction between investor sentiment and asset price dynamics in HAM.

- Sentiment indicator captures *memory of sentiment, social interaction and sentiment shock*
- Effect of sentiment on stylized facts, market volatility as well as crises within HAM framework

Literature Review

Sentiment effect on asset pricing

- Theoretical
  - De Long et al. (JEBO, 1990) - *Noise trader model*
  - Lux (EJ, 1995; JEBO, 1998) - *Market mood contagion*
- Empirical
  - Baker and Wurgler (JF, 2006; JEP, 2007) - *Top-down approach*
  - Tetlock (JF, 2007) - *Media effect*
- Experimental
  - Hüsler et al. (JEBO, 2013) - *Over-optimism*
  - Makarewics (Comput. Econ, 2017) - *Friendship network*

Literature Review

Sentiment effect on financial crisis

- Siegel (1992) and Baur et al. (1996) - *U.S. stock market crash of 1987*
- Zouaoui et al. (2011) - *Panel data of international stock markets*

Contributions

Our contributions are mainly threefold:

- Model heterogeneous responses to sentiment under a fundamentalist-chartist framework
- Investor sentiment is a significant source of market volatility
- **The sentiment channel provides an explanation to the mechanism underlying different types of financial crises**

Outline

Fundamentalist

- Fundamental value

$$\mu_t = \mu + e_t \tag{1}$$

where $e_t \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$.

- Demand of fundamentalist

$$D_t^f = A(x_t)(\mu_t - p_t) \tag{2}$$

Where $x_t = \mu_t - p_t$. The reaction function A captures the confidence of the fundamentalist.

Fundamentalist

- Reaction function A

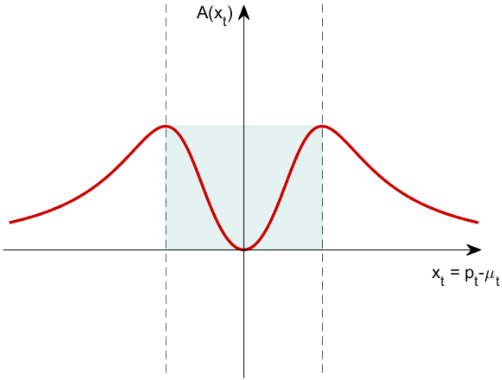

Figure 1: Confidence function of fundamentalist

Chartists: Short Run Asset Value

According to Huang et al. (2010), price domain $\mathbb{P} = [p_{min}, p_{max}]$ can be divided in to n mutually exclusive regimes:

$$\mathbb{P} = \bigcup_{j=1}^{n} \mathbb{P}_j = [\bar{p}_0, \bar{p}_1) \cup [\bar{p}_1, \bar{p}_2) \cup \cdots \cup [\bar{p}_{n-1}, \bar{p}_n] \tag{3}$$

When price falls into a regime, the chartists extrapolate the short run asset value to be in the middle of the regime

$$v_t = (\bar{p}_{j-1} + \bar{p}_j)/2 \quad \textit{if}\, p_t \in [\bar{p}_{j-1}, \bar{p}_j), j = 1, 2 \cdots n \tag{4}$$

Chartists: Momentum Traders

They chase the price trend and are sensitive to sentiment

$$D_t^{mo} = \beta_1 m_t (p_t - v_t) \tag{5}$$

where $\beta_1 > 0$. m_t is the time-varying sentiment factor constructed as

$$m_t = 1 + tanh(\kappa(p_t - v_t)) * h_1 * S_t \tag{6}$$

where S_t is the market sentiment index, and $h_1 \in [0, 1]$ measures sentiment sensitivity. *tanh* function and κ are used to scale the price deviation within $[-1,\ 1]$.

Chartists: Contrarian Traders

They bet on the price reverting to the short-term asset price v_t and are sensitive to sentiment

$$D_t^{co} = \beta_2 c_t (p_t - v_t) \tag{7}$$

where $\beta_2 < 0$. c_t is the time-varying sentiment factor constructed as

$$c_t = 1 - tanh(\kappa(p_t - v_t)) * h_2 * S_t \tag{8}$$

where S_t is the market sentiment index, and $h_2 \in [0, 1]$ measure sentiment sensitivity.

