The Pricing of the Illiquidity Factor's Conditional Risk with Time-varying Premium

Yakov Amihud*
NYU-Stern School of Business
Joonki Noh**
Case Western Reserve University—Weatherhead School of Management

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* Ira Leon Rennert Professor of Finance. New York University Stern School of Business, 44 West 4th Street, New York, NY 10012. Tel. (212) 998-0720. Email: yamihud@stern.nyu.edu.
** Assistant Professor of Banking and Finance, Weatherhead School of Management, Case Western Reserve University, Cleveland, OH 44106. Tel. (216) 368-3737. E-mail: joonki.noh@case.edu.

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Abstract

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1. Introduction

Illiquidity is known to be priced both as a characteristic and as a systematic risk.[1] This paper studies the pricing of the *conditional* systematic risk of a *traded* liquidity premium factor of Illiquid-Minus-Liquid stock portfolios, denoted *IML*, when its risk premium is *time-varying* depending on financial distress. The mean risk-adjusted *IML* is about 4% per year and significant for our sample period of January 1947 through December 2017.[2] Notably, our conditional risk of *IML*, denoted β_{IML}, is different from the illiquidity risks studied by Pastor and Stambaugh (2003), Acharya and Pedersen (2005), and Watanabe and Watanabe (2008), denoted β_{IL}. *IML* is a traded return factor representing the return premium on illiquidity while these three studies use non-traded factors of innovations in the level of market illiquidity. To illustrate the difference between these two βs, consider the standard CAPM. Our β_{IML} is analogous to the β of the risk premium factor of market excess return *RMrf* while β_{IL} is analogous to the β of innovations in the level of market volatility (risk) studied by Ang, Hodrick, Xing, and Zhang (2006).

We hypothesize that investors demand a positive premium on the *conditional* systematic risk of *IML* that *varies* as a function of market conditions, being higher in times of financial distress. Our hypothesis follows from Brunnermeier and Pedersen's (2009) theory that higher funding illiquidity and binding financial conditions raise both market illiquidity and the shadow price of liquidity. Investors who become financially constrained must liquidate their holdings and are willing to bear higher costs of illiquidity or pay more for liquidity. This results in higher illiquidity – the price impact in liquidation – and in higher shadow price of liquidity.[3] We propose that investors thus demand a risk premium on stocks with greater exposure to – or β of – the illiquidity premium factor *IML* in times of financial distress and funding constraint, when both illiquidity and its shadow price are higher. This prediction is analogous to the theory and findings of Lettau and Ludvigson (2001, henceforth LL) that investors price the conditional β of the market factor in times of higher risk or risk aversion captured by higher consumption/asset

[1] Amihud and Mendelson (1986) and Brennan and Subrahmanyam (1996) find that stock expected return increases in stock illiquidity and Pastor and Stambaugh (2003) and Acharya and Pedersen (2005) find that the expected return is an increasing function of illiquidity systematic risk unconditionally, using a non-traded illiquidity factor. See a review of research on the pricing of liquidity and liquidity risk in Amihud, Mendelson, and Pedersen (2005, 2013).

[2] The *IML* was found to be positive and significant and to co-move strongly across 45 countries (Amihud, Hameed, Kang, and Zhang, 2015).

[3] Acharya, Amihud, and Bharath (2013) find that illiquidity shocks affect corporate bond returns in times of financial and economic distress more strongly.

ratio denoted *cay*. Testing our hypothesis, we follow LL in conditioning both *IML* risk and its risk premium on financial distress, allowing them to change over time.

Specifically, following Cochrane (1996, 2005) and LL,[4] we estimate and test a cross-sectional model of conditional stock expected return as a function of conditional β_{IML} using the *IML* factor that is scaled by lagged instrumental variable which summarizes the investors' conditioning information on financial distress. Our conditioning variable is the yield spread between BAA- and AAA-rated corporate bonds, denoted *SP*, a known proxy for financial distress which we find to forecast our key factor *IML*. This approach allows both conditional β_{IML} and its risk premium to vary as functions of the state of the market, being higher in times of anticipated financial distress.

Our main finding is that the *conditional* β_{IML} is positively and significantly priced in the cross-section of stock returns, meaning that expected returns are higher for stocks with greater exposure to the illiquidity factor *IML* in times of financial distress when investors' aversion to illiquidity and thus its risk premium is higher. In contrast, the *unconditional* β_{IML} (with a constant risk premium) is not significantly priced in the cross-section. This finding is similar to those of Cochrane (1996) and LL on the significant pricing of the conditional β of their pricing factors (with conditional risk premiums that are allowed to vary) while the unconditional β is not priced.[5]

Our analysis differs from that of Watanabe and Watanabe (2008) who study the pricing of a conditional illiquidity-based systematic risk of Acharya and Pedersen (2005) and employ the market trading volume as a conditioning variable. First, as pointed out above, we differ in that we study the systematic risk of a traded factor that reflects the illiquidity return premium while Watanabe and Watanabe (2008) employ a non-traded illiquidity factor of the shocks in the level of illiquidity of the market and of individual stocks. Second, while their conditioning variable – aggregate market volume – relates to the market trading condition (which is motivated by investors' preference uncertainty), our conditioning variable indicates financial distress and investors' aversion to illiquidity, based on the theory of Brunnermeier and Pedersen (2009).

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[4] Similar models of the market factor's conditional β using macroeconomic conditioning variables are proposed by Shanken (1990), Ferson and Schadt (1996), Jagannathan and Wang (1996), and Ferson and Harvey (1999). Ferson, Sarkissian, and Simin (2008) show that asset pricing models that employ conditional betas estimated from conditional asset pricing regressions are robust to data mining and that the pricing result is not spurious.

[5] In general, Cochrane (1996, p. 617) concludes that "[T]he scaled factor models typically perform substantially better than the nonscaled factor models."

We highlight the unique significant pricing of the conditional β_{IML} in the cross-section of stock returns: none of the β risks of the factors of Fama and French (1993) and Carhart (1997) (henceforth denoted FFC factors) is significantly priced in a consistent manner. Specifically, we control for the βs of the FFC factors and find no consistent significant pricing for either their unconditional βs or for their conditional βs using financial distress as a conditioning variable, as we do for the *IML* factor. Also, the pricing of the conditional β_{IML} remains significant after controlling for the conditional β risks of commonly used liquidity-based factors – Pastor and Stambaugh (2003), Liu (2006), and shocks to market illiquidity as used by Watanabe and Watanabe (2008) – while none of these factors' conditional β risks is positively and significantly priced.[6]

Notably, our test assets are *individual* stocks instead of the often-used stock portfolios due to the well-known potential pitfalls in using portfolios for testing asset pricing models.[7] Lewellen, Nagel, and Shanken (2010) show that using test portfolios sorted on stock characteristics can impart a strong factor structure across them, providing a hurdle that is too low for testing whether factor risks are priced. Jegadeesh et al. (2019) re-examine prominent asset pricing models and find that the pricing ability of popular factor loadings are disappointing in the cross-section of individual stocks compared to stock characteristics, casting doubt on the pricing of existing popular measures of systematic risk.[8] In addition, the estimated pricing of factor risks may be sensitive to a subjective choice of sorting variables.[9] The main drawback of employing individual stocks as test assets is the well-known bias due to errors-in-variables (EIV). We attend to this problem by employing the EIV bias correction procedure of Litzenberger and Ramaswamy (1979) and Chordia, Goyal, and Shanken (2017).

[6] Our tests also differ from those of Pastor and Stambaugh (2003), Liu (2006), Korajczyk and Sadka (2008), and Watanabe and Watanabe (2008) in that our tests employ individual stocks as test assets while the other tests employ characteristics-sorted portfolios as test assets. In addition, these studies do not allow for time-varying risk premiums except for Watanabe and Watanabe (2008).

[7] For a review of the potential problems in using stock portfolios in tests of asset pricing models, see Gagliardini, Ossola, and Scaillet (2016) and Jegadeesh, Noh, Pukthuanthong, Roll, and Wang (2019).

[8] Recent studies question some earlier studies on the pricing of liquidity-related risks. See Holden and Nam (2019), Kazumori et al. (2019), Li, Novy-Marx, and Velikov (2019), and Pontiff and Singla (2019).

[9] Also, the power of an asset pricing test becomes lowered by the low dimensionality issue that can occur when employing stock portfolios and including a number of systematic risks and characteristics in the model. This problem can hinder identifying priced systematic risks significantly.

The selection of our scaling instrument *SP*, the corporate bond yield spread, follows Cochrane (1996) and LL who chose scaling instruments that forecast their pricing factors.[10] First, we find that *SP* forecasts the illiquidity premium factor *IML*. Second, *SP* is an indicator of financial distress, suitably representing the state of funding illiquidity in Brunnermeier and Pedersen's (2009) theory which instigates a rise in the shadow price of illiquidity. Importantly, the corporate bond yield spread is known to reflect an illiquidity premium in addition to the default premium, see Chen, Lesmond, and Wei (2007), Dick-Nielsen, Feldhutter, and Lando (2012), and Bongaerts, de Jong, and Driessen (2017). The corporate bond yield spread is also found to forecast adverse economic conditions. Fama and French (1989, p. 43) suggest that the default spread is "higher when times are poor". Gilchrist and Zakrajšek (2012) find that the BAA-AAA corporate bond yield spread significantly forecasts adverse economic conditions (increase in unemployment and decline in industrial production). We also employ an alternative conditioning variable that proxies for financial distress and funding constraint: the series of broker-dealers' loans that importantly includes margin loans, which is the basis for the Brunnermeier and Pedersen (2009) analysis, relative to their total loans. The test results are similar for both measures of funding illiquidity: the conditional β_{IML} on funding illiquidity is positively and significantly priced in the cross-section of individual stock returns in times of anticipated financial distress.

Earlier studies on the pricing of conditional liquidity-based systematic risk include Martinez, Nieto, Rubio, and Tapia (2005) for Spanish stock market and Acharya, Amihud, and Bharath (2013) for corporate bond market, both of which use non-traded illiquidity factors. In contrast, in our conditional asset pricing model with time-varying risk premium, the conditional β_{IML} employs a *traded* return factor that captures the return premium on illiquid stocks over liquid ones. Jensen and Moorman (2010) show that monetary expansion or contraction, which affects funding illiquidity, affects stock illiquidity and also affects the return spread between illiquid and liquid stocks but without performing asset pricing tests.

[10] Cochrane's (1996) reasons the selection of his scaling variables – the term yield spread on Treasury bonds, the dividend/price ratio and the corporate bond default spread which we denote *SP* – in that "[T]hese instruments are popular forecasters of stock returns" (p. 588). Lettau and Ludvigson (2001) employ a scaling variable *cay*, the consumption/asset ratio, which forecasts the market return factor. They say (p. 1243) that *cay* "has striking forecasting power for excess returns on aggregate stock market indexes." Ferson and Harvey (1999) include *SP* among their instrumental variables that proxy for time variation in expected return.

We proceed as follows. In Section 2, we introduce the *IML* illiquidity return premium factor and examine its risk-adjusted mean, its relation to other factors, and its behavior over time. Section 3 presents our main test results on the pricing of the conditional *IML* factor risk under the FFC four-factor model and six prominent stock characteristics controlled. In Section 4, we present a number of robustness tests and show that the pricing of the conditional β_{IML} remains positive and significant in all these tests. Concluding remarks are presented in Section 5.

