The Global Credit Spread Puzzle*

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Abstract

Using security-level credit spread data in eight developed economies, we document a large cross-country difference in credit spreads conditional on credit ratings and other default risk measures. The standard benchmark structural models not only have difficulty matching credit spreads but also fail to explain the cross-country variation in spreads as well as the dynamic behavior of credit spreads. Since this cross-country variation is positively related to illiquidity measures, we implement an extended structural model that incorporates endogenous liquidity in the secondary market, and find that this model largely explains credit spreads in cross sections and over time. Therefore, default risk itself unlikely explains corporate credit spreads.

JEL Classification: G12, G13

Keywords: Corporate credit spreads, Credit spread puzzle, Structural credit risk models, the Merton model, the Black and Cox model, CDS, Fixed income asset pricing

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1 Introduction

How are corporate bonds outside the U.S. priced? Despite the rapid growth in the corporate bond markets in developed economies (see Figure 1), we know little about the pricing of these bonds. Among the numerous factors that potentially affect credit spreads, default risk is a natural starting point, and structural models of risky debt stemming from Merton (1974) provide a sensible estimate of default risk based on observed proxies for risk. Thus, in this paper, we study the performance of structural models to explore how they may explain non-U.S. corporate credit spreads.

We focus on the standard Black and Cox (1976) model as well as its extension with endogenous (corporate bond) illiquidity in this analysis.[1] We empirically test these two models using security-level bond price data on domestic corporate bonds in eight developed countries—Australia, Canada, France, Germany, Italy, Japan, U.K., and the U.S.—from 1997 to 2017.

There are three main empirical findings from our analysis of the standard Black-Cox model. First, we find strong evidence that the Black-Cox model has difficulty matching individual corporate bond spreads. For example, the average pricing error of the model is all below -40 basis points (bps) for AA+ bonds except for Japan. The average percentage pricing error of the model for BBB bonds is mostly negative, and below -31% except for Japan and France. For Australia, the average percentage pricing error ranges from -74% for BBB bonds to -97% for A bonds, depending on how default boundary is determined. In other words, the Black-Cox model clearly has an accuracy problem in predicting spreads, regardless of the countries considered in our sample.

Second, we also observe that there is significant heterogeneity in credit spreads across the eight countries controlling for credit ratings. For example, on average, credit spreads for BBB-rated bonds are 41 bps in Japan, while they are 121 bps in Germany, 166 bps in the U.S., and 231 bps in Australia. On the other hand, inputs to the Black-Cox model that measure issuer's default risk (e.g., leverage, asset volatility and payout ratio) are similar across countries.

Importantly, we find that the Black-Cox model fails to explain the cross-section of investment-grade (IG) credit spreads, regardless of the methods used to determine default boundary. For instance, if we implement one method that can generate large enough credit spreads to match the data in the U.S., then the BC model overpredicts corporate spreads in Japan, but underpredicts those in Australia. The key reason for this failure is that, conditional on credit ratings, firms in each country have relatively similar fundamentals to each other, despite the difference in credit spreads.

Third, we find that the model has difficulty capturing the time series dynamics of corporate bond spreads. For instance, correlations between the model and observed median bond spreads at the country level vary considerably across the countries, ranging from 0.06 for Canada to around

[1] The Black-Cox model is examined in several recent studies that use the U.S. data, including Bao (2009), Bao and Hou (2017), Feldhütter and Schaefer (2018), Bai, Goldstein, and Yang (2019), and Huang, Shi, and Zhou (2019). We also consider the Merton (1974) model in the Internet Appendix.

0.86 for UK and the U.S. in the baseline case.

Overall, we find robust evidence that the standard Black-Cox model not only has substantial pricing errors in predicting individual corporate bond spreads but also has difficulty explaining the differences in these spreads across countries. These findings imply that default risk alone is unlikely to sufficiently explain credit spreads at least in our sample of global corporate bonds—and collectively are referred to as the "global credit spread puzzle."

To ensure that our results are not driven by our specific bond data, we repeat our empirical evaluation of the Black-Cox model using 5-year CDS spreads outside the U.S. Despite this smaller sample, we confirm that qualitatively similar results hold for CDS spreads, although quantitatively, the cross-country difference in CDS spreads is smaller than that in corporate credit spreads.

To determine what else might explain cross-country differences in credit spreads aside from default risk, we test several alternative explanations. We rule out across country heterogeneity in loss given default, the probability of default, or investors' risk aversion as potential explanations. We also do not find evidence that prominent factors in international finance can explain the difference in credit spreads in our sample.

However, we find evidence that differences in corporate bond illiquidity explain some of the cross-country difference in credit spreads. Specifically, using the country-specific illiquidity measures of bid-ask spreads, TED spreads and yield-curve fitting errors, we find that the difference between observed credit spreads and the Black-Cox model's prediction is positively correlated with these three illiquidity measures.

Given the empirical evidence for the importance of secondary market liquidity, we propose an extended Black-Cox model that incorporates illiquidity in bond pricing. Specifically, we aim to capture the over-the-counter (OTC) structure of corporate bond markets by using the search and bargaining mechanism of Duffie, Gârleanu, and Pedersen (2005) and He and Milbradt (2014). The resultant endogenous liquidity is incorporated into the baseline Black-Cox model with a novel approach, such that the model-implied default probability is not altered. As such, we can quantify the incremental contribution of search friction under the notion of the credit spread puzzle.

We then conduct an empirical analysis of this extended Black-Cox model with endogenous illiquidity using the same sample of corporate bonds as in the analysis of the standard Black-Cox model. We find that incorporating endogenous illiquidity in the standard model improves the model performance significantly. Specifically, we find that the model-implied liquidity component in yield spreads performs remarkably well in filling the gap between the observed corporate bond spreads and the Black-Cox model-implied spreads in terms of both overall levels and time-series. Thus, our results suggest that incorporating search frictions into standard structural models can help explain the global credit spread puzzle.

To summarize, this paper contributes to the literature in at least three aspects. First, we conduct among the first an empirical analysis of structural credit risk models using a sample of individual corporate bonds from eight developed countries. Second, we provide evidence that credit risk alone unlikely can explain individual corporate bond spreads. Third, we incorporate search into the standard Block-Cox model within the framework of He and Milbradt (2014) and present the first empirical analysis of this type of models in the literature. Importantly, we find that incorporating search improves the model performance significantly and helps resolve the global credit spread puzzle.

The rest of the paper is organized as follows. In the next section we review the related literature. In Section 3, we describe the data sets for our empirical analysis. In Section 4, we introduce the Black-Cox model and its extension allowing for endogenous illiquidity. In Section 5 we discuss the implementation of these two structural models as well as the estimation of model parameters. We compare the predicted credit spreads with observed spreads, and evaluate the model's performance in Section 6. Section 7 concludes.

2 Related Literature

This paper relates to the literature that explains corporate credit spreads using structural models of risky debt in the U.S. market. One stream of this literature, going back to Jones, Mason, and Rosenfeld (1984), focuses on implications of structural models under the risk-neutral measure using alternative empirical methodologies. See, e.g., Jones, Mason, and Rosenfeld (1984); Eom, Helwege, and Huang (2004); Ericsson and Reneby (2005); Schaefer and Strebulaev (2008); Bao and Pan (2013); Bao and Hou (2017); Culp, Nozawa, and Veronesi (2018); Huang, Shi, and Zhou (2019).

Another stream of research explores model implications under both the risk-neutral and physical measures, such as studying the pricing performance of structural models by calibrating them to historical default losses. To resolve the credit spread puzzle documented in Huang and Huang (2012), many studies propose various economic channels to account for the credit component of yield spreads by incorporating additional sources of default premium. Examples include Bao (2009); Chen, Collin-Dufresne, and Goldstein (2009); Chen (2010); Bhamra, Kuehn, and Strebulaev (2010); Christoffersen, Du, and Elkamhi (2017); Du, Elkamhi, and Ericsson (2019); McQuade (2018); Shi (2019).[2] On the other hand, He and Milbradt (2014) and Chen, Cui, He, and Milbradt (2018) incorporate OTC market search frictions into structural models to capture the non-credit component of yield spreads.

We contribute to the above two streams of literature in at least two aspects. First, we examine both corporate bond and CDS markets (in seven developed economies) outside the U.S. Second, we consider not only standard structural models but also the type of models developed by He and Milbradt (2014) and Chen, Cui, He, and Milbradt (2018), the latter of which have not been

[2] Gourio (2012), Boyarchenko (2012), and Albagli, Hellwig, and Tsyvinski (2013) consider alternative economic mechanisms to generate credit spread levels consistent with historical data. They do not adopt the contingent-claim approach to modeling credit risk and thus carry limited implications for the credit spread puzzle.

empirically examined in the literature. Among other things, doing so allows us to obtain a model-based illiquidity measure across different credit markets globally.

One recent, notable development in the credit spread puzzle literature is the debate on the performance of the Black-Cox model in the U.S. credit market. Interestingly, while Bao (2009) finds that the Black-Cox model underestimates the U.S. corporate credit spreads, in a recent intriguing study Feldhütter and Schaefer (2018) report that the model performs well in matching spreads on investment-grade (IG) bonds in the U.S. However, Bai, Goldstein, and Yang (2019) argue that Feldhütter and Schaefer (2018)'s main findings are sensitive to changes in the model's calibration method. Specifically, they argue that the Feldhütter-Schaefer calibration method overestimates the market leverage ratio for firms in the CCC-C rating category and, as a result, also overestimates default boundary. Feldhütter and Schaefer (2019) address this issue by excluding C-rated bonds from their estimation sample.

We contribute to this debate by providing evidence based on seven different credit markets outside the U.S. Furthermore, in addition to the methods of Feldhütter and Schaefer (2018) and Bai et al. (2019), we propose and implement an alternative method for determined default boundary. Importantly, we show that the main conclusions of this study hold regardless of which one of these three methods is used.

Several studies empirically decompose observed corporate yield spreads into default-premium and liquidity-premium components using a reduced-form approach (Longstaff et al. 2005) or linear regressions (Dick-Nielsen et al. 2012).[3] Our decomposition is based on the structural model and differs from the prior studies in at least two aspects. First, these studies take observed yield spreads as inputs to the model, so the resulting estimates of liquidity components incorporate the bond pricing information. The structural approach adopted here generates these estimates without using such information and thus is not biased in favor of the liquidity-based explanation of the credit spread puzzle. Second, this approach can accommodate the interdependence between liquidity and default components (Friewald et al. 2012) in a unified manner.

There are fewer papers that examine corporate bond markets outside the U.S. Liu (2016) uses international corporate bond data to study the diversification benefit across countries. Valenzuela (2016) examines the role of rollover risk in international bonds using regression models, while Kang and Pflueger (2015) provide evidence for the link between inflation risk and corporate bond prices using data on international bond indexes. None of these papers, however, test structural credit risk models for domestic issuers outside the U.S., which is the focus of this paper.

Liao (2019) studies the relationship in corporate bond yields of the same issuer in different currencies, and finds that the cross-currency variation in spreads is related to transaction costs measured by the violation of the covered interest parity. Though our finding on the cross-country variation in credit spreads is consistent with Liao (2019), our explanation for the variation is

[3] Beber et al. (2009) and Schwarz (2018) perform similar decompositions of sovereign yield spreads.

different. Instead of relying on limits to arbitrage, we calibrate an extended structural model based on no-arbitrage condition, and show that our model with the domestic bond market search friction works well in explaining the observed credit spreads. Therefore, from our perspective, corporate bonds in both Australia (that have high spreads) and Japan (that have low spreads) are fairly priced once the pricing model is correctly specified, and the observed difference in credit spreads does not imply an arbitrage opportunity.

3 Data

We use month-end prices for corporate bonds in the ICE Bank of America Merrill Lynch Global Corporate Index and High Yield Index ("Merrill Lynch data") from Mercury, the client portal of Bank of America Merrill Lynch (ML). The sample period is from January 1997 to December 2017 except for Italy and Australia whose samples start from 2003 and 2007, respectively. Merrill Lynch data covers corporate bonds denominated in six international currencies: Australian dollars, British pounds, Canadian dollars, Euro (and former Euro-area currencies such as Deutsche Mark), Japanese yen and U.S. dollars. We focus on domestic issues in domestic currencies, which yields seven advanced, non-U.S. economies for our analysis: Australia, Canada, France, Germany, Italy, Japan, and the U.K. As of December 2017, these seven countries account for 30% and 19% of the market values of corporate bonds in the ML Global Corporate Index and ML High Yield Index, respectively; the U.S. accounts for about 50% and 51%, respectively. Note that we focus on domestic issues in each country to provide out-of-sample evidence for the credit spread puzzle; if we include foreign currency-denominated bonds issued by U.S. firms, then our empirical results will resemble the ones in the U.S. mechanically.

The ML database imposes the minimum maturity of one year and minimum face values, which varies across currencies.[4] For bond characteristics, Merrill Lynch data provides the credit rating, maturity date, and coupon of each issue.

We then merge the bond data with firm and stock data from Compustat Global or Compustat NA for Canada, which provides balance sheet information and stock return volatility. We link the bond-level observations and firm-level observations based on each issuer's name. We then use Compustat name history data to track the history of names for each identifier (gvkey), then use the Levenshtein Algorithm to find a candidate match, and manually verify each match. For firms with multiple stock issues, we remove duplicate observations for shares listed in multiple stock exchanges. If a firm has multiple share classes, we add them up to compute the market value of firm equity, but we take the value-weighted average across shares in computing stock returns (which we use in computing volatility). To reduce the effect of outliers, we drop the observation if the

[4] For the investment-grade index, the minimum face values are AUD 100 million, CAD 100 million, EUR 250 million, JPY 20 billion, GBP 100 million, and USD 250 million. For the high-yield index, the minimum are USD 250 million, EUR 250 million, GBP 100 million, or CAD 100 million. The high-yield index does not include Australia and Japan, given the lack of the market activity.

book-to-market ratio of the stock is more than 8 (the 99 percentile of the distribution) or less than 0.05 (the 1 percentile).

Next, we use Bloomberg to identify callability, seniority, and security of the bonds. We choose senior, unsecured, noncallable bonds issued by nonfinancial issuers. Bloomberg also provides information on large shareholders of the bond issuers, which allows us to screen out state-owned firms. Specifically, we drop firms for which government equity ownership is more than 50%. We also decrease a firm's credit rating by one notch (e.g., change from AA to AA-) if the ownership ratio is between 20% and 50%, following Moody's (2014).[5]

To compare our results with those in the U.S., we merge the Lehman Brothers Fixed Income database and Merrill Lynch U.S. Corporate Bond database to obtain month-end prices of U.S. corporate bonds from 1987 to 2015. The choice of the beginning of the sample period follows Feldhütter and Schaefer (2018). We use CRSP for stock price and Compustat NA for accounting information for U.S. bonds.

In Table A1 in Appendix IV, we describe our sample selection process. In the original data, there are 8,610 bonds that are offered in seven (non-U.S.) countries of our interest, and that have at least 24 monthly observations. Among those, 3,983 bonds are issued by public firms appearing in Compustat. Of those bonds, we focus on noncallable, senior unsecured bonds in the nonfinancial sector, which yields our final sample of 2,173 bonds issued by 364 firms with 133,993 bond-month observations. It is noteworthy that C-rated issues become rather rare as the focus is shifted from the US to international bond markets. Outside of US, there are merely 36 issues and totally 579 monthly observations in our sample falling into the CCC-C rating category. Therefore, the debate about the market-to-book ratios for C-rated bonds does not cause a serious issue in our estimation, which is confirmed in Section 5.3.

We use government bond yields (0.25, 1, 5, 10, and 20 years to maturity) as risk-free rates, which are retrieved from Mercury as well. Following Eom et al. (2004), for each corporate bond we compute the (continuously compounded) yield of a (hypothetical) Treasury bond with exactly the same coupon rate and time to maturity, which constitutes out estimate of r in Eq. (8).[6] An alternative risk-free benchmark would be constructed from swap rates: from borrowers' perspective, it makes sense to compute credit spreads against swap rates, so they can compare the borrowing cost of corporate bonds with bank loans. However, as researchers, we are primarily interested in measuring the compensation for bearing default (and potentially liquidity) risk. Thus, in our main analysis, we use government bond yield as a risk-free benchmark, but reproduce the key results using swap rates listed in Internet Appendix V.

We use stock market index[7] data from each country obtained from Global Financial Data. We

[5] This adjustment leads to our removing one firm (Areva S.A.) and downgrading for five firms (Engie S.A., ENBW Energie Baden, Deutsche Telekom, Thales, and Deutsche Post A.G.).

[6] We use German Bund yields for risk-free rates in all Euro-area countries.

[7] We use TOPIX for Japan, FTSE100 Index for UK, DAX for Germany, Paris CAC40 Index for France, FTSE MIB Index for Italy, Toronto Stock Exchange Composite Index for Canada, and S&P/ASX200 index for Australia.

obtain macroeconomic data from the OECD website and FRED. We also use month-end single-name CDS spreads from 2002 to 2015 obtained from Markit. Finally, we obtain historical probability of default and recovery rates (including those of U.S. and non-U.S. issuers) from Moody's Default and Recovery Database.

In Table 1, we present the summary statistics for our sample of corporate bonds. We take (simple) average across bonds for each portfolio formed on credit ratings and maturity. For credit ratings, we form four portfolios: AA+ (which include AAA and AA), A, BBB, and HY (high-yield bonds rated BB and below).

We find that, conditional on credit ratings, credit spreads vary substantially across countries. For AA+ bonds, average credit spreads range from 17 bps in Japan to 146 bps in Australia, while BBB credit spreads range from 41 bps in Japan to 231 bps in Australia. When we compare BBB spreads across countries, Japan has the lowest credit spreads, followed by Germany and France. Italy, Canada, and the U.S. sit in the middle, while the U.K. has slightly higher credit spreads. Australia has the highest credit spreads in our sample.

Years to maturity vary across countries as well. The U.K. and Canada have long maturity bonds, ranging from 8.9 years (Canadian BBB bonds) to 16.6 years (Canadian A bonds) for IG bonds. In contrast, Australia and Germany have shorter maturity on average, with 3.8 years for German AA+ bonds and 6.3 years for Australian AA+ bonds.

Regarding the issue size (face value of bonds), Canada has the smallest average issue size, ranging from 100 to 220 million U.S. dollars, while the European countries have a large average issue size.

In Table 1, we also show the average number of bond issues per month as well as the average number of bonds per issuer. Regarding the number of bonds, Japan is the largest country in our international bond sample, even though the data include no Japanese HY bonds. France has the second-largest number of issues per month, followed by Canada and the U.K.

Regarding the concentration of issuers, IG bonds in Japan and Canada are dominated by large issuers: the average number of bonds per issuer ranges from 5.4 to 13.5 in Japan, and from 4.4 to 12.9 issues in Canada. The average number of bonds per issuer is lower in other countries, with Australia being the lowest (1.0, 3.2 and 2.2 bonds per issuer for AA+, A, and BBB firms, respectively).

There might be a concern that our Merrill Lynch data are based on quotes from a single dealer. However, such data for the U.S. sample have been used in several studies. For example, Goldberg and Nozawa (2018) find that transaction price in TRACE and Merrill Lynch quotes are similar to each other. Though we do not have transaction prices for international corporate bonds, we can compare them against other variables, such as stock prices. In Section I of the Internet Appendix, we follow Collin-Dufresne, Goldstein and Martin (2001) and run regressions of monthly changes in credit spreads on issuers' stock returns, changes in volatility, the level and slope in risk free rates, stock market indices, and skewness. Our estimation results are similar to those when we use the U.S. data. For example, monthly stock returns both at the security and index level are significantly negatively related to credit spread changes, while stock volatility is positively related to credit spreads; the regression R-squared ranges from 0.09 in Japan to 0.31 in Italy. These findings underscore the reliability of our corporate bond data. Lastly, we use CDS spreads and find that our main empirical results are robust.

4 Structural Credit Risk Models

In this study, we focus on two structural models of corporate debt pricing, the Black and Cox (1976) model and an extension of this model. We use the former as the benchmark, given the recent literature on the credit spread puzzle (see, e.g., Bao 2009; Huang and Huang 2012; Feldhütter and Schaefer 2018; Bai, Goldstein, and Yang 2019). Another reason for doing so is that as shown below, the Black-Cox model can be nested in a reduced form of the He and Milbradt (2014) model. In this section we describe the latter first and then review the Black-Cox model.

4.1 Structural Models with Search Frictions

To quantify the contribution of market illiquidity to the credit spread puzzle, we must estimate the liquidity component without referring to bond pricing information or altering the measured credit component in yield spreads (as captured by the Black-Cox model). To this end, we incorporate an OTC search friction into the Black-Cox model, based on the insights of He and Milbradt (2014). They endogenously derive how secondary market illiquidity is priced in yield spreads and how it interacts with an issuer's credit worthiness. More importantly, their model-based expression for proportional bid-ask spreads offers non-pricing metrics for our calibration of search-related parameters. Meanwhile, by retaining our baseline estimates of firm fundamental parameters, we ensure that the extended model is still consistent with the notion of the credit spread puzzle, as model-implied $\mathbb{P}$-measure default probabilities remain unchanged.

Consider a corporate bond of fixed maturity T and face value K which pays continuous coupon at a constant rate c. Default occurs due to covenant violation. The idea is, if the firm value falls enough relative to the face value of debt, firm may default even before the maturity of the debt. The firm value threshold at which firms choose to or are forced to default is called default boundary.

As initially proposed by Duffie, Gârleanu, and Pedersen (2005), the endogenous bond liquidity can be derived from the valuation wedge between L-type investors, who have been hit by liquidity shocks and thus face costs for holding bonds, and H-type investors who have not. Also, the corresponding bond valuation functions follow Proposition 1 in He and Milbradt (2014):

$$\begin{aligned}\begin{bmatrix} D_H(t,T) \\ D_L(t,T) \end{bmatrix} &= Z^{-1}\begin{bmatrix} c \\ c-\chi \end{bmatrix} + e^{-Z(T-t)}\left(\begin{bmatrix} K \\ K \end{bmatrix} - Z^{-1}\begin{bmatrix} c \\ c-\chi \end{bmatrix}\right)(1-\widetilde{\pi}^Q(T-t)) \\ &\quad + U\widetilde{G}(t,T)U^{-1}\left(\begin{bmatrix} R_H K \\ R_L K \end{bmatrix} - Z^{-1}\begin{bmatrix} c \\ c-\chi \end{bmatrix}\right), \end{aligned} \tag{1}$$

where χ is the holding cost, $\{R_H, R_L\}$ are state-dependent recovery rates, Z a 2-by-2 matrix of the liquidity-adjusted discount factors, U the matrix that diagonalizes Z, $\widetilde{\pi}^Q(t,T)$ the risk-neutral default probability over $(t,T]$, and $\widetilde{G} = \text{diag}[\widetilde{G}_1, \widetilde{G}_2]$ the state-dependent time-t price of the Arrow-Debreu default claim. As will be shown in Section 4.2, the bond valuation formula under search friction essentially maps that under the Black-Cox modeling of corporate bond prices, with the discount rate adjusted by transition intensity and the bond cash flows adjusted by holding costs.

Specifically, Z, $\pi^Q(t,T)$, and $\widetilde{G}$ are

$$Z = \begin{bmatrix} r+\xi & -\xi \\ -\lambda\beta & r+\lambda\beta \end{bmatrix} = U \cdot \begin{bmatrix} \widetilde{r}_1 & 0 \\ 0 & \widetilde{r}_2 \end{bmatrix} \cdot U^{-1} \tag{2}$$

$$\pi^Q(t,T) = N[x^-(\nu)] + \left(\frac{dK}{A_t}\right)^{\frac{2\nu}{(\sigma^A)^2}} N[x^+(\nu)], \tag{3}$$

$$\widetilde{G}_j(t,T) = \left(\frac{dK}{A_t}\right)^{\frac{\nu+\zeta_j}{(\sigma^A)^2}} N\left[x^+(\zeta_j)\right] + \left(\frac{dK}{A_t}\right)^{\frac{\nu-\zeta_j}{(\sigma^A)^2}} N\left[x^-(\zeta_j)\right], \quad j=1,2, \tag{4}$$

$$x^\pm(z) \equiv \frac{\ln(dK/A_t) \pm z\cdot(T-t)}{\sigma^A\sqrt{T-t}}, \tag{5}$$

where r denotes the risk-free rate, $\widetilde{r}_1 = r+\xi+\lambda\beta > \widetilde{r}_2 = r$, ξ represents the intensity of transforming a H-type investor to a L-type one, and $\lambda\beta$ represents the intensity of "backward" transition from the L state to the H state. Additionally, d is the default boundary, K/A_t the time-t leverage, σ^A asset volatility, δ the payout rate, $\nu = r - \delta - 0.5(\sigma^A)^2$, $\zeta_j = \sqrt{\nu^2 + 2\widetilde{r}_j(\sigma^A)^2}$, and $N[\cdot]$ the cumulative standard normal density function.