Belief Switching Regime

- Agents are allowed to switch their belief type conditional on the performance of three rules measured as

$$U_{n,t} = \varphi U_{n,t-1} + \pi_{n,t} \tag{9}$$

where $0 \leq \varphi \leq 1$ represents the strength of memory, n is the type of trader.

- Profit can be calculated as

$$\pi_{n,t} = (p_t - p_{t-1}) D^n_{t-1} \tag{10}$$

Belief Switching Regime

Market fractions $\omega_{i,t}$ are updated according to performance $U_{n,t}$ by following a discrete choice probability

$$\omega_{i,t}(p_t) = \frac{exp(\rho U_{i,t}(p_t))}{\sum_k exp(\rho U_{k,t}(p_t))} \tag{11}$$

ρ measures the intensity of the choice as in Brock and Hommes (1998).

Sentiment Indicator

- Three main sources of sentiment: last-period sentiment, investor mood from social interaction, and sentiment shock.

$$S_t = \eta_1 S_{t-1} + \eta_2 SI_t + \eta_3 \epsilon_t \tag{12}$$

where η_1, η_2, η_3 are the weights. $\epsilon_t \sim U(-1, 1)$.

- Social interaction measurement based on majority opinion formation in Kirman (1993), Lux (1995)

$$SI_t = tanh\left[\kappa(\mu_t - p_t)\right] * \omega_t^f + tanh\left[\kappa(p_t - v_t)\right] * (\omega_t^{mo} - \omega_t^{co}) \tag{13}$$

Price Formation: Market Maker

We assume net zero supply of the risky asset, and the market price is determined by a market maker as

$$p_{t+1} = p_t + \gamma(\omega_t^f D_t^f + \omega_t^{mo} D_t^{mo} + \omega_t^{co} D_t^{co}) \tag{14}$$

where γ represents the speed of price adjustment by the market maker.

Outline

Stylized Facts

We calibrate our model based on some well-documented stylized facts of financial markets as summarized by Westerhoff and Dieci (2006):

1. price distortions in the forms of bubbles and crashes
2. excess returns
3. leptokurtic distribution of returns
4. negligible autocorrelation of daily returns
5. strong autocorrelation of absolute daily returns

Stylized Facts: Actual Market

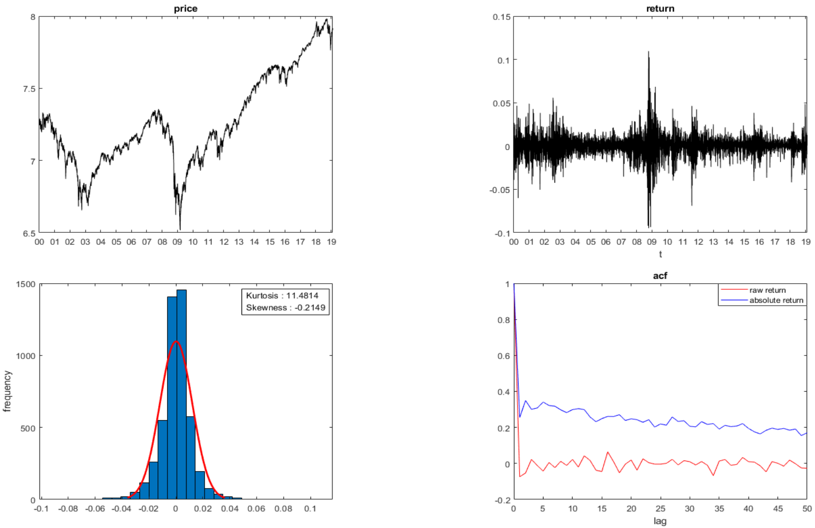

Figure 2: Daily S&P500 index between Jan 3, 2000 and Feb 12, 2019

Stylized Facts: Standard Parameter Setting

Parameter	Value	Definition
μ	1014	Mean of fundamental prices
σ	1	SD of fundamental prices
β_1	1.75	Momentum extrapolation rate
β_2	-1.25	Contrarian extrapolation rate
ϕ	0.1	Performance memory strength
ρ	0.5	Intensity of choice
γ	0.845	Speed of price adjustment
η_1	0.4	Last-period sentiment weight
η_2	0.5	Social interaction weight
η_3	0.1	Sentiment shock weight
λ	12	Support and resistance interval
a	1.11×10^{-5}	Confidence function factor
b	1×10^{-8}	Confidence function factor
κ	1000	Scaling factor
$h = h_1 = h_2$	$0/1$	Without sentiment/with sentiment