2. *IML*, the return on *Illiquid-Minus-Liquid* stocks

2.1. The construction of *IML*

We construct a return factor *IML* of *Illiquid-Minus-Liquid* stocks, which we expect to have a positive risk-adjusted return. This follows Amihud and Mendelson's (1986) proposition that stock illiquidity is positively priced across stocks, and the evidence since then (see summary in Amihud, Mendelson, and Pedersen, 2005, 2013). The construction of *IML* follows Fama and French's (1993) procedure in constructing their return factors *SMB* (small-minus-big firm size) and *HML* (high-minus-low book-to-market ratio), after having found that the characteristics size and book-to-market ratio are priced across stocks. We employ NYSE/AMEX[11] stocks with codes 10 and 11 (common stocks). The sample period covers 71 years, January 1947 through December 2017, 852 months in total. We begin the sample period in 1947 because book values of stocks (to calculate book-to-market ratios) which we need in the cross-sectional analysis are available on Compustat since the middle of 1951, and we need 60 months before that to estimate the β coefficient of *IML*.[12]

To construct the *IML*, we employ two illiquidity measures, *ILLIQ* and *ZERO*, proposed respectively by Amihud (2002) and by Lesmond, Ogden, and Trzcinka (1999). The first measure is based on the full information on return, price, and trading volume and the second one relies on the counts of days with zero returns or no trading. These measures are found by

[11] These are the New York Stock Exchange and American Stock Exchange, where trading could be done directly between investors with the intermediation of specialists. The convention in the literature is to exclude Nasdaq stocks because, during much of the sample period, Nasdaq trading was conducted through market makers so the trading volume was counted twice (e.g., see Amihud (2002); Ben Rephael, Kadan, and Wohl (2013)).
[12] Also, since 1947, we have more than 40 stocks on average in each of the three high-*ILLIQ* and three low-*ILLIQ* portfolios. While there is no rule for the minimum number of stocks, it is clear that the efficiency and accuracy of the analysis are lower with a smaller number of stocks. For example, in May 1933 which is in the middle of the Great Depression, there are only 10 stocks in each of the three high-*ILLIQ* and low-*ILLIQ* portfolios.

Hasbrouck (2009) and by Goyenko et al. (2009) to be strongly correlated with high-frequency (intraday) measures of price impact and the bid-ask spread, respectively.[13] $ILLIQ_{j,t}$ and $ZERO_{j,t}$ are calculated for each stock *j* over a rolling twelve-month window that ends in month *t*. $ILLIQ_{j,t}$ is the average daily values of $ILLIQ_{j,d} = |return_{j,d}|/dollar\ volume_{j,d}$ (in $million). We delete stock-days with trading volume below 100 shares or with return of less than -100% and we also delete the highest daily value of $ILLIQ_{j,d}$ in each twelve-month period. A stock is included if its price remains between $5 and $1000 and it has more than 200 days of valid return and volume data during the twelve-month period. $ZERO_{j,\mathrm{t}}$ is the ratio of the number of days with zero returns or no trade divided by the total number of days in the rolling twelve-month window, which is used to calculate $ILLIQ_{j,t}$. Notably, *ZERO* is not based on trading volume or volatility which are components of *ILLIQ*. Finally, for each month *t*, we delete the stocks with the highest 1% values of $ILLIQ_{j,t}$ or $ZERO_{j,t}$.

Using $ILLIQ_{j,t}$ and $ZERO_{j,t}$, we construct two return factors $IML_{ILLIQ,t}$ and $IML_{ZERO,t}$ separately, employing the same methodology and the same sample of stocks. We sort stocks first by return volatility and then by each illiquidity measure in order to mitigate a possible confounding between their effects given the positive illiquidity-volatility correlation (Stoll, 1978) and the evidence on the effect of volatility on expected returns.[14] Stocks are sorted into three portfolios by $StdDev_{j,t}$, the standard deviation of daily returns, and within each volatility portfolio we sort stocks into five portfolios by either $ILLIQ_{j,t}$ or $ZERO_{j,t}$, all calculated over a period of twelve months. This produces 15 (3×5) portfolios for each illiquidity measure.[15] We calculate the month-*t* value-weighted average return of the stocks included in the portfolio on month *t*-2 (i.e., skipping month *t*-1) in order to avoid the effect of short-term reaction of stock

[13] *ILLIQ* is found by Hasbrouck (2009) and by Goyenko et al. (2009) as the best low-frequency proxy for Kyle's (1985) price impact measure that is estimated from intraday data. *ZERO* is found Lesmond et al. (1999) and by Goyenko et al. (2009) to be highly correlated with realized spread. Goyenko et al. (2009, p. 155) state that "… in more recent years, during the decimals regime, the performance of all measures deteriorates with the exception of Zeros and the Amihud measures." *ZERO* is used by Lesmond (2005) and by Bekaert, Harvey, and Lundblad (2007) to measure illiquidity in global markets.

[14] Levy (1978) and Merton (1987) propose that expected stock return is *positively* related to idiosyncratic volatility because of limited diversification, while Amihud (2002), and Ang, Hodrick, Xing, and Zhang (2006, 2009) find a negative effect of (idiosyncratic) volatility on expected return.

[15] This procedure follows the procedure in Fama and French (1993) when constructing their *HML* factor. They first sort stocks by size and then by book-to-market ratio within each size portfolio.

returns following unusually large shocks of illiquidity or volatility.[16] We adjust returns by Shumway's (1997) method to correct for delisting bias.[17] $IML_{ILLIQ,t}$ and $IML_{ZERO,t}$ are the averages of the returns on the highest-illiquidity quintile portfolios minus the averages of the returns on the lowest-illiquidity quintile portfolios across the three corresponding *StdDev* portfolios. Finally, we calculate IML_t for each month t as the average of $IML_{ILLIQ,t}$ and $IML_{ZERO,t}$.

2.2. Analysis of *IML*

INSERT TABLE 1

Table 1 presents statistics for the *IML* return series for the entire sample period of 71 years or 852 months, from January 1947 to December 2017. For robustness, we also present results for two equal subperiods of 35.5 years, 1/1947-6/1982 and 7/1982-12/2017. In Panel A, the average *IML* is 0.319% per month, nearly 4% a year, and it is statistically significant with t = 3.43. The median is 0.277% and the fraction of months with positive *IML* values is 0.550, which is significantly greater than 0.50, the chance result. The mean *IML* is positive and significant in both subperiods.

Panel B presents $alpha_{IML}$, the risk-adjusted *IML* after controlling for the FFC risk factors, estimated from the following model:

$$IML_t = alpha_{IML} + \beta_{RMrf}*RMrf_t + \beta_{SMB}*SMB_t + \beta_{HML}*HML_t + \beta_{UMD}*UMD_t + \varepsilon_t \,. \quad (1)$$

RMrf, *SMB*, *HML,* and *UMD* are, respectively, the market excess return over the T-bill rate, and the returns on small-minus-big firms (size factor), high-minus-low book-to-market ratio firms (value-growth factor), and winner-minus-loser stocks (momentum factor).

We find that $alpha_{IML}$ is positive and significant for the entire sample period of 1947 through 2017 and for each of the two subperiods. For the entire period, $alpha_{IML}$ is 0.341% per month with t = 5.47 and for the first and second subperiods, it is 0.441% with t = 4.94 and

[16] This follows Brennan, Chordia, and Subrahmanyam (1998) and Brennan, Chordia, Subrahmanyam, and Tong (2012), who discuss the merit of skipping one month. Fu (2009) notes the effect of return reversal on the relation between stock return and lagged idiosyncratic volatility.

[17] The last month return of a delisted stock is either the last return available from the Center for Research in Security Prices (CRSP), RET, or the delisting return DLRET, if available. If both are available, the calculated last month return proposed by CRSP is (1+RET)*(1+DLRET)-1. If neither the last return nor the delisting return is available and the deletion code is in the 500s—which includes 500 (reason unavailable), 520 (became traded over the counter), 551–573 and 580 (various reasons), 574 (bankruptcy), 580 (various reasons), and 584 (does not meet exchange financial guidelines)—the delisting return is assigned to be -30%. If the delisting code is not in the 500s, the last return is set to -1.0.

0.288% with t = 3.33, respectively.[18] The positive and significant $alpha_{IML}$ after controlling for *SMB*, whose slope coefficient is positive and highly significant, means that the illiquidity premium is in excess of the size premium, which itself is partially due to small stocks' illiquidity. *HML*'s positive slope coefficient suggests a positive relation between illiquidity and the book-to-market ratio. This is consistent with the finding of Fang, Noe, and Tice (2009) on the negative relation between illiquidity and the inverse of book-to-market ratio.

Panel C presents *out-of-sample* estimates of one-month-ahead $alpha_{IML,t}$. We first estimate the coefficients of Model (1) over a rolling window of past 60 months up to month t - 1 and then use the estimated factors' coefficients β_{t-1} to calculate $alpha_{IML,t}$ using the realized factors returns in month t:

$$alpha_{IML,t} = IML_t - [\beta_{RMrf,t-1}*RMrf_t + \beta_{SMB,t-1}*SMB_t + \beta_{HML,t-1}*HML_t + \beta_{UMD,t-1}*UMD_t] .$$

This procedure is repeated by rolling forward the 60-month estimation window one month at a time. The statistics of the series $alpha_{IML,t}$ are presented for January 1952 through December 2017 since the first five years are used to estimate the first set of β values. The mean of out-of-sample $alpha_{IML,t}$ is 0.356% per month with t = 5.87. For the first and second subperiods, the mean $alpha_{IML,t}$ is 0.487% with t = 5.36 and 0.242% with t = 3.00, respectively, all statistically significant. The medians are close to the means, and the fraction of positive values of $alpha_{IML,t}$ is significantly greater than 0.50, the chance result, for the entire period and for each of the two subperiods.

In Panel D we estimate Model (1) separately for each of the two illiquidity factors, $IML_{ILLIQ,t}$ and $IML_{ZERO,t}$. The respective risk-adjusted returns are $alpha_{ILLIQ}$ = 0.391% per month (t = 6.00) and $alpha_{ZERO}$ = 0.291% (t = 4.03), both highly significant, for the entire sample period. The estimated $alpha_{ILLIQ}$ and $alpha_{ZERO}$ are also positive and significant for each of the two subperiods. This result indicates that the positive and significant risk-adjusted *IML* return is not confined into one particular measure of illiquidity.[19]

3. The pricing of conditional *IML* risk

[18] The decline in the illiquidity premium over time is shown in Amihud (2002) and Ben Refael, Kadan, and Wohl (2015).

[19] Our main empirical results are also robust to choosing $IML_{ILLIQ,t}$, $IML_{ZERO,t}$, or their average, i.e., IML_t. See Section 4.2.

We test an asset pricing model that includes the *conditional* systematic risk of the illiquidity return premium factor *IML*. The test is whether investors require higher expected returns on stocks whose conditional β_{IML}s rise in times of higher anticipated funding illiquidity and distress. This prediction follows from Brunnermeier and Pedersen's (2009) proposition that the opportunity cost of illiquidity rises in times of funding illiquidity or financing constraints.

