Procyclical Markups

Directed Search, Nominal Rigidities and Markup Cyclicality

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Issue in New Keynesian Models

- The key transmission mechanism of demand shocks

$$\text{Aggregate demand} \uparrow \Longrightarrow \text{real wage } \frac{W}{P} \uparrow \Longrightarrow \text{price markup } \left(\frac{P}{W} - 1\right) \downarrow \Longrightarrow \text{inflation} \uparrow$$

- However, a vast literature, Nekarda and Ramey (2019); Stroebel and Vavra (2019); Anderson et al. (2018); Gottfries et al. (2018); Cantore et al. (2019), find evidence supporting

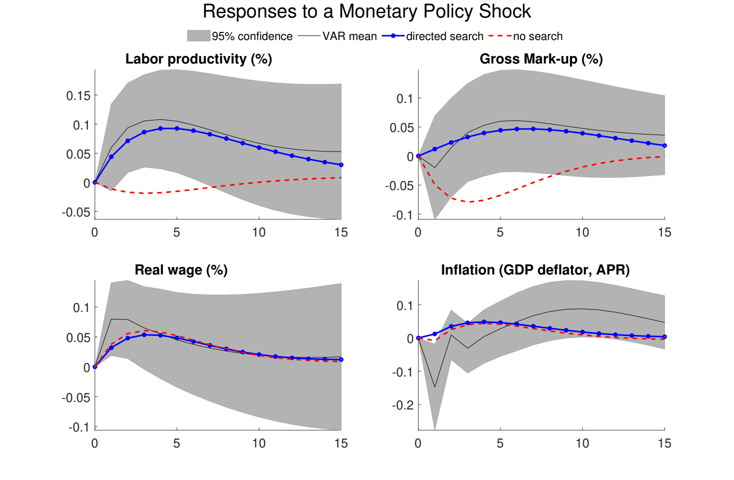
$$\text{Aggregate demand} \uparrow \Longrightarrow \text{price markup} \uparrow.$$

- We provide a theory of procyclical markup based on directed search where:

$$\text{Aggregate demand} \uparrow \Longrightarrow \left\{ \begin{array}{r} \text{real wage} \uparrow \\ \text{productivity} \uparrow \\ \text{desired markup} \uparrow \end{array} \right. \Longrightarrow \left\{ \begin{array}{r} \text{price markup} \uparrow \\ \text{inflation} \uparrow \end{array} \right.$$

- Our theory has an analytical solution in a static model, and good performance in a full-fledged estimated medium scale DSGE as in Christiano et al. (2016).

Preview of Puzzle and Our Solution



"No search" is our replication of the baseline model in Christiano et al. (2016), while "directed search" is our re-estimated DSGE model with directed search in goods market on top of that. Following Nekarda and Ramey (2019), we measure gross mark-up fluctuations simply by the inverse of labor share.

Related Literature

- Bils et al. (2018): Alternative measures may suggest countercyclical mark-ups. Conditional cyclicality is not discussed due to the lack of quarterly data.
- Cantore et al. (2019): Labor search may break the link between inverse labor share and mark-up. However, real wage has too be countercyclical.
- Anderson et al. (2018): Higher mark-up firms are more sensitive to cycles. Yet, individual firms cannot have higher mark-ups in monetary expansion.
- Kaplan and Menzio (2016), Huo and Rios-Rull (2015): Mark-ups are higher when consumers spend less time comparing prices. The model is not built for monetary analysis.
- Meier and Reinelt (2019): Monetary expansion raises productivity like us, but reduces mark-up as in the puzzle.

Road map

We proceed gradually in four steps by showing

1. In Dixit-Stiglitz models with sticky prices, exogenous nominal expenditure, and exogenous nominal wage rate, we must have

$$\text{Nominal wage} \uparrow \implies \text{real wage} \uparrow \implies \text{mark-up} \downarrow \implies \text{inflation} \uparrow,$$

$$\text{Nominal expenditure} \uparrow \implies \text{real expenditure} \uparrow \implies \text{constant} \begin{cases} \text{markup} \\ \text{inflation} \end{cases}$$

2. We introduce Directed Search on top of Dixit-Stiglitz in goods market to show that the following is true

$$\begin{cases} \text{Real wage} \uparrow \\ \text{Markup} \uparrow \end{cases} \text{can coexist},$$

$$\begin{cases} \text{Markup} \uparrow \\ \text{Inflation} \uparrow \end{cases} \text{can coexist}, \quad (\text{when real expenditures} \uparrow).$$

3. We endogenize nominal expenditures and nominal wages via
   - cash-in-advance,
   - Calvo wage,
   - Money market clearing, to have

$$\text{Money supply} \uparrow \Longrightarrow \begin{cases} \text{nominal expenditures} \uparrow \\ \text{nominal wage} \uparrow \end{cases}$$

   so that we can derive the necessary and sufficient condition for

$$\text{Money supply} \uparrow \Longrightarrow \text{Agg Demand} \uparrow \Longrightarrow \text{Real expenditures} \uparrow \Longrightarrow \begin{cases} \text{real wage} \uparrow \\ \text{inflation} \uparrow \\ \text{markup} \uparrow \end{cases}$$

4. We introduce directed search into an estimated medium scale DSGE model as in Christiano et al. (2016) to show that
   - Mark-ups become procyclical conditioning on monetary shocks, and
   - Other parts of the model perform at least equally well if not better (productivity).

Dixit-Stiglitz

Let's look at the Dixit-Stiglitz model with

1. **Exogenous nominal expenditure**:
   - Follows the standard Dixit-Stiglitz model.
2. **Exogenous nominal wage**:
   - Unlimited labor supply at exogenous nominal wage rate (Huo and Ríos-Rull (2019))
3. **Sticky prices**:
   - Rotemberg pricing (easier than Calvo)

Goal: to show

$$\text{Nominal wage} \uparrow \implies \text{real wage} \uparrow \implies \text{mark-up} \downarrow \implies \text{inflation} \uparrow,$$

$$\text{Nominal expenditure} \uparrow \implies \text{real expenditure} \uparrow \implies \text{constant} \begin{cases} \text{markup} \\ \text{inflation} \end{cases}$$

Environment

- Measure one of firms $j \in [0, 1]$, producing differentiated variety j with identical technology.
- Each firm is a monopoly in its own variety, setting price p_j, and producing whatever is demanded. (Another New Keynesian Problem)
- Given the price function $\{p_i\}$, the representative household allocates expenditure e among these goods varieties optimally.
- Knowing households' demand on each variety j that depends on p_j as well as $\{p_i\}_{i \neq j}$, firm j sets price p_j optimally.
- Firms' price decisions are consistent with the price distribution $\{p_i\}$.

- Given utility function $\widetilde{u}(\cdot)$, and expenditure e, the representative household chooses the purchase of varieties $\{c_i\}$ to solve

$$V(e,\{p_i\}) = \max_{\{c_i\}} \widetilde{u}\left(\left(\int_0^1 c_i^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}\right), \qquad \text{with } \varepsilon > 1,$$

$$\text{s.t.} \qquad e \geq \int_0^1 p_i \; c_i \;\; di.$$

- The solution is a decision rule

$$c(e,\{p_i\}_{i\neq j}, p_j) = \left(\frac{p_j}{P}\right)^{-\varepsilon} \frac{e}{P}, \qquad \text{where } P \equiv \left(\int_0^1 p_i^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}},$$

which aggregates to $c^h(e,\{p_i\}_{i\neq j}, p_j) = c(e,\{p_i\}_{i\neq j}, p_j)$.

Firms' Problem

- Firm j purchases labor n_j from a competitive market at nominal wage W, and produces output y_j via technology $y_j = n_j$.

- With inherited price p_-, firm j sets price p_j to produce $y_j = c^h(e, \{p_i\}_{i \neq j}, p_j)$, at (ridiculously proportional to expenditures for simplicity) cost $\chi\left(p_j/p_-\right)e$, to solve

$$\Omega(e, W, \{p_i\}_{i \neq j}, p_-) = \max_{p_j} (p_j - W)c^h(e, \{p_i\}_{i \neq j}, p_j) - \chi\left(\frac{p_j}{p_-}\right)e.$$

- The F.O.C. is

$$\begin{aligned} 0 &= \left(1 + \frac{p_j c^h_{p_j}}{c^h} - \frac{W}{p_j}\frac{p_j c^h_{p_j}}{c^h}\right)c^h - \chi_p\left(\frac{p_j}{p_-}\right)\frac{e}{p_-}, \\ &= (\varepsilon - 1)\left(\frac{\varepsilon}{\varepsilon - 1}\frac{W}{p_j} - 1\right)\left(\frac{p_j}{P}\right)^{1-\varepsilon} - \chi_p\left(\frac{p_j}{p_-}\right)\frac{p_j}{p_-}. \end{aligned}$$

- The solution of p_j is a decision rule $p^f(e, W, \{p_i\}_{i \neq j}, p_-)$.