Given that the Merrill Lynch data consists of bid quotes instead of mid prices, we focus on the bid price in our empirical analysis of the model in Eq. (1). Under this model the bid price is

$$D_B(t,T) = \beta D_H(t,T) + (1-\beta)D_L(t,T). \tag{6}$$

The model-implied yield to maturity $y_S(t,T)$ solves the following equation:

$$D_B(t,T) = \frac{1}{y_S} cK\left(1 - e^{-y_S(T-t)}\right) + Ke^{-y_S(T-t)}. \tag{7}$$

4.2 The Black-Cox Model

If we shut down the search component in Eq. (1), the model degenerates to the Black-Cox model. Equivalently, once the H-type investors are assumed to be immune to liquidity shocks ($\xi \equiv 0$), their valuation for the bond converges to the following pricing formula under the Black-Cox model:

$$\begin{aligned} D_{BC}(t,T) &= \frac{cK}{r} + e^{-rt}K\left(1-\frac{c}{r}\right)(1-\pi^Q(t,T)) + K\left(R-\frac{c}{r}\right)G(t,T) \\ &= \frac{cK}{r}(1-e^{-rt})(1-\pi^Q(t,T)) + e^{-rt}K(1-\pi^Q(t,T)) + K\left(R-\frac{c}{r}\right)G(t,T), \end{aligned} \tag{8}$$

where R denotes the recovery rate. The three terms in Eq. (8) captures different components in corporate bond pricing: the present value of expected coupon payments, the present value of expected principal repayment, and the expected recovery value upon default. The model-implied yield to maturity $y(t,T)$ solves the following equation:

$$D_{BC}(t,T) = \frac{cK}{y}\left(1-e^{-y(T-t)}\right) + Ke^{-y(T-t)}. \tag{9}$$

The Black-Cox credit spread is given $s_{BC}(t,T) = y(t,T) - r$.

Note that Eq. (8) coincides with Eq. (3) of Leland and Toft (1996). Compared with the Black-Cox price of zero-coupon bonds as considered by Feldhütter and Schaefer (2018), it takes into account the coupon rates and serves as a special case of Eq. (1). Internet Appendix II considers three alternative specifications of the Black-Cox model, including the ones with zero-coupon bonds and bonds with discrete coupon payments (see, e.g., Bao 2009). We find that they lead to fairly similar model-implied credit spreads with the same set of parameter values.

5 Implementation

In this section, we describe the methodology we use to estimate the parameters of the models introduced in Section 4. We consider the parameters of the Black-Cox model first and then those of the search model.

Let θ^Q_{BC} and θ^P_{BC} denote the vectors of $\mathbb{Q}$ and $\mathbb{P}$ parameters under the BC model, respectively. It follows that $\theta^Q_{BC} = (K/A_t, \delta, \sigma^A, R, d)$ and $\theta^P_{BC} = (SR)$. We obtain all parameters except d from the data, and then set d to match the model-implied probability of default under the $\mathbb{P}$-measure to historical default frequency. We estimate asset volatility (σ^A), leverage (K/A_t) and the payout rate (δ) at the firm level. For the Sharpe ratio, recovery rate, and the probability of default for a given rating category, we use the fixed values across firms.

5.1 Firm-Level Inputs

We compute leverage as the ratio of book value of debt to the value of asset, defined as the sum of book value of debt and market value of equity. Payout ratio is the ratio of payment to outside stakeholders (dividend payment, share repurchases, and net interest payment) over the past one year divided by the asset value. For firms with extremely high payout ratio (more than three times the median payout ratio in each country), we set the payout ratio to be three times the median payout ratio.

Following Schaefer and Strebulaev (2008), we estimate asset volatility as:

$$\sigma^A_{i,t} = \sqrt{(1 - L_{i,t})^2(\sigma^E_{i,t})^2 + L^2_{i,t}(\sigma^D_{i,t})^2 + (1 - L_{i,t})L_{i,t}\sigma^E_{i,t}\sigma^D_{i,t}\rho^{ED}}, \tag{10}$$

where $L_{i,t}$ is leverage, $\sigma^E_{i,t}$ is equity volatility, $\sigma^D_{i,t}$ is debt volatility, and ρ^{ED} is correlation across debt and stock returns. We estimate $\sigma^E_{i,t}$ using daily stock returns with a 1-year rolling window. Estimating debt volatility and correlation is more challenging. To strike a balance between accuracy and transparency, we take the following steps. First, we compute the constant volatility for each bond using monthly returns. Second, we take the simple average across bonds within each rating category for each country to compute the average debt volatility. Third, we assign the same debt volatility for bonds in each rating/country bin. For correlation, we repeat the similar steps by computing correlation using monthly stock and bond returns for each bond, then take the average for each rating and in each country. After computing asset volatility for all firms for every month, we take the average over time to obtain the constant asset volatility.

Table 2 reports summary statistics of the firm-level inputs to the model. To compare across countries, we focus upon BBB firms, for which we have the largest number of observations. The average leverage is 0.52 for Japan, 0.33 for the U.K., 0.37 for Germany, 0.39 for France, 0.53 for Italy, 0.33 for Canada, 0.24 for Australia, and 0.31 for the U.S. Aside from Japan and Italy, the leverage of average BBB firms is similar across countries. Median firms have similar leverage to average firms.

The average payout ratio is the lowest in Japan, followed by the U.K., France, and Germany. Australia, Canada, and the U.S. have higher payout ratios. All else equal, a higher payout ratio pushes down the growth of asset value, and thus increases the probability of default of the issuer under both the $\mathbb{P}$- and $\mathbb{Q}$-measures.

Asset volatility for average BBB-rated firms is 0.18 in Japan, 0.19 in the U.K., 0.18 in Germany and France, 0.13 in Italy, 0.15 in Canada, 0.20 in Australia, and 0.26 in the U.S. Overall, asset volatility is quite similar across countries.

In summary, the main takeaway from Table 2 is that, conditional on credit ratings, there is no clear pattern in fundamental riskiness of firms across countries. We use a structural model to formally evaluate whether the variation in fundamentals aligns with the variation in credit spreads across countries in Section 6.

5.2 Country-Level Inputs

The Sharpe ratio of asset is needed to match a model-implied $\mathbb{P}$-measure default probability to historical default frequency. As we evaluate a structural model using bond-level data, ideally we need Sharpe ratios of individual firms. Following Chen, Collin-Dufresne, and Goldstein (2009) and Feldhütter and Schaefer (2018) who use one single Sharpe ratio for U.S. firms, however, we use one Sharpe ratio estimated separately for each country. Specifically, we compute average annual returns and average volatility for each stock using all Compustat firms from 1987 to 2017. We then compute the Sharpe ratio for each stock and take the median value in each country for the country-level Sharpe ratio.

In Panal A1 of Table 3, the estimated median Sharpe ratios are 0.20 for Japan, 0.29 for the U.K., 0.23 for Germany, 0.29 for France, 0.18 for Italy, and 0.23 for Canada and Australia. In the U.S., the commonly used value in the literature is 0.22 (e.g., Chen, Collin-Dufresne, and Goldstein 2009; Feldhütter and Schaefer 2018), and thus we use this number. The Sharpe ratios across countries are reasonably similar to each other. If high credit spreads in Australia reflect the high risk aversion for Australian investors, then the Sharpe ratio in Australia must be much higher than in other countries, which we do not see in the data. We check the robustness of these estimates by examining a subsample of firms that are matched to our bond data sets. As shown in Panel A2, the median values are generally similar to the estimates using all firms. Therefore, we use the latter estimates for the rest of the analysis.

Given the estimated Sharpe ratio SR, we compute the drift of a firm's asset value by:

$$\mu_{i,t} = r_t + SR \cdot \sigma_i^A.$$

By replacing the risk-free rate in Eq. (3) with $\mu_{i,t}$, we compute the model-implied probability of default under the $\mathbb{P}$-measure.

The recovery rate, the fraction of a firm's asset that investors recover upon default, is often assumed to be constant across countries, and the previous literature relies on Moody's estimate for recovery rate at the global level (including both U.S. and non-U.S. bonds) in analyzing U.S. corporate bond prices (e.g., Chen et al. 2009; Huang and Huang 2012; Feldhütter and Schaefer 2018). This assumption is justified as long as bankruptcy laws and the definition of seniority and collateral security are common across countries.

In practice, bankruptcy laws and covenants may differ across countries, leading to a potential difference in recovery rates across countries. We investigate this possibility using the recovery data for each default case since 1983 when Moody's recovery data start. However, we find that, though Moody's data cover default events across countries, the recovery rate is mostly missing in countries outside the U.S., Canada, and the U.K., possibly reflecting the lack of active distress debt markets outside these three countries. Thus, we aggregate all seven countries (Japan, the U.K., Germany, France, Italy, Canada, and Australia) compute average international recovery rates, and compare them against the values in the U.S.

The average recovery rate for senior unsecured debt is estimated at 37.3% for the seven countries, which is very close to the U.S. average of 38.0% in the sample period. The difference across countries is negligible compared with the relatively large countercyclical variation in recovery over time (Chen, 2010). Thus, we use the five-year moving average recovery rate (shown in Figure 2) at the global level to price corporate bonds in non-U.S. markets. In section 6.3.1, we use alternative measures of heterogeneous recovery rates to show that the potential difference in recovery does not drive our main findings.

To estimate the structural model of debt, we match the probability of default under the $\mathbb{P}$-measure to historical default frequency. The previous research in the literature (e.g., Huang and Huang 2012; Feldhütter and Schaefer 2018) uses Moody's probability of default estimated at the global level. If Moody's credit rating standard is consistent across countries, this choice is justified as we measure the probability of default for a *given* credit rating.

To verify the consistency, we compute cumulative default probabilities using Moody's event-level default data separately for U.S. firms and non-U.S. firms in the seven countries that we study. In Table A2, we show that the cumulative default frequency given credit ratings are similar between the U.S. and other countries. Thus, we use the historical default probability at the global level. Since credit spreads in Japan are lower than those in other countries, we also compute the default probabilities for Japanese firms alone. For AAA- and AA-rated Japanese firms, there is no default in the data, reflecting the smaller sample. For A- and BBB-rated firms, the 10-year cumulative default probability in Japan is 0.89% and 2.75%, not statistically significantly different from the estimates in other countries (2.66% and 2.38%, respectively).

Regarding the sample period, Feldhütter and Schaefer (2018) emphasize the importance of using the longer history of default data. We follow their approach and use the global default frequency from 1920 to 2017.[8]

5.3 Default Boundary

As we do not observe default boundary parameter, d, in the data (except at the bond maturity), we need to estimate it. Given that the credit spread puzzle is about the structural model's inability to match credit spreads and the historical default frequency at the same time, it seems reasonable to choose d to match the model-implied $\mathbb{P}$-measure default probability to the historical data. However, there is no consensus in the literature so far on how to "best" estimate d. We implement four

[8] The micro-level data is available after 1970, but Moody's publishes the historical default frequencies at the aggregate global level averaged since 1920.

different estimation methods in this study—and show later that the main conclusion of our study is robust to the use of different estimates of d provided that they are empirically reasonable. Below we briefly describe these four methods, which are detailed in Appendix B.

The first method is to follow Feldhütter and Schaefer (2018) and back out d by minimizing the distance between Moody's default frequencies and the Black-Cox default probabilities at the rating and maturity bin level. The second method we use is the one proposed by Bai, Goldstein, and Yang (2019), who show that Feldhütter and Schaefer (2018)'s estimates of d for the U.S. firms are sensitive to proxies of firms' asset market values used. Bai et al. (2019) propose to add an estimated market value of debt rather than its book value to the market value of equity to obtain the asset value. The third method is a model-based one that need not approximate the asset value with some observable proxies. Instead, we determine the asset (market) value as well as asset volatility via the Black-Cox model based on equity value and equity volatility in the spirit of Jones et al. (1984); Bao (2009). In these three methods the default boundary is assumed to be the same for all firms over time in each country. The fourth method we implement is to use firm-specified default boundaries. Specifically, we vary d such that each firm exactly matches the historical default frequency every month.

In Panel B of Table 3, we present the estimated default boundaries based on the above four different methods for each country, which are denoted d^{FS}, d^{BGY}, d^{HNS}, and d^{firm}, respectively. We see that d^{FS} is 0.82 for Japan, 0.85 for Italy, 0.99 for Germany and Australia, and is greater than 1.0 for UK (1.05), Canada (1.07), and France (1.10). As expected, $d^{BGY} \leq d^{FS}$ regardless of the countries considered, although they are close to each other except for Italy whose d^{BGY} is 0.63.[9] Interestingly, $d^{HNS} \leq d^{BGY}$ except for Italy. Note also that $d^{HNS} < 1.0$ except for France (1.03). Lastly, we report the average and median values of heterogeneous d^{firm} in each country. The firm-level estimates are much higher than d^{FS}, d^{BGY} or d^{HNS}. In particular, the average d^{firm} is above 1.0 for all countries, and as high as 2.34 for Australia. The median values of d^{firm} are lower than their averages, indicating that the distribution of d^{firm} is skewed to the right.

The fact that the default boundary is above 1.0 for some countries implies that our measure of market leverage is only a proxy for true leverage. If a true measure of leverage is available in the data, then $d < 1.0$ as there is no reason for a firm to default when its equity value is positive. However, there may be debt-like obligations that are missing in the book value of debt in balance sheets. For example, firms with higher operating leverage are more likely to default than firms with low operating leverage, even if the financial leverage is the same.[10] Thus, we focus only on d^{FS}, d^{BGY} or d^{HNS} and keep even those estimates greater than 1.0 in the analysis that follows—and we do not consider d^{firm} as it is too high.

[9] For the U.S. Feldhütter and Schaefer (2018) report that $d^{FS} = 0.89$ and Bai et al. (2019) report that $d^{BGY} = 0.62$.

[10] We thank Bob Goldstein for pointing this out.

5.4 Search Related Parameters

The parameters of the BC model with search friction in Eq. (1) include type-dependent recovery rates $\{R_H, R_L\}$ and four parameters on the secondary market search $\{\xi, \lambda, \beta, \chi\}$.

To determine $\{R_H, R_L\}$, we adopt the assumption in He and Milbradt (2014) that the historical discovery rate corresponds to the bid in the post-default market. It follows that $\{R_H, R_L\}$ can be obtained by solving the following system of two equations for each country:

$$\begin{aligned} R &= (1-\beta)R_L + \beta R_H; \\ \phi_d &= \frac{2(1-\beta)(R_H - R_L)}{(1+\beta)R_H + (1-\beta)R_L}. \end{aligned}$$

We set the historical average recovery rate R as similar to the one specified in Section 5.2, and set the bid-ask spread of defaulted bonds ϕ_d to 2.8% as reported by Jankowitsch, Nagler, and Subrahmanyam (2014). As a result, $\{R_H, R_L\}$ depend on β only.

We use $\chi = \chi_c c + \chi_F F$, following the parameterization of He and Milbradt (2014).[11] Let $\theta^S = \{\xi, \lambda, \beta, \chi_c, \chi_F\}$. We determine θ^S based on percentage bid-ask spreads

$$\phi(t, T; \theta^S) = \frac{Ask(t,T) - Bid(t,T)}{(Ask(t,T) + Bid(t,T))/2} = \frac{(1-\beta)(D_H(t,T) - D_L(t,T))}{[(1+\beta)D_H(t,T) + (1-\beta)D_L(t,T)]/2}.$$

Specifically, for each country, θ^S is estimated by minimizing the summed square of fitting errors over the entire sample:

$$\theta^S = \arg\min \sum_t \sum_i \left(\phi(t, T_i; \theta^S) - \phi_{i,t}^{obs}\right)^2, \tag{11}$$

where T_i is the maturity of bond i, and $\phi_{i,t} \equiv \phi(t, T_i; \theta^S)$ and $\phi_{i,t}^{obs}$ denote the model-implied and observed bid-ask spreads, respectively.[12]

Given that the bond-level measures of bid-ask spreads are inevitably rather noisy, we obtain $\phi_{i,t}^{obs}$ in two steps. First, in each month, we assign the bid-ask spreads of individual bonds to one of 12 credit rating-and-maturity bins constructed from four credit ratings, (AA+, A, BBB, HY), and three maturity groups, (< 5 years, 5–10 years, 10+ years). Next, we calculate the median bid-ask spread with bonds in the same category and then use this smoothed bid-ask spread as our measure of $\phi_{i,t}^{obs}$ in Eq. (11).[13]

[11] Chen et al. (2018) present an alternative way to parameterize χ.

[12] One might be concerned with the downward bias of BGN quoted bid-ask spreads as a measure of transaction costs, as documented in Bao et al. (2011) and Schestag et al. (2016). However, in Appendix A, we show that this finding is confined to the U.S. corporate bond market. Indeed, in the international setting, quoted bid-ask spreads are more closely correlated with benchmark liquidity measures, and overall their magnitude is even slightly greater than benchmarks. Given that BGN quoted bid-ask spreads have the best coverage among monthly measures of bond-level transaction costs, we focus on this measure in our model estimation.

[13] This cross-sectional smoothing within each rating-maturity group is motivated by the key implication of He and Milbradt (2014): the endogenous bid-ask spread depends not only on the issuer's default risk but also on the bond's time-to-maturity. Untabulated results show that, compared to unsmoothed bid-ask spreads, smoothed ones have

We report the estimates of θ^S in the Appendix. Consistent with search model implications, L-type investors in countries with large pricing errors from the Black-Cox model have either high holding costs (Australia) or low intensity to meet dealers (Italy).

6 Empirical Results

In this section, we examine the empirical performance of the structural models described in Section 4 in matching corporate bond and CDS spreads. We consider the Black-Cox model first and present the evidence that calls for models with illiquidity. We then show that the reduced He-Milbradt model improves the model performance significantly.

6.1 Performance of the Black-Cox Model in Matching Bond Spreads

Let's examine the ability of the Black-Cox model to match individual corporate bond spreads first. We implement the Black-Cox model in Eq. (8) with three different default boundaries, (d^{FS}, d^{BGY},d^{HNS}), and compare the model-implied corporate bond spreads with observed spreads for each country in our sample.

6.1.1 Can the Model Match Individual Bond Spreads?

Given that the analysis is based on a sample of individual bonds, one standard way to evaluate the model is to examine its pricing errors. Table 4 reports the average pricing errors on corporate bond spreads by credit ratings under each of (d^{FS}, d^{BGY}, d^{HNS}) for each country.

Consider the baseline case (d^{FS}) first. The mean pricing error on the spread level (panel A) for IG bonds is all negative except for A and BBB bonds in Japan and France. That is, the model underestimates the IG credit spreads for Italy, UK, Canada, Germany, Australia, and the U.S., as well as for AA+ bonds in Japan and France. The magnitude of the underestimation is also substantial. For HY bonds, the model underestimates (overestimates) the spreads for UK, Canada, and German (for Italy, France, and the U.S.). The results on the mean percentage pricing errors (panel B) display similar patterns to those in panel A. The only exception is that the model now also underestimates the spread for A bonds in Japan. Note that the average percentage pricing error for IG bonds is mostly very negative. For example, for AA+ bonds, it is lower than -70% for all countries except Canada (-27%). For A bonds, the mean percentage pricing error is 58% for France, -9% for Germany, and below -35% for the other six countries. For BBB bonds, it is 33% for Japan, 21% for France, and below -35% for the other six countries. For HY bonds, it is below -55% for UK, Canada, and Germany; about zero for the U.S., and above 15% for Italy and France.

stronger explanatory power for individual bonds' pricing errors in panel regressions.

Consider d^{HGY} next. Replacing d^{FS} with d^{HGY} tends to lower the model spread. Indeed the pricing errors generally become more negative are largely similar to those based on d^{FS}. Nonetheless, the results based on d^{HGY} are qualitatively similar to the baseline case, except for HY bonds in Italy and the U.S. For these bonds, while the average percentage pricing error under d^{FS} is 16% and about 0% for Italy and the U.S., respectively, its counterpart under d^{HGY} is -77% and -45%, respectively. The reason for such a big difference between these two sets of the results is the big gap between d^{FS} and d^{HGY}. Recall from Section 5.3 that $d^{FS} = 0.85$ and $d^{BGY} = 0.63$ for Italy (Table 3) and $d^{FS} = 0.89$ and $d^{BGY} = 0.67$ for the U.S. Note, however, that in spite of the big differences between d^{FS} and d^{BGY} for Italy and the U.S., the average percentage pricing errors for these two countries show similar pattern; namely, the model performance improves as the credit rating is lower.

Lastly, consider d^{HNS}. The results are qualitatively similar to the baseline case.

To summarize, we make three observations from Table 4 . First, the results for the U.S. bonds are consistent with Huang and Huang (2012). Second, the other 7 countries, the BC model underestimates the IG bond spreads except for A and BBB bonds in France (whose average model spreads are very high mainly due to a few SOEs in the sample); however, the model overestimates the HY bond spreads for Italy and France. Lastly, the Black-Cox model has difficulty matching individual bond spreads, regardless of the default boundary estimates used and the countries considered in our sample. This finding is consistent with the evidence in the U.S. based on individual bonds (e.g., Eom, Helwege, and Huang 2004).

6.1.2 Can the Model Match the Average or Median Spreads?

Let's first look at the distributions of both observed and predicted spreads. Table 5 presents their means, the 10-, 25-, 50-, 75-, and 90-percentiles based on a variety of default boundary parameter, d, by four different rating groups. Consider AA+ bonds (panel A) for example. While the mean observed spreads are only slightly greater than their median counterparts, the model with the FS- and BGY-methods of d generates positively skewed distributions of model-implied credit spreads—and, as a result, the average model spreads with fixed d are much greater than the medians.[14] This finding indicates that comparing average model spreads with average observed spreads may lead to a wrong conclusion about the performance of the model: the model can overpredict credit spreads for a few firms with very high leverage or volatility to compensate for the underprediction of the credit spreads for the remaining firms. That said, comparing the median credit spreads in the data and the model helps detect such problems.

With the left four panels of Figure 3, we compare the model mean or median spreads with d^{FS} (the x-axis) and the data (the y-axis). These plots summarize the key takeaway from Table 5. If the model explains the credit spreads in the data well, then we should see observations plot along

[14] Feldhütter and Schaefer (2019) confirm this finding in the US data.

the 45-degree line. If the model underpredicts credit spreads, then observations plot in the upper left triangular region.

In most countries and credit ratings, the mean credit spreads lie north and east to the median, reflecting the positive skewness in each country. However, the horizontal distance between mean and median is much greater than the vertical distance, showing that average model-based credit spreads are more affected by outliers than are actual credit spreads.

Figure 3 shows that the model matches the average spread better than the median spread. Since the model-based IG credit spreads are much lower for the median than for the mean, all countries plot above the 45-degree line, showing that the model underpredicts the IG spreads in the data.

It is striking that in Figure 3, the median IG firms across countries have quite different credit spreads, conditional on the credit rating. However, the model-implied credit spreads are quite similar to each other, reflecting the fact that these median firms share similar fundamentals in terms of leverage, volatility, and payout ratio. Therefore, we see median observations distributed widely across the vertical axis, but not across the horizontal axis. When we use d^{FS}, the Black-Cox model does not explain the cross-sectional variation in credit spreads well.

Using the right four panels of Figure 3, we compare the model-implied median credit spreads (the x-axis) and the data (the y-axis) with different specifications of default boundary, d. The diamond dots present the BGY-method of d. Comparing the results from d^{FS}, we see a large decline in model-implied credit spreads for the U.S. For example, the median BBB credit spreads decrease from 51 bps (with d^{FS}) to 14 bps (with d^{BGY}), reflecting the lower values of default boundary under the BGY-method. However, the difference between two methods is not large for other countries. Comparing the median in the left panels (with d^{FS}) and the diamond dots in the right panels (d^{BGY}), the non-U.S. credit spreads remain largely unchanged.

We emphasize that the global credit spread puzzle is not entirely driven by Japan, which always has the lowest credit spreads. For example, the median of BBB bonds in Canada has credit spreads of 154 bps, and the model (with d^{FS}) predicts 3 bps. In Germany, the median BBB spreads are 105 bps, and the model predicts 14 bps. Thus, of the 49 bps differences in credit spreads between Canada and Germany, the model explains -11 bps, generating the opposite prediction. When we use d estimated at the individual level, the model generates 89 bps for Canada and 83 bps for Germany, which still explains only 6 bps of the difference. Thus, the variation in credit spreads excluding Japan is quite substantial, relative to the variation in firm fundamentals.

For HY bonds, the figures are less conclusive about the performance of the Black-Cox model. Both d^{FS} and d^{BGY} underpredict HY median credit spreads for all countries except France. However, unlike IG bonds, the median U.S. firm, which has the highest HY credit spreads in the data, has the second highest model-implied credit spreads (behind France). In contrast, HY credit spreads are lowest in Germany, both in the data and in the model with fixed d. Therefore, the model qualitatively generates the right prediction for the cross-section of HY credit spreads.

The fact that credit spreads are different across countries, but the leverage, volatility, and payout ratio are similar to each other, poses a challenge not only on the Black-Cox model, but also on any structural models of debt based only on these inputs. However elaborate the calibration method for d is, the target is to match the historical default frequency that is fixed for each credit rating. Therefore, our results suggest that we require a model that includes additional inputs that vary substantially across countries.

6.2 Performance of the Black-Cox Model in Matching CDS Spreads

In this section, we examine the performance of the Black-Cox model using CDS spreads. It is known that CDS spreads may be less affected by illiquidity and better reflect the underlying name's credit risk than corporate bonds do. If the Black-Cox model is unable to explain bond credit spreads primarily due to missing liquidity premium, then the model should perform better in pricing CDS spreads.

