The Macroeconomic Implications of Limited Arbitrage

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Motivations I

- real sector disturbance $\rightarrowtail$ arbitrage crashes: GFC
  - 2007 subprime: collateral value collapse $\rightarrowtail$ arbitrageurs unwind
  - price gap of similar assets $\uparrow$, arbitrage crashes
- arbitrage failure $\rightarrowtail$ real contractions: European banking crisis
  - "carry trade" by Eurozone banks: high-yield GIPSI & low-yield German sovereign bond (Acharya & Steffen (2015))
  - yield diverge — 70% bank losses — firm lending and output plummet
- slow, incomplete recoveries in real and financial sectors
  - mispricing skyrocketed and remained large after crises
  - e.g., violation of CIP, CDS-bond basis

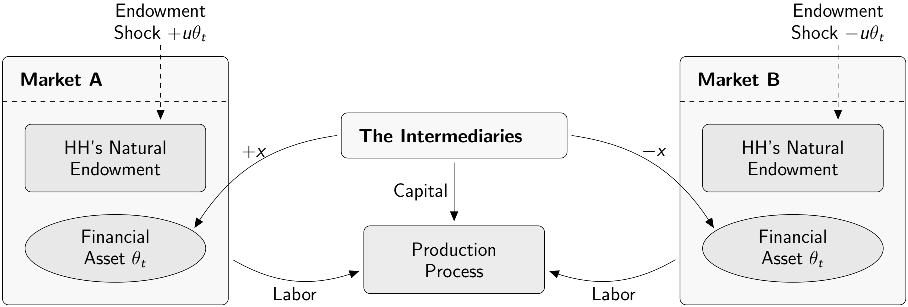
Literature on Financial Frictions and Crises

- finance: limits of arbitrage in financial markets
  - e.g., Vishny & Shleifer (1997), Gromb & Vayanos (2002, 2018), Krishnamurthy (2002), Brunnermeier & Pedersen (2008), Kondor (2009)
- macro: limits of arbitrage in production
  - e.g., Kiyotaki & Moore (1997), Bernanke, Gertler & Gilchrist (1999), Brunnermeier & Sannikov (2014), Kiyotaki & Gertler (2015)
- links between arbitrage trading & macroeconomy, role in crises
  - ???

Overview

- unified and tractable framework
  - link real investments & mispricing in segmented markets
- macroeconomic impacts of limited arbitrage
  - boost aggregate investments and output
  - increase systemic risk
- analytical solutions to multiple equilibria
  - regime shifts: crisis & policy indications
  - slow & incomplete recovery from Great Recession

Baseline Model



Households

- HH's natural endowment

$$y_{i,t} = b + u_i\theta_t, \quad i \in \{\mathrm{A}, \mathrm{B}\}, \quad t \in \{1, 2, \ldots\}$$

  - θ_t follows a symmetric distribution around zero on $\left[-\bar{\theta}, \bar{\theta}\right]$
  - shock intensities: $u_A = -u_B =: u$

- opposite shocks, opposite hedging demand

Intermediaries

- both arbitrageurs and entrepreneurs
  - take identical but opposite positions $x_{\mathrm{A},t} = -x_{\mathrm{B},t} = x_t$
  - convert perishable goods one-to-one into durable goods
  - invest capital & hire HH as labor

$$\begin{aligned} Y_t &= F(K_{t-1}) + (1-\delta)K_{t-1} \\ &= a\, K_{t-1}^{\alpha} L^{1-\alpha} + (1-\delta)K_{t-1} \end{aligned}$$

Financial Assets

- Gromb and Vayanos (2002, 2017)
  - long-lived, in zero net supply
  - settlement of previous positions: $x_{t-1}(P_t^A - P_t^B)$
  - IM's liability—net payment from IM to HH

Collateral Constraints

- post capital input as collateral
  - cover IM's next period liability in case of default
  - depreciated capital as limit: $(1 - \delta)K_t$
- real-world securitization
  - securitized products as collateral