Stylized Facts: Artificial Market

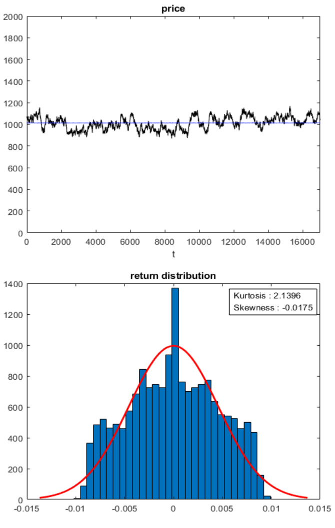

Figure 3: Dynamics of model without sentiment ($N = 17000$).

Stylized Facts: Artificial Market

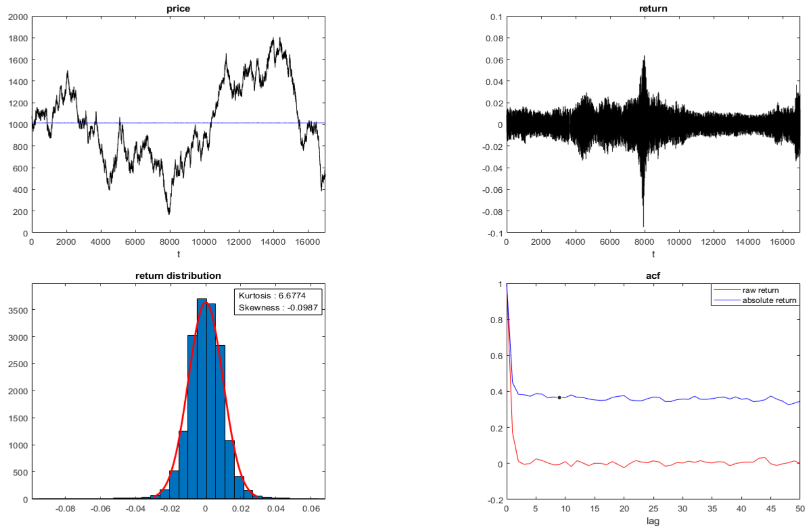

Figure 4: Dynamics of model with sentiment ($N = 17000$).

Stylized Facts for 1000 Simulations

To check the robustness, we run our models 1000 times by using Monte Carlo simulation for a range of sentiment sensitivity

	h=0	h=0.2	h=0.4	h=0.6	h=0.8	h=1
kurtosis	2.155	2.075	3.396	4.615	4.730	5.804
skewness	-0.010	-0.015	-0.026	-0.036	-0.037	-0.040
AC r_1	0.010	0.041	0.166	0.219	0.189	0.166
AC r_5	-0.006	-0.004	0.004	0.001	0.007	0.004
AC r_{10}	-0.003	-0.004	-0.002	-0.003	-0.002	-0.003
AC $\|r_1\|$	-0.014	-0.003	0.172	0.344	0.372	0.412
AC $\|r_5\|$	0.070	0.075	0.179	0.252	0.253	0.298
AC $\|r_{10}\|$	0.039	0.046	0.167	0.241	0.243	0.290

Stylized Facts for 1000 Simulations

To check the robustness, we run the three-type model 1000 times by using Monte Carlo simulation for a range of sentiment sensitivity

	h=0	h=0.2	h=0.4	h=0.6	h=0.8	h=1
kurtosis	2.155	2.075	3.396	4.615	4.730	5.804
skewness	-0.010	-0.015	-0.026	-0.036	-0.037	-0.040
AC r_1	0.010	0.041	0.166	0.219	0.189	0.166
AC r_5	-0.006	-0.004	0.004	0.001	0.007	0.004
AC r_{10}	-0.003	-0.004	-0.002	-0.003	-0.002	-0.003
AC $\|r_1\|$	-0.014	-0.003	0.172	0.344	0.372	0.412
AC $\|r_5\|$	0.070	0.075	0.179	0.252	0.253	0.298
AC $\|r_{10}\|$	0.039	0.046	0.167	0.241	0.243	0.290