Following Bai, Goldstein, and Yang (2019), the model-implied CDS spread is:

$$CDS(T) = \frac{4(1-R)\sum_{i=1}^{4T} DF(\frac{t_{i-1}+t_i}{2})[\pi^Q(t_i) - \pi^Q(t_{i-1})]}{\sum_{i=1}^{4T} DF(t_i)(1-\pi^Q(t_i)) + \frac{1}{2}\sum_{i=1}^{4T} DF(\frac{t_{i-1}+t_i}{2})[\pi^Q(t_i) - \pi^Q(t_{i-1})]} \tag{12}$$

where $DF(t) = e^{-r_t t}$, $\pi^Q(\cdot)$ is the Black-Cox model-based $\mathbb{Q}$-measure default probability in Eq.(3), and it is assumed that if a credit event occurs between two payment dates, then the CDS buyer always pays half of the periodic premium.[15]

In our implementation of Eq. (12), we use swap spreads for $\{r_t\}$. Also, we use the same values of d as we do for corporate bonds, as they are calibrated to the $\mathbb{P}$-measure default probabilities and do not depend on asset prices. Since d^{FS} and d^{BGY} yield very similar results, we focus on the former and individual default boundary, which varies across firms and over time.

We begin our analysis by studying the 5-year CDS contract, the most liquid one. Note that the CDS samples are mostly less than one third of their corporate bond counterparts in most countries (see Table 1). For instance, in the CDS samples, there are only a few AA+ firms in Italy and Australia, and a few HY-rated firms in Italy and Canada, while there are no AA+ firms in Canada and no HY firms in Australia. Table 6 reports the pricing error of the BC model in the CDS market. Similar to the bond case, the pricing error is mostly negative.

In Figure 4, we plot for each credit rating the median model-implied CDS spreads on the x-axis and the observed ones on the y-axis. When compared with corporate bonds, the cross-country difference in median CDS spreads is relatively small. For example, the range for the median CDS spreads is 10, 31, and 86 bps for AA+, A, and BBB-rated names, respectively. Thus, part of the large cross-country difference in corporate credit spreads likely reflects the difference in liquidity

[15] Numerically, ignoring the final accrual payment makes little difference in the model CDS spread.

premium across countries. Still, the performance of the Black-Cox model for CDS is qualitatively similar to that of corporate bonds: the model generates a smaller variation in CDS spreads than in the data. As a result, the median spreads in Figure 4 lie on top of each other rather than along the 45-degree line. Moreover, the choice of default boundary shifts the observations horizontally, but does not affect the model's ability to explain the difference in CDS spreads across countries.

We also examine the term structures of CDS spreads for IG firms in each country (see the Internet Appendix for the details of the analysis). We find that while both the observed and Black-Cox (with d^{FS}) implied CDS curves (of median spreads) are upward sloping, the latter tends to be steeper than the former. As a result, the model underestimates the short-term CDS spreads more than long-term ones for IG names. The underprediction of the short-term IG credit spreads is not surprising, however, given that the Black-Cox model does not include a jump in a firm's asset value process.

Overall, our analysis of CDS spreads confirms findings with respect to the corporate bond market that the Black-Cox model does not explain the cross-country difference in credit spreads, aside from the smaller cross-sectional variation in CDS spreads.

6.3 Dissecting Pricing Errors

Sections 6.1–6.2 have shown the evidence that the Black-Cox model has difficulty matching both corporate bond and CDS spreads, as well as explaining their cross-sectional variations across eight countries. In this subsection we analyze the fitting errors and determine whether time-series patterns in the errors exist.

6.3.1 Loss Given Default

The same recovery rate is used across different countries in the baseline analysis, based on the evidence in the Moody's default database (see Section 5.2). However, recovery rates may vary at the country level, due to potential differences in legal environments across countries. In this subsection we consider two alternative estimates for loss given default (LGD).

The first one is the forward-looking LGD implied from HY CDS spreads.[16] For distressed names, Markit backs out the implied recovery rate from the observed CDS spreads under an assumed term structure of default intensities. We use the average of the Markit implied LGD values from all HY single-name CDS contracts in each country as its country-specific LGD except for Australia.[17] The second alternative measure is based on the country-specific bankruptcy efficiency scores (equivalent to the recovery rates times 100) estimated by Djankov, Hart, McLiesh, and Shleifer (2008), who

[16] Implied recovery rates from IG CDS spreads are not used here because these recoveries from Markit have very low cross-sectional variations.

[17] There are no HY Australian CDS contracts denominated in Australian dollars available in our sample.

utilize a survey that they conduct among lawyers on typical bankruptcy proceedings in each country using a case study of a distressed firm.

We redo the analysis in Section 6.1 using the Black-Cox model with d^{FS} and each of the above two alternative measures of LGD, and plot the median pricing errors (the difference between the observed and model median credit spreads) for IG bonds in each country in Figure 5. If LGD is the driver of the cross-country difference in corporate bond spreads, we should see a positive relationship between the pricing errors and LGD. The results based on the CDS-implied LGD estimates, however, display a negative relationship between the pricing errors and LGD (panel A of Figure 5). Specifically, corporate bond spreads in Japan are lower than in other countries but its LGD is in fact slightly higher than those in other countries, which are very similar to each other. The results using the survey-based LGD estimates also display a slightly negative relationship between the pricing errors and LGD (panel B). Note from the figure that Australia, Canada, Japan, the U.K. and the U.S. have relatively high bankruptcy efficiency scores (and thus low LGD estimates) while France, Germany, and Italy have low efficiency scores.

In sum, our results indicate that neither measure of the heterogeneous LGD estimates can likely explain the large gap in corporate bond spreads.

6.3.2 Country-Level Pricing Errors

In this subsection we consider other potential drivers of the country-level pricing errors, such as financial market conditions and liquidity factors. For the former, we use the level and slope of the risk-free rates and option-based uncertainty measures in each country. Specifically, we use each country's stock index options and construct the country-specific option-implied volatility and skewness measure following Collin-Dufresne, Goldstein, and Martin (2001).[18]

For liquidity factors we explore proxies for corporate bond illiquidity, which is not captured by structural models of debt such as the Black-Cox model. Given that we have no transaction data for non-U.S. corporate bonds available, we consider three alternative liquidity measures of international corporate bonds instead of those transaction-based illiquidity measures devised for the U.S. market. The first one is yield curve fitting errors (the "noise") of corporate bonds (a measure of illiquidity arising from dealers' inventory frictions) proposed by Goldberg and Nozawa (2018) in the spirit of Hu, Pan, and Wang (2013) (see the Internet Appendix for the details on the construction of this measure). The second measure of illiquidity is TED spreads for each country given that they capture the information about the funding market conditions for dealers. We use German TED spreads for all Euro-area countries. The third measure is the average bid-ask spread for each country, computed using bid and ask prices from the Bloomberg Generic Quote (BGN)

[18] We fit a quadratic function on option implied volatility for one month options:

$$\sigma^{IV}(m_k) = b_0 + b_1 m_k + b_2 m_k^2 + u_k,$$

where m_k is the moneyness of option k, and compute the skew by $\hat{\sigma}^{IV}(0.9) - \hat{\sigma}^{IV}(1.0)$.

pricing source for the non-U.S. corporate bonds in our sample.

Next, we examine the explanatory power of the aforementioned four indicators of financial market conditions and three illiquidity measures for the pricing errors. First, we run monthly cross-sectional univariate regressions of pricing errors on each of these seven variables in the spirit of Fama and MacBeth (1973), given that there are only eight observations in each month. Panel A of Table 7 reports the average slope coefficients, associated t-statistics, and R-squared. The estimated coefficients suggest that the level of the government yield curve and illiquidity measures are positively related with pricing errors. The positive correlation between the illiquidity measures and pricing errors suggests that liquidity premiums partly explain the variation in credit spreads that is missed by the Black-Cox model.[19] On the other hand, the high R-squared for the level of yield curve ($R_t^f(1)$) needs to be treated with caution, since much of the cross-sectional variation reflects the difference in risk-free rates between Australia and Japan. Aside from these two countries, little cross-sectional variation in risk-free rates exists among other countries. Furthermore, we find little evidence that option-based uncertainty measures explain the cross-section of the pricing errors.

6.3.3 Security-Level Analysis

To further explore the sources of pricing errors, we estimate a panel regression of security-level pricing errors and report the results in Table 8. First, if the Black-Cox model does not accurately characterize the functional form of credit spreads, pricing errors should be correlated with the model inputs. Column (1) examines three key inputs: the risk-free rate, leverage, and equity volatility.[20] We find that leverage is strongly and negatively associated with pricing errors, reflecting the fact that firms with low leverage and better credit quality have a more pronounced gap between data and the model. Column (2) considers time to maturity and issue size, showing that the latter is significantly negative. Larger issues tend to be more liquid and thus have lower pricing errors than smaller issues.

Next, we focus on various explanations for the credit spread puzzle (CSP) in the U.S. market as proposed in previous studies. One is macroeconomic risk (Chen et al. 2009; Bhamra et al. 2010; Chen 2010). Column (3) reports the results from two proxies for macro conditions, the real GDP growth rate (seasonally adjusted) and the slope of yield curve. Note that the pricing error widens in economic downturns, which is consistent with the prediction of macro-based theories. On the other hand, Du et al. (2019) reconcile the CSP (in the U.S. single-name CDS market) by incorporating both stochastic asset volatility and jumps, and emphasize the important role played by the asset variance risk premium (VRP) in doing so. As such, we include equity VRP estimator of Bollerslev et al. (2009), $skew_t$ (a proxy for jump magnitudes/probabilities), and IV_t (a proxy for time-varying

[19]In Figure IA3 in the Internet Appendix, we show that shares of various types of corporate bond investors are quite different across countries, corroborating our argument that corporate bond liquidity varies across countries.

[20]Asset volatility is not used here because it is assumed to be constant and thus does not reflect time variations in volatility.

volatility) in Columns (4), (5), and (6), respectively. The results indicate that all three variables are significant and that although the first two have a counter intuitive, negative sign, IV_t has a positive sign and nontrivial explanatory power for pricing errors. Another often cited driver of the CSP is bond illiquidity. Consistent with results from country-level regressions, liquidity measures capture a sizable portion of variations in pricing errors, as shown in Column (7).

We then consider four standard equity market factors, $\{MKT_t, SMB_t, HML_t, UMD_t\}$, given the evidence that these factors hold some explanatory capacity for credit spreads in the U.S. (e.g., Collin-Dufresne et al. 2001; Avramov et al. 2007). Results reported in Column (8) indicate that the four equity market factors are all insignificant, thereby exhibiting very limited correlations with pricing errors in our sample of international corporate bonds.

Lastly, we run a "kitchen-sink" regression that pools all the aforementioned explanatory variables, with (Column (10)) and without (Column (9)) country fixed effects to account for other country-specific factors yet considered (e.g., debt enforcement and account transparency). Taking the results shown in Columns (1)–(10) together, we find that the firm leverage, implied volatility, and market liquidity display the most robust statistical significance in explaining the pricing errors, consistent with their unconditional explanatory power as measured by the R^2 values.

6.3.4 International Finance

In this subsection, we consider some factors documented in the international finance literature. We examine first whether the currency risk factors of Lustig, Roussanov, and Verdelhan (2011) explain the difference in credit spreads across countries. Their currency risk factors are based on the difference in currency returns between high risk-free rate countries and low risk-free rate ones. Since credit spreads are differences in yields between corporate and government bonds, there is no mechanical link between credit spreads and asset pricing factors based on risk-free rates. Indeed, we find little evidence empirically that the difference in exposure to currency factors explains the difference in corporate credit spreads across those countries in our sample (see the Internet Appendix for the details of this analysis).

Next, we consider the potential role played by the violation of covered interest parity (CIP). Liao (2019) argues that the difference in credit spreads across currencies is due to the violation of CIP, which prohibits arbitragers from correcting mispricing. One main objective of our study is to understand the source of this "mispricing." To this end, we can directly test this mispricing-based explanation for credit spreads by comparing credit spreads before and after the 2008 financial crisis. Under this explanation, we would not see much cross-country variation in credit spreads before the financial crisis, as the CIP holds quite well before the crisis. In the data, the standard deviations of median A- and BBB-rated firms across eight countries are respectively 25 and 31 bps in the first half of the sample (before December 2007), but are 43 and 51 bps in the second half, respectively. As the difference in credit spreads before the crisis is substantial, the violation of the covered interest parity does not fully explain the credit spreads in our sample of non-U.S. domestic issuers. We provide evidence in Section 6.5 that illiquidity can largely explain the difference in spreads in our sample.

6.4 Time-Series Credit Spread Puzzle

Having examined the variation in credit spreads across countries, we now focus on their variation in time given that both credit spreads and pricing errors vary substantially over time. To this end, we focus on monthly median credit spreads for each country.

Figure 6 plots the observed median spreads (in blue) as well as their counterparts implied from the Black-Cox model with d^{FS} (in red). Note that the four European countries share a common variation in credit spreads, peaking either during the 2008 financial crisis or the 2012 sovereign debt crisis. The U.S., Canada, and Australia all have a huge spike during the 2008 crisis. On the other hand, the credit spreads in Japan are relatively stable after the 1998 Asian financial crisis. For all countries except Japan, the gap between the observed and model spreads seems to spike around the 2008 crisis. This time series pattern is consistent with the liquidity component in yield spreads as estimated by Chen, Cui, He, and Milbradt (2018) using the U.S. data.

To evaluate the model's ability to explain time-varying credit spreads, we report correlation coefficients between the model and observed median spreads in each of the eight panels of Figure 6. While the correlations are high in the U.K. (0.87), the U.S. (0.86), and Italy (0.74), they are much lower in other countries: the correlation is around 0.5 in Japan, Germany, France, and Australia, and essentially zero in Canada.[21] Thus, the global credit spread puzzle exists not only in the cross-section, but also over time. Using different estimates of default boundary does not resolve the time-series puzzle: as reported in Figure 6, if we use d^{firm} (a time-varying default boundary), the correlation coefficients between the model and observed median spreads, Corr(Indiv), are generally lower Corr(FS).

To explain the time-variation in pricing errors, we run a panel regression of median mispricing in each country on the explanatory variables and country fixed effects:

$$s_{c,t} - s_{c,t}^{BC} = b_0 + b_1 X_{c,t} + D_c + \eta_{c,t} \tag{13}$$

where $s_{c,t}$ and $s_{c,t}^{BC}$ denote the observed and model-implied median spreads for country c, respectively, $X_{c,t}$ denotes the set of country-level explanatory variables used, and D_c is the dummy variable for country c.

In Panel B of Table 7, we present the estimated slope coefficients and adjusted R-squared. The estimated slope coefficients show that higher *noise*, TED spreads and bid-ask spreads, and higher option-implied uncertainty are all associated with a greater gap between observed credit

[21] The estimated correlation in the U.S. is consistent with Feldhütter and Schaefer (2018).

spreads and the model. The R^2_{adj} is 0.69 for the three illiquidity measures and is 0.67 for the two option-implied uncertainty measures. The link between the uncertainty measure and global credit spreads is consistent with the finding of Culp, Nozawa, and Veronesi (2018), who document a strong link between option prices in the U.S. and credit spreads. The R^2_{adj} of the kitchen-sink regression that includes the level and slope of the risk-free yield curve, three illiquidity measures, and two option-implied uncertainty measures is as high as 0.79. Thus, this set of explanatory variables seems to capture the time-variation in country-level fitting errors well. In particular, illiquidity most significantly explains both cross-section and time-series variations in credit spreads.

6.5 Performance of the Model with Endogenous Liquidity

Our empirical results so far have provided strong evidence on the inability of the baseline model (based only on default risk) in matching credit spreads and the cross-country variation in credit spreads, as well as in explaining the dynamic behavior of credit spreads. In this section, we quantitatively assess the validity of market illiquidity as a potential solution to these issues.

6.5.1 Pricing Errors and Endogenous Liquidity

To ensure that the model-implied default probability is identical to that in Section 5.3, we implement Eq. (1) with the same set of firm-specific parameters (as well as d^{FS}) as summarized in Section 5.4. We then calculate the model-implied yield, y_S, based on the bid price, using Eq. (7).[22] The model-implied liquidity component is defined as $y_S - y_{BC}$.

To what extent the incorporated search friction can fill the gap between observed and Black-Cox credit spreads? Let's examine the pricing errors first. Rows labeled "d^{FS}+search" in Table 4 report the pricing errors of the baseline model (the Black-Cox model with d^{FS}) with search. The results on the mean pricing error, reported in panel A, indicate that incorporating search significantly improves the pricing performance of the model for IG bonds, except for Japan and A and BBB bonds in France. For example, for Italy the pricing error reduces from -92 bps (AA+), -116 bps (A), and -48 bps (BBB) to -38, -41, and -10 bps, respectively. For the U.S., the pricing error changes from (-53, -42, -63) bps to (-26, 17, 16) bps for (AA+, A, BBB). The most striking improvement occurs for Australian bonds: the pricing error drops from (-127, -163, -186) to (-12, -13, -11). For Japan and A and BBB bonds in France, the baseline model already overestimate spreads—and incorporating search worsens the problem. For HY bonds, the evidence is mixed. While the search component dramatically reduces the pricing error for UK, Germany, and especially Canada, it has little effect for France and substantially increases the pricing error for Italy and the U.S.

The results on the mean percentage pricing errors (panel B) show similar patterns except for

[22] Feldhütter and Schaefer (2018) examine the bid bias in U.S. dealer quotes from Merrill Lynch. We extend the evidence to the global setting by comparing Merrill Lynch data with alternative pricing sources. Merrill Lynch quotes are very close to BGN bid prices and are, on average, 0.14% lower than Markit composite prices.

AA+ and A bonds in Japan. For these two groups of bonds, the baseline model has low mean pricing errors (-4 bps and 3 bps, respectively) but very high mean percentage pricing errors (-94% and -71%, respectively). Incorporating search improves the latter to 21% and 37%, respectively.

Overall, the search component generally improves the model performance for UK, Canada, Germany, and Australia. The component also improves the performance for IG bonds for Italy, the U.S. and Japan except for BBB bonds, but it lowers the performance for the most risky bonds for these three countries. For France, the search component helps for AA+ bonds only.

6.5.2 Evidence Based on Median Pricing Errors

Figure 7 illustrates the search model's performance in explaining median credit spreads across countries. Incorporating search and bargaining shows remarkable improvement for high-quality (AAA-A) bonds. Also, the model-implied liquidity component is largest in countries with extreme estimates of liquidity related parameters (e.g., Australia and Italy). In terms of fractional yield spreads, the liquidity component plays a less important role among HY bonds, as the Black-Cox model already shows a good fit of the U.S. spreads and even overshoots the Italian spreads. Overall, Figure 7 is consistent with the calibration results of He and Milbradt (2014): (1) the model with endogenous liquidity delivers a satisfactory fit for the overall level of credit spreads; (2) the liquidity component takes up a larger fraction of IG spreads, compared to HY ones (it is 44% versus 31% in He and Milbradt 2014); and (3) its pattern across rating classes is reversed if we focus on its absolute magnitude (e.g., in UK the median liquidity component is 21 bps, 70 bps, 72 bps and 147 bps, which correspond to the four rating categories).

However, a few caveats need to be raised when we collectively interpret these three strands of findings. First, the He-Milbradt model is built and examined in the risk-neutral setting. We complement their quantitative analysis by calibrating the model to historical default rates, such that its implications for the credit spread puzzle can be derived.[23]

Second, in the He-Milbradt decomposition of credit spreads, one out of four components—the liquidity-driven default component—is missing in the search model that we examine in our study. In other words, illiquidity of the secondary market would not feed back to the distance-to-default of corporate bonds, as the default boundary is exogenously set by matching the historical default experiences. Nevertheless, this feature does not necessarily lead to lower credit spreads than the He-Milbradt model's prediction: if the He-Milbradt model exactly holds and the FS calibration method perfectly identifies the average default boundary in the history, then the boundary estimates obtained through Eq. (15) should generally reflect endogenous corporate default decisions with the search friction taken into account.[24]

[23] Chen et al. (2018) introduce macroeconomic risk into the He-Milbradt model and shed light on the credit spread puzzle using the Monte-Carlo method.

[24] To test this hypothesis, we use d^{FS} in He and Milbradt (2014)'s optimal boundary formula to back out the tax benefits of debt. As we show in Table A3 in Appendix A, the model-implied magnitude of tax benefits is too low, or

Finally, we implement the search model firm by firm in our study, which allows us to examine model implications not only about the aggregate level of credit spreads but also about their cross-sectional and time-series variations. The "LiqComp" column in Table 7 shows that the model-implied liquidity component delivers a close matching of both cross-sectional and time-series patterns in Black-Cox pricing errors. Specifically, its slope coefficients are close to one for different regression specifications, and the univariate R^2 is 0.86 and 0.92 in Fama-MacBeth and panel regressions.

6.5.3 Time Series Correlations

In Figure 8, we map the search model's predictions on the time series of median spreads to their counterparts in the data. Compared to our results in Figure 6, incorporating market friction significantly improves the model's performance in tracking the historical variations in credit spreads. Indeed, the correlation coefficient ranges from 0.72 (France) to 0.95 (U.K.). The country-specific liquidity component mirrors our previous findings on the aggregate spread level: in Japan, it is kept at a reasonably low level throughout the sample period, even if the observed and modeled total yield spreads spike during the Asian financial crisis; in Australia, meanwhile, the temporal variation in modeled yield spreads seems exclusively driven by market liquidity.

Figure 8 also sheds light on the role of liquidity crunch in crisis episodes. For the four European countries in our sample, the response of their liquidity component to the 2008–2009 global financial crisis seemed of the same order of magnitude, roughly around 100 basis points. However, U.K., Germany, and France witnessed a rather modest increase in their respective liquidity components during the European debt crisis, suggesting the importance of credit risk and risk premium in the latter episode. In contrast, the liquidity component in Italy spiked during the 2010–2013 period and dominated the total reaction of yield spreads to the eurozone crisis. This pattern distinguishes Italy, the country more directly affected by the eurozone crisis, from the other three.

7 Conclusion

While the widely used structural approach to credit risk modeling has been studied extensively using the U.S. data, very few studies have examined its performance in credit markets outside the U.S. In this paper, we empirically examine two well-known structural models, those of Merton (1974) and Black and Cox (1976), using a sample of individual corporate bonds issued in seven developed economies: Japan, U.K., Germany, France, Italy, Canada, and Australia. In addition, we test the Black-Cox model (the benchmark model in this study) using a sample of single-name CDS spreads from these countries.

We find that both models have substantial credit spread prediction errors, although the average

equivalently, d^{FS} is too high, compared to the empirically measured effective tax rates.

percentage pricing errors of CDS tend to be lower than those of corporate bonds in magnitude. Bond spread prediction errors mostly have the same sign for investment-grade (IG) names: the models underpredict IG spreads except for BBB names in Japan and A and BBB names in France. While Japan is a unique case worth further inquiry, positive spread prediction errors for those names in France are mainly driven by a few SOEs with high leverage and volatility yet low (observed) spreads. For high-yield bonds, the evidence is mixed. The models overpredict spreads for Italy and France but underpredicts spreads for UK, Canada, and Germany. For the U.S. HY bonds, the sign of the prediction errors depends on the method used for determining default boundary. CDS spread prediction errors display similar patterns.

We also document a large heterogeneity in credit spreads across countries given the observed proxies for default risk, including credit rating, leverage, and volatility. In particular, we observe that the median firm in each country has very different credit spreads yet has similar leverage and volatility. As a result, the models have difficulty explaining the cross-section of credit spreads across countries.

We further document that correlations between the Black-Cox model-implied credit spreads and the observed bond spreads vary substantially across countries. The correlations for median firms range from around 0.86 in UK and the U.S. to 0.06 in Canada. This finding is striking, as we use the same bond data base for the international corporate bonds as we do for the U.S. bonds.

In other words, we find that the standard benchmark model clearly has difficulty in accurately predicting credit spreads, that it tends to underpredict spreads on IG names, and that the model has difficulty capturing the dynamic behavior of spreads.

Our analysis of pricing errors indicates that in order to better explain the cross-country difference in credit spreads, we need to incorporate illiquidity in the corporate bond market into the model. We consider and implement an extended Black-Cox model that incorporates search frictions, and find that the new model significantly improves the model performance in predicting credit spreads.

To summarize, this paper contributes to the literature in at least three aspects. First, we conduct an empirical analysis of structural credit risk models using data on both corporate bond and CDS spreads from eight developed countries. Second, we provide evidence that credit risk alone unlikely can explain credit spreads. Third, we incorporate search into the standard Black-Cox model within the framework of He and Milbradt (2014) and find that doing so improves the model performance significantly and helps resolve the global credit spread puzzle.