An equilibrium is a set of functions $\{\overline{c}, \overline{p}\}$ on (e, W, p_-) s.t.

- Households spend

$$\overline{c}(e, W, p_-) = c^h(e, \overline{p}(e, W, p_-), \overline{p}(e, W, p_-)),$$

- Firms price and produce

$$\overline{p}(e, W, p_-) = p^f(e, W, \overline{p}(e, W, p_-), p_-).$$

1. In equilibrium, $\overline{p}(e, W, p_-)$ solves (mark-up is $\frac{\overline{p}}{W} - 1$)

$$0 = \left[\varepsilon \frac{W}{\overline{p}} - (\varepsilon - 1)\right] - \chi_p\left(\frac{\overline{p}}{p_-}\right)\frac{\overline{p}}{p_-}. \qquad (1)$$

2. The corresponding aggregate consumption for each variety $\overline{c}(e, W, p_-)$ satisfies

$$\overline{c}(e, W, p_-) = \frac{e}{\overline{p}(e, W, p_-)}.$$

3. The corresponding indirect utility function satisfies

$$V(e, \overline{p}) = \widetilde{u}\left(\frac{e}{\overline{p}}\right).$$

- As long as $p\,\chi_p(p)$ is strictly increasing in p (assumption), (1) implies that

$$\text{Nominal wage}\uparrow \implies \text{real wage}\uparrow \implies \text{mark-up}\downarrow \implies \text{inflation}\uparrow,$$

- Since expenditures, e, do not show up in (1), we have

$$\text{Nominal expenditure}\uparrow \implies \text{real expenditure}\uparrow \implies \textit{constant}\ \begin{cases}\text{markup}\\ \text{inflation}\end{cases}$$

- So we are done with Step 1

Directed Search

We introduce directed search in goods market (shopping friction) on top of the Dixit-Stiglitz model to get two additional channels:

- **Directed search in goods market yields endogenous productivity and endogenous desired mark-up:**
  1. Each firm/variety has many locations (a consumer is in each location)
  2. Firms post prices,
  3. Households choose shopping effort, and look for varieties in different submarkets, obtaining endogenous number of varieties
  4. Firms get different number of consumers (consumers determine productivity)
- **Channel 1 - Endogenous productivity**:
  Real wages and markups can move together (workers shop more in monetary expansions increasing productivity)
- **Channel 2 - Endogenous desired mark-up**:
  Firms trade off prices and matching probabilities. When matching probability gets closer to one, less of it needs to be traded off for price raise. In expansions, higher markups and higher inflation can both go up when real expenditures go up.

Environment (1/2)

- Firm are still monopolies in their own varieties as in Dixit-Stiglitz.
- Each firm operates a continuum of locations, each with its own pre-installed inputs and identical production technology.
- A directed search protocol determines the coordination of households and firms via submarkets indexed by price and tightness $\{p, q\}$.
- Households have to search and find varieties, and they value both the number of varieties and the quantity of each. To obtain varieties, each household chooses shopping effect $d(p, q)$ in each submarket $\{p, q\}$.
- Households that find a variety are randomly allocated to one and only one of its locations. Each location can be filled with at most one household.

Environment (2/2)

- Each firm can only go to one submarket.
- A change in price is implemented via switching to a different submarket that has a different price, market tightness and demand for each variety.
- When a firm goes to one submarket, it moves all locations to it.
- Denote $J(p,q)$ as the measure of firms, and $D(p,q)$ as the total shopping effort in submarket $\{p,q\}$. The total number of matches is given by a CRS, continuously differentiable, strictly increasing, and strictly concave matching function $\psi(J(p,q),D(p,q))$. we have $q=\frac{D(p,q)}{J(p,q)}$.
- Denote the number of matches by each unit of D as $\psi^h(q)\equiv\frac{\psi(J(p,q),D(p,q))}{D(p,q)}$, and that by each firm as $\psi^f(q)\equiv\frac{\psi(J(p,q),D(p,q))}{J(p,q)}$.

1. Use $\{p, q\} \in \Phi$ to denote a submarket. Households choose shopping effort allocation $d(p, q)$ across all active submarkets, as well as the quantity to purchase $c(p, q)$ for each variety in each of them, given expenditure e.

2. For a *market utility* $\overline{V}$, We can solve for the set of submarkets $\Phi(e, \overline{V})$, such that a household in $\forall\{p, q\} \in \Phi(e, \overline{V})$ has utility $\overline{V}$.

3. Prove that households going to different elements of $\Phi(e, \overline{V})$ also have utility $\overline{V}$. It allows us to solve for the tightness $q^h(e, \overline{V}, p)$ and demand $c^h(e, \overline{V}, p)$ for firms.

4. Firms that take $\{c^h(\cdot), q^h(\cdot)\}$ as given post a price p optimally with decision rule $p^f(e, W, \overline{V}, p_-)$.

5. Consistency conditions must be satisfied in competitive search equilibrium.

Step 1: Households' Optimality Conditions Given $\{e, \Phi\}$

- The representative household chooses the purchase of each variety $c(p, q)$ and the total shopping effort $d(p, q)$ in each submarket $\{p, q\} \in \Phi$ to solve

$$V(e, \Phi) = \max_{\{c(p,q), d(p,q)\}} u\left(c^A, d^A\right),$$

$$\text{s.t. } e \geq \int_\Phi d(p, q)\, \psi^h(q)\, p\, c(p, q)\; dpdq, \tag{2}$$

$$c^A \equiv \left(\int_\Phi d(p, q)\, \psi^h(q)\; c(p, q)^{\frac{\varepsilon-1}{\varepsilon}}\; dpdq\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

$$d^A \equiv \int_\Phi d(p, q)\; dpdq. \tag{4}$$

with solution $\{c(e, \Phi, p, q), d(e, \Phi, p, q)\}$, and $V(e, \Phi)$.

Step 1: Households' Optimality Conditions Given $\{e, \Phi\}$

- Use λ to denote the Lagrange multiplier on budget. The F.O.C.s are

$$0 = \left(\frac{c(p,q)}{c^A}\right)^{-\frac{1}{\varepsilon}} u_{c^A} - \lambda p, \tag{5}$$

$$0 = \frac{1}{\varepsilon - 1}\left(\frac{c(p,q)}{c^A}\right)^{-\frac{1}{\varepsilon}} u_{c^A} + \frac{u_{d^A}}{\psi^h(q)c(p,q)}. \tag{6}$$

- Rearranging equation (6) yields

$$d(p,q)\psi^h(q)\left(\frac{c(p,q)}{c^A}\right)^{\frac{\varepsilon-1}{\varepsilon}} c^A = -(\varepsilon - 1)\frac{u_{d^A}}{u_{c^A}} d(p,q).$$

- Under GHH utility $u(c^A, d^A) \equiv \widetilde{u}\left(c^A - \zeta\frac{(d^A)^{1+\nu}}{1+\nu}\right)$, taking integrals yields

$$c^A = (\varepsilon - 1)\zeta(d^A)^{1+\nu}. \tag{7}$$

- Consider a degenerate set of submarkets $\Phi = \{p, q\}$. (2) (binding) becomes

$$e = d(e, \{p, q\}, p, q)\psi^h(q)p\, c(e, \{p, q\}, p, q). \tag{8}$$

- At the same time, (7) becomes

$$\psi^h(q)^{\frac{\varepsilon}{\varepsilon-1}} c(e, \{p, q\}, p, q) = (\varepsilon - 1)\zeta\, d(e, \{p, q\}, p, q)^{1+\nu-\frac{\varepsilon}{\varepsilon-1}}. \tag{9}$$

- Combining equation (8) and (9), we can solve for

$$d(e, \{p, q\}, p, q) = \left[\left(\frac{\zeta^{-1}}{\varepsilon - 1}\frac{e}{p}\right)^{\varepsilon-1} \psi^h(q)\right]^{\frac{1}{(1+\nu)(\varepsilon-1)-1}}, \tag{10}$$

$$c(e, \{p, q\}, p, q) = \frac{1}{d(e, \{p, q\}, p, q)\psi^h(q)}\frac{e}{p}. \tag{11}$$