3.1 Methodology and test procedure

We follow the methodology of Cochrane (1996; 2005, Ch. 8) and LL, which estimates and tests asset pricing models with conditioning information proxied by a lagged instrument variable.[20] These studies propose the following *conditional* asset pricing model:[21]

$$m_t = a_{t-1} + b_{t-1}*F_t = a_0 + a_1*z_{t-1} + (b_0+b_1*z_{t-1})*F_t = a_0 + a_1*z_{t-1} + b_0*F_t + b_1*(z_{t-1}*F_t), \quad (2)$$

where m_t is the stochastic discount factor, F_t is the pricing factor, and a_{t-1} and b_{t-1} are parameters that can vary over time and are modeled to depend on z_{t-1}, a conditioning variable that summarizes the investors' information set in time *t-1*. In Model (2), Cochrane and LL transform a one-factor *conditional* model with time-varying coefficients (the first equality) into a three-factor *unconditional* model of z_{t-1}, F_t, and $z_{t-1}*F_t$ with fixed coefficients (the last equality), where $z_{t-1}*F_t$ is a *scaled factor*.[22] The scaling instrument z_{t-1} is selected among instruments that forecast the pricing factor F_t and enter the model significantly. For z_t, researchers employ variables that forecast the market excess return: Cochrane (1996) and Ferson and Harvey (1999) employ the dividend/price ratio, the term spread of interest rates, and the corporate bond default yield spread (similar to our *SP*), and LL employ *cay*, the consumption/asset ratio.

The conditional asset pricing Model (2) is equivalent to the following *conditional* beta representation (based on the first equality):

$$\mathrm{E}_{t-1}[(r_j - rf)_t] = \gamma_{0,t-1} + \gamma_{F,t-1}*\beta_{F,j,t-1}, \quad (3)$$

where E_{t-1} is conditional expectation in time *t-1*, $(r_j - rf)_t$ is stock *j*'s excess return over the risk-free rate rf_t, $\gamma_{0,t-1}$ is the excess return of a zero-beta portfolio over the risk-free rate, and $\gamma_{F,t-1}$ and $\beta_{F,j,t-1}$ are, respectively, the conditional price of risk factor *F* and conditional exposure of stock *j*

[20] See also Ferson and Schadt (1996), Jagannathan and Wang (1996), and Ferson and Harvey (1999).
[21] For the ease of exposition, we use here a single-factor asset pricing model. Our implementation of the asset pricing tests in Section 3.2 employs a multi-factor asset pricing model with the *FFC* and the *IML* factors.
[22] Cochrane (2005, p. 144) proposes: "To express the conditional implications of a given model, all you have to do is include some well-chosen scaled … portfolio returns… [y]ou can just add new factors, equal to the old factors scaled by the conditioning variables and … forget that you ever heard about conditioning information."

to F. Model (2) is also equivalent to the following *unconditional* multifactor beta representation (based on the last equality with the scaled factor $z_{t-1}*F_t$):

$$\mathrm{E}[(r_j - rf)_t] = \mathrm{E}[\gamma_{0,t}] + \gamma_z*\beta_{z,j} + \gamma_F*\beta_{F,j} + \gamma_{Fz}*\beta_{Fz,j}\,, \tag{4}$$

where E is unconditional expectation, $\beta_{z,j}$, $\beta_{F,j}$, and $\beta_{Fz,j}$ are the slope coefficients from a time-series regression of $(r_j - rf)_t$ on z_{t-1}, F_t, and $z_{t-1}*F_t$, respectively, and γ_z, γ_F, and γ_{Fz} are their associated coefficients in Model (4). Importantly, testing whether γ_{Fz} is significantly different from zero tests whether the conditioning of F_t by z_{t-1} enters the conditional asset pricing model in Model (2) significantly. After estimating and testing γ_{Fz}, we compute the conditional price of F risk (= $\gamma_{F,t-1}$ in Equation (3)) using the estimates of all γ coefficients from Model (4). LL show (pp. 1247-1249) that the conditional price of F risk can be computed as follows:

$$\gamma_{F,t-1} = -(\gamma_{0,t-1} + rf_{t-1})*\mathrm{Var}_{t-1}[F_t]*(b_0 + b_1*z_{t-1}), \tag{5}$$

$$\boldsymbol{b} = -\{\mathrm{E}[\gamma_{0,t} + rf_t\,]*\mathbf{Cov}[\boldsymbol{SF_t}]\}^{-1}*\boldsymbol{\gamma},$$

where the three column vectors are defined as $\boldsymbol{b} = [a_1, b_0, b_1]'$, $\boldsymbol{SF_t} = [z_{t-1}, F_t, z_{t-1}*F_t]'$, and $\boldsymbol{\gamma} = [\gamma_z, \gamma_F, \gamma_{Fz}]'$, and Var_{t-1} is the conditional variance and **Cov** is the unconditional covariance matrix. (We use bold-face letters to denote vectors and matrices.) Thus the conditional price of the F risk ($=\gamma_{F,t-1}$) is a function of the scaling instrument z_{t-1}. LL also assume that the conditional risk exposure to F is also a function of z_{t-1}. Based on Models (2)-(4), for $z_t = cay_t$ and $F_t = RMrf_t$, LL find that γ_{Fz} is significant and the average of conditional price of F risk is also positive, concluding that their conditional version of the CAPM empirically outperforms the unconditional CAPM. We follow the same procedure using $z_t = SP_t$ and $F_t = IML_t$ with added controls for commonly-used factors' risks and stock characteristics in estimating Model (4).

Following Cochrane (2005) and LL, we select a conditioning variable which forecasts the factor whose conditional systematic risk is being studied and whose value can be observed by investors prior to making their pricing decision. Our scaling instrument *SP*, the BAA-AAA corporate bond yield spread (in percent points), significantly forecasts the illiquidity return premium factor *IML*. *SP* is available for the entire sample period of January 1947 through December 2017 from the St. Louis Federal Reserve Bank database. The scaling variable is $Z1_t = maSP_t$,[23] the mean-adjusted SP_t using the mean over of the preceding ten years. In a regression of IML_t on $Z1_{t-1}$ (and a constant), the slope coefficient of $maSP_{t-1}$ is 0.788 with t = 2.84 for the

[23] We follow here Lettau and Ludvigson (2001) who use the mean-adjusted value of their scaling variable *cay* and Cochrane (1996, p. 588) who similarly transforms his scaling variable, the dividend/price series.

sample period.[24] The positive effect of SP_{t-1} on the expected illiquidity premium is consistent with Brunnermeier and Pedersen's (2009) proposition that in financial distress and lower funding liquidity, the market illiquidity and its shadow price or premium rise. It is also consistent with *SP* forecasting adverse economic conditions (Gilchrist and Zakrajšek, 2012). The value of SP_{t-1} is available continuously thus *SP* satisfies Cochrane's (2005, p. 143) requirement that the conditioning variable should be a part of the investors' information set in time *t-1*.

The testing procedure is as follows. In the first stage, we estimate a time-series factor model for each stock *j* over a rolling window of 60 months beginning with the period of January 1947 to December 1951:

$$(r_j - rf)_t = \beta_{0j} + \beta_{RMrf,j}*RMrf_t + \beta_{SMB,j}*SMB_t + \beta_{HML,j}*HML_t + \beta_{UMD,j}*UMD_t + \beta_{IML,j}*IML_t + \beta_{IMLZ1,j}*IML_t*Z1_{t-1} + \beta_{Z1,j}*Z1_{t-1}, \quad (6)$$

where rf_t is one-month U.S. T-bill rate and the model includes the scaled factor IML_t*Z1_{t-1} to capture the time-variations in conditional *IML* risk exposure and its risk premium. In the second stage, in each month *s* that follows the 60-month estimation window, we employ the Fama-Macbeth (1973) procedure of estimating a cross-sectional regression of stock excess returns $(r_j - rf)_s$ on the seven β coefficients that are estimated in Model (6) and on six lagged stock characteristics:

$$\begin{aligned}(r_j - rf)_s = {} & \gamma_{0,s} + \gamma_{RMrf,s}*\beta_{RMrf,j,s-2} + \gamma_{SMB,s}*\beta_{SMB,j,s-2} + \gamma_{HML,s}*\beta_{HML,j,s-2} + \gamma_{UMD,s}*\beta_{UMD,j,s-2} \\ & + \gamma_{IML,s}*\beta_{IML,j,s-2} + \gamma_{IMLZ1,s}*\beta_{IMLZ1,j,s-2} + \gamma_{Z1,s}*\beta_{Z1,j,s-2} \\ & + \delta_{1,s}*ILLIQma_{j,s-2} + \delta_{2,s}*StdDev_{j,s-2} + \delta_{3,s}*BM_{j,s-2} \\ & + \delta_{4,s}*Size_{j,s-2} + \delta_{5,s}*R12lag_{j,s-2} + \delta_{6,s}*R1lag_{j,s-1}. \quad (7)\end{aligned}$$

The six stock characteristics in Model (7) are: (1) *ILLIQma*, the mean-adjusted stock illiquidity. This variable is the *ILLIQ* value of stock *j* calculated over a twelve-month period divided by the mean of all stocks' *ILLIQ* values for that period.[25] (2) *StdDev*, return volatility, measured by the standard deviation of daily stock returns over a twelve month period. Volatility is known to be positively correlated with *ILLIQ* and it has its own effect on the cross-section of stock returns (Amihud, 2002; Ang et al., 2006). For the calculation of *ILLIQ* and *StdDev*, we require to have more than 200 days of valid return and volume data, as we did in the construction of *IML*. (3)

[24] Conducting the diagnostic test of Amihud and Hurvich (2004) we find no evidence of a finite-sample bias in the slope coefficient due to high autocorrelation in the predictive variable.
[25] This adjustment keeps the mean of *ILLIQma* stable at 1.0 in all months. See Amihud (2002), and Amihud, Hameed, Kang, and Zhang (2015).

BM (in logarithm), the book-to-market ratio, using the book value from the firm's annual financial report (data are from Compustat) known as of the end of the previous fiscal year and the market value as of December of the year before the year of analysis. Following Fama and French (1992), we exclude stocks with negative book values. (4) *Size*, the logarithm of market capitalization. (5) *R12lag*, the lagged return over eleven months from month *s*-2 to month *s*-12 to capture the momentum effect. (6) *R1lag*, the stock return in month *s*-1 to capture the short-term reversal effect.

Model (7) is estimated over 66 years (792 months), January 1952 through December 2017. The estimations and tests employ the following three methods:

(1) Ordinary least squares (OLS), which is commonly used in Fama–Macbeth regressions.

(2) CGS, the method of Chordia, Goyal, and Shanken (2017) that corrects for the bias in the estimation of the γ coefficients which is due to the EIV problem by the estimated β coefficients.[26] This is particularly important since our test assets are individual stocks instead of widely used stock portfolios.

(3) Weighted least squares (WLS), following Asparouhova et al. (2010), to account for possible bias due to microstructure noise which inflates the average return of illiquid stock. The weights are proportional to the prior month's gross return, $1 + r_{j,s-1}$.

Using the estimates of γ (including intercept) from Model (7) we compute the time-series of the monthly conditional price of risk of *IML* employing Equation (5) (in a multi-factor version), and then test whether its mean is significantly different from zero.[27] Our hypothesis is that the mean conditional price of risk is positive.