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Table 1: Summary Statistics for Corporate Bond Data

	Bond characteristics by credit ratings							
	AA+	A	BBB	HY	AA+	A	BBB	HY
	Japan				*Italy*			
Credit Spreads (bps)	17	27	41	-	86	114	161	246
Years to Maurity	6.5	5.1	4.2	-	10	7.7	6.9	6.4
Issue Size (USDmil)	344	311	278	-	1340	1216	1075	751
Average NObs	94.6	56.8	61.8	-	2.0	11.4	24.2	4.4
NBonds/Issuer	13.5	7.6	5.4	-	3.6	5.3	5.6	5.9
	U.K.				*Canada*			
Credit Spreads (bps)	77	129	176	418	82	99	160	344
Years to Maurity	10.6	12.1	9.3	8.3	16.4	16.6	8.9	4.7
Issue Size (USDmil)	609	439	408	401	100	153	220	181
Average NObs	4.0	28.7	20.8	4.2	1.5	26.0	47.9	1.6
NBonds/Issuer	2.5	5.3	2.7	2.3	5.7	12.6	4.6	1.5
	Germany				*Australia*			
Credit Spreads (bps)	52	89	121	278	146	185	231	-
Years to Maurity	3.8	5.6	6.2	4.3	6.3	4.3	3.8	-
Issue Size (USDmil)	726	1077	870	745	222	250	176	-
Average NObs	2.1	17.6	24.0	6.8	0.7	16.0	9.2	-
NBonds/Issuer	1.8	4.5	4.7	3.7	1.0	3.2	2.2	-
	France				*U.S.*			
Credit Spreads (bps)	59	87	136	295	71	99	166	415
Years to Maurity	5.1	6	5.6	4	6.6	6.9	6.9	6.8
Issue Size (USDmil)	896	853	715	689	565	409	358	300
Average NObs	7.8	35.9	39.6	10.5	56.5	212.2	276.0	200.3
NBonds/Issuer	4.0	5.5	3.8	3.8	4.6	4.8	4.1	2.9

Note: We sort bonds into portfolios based on credit rating and time to maturity every month, and compute simple averages of characteristics across bonds every month. We then take averages over time for each portfolio and report the results in this table. Average N refers to how many bonds (per month) we have in each portfolio. The sample is monthly from 1997 to 2017 for the non-U.S. sample except for Australia (starting in 2007) and Italy (starting in 2003), and from 1987 to 2015 for the U.S. bonds.

Table 2: Firm-Level Inputs to the Black-Cox Model

	Rating	NObs	Mean	10%	50%	90%	NObs	Mean	10%	50%	90%
		Japan					*Germany*				
K/A	AA+	31	0.44	0.21	0.42	0.73	9	0.35	0.11	0.21	0.75
	A	64	0.46	0.22	0.46	0.72	28	0.41	0.16	0.43	0.64
	BBB	63	0.52	0.33	0.52	0.71	39	0.37	0.12	0.36	0.60
	HY	0	-	-	-	-	12	0.38	0.23	0.36	0.57
σ^E	AA+	31	0.26	0.16	0.24	0.38	9	0.27	0.17	0.24	0.42
	A	64	0.31	0.19	0.29	0.46	28	0.31	0.18	0.28	0.48
	BBB	63	0.37	0.23	0.36	0.51	39	0.28	0.18	0.26	0.43
	HY	0	-	-	-	-	12	0.32	0.20	0.28	0.45
σ^A	AA+	31	0.15	0.05	0.15	0.22	9	0.19	0.05	0.20	0.29
	A	64	0.17	0.08	0.17	0.24	28	0.18	0.11	0.17	0.28
	BBB	63	0.18	0.10	0.17	0.23	39	0.18	0.11	0.16	0.25
	HY	0	-	-	-	-	12	0.20	0.16	0.20	0.24
δ	AA+	31	0.009	0.004	0.008	0.016	9	0.015	0.000	0.007	0.040
	A	64	0.008	0.000	0.007	0.016	28	0.024	0.006	0.018	0.053
	BBB	63	0.005	0.000	0.004	0.012	39	0.036	0.011	0.035	0.063
	HY	0	-	-	-	-	12	0.030	0.020	0.027	0.045
		UK					*France*				
K/A	AA+	15	0.19	0.07	0.16	0.34	9	0.24	0.07	0.21	0.48
	A	42	0.36	0.15	0.34	0.57	24	0.36	0.08	0.35	0.65
	BBB	40	0.33	0.16	0.32	0.57	39	0.39	0.16	0.39	0.59
	HY	13	0.39	0.18	0.40	0.61	18	0.54	0.27	0.54	0.78
σ^E	AA+	15	0.26	0.17	0.26	0.37	9	0.30	0.19	0.27	0.48
	A	42	0.24	0.14	0.22	0.40	24	0.27	0.17	0.25	0.43
	BBB	40	0.27	0.17	0.24	0.44	39	0.29	0.18	0.26	0.45
	HY	13	0.37	0.22	0.33	0.63	18	0.38	0.23	0.36	0.57
σ^A	AA+	15	0.21	0.16	0.20	0.28	9	0.21	0.15	0.22	0.29
	A	42	0.16	0.12	0.15	0.22	24	0.18	0.11	0.16	0.26
	BBB	40	0.19	0.14	0.18	0.25	39	0.18	0.11	0.18	0.25
	HY	13	0.22	0.17	0.20	0.31	18	0.19	0.11	0.17	0.26
δ	AA+	15	0.011	0.000	0.003	0.037	9	0.024	0.008	0.022	0.045
	A	42	0.021	0.000	0.004	0.050	24	0.026	0.000	0.022	0.055
	BBB	40	0.025	0.000	0.030	0.051	39	0.025	0.003	0.021	0.049
	HY	13	0.034	0.000	0.036	0.060	18	0.020	0.005	0.016	0.044

This table presents summary statistics for firm-level inputs to the Black-Cox model for each country and for each credit rating. The statistics are computed using the panel data of bond issuers, and NObs is the number of firms that are in each category. The sample is from 1997 to 2017 ...

Table 2 − Continued

	Rating	NObs	Mean	10%	50%	90%	NObs	Mean	10%	50%	90%
		Italy					*Australia*				
K/A	AA+	3	0.26	0.17	0.27	0.33	1	0.78	0.68	0.78	0.88
	A	11	0.40	0.22	0.41	0.58	10	0.26	0.12	0.22	0.55
	BBB	18	0.53	0.40	0.53	0.70	17	0.24	0.11	0.24	0.36
	HY	6	0.61	0.41	0.66	0.70	0	-	-	-	-
σ^E	AA+	3	0.24	0.12	0.21	0.48	1	0.25	0.18	0.25	0.34
	A	11	0.24	0.16	0.22	0.34	10	0.21	0.14	0.19	0.28
	BBB	18	0.26	0.18	0.25	0.33	17	0.27	0.19	0.24	0.37
	HY	6	0.34	0.28	0.33	0.44	0	-	-	-	-
σ^A	AA+	3	0.17	0.11	0.19	0.22	1	0.06	0.06	0.06	0.06
	A	11	0.15	0.12	0.14	0.19	10	0.15	0.09	0.15	0.22
	BBB	18	0.13	0.11	0.13	0.15	17	0.20	0.15	0.20	0.26
	HY	6	0.15	0.12	0.13	0.20	0	-	-	-	-
δ	AA+	3	0.052	0.042	0.054	0.064	1	0.000	0.000	0.000	0.000
	A	11	0.045	0.020	0.050	0.060	10	0.033	0.000	0.040	0.071
	BBB	18	0.048	0.024	0.046	0.073	17	0.034	0.000	0.037	0.057
	HY	6	0.065	0.016	0.089	0.101	0	-	-	-	-
		Canada					*U.S.*				
K/A	AA+	3	0.37	0.19	0.41	0.48	79	0.16	0.06	0.15	0.25
	A	18	0.36	0.19	0.36	0.46	312	0.23	0.09	0.19	0.42
	BBB	51	0.33	0.14	0.32	0.50	544	0.31	0.13	0.29	0.50
	HY	5	0.39	0.22	0.36	0.60	661	0.48	0.22	0.46	0.77
σ^E	AA+	3	0.24	0.11	0.27	0.34	79	0.24	0.15	0.23	0.34
	A	18	0.19	0.13	0.17	0.29	312	0.29	0.18	0.27	0.41
	BBB	51	0.22	0.13	0.19	0.33	544	0.33	0.21	0.31	0.49
	HY	5	0.33	0.19	0.30	0.49	661	0.49	0.28	0.44	0.78
σ^A	AA+	3	0.14	0.12	0.13	0.18	79	0.23	0.19	0.22	0.26
	A	18	0.13	0.10	0.12	0.15	312	0.25	0.19	0.23	0.35
	BBB	51	0.15	0.08	0.14	0.23	544	0.26	0.19	0.25	0.35
	HY	5	0.20	0.15	0.20	0.25	661	0.30	0.21	0.28	0.41
δ	AA+	3	0.048	0.025	0.046	0.079	79	0.040	0.010	0.041	0.074
	A	18	0.038	0.017	0.039	0.055	312	0.046	0.015	0.041	0.082
	BBB	51	0.034	0.000	0.035	0.061	544	0.047	0.014	0.041	0.091
	HY	5	0.045	0.022	0.044	0.066	661	0.043	0.012	0.037	0.079

for non-U.S. firms, and from 1987 to 2015 for the U.S. firms. K/A is leverage defined by the ratio of the book value of debt to the sum of the book value of debt and the market value of equity. σ^E is annualized equity volatility, σ^A is annualized asset volatility, and δ is the payout ratio.

Table 3: Estimates for the Sharpe Ratio and Default Boundary

		Country						
		Japan	UK	Germany	France	Italy	Canada	Australia
Panel A: Sharpe Ratios								
A1. Security Level Averages: All firms								
Number of Firms		4829	3037	1108	1077	565	3664	1388
Mean		0.24	0.36	0.28	0.29	0.20	0.28	0.30
Median		0.20	0.29	0.23	0.29	0.18	0.23	0.23
Sample begins		1987	1987	1987	1987	1987	1984	1987
A2. Security Level Averages: Bond Issuers Only								
Number of Firms		174	240	130	150	69	417	64
Mean		0.22	0.59	0.35	0.32	0.22	0.34	0.37
Median		0.22	0.34	0.31	0.29	0.20	0.32	0.34
Sample begins		1987	1987	1987	1987	1987	1984	1987
Panel B: Default Boundary Estimates								
d^{FS}		0.82	1.05	0.99	1.10	0.85	1.07	0.99
d^{BGY}		0.81	0.98	0.95	1.10	0.63	1.03	0.99
d^{HNS}		0.80	0.98	0.92	1.03	0.78	0.90	0.92
d^{firm}	Mean	1.14	1.81	1.57	1.78	1.06	1.68	2.34
	Median	1.02	1.55	1.24	1.28	0.96	1.49	2.01

Panel A presents the estimate for the Sharpe ratio on individual stocks in each country. We compute average annual returns and average volatility for each stock using the full sample of stock returns until 2017. We then compute the Sharpe ratio for each stock and compute the mean and median across firms for each country. Panel A1 shows the results using all Compustat firms. Panel A2 shows the results using a subset of firms that are matched to our corporate bond sample.

Panel B reports the estimated default boundaries based on four different methods for each country, using the sample of firms that have at least one bond in the Merrill Lynch data (including callable bonds). The four boundaries include d^{FS} (the Feldhütter and Schaefer 2018 approach), d^{BGY} (the Bai, Goldstein, and Yang 2019 approach), d^{HNS} (the JMR approach), and d^{firm} (the individual firm-level default boundary by matching $\mathbb{P}$-measure default probability exactly every month to the historical default frequency). The table reports the mean and median using the panel data of d^{firm}.

Table 4: Bond-Level Pricing Errors of the Black-Cox Model with and without Search

	Panel A: Mean Pricing Error (bps)								Panel B: Mean Percentage Pricing Error (%)							
Models	AA+	A	BBB	HY	AA+	A	BBB	HY	AA+	A	BBB	HY	AA+	A	BBB	HY
	Japan				Italy				Japan				Italy			
d^{FS}	-4	3	11	-	-92	-116	-48	32	-94	-71	33	-	-94	-79	-43	16
d^{BGY}	-6	1	7	-	-97	-145	-127	-160	-95	-73	18	-	-99	-95	-89	-77
d^{HNS}	3	21	1	-	-66	-101	-51	73	-95	-90	-2	-	-70	-67	-45	35
$d^{FS} + search$	16	22	32	-	-38	-41	-10	82	21	37	117	-	-32	-25	-14	40
	UK				Canada				UK				Canada			
d^{FS}	-67	-84	-131	-231	-26	-77	-131	-140	-90	-67	-73	-61	-27	-76	-78	-58
d^{BGY}	-70	-99	-144	-243	-54	-88	-140	-188	-94	-80	-82	-66	-70	-87	-85	-70
d^{HNS}	-55	-59	-88	-199	-46	-90	-147	-258	-71	-49	-55	-55	-56	-89	-87	-86
$d^{FS} + search$	-24	-22	-56	-82	27	-18	-28	-1	-16	-12	-32	-22	58	-12	-13	-7
	Germany				Australia				Germany				Australia			
d^{FS}	-52	-5	-37	-157	-127	-163	-186	-	-93	-9	-31	-69	-90	-96	-97	-
d^{BGY}	-53	-21	-51	-173	-130	-164	-184	-	-95	-28	-47	-77	-92	-97	-96	-
d^{HNS}	-42	-38	-30	-150	-141	-158	-124	-	-76	-61	-34	-73	-96	-95	-74	-
$d^{FS} + search$	7	28	9	-49	-12	-13	-11	-	30	35	9	-23	-17	-7	-6	-
	France				US				France				US			
d^{FS}	-56	54	11	344	-53	-42	-63	16	-77	58	21	108	-84	-38	-31	0
d^{BGY}	-61	25	-6	360	-62	-78	-130	-137	-81	19	-4	94	-96	-81	-73	-45
d^{HNS}	-47	22	15	458	-51	-60	-70	-23	-63	13	8	159	-78	-61	-38	-9
$d^{FS} + search$	-17	96	66	358	-26	17	16	110	-12	109	69	123	-41	28	15	26

The table summarizes the pricing errors of corporate bond yield spreads under the Black-Cox model with and without search. Pricing errors are reported as the differences in bps (panel A), $s^{BC}_{k,t} - s_{k,t}$, or as the percentage differences in % (panel B), between the model implied and observed spreads. Bond observations are grouped into categories where the issued bond is rated as $AAA\&AA$, A, BBB or high-grade (HY). All entries in the table are the average across bonds in a country/rating group, over the 1987–2015 sample period for the U.S. and the 1997–2017 period for other countries.

Table 5: Performance of the Black-Cox Model, AA+ Rating

		Mean	10%	25%	50%	75%	90%
Japan	Credit Spreads (bps)	15	4	8	13	20	26
	d^{FS}	10	0	0	1	11	38
	d^{BGY}	9	0	0	1	9	34
	d^{HNS}	17	0	0	1	6	48
UK	Credit Spreads (bps)	74	36	49	72	92	113
	d^{FS}	6	0	0	0	2	9
	d^{BGY}	4	0	0	0	1	6
	d^{HNS}	18	12	14	17	23	30
Germany	Credit Spreads (bps)	57	28	41	53	68	79
	d^{FS}	5	0	0	0	3	10
	d^{BGY}	3	0	0	0	2	8
	d^{HNS}	15	6	8	11	14	17
France	Credit Spreads (bps)	81	34	48	73	100	137
	d^{FS}	24	0	0	1	20	72
	d^{BGY}	19	0	0	2	19	55
	d^{HNS}	33	1	3	10	26	102
Italy	Credit Spreads (bps)	98	53	72	95	120	141
	d^{FS}	6	0	0	3	8	16
	d^{BGY}	1	0	0	0	1	3
	d^{HNS}	30	3	6	10	13	120
Canada	Credit Spreads (bps)	78	39	44	61	79	133
	d^{FS}	52	3	6	19	97	137
	d^{BGY}	24	1	3	5	39	69
	d^{HNS}	31	8	10	18	38	87
Australia	Credit Spreads (bps)	146	74	85	123	220	241
	d^{FS}	19	0	2	6	24	63
	d^{BGY}	16	0	1	4	30	54
	d^{HNS}	5	2	2	4	7	9
US	Credit Spreads (bps)	65	24	37	54	78	117
	d^{FS}	12	0	0	1	5	24
	d^{BGY}	4	0	0	0	1	7
	d^{HNS}	14	0	3	8	13	23

Table 5 – Continued, A Rating

		Mean	10%	25%	50%	75%	90%
Japan	Credit Spreads (bps)	22	8	12	18	27	40
	d^{FS}	24	0	0	5	29	72
	d^{BGY}	22	0	0	5	27	67
	d^{HNS}	42	0	0	1	24	120
UK	Credit Spreads (bps)	135	64	84	117	155	225
	d^{FS}	51	0	0	9	46	136
	d^{BGY}	36	0	0	4	27	83
	d^{HNS}	76	3	12	22	65	237
Germany	Credit Spreads (bps)	98	45	59	85	114	157
	d^{FS}	92	0	1	23	120	264
	d^{BGY}	76	0	0	14	82	214
	d^{HNS}	59	0	2	8	32	137
France	Credit Spreads (bps)	100	49	64	90	119	162
	d^{FS}	154	0	0	15	242	508
	d^{BGY}	125	0	0	13	186	400
	d^{HNS}	122	0	3	34	176	360
Italy	Credit Spreads (bps)	155	63	86	131	190	300
	d^{FS}	39	0	1	10	54	116
	d^{BGY}	10	0	0	1	7	29
	d^{HNS}	54	1	3	11	55	183
Canada	Credit Spreads (bps)	102	49	65	89	127	165
	d^{FS}	25	0	1	9	28	60
	d^{BGY}	14	0	0	4	15	33
	d^{HNS}	12	1	2	5	11	31
Australia	Credit Spreads (bps)	170	89	111	150	209	265
	d^{FS}	8	0	0	0	1	19
	d^{BGY}	6	0	0	0	1	16
	d^{HNS}	13	1	2	3	8	14
US	Credit Spreads (bps)	99	39	56	79	116	179
	d^{FS}	57	0	1	8	49	169
	d^{BGY}	21	0	0	2	15	58
	d^{HNS}	39	1	6	12	32	104

Table 5 – Continued, BBB Rating

		Mean	10%	25%	50%	75%	90%
Japan	Credit Spreads (bps)	35	14	20	29	42	64
	d^{FS}	49	0	1	12	55	129
	d^{BGY}	44	0	1	11	49	117
	d^{HNS}	38	0	0	4	34	118
UK	Credit Spreads (bps)	188	95	121	155	210	295
	d^{FS}	57	0	3	20	62	163
	d^{BGY}	45	0	1	12	40	113
	d^{HNS}	100	3	13	28	124	303
Germany	Credit Spreads (bps)	122	57	74	105	145	207
	d^{FS}	85	0	0	14	84	244
	d^{BGY}	72	0	0	9	57	172
	d^{HNS}	92	0	2	9	45	215
France	Credit Spreads (bps)	147	61	80	117	178	267
	d^{FS}	159	0	2	35	185	457
	d^{BGY}	142	0	2	30	156	389
	d^{HNS}	163	1	7	56	191	480
Italy	Credit Spreads (bps)	152	63	80	112	190	304
	d^{FS}	105	0	3	37	137	339
	d^{BGY}	26	0	0	1	16	79
	d^{HNS}	101	1	6	32	120	309
Canada	Credit Spreads (bps)	172	83	111	154	207	269
	d^{FS}	43	0	0	3	28	78
	d^{BGY}	34	0	0	2	18	59
	d^{HNS}	26	0	1	2	6	39
Australia	Credit Spreads (bps)	195	103	130	176	233	312
	d^{FS}	15	0	0	0	3	19
	d^{BGY}	18	0	0	0	3	18
	d^{HNS}	86	2	3	11	47	151
US	Credit Spreads (bps)	183	66	93	140	221	344
	d^{FS}	120	1	8	46	148	335
	d^{BGY}	53	0	1	12	57	148
	d^{HNS}	113	4	10	32	136	314

Table 5 – Continued, HY Rating

		Mean	10%	25%	50%	75%	90%
UK	Credit Spreads (bps)	406	225	288	363	468	681
	d^{FS}	217	1	37	128	283	524
	d^{BGY}	225	1	21	92	245	557
	d^{HNS}	273	5	25	124	287	789
Germany	Credit Spreads (bps)	244	120	151	205	298	449
	d^{FS}	110	0	2	40	149	288
	d^{BGY}	109	0	1	24	104	234
	d^{HNS}	142	0	1	10	102	311
France	Credit Spreads (bps)	287	114	167	257	375	519
	d^{FS}	642	5	84	369	787	1425
	d^{BGY}	668	3	61	314	809	1555
	d^{HNS}	759	34	151	488	874	1545
Italy	Credit Spreads (bps)	217	100	139	205	273	361
	d^{FS}	251	3	23	181	424	659
	d^{BGY}	60	0	1	9	83	219
	d^{HNS}	289	3	22	115	470	834
Canada	Credit Spreads (bps)	315	174	221	287	405	491
	d^{FS}	214	0	2	38	314	766
	d^{BGY}	173	0	1	24	228	631
	d^{HNS}	68	2	4	13	46	186
US	Credit Spreads (bps)	480	159	266	424	604	867
	d^{FS}	496	33	114	324	707	1231
	d^{BGY}	343	7	39	145	382	803
	d^{HNS}	457	19	70	266	673	1165

These tables report the summary statistics of the distribution of credit spreads in the data and the Black-Cox model. The statistics are computed using the panel data from 1997 to 2017 outside of the U.S., while using the data from 1987 to 2015 for the U.S. d^{FS} refers to the Black-Cox model-based estimates in which the default boundary is estimated following Feldhütter and Schaefer (2018). d^{BGY} is estimated following Bai, Goldstein, and Yang (2019). d^{HNS} is based on JMR-type estimates of asset value and volatility.

Table 6: CDS Pricing Errors of the Black-Cox Model

	Panel A: Mean Pricing Error (bps)								Panel B: Mean Percentage Pricing Error (%)							
Models	AA+	A	BBB	HY	AA+	A	BBB	HY	AA+	A	BBB	HY	AA+	A	BBB	HY
	Japan				Italy				Japan				Italy			
d^{FS}	-19	-11	-16	-156	-24	-50	-32	-1097	-18	-7	-7	5	-24	-38	-21	-66
d^{BGY}	-19	-11	-16	-158	-26	-69	-90	-1331	-18	-7	-8	4	-26	-52	-62	-93
d^{HNS}	-11	1	-24	-42	1	-47	-1	-953	-12	1	-13	-29	2	-34	10	-48
	UK				Canada				UK				Canada			
d^{FS}	-21	-35	-32	-135	-	-63	-39	-97	-21	-35	-31	-34	-	-45	-31	-47
d^{BGY}	-21	-39	-47	-160	-	-65	-45	-148	-21	-39	-40	-44	-	-47	-35	-57
d^{HNS}	-19	-2	-1	-61	-	-80	-76	-241	-19	-3	-5	-8	-	-65	-44	-98
	Germany				Australia				Germany				Australia			
d^{FS}	-8	-6	19	-105	-28	-74	-115	-	-6	-4	21	-43	-27	-57	-73	-
d^{BGY}	-12	-17	-8	-141	-28	-75	-115	-	-10	-14	-1	-54	-27	-58	-73	-
d^{HNS}	-12	-25	52	-63	-19	-62	-67	-	-12	-24	53	-36	-23	-60	-47	-
	France				US				France				US			
d^{FS}	-27	69	31	175	-15	-18	-19	26	-27	61	16	48	-18	-16	-14	6
d^{BGY}	-26	73	36	199	-34	-31	-54	-111	-27	65	20	55	-31	-29	-39	-43
d^{HNS}	-16	60	56	274	-32	-21	-16	19	-16	54	31	82	-28	-20	-15	17

The table summarizes the fitting errors of CDS spreads under the Black-Cox model with alternative approaches to identify the default boundary. Pricing errors are reported as the differences in bps (panel A), $cds^{BC}_{k,t} - cds_{k,t}$, or as the percentage differences in % (panel B), between the model implied and observed spreads. Bond observations are grouped into categories where the issued bond is rated as $AAA\&AA$, A, BBB or high-grade (HY). All entries in the table are the average across bonds in a country/rating group over the 2001.01-2015.05 sample period.