- With (3)(4)(7), the utility of households in $\Phi = \{p, q\}$ becomes

$$V(e, \{p,q\}) = \widetilde{u}\left([(1+\nu)(\varepsilon-1)-1]\zeta \frac{d(e, \{p,q\}, p, q)^{1+\nu}}{1+\nu}\right).$$

- For $V(e, \{p,q\}) = \overline{V}$, we have

$$d(e, \{p,q\}, p, q) = \left\{\frac{(1+\nu)\widetilde{u}^{-1}(\overline{V})}{\zeta[(1+\nu)(\varepsilon-1)-1]}\right\}^{\frac{1}{1+\nu}}. \tag{12}$$

- Substituting (12) into (10) yields

$$\psi^h(q)p^{1-\varepsilon} = \zeta^{\frac{1}{1+\nu}}\left(\frac{\varepsilon-1}{e}\right)^{\varepsilon-1}\left[\frac{(1+\nu)\widetilde{u}^{-1}(\overline{V})}{(1+\nu)(\varepsilon-1)-1}\right]^{\frac{(1+\nu)(\varepsilon-1)-1}{1+\nu}}. \tag{13}$$

For monotonicity, the solution for q in terms of p is denoted as $q^h(e, \overline{V}, p)$. We denote $\Phi(e, \overline{V}) \equiv \left\{\{p,q\} \in \mathbb{R}^2_{\geq 0} | q = q^h(e, \overline{V}, p)\right\}$. This is the set of submarkets in which each single submarket alone induces household utility $\overline{V}$.

- Denote the following individual price aggregate

$$\widetilde{p} \equiv \left[\int_{\Phi} d(p,q) \psi^{h}(q) p^{1-\varepsilon} dpdq \right]^{\frac{1}{1-\varepsilon}}. \tag{14}$$

- Combining (2)(5) with (14), we can obtain

$$e = \widetilde{p} c^{A}, \tag{15}$$

$$c(p,q) = \left(\frac{p}{\widetilde{p}} \right)^{-\varepsilon} \frac{e}{\widetilde{p}}. \tag{16}$$

- Under $\Phi(e, \overline{V})$, combing (4)(7)(13)(14)(15) yields

$$V(e, \Phi(e, \overline{V})) = \widetilde{u} \left([(1+\nu)(\varepsilon - 1) - 1] \zeta \frac{d^{A}(e, \Phi(e, \overline{V}))^{1+\nu}}{1+\nu} \right) = \overline{V}. \tag{17}$$

- (17) implies that the coexistence of any subset of submarkets in $\Phi(e, \overline{V})$ does not change household utility $\overline{V}$ induced by any single submarket in $\Phi(e, \overline{V})$. Hence, $q^h(e, \overline{V}, p)$ below solved from (13) will be taken as given by firms.

$$q^h(e, \overline{V}, p) = \psi^{h,-1}\left(\zeta^{\frac{1}{1+\nu}}\left(\frac{\varepsilon-1}{e}\right)^{\varepsilon-1}\left[\frac{(1+\nu)\widetilde{u}^{-1}(\overline{V})}{(1+\nu)(\varepsilon-1)-1}\right]^{\frac{(1+\nu)(\varepsilon-1)-1}{1+\nu}} p^{\varepsilon-1}\right).$$

- Combing (7)(15)(17) yields

$$\widetilde{p} = \frac{(1+\nu)(\varepsilon-1)-1}{(1+\nu)(\varepsilon-1)\widetilde{u}^{-1}(\overline{V})}e. \tag{18}$$

- Combining (16)(18) yields

$$c(p, q) = \left[\frac{(1+\nu)(\varepsilon-1)-1}{(1+\nu)(\varepsilon-1)\widetilde{u}^{-1}(\overline{V})}\right]^{\varepsilon-1}\left(\frac{e}{p}\right)^{\varepsilon} \equiv c^h(e, \overline{V}, p).$$

$c^h(e, \overline{V}, p)$ will also be taken as given by firms.

Step 4: Firms' Optimality Condition Given $(e, W, \overline{V}, p_-)$

- A firm purchases labor n from a competitive market at nominal wage W, and produces output y via technology $y = n$.

- With initial price p_-, a firm sets a price p to produce $y = c^h(e, \overline{V}, p)$ in $\psi^f\left[q^h(e, \overline{V}, p)\right]$ of its locations, at non-pecuniary cost $\chi(p/p_-)\,e$, to solve

$$\Omega(e, W, \overline{V}, p_-) = \max_p \left(p\psi^f\left[q^h(e, \overline{V}, p)\right] - W\right) c^h(e, \overline{V}, p) - \chi\left(\frac{p}{p_-}\right) e.$$

- The F.O.C. is

$$\begin{aligned} 0 &= \left(\psi^f + \frac{q^h\psi^f_q}{\psi^f}\frac{pq^h_p}{q^h}\psi^f + \frac{pc^h_p}{c^h}\psi^f - \frac{W}{p}\frac{pc^h_p}{c^h}\right) c^h - \chi_p\left(\frac{p}{p_-}\right)\frac{e}{p_-}, \\ &= \left[\varepsilon\frac{W}{p} - (\varepsilon - 1)\frac{\psi^f(q^h)}{1 - \mathcal{E}(q^h)}\right]\frac{p\,c^h}{e} - \chi_p\left(\frac{p}{p_-}\right)\frac{p}{p_-}, \end{aligned}$$

where $\mathcal{E}(\cdot) \in (0, 1)$ denotes the elasticity of $\psi^f(\cdot)$ and is decreasing in q. The solution of p is a decision rule $p^f(e, W, \overline{V}, p_-)$.

Definition

An equilibrium is a set of functions $\{\overline{c}, \overline{d}, \overline{p}, \overline{q}, \overline{V}\}$ on (e, W, p_-) s.t.

- aggregate demand for each variety:

$$\overline{c}(e, W, p_-) = c^h(e, \overline{V}(e, W, p_-), \overline{p}(e, W, p_-)),$$

- each household's shopping effort:

$$\overline{d}(e, W, p_-) = d(e, \{\overline{p}(e, W, p_-), \overline{q}(e, W, p_-)\}, \overline{p}(e, W, p_-), \overline{q}(e, W, p_-)),$$

- optimal pricing condition:

$$\overline{p}(e, W, p_-) = p^f(e, W, \overline{V}(e, W, p_-), p_-),$$

- consistent condition for market tightness:

$$\overline{q}(e, W, p_-) = \overline{d}(e, W, p_-),$$

- the market utility for households:

$$\overline{V}(e, W, p_-) = V(e, \{\overline{p}(e, W, p_-), \overline{q}(e, W, p_-)\}).$$

Proposition

In the equilibrium, $\{\overline{p}(e, W, p_{-}), \overline{q}(e, W, p_{-})\}$ solve (mark-up is $\frac{\overline{p}\psi^f(\overline{q})}{W} - 1$)

$$0 = \left[\frac{\varepsilon W}{\overline{p}\psi^f(\overline{q})} - \frac{\varepsilon - 1}{1 - \mathcal{E}(\overline{q})}\right] - \chi_p \left(\frac{\overline{p}}{p_{-}}\right) \frac{\overline{p}}{p_{-}}, \quad (19)$$

$$0 = \frac{(\varepsilon - 1)\zeta \overline{q}^{1+\nu}}{\psi^f(\overline{q})^{\frac{1}{\varepsilon - 1}}} - \frac{e}{\overline{p}}. \quad (20)$$

The corresponding $\{\overline{c}(e, W, p_{-}), \overline{d}(e, W, p_{-})\}$ satisfy

$$\overline{c}(e, W, p_{-}) = \frac{e}{\overline{p}(e, W, p_{-})\psi^f(\overline{q}(e, W, p_{-}))},$$

$$\overline{d}(e, W, p_{-}) = \overline{q}(e, W, p_{-}).$$

The corresponding indirect utility function becomes

$$V(e, \{\overline{p}, \overline{q}\}) = \widetilde{u}\left([(\varepsilon - 1)(1 + \nu) - 1]\,\zeta \frac{d(e, \{\overline{p}, \overline{q}\}, \overline{p}, \overline{q})^{1+\nu}}{1 + \nu}\right).$$

1. (??) allows q to depend on e. When $\psi^f(q)$ goes up more than $\frac{W}{\overline{p}}$, we can have real wage $\frac{W}{\overline{p}}$ and mark-up $\frac{\overline{p}\ \psi^f(\overline{q})}{W} - 1$ going up in the same time,

   real wage $\uparrow$ mark-up $\uparrow$ can coexist.