The detailed procedure for the conditional price of risk of *IML* is as follows. First, using the estimates of $\boldsymbol{\gamma} = [\gamma_{Z1}, \gamma_{RMrf}, \gamma_{SMB}, \gamma_{HML}, \gamma_{UMD}, \gamma_{IML}, \gamma_{IMLZ1}]'$ from Model (7) and based on Equation (5),[28] we compute $\boldsymbol{b} = -\{\mathrm{E}[\gamma_{0,t} + rf_t]^*\mathbf{Cov}[\boldsymbol{SF_t}]\}^{-1}{}^*\boldsymbol{\gamma}$, where the 7x1 column vectors are

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[26] This method follows the bias-correcting method of Litzenberger and Ramaswamy (1979) and Shanken and Zhou (2007). It relies on White's (1980) heteroscedasticity-consistent covariance matrix estimator for the OLS estimates of the β values and corrects for the EIV-induced bias in the OLS cross-sectional estimates of the γ values. In a single-factor model, the EIV problem induces downward bias in the (absolute) value of the γ coefficient. In a multifactor model, the directions of the EIV-induced biases depend not only on the variances of the β estimation errors but also on their covariances. Our estimation involves the covariance matrix of the estimated β values.

[27] For their calculation, LL assume that the conditional variance of market factor and zero-beta rate, which are $\mathrm{Var}_{t-1}[F_t]$ and $\gamma_{0,t-1}+rf_{t-1}$ in Equation (5), respectively, are approximately constants. We allow the zero-beta rate to vary over time and compute rolling sample covariance matrix for conditional covariance matrix of the FFC and *IML* factors for each month using the realizations of those factors over the preceding 36 months.

[28] We use the intercept in Model (7) for $\gamma_{0,t}$ in Equation (5).

defined as $\boldsymbol{b}$ = [a_1, $b_{0,RMrf}$, $b_{0,SMB}$, $b_{0,HML}$, $b_{0,UMD}$, $b_{0,IML}$, $b_{1,IML}$]' and $\boldsymbol{SF_t}$ = [$Z1_{t-1}$, $RMrf_t$, SMB_t, HML_t, UMD_t, IML_t, $Z1_{t-1}$*IML_t]', unconditional E[$\gamma_{0,t}$+ rf_t] is estimated by taking its average, and unconditional **Cov**[$\boldsymbol{SF_t}$] is estimated by taking the sample covariance matrix of $\boldsymbol{SF_t}$ over the entire sample period. Then, defining the two 5x1 column vectors $\boldsymbol{b_0}$= [$b_{0,RMrf}$, $b_{0,SMB}$, $b_{0,HML}$, $b_{0,UMD}$, $b_{0,IML}$]' and $\boldsymbol{b_1}$= [0, 0, 0, 0, $b_{1,IML}$]' from the estimated $\boldsymbol{b}$, we compute the conditional price of $\boldsymbol{F}$ risk (= $\boldsymbol{\gamma}_{\boldsymbol{F},t-1}$); the 5x1 column vector $\boldsymbol{\gamma}_{\boldsymbol{F},t-1}$ = -($\gamma_{0,t-1}$+ rf_{t-1})*$\mathbf{Cov}_{t-1}$[$\boldsymbol{F_t}$]*($\boldsymbol{b_0}$+$\boldsymbol{b_1}$*$Z1_{t-1}$) as in Equation (5), where $\boldsymbol{F_t}$ = [$RMrf_t$, SMB_t, HML_t, UMD_t, IML_t]' and conditional $\mathbf{Cov}_{t-1}$[$\boldsymbol{F_t}$] is estimated by taking the sample covariance matrix of $\boldsymbol{F_t}$ over the preceding 36 months in month *t-1*. Finally, the last element in the calculated vector $\boldsymbol{\gamma}_{\boldsymbol{F},t}$ is the monthly conditional price of the *IML* risk in month *t*. While Models (4) and (5) include no stock characteristics, our Model (7) controls for the effects on expected return of six commonly-used stock characteristics. As a robustness check, we also calculate the conditional price of *IML* risk with the γ estimates obtained from Model (7) without the stock characteristics and find that the results are qualitatively similar.

We employ NYSE/AMEX-listed stocks that satisfy our requirement of having data for all the variables in Model (7) and their prices in month *s*-1 are between \$5 and \$1000. We then trim stocks whose *ILLIQ* is in the extreme 1% or whose estimated βs are in the 0.5% of each tail of the distribution (for each β). We end up having on average 718.8 stocks for the monthly cross-sectional regressions in Model (7), ranging between 174 and 1084 stocks. Stock returns in the monthly cross-sectional regressions are corrected for potential bias due to delisting (or survivorship) using Shumway's (1997) procedure.

Table 2 presents summary statistics for the 13 explanatory variables in Model (7) that include seven β coefficients and six stock characteristics. In the left two columns, we present the average of the monthly means and monthly standard deviations that are calculated across all stocks in each month *s* over the 792 months of our sample period, 1952-2017. The right panel presents the averages of the monthly pairwise cross-stock correlations among some variables.

INSERT TABLE 2

3.2. Tests results of the pricing of conditional *IML* risk

Our hypothesis is that γ_{IMLZ1} in Model (7) is positive[29] and significant and that the average of the conditional price of *IML* risk in Equation (5) is also positive and significant, implying that stock expected return is positively affected by its conditional exposure (β_{IML}) to the *IML* factor in times of financial distress. Similar to LL (p. 1266) who suggest that the pricing of the *conditional* market *beta* "… may be attributable to time variation in risk aversion… or time variation in risk itself," the pricing of *conditional* β_{IML} can be attributable to the time variation in illiquidity premium or variation in illiquidity risk itself.

The tests of Model (7) that are estimated by the three methods: OLS, CGS, or WLS, employ the following two statistics:

(i) The mean of the monthly slope coefficients and its *t*-statistic

(ii) The precision-weighted mean and the respective *t*-statistic. The weights are proportional to the reciprocal of the standard errors of the slope coefficients from the monthly cross-sectional regressions, thus more precisely estimated monthly slope coefficients have greater weights.[30] Ferson and Harvey (1999) propose this weighting method to improve the efficiency of the slope coefficients estimated by the Fama-Macbeth procedure and mitigate the problem of heteroskedasticity.

INSERT TABLE 3

Consistent with our hypothesis, we find in Table 3 that γ_{IMLZ1} is positive and highly significant, suggesting that the *IML* systematic risk *conditional* on times of financial distress enters the asset pricing model significantly. These results hold under all three estimation methods: OLS, CGS, and WLS. Specifically, we find that the OLS estimated mean of γ_{IMLZ1} is 0.062% (t = 3.17) and its precision-weighted mean is 0.040% (t = 3.20). For the CGS method, the mean and precision-weighted mean of γ_{IMLZ1} are 0.060% (t = 3.52) and 0.043% (t = 3.72), respectively. For the WLS method, the mean and precision-weighted mean of γ_{IMLZ1} are close to those under the OLS method.

[29] Suppose that the conditional β_{IML} (= $\beta_{IML,j,t-1}$) is modeled as $\beta_{IML,j,t-1} = \beta_{IML0,j} + \beta_{IMLZ1,j} * Z1_{t-1}$ for stock j and that a bad state is signified by $Z1_{t-1} > 0$ in time t-1. Then a higher value of $\beta_{IMLZ1,j}$ implies a higher value of $\beta_{IML,j,t-1}$ across stocks in the bad state. Thus a positive value of γ_{IMLZ1} in Model (7) is consistent with the positive pricing of conditional β_{IML} in the bad state.

[30] For the weighted mean of the monthly slope coefficients estimated by the CGS method, we use as weights the reciprocals of the standard errors of monthly OLS cross-sectional regressions. Chordia et al. (2015) find, through simulations, that the Fama–MacBeth standard error estimates by OLS method are practically identical to the true standard deviations of the EIV-corrected slope coefficients by the CGS method.

We now compute the time-series of the monthly conditional price of risk of *IML* (following the methodology in Equation (5)) using the CGS estimates of Model (7) that includes the βs of the four FFC factors as well as the βs of *IML*, *IML* scaled by the instrument *Z1*, and *Z1*. Using the procedure whose details are presented in Section 3.1 above, we find that the average conditional price of *IML* risk is positive at 0.108% per month (= 1.30% per year) with t = 3.34, highly significant. We then estimate a time-series regression of the conditional price of *IML* risk on $HimaSP_{t-1}$, a constant, and $RMrf_t$ (as a control for the state of the overall market) where $HimaSP_t = 1$ when $maSP_t > 0$ (= 0 otherwise). We find that the coefficient of $HimaSP_{t-1}$ is positive and highly significant at 0.239% per month (= 2.87% per year) with t = 3.73.[31] That is, the conditional price of risk of the *IML* factor rises significantly in times of financial distress, as we propose.

As a robustness check we calculate the monthly time-series of conditional price of *IML* risk using the CGS estimates of γ coefficients in Model (7) without the six stock characteristics and find that the test results are stronger. The average conditional price of *IML* risk is 0.219% per month (= 2.63% per year) with t = 4.60, highly significant. In a time-series regression of the monthly conditional price of *IML* risk on $HimaSP_{t-1}$ and $RMrf_t$, the coefficient of $HimaSP_{t-1}$ is 0.376% (= 4.51% per year) with t = 4.38, indicating again that the price of *IML* systematic risk rises significantly in times of financial distress.

Notably, *none* of the unconditional systematic risks of the four FFC factors – β_{RMrf}, β_{SMB}, β_{HML}, and β_{UMD}— is significantly priced. It is also notable that the positive and significant price of *IML* risk is estimated after controlling for illiquidity as a characteristic. We find that the slope coefficient δ_1 of *ILLIQma* is positive and significant: By the OLS estimate, δ_1 is 0.041% with t = 2.58. The other stock characteristics are all significantly priced with signs that have been observed in earlier studies. By the OLS estimate, the slope coefficients of *StdDev*, *BM*, *Size*, *R12lag*, and *R1lag* are -27.935% (t = -4.25), 0.098% (t = 2.45), -0.086% (t = -3.82), 1.116% (t = 6.77), and -5.047% (t = -14.99), respectively.

To test whether the *unconditional* β_{IML} is priced across stocks we estimate a special case of Models (6) and (7) where we set to zero the β coefficients of the *Z1*-related variables in Model (6) and the corresponding γ coefficients in Model (7). The estimations of these models provide

[31] When excluding $RMrf_t$, we find that the coefficient of $HimaSP_{t-1}$ remains positive and highly significant at 0.238% with t = 3.70. Conducting the diagnostics based on Amihud and Hurvich (2004), we find no evidence of small sample bias in the estimated slope coefficient of $HimaSP_{t-1}$.

five time-series of monthly risk prices on the unconditional factor βs: $\gamma_{RMrf,s}$, $\gamma_{SMB,s}$, $\gamma_{HML,s}$, $\gamma_{UMD,s}$, and $\gamma_{IML,s}$. We find that the means and *t*-statistics of these γ coefficients by the CGS method for the FFC factors are as follows: $\gamma_{RMrf,s}$ is 0.210% (t = 2.44); $\gamma_{SMB,s}$ is 0.618% (t = 1.22); $\gamma_{HML,s}$ is 0.093% (t = 1.94); and $\gamma_{UMD,s}$ is -0.039% (t = -0.58). Importantly, for the pricing of the unconditional β_{IML} we find that $\gamma_{IML,s}$ is 0.043% (t = 0.94) which is positive but insignificant. This indicates that investors do not require significantly higher expected returns on stocks whose β_{IML}s are *unconditionally* higher. This result is similar to the findings of Cochrane (1996) and LL on the significant pricing of the conditional β of their pricing factors while the unconditional β is not significantly priced.

We conclude that across stocks there is a positive and significant risk premium on the conditional systematic risk of the illiquidity return factor *IML* which rises in times of financial distress.