Sentiment and Excess Volatility

We use the standard deviation of the market prices from the fundamental values as a quantitative measure of excess volatility

$$SD_{p-\mu} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(p_t - \mu_t)^2}$$

Sentiment and Excess Volatility

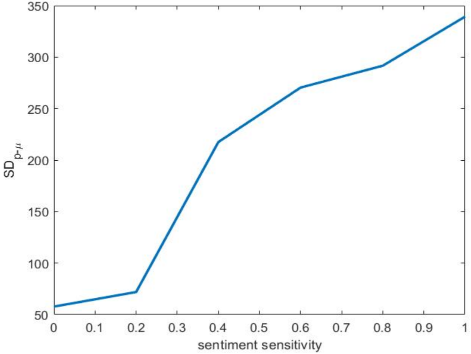

Figure 5: Average standard deviation of the market prices from the fundamental values for 1000 simulations

Sentiment and Crisis

- Following Huang et al. (2010), we replicate 3 different types of crisis
  - Sudden crisis
  - Smooth crisis
  - Disturbing crisis
- Differences between our model and Huang's model
  - 3 agent types vs 2 agent types
  - with sentiment vs without sentiment
  - stochastic vs deterministic

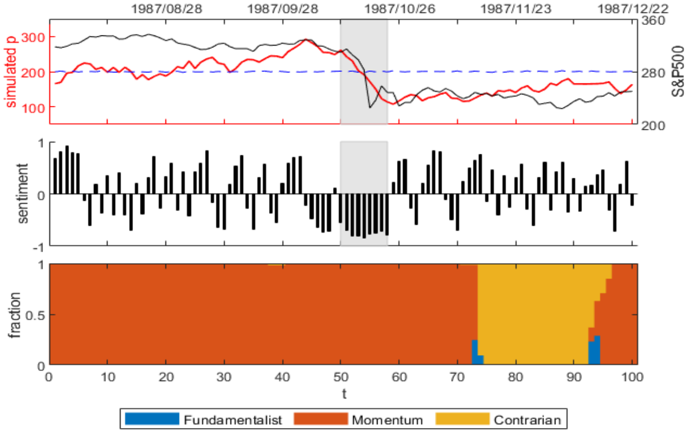

Figure 6: Sudden crisis modelling of S&P500 index from 1987/8/3 to 1987/12/22

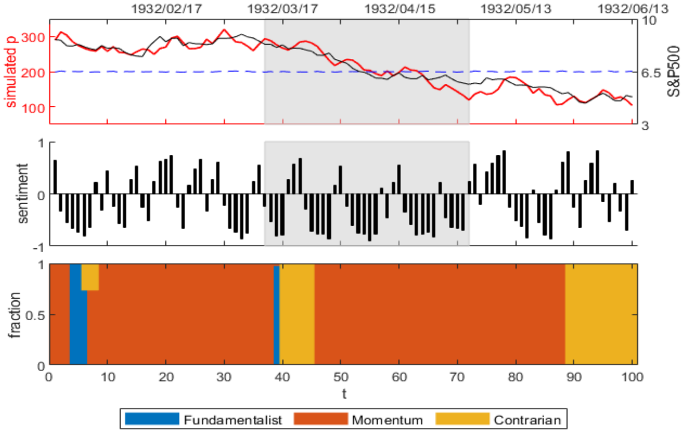

Figure 7: Smooth crisis modelling of S&P500 index from 1932/1/20 to 1932/6/13

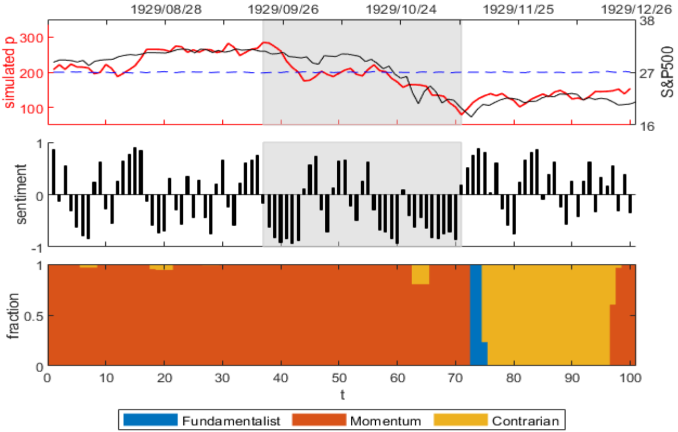

Figure 8: Disturbing crisis modelling of S&P500 index from 1929/8/1 to 1929/12/26