2. When $\frac{\varepsilon-1}{1-\mathcal{E}(\overline{q})}$ goes down more than $\frac{W}{\overline{p}\psi^f(\overline{q})}$ in (19) as a result of $\frac{e}{\overline{p}}$ and $\overline{q}$ going up, we can still have $\overline{p}$ going up, when $\chi_p(p)p$ is increasing in p, i.e.

   mark-up $\uparrow$ inflation $\uparrow$ can coexist (when real expenditures $\uparrow$) .

   Models with capacity utilization responding would have a problem here

3. The exact condition for what we want can be found in the next section.

Endogenous (e, W)

We endogenize (e, W) with the following features:

- **Cash-in-advance** To get monetary policy to matter: a bit old fashioned but
  - Money demand↑ $\Longrightarrow$ nominal expenditure↑.
- **Stickys wages a la Calvo so**:
  - Nominal expenditure↑ $\Longrightarrow$ nominal wage↑.
- **Goal**: To derive necessary and sufficient conditions for

$$\begin{array}{r} \text{Expansionary} \\ \text{Monetary} \\ \text{Policy} \end{array} \Longrightarrow \begin{array}{c} \text{Aggregate} \\ \text{demand} \uparrow \end{array} \Longrightarrow \left\{ \begin{array}{r} \text{real wage} \uparrow \\ \text{productivity} \uparrow \\ \text{desired markup} \uparrow \end{array} \right. \Longrightarrow \left\{ \begin{array}{r} \text{price markup} \uparrow \\ \text{inflation} \uparrow \end{array} \right.$$

Environment

- Before the directed search stage, households choose nominal expenditure e and money balance M subject to cash-in-advance constraint $\iota e \leq M$.
- Money market clearing requires demand to be equal to supply $M = M^s$.
- Wage is set before expenditure takes place.
- Competitive labor packers produce labor aggregates from labor varieties. Labor varieties with higher wages will be demanded less.
- A variety specific labor union sets wage on behalf of households.
- Each household supplies all types of labor varieties, and the same amount of labor in each variety such that it aggregates to the aggregate variety demand.
- Each labor union internalizes households' optimal expenditures decisions, but not any equilibrium aggregate objects..

- Before search, each household with indirect utility $V(e, \overline{p})$ or $V(e, \{\overline{p}, \overline{q}\})$, nominal wage income WL, firm profit transfer Π^f, and endowed money supply M^s chooses nominal expenditure e and money demand M to solve

$$\max_{e,M} V(e, \cdot),$$
$$\text{s.t. } e + M \leq WL + \Pi^f + M^s,$$
$$\iota e \leq M.$$

- The solution is a set of functions $\{e, M\}$ on $WL + \Pi^f + M^s$ such that

$$e(WL + \Pi^f + M^s) = \frac{WL + \Pi^f + M^s}{1 + \iota},$$
$$M(WL + \Pi^f + M^s) = \iota \, e(WL + \Pi^f + M^s).$$

- Market market clearing requires that "money demand = money supply"

$$M(WL + \Pi^f + M^s) = M^s.$$

- Labor aggregate L comes from both households i and varieties j, $\ell_{i,j}$ via

$$L = \left[\int_0^1 \left(\int_0^1 \ell_{i,j} di\right)^{\frac{\varepsilon_W - 1}{\varepsilon_W}} dj\right]^{\frac{\varepsilon_W}{\varepsilon_W - 1}}.$$

- Taking nominal wage W for L and nominal wages W_j for $\ell_{i,j}$ as given, the optimality of the labor packer requires that

$$\int_0^1 \ell_{i,j} di = \left(\frac{W_j}{W}\right)^{-\varepsilon_W} L, \text{ and } W = \left(\int_0^1 W_j^{1-\varepsilon_W} dj\right)^{\frac{1}{1-\varepsilon_W}}.$$

- For $j \in [0, \theta_w]$, $W_j = W_-$. For $j \in [\theta_w, 1]$, W_j is set by a union j. The union requires household i to supply $\ell_j = \int_0^1 \ell_{i,j} di$ units of labor and it complies.

$$\max_{W_j} \left\{ V\left(e\left(\int_0^1 W_j \ell_j dj + \Pi^f + M^s\right), \cdot\right) - \int_0^1 \ell_j dj \right\}, \text{s.t. } \ell_j = \left(\frac{W_j}{W}\right)^{-\varepsilon_W} L.$$

- The solution is a function $W^\#$ on e such that

$$W^\#(e) = \frac{\varepsilon_W}{\varepsilon_W - 1} \frac{1+\iota}{V_e(e, \cdot)}.$$

Definition

An equilibrium is a set of functions $\{\overline{\overline{W}}, \overline{\overline{e}}, \overline{\overline{M}}, \overline{\overline{L}}, \overline{\overline{\Pi}}^f\}$ on (M^s, W_-, p_-) solving

- aggregate wage determined by union's optimality

$$\overline{\overline{W}} = \left[\theta_w W_-^{1-\varepsilon_w} + (1-\theta_w) W^{\#}(\overline{\overline{e}})^{1-\varepsilon_w}\right]^{\frac{1}{1-\varepsilon_w}}$$

- household's optimality:

$$\overline{\overline{e}} = e(\overline{\overline{WL}} + \overline{\overline{\Pi}}^f + M^s),$$
$$\overline{\overline{M}} = \iota \overline{\overline{e}},$$

- money and goods market clearing conditions:

$$\overline{\overline{M}} = M^s,$$
$$\psi^f(\overline{q}(\overline{\overline{e}}, \overline{\overline{W}}, p_-))\overline{c}(\overline{\overline{e}}, \overline{\overline{W}}, p_-) = \psi^f(\overline{q}(\overline{\overline{e}}, \overline{\overline{W}}, p_-))\overline{\overline{L}},$$

- firm profit transfer:

$$\overline{\overline{\Pi}}^f = \left\{\overline{p}(\overline{\overline{e}}, \overline{\overline{W}}, p_-)\psi^f\left[\overline{q}(\overline{\overline{e}}, \overline{\overline{W}}, p_-)\right] - \overline{\overline{W}}\right\}\overline{\overline{L}}.$$

Lemma

In equilibrium, the law of motion for the aggregate wage can be described by

$$\overline{\overline{W}}(M^s, W_-, p_-) = \left\{\theta_w W_-^{1-\varepsilon_w} + (1-\theta_w)\left[\frac{\varepsilon_w}{\varepsilon_w - 1}\frac{1+\iota}{V_e(M^s/\iota, \cdot)}\right]^{1-\varepsilon_w}\right\}^{\frac{1}{1-\varepsilon_w}}.$$

For log utility $\widetilde{u}(\cdot) = \ln(\cdot)$ in Dixit-Stiglitz, and $\widetilde{u}(\cdot) = \frac{(1+\nu)(\varepsilon-1)-1}{(1+\nu)(\varepsilon-1)}\ln(\cdot)$ in directed search, we have

$$V_e(e, \overline{p}) = V_e(e, \{\overline{p}, \overline{q}\}) = \frac{1}{e},$$

which implies that

$$\overline{\overline{W}}(M^s, W_-, p_-) = \left\{\theta_w W_-^{1-\varepsilon_w} + (1-\theta_w)\left(\frac{\varepsilon_w}{\varepsilon_w - 1}\frac{1+\iota}{\iota}M^s\right)^{1-\varepsilon_w}\right\}^{\frac{1}{1-\varepsilon_w}}.$$

Definition

An equilibrium is a set of functions $\{e^*, W^*, p^*, q^*\}$ on (M^s, W_-, p_-) s.t.

$$
\begin{aligned}
e^*(M^s, W_-, p_-) &= \overline{\overline{e}}(M^s, W_-, p_-), \\
W^*(M^s, W_-, p_-) &= \overline{\overline{W}}(M^s, W_-, p_-), \\
p^*(M^s, W_-, p_-) &= \overline{p}\left(e^*(M^s, W_-, p_-), W^*(M^s, W_-, p_-), p_-\right), \\
q^*(M^s, W_-, p_-) &= \overline{q}\left(e^*(M^s, W_-, p_-), W^*(M^s, W_-, p_-), p_-\right).
\end{aligned}
$$

Note that this definition we are combining the equilibrium that ignores search with the competitive search equilibrium, so that all equilibrium conditions can be combined. We only focus on the smallest relevant fixed point problem that only contains 4 objects.