4. Robustness tests

We present six robustness tests of the positive pricing of the conditional β_{IML} that rises in times of financial distress.

1) Using the baseline Model (7), we examine whether the test results are consistent over time by splitting the sample period of 66 years into two non-overlapping subperiods of 33 years each and testing our hypothesis separately in each subperiod.

2) We test whether the results are robust to the measure of illiquidity by separately testing, using *either* IML_{ILLIQ} or IML_{ZERO}, whether γ_{IMLZ1} and the average of conditional price of *IML* risk are positive and significant when doing the analysis with *either* IML_{ILLIQ} or IML_{ZERO}.

3) We test whether the pricing of conditional β_{IML} remains positive and significant after expanding the baseline model by conditioning each of the FFC factor βs on $Z1_{t-1}$ as we do for *IML*. We also test whether the conditional β_{IML} is uniquely priced compared to the other conditional βs.

4) We test whether the pricing of conditional β_{IML} remains positive and significant after expanding the baseline model with three liquidity-based factors that were proposed in earlier studies, allowing their βs and risk premiums to be conditional on $Z1_{t-1}$ as we do for *IML*.

Next, we do two tests in which we replace the scaling variable $Z1_t$ by other scaling variables that are related to financial distress and re-estimate the baseline models of Table 3. We use the following alternative scaling variables:

5) We use as a conditioning variable $Z2_t = dSP_t^+$, the positive change or the rise in SP_t. Then, finding that the coefficient of β_{IMLZ2} and the average of associated conditional price of *IML* risk are positive and significant implies a positive pricing of the conditional *IML* risk when financial conditions worsen.

6) We use for the conditioning variable $Z3_t$ the broker-dealer loans series that includes their margin loans in excess of total loans. This choice of scaling variable is motivated by Brunnermeier and Pedersen's (2009) theory which is based on the effect of margin loan constraint on the rise in illiquidity and in its premium.

In these robustness tests, we find that the slope coefficient of β_{IMLZ} is positive and highly significant and that the average of conditional price of *IML* risk is also positive and significant as we find them to be for the baseline model in Table 3, indicating that the pricing of conditional β_{IML} in times of financial distress or funding illiquidity is fairly robust.

4.1 Testing the baseline model over two subperiods.

We test whether the positive and significant pricing of conditional β_{IML} in times of financial distress is consistent over time. We split the sample period of 66 years (January 1952 through December 2017) into two equal subperiods of 33 years and repeat our tests for each subperiod separately. This can be viewed as an out-of-sample test of the pricing of conditional β_{IML} over the second subperiod after having observed its pricing in the first subperiod. The test results for γ_{IMLZ1} are presented in Table 4 using the CGS bias-correcting method. We find that in both subperiods, the means and precision-weighted means of γ_{IMLZ1} are positive and significant. Then for each of the two subperiods, we compute the time-series of the monthly conditional price of risk of *IML* (following Equation (5)) using the corresponding CGS estimates of γs.[32]

We find that the averages of the monthly conditional price of *IML* risk is 0.111% with t = 2.38 and 0.113% with t = 2.33 for the first and second subperiod, respectively. Notably, the economic magnitudes of those average conditional prices of *IML* risk are similar in both subperiods indicating consistency over time in the pricing of conditional β_{IML}. The power of the

[32] We apply the procedure detailed in Section 3.1 for each subperiod separately.

subperiod tests is naturally lower than it is when we use the entire sample period. In time-series regressions of the monthly conditional price of *IML* risk on $HimaSP_{t-1}$ and $RMrf_t$ we find that the coefficient of $HimaSP_{t-1}$ is 0.192% (t = 2.01) and 0.334% (t = 3.30) for the first and second subperiod, respectively.

INSERT TABLE 4

4.2. Separate tests using IML_{ILLIQ} or IML_{ZERO}.

We test whether the results are robust to the illiquidity measures that we use in constructing the illiquidity return factor *IML*. We replace IML_t in Model (6) by either $IML_{ILLIQ,t}$ or $IML_{ZERO,t}$ and then use the estimated βs of either of these factors in Model (7). The test results are qualitatively similar to those reported in Table 3, that is, we find positive and significant slope coefficients of β_{IMLZ1} with either IML_{ILLIQ} or IML_{ZERO}. For example, under the CGS method, the mean γ_{IMLZ1} is 0.054% (t = 3.06) or 0.060% (t = 3.18) when using IML_{ILLIQ} or IML_{ZERO}, respectively. The average conditional price of risk of *IML* is positive and significant for either IML_{ILLIQ} or IML_{ZERO} being 0.091% (t = 3.15) for IML_{ILLIQ} or 0.143% (t = 4.06) for IML_{ZERO}. In time-series regressions of the conditional price of *IML* risk on $HimaSP_{t-1}$ (and $RMrf_t$ as a control), we find that the coefficient of $HimaSP_{t-1}$ is 0.203% (t =3.56) when using IML_{ILLIQ} or 0.273% (t = 3.91) when using IML_{ZERO}. These results demonstrate that the significant pricing of conditional β_{IML} in times of financial distress is robust to the measures of illiquidity used.

4.3. Tests using the conditional βs of the four FFC factors

This section provides two tests. First, we test whether the pricing of the conditional β_{IML} remains positive and significant when we allow for the β of each of the four FFC factors to be estimated conditional on *Z1* in the same way as we estimate conditional β_{IML}. Second, we test whether the pricing of conditional β with *maSP* as a conditioning variable is unique to the *IML* factor by testing whether the conditional βs of the FFC factors are also consistently and significantly priced.

The test is conducted as follows. For each factor *FF*, *FF* = *RMrf*, *SMB*, *HML,* or *UMD,* we add to Model (6) the term $\beta_{FFZ1}*FF_t*Z1_{t-1}$ and estimate β_{FFZ1} together with all the other βs. Then, we add β_{FFZ1} to Model (7) and estimate its slope coefficient γ_{FFZ1} together with the coefficients of all the other variables in Model (7). We first test whether the slope coefficient of β_{IMLZ1} or β_{FFZ1} is positive and significant. Second, if conditioning of IML_t or FF_t on $maSP_{t-1}$ enters the asset pricing model significantly, we conduct the estimations and tests on the conditional price of *IML* or *FF* risk.

We have two important findings that are presented in the Appendix Table A.1. First, the pricing of conditional β_{IML} remains positive and highly significant regardless of which of the FFC factors' *conditional* βs is added to Model (7). For example, in the model that includes the conditional β of *FF* which is either *RMrf*, *SMB*, *HML*, or *UMD*, the precision-weighted mean of γ_{IMLZ1} is 0.039% (t = 3.28), 0.043% (t = 3.66), 0.042% (t = 3.55), or 0.041% (t = 3.49), respectively, using the CGS method. Then we estimate the average of the time-series of monthly conditional price of risk of *IML* (following Equation (5)) in a model that includes the conditional β_{FF} for *FF* = *RMrf*, *SMB*, *HML*, or *UMD*. We find that the average price of the conditional *IML* risk remains positive and significant being, respectively, 0.120% (t = 3.70), 0.128% (t = 3.42), 0.097% (t = 3.16), or 0.092% (t = 2.96). In a time-series regression of the conditional price of *IML* risk on $HimaSP_{t-1}$ and $RMrf_t$, we find that the corresponding coefficient of $HimaSP_{t-1}$ is 0.242% per month (t = 3.74), 0.264% (t = 3.56), 0.232% (t = 3.81), or 0.230% (t = 3.73), which demonstrates the robustness of positive and significant pricing of conditional β_{IML} in the presence of the conditional risk of the FFC factors.

Second, the significant and positive pricing of the conditional β is unique to *IML* as none of the conditional βs of the FFC factors is consistently priced.[33] This supports our proposition on the link between the pricing of the exposure to the illiquidity factor *IML* and financial distress or funding illiquidity, following Brunnermeier and Pedersen (2009). Detailing the slope coefficient of β_{FFZ1} for $FF_t = RMrf_t, SMB_t, HML_t$, or UMD_t, we find under the CGS method that the precision-weighted means of γ_{FFZ1} are as follows: γ_{RMrfZ1} = -0.041% (t = -1.47), γ_{SMBZ1} = 0.010% (t = 0.91), γ_{HMLZ1} = 0.021% (t = 1.79), and γ_{UMDZ1} = -0.008% (t = -0.50). This indicates that conditioning of FF_t on $maSP_{t-1}$ enters none of the corresponding asset pricing models significantly for $FF_t = RMrf_t, SMB_t, HML_t$, or UMD_t.[34] In addition, the test results in Table 3 show that the unconditional βs of these FFC factors are not significantly priced.

[33] These results hold both for the entire sample period and for each of the two subperiods.
[34] We further investigate the model which includes $SMB_t{*}Z1_{t-1}$ given the positive correlation between illiquidity and size. For the two equally split subperiods, the precision-weighted means for γ_{IMLZ1} and γ_{SMBZ1} are, respectively, 0.033% (t = 2.12) and -0.006% (t = -0.40) for the first subperiod and 0.052% (t = 3.01) and 0.026% (t = 1.54) for the

We conclude that the pricing of the conditional β_{IML} in times of financial distress remains positive and significant in the presence of the conditional βs of the FFC factors and that this pricing of conditional β is unique to *IML*.

4.4. The pricing of the conditional β_{IML} and the conditional β*s* of other liquidity factors

We add to our model liquidity-based factors that were used in earlier studies and test whether the pricing of the conditional β_{IML} remains positive and significant in the presence of the conditional βs of other liquidity-based factors.[35] For each of these factors, denoted LF_t, we add to Model (6) $\beta_{LF}*LF_t + \beta_{LFZ1}*LF_t*Z1_{t-1}$. We estimate this augmented model over a rolling 60-month period, and add the estimated coefficients β_{LF} and β_{LFZ1} to the cross-sectional regression in Model (7). Finally, we estimate the slope coefficients of these βs, γ_{LF} and γ_{LFZ1}, together with the slope coefficients of all the other βs and stock characteristics in Model (7).

We use three liquidity-based factors that were presented in earlier studies. The first two are traded factors that represent a liquidity-based return premium as does *IML*, and the third is a non-traded factor that represent shocks to market-wide (il)liquidity. The three liquidity factors are:

(i) *PS*, a traded liquidity risk factor due to Pastor and Stambaugh (2003). It is the value-weighted average return on the high-minus-low decile portfolios obtained by sorting stocks on the β values which are obtained from a regression of each stock return series on innovations in the aggregate liquidity index that they propose. This factor has a positive and significant excess return. *PS*, available from Lubos Pastor's homepage,[36] begins on January, 1968.

(ii) *LIU*, a traded illiquidity premium factor proposed by Liu (2006). It is the differential return between illiquid and liquid stocks, using Liu's liquidity measure that is based on non-trading days and turnover. The time series of *LIU*, kindly provided by the author, is available from January, 1947 to December, 2014. The correlation of *LIU* and *IML* is 0.46. While both *IML* and *LIU* measure the return premium on illiquid-minus-liquid stocks, they differ not only in

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second subperiod. Also, while γ_{HMLZ1} is positive and significant at the 10% level for the entire sample period as reported above, this result is not robust across the two subperids. Under the CGS method, the precision-weighted means of γ_{HMLZ1} are 0.006% (t = 0.39) and 0.034% (t = 1.87) for the first and second subperiods, respectively. In contrast, the precision-weighted means of γ_{IMLZ1} are 0.034% (t = 2.14) and 0.050% (t = 2.84), respectively, both being positive and significant.