Crisis Identification

To identify the crisis in financial market, we adopt a crisis indicator called *CMAX* used in Patel and Sarkar (1998) and Zouaoui (2011)

$$CMAX_t = \frac{p_t}{max(p_{t-T}, ..., p_t)}$$

usually T is 12 to 24 months

Crisis Identification

- A crisis is identified if

$$
\begin{aligned}
&(1) \quad CMAX_t < \overline{CMAX} - 2\sigma \\
&(2) \quad p_t < \tau * \mu_t \qquad (\tau < 1)
\end{aligned}
$$

- A crisis is eliminated if detected twice over T periods

Crisis Identification with S&P500

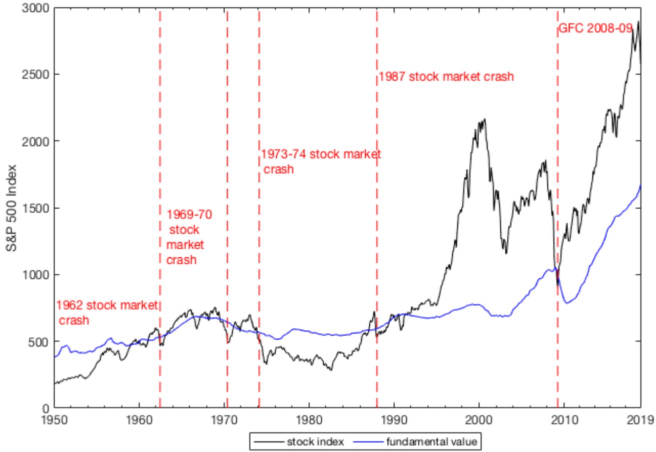

Figure 9: Crises detected from 1950 using real S&P500 monthly data, $\tau = 0.9$

Crisis Identification with Simulated Data

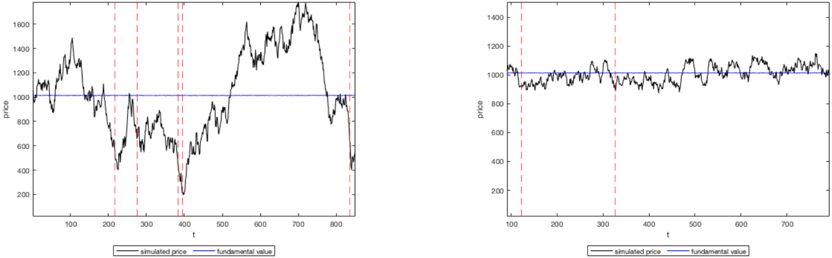

Figure 10: Crisis identified in simulated monthly data with sentiment (left) and without sentiment (right), $\tau = 0.9$

Magnitude of Crisis

The magnitude of a crisis is defined as the percentage drop in price from the peak to the trough

- Peak: maximum price over T periods prior to crisis identification
- Trough: minimum price during the crisis

Crisis Identification & Magnitude

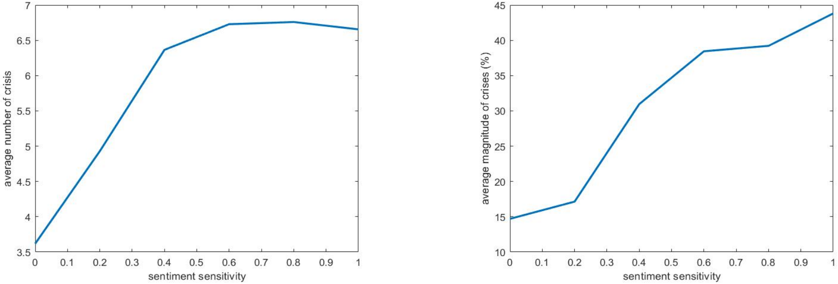

Figure 11: Average number & average magnitude of crisis with different *sentiment sensitivity* for 1000 simulations