Proposition

The equilibrium objects $\{e^*, W^*, p^*, q^*\}$ on (M^s, W_-, p_-) solve

$$0 = \iota M^s - e^*,$$

$$0 = \left\{ \theta_w W_-^{1-\varepsilon_w} + (1-\theta_w) \left(\frac{\varepsilon_w}{\varepsilon_w - 1} \frac{1+\iota}{\iota} M^s \right)^{1-\varepsilon_w} \right\}^{\frac{1}{1-\varepsilon_w}} - W^*,$$

$$0 = \frac{\varepsilon W^*}{p^* \psi^f(q^*)} - \frac{\varepsilon - 1}{1 - \mathcal{E}(q^*)} - \chi_p \left(\frac{p^*}{p_-} \right) \frac{p^*}{p_-},$$

$$0 = \frac{e^*}{p^*} - \frac{(\varepsilon - 1)\zeta {q^*}^{1+\nu}}{\psi^f(q^*)^{\frac{1}{\varepsilon - 1}}}.$$

Note that $\varepsilon > 1$ (demand elasticity), $\zeta > 0$ (slope of shopping disutility), $\nu \geq 0$ (curvature of shopping disutility), $\psi^f(\cdot)$ is concave and bounded above by 1, and $\mathcal{E}(q) \equiv \frac{q\psi^{f\prime}(q)}{\psi^f(q)}$. If $(1+\nu)(\varepsilon - 1) > 1$, search friction disappears as $\zeta \to 0$.

Definition

A non-inflationary (general) equilibrium is a set of functions $\{e^*, W^*, p^*, q^*\}$ on (M^s, W_-, p_-) such that

$$W^*(M^s, W_-, p_-) = W_-,$$
$$\chi_p\left(\frac{p^*(M^s, W_-, p_-)}{p_-}\right)\frac{p^*(M^s, W_-, p_-)}{p_-} = 0.$$

Corollary

For $\forall p_- > 0$, $\exists W_-, M^s > 0$ such that the non-inflationary general equilibrium uniquely exists. $\frac{W_-}{p_-}$ needs to be the equilibrium real wage under no nominal rigidities, while $\frac{M^s}{p_-}$ needs to be the corresponding real expenditure.

Corollary

Use $_{SS}$ to denote the corresponding objects in a non-inflationary equilibrium. Log-linearization around it yields equilibrium objects $\{\widehat{e}, \widehat{W}, \widehat{p}, \widehat{\mathcal{I}}\}$ solving

$$\begin{aligned}
\widehat{e} &= \widehat{M}^s, \\
\widehat{W} &= (1-\theta_w)\widehat{M}^s, \\
\widehat{p} &= -\frac{\varepsilon - 1}{\kappa_{SS}(1-\mathcal{E}_{SS})}(\widehat{p} + \widehat{\mathcal{I}} - \widehat{W} - \gamma_{SS}\widehat{\mathcal{I}}), \\
\widehat{\mathcal{I}} &= \left(\frac{1+\nu}{\mathcal{E}_{SS}} - \frac{1}{\varepsilon - 1}\right)^{-1}(\widehat{e} - \widehat{p}).
\end{aligned}$$

The term in blue is mark-up, and in red is desired mark-up.
$\mathcal{E}_{SS}$ denotes the non-inflationary level of $\mathcal{E}(q^*)$,
κ_{SS} denotes the non-inflationary slop of $\chi_p(\cdot)$, and
γ_{SS} denotes the non-inflationary elasticity of $1 - \mathcal{E}(q^*)$ w.r.t. $\mathcal{I}^* \equiv \psi^f(q^*)$.
So far, there is no need to know the functional forms of $\chi(\cdot)$ or $\psi^f(\cdot)$.

- Consider the log-linearized pricing equation

$$\widehat{p} = -\frac{\varepsilon - 1}{\kappa_{SS}(1 - \mathcal{E}_{SS})}(\widehat{p} + \widehat{\mathcal{I}} - \widehat{W} - \gamma_{SS}\widehat{\mathcal{I}}).$$

  - standard mark-up channel: $\widehat{p} + \widehat{\mathcal{I}} - \widehat{W}$ (mark-up$\uparrow \implies$ inflation$\downarrow$),
  - desired mark-up channel: $\gamma_{SS}\widehat{\mathcal{I}}$ (desired mark-up$\uparrow \implies$ inflation$\uparrow$),
  - mark-up$\uparrow \cap$ inflation$\uparrow$ happens only if the second channel dominates.
  - Denote $\frac{1-\theta_p}{\theta_p} \equiv \frac{\varepsilon-1}{\kappa_{SS}(1-\mathcal{E}_{SS})} > 0$ to bridge Calvo pricing and Rotemberg pricing. When search is turned off, this equation is identical to Calvo with rigidity θ_p.

- Consider the log-linearized matching equation

$$\widehat{\mathcal{I}} = \left(\frac{1+\nu}{\mathcal{E}_{SS}} - \frac{1}{\varepsilon - 1}\right)^{-1}(\widehat{e} - \widehat{p}).$$

  - $\Psi_{SS} \equiv \left(\frac{1+\nu}{\mathcal{E}_{SS}} - \frac{1}{\varepsilon-1}\right)^{-1} > 0$ to capture the endogenous productivity channel.
  - shopping disutility: ν (dampening Ψ_{SS}),
  - matching elasticity: $\mathcal{E}_{SS}$ (amplifying Ψ_{SS}),
  - variety preference: $\frac{1}{\varepsilon-1}$ (amplifying Ψ_{SS}).

Corollary

The solution of $\{\widehat{e}, \widehat{W}, \widehat{p}, \widehat{\mathcal{I}}\}$ is

$$\widehat{e} = \widehat{M}^s,$$
$$\widehat{W} = (1-\theta_w)\widehat{M}^s,$$
$$\widehat{p} = \left[1 - \frac{1-(1-\theta_p)(1-\theta_w)}{1-(1-\theta_p)(1-\gamma_{SS})\Psi_{SS}}\right]\widehat{M}^s,$$
$$\widehat{\mathcal{I}} = \frac{1-(1-\theta_p)(1-\theta_w)}{1-(1-\theta_p)(1-\gamma_{SS})\Psi_{SS}}\Psi_{SS}\widehat{M}^s.$$

The solution for real expenditure, real wage and mark-up is

$$\widehat{e} - \widehat{p} = \frac{1-(1-\theta_p)(1-\theta_w)}{1-(1-\theta_p)(1-\gamma_{SS})\Psi_{SS}}\widehat{M}^s,$$
$$\widehat{W} - \widehat{p} = \left[\frac{1-(1-\theta_p)(1-\theta_w)}{1-(1-\theta_p)(1-\gamma_{SS})\Psi_{SS}} - \theta_w\right]\widehat{M}^s,$$
$$\widehat{p} + \widehat{\mathcal{I}} - \widehat{W} = \left[\frac{1-(1-\theta_p)(1-\theta_w)}{1-(1-\theta_p)(1-\gamma_{SS})\Psi_{SS}}(\Psi_{SS}-1) + \theta_w\right]\widehat{M}^s.$$

Proposition

Conditioning on monetary expansion $\widehat{M}^s > 0$ (given $\theta_p, \theta_w \in (0,1)$),

$$\widehat{e} - \widehat{p} > 0 \iff (1-\theta_p)(1-\gamma_{SS})\Psi_{SS} < 1.$$

Conditioning on $\widehat{M}^s > 0$ and $\widehat{e} - \widehat{p} > 0$,

$$\widehat{\mathcal{I}} > 0 \iff \Psi_{SS} > 0,$$
$$\widehat{W} - \widehat{p} > 0 \iff (1-\gamma_{SS})\Psi_{SS} < \frac{\theta_p(1-\theta_w)}{(1-\theta_p)\theta_w},$$
$$\widehat{p} > 0 \iff (1-\gamma_{SS})\Psi_{SS} < 1-\theta_w,$$
$$\widehat{p} + \widehat{\mathcal{I}} - \widehat{W} > 0 \iff [1-(1-\theta_p)(1-\gamma_{SS}\theta_w)]\Psi_{SS} > \theta_p(1-\theta_w).$$

When endogenous productivity Ψ_{SS} or endogenous desired mark-up γ_{SS} is off,

$$\Psi_{SS} = 0 \implies \widehat{\mathcal{I}} = 0 \implies (\widehat{p} + \widehat{\mathcal{I}} - \widehat{W} > 0) \cap (\widehat{W} - \widehat{p} > 0) = \varnothing,$$
$$\gamma_{SS} = 0 \implies (\widehat{p} + \widehat{\mathcal{I}} - \widehat{W} > 0) \cap (\widehat{p} > 0) = \varnothing.$$

- Consider a quadratic function for adjustment cost

$$\chi\left(\frac{p}{p_-}\right) = \frac{\kappa}{2}\left(\frac{p}{p_-} - 1\right)^2.$$

This implies that $\kappa_{SS} = \kappa$.