[35] Notably, while we use individual stocks as test assets, earlier studies employ as test assets stock *portfolios* sorted on some characteristics.

[36] http://faculty.chicagobooth.edu/lubos.pastor/research/liq_data_1962_2017.txt

underlying illiquidity measures but also in their construction. Compared to *IML*, *LIU* reflects the returns on more extremely illiquid and liquid stocks without controlling for stock return volatility (*StdDev*).[37]

(iii) *dMILLIQ*, the first-order difference of the logarithm of monthly market illiquidity *MILLIQ*, a non-traded factor. *MILLIQ* is the average over the days of each month of the value-weighted average of daily *ILLIQ* across the stocks that satisfy our data requirements.[38] This series is available for the entire sample period of our analysis. Testing whether the systematic risk of market-wide illiquidity shocks is priced is related to the analyses of Pastor and Stambaugh (2003), Acharya and Pedersen (2005), and Watanabe and Watanabe (2008).

INSERT TABLE 5

We find that the conditional β_{IML} is positively and significantly priced in all model specifications with any of the alternative liquidity factors. Table 5 presents the test results of the slope coefficient of β_{IMLZ1} in the presence of β_{LF} and β_{LFZ1}, the βs of the other liquidity factors and their scaled factors, *LF* = *PS*, *LIU*, or *dMILLIQ*. To save space we present the results only under the CGS estimation method for the slope coefficients of the liquidity-related βs. The cross-sectional regression models include all the other βs and stock characteristics in Model (7). We find that the slope coefficient of β_{IMLZ1} remains positive and highly significant in the presence of the three alternative liquidity-related βs. Then, using the CGS estimates of γs in Model (7), we compute the conditional price of *IML* risk (following Equation (5)) in the presence of the βs of *LF* = *PS*, *LIU*, or *dMILLIQ*. We find that the average of monthly conditional price of *IML* risk is 0.140% (t = 3.41), 0.079% (t = 2.61), or 0.091% (t = 2.84), respectively. Estimating time-series regressions of the conditional price of *IML* risk on $HimaSP_{t-1}$ and $RMrf_t$, the corresponding coefficient of $HimaSP_{t-1}$ is 0.342% per month (t = 4.20), 0.207% (t = 3.43), or 0.216% (t = 3.37).

In contrast, none of conditional βs of the other liquidity factors is significantly and consistently priced. In Table 5, none of the means of the γ_{LFZ1} coefficients is significant, indicating that conditioning of LF_t by $maSP_{t-1}$ does not enter the corresponding asset pricing model significantly. In addition, we find that the average of monthly conditional price of *LF* risk

[37] *LIU* is based on extreme decile portfolios sorted on his illiquidity measure with equally weighted returns, while *IML* is based on quintile portfolios double sorted on *ILLIQ* and *StdDev* with value-weighted returns. The mean return of *LIU* is nearly halved when using *value*-weighted returns, see Liu (2006, p. 642).

[38] The value weighting employs the stock capitalization at the end of the preceding month. Included are common stocks (codes 10 and 11) that trade on NYSE/AMEX whose price is between $5 and $1,000 at the end of the preceding month. For each day, we delete the 1% of stocks with the highest *ILLIQ*, which are possible outliers.

is -0.103% per month (t = -3.42), -0.019% (t = -0.91), or -0.027% (t = -1.11), respectively, for $LF = PS$, LIU, or $dMILLIQ$. When running time-series regressions of the conditional price of LF risk on $HimaSP_{t-1}$ and $RMrf_t$, we find that the coefficient of $HimaSP_{t-1}$ is -0.081% (t = -1.32), 0.065% (t = 1.39), or -0.317% (t = -1.08), respectively, for $LF = PS$, LIU, or $dMILLIQ$.

A question that comes up is why the conditional β_{IML} outperforms the conditional βs of the other liquidity-based factors, PS and LIU, in the cross-sectional asset pricing tests. We note that Cochrane (1996) and LL select conditioning variables that predict their pricing factors. The reason may be that our conditioning variable, lagged $maSP$, does not predict the factors PS and LIU while it does predict IML as shown earlier. Notably, among the liquidity-based factors tested only IML satisfies the suggested link by Cochrane (1996) and LL between pricing the factor whose conditional risk is being tested and our conditioning variable $maSP$ which captures financial distress. Estimating a time-series regression of $LF_t = PS_t$ or LIU_t on $maSP_{t-1}$, the slope coefficient of $maSP_{t-1}$ is insignificant being, respectively, 0.162 with t = 0.41 and -0.114 with t = -0.33. The slope coefficient of $maSP_{t-1}$ is similarly insignificant when adding $RMrf_t$ as a control variable.

In summary, we find that the positive and significant pricing of the conditional β_{IML} in times of financial distress survives a "horse race" with the conditional βs of other liquidity-based factors and that none of them is significantly priced as the conditional β_{IML} is priced.

4.5. Using as a conditioning variable the *positive* change in *SP*

We employ a different scaling variable, the *rise* in SP_t which indicates worsening of a financial and economic state. Denoting by dSP_t the first-order difference in SP_t, the new conditioning variable is $Z2_t = dSP_t^+$, which equals dSP_t when $dSP_t > 0$ and zero otherwise. We estimate Models (6) and (7) using this scaling variable. The associated cross-sectional test results on β_{IMLZ2}, presented in Table 6, Panel A, are qualitatively similar to those in Table 3 with $Z1_t$.[39]

By the CGS method the mean and precision-weighted mean of γ_{IMLZ2} are 0.013% (t = 3.50) and 0.005% (t = 2.56), respectively. Under the OLS and WLS methods, the mean and precision-weighted mean are also significantly positive. Next, we compute the monthly time-

[39] Employing both dSP_t^+ and dSP_t^- as conditioning variables does not affect the significance of the slope coefficient of β_{IMLZ2}, while the slope coefficient of $\beta_{IMLZ2'}$ based on $Z2_t' = dSP_t^-$ is insignificant, where dSP_t^- equals dSP_t when $dSP_t \leq 0$ and zero otherwise. This indicates that the conditioning of IML_t only by $Z2_{t-1}$ enters the associated conditional asset pricing model significantly.

series of the conditional price of risk of *IML* using the CGS estimates of γs in Model (7) that employs *Z2*. We find that the average conditional price of *IML* risk is 0.080% per month (= 0.96% per year) and significant with t = 2.25. In a time-series regression of the monthly conditional price of *IML* risk on $HidSP_{t-1}^{+}$ and $RMrf_t$, we find that the coefficient of $HidSP_{t-1}^{+}$ is positive at 0.280% per month (=3.36% per year) and highly significant with t = 3.99, where $HidSP_t^{+}$ = 1 if $dSP_t > 0$ and zero otherwise.

In summary, with this scaling variable that proxies for worsening financial distress, the pricing of the conditional β_{IML} remains positive and significant.

INSERT TABLE 6

4.6. Using broker-dealer loans series as a conditioning variable

We employ a proxy measure of funding illiquidity or financial constraint based on loans made by brokers and dealers that include their margin loans. The use of this proxy is motivated by Brunnermeier and Pedersen (2009, p. 2202) who link margin requirements and dealer funding to market liquidity. Here, funding illiquidity is indicated by a decline in broker-dealer loans, which include margin loans, relative to a benchmark series of total loans of brokers and dealers. We use the following series, available from the Federal Reserve Bank of St. Louis. *S1* is the series "Security brokers and dealers; other loans and advances; assets" (SBDOLAA)[40] that includes "margin accounts at brokers and dealers." *S2* is a benchmark loan series, defined as "Security brokers and dealers; loans; liability" (series SBDLL). These series are quarterly, available for the period Q1/1952 to Q4/2017. They are generally upward trending with a sharp decline at the end of 2008 during the most recent financial crisis. The mean ratio *S1*/*S2* is 0.604, the median ratio is 0.619, and the interquartile range is 0.447 to 0.741. The series is highly persistent with a first-order serial correlation of 0.88.

We construct a series of the quarterly change in the ratio of these two series: $dS12_q = (S1/S2)_q - (S1/S2)_{q-1}$ in quarter q. Next, we examine the economic significance of this series by relating it to other economic series. We find the following results. First, $dS12_q$ is negatively correlated with dSP_q, the quarterly change in the yield spread between BAA- and AAA-rated

[40] The definitions are available in the web site of the Federal Reserve Bank of St. Louis for the respective series. The components of the series SBDOLAA are available from this web site https://www.federalreserve.gov/apps/fof/SeriesAnalyzer.aspx?s=FL663069005&t=

corporate bonds (using the average spread over the quarter). In a regression of $dS12_q$ on dSP_q, the slope coefficient is -0.087 with t = -3.92. This means that in times of financial distress, the series that includes broker-dealer margin loans declines relative to the benchmark loan series. Second, broker-dealer margin loans increase (decrease) following a rise (fall) in stock prices. In a regression of $dS12_q$ on $RMrf_{q-1}$, the quarterly market excess return, (and an intercept), the slope coefficient is 0.192 with t = 3.60. Finally, lagged $dS12_q$ negatively and significantly forecasts IML_q, the quarterly compounded monthly IML_t. Regressing IML_q on $dS12_{q\text{-}1}$ (and an intercept), the slope coefficient of $dS12_{q\text{-}1}$ is -0.119 with t = -2.90 and when adding to the model $RMrf_q$ as a control, the slope coefficient of $dS12_{q\text{-}1}$ is -0.112 with t = -2.72. This suggests that a decline in broker-dealer margin loans, which indicates financing constraint, forecasts a rise in the expected illiquidity premium. These results are consistent with Brunnermeier and Pedersen's (2009) theory on the effect of the margin loans and financial constraint on the shadow price of liquidity and on the effect of shocks to market price on subsequent margin loans.

We now estimate Model (6) replacing $Z1_{t-1}$ by $Z3_{t-1} = -dS12_{q-1}$, the value of the series in the quarter that precedes the quarter of month t. We multiply $dS12$ by -1 to make $Z3$ positively related to funding illiquidity and financial distress as are $Z1$ and $Z2$ above. Since the data on $S1$ and $S2$ are available from Q1/1952, the cross-sectional monthly estimation of Model (7) is conducted over the period of August 1957 through December 2017 (725 months). We expect that γ_{IMLZ3} is positive and significant and the average of the associated conditional price of IML risk is positive and significant as in our earlier analyses.

The cross-sectional test results in Table 6, Panel B for γ_{IML} and γ_{IMLZ3} show that γ_{IMLZ3} is positive and significant.[41] The mean and precision-weighted mean of γ_{IMLZ3} under the CGS method are, respectively, 0.019% with t = 2.15 and 0.012% with t = 2.02. Under the OLS method, the mean and precision-weighted mean of γ_{IMLZ3} are 0.018% (t = 2.13) and 0.012% (t = 2.03), respectively. The results are similar under the WLS method. With the CGS estimates of γs in Model (7), we compute the monthly conditional price of IML risk as in Equation (5) and find that its average is positive at 0.147% per month (=1.76% per year) and is highly significant with t = 4.28. When a running time-series regression of the conditional price of IML risk on $HiZ3_{t-1}$

[41] Regarding the pricing of the other factors' βs, we find that only the means and precision-weighted means of γ_{HML} are positive and marginally significant at the 10% level under all three estimation methods.

and $RMrf_t$, we find that the coefficient of $HiZ3_{t-1}$ is positive at 0.146% per month (=1.75% per year) and significant with t = 2.13, where $HiZ3_t$ = 1 if $Z3_t$ is positive and zero otherwise.