Table 7: Panel Regressions of Country-Level Pricing Errors on Aggregate Variables

	$R_t^f(1)$	$R_t^f(10) - R_t^f(1)$	$Noise_t$	TED_t	$BidAsk_t$	IV_t	$SKEW_t$	$LiqComp_t$	$\bar{R}^2_{Panel}$
Panel A: Fama-MacBeth cross-sectional regressions (univariate)									
b	0.25	0.01	3.00	0.50	0.68	2.77	-0.26	0.91	
$t(b)$	(7.84)	(0.10)	(5.36)	(1.40)	(3.59)	(1.58)	(-0.09)	(8.95)	
R^2_{CX}	0.92	0.22	0.16	0.01	0.04	0.19	0.03	0.86	
Panel B: Multivariate pooled OLS regressions with country fixed effects									
	-0.03	0.03							0.47
	(-0.96)	(0.40)							
			0.48	0.79	0.49				0.69
			(1.21)	(10.63)	(5.11)				
						3.14	-1.40		0.67
						(4.51)	(-2.59)		
	0.02	0.10	1.05	0.56	0.42	1.48	-1.97		0.79
	(0.65)	(3.92)	(2.77)	(6.69)	(4.30)	(3.08)	(-2.57)		
								1.11	0.84
								(13.49)	
	0.01	0.06	1.23	0.33	0.17	0.40	-0.45	0.89	0.92
	(1.02)	(3.04)	(1.24)	(5.45)	(4.00)	(2.46)	(-1.32)	23.22	

The table reports the regression of country-level (median) pricing errors on explanatory variables. The first line in Panel A shows the average slope coefficients (b) from univariate monthly cross-sectional regressions of pricing errors on an explanatory variable. The second line shows t-statistics adjusted for serial correlation up to Newey-West 12 lags, and the third line shows the R-squared (R^2_{CX}) of the cross-sectional regression of (time-series) average pricing errors on average explanatory variables. $R_t^f(1)$ is the one-year yield on government bonds, $R_t^f(10) - R_t^f(1)$ is the difference in yields between ten- and one-year government bonds, $Noise_t$ is the average fitting errors of corporate bond yields, TED_t is the TED spreads in each country, $BidAsk_t$ is the average bid-ask spreads, IV_t is the option-implied volatility, $SKEW_t$ is the difference in option implied volatility between out-of-the-money put options and at-the-money put options on the country's stock index, and $LiqComp_t$ is the model-based estimates for the liquidity premium described in Section 6.5. Panel B shows the slope coefficients b_1 in the multivariate panel regressions with country fixed effects:

$$s_{c,t} - s_{c,t}^{BC} = b_0 + b_1 X_{c,t} + D_c + \eta_{c,t}$$

and $\bar{R}^2_{Panel}$ is the adjusted R-squared for this panel regression. t-statistics in parentheses are adjusted for cross-sectional correlation and serial correlation up to Newey-West 12 lags.

Table 8: Panel Regressions of Security-Level Pricing Errors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12
rf_t	0.02								0.15	0.12		0.17
	(0.47)								(4.43)	(3.69)		(6.59)
$lev_{i,t}$	−4.88								−5.83	−5.97		−5.44
	(−10.30)								(−12.97)	(−12.44)		(−10.18)
$\sigma^E_{i,t}$	−0.66								−1.15	−1.21		−2.07
	(−1.17)								(−2.92)	(−3.18)		(−5.72)
$\log Mat_{i,t}$		0.03							−0.03	−0.07		0.22
		(0.96)							(−0.97)	(−2.56)		(5.72)
$\log size_i$		−0.03							−0.13	−0.41		−0.12
		(−2.33)							(−13.11)	(−6.27)		(−2.59)
GDP_t			−0.20						−0.06	−0.05		0.02
			(−2.44)						(−1.19)	(−1.12)		(0.91)
$slope_t$			0.03						0.29	0.31		0.27
			(0.54)						(8.11)	(4.79)		(4.91)
VRP_t				−0.52					0.26	0.20		0.17
				(−2.81)					(3.19)	(2.31)		(2.91)
$skew_t$					−0.04				−0.00	−0.00		−0.02
					(−1.90)				(−0.07)	(−0.05)		(−1.11)
IV_t						3.91			4.98	3.88		2.63
						(4.39)			(8.14)	(7.97)		(5.16)
$BidAsk_{i,t}$							0.21		0.30	0.35		−0.13
							(1.91)		(2.43)	(3.16)		(−2.36)
$Noise_t$							1.18		0.15	0.93		−0.14
							(1.88)		(0.21)	(1.24)		(−0.21)
TED_t							0.78		0.62	0.82		0.13
							(4.69)		(3.44)	(4.56)		(1.36)
MKT_t								−2.44	0.83	0.82		0.67
								(−1.55)	(1.55)	(1.58)		(3.97)
SMB_t								0.04	0.74	0.90		0.76
								(0.05)	(1.10)	(1.71)		(2.05)
HML_t								−1.03	0.57	−0.05		0.39
								(−0.75)	(0.67)	(−0.05)		(0.56)
UMD_t								−1.77	−0.79	−1.01		−0.33
								(−1.19)	(−1.33)	(−1.94)		(−1.10)
$LiquidComp_{i,t}$											1.42	1.67
											(13.23)	(37.66)
FEs										X		X
$\bar{R}^2$	0.17	0.00	0.00	0.00	0.00	0.02	0.03	0.00	0.27	0.29	0.14	0.44
Obs	336926	340156	340156	303456	280619	275007	300013	340156	257516	257516	302055	257516

The table presents the estimated panel regression of pricing errors on explanatory variables and (potentially) country-fixed effects:

$$s_{i,t} - s^{BC}_{i,t} = b_0 + b_1 X_{i,t} + D_c + \xi_{i,t}.$$

rf_t is the risk-free rate used in model estimation; lev and σ^E denote the issuers' leverage and equity volatility; $\log Mat$ the log of years to maturity; $\log size$ the log face value of the bond; GDP real GDP growth rate in local currency; $slope$ is the difference between 10-year and 1-year risk-free rate; $skew$ is the difference in option-implied volatility between out-of-the-money put options and at-the-money put options on the country's stock index; VRP is measured by the spread between implied and realized volatility for index options; IV is the volatility index constructed from index options; $BidAsk$ is the percentage bid-ask spread; $Noise$ is country-level noise measure based on individual issuers' yield curve fitting errors; TED_t is the TED spread; MKT, SMB, HML and UMD are equity market risk factors; $\widehat{Liq}_{k,t}$ is the liquidity component in the yield spread as implied by the search model. Standard errors in parentheses are adjusted for cross-sectional correlation and serial correlation up to Newey-West 12 lags.

Figure 1: Outstanding Debt Securities Issued by Non-Financial Corporations as a Fraction of GDP

This figure shows outstanding debt securities issued by non-financial corporations as a fraction of GDP in 1997 (in black) and 2017 (in purple) for eight countries. The data is from the Bank of International Settlements. The debt securities are debt instruments designed to be traded in financial markets, including commercial paper, bonds, debentures, and asset-backed securities.

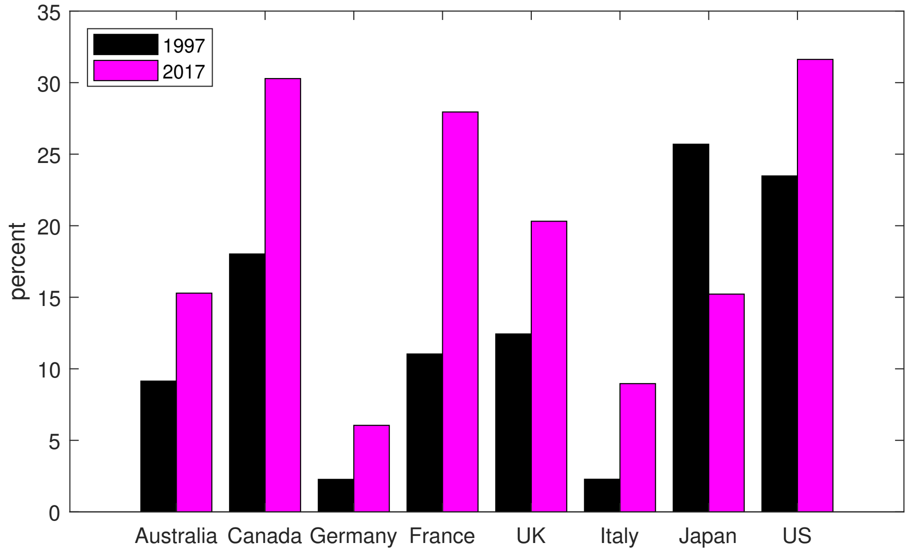


Figure 2: 5-Year Moving Average Recovery Rates

This figure plots the 5-year moving average (solid line) and one-year recovery rate (dotted line) of Moody's recovery rate for senior unsecured bonds at the global level.

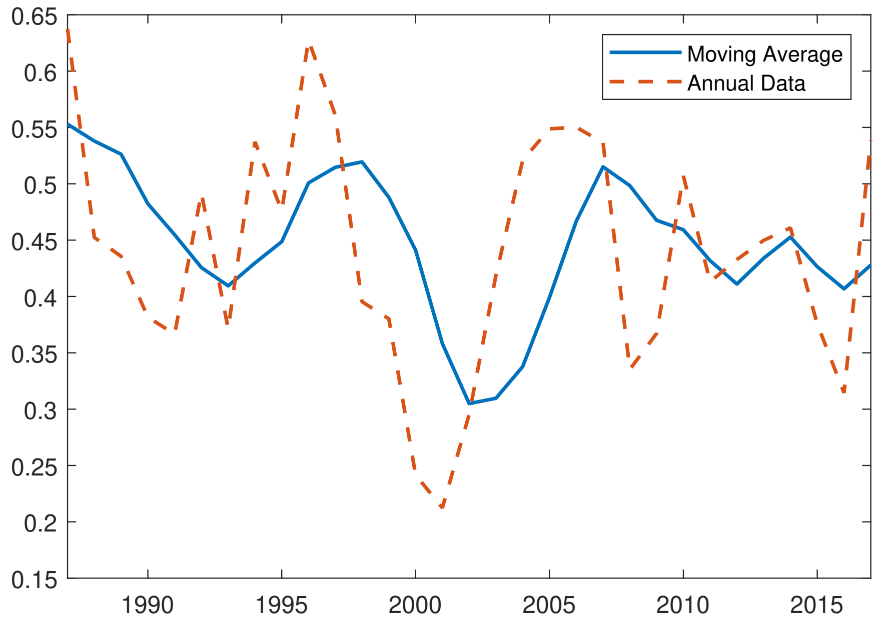


Figure 3: Black-Cox Model and Corporate Credit Spreads (bps)

The figures on the left column show the mean (dot) and median (star) credit spreads in the data and in the Black-Cox model, in which default boundary is estimated using the Feldhütter and Schaefer (2018) approach. The figures on the right compare the median credit spreads using two approaches to estimate d: i) Bai, Goldstein, and Yang (2019) (diamond) and ii) using JMR-type estimates of asset value and asset volatility (circle).

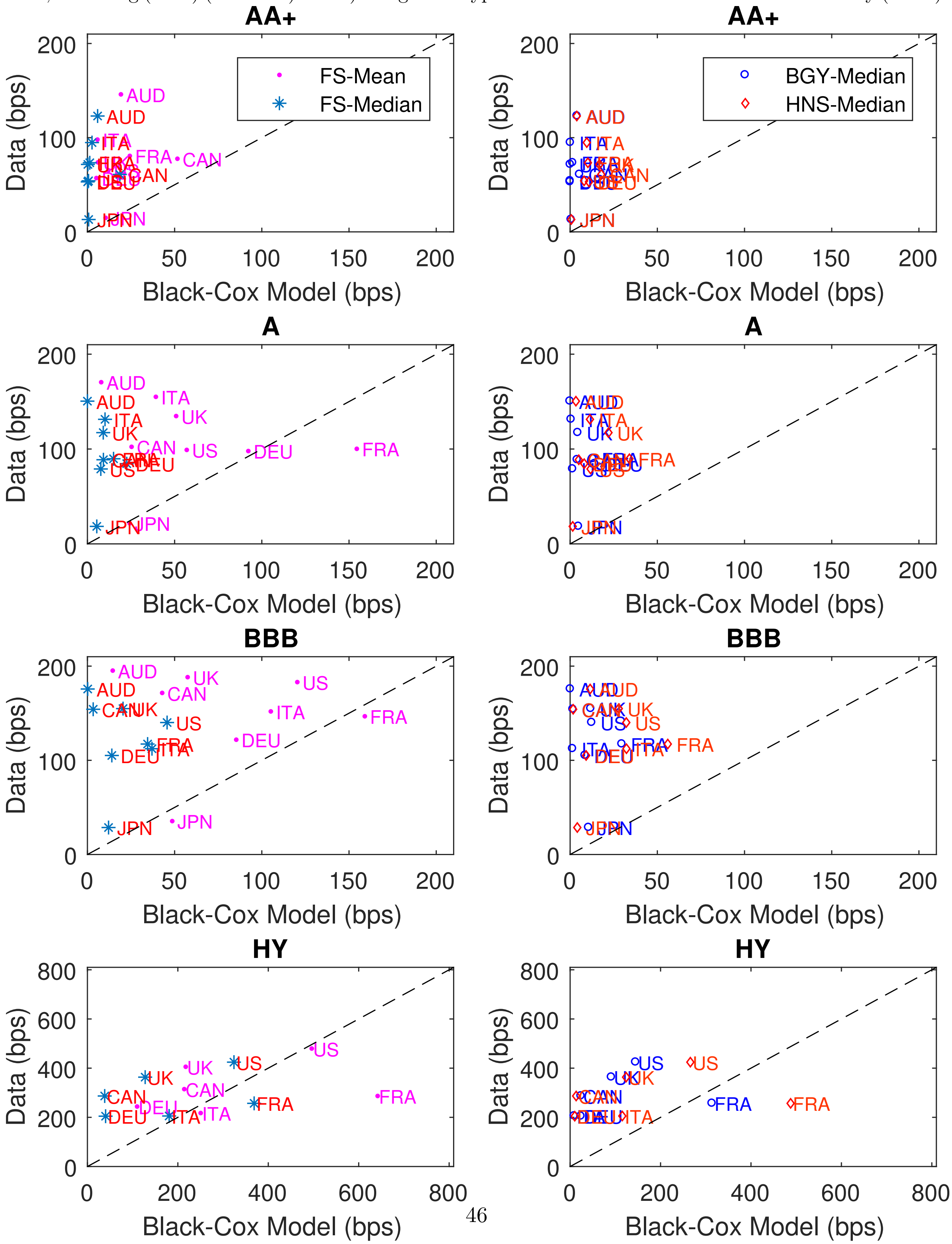


Figure 4: Black-Cox Model and CDS Spreads (bps)

The figures compare the median CDS spreads using three approaches to estimate d: i) Feldhütter and Schaefer (2018)'s approach (star), ii) Bai et al. (2019)'s approach (circle), and iii) an approach based on JMR-type estimates of asset value and asset volatility (diamond).

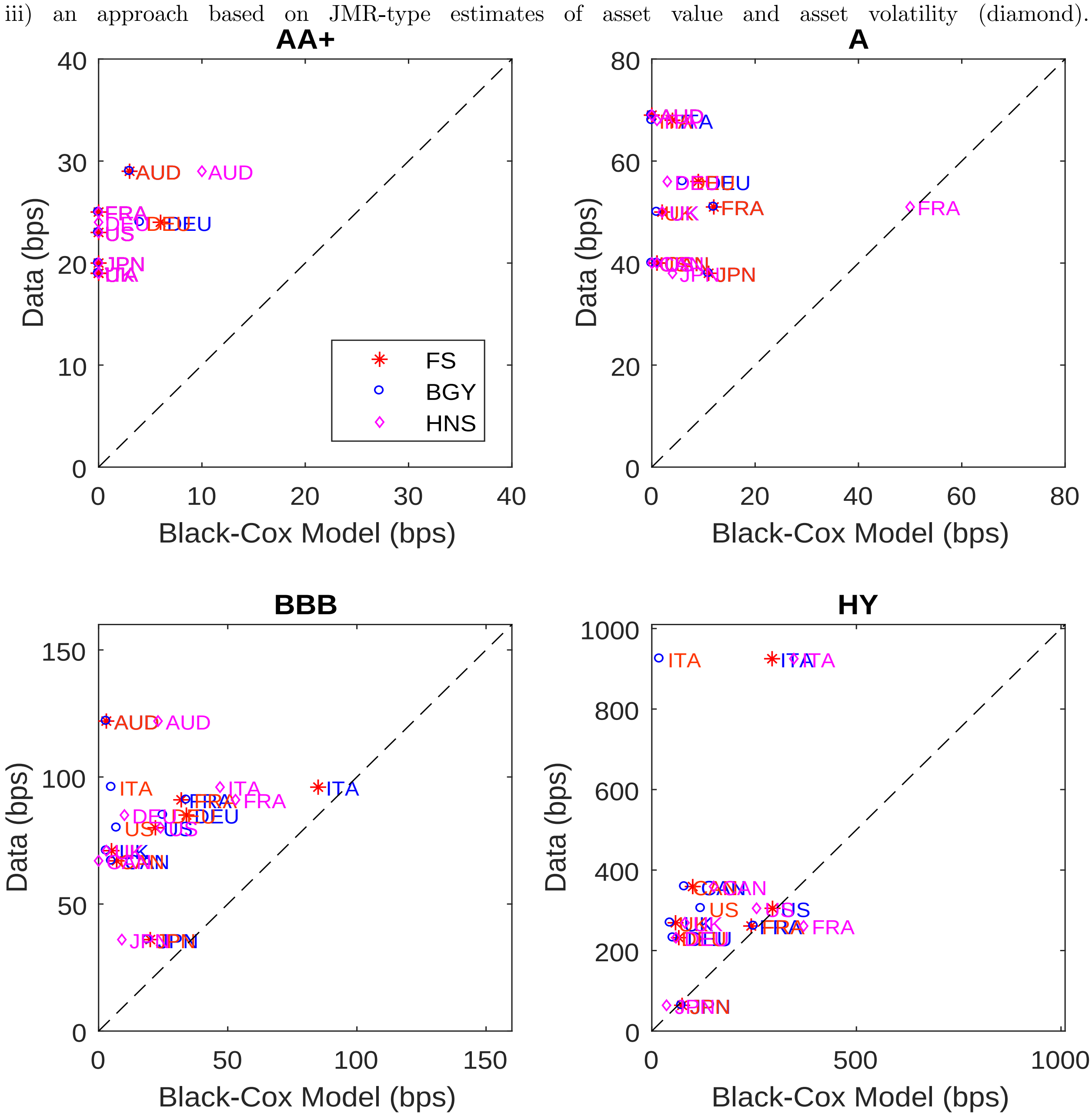
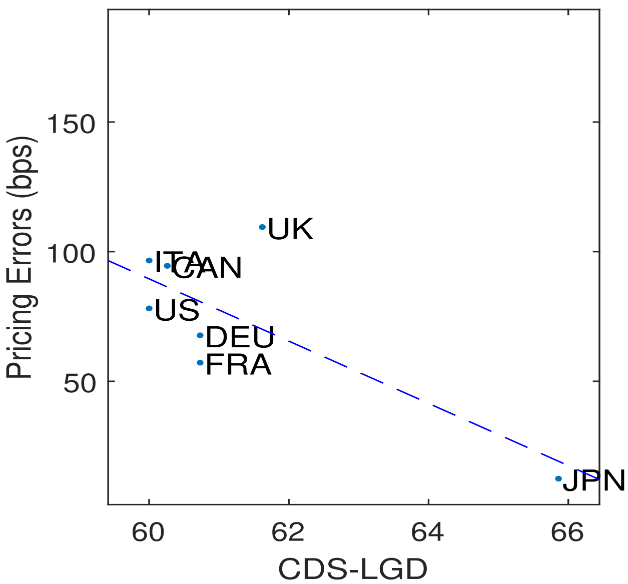


Figure 5: Loss Given Default and Pricing Errors

The figure on the left column plots the median pricing errors (the difference in credit spreads between the data and the Black-Cox model with the FS-method of d) against the average loss given default implied from the HY CDS contracts in each country. The figure on the right plots the pricing errors against the survey-based measure of loss given default in Djankov et al. (2008).



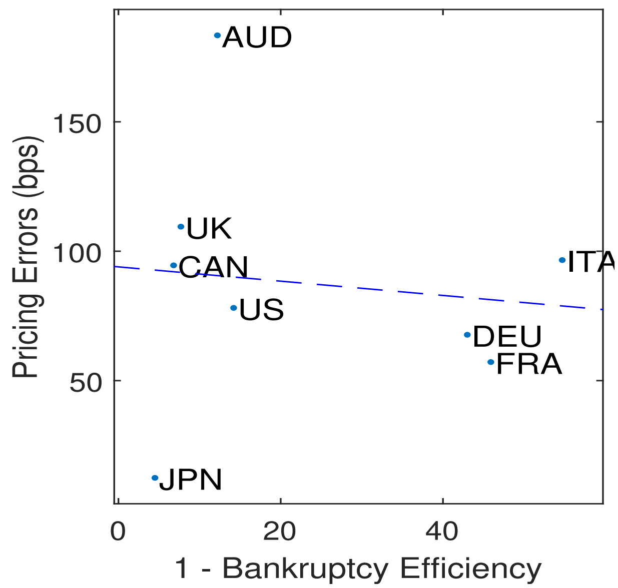


Figure 6: Time Series of Observed and Black-Cox Yield Spreads

These figures plot the monthly observed (blue solid line) and Black-Cox model-implied (orange line with crosses, with the FS-method of d) median credit spreads. Corr(FS) shows the correlation between the two series, while Corr(Indiv.) shows the correlation between the observed credit spreads and the Black-Cox model with individual d.

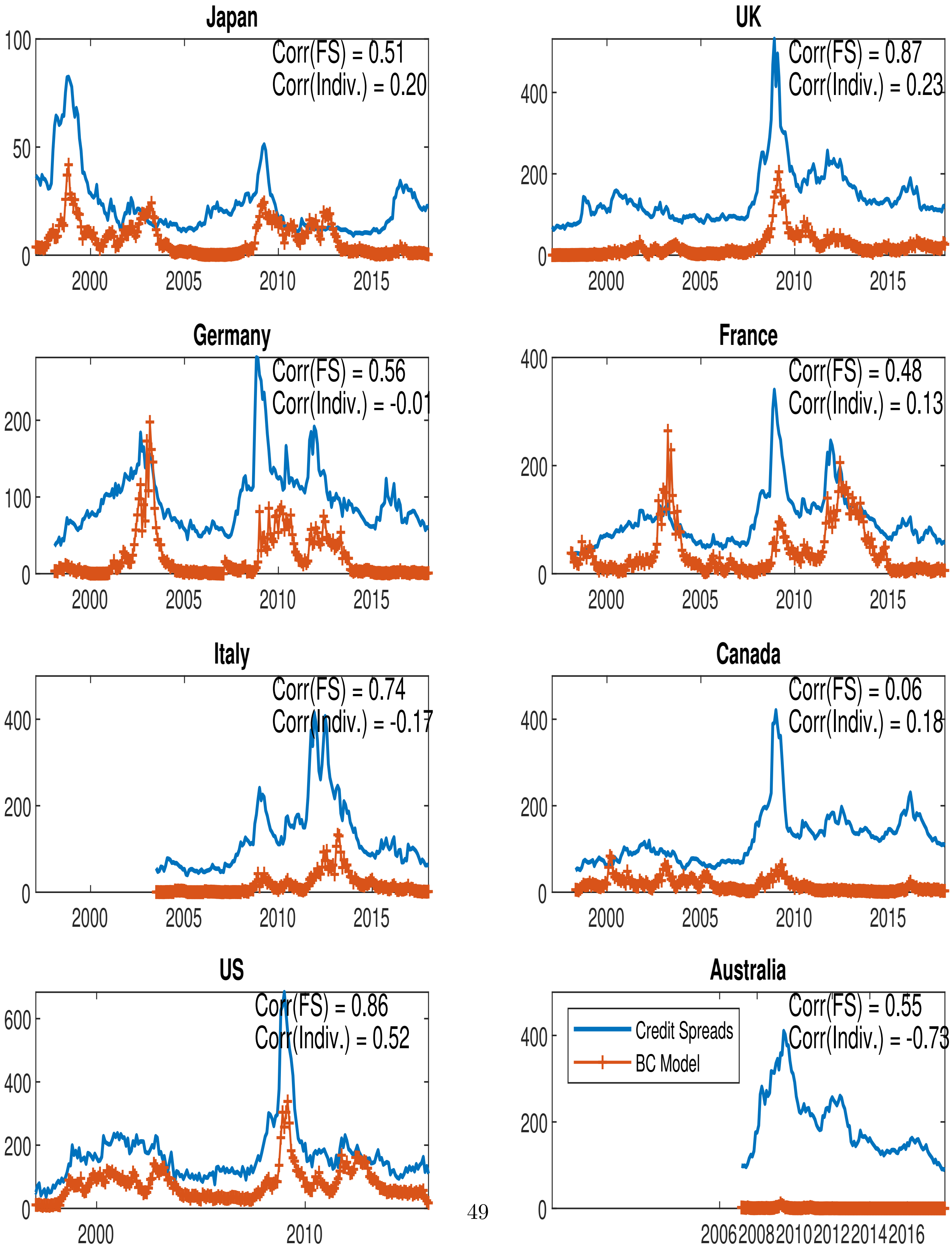


Figure 7: Median Credit Spreads from the Search Friction Model

This figure plots the median credit spreads (1) in the data, (2) produced by the Black-Cox model, and (3) produced by a search friction model with firm fundamental parameters identical to the benchmark Black-Cox model. The medians are computed using the panel data from 1997 to 2017 outside of the U.S., while using the data from 1987 to 2015 for the U.S. The default boundary in structural models is estimated following Feldhütter and Schaefer (2018). In each panel, countries are displayed in order of the median of observed credit spreads.