- Consider a bounded matching function following den Haan et al. (2000)

$$\psi^f(q) = \left(1 + q^{-\gamma}\right)^{-\frac{1}{\gamma}}.$$

This implies $1 - \mathcal{E}(q) = \psi^f(q)^\gamma = \mathcal{I}^\gamma$ and $\gamma_{SS} = \gamma$.

Corollary

When $\gamma = 1$, we have $\widehat{p} = -\frac{1-\theta_p}{\theta_p}(\widehat{p} - \widehat{W})$, which is identical to the equation with no search. However, mark-up with no search $\widehat{p} - \widehat{W}$ is countercyclical, while mark-up with search $\widehat{p} + \widehat{\mathcal{I}} - \widehat{W}$ can possibly be procyclical.

- Three parameters on directed search: ζ for the slop of shopping disutility, ν for the curvature of it, and γ for the curvature of matching function.
- Two equilibrium channels affected by these three parameters: Ψ_{SS} for the endogenous productivity and γ_{SS} for the endogenous desired mark-up.
- The endogenous desired mark-up channel operating via the following equation

$$\widehat{p} = -\frac{1-\theta_p}{\theta_p}(\widehat{p} + \widehat{\mathcal{I}} - \widehat{W} - \gamma_{SS}\widehat{\mathcal{I}}), \text{ in which } \gamma_{SS} = \gamma$$

  does not allow us to separately identify γ and θ_p.

- The endogenous productivity operating via the following equation

$$\widehat{\mathcal{I}} = \Psi_{SS}(\widehat{e} - \widehat{p}), \text{ in which } \Psi_{SS} = \left(\frac{1+\nu}{1-\mathcal{I}_{SS}^{\gamma}} - \frac{1}{\varepsilon - 1}\right)^{-1}.$$

  only allows us to identify one parameter in $\{\zeta, \nu, \gamma\}$. We choose ζ to target on $\mathcal{I}_{SS}$, and $\nu = 0$ to reduce the degree of freedom. γ is to target on Ψ_{SS}.

Upper Bound for γ

- The optimal problem with GHH utility function is valid only if

$$(1+\nu)(\varepsilon-1)-1>0.$$

- This implies that

$$\frac{\varepsilon-1}{\varepsilon}>\frac{1}{\nu+2}.$$

- Denote τ as mark-up. The non-inflationary equilibrium objects satisfy

$$\frac{\mathcal{I}_{SS}^{\gamma}}{1+\tau_{SS}}=\frac{\varepsilon-1}{\varepsilon}>\frac{1}{\nu+2}.$$

- As $\mathcal{I}_{SS}\in(0,1)$, the condition above cannot hold if γ is too large.

- Following the estimation of Christiano et al. (2016), we choose $\theta_p = \theta_w = \frac{3}{4}$.
- Choose $\nu = 0$ and $\gamma = 2$ to get strong enough search friction.
- Choose ζ such that $\mathcal{E}_{SS} = \frac{3}{8}$, and hence $\mathcal{I}_{SS} = (1 - \mathcal{E}_{SS})^{\frac{1}{2}} = 79\%$.
- Choose $\varepsilon = \frac{5}{2}$ such that $\frac{p_{SS}\mathcal{I}_{SS}}{W_{SS}} - 1 = \frac{1}{9}$, which ultimately implies that $\Psi = \frac{1}{2}$.
- Now we can get

$$\left(\frac{\widehat{M}^s}{\widehat{M}^s - \widehat{p}}, \frac{\widehat{\mathcal{I}}}{\widehat{M}^s - \widehat{p}}, \frac{\widehat{W} - \widehat{p}}{\widehat{M}^s - \widehat{p}}, \frac{\widehat{p}}{\widehat{M}^s - \widehat{p}}, \frac{\widehat{p} + \widehat{\mathcal{I}} - \widehat{W}}{\widehat{M}^s - \widehat{p}}\right) = \left(\frac{6}{5}, \frac{1}{2}, \frac{1}{10}, \frac{1}{5}, \frac{2}{5}\right).$$

- This implies that 1% real money supply (real expenditure) increase can be induced by 1.2% nominal money supply (monetary expansion), which induces productivity increase by 0.5%, real wage 0.1%, price 0.2%, mark-up 0.4%.
- This is an example for

money supply $\uparrow \Longrightarrow$ real expenditure $\uparrow \cap$ real wage $\uparrow \cap$ inflation $\uparrow \cap$ mark-up $\uparrow$.

Medium Scale DSGE

Outline

We exam the model performance in an estimated medium scale DSGE model:

- **Baseline model**:
  - **Model**: the baseline model of Christiano et al. (2016) except (1) Rotemberg pricing instead of Calvo pricing, (2) no government spending.
  - **Purpose**: tractable when introducing directed search.
- **Directed search**:
  - **Model**: consumption and investment goods produced by aggregated varieties which need to be found in goods market with directed search friction.
  - **Purpose**: tractable New Keynesian Phillips Curve.
- **Quantitative work**:
  - **Structural VAR**: Christiano et al. (2016) without labor search variables.
  - **Impulse response matching**: target on the same set of moments (9 variables and 3 shocks) as the baseline of Christiano et al. (2016).
  - **Untargeted moments**: (1) labor productivity (2) labor share (inverse mark-up).
  - **Estimation**: Bayesian estimation for both baseline and directed search models.

Goal: to show that directed search improves the match of mark-up (labor share) cyclicality under monetary shocks without hurting other parts of the model.

Impulse Response Matching

- Following Christiano et al. (2016), we estimate 14 parameters on
    - curvature of matching function, capital share,
    - Frisch elasticity of labor supply, consumption habit,
    - cost of capacity utilization, investment adjustment, and Rotemberg pricing,
    - Taylor rule parameters,
    - shock standard deviation and persistence,
- to match 9 SVAR impulse responses of
    - real GDP, real consumption, real investment, hours worked,
    - capacity utilization, relative price of investment,
    - real wage, inflation, and Fed Fund Rates.
- under the following 3 structural shocks
    - monetary shock
    - neutral technology shock
    - investment specific technology shock

Modeling Directed Search with Capital

- Households exert shopping effort $\{d_t(p,q)\}$ in the whole submarket $\{p,q\}$ to find and purchase $\{y_t(p,q)\}$ of each goods variety to produce y_t^A via

$$y_t^A = \left(\int_{\{p,q\}\in\Phi_t} d_t(p,q)\psi^h(q) y_t(p,q)^{\frac{\varepsilon-1}{\varepsilon}} dpdq \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1,$$

$$d_t^A = \int_{\{p,q\}\in\Phi_t} d_t(p,q) dpdq,$$

- Households use unconsumed output $y_t^A - c_t^A$ to produce investment goods i_t^A, and maintain capital utilization u_t^k at cost $a(u_t^k)k_{t-1}$ via the following technology with z_t^i denoting the level of investment specific technology

$$i_t^A + a(u_t^k)k_{t-1} = z_t^i(y_t^A - c_t^A),$$

- Households install capital k_t via a technology with adjustment cost $S(\cdot)$

$$k_t = (1-\delta_k)k_{t-1} + \left[1 - S\left(\frac{i_t^A}{i_{t-1}^A}\right)\right] i_t^A.$$

- Capital cannot be sold, but $u_t^k k_{t-1}$ is rented to firms at gross nominal rate R_t^k.