In conclusion, using a loan-based conditioning variable that includes broker-dealer's margin loans, we find that the conditional systematic risk of the *IML* factor is positively and significantly priced in times of financial distress and funding illiquidity, which is consistent with its pricing evidence when using the corporate bond yield spread.

5. Conclusion

This paper tests whether the market is pricing the *conditional* systematic risks of the illiquidity return premium factor, denoted *IML*. The conditional *IML* systematic risk (= β_{IML}) and its conditional premium are modeled to be functions of financial distress and funding illiquidity using as a proxy the yield differential between BAA- and AAA-rated corporate bonds. We find that expected returns are higher for stocks with greater sensitivity to the illiquidity return premium factor *IML* in times of greater financial distress and funding illiquidity. This pricing evidence of conditional β_{IML} remains robustly positive and significant after controlling for the conditional and unconditional βs of the FFC return factors. Further, we find that the pricing of the conditional β_{IML} is positive and significant in the presence of the conditional βs of several commonly used illiquidity-based factors and of stock characteristics including size and illiquidity. Our finding also holds when using an alternative proxy for financial distress and funding illiquidity: the (negative of the) difference between the loans made by brokers and dealers, which include their margin loans, and a benchmark of their total loans. In all, stock's greater exposure to the *IML* factor in times of financial distress is positively and significantly priced.

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Table 1: Time-series estimation results for the illiquidity return premium factor *IML* (*Illiquid-Minus-Liquid)*

IML is the differential return between the highest-illiquidity and lowest-illiquidity quintile portfolios of stocks. We sort stocks by either one of the two measures of illiquidity: (1) *ILLIQ*, the average daily values of |return|/dollar volume, or (2) *ZERO*, the proportion of zero-return or no-trading days. Both measures are calculated over a rolling window of twelve months. In each month, stocks are first sorted into three portfolios by the standard deviation (*StdDev*) of their daily returns, and within each tercile portfolio, stocks are sorted into five portfolios by *ILLIQ* or by *ZERO*. This produces 15 (3x5) portfolios for each illiquidity measure. Value-weighted average returns are calculated for each portfolio for each month *t* using the ranking done in month *t*-2 (i.e., skipping one month after the portfolio formation period). The *IML* for each illiquidity measure is the average return on the three highest-illiquidity quintile portfolios (across the volatility portfolios) minus the average return on the three lowest-illiquidity quintile portfolios. This produces IML_{ILLIQ} and IML_{ZERO}. Finally, we define $IML = (IML_{ILLIQ} + IML_{ZERO})/2$. The returns are in monthly percentage points. We use NYSE/AMEX stocks and apply some filters (details are provided in the text). Estimations are performed for the entire sample period of 71 years (852 months), January 1947 to December 2017, and for each of its two equal subperiods.

Panel A: Statistics on *IML* returns. The *p*-values are from tests of whether the fraction of positive returns is 0.50, the result due to chance. All other numbers in parentheses (in all panels) are *t*-statistics, employing robust standard errors (White, 1980).

Panel B: The intercept $alpha_{IML}$ and the β coefficients of the FFC factors obtained from the regression model

$$IML_t = alpha_{IML} + \beta_{RMrf}*RMrf_t + \beta_{SMB}*SMB_t + \beta_{HML}*HML_t + \beta_{UMD}*UMD_t + \varepsilon_t\,, \quad (1)$$

RMrf is the market return in the excess of the risk-free rate, *SMB* and *HML* are the Fama and French (1993) factors of size and the book-to-market (BE/ME) ratio, and *UMD* is the Carhart (1997) momentum factor. (We denote them as FFC factors.) The calculation of *t*-statistics (in parentheses) employs robust standard errors (White, 1980).

Panel C: Out-of-sample, one-month-ahead rolling $alpha_{IML,t}$. Model (1) is estimated over a rolling window of 60 months beginning in January 1947. For month 61, $alpha_{IML,t} = IML_t - [\beta_{RMrf,t-1}*RMrf_t + \beta_{SMB,t-1}*SMB_t + \beta_{HML,t-1}*HML_t + \beta_{UMD,t-1}*UMD_t]$, using the β values estimated from the previous 60-month estimation window. The values of the out-of-sample $alpha_{IML,t}$ begin in January 1952.

Panel D: Estimates of Model (1) separately for IML_{ILLIQ} and IML_{ZERO}.

	1947–2017	**1947-6/1982**	**7/1982–2017**
Panel A: Statistics on *IML*			
Mean	0.319 (3.43)	0.385 (2.77)	0.254 (2.05)
Median	0.277	0.295	0.227
Fraction positive	0.550	0.549	0.552
Serial correlation	-0.057	-0.040	-0.079
N	852	426	426
Panel B: Regression of *IML* on the FFC factors			
$\boldsymbol{alpha_{IML}}$	**0.341 (5.47)**	**0.441 (4.94)**	**0.288 (3.33)**
β_{RMrf}	-0.287 (-15.59)	-0.328 (-13.00)	-0.234 (-10.08)
β_{SMB}	0.606 (18.87)	0.595 (13.56)	0.574 (12.88)
β_{HML}	0.404 (12.34)	0.468 (8.02)	0.366 (10.00)
β_{UMD}	-0.078 (-3.77)	-0.206 (-5.37)	-0.006 (-0.26)
R^2	0.61	0.66	0.61
Panel C: One-month-ahead rolling $alpha_{IML,t}$			
Mean $\boldsymbol{alpha_{IML,t}}$	**0.356 (5.87)**	**0.487 (5.36)**	**0.242 (3.00)**
Median	0.330	0.480	0.293
Fraction positive	0.587	0.617	0.561
Serial correlation	0.073	0.124	0.017
N	792	366	426
Panel D: Estimated intercepts (*alpha*) of Model (1) for IML_{ILLIQ} and IML_{ZERO}			
$alpha_{ILLIQ}$	0.391 (6.00)	0.500 (5.66)	0.328 (3.57)
$alpha_{ZERO}$	0.291 (4.03)	0.382 (3.75)	0.247 (2.41)
Both include FFC factors	Yes	Yes	Yes

Table 2: Summary statistics of the variables

This table presents summary statistics for the seven β coefficients and six stock characteristics that are calculated for each stock for each month over 66 years, from January 1952 through December 2017. The β coefficients are estimated from the following time-series regression model over a rolling window of 60 months for each stock *j*, with the first window being January 1947 to December 1951:

$$(r_j\text{-}rf)_t = \beta_{0j} + \beta_{RMrf,j}*RMrf_t + \beta_{SMB,j}*SMB_t + \beta_{HML,j}*HML_t + \beta_{UMD,j}*UMD_t + \boldsymbol{\beta_{IML,j}*IML_t}$$
$$+ \boldsymbol{\beta_{IMLZ1,j}*IML_t*Z1_{t-1} + \beta_{Z1,j}*Z1_{t-1}}. \quad (6)$$

The dependent variable is the monthly return on stock *j*, $r_{j,t}$, in excess of the risk-free rate rf_t. The first four factors are those of Fama and French (1993) and Carhart (1997) (see Table 1). The variable *IML* is the return on the illiquid-minus-liquid portfolios (see Table 1). $Z1_t$ is the differential yield between BAA- and AAA-rated corporate bonds (denoted SP_t) in excess of the moving average over the preceding ten years. As for stock characteristics, *ILLIQma* is stock illiquidity (see Table 1), mean adjusted by division by the mean of *ILLIQ* values across all the stocks used in the monthly cross-sectional regressions, and *StdDev* is return volatility, measured by the standard deviation of daily returns. Both *ILLIQ* and *StdDev* are calculated from daily data over a twelve-month rolling window. The variable *BM* (in logarithm) is the book-to-market ratio, using the book value from the firm's annual financial report known as of the end of the previous fiscal year and the market value as of December of the year before the year of analysis. The variable *Size* (in logarithm) is the market capitalization and *R12lag* is the lagged cumulative stock return over past eleven months. These stock characteristics are lagged, skipping one month, so that, e.g., the observation for January, 1952 is obtained from the period that ends on November, 1951. The variable *R1lag* is the one-month lagged stock return.

The table presents the averages of the monthly cross-stock mean and standard deviation, and of the monthly pairwise cross-stock correlations among the variables that are used in that month's cross-sectional regression. For the right panel, we focus on the three liquidity-based variables: β_{IML}, β_{IMLZ1}, and *ILLIQma*.

	Average of cross-sectional…		Average of cross-sectional pairwise correlations between…		
Variable	Mean	Std. Dev.	β_{IML}	β_{IMLZ1}	*ILLIQma*
β_{RMrf}	1.023	0.426	0.261	0.030	-0.056
β_{SMB}	0.355	0.926	-0.587	-0.006	0.186
β_{HML}	0.199	0.727	-0.394	-0.017	0.050
β_{UMD}	-0.055	0.430	0.175	-0.054	-0.011
β_{IML}	0.024	1.104	1.000	0.070	0.080
β_{IMLZ1}	0.054	2.378	0.070	1.000	0.015
β_{Z1}	0.000	0.053	0.016	-0.136	-0.016
ILLIQma	1.000	1.757	0.080	0.015	1.000
StdDev	0.019	0.006	-0.195	0.015	0.214
BM	-0.439	0.629	0.046	0.013	0.231
Size	20.295	1.450	-0.058	-0.018	-0.642
R12lag	0.131	0.240	0.016	0.015	0.014
R1lag	0.011	0.077	0.003	0.011	0.012

Table 3: Pricing of stock systematic risks and characteristics in the cross-section

This table presents the test results of the Fama-Macbeth monthly cross-sectional regressions of Model (7) with individual stock returns. For each month *s*, we estimate a cross-sectional regression of stock excess returns $(r_j - rf)_s$ on the seven β coefficients that are estimated by Model (6) and on the six stock characteristics (see Table 2):

$$(r_j - rf)_s = \gamma_{0,s} + \gamma_{RMrf,s}*\beta_{RMrf,j,s-2} + \gamma_{SMB,s}*\beta_{SMB,j,s-2} + \gamma_{HML,s}*\beta_{HML,j,s-2} + \gamma_{UMD,s}*\beta_{UMD,j,s-2} + \boldsymbol{\gamma_{IML,s}*\beta_{IML,j,s-2}} \\ + \boldsymbol{\gamma_{IMLZ1,s}*\beta_{IMLZ1,j,s-2} + \gamma_{Z1,s}*\beta_{Z1,j,s-2}} + \delta_{1,s}*ILLIQma_{j,s-2} + \delta_{2,s}*StdDev_{j,s-2} + \delta_{3,s}*BM_{j,s-2} \\ + \delta_{4,s}*Size_{j,s-2} + \delta_{5,s}*R12lag_{j,s-2} + \delta_{6,s}*R1lag_{j,s-1}. \quad (7)$$

The model is estimated across stocks over the period from January 1952 through December 2017, that is, 792 months. We present the mean of each slope coefficient and the precision-weighted ("wtd") mean where the weight is the reciprocal of the standard error of the slope coefficient. We employ three estimation methods: (1) ordinary least squares (OLS), (2) the bias-correcting method of Chordia, Goyal and Shanken (2017) (CGS), and (3) weighted least square (WLS), following Asparouhova et al. (2010). The slope coefficients are in percentages. The corresponding *t*-statistics are presented in parentheses. The *Avg adj.* R^2 is the average of monthly *adjusted* R^2 values of cross-sectional regressions.