IM's Optimization Problem

$$\max_{c_s^{\text{IM}}, x_s, K_s} \mathbb{E}\left[\sum_{s=t}^{\infty} \rho^s \log\left(c_s^{\text{IM}}\right)\right],$$

subject to

$$c_t^{\text{IM}} + K_t = \underbrace{-x_{t-1}(P_t^B - P_t^A)}_{\text{obligation}} + \underbrace{x_t(P_t^B - P_t^A)}_{\text{arbitrage gain}} + F(K_{t-1}) + (1-\delta)K_{t-1},$$

$$\underbrace{-x_t(P_{t+1}^B - P_{t+1}^A)}_{\text{next period obligation}} + (1-\delta)K_t \geq 0.$$

HH's Optimization Problems

$$\max_{c_s^i, y_s^i} \mathbb{E}\left[\sum_{s=t}^{\infty} \beta^s \log\left(c_s^i\right)\right], \quad i \in \{\text{A, B}\},$$

subject to

$$c_t^i = \underbrace{y_{t-1}^i(P_t^i + \theta_t) - y_t^i P_t^i}_{\text{income from trading assets}} + \underbrace{a(1-\alpha)K_{t-1}^{\alpha} L^{-\alpha}}_{\text{labor income}} + \underbrace{(b + u_i \theta_t)}_{\text{endowment}}.$$

Market Liquidity & Mispricing in Equilibrium

Market Liquidity & Mispricing In Equilibrium

- $\rho > \bar{\rho}$, patient IM
  - full liquidity, no price discrepancy.
  - neoclassical growth model with frictionless financial markets
- $0 < \rho \leq \bar{\rho}$, impatient IM, collateral constrained
  - mispricing with limited arbitrage

$$x_t \in (0, u) \quad \text{and} \quad \phi_t =: P_t^B - P_t^A = \frac{(1-\delta)K_{t-1}}{x_{t-1}} > 0.$$

Dynamics with Binding Constraints I

Dynamics of IM's Wealth, Capital Accumulation and Consumption

Under binding collateral constraints, IM's consumption and capital evolves according to

$$C_t = (1 - \alpha\rho) W_t, \quad K_t = \alpha\rho W_t S_t.$$

where W_t is IM's wealth at the beginning of t,

$$W_t := F(K_{t-1}) + (1-\delta)K_{t-1} - x_{t-1}\phi_t = F(K_{t-1})$$

and the leverage ratio: $S_t := \dfrac{\phi_{t+1}}{\phi_{t+1} - (1-\delta)\phi_t} > 1.$

Dynamics with Binding Constraints II

- arbitrage gain serves as leverage to production
  - $K_t = \alpha\rho W_t + x_t\phi_t = \alpha\rho W_t S_t$
  - negative interest loan to IM
  - loan: immediate arbitrage gains
  - repayment: next period obligated settlement
- capital's collateral premium, marginal return $\uparrow$

Steady States With Binding Collateral Constraints

- steady states: $K_t = K^*$, $x_t = x^*$, $\phi_t = \phi^*$
- collateral premium boosts capital: $K^* = F'^{-1}\left(\frac{\delta}{\rho}\right) > F'^{-1}\left(\frac{1}{\rho}\right)$
  - depreciation δ, inverse measure of collateral value
- fixed "loan" size: $x^*\phi^* = x_t\phi_t = x_{t-1}\phi_t$
  - zero-interest, roll over infinitely

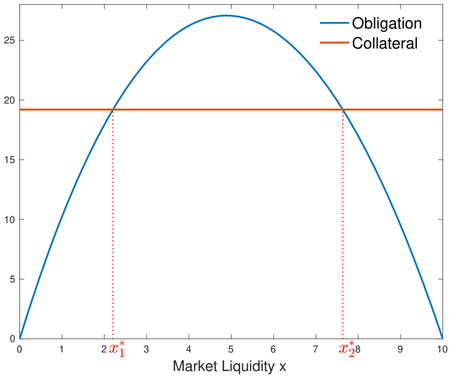
Steady States With Binding Collateral Constraints

- binding collateral constraints

$$\underbrace{(1-\delta)\,K^*}_{\text{collateral value}} = \underbrace{x^*\phi^*}_{\text{obligation}}$$