- Households' nominal expenditure is

$$e_t = \int_{\Phi_t} d_t(p,q)\psi^h(q)p\, y_t(p,q)dpdq.$$

- Denote W_t as nominal wage, L_t as labor supply, u_t^k as utilization, R_t as gross federal funds rate, b_t as nominal bond position, and Π_t^f as nominal transfer of firm profits. Then, households' budget constraint is

$$e_t \leq W_t L_t + R_t^k u_t^k k_{t-1} + R_{t-1} b_{t-1} - b_t + \Pi_t^f.$$

- Households' utility function combines internal habit, GHH shopping disutility, and additively separable labor disutility.

- We first focus on the problem with given $W_t L_t$, in which households choose $\{y_t(p,q).d_t(p,q), y_t^A, c_t^A, i_t^A, u_t^k, k_t, b_t\}$. This allows us to get a tightness and demand function $\{q_t^h(p), y_t^h(p)\}$ as what we did before.

- We specify the standard Calvo wage problem on top of that (omitted).

- Each of the firms operates a continuum of locations.
- Firm $j \in [0, 1]$ uses capital $k_{j,t}$ (from effective capital stock $u_t^k k_{t-1}$) and labor $\ell_{j,t}$ to produce goods variety $y_{j,t}$ at each of its locations.
- For comparison with Christiano et al. (2016), we also assume that firms need to take a within period loan at nominal interest rate R_{t-1} to pay labor cost.
- The cost minimization problem with neutral technology level z_t^n is

$$\max_{\{k_{j,t}, \ell_{j,t}\}} \left\{ -R_t^k k_{j,t} - R_{t-1} W_{j,t} \ell_{j,t} \right\}, \quad \text{s.t.} \quad k_{j,t}^{\alpha} (z_t^n \ell_{j,t})^{1-\alpha} \geq y_{j,t}.$$

- Denote λ_t^f as the Lagrange multiplier on constraint. We have

$$\lambda_{j,t}^f = \frac{R_{t-1}}{1-\alpha} \frac{W_t \ell_{j,t}}{k_{j,t}^{\alpha} (z_t^n \ell_{j,t})^{1-\alpha}}.$$

Markup

- For a price $p_{j,t}$, the matching probability of each locations is $\psi^f[q_t^h(p_{j,t})]$.
- Goods produced in unmatched locations does not get sold and is perished.
- The marginal cost of inputs for one additional unit of goods variety getting sold is $\frac{\lambda^f_{j,t}}{\psi^f[q_t^h(p_{j,t})]}$. Gross markup in theory should be

$$\text{gross markup}_{j,t} = \frac{p_j \psi^f[q_t^h(p_{j,t})]}{\lambda^f_{j,t}} = \frac{1-\alpha}{R_{t-1} \cdot \text{labor share}_{j,t}}.$$

- Whether lower federal funds rate does affect marginal cost or markup in this way is still an open empirical question.
- We use the following measure of markup in both model and data to make our results easy to compare with other work such as Nekarda and Ramey (2019).

$$\text{gross markup}_{j,t} = \frac{1-\alpha}{\text{labor share}_{j,t}}.$$

- Doing so in fact makes it more difficult for our model to match markup data.

- Each firm produces one goods variety but sells it in two separated markets.
    - One market with directed search friction has variety demand $y_t^h(p)$.
    - Another with no search has variety demand $x_t^f(p)$ to cover price adj. cost.
    - Both markets are monopolistically competitive with demand elasticity ε.
    - The price of the same goods variety in two markets are assumed to be the same.

- Price adjustment cost function is $\chi_t\left(\frac{p_t}{p_{t-1}}\right)$. It is a time varying function normalized by the nominal value of gross aggregate output.

- Denote $\{\lambda_t^e\}$ as the marginal value of one dollar for households. Each of the firms with marginal cost $\{\lambda_t^f\}$ chooses prices $\{p_t\}$ to maximize the present value of profits for household owners of the firms

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t \lambda_t^e \left\{ \left[p_t \psi^f(q_t^h(p_t)) - \lambda_t^f \right] y_t^h(p_t) + (p_t - \lambda_t^f) x_t^f(p_t) - \chi_t \left(\frac{p_t}{p_{t-1}} \right) \right\}.$$

λ_t^e can be obtained from households' optimization problem.

- Consistency condition:
  - Market price for goods varieties is consistent with firms' pricing choice.
  - Market tightness in submarkets is consistent with households' shopping decision.
  - Market clearing conditions hold in each of the two goods markets.

- The allows us to obtain New Keynesian Phillips Curve for gross inflation Π_t:

$$(\Pi_t - \Pi_{SS})\Pi_t = \frac{\varepsilon - 1}{\kappa}\left\{\frac{\varepsilon}{\varepsilon - 1}\frac{\lambda_t^f}{p_t} - \left[\widetilde{\chi}_t + (1 - \widetilde{\chi}_t)\mathcal{I}_t^{1-\gamma}\right]\right\} + \beta\mathbb{E}_t\left[\mathcal{M}_{t+1}(\Pi_{t+1} - \Pi_{SS})\Pi_{t+1}\right],$$

  where $\widetilde{\chi}_t$ denotes the fraction of price adjustment costs in gross output (very small), and $\mathcal{M}_{t+1}$ is the stochastic discount factor of households.

- $\mathcal{I}$ is endogenous productivity, and $\frac{p_t\mathcal{I}}{\lambda_t^f} - 1$ is markup.
  - $\mathcal{I}$ is needed to make markup procyclical.
  - $\mathcal{I}$ affects inflation through (1) endogenous productivity $\mathcal{I}$, and (2) endogenous desired markup $\mathcal{I}^{-\gamma}$ in opposite directions. When $\gamma \geq 1$, the latter dominates.

Parameter	Value	Description
	Common in All Models	
δ_k	0.025	Depreciation rate of physical capital
θ_w	0.75	Quarterly frequency of not adjusting nominal wage
ε_w	$\frac{1.2}{1.2-1}$	Labor demand elasticity by contractors
ω	1.0	Log utility
$400 \ln \mu^y_{SS}$	1.7	Annual output per capita growth rate
$400 \ln \mu^k_{SS}$	2.9	Annual investment per capita growth rate
$400(\Pi_{SS} - 1)$	2.5	Annual net inflation rate
$400(R_{SS}/\Pi_{SS} - 1)$	3.0	Annual net real interest rate
	Only in the Model with Directed Search	
ν	0.0	Curvature of shopping disutility

Calibrated Parameters

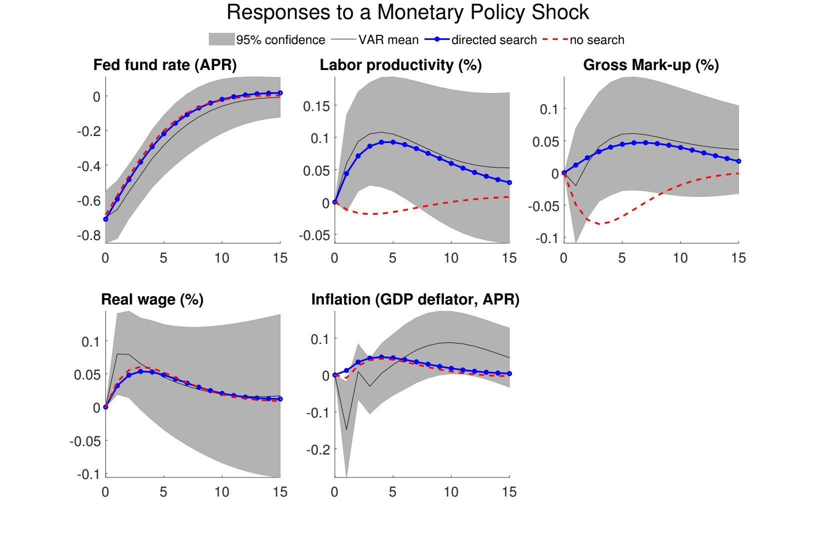
Parameter	CET	no search	directed search	Target
Discount factor β	0.9968	0.9968	0.9968	$400(R_{SS}/\Pi_{SS}-1)=3.0$
Slop of utilization cost σ_b	0.036	0.040	0.040	Utilization $u^k_{SS}=1$
Slop of working disutility η	did not find	0.843	mean=4.247	Labor $L_{SS}=0.945$
Demand elasticity ε	$\frac{1.24}{1.24-1}=5.17$	-	-	Estimated
	-	21	mean=2.63	Mark-up $\tau_{SS}=0.05$
Slop of shopping disutility ζ	-	0.0000	mean=0.4075	Matching prob $\mathcal{I}_{SS}=0.70$