Outline

Alternative Fundamentalist Chance Functions

	Linear chance function		Day & Huang (1990)	
	h=0	h=1	h=0	h=1
kurtosis	2.378	3.377	2.537	5.309
skewness	-0.018	-0.109	-0.009	-0.106
AC r_5	-0.004	0.004	-0.003	-0.002
AC r_{10}	-0.001	-0.002	0.000	-0.003
AC r_{20}	0.000	-0.001	0.000	-0.001
AC $\|r_5\|$	0.103	0.238	0.134	0.315
AC $\|r_{10}\|$	0.074	0.227	0.086	0.298
AC $\|r_{20}\|$	0.058	0.223	0.058	0.290
$SD_{p-\mu}$	141.601	293.900	145.606	336.214
# of crisis	5.560	7.353	5.452	6.800
crisis magnitude (%)	20.166	38.883	20.927	42.916

Alternative Chartist Strategies

	Exponential MA		Extrapolative trend	
	h=0	h=1	h=0	h=1
kurtosis	3.500	4.259	3.557	4.375
skewness	-0.002	-0.017	-0.006	-0.033
AC r_5	0.021	0.086	0.002	0.049
AC r_{10}	0.000	0.017	-0.002	0.001
AC r_{20}	-0.003	-0.006	-0.002	-0.006
AC $\|r_5\|$	0.055	0.096	0.059	0.090
AC $\|r_{10}\|$	0.053	0.084	0.057	0.082
AC $\|r_{20}\|$	0.051	0.079	0.056	0.078
$SD_{p-\mu}$	169.035	196.864	176.808	197.421
# of crisis	9.162	9.596	9.078	9.564
crisis magnitude (%)	39.815	49.425	40.234	49.301

Outline

1. Introduction
2. The Model
3. Numerical Simulation with Stochastic Model
4. Robustness Checks
5. Conclusion

Conclusion

The key findings of this paper are

- Investor sentiment contributes to more realistic stylized facts and excess market volatility.
- With presence of investor sentiment, financial crisis can be triggered even without mean-reverting action of fundamentalist.

Thank you!

Appendix: Deterministic Models

- 2-type models
  Fundamentalist versus momentum traders.
  Fundamentalist versus contrarian traders.
  Momentum versus contrarian traders.
- 3-type model
  Fundamentalist, momentum and contrarian traders.

2-type Model: Fundamentalist vs Momentum Traders

The system can be modelled as a three-dimensional nonlinear map:

$$\begin{cases} p_{t+1} = p_t + \gamma[\omega_t^f A_t(\mu_t - p_t) + \beta_1 \omega_t^{mo} m_t \left(p_t - v_t\right)] \\ U_{t+1} = \varphi U_t + \left(p_{t+1} - p_t\right)\left[\left(A_t(\mu_t - p_t\right) - \beta_1 m_t \left(p_t - v_t\right)\right] \\ S_{t+1} = \eta_1 S_t + \eta_2 \left[\tanh\left(\kappa(\mu_t - p_{t+1})\right)\omega_{t+1}^f + \tanh\left(\kappa\left(p_{t+1} - v_{t+1}\right)\right)\omega_{t+1}^{mo}\right] \end{cases}$$

where

$A_t = \frac{a(\mu_t - p_t)^2}{1 + b(\mu_t - p_t)^4}$,

$\omega_t^f = \frac{\exp\left(\rho U_t\right)}{\exp\left(\rho U_t\right) + 1}$, $\omega_t^{mo} = \frac{1}{\exp\left(\rho U_t\right) + 1}$,

$m_t = 1 + \tanh(\kappa(p_t - v_t)) * h_1 * S_t$,

$U_t = U_t^f - U_t^{mo}$

2-type Model: Fundamentalist vs Momentum Traders

Proposition 1

The system has

1. *an unstable fundamental steady state (FSS) with* $(p^*, U^*, S^*) = (\mu, 0, 0)$ *if* $\mu = v$ *; two types of non-fundamental steady states (NFSS) with the form* $(p, U, S) = (p_1^*, 0, 0)$ *,* $(p, U, S) = (p_2^*, 0, 0)$ *and* $p_1^* < \mu, p_2^* > \mu$ *if* $\mu = v$*.*
2. *two types of non-fundamental steady states (NFSS) with the form* $(p^*, U^*, S^*) = (p_1^*, 0, S_1^*)$ *,* $(p, U, S) = (p_2^*, 0, S_2^*)$ *and* $p_1^* < \mu, p_2^* > \mu$ *if* $\mu \neq v$ *.*

2-type Model: Fundamentalist vs Momentum Traders

Lemma 1

For NFSS, the system could achieve positive sentiment equilibria ($S^* > 0$) *if* $\mu - v > 0$, *negative sentiment equilibria* ($S^* < 0$) *if* $\mu - v < 0$, *zero sentiment equilibria* ($S^* = 0$) *if* $\mu - v = 0$ *and* $\beta_1 \leq \frac{1}{2}ab^{-\frac{1}{2}}$.