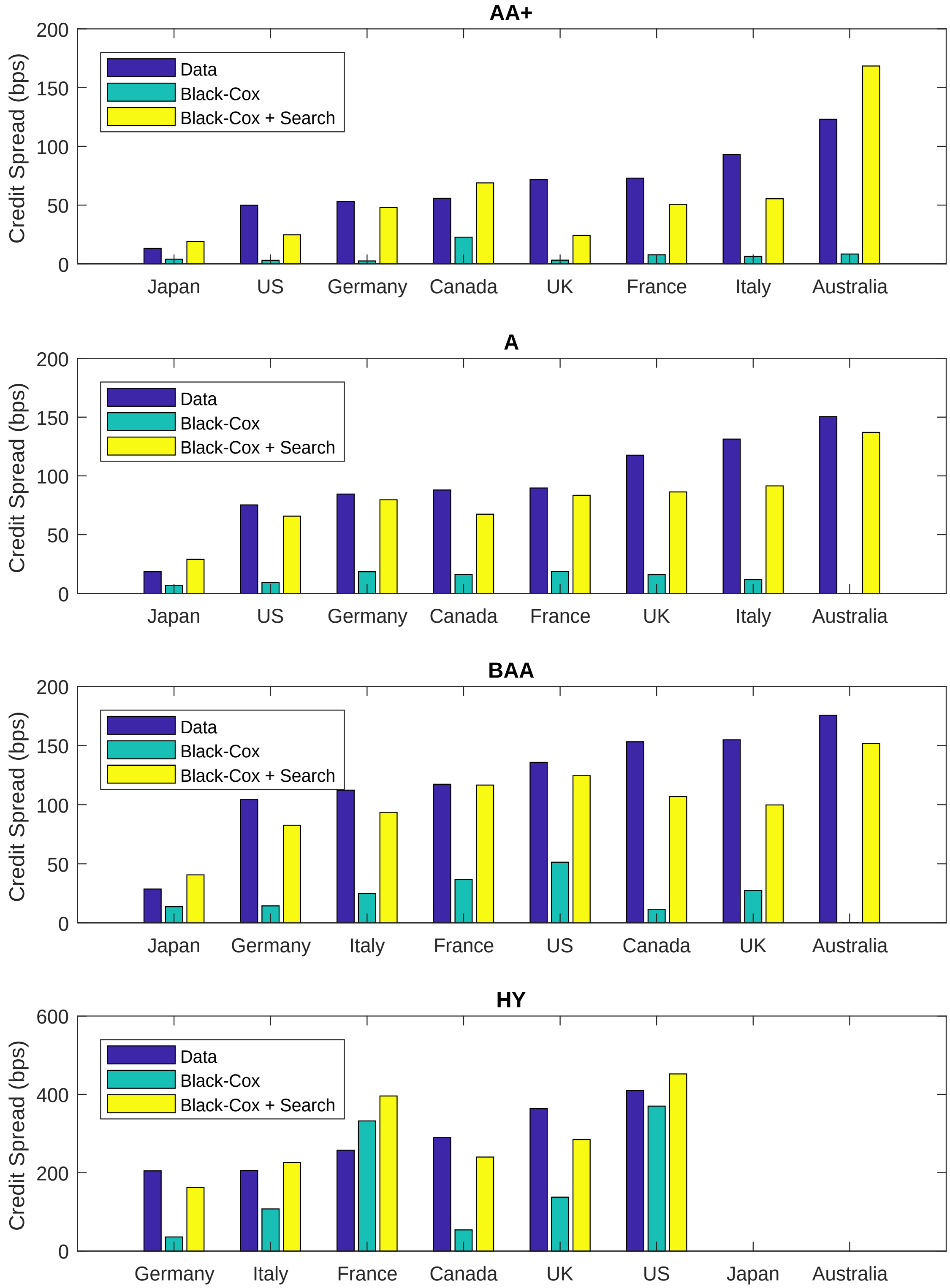
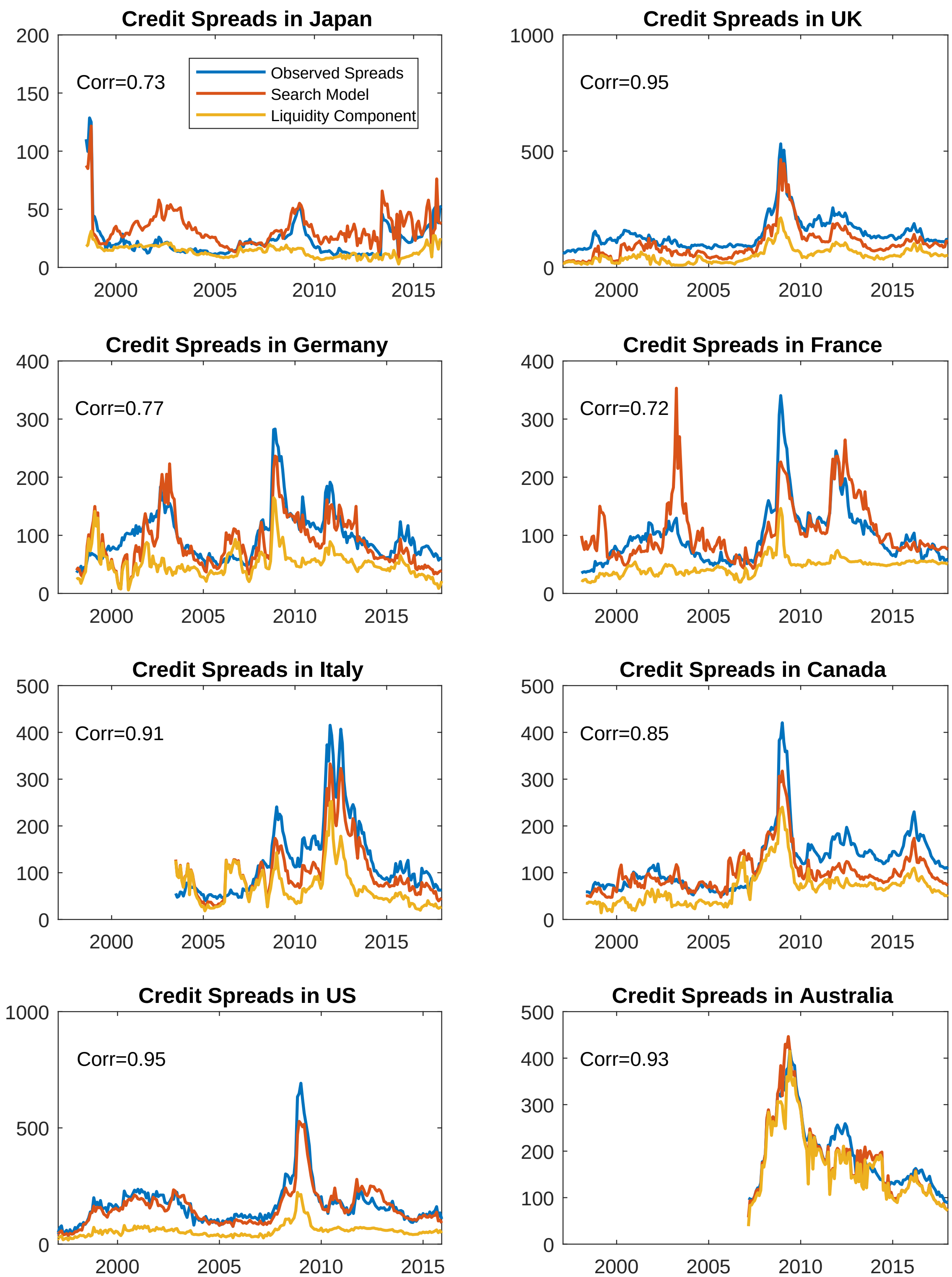


Figure 8: Time Series of Observed and Search Model Yield Spreads

This figure plots the monthly observed and search-model-implied median credit spreads over time. The blue line shows the median corporate yield spreads of senior unsecured bonds in each country, the red line shows the prediction of the model incorporating search frictions, and the yellow line shows the model-implied liquidity component in yield spreads.



Appendix A The Black-Cox Model with Liquidity Frictions

A.1 Bond Valuation and Endogenous Default Boundary

In Section 6.5, we introduce secondary market search costs into the Black-Cox model, and the resultant liquidity discount in bond prices is reflected in Eq. (1).

The bond valuation formula as presented by Eq. (1) is for a given default boundary d. He and Milbradt (2014) derive the endogenous default boundary d^* that maximizes the equity value:

$$d^* = \frac{-\eta v_0^E + \iota_1' \left(\exp(-Z(T-t)) v_1^D h_F - U h_G U^{-1} \cdot v_2^D\right) / (T-t)}{K(\eta - 1)} \tag{14}$$

where

$$\begin{aligned}
\iota_1 =& [1, 0]', \\
\eta =& -\frac{\nu + \sqrt{\nu^2 + 2r(\sigma^A)^2}}{(\sigma^A)^2}, \\
v_1^D =& \begin{bmatrix} K \\ K \end{bmatrix} - Z^{-1} \begin{bmatrix} cT \\ (c-\chi)T \end{bmatrix}, \\
v_2^D =& \begin{bmatrix} R_H K \\ R_D K \end{bmatrix} - Z^{-1} \begin{bmatrix} cT \\ (c-\chi)T \end{bmatrix}, \\
v_0^E =& \frac{1}{r} \left[\iota_1' \left(Z^{-1} \begin{bmatrix} c \\ c-\chi \end{bmatrix} + \frac{1}{T} \exp(-Z(T-t)) v_1^D \right) - K/T - (1-\tau) cT \right].
\end{aligned}$$

Similar to Leland and Toft (1996), the tax benefit of debt, as represented by parameter τ, is an important determinant of optimal default boundary. To examine how the Feldhütter-Schaefer boundary estimation is aligned with the endogenous default model, we take into Eq. (14) the boundary values estimated from Eq. (15) to back out the marginal tax rate. As we show in Table A3, the model-implied tax rate is substantially lower than the empirical measure for all countries. For issuers in the U.K., their median of model-implied τ is even negative, which is not surprising given that the estimate of the U.K. default boundary is above one. When we replace the FS boundary estimates with the ones obtained when we use the BGY approach, the model-implied tax rates move closer to their empirical counterparts for all but two countries, but the overall gap is still large. On the positive side, the cross-country variation in both FS and BGY boundary estimates is generally in line with empirical effective tax rates, with correlations of 46% and 42%, respectively.

Why does the calibration methodology to minimize the $\mathbb{P}$-measure fitting errors lead to boundary values that are much higher than, but reasonably correlated with, what is implied by the endogenous default assumption? One reason to explain this observation is suggested by Bai et al. (2019), who find that a homogenous location of the default boundary does not fit the data well if the asset value is assumed to follow geometric Brownian motion. Another possibility is that the search model examined in this paper does not consider macroeconomic dynamics: as illustrated by Chen (2010) and Chen et al. (2018), firms tend to increase their optimal default boundary in response to regressions. Overall, both interpretations suggest that once we allow for heterogenous and time-varying default boundaries, the model implications might become more consistent with the endogenous default assumption.

A.2 Inference of Search-Based Liquidity

To ensure the search model is qualitatively in line with the notion of the credit spread puzzle, we adopt the same firm-level inputs and specific-level parameters (default boundary and Sharpe ratio) as presented in Section 5. It follows that the model-implied default intensity is identical to the baseline Black-Cox case, regardless of the search parameter values that we use. On the other hand, search parameters θ are identified by minimizing the fitting errors with respect to percentage bid-ask spreads, such that no bond pricing information is involved in the model estimation.

Schestag et al. (2016) empirically compare various approaches to measuring bond liquidity and find that BGN quoted bid-ask spreads are downward biased and relatively inaccurate in the U,S. Since our measure of bid-ask spreads is based on the same data source, we examine this measure's performance in a global setting in the Internet Appendix, Section **??**.

We find that overall individual bonds enjoy better liquidity and that the performance of *Spread_BGN* is significantly improved. Given that the BGN database offers much wider coverage for international bonds than Markit, we stick to *Spread_BGN* in our estimation of liquidity-related parameters. Meanwhile, we keep in mind the directions and magnitude of its potential bias when discussing the model's pricing performance.

The estimation results are summarized in Table A4. Among countries with large overall pricing errors and bid-ask spreads, Italy has a fairly low estimate of meeting intensity, with $\hat{\lambda} = 0.1.76$, and Australia has an extremely high estimate of holding cost parameters. In contrast, the corresponding estimates for Japan are all on the "high liquidity" side, which is consistent with its relatively small pricing errors. In summary, estimates of search parameters suggest that liquidity frictions in the secondary markets have great promise in explaining the cross-country variation in credit spreads above and beyond conventional structural models.

Appendix B Default Boundaries

We determine the default boundary using four different methods in this study. We describe these methods in detail in this appendix.

B.1 Feldhütter and Schaefer (2018)'s Approach

Following Feldhütter and Schaefer (2018), we back out the values for default boundary by minimizing the distance between Moody's default probability and the Black-Cox model prediction at the rating and maturity bin level:

$$d^{FS} = \text{argmin} \sum_{T=1}^{20} \sum_{R=AA+}^{HY} \frac{1}{T} \left| \pi_{T,R}^{Model}(d|\sigma_i^A, A_{i,t}) - \pi_{T,R}^{Moody's} \right| \tag{15}$$

where $\pi_{T,R}(d)$ is the probability of default for T-year bonds with rating R under the $\mathbb{P}$-measure, R ={AA+,A,BBB,HY} for countries other than the U.S., and R ={AAA,AA,A, BBB,BB,B,C} for the U.S. We estimate Eq.(15) separately for each country, allowing d to vary across countries, but holding it constant within a country.

To maximize the sample size, we use all nonfinancial bond issuers, regardless of whether these bonds are senior, unsecured non-callable bonds or not. We also assume that all firms have debt maturing from 1 to 20 years, regardless of actual maturity of the bond issued by these firms.[25]

In order to quantify the magnitude of estimation errors in historical default boundaries, we would need, in principle, micro-level data of default dating back to 1920. Since Moody's Default and Recovery Database covers the default since 1970, the micro-level data is not available to us. Thus, we follow Feldhütter and Schaefer (2018) and use simulation-based methods to compute confidence intervals for historical default frequency (see Internet Appendix IV..1 for details).

In Panel B of Table 3, we present the estimated default boundary for each country. The boundary ranges from 0.85 (Italy) to 1.10 (France). The fact that some countries have the optimal boundary above 1 implies that our measure of market leverage is only a proxy for true leverage. If a true measure of leverage is available in the data, d should not be greater than 1 since there is no reason for a firm to default when firms' equity value is positive. However, there may be debt-like obligations to firms that are missed in the book value of debt in balance sheets. For example, firms with higher operating leverage are more likely to default than firms with low operating leverage, even if the financial leverage is the same.[26] Thus, we use the estimated optimal value of d even when they are above 1.

By letting d vary across countries, we account for the heterogeneity in accounting, legal, and business environments for firms in different countries. Ultimately, what matters for our test of structural models is that we match the model-implied $\mathbb{P}$-measure default probability to the historical data.

In the top panel of Figure A1, we compare the Moody's historical default frequency with the

[25] In Table IA4 in Appendix IV, we present the summary statistics of inputs of all nonfinancial firms in the bond data that we use to evaluate the $\mathbb{P}$-measure default probability. The tables show that firms' characteristics are similar to the smaller sample of noncallable bond issuers in Table 2.

[26] We thank Bob Goldstein for pointing it out.

Black-Cox implied default probability under the $\mathbb{P}$-measure with the optimal default boundary. To construct these figures, we average all international firms in the seven countries to compute the average probability of default. We observe that the confidence band at the long horizon is wide even with 98 years of data, especially for IG bonds. As a result, the model-implied $\mathbb{P}$-measure default probability lies within the confidence band. However, we observe that AA+ rated firms have a lower model-based default probability than the data. Thus, we must be cautious in interpreting the model-based credit spreads for these bonds: rather than argue that these estimates of d are the best estimates for matching historical default frequency, our aim instead is to follow the literature to test a variety of d and look for a robust pattern in the data.

B.2 Bai, Goldstein, and Yang (2019)'s Approach

Bai, Goldstein, and Yang (2019) show that Feldhütter and Schaefer (2018)'s estimates for the default boundary in the U.S. change dramatically if one uses an alternative measure of the market value of firm assets. Instead of adding book value of debt to the market value of equity, Bai, Goldstein, and Yang (2019) propose to add an estimated market value of debt to the market value of equity to obtain the asset value. Since the market value of debt for HY firms is typically lower than the book value, HY firms demonstrate higher leverage with this alternative measure than they do with the standard measure, which uses the book value of debt in the denominator. Since observations of HY firms influence the optimization problem in Eq.(15), Bai, Goldstein, and Yang (2019) report that the alternative measure for market value of assets leads to significantly lower estimates of default boundary than that of Feldhütter and Schaefer (2018).

Following the spirit of Bai, Goldstein, and Yang (2019), we multiply the book value of debt with the average bond price of the firm to obtain the estimate for the market value of debt. By doing so, we implicitly assume that all debts of the firm, including bank loans, have the same market price as average corporate bonds issued by the firm. Using the alternative asset value, we reestimate Eq.(15) and find the optimal value for d in each country.

In Panel B of Table 3, we report the estimates for d for the seven countries. When compared with Feldhütter and Schaefer (2018)'s approach (FS-method), Bai, Goldstein, and Yang (2019)'s approach (BGY-method) yields somewhat lower estimates of default boundaries. In the U.S., we find that default boundary d goes down sharply to 0.67 from 0.89 in Feldhütter and Schaefer (2018)'s estimates (not reported in the table). In contrast, the difference between the FS- and BGY-methods is not as dramatic in international bond markets because we use only four rating categories (AA+, A, BBB, and HY) for international bonds due to the limited sample size, while we have seven categories (AAA, AA, A, BBB, BB, B and CCC) in the U.S. Therefore, the weight of HY bonds is smaller for non-U.S. countries than for the U.S., which explains why both methods yield similar estimates in non-U.S. countries.

In the bottom panel of Figure A1, we compare the model-based $\mathbb{P}$-measure default probability with historical default frequency for the seven countries. Since default boundaries do not change much from the FS-method, the resulting $\mathbb{P}$-measure default probability is also similar to the FS-method.

B.3 A Model-Based Approach

In response to the critics of Bai et al. (2019), Feldhütter and Schaefer (2019) point out that the BGY adjustment for the market-to-book ratio could create bias in the opposite direction, as bank loans tend to have higher seniority and tighter covenants compared to corporate bonds. We consider an alternative approach that is based on the model under consideration and that, as a result, need not approximate the unobservable asset value with some measurable proxies.

Specifically, following Jones, Mason, and Rosenfeld (1984) we note that the market value of asset A_t and asset volatility σ^A in Eq. (3) can be expressed as implicit functions of two variables that can be directly measured in the data—the (quasi-market) leverage ratio L_t^q and equity volatility σ^E,

$$L_t^q = \frac{K}{E(A_t, \sigma_A) + K}, \tag{16}$$

$$\sigma^E = \frac{A_t}{E_t} E_A(A_t, \sigma_A)\sigma_A. \tag{17}$$

With functional form of $E()$ and $E_A()$ as directly derived from the Black-Cox model, we can ensure that parameter estimates are internally consistent with the model and no additional assumptions is imposed. We empirically implement Eqs. (16) and (17) and then insert the estimator of A_t and σ^A into the Black-Cox default probability function $\pi^P()$.[27] Finally, the optimization in Eq. (15) is re-performed with the new model-implied default rates to solve for the default boundary.

$$d^{JMR} = \text{argmin} \sum_{T=1}^{20} \sum_{R=AA+}^{HY} \frac{1}{T} \left| \pi_{T,R}^{Model} \left(d|\sigma_i^A(d), A_{i,t}(d) \right) - \pi_{T,R}^{Moody's} \right| \tag{18}$$

Details of this alternative approach to boundary identification are described in Internet Appendix III.

B.4 Firm-Level Heterogeneous Default Boundary

Thus far, we have been studying the case of constant default boundary, which is assumed to be the same for all firms over time in each country. These estimates imply that at the firm level, the probability of default deviates from the target historical default frequency. Furthermore, as we minimize the average difference between the model and historical data, researchers have considerable

[27] The original Jones-Mason-Rosenfeld estimator applies to the Merton model (Also see Campbell, Hilscher, and Szilagyi (2008), Hillegeist, Keating, Cram, and Lundstedt (2004), and Bai and Wu (2016)). Bao (2009) firstly extends this estimation method the Black-Cox model to identify the values of A_t and σ_A for a given default boundary.

discretion in estimating d: paying more attention to IG bonds versus HY bonds in the optimization problem in Eq.(15) would yield rather different estimates for default boundary (and thus credit spreads).

To address such problems, we test another estimate of heterogeneous default boundary, in which we vary d such that each firm exactly matches the historical default frequency every month. In addition, we allow the firm-specific d to vary over the maturity of the debt such that the model-implied $\mathbb{P}$-measure default probability for each bond matches Moody's historical data. This way, we obtain d such that the process involves no optimization; we can simply replace r_t with μ_t in Eq.(3), equate the probability to the corresponding value in Moody's data, and then numerically back out d. On the other hand, we are making an extreme assumption that all firms with the same credit rating have the same probability of default; nonetheless, we use this approach to give the model the best chance to explain the heterogeneity across countries.

Indeed, if we let d vary across firms and maturity, then the mean and median are close to each other. In such cases, evaluating the model based on the mean or median does not matter.

In an untabulated analysis, we find that model-based credit spreads increase dramatically for all countries, and now the model overpredicts credit spreads for Japan. However, even with the extreme flexibility in default boundary, the model fails to generate large enough variations *across* countries. When we change specification for d, the model generates large (or small) credit spreads for all countries, but changing d does not help explain why credit spreads in Australia, Italy, and Canada are higher than those in Germany and Japan. We also find that although using a firm-level d does not help match HY credit spreads in the data and the model, the model at least generates substantial cross-sectional variation in credit spreads.

Table A1: Sample Selection

Firm Type	Bond Type	Industry	Count	Japan	UK	Germany	France	Italy	Canada	Australia	All
All			# Bonds	2,824	1,086	1,149	1,054	388	1,774	335	8,610
			# Obs	178,139	79,948	61,576	63,764	21,167	127,403	15,005	547,002
Private			# Bonds	1,103	665	788	484	254	998	127	4,627
Firms			# Obs	55,894	55,395	43,618	30,764	13,191	79,523	5,985	293,390
Public			# Bonds	1,721	421	361	570	134	776	208	3,983
Firms			# Obs	122,245	24,553	17,958	33,000	7,976	47,880	9,020	253,612
Within	Noncallable, senior, unsecured bonds		# Bonds	976	278	294	436	121	396	164	2,665
Public		Nonfinancials	# Bonds	925	219	233	364	121	245	66	2,173
Firms		Financials	# Bonds	56	59	61	78	1	155	98	508
	Others		# Bonds	745	143	67	134	13	380	44	1,318
	Total		# Bonds	1,721	421	361	570	134	776	208	3,983
Final Sample			# Bonds	925	219	233	364	121	245	66	2,173
			# Obs	55,173	14,601	12,240	22,737	7,406	18,338	3,498	133,993
			# Firms	106	60	52	50	18	55	23	364

Note: Table presents the sample selection process. # Obs is the number of bond-month observations. The sample is monthly from January 1997 to December 2017.

Table A2: Cumulative Default Frequency: 1970–2016

Panel A: Outside the U.S.

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
AAA	0.00	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18
AA	0.04	0.05	0.12	0.17	0.26	0.38	0.50	0.57	0.64	0.72	0.79	0.88	0.93	0.96	0.96	0.96	1.00	1.13	1.26	1.34
A	0.08	0.24	0.44	0.70	1.08	1.43	1.77	2.11	2.40	2.66	2.87	3.10	3.33	3.54	3.80	4.04	4.26	4.41	4.49	4.56
BBB	0.20	0.45	0.79	1.07	1.30	1.48	1.69	1.97	2.22	2.38	2.57	2.72	2.81	2.85	2.89	2.94	2.97	2.97	2.97	2.97
Ba	0.84	2.22	3.47	5.02	6.32	7.35	8.13	8.93	9.66	10.28	10.57	10.78	10.85	10.90	10.90	10.90	10.90	11.05	11.17	11.25
B	2.78	6.91	10.45	13.74	16.23	17.91	19.46	20.58	21.36	21.59	21.70	21.70	21.70	21.91	22.26	22.82	23.29	23.60	23.60	23.60
Caa-	16.78	26.45	32.83	35.97	38.08	39.55	41.86	43.59	44.90	46.15	47.47	47.81	48.29	48.29	48.29	48.29	48.29	48.29	48.29	48.29

Panel B: U.S.