	Estimation method					
	OLS		CGS		WLS	
Coefficient of	Mean	Wtd mean	Mean	Wtd mean	Mean	Wtd mean
β_{RMrf}	0.206 (1.33)	0.138 (0.72)	0.163 (0.99)	0.111 (0.47)	0.203 (1.31)	0.135 (0.69)
β_{SMB}	0.048 (0.92)	0.033 (0.73)	0.079 (1.62)	0.052 (1.23)	0.049 (0.92)	0.032 (0.71)
β_{HML}	0.082 (1.74)	0.071 (1.72)	0.080 (1.75)	0.075 (1.84)	0.071 (1.51)	0.063 (1.54)
β_{UMD}	-0.027 (-0.40)	0.009 (0.16)	0.020 (0.29)	0.034 (0.61)	-0.020 (-0.30)	0.014 (0.25)
$\boldsymbol{\beta_{IML}}$	**0.011** **(0.23)**	**0.005** **(0.13)**	**0.039** **(0.89)**	**0.026** **(0.65)**	**0.005** **(0.12)**	**0.001** **(0.03)**
$\boldsymbol{\beta_{IMLZ1}}$	**0.062** **(3.17)**	**0.040** **(3.20)**	**0.060** **(3.52)**	**0.043** **(3.72)**	**0.062** **(3.21)**	**0.041** **(3.26)**
$\boldsymbol{\beta_{Z1}}$	**-0.330** **(-0.49)**	**-0.572** **(-1.31)**	**-0.441** **(-1.15)**	**-0.333** **(-0.89)**	**-0.221** **(-0.33)**	**-0.517** **(-1.19)**
ILLIQma	0.041 (2.58)	0.029 (2.52)	0.040 (2.57)	0.029 (2.51)	0.036 (2.30)	0.025 (2.23)
StdDev	-27.935 (-4.25)	-29.989 (-4.98)	-27.536 (-4.12)	-29.931 (-4.93)	-26.834 (-4.07)	-29.002 (-4.81)
BM	0.098 (2.45)	0.090 (2.52)	0.093 (2.32)	0.083 (2.29)	0.107 (2.68)	0.097 (2.71)
Size	-0.086 (-3.82)	-0.077 (-3.67)	-0.081 (-3.61)	-0.074 (-3.48)	-0.087 (-3.85)	-0.079 (-3.70)
R12lag	1.116 (6.77)	0.946 (6.16)	1.036 (6.31)	0.874 (5.80)	1.149 (6.99)	0.976 (6.36)
R1lag	-5.047 (-14.99)	-4.811 (-15.15)	-5.151 (-15.48)	-4.884 (-15.44)	-4.907 (-14.68)	-4.650 (-14.87)
Avg adj. R^2	10.96%		10.52%		10.98%	

Table 4: Pricing over two subperiods of stock systematic risks and characteristics
This table replicates the asset pricing tests presented in Table 3 with the statistics for the slope coefficients presented separately for two equal subperiods. The results are based on the estimations according to the CGS bias-correcting method. The slope coefficients are in percentages and their *t*-statistics are presented in parentheses. The variables and the estimation procedure are the same as those in Table 3 and are explained in the legend there.

	Subperiod I: 1952 through 1984		**Subperiod II:** 1985 through 2017	
Coefficient of	Mean	Wtd mean	Mean	Wtd mean
β_{IML}	0.034 (0.51)	0.013 (0.25)	0.044 (0.77)	0.038 (0.64)
$\boldsymbol{\beta_{IMLZ1}}$	**0.051 (2.00)**	**0.037 (2.38)**	**0.069 (3.07)**	**0.049 (2.86)**
Avg adj. R^2	11.00%		10.42%	

Table 5: Pricing of the conditional $\boldsymbol{\beta_{IML}}$ in the presence of other liquidity-based $\boldsymbol{\beta}$s

This table presents the test results of Fama–MacBeth monthly cross-sectional regressions of stock returns. We first add to Model (6) $\beta_{LF}*LF_t + \beta_{LFZ1}*LF_t*Z1_{t-1}$ where LF_t is one of the following three liquidity-based factors: (1) *PS*, the traded factor of Pastor and Stambaugh (2003) (available from Lubos Pastor's homepage), the value-weighted average return on stocks with high exposure to innovations in their aggregate liquidity relative to that on stocks with low exposure (using decile portfolios); (2) *LIU*, the traded factor of the return premium on high-minus-low illiquidity portfolio using Liu's (2006) measure based on non-trading days and turnover and obtained from the author; and (3) *dMILLIQ*, a non-traded factor of the first-order changes in the monthly value-weighted market illiquidity (in logarithm). Then, we add to Model (7) β_{LF} and β_{LFZ1} and estimate their slope coefficients γ_{LF} and γ_{LFZ1} in cross-sectional regressions.

To save space, the table presents only the slope coefficients that are related to the liquidity βs. The estimation of those slope coefficients includes all the other βs and six stock characteristics in Model (7). The estimation employs the CGS bias-correcting method. Explanations of the estimation method and the test statistics are given in the legend of Table 3.

	Coefficient of	Mean	Wtd mean
$LF_t = PS_t$. Data period: 2/1973–2017	β_{IML}	0.059 (1.15)	0.046 (1.03)
	$\boldsymbol{\beta_{IMLZ1}}$	**0.087 (3.83)**	**0.059 (3.54)**
	B_{LF}	-0.076 (-0.96)	-0.064 (-0.90)
	$\boldsymbol{B_{LFZ1}}$	**-0.039 (-1.29)**	**-0.028 (-1.30)**
$LF_t = LIU_t$. Data period: 1952–2014	β_{IML}	0.014 (0.31)	0.002 (0.05)
	$\boldsymbol{\beta_{IMLZ1}}$	**0.055 (3.13)**	**0.042 (3.49)**
	B_{LF}	-0.045 (-0.74)	-0.020 (-0.40)
	$\boldsymbol{B_{LFZ1}}$	**0.012 (0.57)**	**0.026 (2.12)**
$LF_t = dMILLIQ_t$. Data period: 1952–2017	β_{IML}	0.041 (0.90)	0.029 (0.71)
	$\boldsymbol{\beta_{IMLZ1}}$	**0.062 (3.42)**	**0.042 (3.48)**
	B_{LF}	-0.348 (-1.08)	-0.177 (-0.60)
	$\boldsymbol{B_{LFZ1}}$	**-0.057 (-0.55)**	**-0.052 (-0.71)**

Table 6: Pricing of the conditional β_{IML} with alternative conditioning variables

This table presents the test results of Fama–MacBeth monthly cross-sectional regressions using two alternative conditioning variables that replace *Z1* in Models (6) and (7) described in the legends of Tables 2 and 3.

Panel A: $Z2_t = dSP_t^+$, the positive value of $dSP_t = SP_t - SP_{t-1}$ (it is zero otherwise), where SP_t is the yield spread between BAA- and AAA-rated corporate bonds.

Panel B: $Z3_q = -dS12_q = -\left((S1/S2)_q - (S1/S2)_{q-1}\right)$, the change in the ratio of two quarterly loan series for quarter *q*. $S1_q$ is broker-dealers loans that include their margin loans (SBDOLAA) and $S2_q$ is the benchmark loans (SBDLL). These series are available since Q1/1952.

The data source for all series is the Federal Reserve Bank of St. Louis. To save space, we present only the test results for the slope coefficients of the βs of *IML*-related variables. Explanations of the estimation methods and the test statistics are given in the legend of Table 3.

	Estimation method					
	OLS		CGS		WLS	
Coefficient of	Mean	Wtd mean	Mean	Wtd mean	Mean	Wtd mean
Panel A: $Z2_t = dSP_t^+$, the value of the rise in the corporate bond yield spread.						
β_{IML}	0.040 (0.85)	0.030 (0.74)	0.033 (0.69)	0.017 (0.42)	0.031 (0.66)	0.024 (0.59)
$\boldsymbol{\beta_{IMLZ2}}$	**0.014** **(3.26)**	**0.005** **(2.57)**	**0.013** **(3.50)**	**0.005** **(2.56)**	**0.014** **(3.20)**	**0.005** **(2.41)**
Avg Adjusted R^2	11.04%		10.63%		11.07%	
Panel B: $Z3_q$ is the (negative of the) change in broker-dealers loans that include their margin loans relative to the benchmark of broker-dealers all loans series.						
β_{IML}	0.084 (1.79)	0.066 (1.59)	0.088 (1.85)	0.070 (1.69)	0.079 (1.67)	0.062 (1.50)
$\boldsymbol{\beta_{IMLZ3}}$	**0.018** **(2.13)**	**0.012** **(2.03)**	**0.019** **(2.15)**	**0.012** **(2.02)**	**0.018** **(2.12)**	**0.012** **(2.02)**
Avg adj. R^2	10.82%		10.44%		10.83%	

Table A.1: Pricing of the conditional β_{IML} in the presence of conditional βs of the FFC factors

This table presents the test results of an extended model of Model (7) with the conditional βs of the FFC factors. Explanations of the estimation method and the test statistics are provided in the legends of Tables 2 and 3. First, Model (6) is augmented by $\beta_{FFZ1,j}{*}FF_t{*}Z1_{t-1}$, where $FF_t = RMrf_t$, SMB_t, HML_t, or UMD_t. The estimated β_{FFZ1} is then added to Model (7), and finally its slope coefficient γ_{FFZ1} is estimated with all the other variables in Model (7).

To save space, the table presents only the slope coefficients that are related to the βs of *IML* and *FF* factors. The estimation employs the CGS bias-correcting method.

	Coefficient of	Mean	Wtd mean
$FF_t = RMrf_t$	β_{IML}	0.042 (0.93)	0.034 (0.85)
	$\boldsymbol{\beta_{IMLZ1}}$	**0.054 (3.12)**	**0.039 (3.28)**
	B_{FF}	0.181 (2.13)	0.117 (1.51)
	$\boldsymbol{B_{FFZ1}}$	**-0.017 (-0.47)**	**-0.041 (-1.47)**
$FF_t = SMB_t$	β_{IML}	0.052 (1.14)	0.041 (1.04)
	$\boldsymbol{\beta_{IMLZ1}}$	**0.061 (3.50)**	**0.043 (3.66)**
	B_{FF}	0.077 (1.54)	0.057 (1.31)
	$\boldsymbol{B_{FFZ1}}$	**0.020 (1.35)**	**0.010 (0.91)**
$FF_t = HML_t$	β_{IML}	0.030 (0.66)	0.019 (0.47)
	$\boldsymbol{\beta_{IMLZ1}}$	**0.058 (3.48)**	**0.042 (3.55)**
	B_{FF}	0.069 (1.46)	0.066 (1.57)
	$\boldsymbol{B_{FFZ1}}$	**0.030 (1.78)**	**0.021 (1.79)**
$FF_t = UMD_t$	β_{IML}	0.026 (0.58)	0.020 (0.51)
	$\boldsymbol{\beta_{IMLZ1}}$	**0.059 (3.41)**	**0.041 (3.49)**
	B_{FF}	0.002 (0.03)	0.031 (0.53)
	$\boldsymbol{B_{FFZ1}}$	**-0.058 (-1.83)**	**-0.008 (-0.50)**