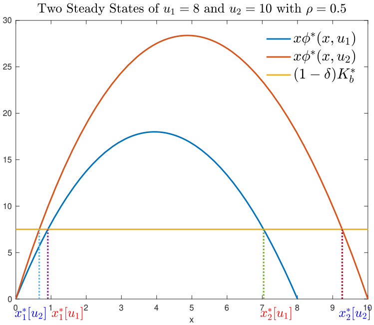
- trading volume $x^* \uparrow$, price gap $\phi^* \downarrow$
- given unique K^*, 2 equilibria: bad vs good regime
  - small (big) trading vol x^*, large (small) price gap ϕ^*
  - market microstructure: transaction costs, market-making rebate; collateral policy: re-use limits, eligibility scope, velocity, etc
  - heavily (lightly) regulated trading environment

Two Steady States with Binding Collateral Constraints



- IM indifferent: $C^*_{\text{IM}} = (1 - \alpha\rho)F(K^*)$
- HH prefers the good regime
  - higher trading volume x^*, better risk sharing

Comparative Statics



Multiple Equilibria and Asset Demand u

All else equal, shock intensity $u_1 < u_2$, binding collateral constraint:

- $K^*[u_1] = K^*[u_2]$;
- $x_1^*[u_1] > x_1^*[u_2]$, $\phi_1^*[u_1] < \phi_1^*[u_2]$;
- $x_2^*[u_1] < x_2^*[u_2]$, $\phi_2^*[u_1] > \phi_2^*[u_2]$

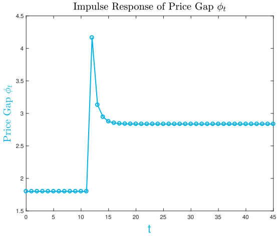
Crises from Regime Shifts

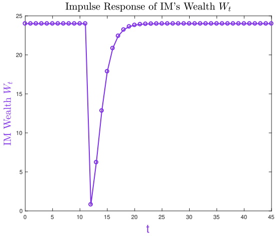
- regime shifts
  - sudden changes in regulation, trading platform, market sentiment, macro/micro factors, etc
- crises arise when shifting from good to bad
  - price gap widens to fit the bad regime
  - large initial positions inherited from the good
  - financial distress or insolvency

Crisis Scenario & Incomplete Recovery I

Markets panic at the good regime :

(i) immediate reaction





- price gap ↑ & big initial position → IM's obligation ↑
- financial distress → K ↓ & liquidity ↓

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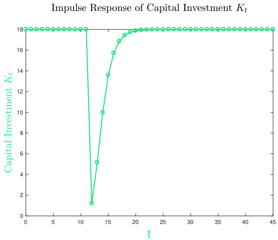

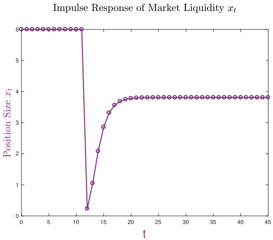

- price gap ↑ & big initial position → IM's obligation ↑
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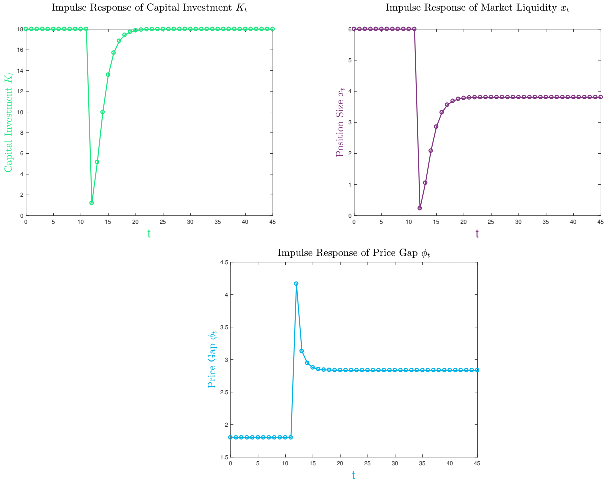
Crisis Scenario & Incomplete Recovery I