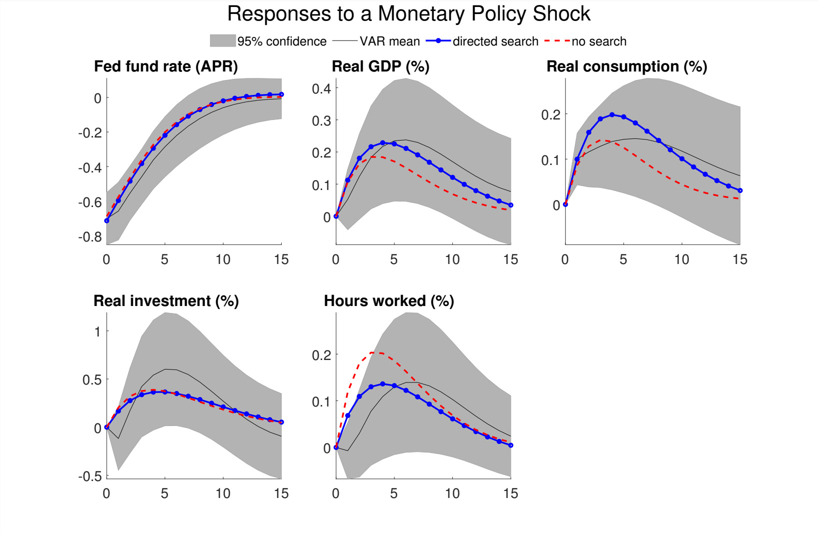
* Parameters $\{\eta, \varepsilon, \eta\}$ are all related to γ, and hence not constant.
* ε is chosen to target on mark-up to avoid negative mark-up.
* ε does not affect model observations in a Calvo pricing model.
* CET refers to the baseline model in Christiano et al. (2016).

Estimated Parameters

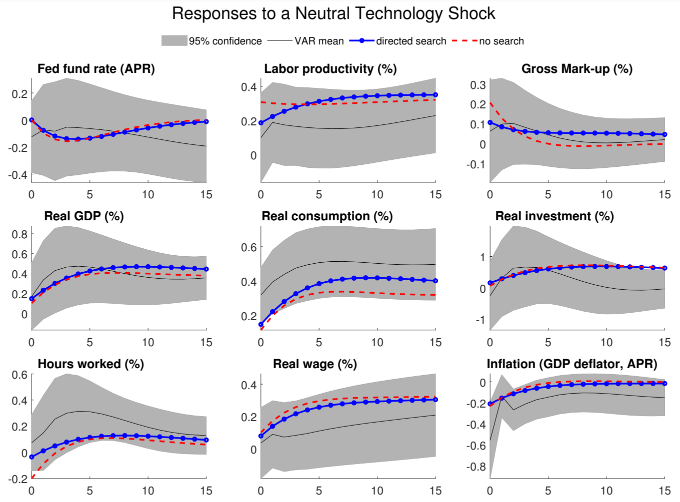
		CET	No Search	Directed Search
	Prior Dist. $\mathcal{D}$,Mode,[2.5-97.5%]		Posterior Dist. Mode,[2.5-97.5%]	
	Preference and Technology Parameters			
Curvature of matching function, γ	$\mathcal{U}$,1.00,[0.53-1.48]			1.22,[1.10-1.32]
Capital Share, α	$\mathcal{B}$,0.33,[0.28-0.38]	0.33,[0.27-0.34]	0.24,[0.21-0.27]	0.25,[0.22-0.28]
Inverse Labor Supply Elasticity, ξ	$\mathcal{G}$,0.94,[0.57-1.55]	0.92,[0.33-1.01]	0.38,[0.28-0.51]	0.50,[0.40-0.62]
Consumption Habit, h	$\mathcal{B}$,0.50,[0.21-0.79]	0.68,[0.65-0.74]	0.76,[0.72-0.79]	0.80,[0.73-0.86]
Capacity Utilization Ajd. Cost, σ_a	$\mathcal{G}$,0.32,[0.09-1.23]	0.03,[0.01-0.16]	1.16,[0.78-1.70]	2.17,[1.50-3.28]
Investment Adjustment Cost, S''	$\mathcal{G}$,7.50,[4.57-12.4]	5.03,[4.15-7.95]	12.5,[9.51-17.0]	14.1,[10.0-17.4]
	Price Stickiness Parameters			
Rotemberg Adjustment Cost, κ	$\mathcal{G}$,139,[5.06-778]		181,[177-185]	
	$\mathcal{G}$,13.9,[0.51-77.8]			33.6,[30.8-37.3]
Calvo Price Stickiness, θ_p	$\mathcal{G}$,0.68,[0.45-0.84]	0.74,[0.67-0.77]		
$\frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}$ **or** $\frac{\varepsilon-1}{\kappa(1-\mathcal{E}_{SS})}$		0.09,[0.07-0.16]	0.11,[0.11-0.11]	0.07,[0.07-0.08]
	Monetary Authority Parameters			
Taylor Rule: Inflation, ϕ_π	$\mathcal{G}$,1.69 [1.42-2.00]	2.02 [1.82-2.39]	1.99 [1.76-2.23]	1.93 [1.69-2.18]
Taylor Rule: GDP, ϕ_y	$\mathcal{G}$,0.08 [0.03-0.22]	0.01 [0.00-0.02]	0.19 [0.12-0.26]	0.15 [0.10-0.20]
Taylor Rule: Smoothing, ρ_R	$\mathcal{B}$,0.76 [0.37-0.94]	0.77 [0.75-0.81]	0.85 [0.83-0.88]	0.86 [0.83-0.88]
	Exogenous Processes Parameters			
Std. Dev. Monetary Policy, $400\sigma_R$	$\mathcal{G}$,0.65 [0.56-0.75]	0.64 [0.57-0.71]	0.67 [0.60-0.74]	0.71 [0.65-0.78]
Std. Dev. Neutral Tech., $100\sigma_n$	$\mathcal{G}$,0.08 [0.03-0.22]	0.32 [0.28-0.35]	0.36 [0.32-0.39]	0.36 [0.32-0.39]
Std. Dev. Invest. Tech., $100\sigma_i$	$\mathcal{G}$,0.08 [0.03-0.22]	0.15 [0.12-0.19]	0.33 [0.27-0.41]	0.32 [0.25-0.39]
AR(1) Invest. Technology, ρ_i	$\mathcal{B}$,0.75 [0.53-0.92]	0.57 [0.44-0.66]	0.49 [0.38-0.59]	0.49 [0.36-0.60]
	Overall Goodness of Fit			
Log Marginal Likelihood (9 Observables):		-	**105.6**	**162.5**

Main Results (puzzle solved)

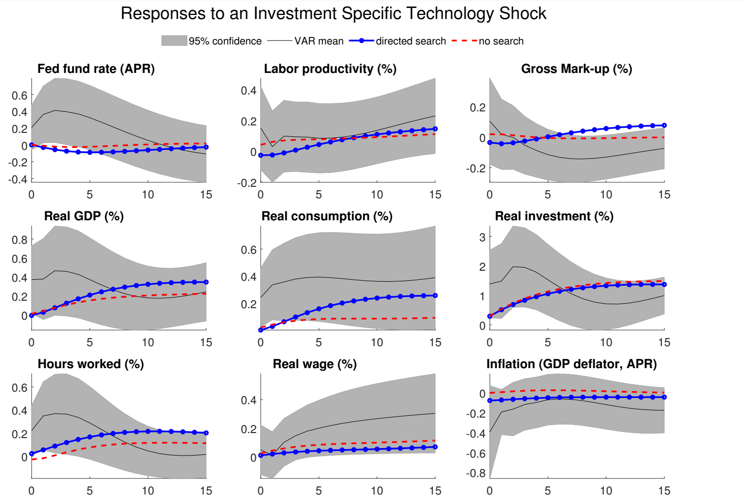



Responses to a Monetary Policy Shock
95% confidence
VAR mean
directed search
no search
Fed fund rate (APR)
Real GDP (%)
Real consumption (%)
Real investment (%)
Hours worked (%)
0
-0.2
-0.4
-0.6
-0.8
0.4
0.3
0.2
0.1
0
0.2
0.1
0
1
0.5
0
-0.5
0.2
0.1
0
0
5
10
15

Other Shocks (equally well)



Other Shocks (equally well)



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