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
AAA	0.00	0.00	0.00	0.06	0.17	0.30	0.42	0.54	0.66	0.77	0.89	1.00	1.11	1.17	1.22	1.28	1.33	1.33	1.33	1.33
AA	0.01	0.03	0.13	0.31	0.47	0.61	0.72	0.83	0.92	1.01	1.13	1.30	1.47	1.64	1.78	1.89	2.01	2.15	2.40	2.59
A	0.04	0.13	0.32	0.49	0.68	0.89	1.14	1.39	1.65	1.88	2.10	2.30	2.52	2.73	2.96	3.20	3.43	3.69	3.95	4.23
BBB	0.18	0.48	0.86	1.28	1.71	2.13	2.50	2.86	3.29	3.77	4.26	4.73	5.19	5.63	6.07	6.47	6.87	7.22	7.49	7.76
Ba	1.17	3.19	5.36	7.67	9.75	11.62	13.25	14.83	16.35	17.83	19.29	20.77	22.11	23.53	24.81	25.93	26.83	27.80	28.78	29.52
B	3.83	8.64	13.10	17.01	20.45	23.56	26.27	28.55	30.73	32.74	34.33	35.58	36.81	37.87	38.82	39.50	40.20	40.77	41.22	41.73
Caa-	17.60	27.14	33.55	38.31	42.06	44.72	46.55	48.44	50.21	51.72	52.84	53.68	54.60	54.60	54.60	55.62	56.94	56.94	56.94	56.94

This table reports the cumulative default frequency for the regions outside the U.S. (Panel A) and U.S. (Panel B) over the period 1970–2016. Every year, we form a cohort of firms with the same credit rating and keep track of the fraction of firms default for the subsequent 20 years. We then take the average across cohorts to estimate the cumulative default frequency. We compute using corporate credit ratings, excluding structured finance and real estate finance.

Table A3: Estimates of the Tax Benefits of Debt

	Japan	UK	Germany	France	Italy	Canada	US	Australia
$\hat{\pi}_{FS}$	6.15	-5.97	3.08	0.03	1.49	1.74	6.82	3.73
$\hat{\pi}_{BGY}$	5.97	-0.48	2.20	0.28	1.85	2.16	9.89	3.73
π_{eff}	16.80	10.02	21.02	17.39	22.23	11.34	23.45	8.60

This table reports the marginal tax benefit of corporate debt as implied by the He-Milbradt model. The rows labeled $\hat{\pi}_{FS}$ and $\hat{\pi}_{BGY}$ show the model-implied median tax rate for each country when the endogenous default boundary is set identical to the default boundary estimated with Feldhütter and Schaefer (2018)'s and Bai et al. (2019)'s approaches. π_{eff} denotes the effective tax rate calculated as $\pi_{eff} = 1-(1-\pi_c)(1-\pi_d)/(1-\pi_i)$, where π_c is the tax rate for corporate earnings, π_d is the tax rate for dividend income, and π_i is the tax rate for personal interest income. The underlying annual tax rates are retrieved from KPMG Tax Rates Online (https://home.kpmg/xx/en/home/services/tax/tax-tools-and-resources/tax-rates-online.html). For each country, the reported effective tax rate is the median over its bond sample period.

Table A4: Estimates of Bond Illiquidity in Secondary Markets

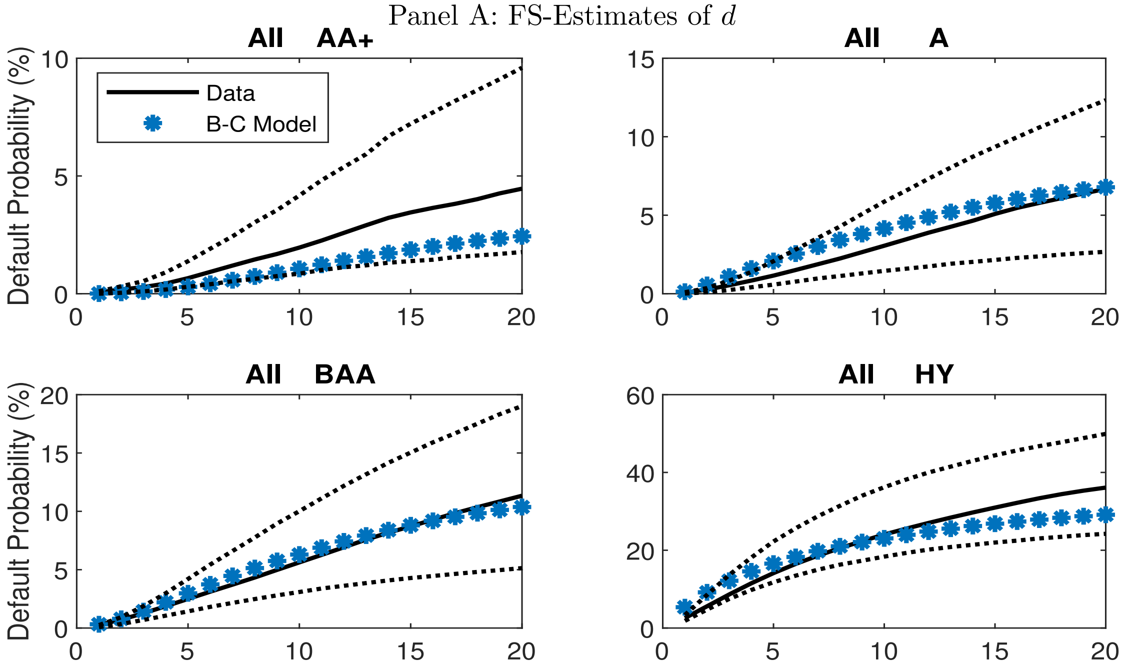
	$\chi_p \times 10^2$	χ_c	ξ	λ	β
Japan	0.09	0.05	0.11	5.32	0.02
	[0.03]	[0.02]	[0.01]	[0.09]	[0.00]
UK	0.09	0.04	0.27	4.09	0.07
	[0.04]	[0.03]	[0.04]	[0.40]	[0.02]
Germany	0.61	0.11	0.14	5.94	0.02
	[0.07]	[0.05]	[0.02]	[0.13]	[0.01]
France	0.26	0.08	0.25	6.09	0.05
	[0.46]	[0.15]	[0.03]	[0.11]	[0.03]
Italy	0.19	0.01	0.21	1.76	0.12
	[0.03]	[0.03]	[0.01]	[0.12]	[0.00]
Canada	0.23	0.13	0.11	4.44	0.02
	[0.11]	[0.09]	[0.01]	[0.48]	[0.01]
US	0.03	0.03	0.18	5.03	0.03
	[0.03]	[0.14]	[0.03]	[1.40]	[0.02]
Australia	1.06	0.40	0.18	4.74	0.02
	[0.24]	[0.09]	[0.04]	[0.23]	[0.00]

The table presents the estimated parameters on secondary market search frictions as modeled by He and Milbradt (2014). They include holding cost per unit of principal χ_p, holding cost per unit of coupon χ_c, liquidity shock intensity ξ, the intensity to meet dealers λ, and the bargaining power of investors β. These search model parameters are estimated by minimizing the mean squared fitting errors to the observed bid-ask spreads. Standard errors are reported in parentheses.

Figure A1: P-Measure Default Probabilities for Different Default Boundaries

These figures show the Black-Cox model-implied $\mathbb{P}$-measure probability of default (star), which is computed by taking the average across firms and time for each rating and maturity bin. The lines show the Moody's historical default frequency from 1920 to 2017. The 95% confidence interval (dotted line) is computed based on the simulation method described in Section 5.3. Panel A is based on the default boundary estimate following Feldhütter and Schaefer (2018), while Panel B corresponds to the Bai, Goldstein, and Yang (2019) method of boundary estimation. Panel C presents the performance of an alternative estimator of default boundary in which the asset value and volatility are literally derived from the Black-Cox model.

Panel A: FS-Estimates of d



Panel B: BGY-Estimates of d

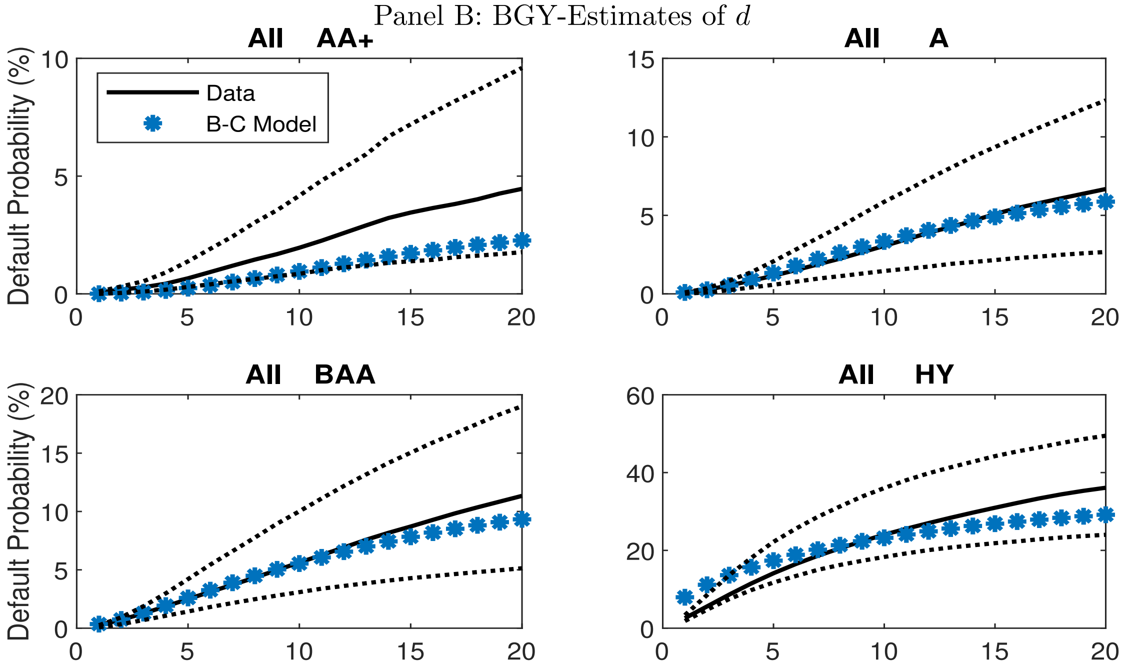
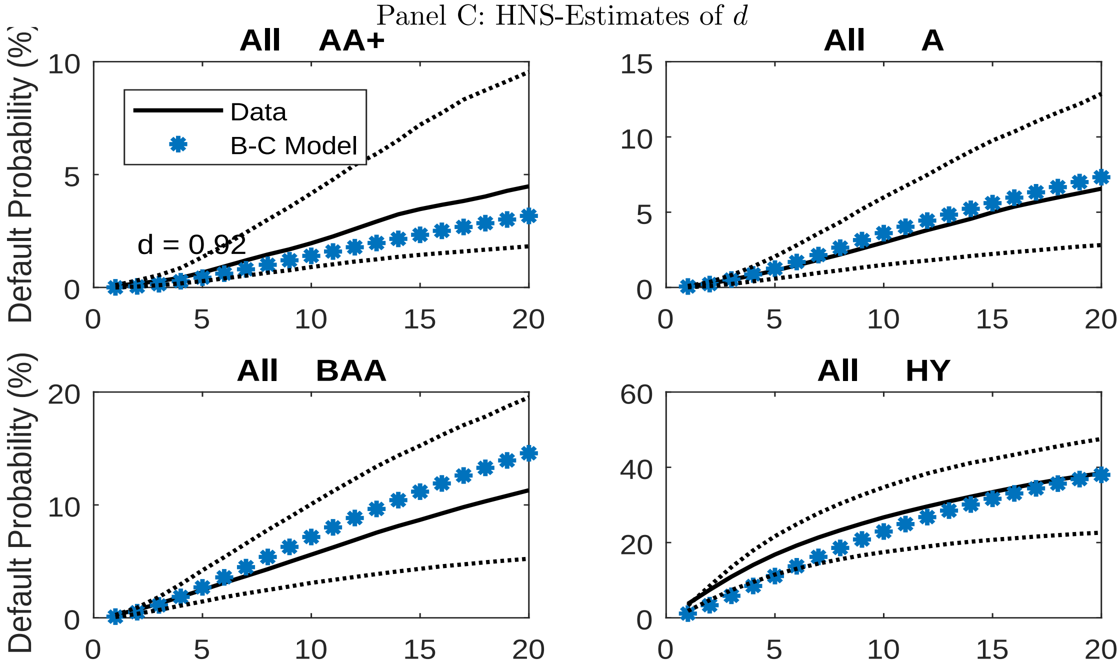


Figure A1 − Continued

Panel C: HNS-Estimates of d



Internet Appendix to "The Global Credit Spread Puzzle"

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I Explaining Changes in Credit Spreads

Collin-Dufresne, Goldstein and Martin (2001) attempt to explain changes in credit spreads using the inputs to the Merton (1974) model. Rather than estimating the Merton model, they run regressions of monthly changes in credit spreads, effectively freeing the parameters of the model to increase the chance to fit the data. Using the sample of U.S. bonds, Collin-Dufresne, Goldstein and Martin (2001) find that the regression R-squared is quite low, suggesting that there may be a bond-market specific factor driving credit spreads.

Now we turn to the international evidence using the bond-level regressions of monthly credit spread changes in the spirit of Collin-Dufresne, Goldstein and Martin (2001),

$$\Delta CS_{k,t} = b_{k,0} + b_{k,1}R_{k,t} + b_{k,2}\Delta r_t^{10} + b_{k,3}(\Delta r_t^{10})^2 + b_{k,4}\Delta slope_t + b_{k,5}\Delta vol_{k,t} + b_{k,6}R_{INDEX,t} + b_{k,7}\Delta skew_{k,t} + \nu_{k,t} \tag{19}$$

where R_k is a stock return on the bond issuer, r^{10} is 10-year risk-free yields in each currency, *slope* is the difference between 10 and 2 year yields, *vol* is the issuer's stock volatility, R_{INDEX} is the return on the country's major stock index, and *skew* is the skewness of issuer's stock return.[28] and examine whether the regression R-squared is sufficiently large.

Table IA1 reports the estimates for Eq.(19) averaged across bonds together with t-statistics. Following Strebulaev and Schaefer (2007), we account for cross-sectional correlation in credit spread changes in computing standard errors for slope estimates. For each country, we reports the coefficients averaged across all bonds. In addition, we report the results for bonds with below-median leverage and above-median leverage separately.

We find that the loading on each factor is generally sensible: higher stock returns on issuer's stock are negatively correlated with credit spread changes as they reflect improving firm value. Except for Japan and Canada, a rise in 10-year risk-free rate is negatively related with credit spread changes, while a rise in yield curve slope is positively associated. Rising volatility leads to an increase in credit spreads as they reflect increasing risk of firm values. A positive overall stock market returns are negatively correlated with credit spread changes even after controlling for individual stock returns.

For non-Japanese bonds, the adjusted R-squared averaged across bonds are comparable to the levels in the U.S., ranging from 0.22 to 0.33. If the Merton model holds, these R-squared must be close to one, and they are clearly below one. The average R-squared for bonds in Japan is unusually low, estimated at 0.06 using all bonds.

Since we are using the same data source for all countries, low R-squared for Japanese bonds

[28] Collin-Dufresne, Goldstein and Martin (2001) use option-based volatility and skewness measures as right-hand side variables. As we do not have reliable option data for those six countries, we rely on realized volatility and skewness from daily stock returns.

cannot be explained by the difference in data quality. One potential reason is that the level of credit spreads in Japan is generally much lower than other countries, and that monthly changes are small and dominated by measurement errors. In addition, the average number of issues per issuer is much higher in Japan (nearly 10 issues per firm) than other countries, and fragmentation of bonds makes Japanese bonds less liquid than other countries.

Although the low R-squared in regression Eq. (19) is compelling, one may be concerned about the potential nonlinear relationship between credit spreads and their determinants, which can be missed by regression in Eq. (19). To address this concern, we run a complimentary regression of credit spread changes on changes in distance to default,

$$\Delta CS_{k,t} = b_{k,0} + b_{k,1}\Delta DD_{k,t} + \nu_{k,t} \tag{20}$$

where $DD_{k,t}$ is distance to default of the bond's issuer.

Table IA1 reports the average coefficients and R-squared for Eq.(20). Consistent with the prediction of the model, an increase in distance to default are negatively correlated with credit spread changes. However, after accounting for a potential nonlinearity, adjusted R-squared is disappointingly low, ranging from 0.02 in Japan to 0.07 in Italy.

Based on the reduced-form analysis, we do not see convincing evidence for the performance of structural models of debt in explaining the time-series variation in credit spreads. The analysis in this section, however, does not answer the question as to whether structural models can match the average level of credit spreads. We will turn to this question in the next section.

II Alternative Specifications of the Black-Cox Model

In uncovering the credit spread puzzle, Huang and Huang (2012) consider two alternative specifications of theoretical bond prices. The first adopts the the discrete-time recovery assumption of Duffie (1998), i.e., the recovery of face value is realized on the first scheduled coupon date after default. For a bond with N coupon payments left, its bond pricing formula is given by

$$D_D(T-t) = \sum_{i=1}^{N} CF_i e^{-r(T_i - t)}(1 - \pi^Q(t, T_i)) + RK \sum_{i=1}^{N} e^{-r(T_i - t)} \left[\pi^Q(t, T_i) - \pi^Q(t, T_{i-1})\right], \tag{21}$$

where T_i denotes the ith coupon date and CF_i the corresponding cash flows to bondholders.[29] The second specification follows Longstaff and Schwartz (1995) by assuming that the constant recovery rate R is measured with respect to an otherwise equivalent Treasury security (with the same coupon

[29] Bao (2009) focuses on the same specification when examining the performance of Black-Cox model in explaining cross-sectional yield spreads.

structure as the defaultable bond). It follows that the according bond pricing formula becomes

$$D_{LS}(T-t) = \sum_{i=1}^{N} CF_i e^{-r(T_i-t)} \left[1-(1-R)\pi^Q(t,T_i)\right]. \tag{22}$$

And the modeled corporate yields under these two specifications can be computed by solving the following equation

$$D_j(T-t) = \frac{cK}{m}\sum_{i=1}^{N} e^{-y_j(T_i-t)} + Ke^{-y_j(T-t)}, \quad j \in \{D, LS\}, \tag{23}$$

where m denotes the coupon frequency.

On the other hand, Feldhütter and Schaefer (2018) focus on the credit spread on a zero-coupon bond that recovers a fixed fraction R of the principal at maturity if default occurs,

$$s_z = -\frac{1}{T-t}\log[1-(1-R)\pi^Q(t,T)]. \tag{24}$$

A comparison with other three specifications reveals that, if we ignore the difference in recovery timing, Eq. (24) is a special case with the coupon rate set to zero.

Table IA2 compares the credit spreads generated by different model specifications, with the same default boundary which is identified using the FS method. We find that switching from one model specification to another has very limited impact on the model-implied spreads—the difference rarely exceeds ten basis points. Also, the pricing performance of our baseline specification with continuous coupon tends to be more in line with the zero-coupon specification, while results of the Longstaff-Schwartz specification share a closer similarity with the case of discrete-time recovery. This finding might be attributable to the fact that latter two specifications take into account the actual frequency of coupon payments. Overall, our baseline results as shown in Section 6 are robust to alternative assumptions of coupon structure and recovery upon default.

III Default Boundary Identification Based on JMR-Type Estimates of Model Parameters

In Section 5.3, the default boundary identified with the Feldhütter and Schaefer (2018) method relies on the following model-free estimation of key parameters,[30]

$$\frac{K}{A_t} = L_t^q = \frac{K}{E_t + K}, \tag{25}$$

$$\sigma_{i,t}^A = \sqrt{(1 - L_{i,t})^2(\sigma_{i,t}^E)^2 + L_{i,t}^2(\sigma_{i,t}^D)^2 + (1 - L_{i,t})L_{i,t}\sigma_{i,t}^E\sigma_{i,t}^D\rho^{ED}}. \tag{26}$$

As argued by Bai et al. (2019), the quasi-market leverage L_t^q could considerably deviate from the true ratio of the book value of debt to the market value of assets for speculative-grade bond issuers. To assess the robustness of the model's pricing results to this issue, we employ a more precise identification of model inputs within the Black-Cox framework. Specifically, we can essentially apply the estimation method of Jones et al. (1984) to the Black-Cox model to identify the values of A_t and σ_A for a given default boundary. Then we search for the optimal location of default boundary which minimize the gap between the and term structure of historical default rates and its model-implied counterpart.

By matching model-implied values of market leverage and equity volatility to observed values, we obtain the following equation set as shown in Appendix B.3,

$$L_t^q = \frac{K}{E(A_t, \sigma_A, d) + K}, \tag{27}$$

$$\sigma^E = \frac{\partial E}{\partial A}\frac{A_t}{E_t}\sigma_A. \tag{28}$$

The modeled equity value $E(A_t, \sigma_A, d)$ is derived as

$$E(A_t, \sigma_A, d) = A_t - (d - R)KG(A_t, \sigma_A, d) - D(A_t, \sigma_A, d). \tag{29}$$

where the first two terms on the right-hand side capture the levered value of the firm, and the modeled market value of debt $D()$ follows Eq (8).

Compared with the estimation method employed by Bao (2009), we take interest payments into account, which introduces two modifications. First, the equity cannot be treated as down-and-out barrier call option on the unlevered asset value anymore. In other words, new equity will

[30] In the original Feldhütter and Schaefer (2018) method, Eq. (26) is replaced by

$$\sigma_A = \alpha(1 - L_t^q)\sigma_A,$$

where the multiplier α is used to adjust for the gap between asset volatility and its lower bound. It is defined as 1 if $L_t^q < 0.25$, 1.05 if $0.25 < L_t^q \leq 0.35$, 1.10 if $0.35 < L_t^q \leq 0.45$, 1.20 if $0.45 < L_t^q \leq 0.55$, 1.40 if $0.55 < L_t^q \leq 0.75$, and 1.80 if $L_t^q > 0.75$. We do not adopt this approximation to estimate asset volatility in Section 5.3, because this multiplier is estimated with the U.S. data and its applicability to international bond issuers is unknown.

issued if the firm's cash flow δA_t is insufficient to cover the coupon payment. Therefore, the total value of equity must be calculated as in Eq (29). Second, a fixed recovery rate R upon default becomes more relevant than a constant recovery rate always received at the maturity date. When we implement Eqs. (27) and (28) at the firm level, the coupon rate c is defined as the annual net interest payment divided by the book value of debt. This is different from its definition when we calculate the modeled corporate yield spread at the bond level.

The identification of A_t and σ_A based on Eqs. (27) and (28) is worth noting in the context of model specification analysis. First, it is literally derived from the implications of the Black-Cox model and does not impose additional assumptions. Second, by relying on firm-level balance sheet information and equity volatility, we can minimize the effect of measurement error and sampling uncertainty and attribute the test results mostly to the model specification error. Finally, this approach need not require the pricing information in corporate bond markets—which is involved in the market-to-book adjustment made by Bai et al. (2019)—and thus ensures that the model spreads are purely out-of-sample predictions.

Eq. (16) and (17) indicate that, for a given default boundary d, we can back out A_t and σ_A from observed quasi-market leverage and equity volatility. It follows that the optimization problem in (15) can be resolved with A_t and σ_A expressed as an implicit function of $\{d, L_t^q, \sigma_E\}$. Table IA3 illustrates the impact of this new estimation method on firm-level inputs. With the new optimal boundary, the model-based estimates, $A_t^\dagger$ and $\sigma_A^\dagger$, are fully determined. As the difference of $A_t^\dagger$ from the approximate asset value $A_t \approx E_t + K$ used in the FS method is entirely driven by the model-implied debt value $D_t^\dagger$, we report summary statistics of the model-implied market-to-book ratio of corporate debts. In addition, we also compare $\sigma_A^\dagger$ with our baseline estimate of σ_A, which is based on Eq. (10).

In each country/rating category, consider a representative firm of which the market-to-book ratio of debt is set equal to the median of that category. Table IA3 indicates that the market value of debt for a representative AA+ firm is substantially greater than its book value in all countries. In contrast, the median market-to-book ratio of speculative-grade debts is lower than one for all countries except for Canada. Both findings are consistent with the evidence for US as documented by Bai et al. (2019) that (1) the market-to-book ratio of AA+ corporate bonds is somewhere between 1.07 and 1.10 and, (2) high-yield corporate bonds could be valued well under par in the market. It follows that the approximation as shown in Eq. (25) overestimates the distance-to-default of high-rating bond issuers and underestimates that of speculative-grade ones. With that being said, the magnitude of underestimation is not as significant as shown in Bai et al. (2019) because corporate debts include bank loans as well, which are generally senior to corporate bonds.

The impact of the JMR-type estimator is not limited to the estimates of asset value. Table IA3 also shows that the ratio of $\sigma_A^\dagger$ to σ_A largely decreases with the credit rating in all countries. This pattern actually complements our earlier finding that $D^\dagger/K$ generally increases with the rating: compared with our baseline estimator based on Eq. (25), the JMR-type estimator leads to lower market leverage and thus greater sensitivity of equity value to asset value if all other parameters stay the same; as shown in Eq. (28), since the equity volatility measured from the data is fixed, the asset volatility has to be depressed given an increase in the equity-to-asset sensitivity. Similarly, for high yield bonds the model-based estimates of asset volatility tend to be higher than the model-free ones. Overall, we find that the newly identified default boundaries match the empirical leverage ratio and equity volatility with fairly reasonable parameter values.

IV Match in P-Measure Default Probability

Consider all issuers of corporate bonds. Tables IA4 present summary statistics for non-financial firms matched to all bonds, including callable bonds. We do not use callable bonds in computing credit spreads, but we still use these firms in estimating default boundary. Comparing Table 2 and Table IA4, we find that the characteristics of the firms are similar between these two samples, which justifies our choice of finding default boundary using the larger sample.