Markets panic at the good regime :

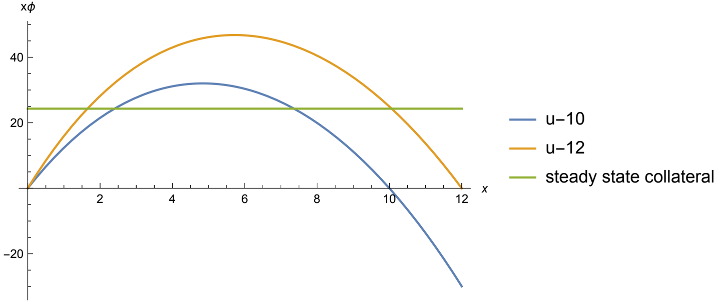
(ii) long term



- IM: slowly recovered; HH: slow & incomplete recovery

Crisis from Regime Shifts II

- crises unavoidable even when switching to a good regime
  - as long as new regime features a bigger price gap
  - example: sudden drop in asset demand u



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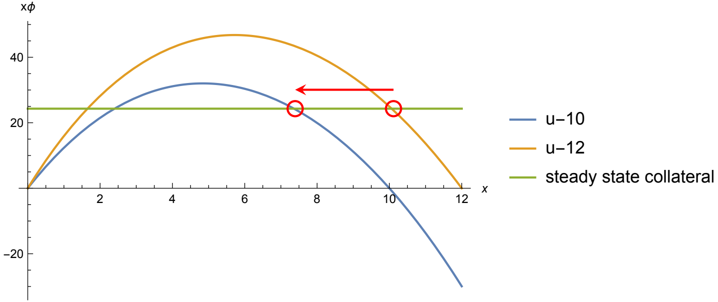
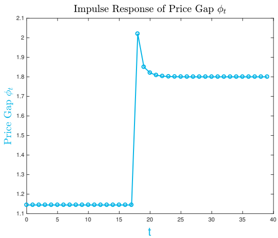
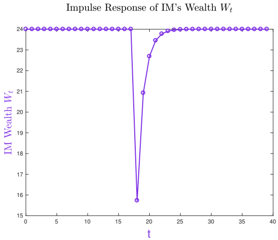


Crisis Scenario & Incomplete Recovery II

switch to a good regime

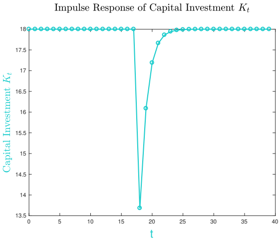


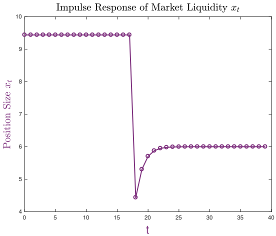


- price gap $\phi_t \uparrow$ & big initial position $x_{t-1} \rightarrow$ IM's liability $x_{t-1}\phi_t \uparrow$
- financial distress $\rightarrow$ K $\downarrow$ & liquidity $\downarrow$, crisis unavoidable

Crisis Scenario & Incomplete Recovery II

switch to a good regime





- price gap $\phi_t \uparrow$ & big initial position $x_{t-1} \rightarrow$ IM's liability $x_{t-1}\phi_t \uparrow$
- financial distress $\rightarrow$ K $\downarrow$ & liquidity $\downarrow$, crisis unavoidable

Policy Trade-off

Welfare vs vulnerability

Given the sudden shock & post-shock regime, the bad-regime economy is (weakly) better off than the good one, with higher post-shock K_t and liquidity x_t before converging to new steady states.

- good regime
  - more vulnerable to systemic risk
  - more negative impact on real sectors and liquidity supply
- policy trade-off: bad to good regime
  - pareto improvement: liquidity, risk sharing & price discovery
  - financial instability, slow recovery & severe contagion to real sectors

Take-away

- interactions of arbitrage and real activities boost production
  - by giving capital investment extra collateral premium
- also increase systemic risks
  - regime shifts trigger crises
- may derail full & fast recoveries
  - policy trade-off