3-type Model: Fundamentalist, Momentum and Contrarian Traders

The system can be modelled to a five-dimensional dynamic map

$$\begin{cases} p_{t+1} = p_t + \gamma \left[\omega_t^f A_t \left(\mu_t - p_t\right) + \left(\beta_1 \omega_t^{mo} m_t + \beta_2 \omega_t^{co} c_t\right) \left(p_t - v_t\right)\right] \\ u_{t+1}^f = \varphi u_t^f + \left(p_{t+1} - p_t\right) A_t \left(\mu_t - p_t\right) \\ u_{t+1}^{mo} = \varphi u_t^{mo} + \left(p_{t+1} - p_t\right) \beta_1 m_t \left(p_t - v_t\right) \\ u_{t+1}^{co} = \varphi u_t^{co} + \left(p_{t+1} - p_t\right) \beta_2 c_t \left(p_t - v_t\right) \\ S_{t+1} = \eta_1 S_t + \eta_2 \left[tanh\left(\kappa \left(\mu_{t+1} - p_{t+1}\right)\right) \omega_{t+1}^f + tanh\left(\kappa \left(p_{t+1} - v_{t+1}\right)\right) \left(\omega_{t+1}^{mo} - \omega_{t+1}^{co}\right)\right] \end{cases}$$

where

$A_t = \frac{a(\mu_t - p_t)^2}{1+b(\mu_t - p_t)^4}$,

$\omega_{h,t} = \frac{\exp\left(\rho U_{h,t}\right)}{\sum_{h=1}^{2} \exp\left(\rho U_{h,t}\right)}$,

$m_t = 1 + \tanh(\kappa(p_t - v_t)) * h_1 * S_t$,

$c_t = 1 - tanh(\kappa(p_t - v_t)) * h_2 * S_t$

3-type Model: Fundamentalist, Momentum and Contrarian Traders

Proposition 2

The system has

1. *a unique FSS with* $\left(p^*, u_f^*, u_{mo}^*, u_{co}^*, S^*\right) = \left(\mu, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0\right)$ *if* $\beta_1 = -\beta_2$ *. The Jacobean matrix of this system has five eigenvalues with* $\lambda_1 = 1, \lambda_2 = \varphi$ *. FSS is asymptotically stable for* $|\lambda_3|, |\lambda_4|, |\lambda_5| < 1$ *.*
2. *a FSS with* $\left(p^*, u_f^*, u_{mo}^*, u_{co}^*, S^*\right) = \left(\mu, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0\right)$ *if* $\beta_1 \neq -\beta_2 \text{and} \mu = v$ *. FSS is asymptotically stable for* $-6 < \gamma\left(\beta_1 + \beta_2\right) < 0$*; Two types of NFSS with the form* $\left(p^*, u_f^*, u_{mo}^*, u_{co}^*, S^*\right) = \left(p_1^*, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, S_1^*\right)$, $\left(p^*, u_f^*, u_{mo}^*, u_{co}^*, S^*\right) = \left(p_2^*, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, S_2^*\right)$, *and* $p_1^* < \mu, S_1^* > 0; p_2^* > \mu, S_2^* < 0$ *if* $\beta_1 \neq -\beta_2$ *and* $\mu \neq v$ *.*

3-type Model: Fundamentalist, Momentum and Contrarian Traders

Lemma 2

For NFSS,the system can achieve both positive and negative sentiment equilibria. If $\beta_1 > -\beta_2$ *, positive (negative) sentiment equilibrium exists at* $p^* < (>)\, \mu$ *and* $p^* < (>)\, v$ *. If* $\beta_1 < -\beta_2$ *, positive (negative) sentiment equilibrium exists at* $p^* < (>)\, \mu$ *and* $p^* > (<)\, v$ *.*