IV..1 Confidence Intervals for Historical Default Frequency

For each country, we select a cohort of identical firms that start their history with values of leverage, payout, and asset volatility in Table IA4. For this simulation, we choose d so that the simulation mean probability of default matches the historical default frequency for each rating and maturity. Here, the goal is to quantify the uncertainty around historical default frequency, not to evaluate the Black-Cox model. The size of the cohort is the same as the number of firms in each rating category.

We then simulate shocks to firms asset value for 20 years at the weekly frequency by:

$$\frac{dA_{i,t}}{A_{i,t}} = (\mu_i - \delta_{i,t})dt + \sigma_i^A dW_{i,t} \tag{30}$$

$$dW_{i,t} = \sqrt{\rho}dW_{s,t} + \sqrt{1-\rho}dW_{i,t} \tag{31}$$

and record firms that touch the default threshold (d times leverage) for the first time. Following Feldhütter and Schaefer (2018), we use a correlation coefficient of $\rho = 0.20$. The number of firms that default in year y as a fraction of remaining firms in the cohort gives an estimate for a hazard rate for the cohort in y-th year.

We repeat the exercise for cohort 1 to 78 (98 years of historical default data minus 20 years of estimation horizon), allowing one time-series of systematic shocks to affect multiple (adjacent) cohorts. Finally, we compute the average hazard rate across cohorts, and use it to compute the cumulative probability of default for maturity of 1 to 20 years. We repeat this process 1,000 times to create the 95 percent confidence interval.

V Using Swap Rates as Risk-Free Benchmark

Figure IA2 plots the Black-Cox model-based credit spreads on the x-axis and the corporate credit spreads on the y-axis, using swap rates as risk-free benchmark. Compared with the main results in Figure 3, the observed credit spreads are somewhat lower, but using different risk-free benchmark does not change the fact that the Black-Cox model does not explain the difference in credit spreads across countries.

VI Currency Factors and Corporate Bond Returns

The fact that corporate credit spreads are higher in Australia than in Japan suggests that there may be a link between the currency risk premiums documented in Lustig, Roussanov, and Verdelhan (2011) and the corporate credit spreads. Specifically, Lustig, Roussanov, and Verdelhan (2011) suggest that Ausutralian dollars load positively on the currency "slope" factor while Japanese Yen loads negatively, which explains why borrowing in Japanese Yen and depositing the proceeds in Australian dollars yields high returns on average.

Though we focus on credit spreads instead of excess returns, they are both driven by risk premiums. Thus, large credit spreads in Australia may be just reflection of large currency risk premiums on Australian-dollar denominated fixed income assets. To test this hypothesis more formally, we compute monthly returns on corporate bonds in each country, multiply them with the growth rate in exchange rates, and take value-weighted average to form IG- and HY-bond portfolios. For each bond, we also compute returns on the government bonds with matching cash flows, convert them into U.S. dollar-denominated returns using exchange rates, and form portfolios using the same weights as the corporate bond portfolios. We then take the difference in monthly returns between the corporate and the matching government bond portfolios to obtain excess returns. This is a U.S. dollar-denominated return on zero-cost portfolio which does not require investors to convert U.S. dollars into foreign currencies. As we work on excess returns, there is no mechanical link between the excess returns and the currency risk factors. For example, if Australian dollars appreciate against U.S. dollars in month $t+1$, it will increase both U.S. dollar-denominated corporate and government bond returns.

We run time-series regression of corporate bond excess returns on the currency risk factors,

$$R^e_{c,t} = \alpha_c + \beta_{1,c} FXPC_{1,t} + \beta_{2,c} FXPC_{2,t} + u_{c,t}$$

where $FXPC_t$ is the principal component of currency returns sorted on interest rates of Lustig, Roussanov, and Verdelhan (2011). If corporate bond excess returns are a simple reflection of currency risk premiums, we expect that α_c is closer to zero than the average excess returns are.

Table IA5 reports the estimated slope coefficients α, β for each country and rating. For IG bonds (Panel A), the currency factors explain excess returns on corporate bonds reasonably well in the U.S. and European countries. The estimated α in U.K., Germany, France, Italy and U.S. is 3.5, 3.3, 2.2, 1.9 and -1.6 basis points per month, respectively. These small alphas are explained by the positive loading on the two currency risk factors. However, the loading of IG corporate bond excess returns in Japan and Australia on these currency factors are small, and the estimated α is close to the average returns.

Panel B of Table IA5 reports the estimates for HY bonds. Though corporate bond excess returns load positively on the currency factors (except for the U.S. bonds on the first principal component), the loading is not large enough, leaving substantial alphas on these investment. For both IG- and HY-bonds, the adjusted R-squared of the regressions are low, ranging from 0.02 to 0.21.

In conclusion, the large variation in credit spreads across countries is not likely to be a simple reflection of currency risk premiums documented in the international finance literature. To explain price of corporate bonds, we need to account for liquidity in each market as we do in the main text.

VII Comparing Bid-Ask Spreads with Other Liquidity Measures

To build a various liquidity measures in comparison, we use daily observations of Markit composite prices to construct two transaction cost measures which, according to Schestag et al. (2016), deliver the best performance among low-frequency measures. One is Roll (1984)'s estimator for effective spreads based on the return autocovariance, and the other is Hasbrouck (2009)'s Gibbs measure.

Table IA6 displays the comparative results. For completeness, we include two proxies measuring other dimension of market liquidity: the run length measure by Das and Hanouna (2010) and the market depth measure embedded in the Markit database. The former is shown to highly correlated with the Amihud (2002) measure,[31] and the latter is defined as the number of distinct contributors at the composite fallback level. The summary statistics in Panel A imply substantial differences between US and international corporate bond markets, as the latter has much lower estimates of effective bid-ask spreads and much greater market depth. They are consistent with the finding of Biais and Declerck (2007).[32]

While the BGN quoted bid-ask spreads (denoted by *Spread_BGN*) appear to slightly underesti-

[31]Bond transaction data has limited availability outside of US, which creates difficulty in implementing high-frequency liquidity measures, like Feldhutter (2012)'s imputed round-trip cost, or other measures involving trading volumes.

[32]Biais and Declerck (2007) focus on the European corporate bond market from 2003 to 2005 and find that the effective bid-ask spreads are lower than the estimates of Edwards et al. (2007) and Goldstein et al. (2007) over the same period. They attribute this finding to the presence of a large pool of potential buyers and sellers in the Euro bond market. Accordingly, our comparative results are mainly driven by the four European countries in our sample, because the Markit composite prices have limited coverage for Japan, Canada and Australia. Our average estimates of effective spreads, 12–15 bps, are generally in line with the range (8 to 22 bps) reported by Biais and Declerck (2007).

mate the transaction costs in US, this measure has a marked upward bias in the international bond markets. To assess the ability of *Spread_BGN* to capture liquidity differences between bonds, we follow Goyenko et al. (2009) by computing average cross-sectional correlations. Panel B shows that *Spread_BGN* appears to have substantially higher correlations for both transaction cost benchmarks outside of US. Finally, results in Panel C reinforce the evidence that (1) *Spread_BGN* is biased in opposite direction for US and international corporate bonds, with magnitude of about -8 and 30+ bps; (2) *Spread_BGN* performs better outside of US in tracking the variation in benchmark measures.

VIII The "Noise" Measure of Corporate Bonds

We construct the noise measure for each country as follows: For each month, we use security-level price data in Merrill Lynch and fit the Nelson-Siegel (Nelson-Siegel-Svensson) curve for each issuer with more than 7 (15) bonds outstanding—and to maximize the sample size, we use all issuers including private firms and financial firms in this exercise. Given our focus on mispricing due to illiquidity, it is important to fit the curve issuer by issuer. We then compute issuer-level, root-mean squared fitting errors as:

$$v_{j,t} = \sqrt{\frac{1}{n_j} \sum_k (ytm_{k,j,t} - ytm_{k,j,t}^{NS})^2}$$

and the country-level fitting errors are:

$$Noise_{c,t} = \frac{1}{N_t} \sum_j v_{j,t}$$

This "noise" measure of corporate bonds is analogous to the "noise" measure in the U.S. Treasury market developed by Hu, Pan, and Wang (2013). Grishchenko and Huang (2012) construct a similar "noise" measure for the TIPS market.

Table IA1: Bond-by-Bond Time-Series Regression of Credit Spread Changes on Their Determinants: Monthly 1997-2017

	Japan	UK	Germany	France	Italy	Canada	Australia
R	-0.10	-0.20	-0.40	-0.58	-0.33	-0.18	-0.01
	(-3.06)	(-1.49)	(-2.60)	(-4.06)	(-1.54)	(-1.49)	(-0.04)
Δr^{10}	0.01	-0.31	-0.33	-0.37	-0.48	-0.15	-0.08
	(0.19)	(-3.84)	(-3.03)	(-3.60)	(-3.61)	(-2.11)	(-1.34)
$(\Delta r^{10})^2$	-0.08	-0.02	0.09	0.23	0.65	0.01	0.20
	(-0.21)	(-0.07)	(0.21)	(0.20)	(1.22)	(0.04)	(1.02)
$\Delta slope$	0.00	0.31	0.33	0.40	0.75	0.07	0.03
	(0.02)	(3.42)	(2.76)	(3.43)	(4.63)	(0.76)	(0.34)
Δvol	0.03	0.32	0.37	0.28	0.38	0.27	0.02
	(1.21)	(3.50)	(2.96)	(2.47)	(2.16)	(2.80)	(0.18)
$StockIndex$	0.01	-1.39	-0.85	-1.06	-1.34	-1.35	-0.48
	(0.06)	(-2.99)	(-2.12)	(-2.53)	(-3.15)	(-2.99)	(-1.38)
$\Delta skew$	0.00	0.00	0.00	0.01	0.01	0.00	0.00
	(0.78)	(-0.11)	(0.10)	(1.28)	(0.69)	(0.50)	(0.11)
$\bar{R}^2$	0.09	0.30	0.25	0.28	0.31	0.19	0.13
N	833	161	202	357	114	177	64
DD	-5.35	-37.25	-35.81	-44.05	-40.88	-18.11	-12.77
	(-2.30)	(-4.55)	(-3.44)	(-4.91)	(-3.67)	(-4.43)	(-3.26)
$\bar{R}^2$	0.03	0.05	0.08	0.07	0.08	0.04	0.04
N	821	161	202	356	114	169	64

Note: We run time-series regression of monthly changes in credit spread (in percent) for bond k as

$$\Delta CS_{k,t} = b_{k,0} + b_{k,1}R_{k,t} + b_{k,2}\Delta r_t^{10} + b_{k,3}(\Delta r_t^{10})^2 + b_{k,4}\Delta slope_t + b_{k,5}\Delta vol_{k,t} + b_{k,6}R_{INDEX,t} + b_{k,7}\Delta skew_{k,t} + \nu_{k,t}$$

where R_k is a stock return on the bond issuer, r^{10} is 10-year risk-free yields, *slope* is the difference between 10 and 2 year yields, *vol* is the issuer's stock volatility, R_{INDEX} is the return on the country's stock index, and *skew* is the skewness of issuer's stock return. This table reports the average slope coefficients and average adjusted R-squared.

Table IA2: Credit Spreads Predicted by Alternative Specifications of the Black-Cox Model with FS Default Boundary, AA+ Rating

		Mean	10%	25%	50%	75%	90%
Japan	Credit Spreads (bps)	15	4	8	13	20	26
	Continuous Coupon	10	0	0	1	11	38
	Discrete-Time Recovery	9	1	0	1	11	38
	Recovery over Treasury	9	1	0	1	11	37
	Zero Coupon	11	0	0	1	11	39
UK	Credit Spreads (bps)	74	36	49	72	92	113
	Continuous Coupon	6	0	0	0	2	9
	Discrete-Time Recovery	19	10	12	15	21	29
	Recovery over Treasury	11	5	6	7	10	18
	Zero Coupon	6	0	0	0	2	9
Germany	Credit Spreads (bps)	57	28	41	53	68	79
	Continuous Coupon	5	0	0	0	3	10
	Discrete-Time Recovery	13	6	8	11	15	21
	Recovery over Treasury	8	2	4	5	8	15
	Zero Coupon	5	0	0	0	3	11
France	Credit Spreads (bps)	80	32	47	73	100	134
	Continuous Coupon	24	0	0	1	21	72
	Discrete-Time Recovery	31	1	4	11	28	79
	Recovery over Treasury	27	1	2	6	24	73
	Zero Coupon	25	0	0	2	23	74
Italy	Credit Spreads (bps)	97	50	71	93	120	141
	Continuous Coupon	6	0	0	3	8	16
	Discrete-Time Recovery	10	4	8	10	17	23
	Recovery over Treasury	6	2	4	5	13	19
	Zero Coupon	7	0	0	3	10	18
Canada	Credit Spreads (bps)	70	34	42	56	72	117
	Continuous Coupon	39	3	4	15	41	122
	Discrete-Time Recovery	49	11	13	23	55	140
	Recovery over Treasury	49	11	13	23	54	138
	Zero Coupon	42	3	5	16	48	136
Australia	Credit Spreads (bps)	146	74	85	123	220	241
	Continuous Coupon	19	0	2	6	24	63
	Discrete-Time Recovery	18	2	3	11	31	70
	Recovery over Treasury	18	2	3	11	31	69
	Zero Coupon	17	0	2	6	22	55
US	Credit Spreads (bps)	65	24	37	54	78	117
	Continuous Coupon	9	0	0	0	3	18
	Discrete-Time Recovery	16	1	4	10	16	28
	Recovery over Treasury	16	1	4	10	16	27
	Zero Coupon	9	0	0	0	4	20

Table IA2 – Continued, A Rating

		Mean	10%	25%	50%	75%	90%
Japan	Credit Spreads (bps)	22	8	12	18	27	40
	Continuous Coupon	24	0	0	5	29	72
	Discrete-Time Recovery	24	0	0	5	29	73
	Recovery over Treasury	24	0	0	5	29	73
	Zero Coupon	25	0	0	5	29	73
UK	Credit Spreads (bps)	135	64	84	118	155	224
	Continuous Coupon	65	0	1	10	48	145
	Discrete-Time Recovery	76	4	13	22	60	164
	Recovery over Treasury	70	2	6	14	55	155
	Zero Coupon	63	0	1	11	50	139
Germany	Credit Spreads (bps)	98	45	59	85	114	157
	Continuous Coupon	92	0	1	22	120	263
	Discrete-Time Recovery	96	1	5	28	129	262
	Recovery over Treasury	94	1	3	25	125	264
	Zero Coupon	93	0	1	23	122	264
France	Credit Spreads (bps)	100	49	64	90	119	163
	Continuous Coupon	154	0	0	15	240	502
	Discrete-Time Recovery	159	0	2	22	255	508
	Recovery over Treasury	156	0	1	18	249	511
	Zero Coupon	152	0	0	16	234	487
Italy	Credit Spreads (bps)	155	63	86	131	190	300
	Continuous Coupon	39	0	1	10	54	116
	Discrete-Time Recovery	44	1	4	16	62	127
	Recovery over Treasury	41	1	3	13	58	122
	Zero Coupon	41	0	1	11	58	122
Canada	Credit Spreads (bps)	102	49	64	88	126	164
	Continuous Coupon	25	0	1	9	28	60
	Discrete-Time Recovery	26	1	5	13	32	68
	Recovery over Treasury	26	1	5	12	32	68
	Zero Coupon	27	0	1	10	31	68
Australia	Credit Spreads (bps)	170	89	111	150	209	265
	Continuous Coupon	8	0	0	0	1	19
	Discrete-Time Recovery	7	1	2	4	9	25
	Recovery over Treasury	6	1	2	3	8	25
	Zero Coupon	9	0	0	0	1	21
US	Credit Spreads (bps)	99	39	56	79	116	179
	Continuous Coupon	43	0	0	5	36	131
	Discrete-Time Recovery	52	2	7	15	46	144
	Recovery over Treasury	52	2	7	15	45	143
	Zero Coupon	46	0	0	6	41	140

Table IA2 – Continued, BBB Rating

		Mean	10%	25%	50%	75%	90%
Japan	Credit Spreads (bps)	35	14	20	29	42	64
	Continuous Coupon	48	0	1	12	54	129
	Discrete-Time Recovery	49	0	1	12	55	130
	Recovery over Treasury	49	0	1	12	55	130
	Zero Coupon	49	0	1	12	55	131
UK	Credit Spreads (bps)	188	95	121	155	210	295
	Continuous Coupon	57	0	3	20	62	162
	Discrete-Time Recovery	67	3	14	32	77	181
	Recovery over Treasury	61	1	7	25	69	172
	Zero Coupon	57	0	3	22	65	163
Germany	Credit Spreads (bps)	122	57	74	105	145	207
	Continuous Coupon	85	0	0	14	83	243
	Discrete-Time Recovery	89	0	4	19	91	252
	Recovery over Treasury	86	0	2	16	87	249
	Zero Coupon	86	0	0	15	87	238
France	Credit Spreads (bps)	147	61	80	117	178	267
	Continuous Coupon	158	0	2	34	184	452
	Discrete-Time Recovery	164	1	6	41	196	467
	Recovery over Treasury	161	1	4	37	190	468
	Zero Coupon	157	0	2	36	185	444
Italy	Credit Spreads (bps)	152	63	80	112	190	304
	Continuous Coupon	105	0	3	37	137	341
	Discrete-Time Recovery	106	0	5	39	142	330
	Recovery over Treasury	105	1	4	38	139	338
	Zero Coupon	107	0	3	38	141	338
Canada	Credit Spreads (bps)	171	83	111	154	207	268
	Continuous Coupon	58	0	0	3	29	81
	Discrete-Time Recovery	77	0	1	6	33	95
	Recovery over Treasury	77	0	1	5	33	94
	Zero Coupon	76	0	0	4	33	95
Australia	Credit Spreads (bps)	195	103	130	176	233	312
	Continuous Coupon	14	0	0	0	3	19
	Discrete-Time Recovery	17	2	2	4	8	23
	Recovery over Treasury	16	1	2	3	8	23
	Zero Coupon	17	0	0	0	3	20
US	Credit Spreads (bps)	183	66	93	140	221	344
	Continuous Coupon	94	0	5	33	116	269
	Discrete-Time Recovery	106	5	13	41	128	288
	Recovery over Treasury	105	5	13	41	127	286
	Zero Coupon	97	0	5	36	124	274

Table IA2 – Continued, HY Rating

		Mean	10%	25%	50%	75%	90%
UK	Credit Spreads (bps)	406	225	288	363	468	681
	Continuous Coupon	216	1	37	127	281	517
	Discrete-Time Recovery	227	4	42	140	309	530
	Recovery over Treasury	222	3	40	133	300	529
	Zero Coupon	207	1	41	127	267	475
Germany	Credit Spreads (bps)	244	120	151	205	298	449
	Continuous Coupon	109	0	2	40	148	286
	Discrete-Time Recovery	103	0	2	36	149	283
	Recovery over Treasury	102	0	2	35	146	284
	Zero Coupon	110	0	3	42	152	282
France	Credit Spreads (bps)	287	114	167	257	375	519
	Continuous Coupon	633	5	84	365	784	1411
	Discrete-Time Recovery	649	10	97	381	781	1432
	Recovery over Treasury	644	7	93	376	786	1426
	Zero Coupon	626	6	87	367	795	1386
Italy	Credit Spreads (bps)	217	100	139	205	273	361
	Continuous Coupon	250	3	23	181	422	656
	Discrete-Time Recovery	252	3	26	188	429	667
	Recovery over Treasury	251	3	26	185	428	664
	Zero Coupon	250	3	23	179	421	652
Canada	Credit Spreads (bps)	317	177	224	290	406	489
	Continuous Coupon	213	0	2	41	337	749
	Discrete-Time Recovery	208	4	7	53	344	693
	Recovery over Treasury	207	4	7	53	343	693
	Zero Coupon	202	0	2	46	325	704
US	Credit Spreads (bps)	482	160	267	425	606	876
	Continuous Coupon	403	22	86	259	581	1025
	Discrete-Time Recovery	432	30	97	277	614	1088
	Recovery over Treasury	430	30	96	275	611	1083
	Zero Coupon	367	25	92	259	532	861

These tables compare the distribution of credit spreads in the data with that generated from different specifications of the Black-Cox model. The statistics are computed using the panel data from 1997 to 2017 outside of the U.S., while using the data from 1987 to 2015 for the U.S. "Continuous Coupon" refers to the specification that the bond issuer continuously pays a constant coupon flow until default occurs. "Discrete-Time Recovery" and "Recovery over Treasury" share the assumption that coupon payments are discrete, with the coupon frequency calibrated to the data; they differ in the definition of recovery rate. "Zero Coupon" assumes that the yield spread of a coupon-bearing bond can be reasonably approximated by its counterpart of a zero-coupon bond with the same maturity. Model-predicted credit spreads under the four specifications are derived from Eq. (8), (21) (22) and (24), respectively. The default boundary d in all model specifications is estimated with the Feldhütter and Schaefer (2018) methodology.

Table IA3: Comparisons of Alternative Estimates of Asset Value and Asset Voaltility

	Rating	NObs	Mean	10%	50%	90%	NObs	Mean	10%	50%	90%
		Japan					*Italy*				
$D^{\dagger}/K$	AA+	31	1.06	1.00	1.04	1.15	3	1.09	1.02	1.08	1.16
	A	64	1.04	1.00	1.03	1.09	11	1.03	0.95	1.03	1.13
	BBB	63	1.04	1.00	1.04	1.09	18	1.01	0.91	0.99	1.14
	HY	0	-	-	-	-	6	1.00	0.93	0.99	1.09
$\sigma_A^{\dagger}/\sigma_A$	AA+	31	0.97	0.63	0.93	1.35	3	1.02	0.67	0.86	1.75
	A	64	1.00	0.68	0.96	1.35	11	1.12	0.77	1.06	1.55
	BBB	63	0.99	0.69	0.97	1.32	18	1.16	0.85	1.13	1.50
	HY	0	-	-	-	-	6	1.13	0.76	1.13	1.45
		UK					*Canada*				
$D^{\dagger}/K$	AA+	15	1.12	1.01	1.10	1.19	3	1.17	0.97	1.16	1.43
	A	42	1.03	0.90	1.00	1.21	18	1.13	0.94	1.05	1.46
	BBB	40	1.03	0.91	1.00	1.18	51	1.08	0.95	1.03	1.38
	HY	13	0.99	0.89	0.97	1.14	5	1.03	0.96	1.03	1.11
$\sigma_A^{\dagger}/\sigma_A$	AA+	15	0.99	0.71	0.97	1.25	3	0.91	0.46	0.96	1.37
	A	42	1.13	0.72	1.05	1.63	18	0.93	0.63	0.90	1.26
	BBB	40	1.12	0.75	1.07	1.57	51	0.94	0.64	0.91	1.28
	HY	13	1.12	0.75	1.07	1.59	5	0.94	0.60	0.91	1.30
		Germany					*Australia*				
$D^{\dagger}/K$	AA+	9	1.04	1.00	1.04	1.08	1	1.11	1.04	1.11	1.16
	A	28	0.99	0.92	0.98	1.06	10	1.09	1.00	1.08	1.20
	BBB	39	0.94	0.85	0.91	1.06	17	1.02	0.95	1.02	1.09
	HY	12	0.90	0.81	0.88	1.03	0	-	-	-	-
$\sigma_A^{\dagger}/\sigma_A$	AA+	9	0.92	0.66	0.89	1.18	1	0.98	0.64	0.94	1.38
	A	28	0.89	0.57	0.84	1.24	10	1.01	0.77	0.99	1.28
	BBB	39	0.90	0.61	0.87	1.23	17	1.47	1.17	1.44	1.77
	HY	12	0.96	0.71	0.93	1.27	0	-	-	-	-
		France					*US*				
$D^{\dagger}/K$	AA+	9	1.09	1.01	1.08	1.19	79	1.09	1.00	1.07	1.20
	A	24	1.04	0.94	1.02	1.14	312	1.07	0.99	1.04	1.18
	BBB	39	1.03	0.94	1.02	1.12	544	1.04	0.97	1.01	1.13
	HY	18	1.01	0.92	1.00	1.10	661	0.98	0.92	0.95	1.04
$\sigma_A^{\dagger}/\sigma_A$	AA+	9	1.03	0.65	0.95	1.58	79	1.00	0.63	0.94	1.56
	A	24	1.16	0.78	1.11	1.60	312	1.00	0.63	0.92	1.58
	BBB	39	1.17	0.82	1.11	1.61	544	1.01	0.64	0.94	1.62
	HY	18	1.27	0.85	1.24	1.74	661	1.06	0.76	1.00	1.64

This table presents summary statistics for JMR-type estimates of model parameters for each country and for each credit rating. The statistics are computed using the panel data of bond issuers, and NObs is the number of firms that are in each category. The sample is from 1997 to 2017 for non-U.S. firms, and from 1987 to 2015 for the U.S. firms. $D^{\dagger}/K$ is the model-implied market-to-book ratio of corporate debts. $\sigma_A^{\dagger}/\sigma_A$ is the ratio of the model-based volatility estimate to the model-free one.

Table IA4: Firm-Level Inputs: All Non-Financial Bond Issuers

	Rating	NObs	Mean	10%	50%	90%	NObs	Mean	10%	50%	90%
		Japan					*Germany*				
K/A	AA+	32	0.40	0.09	0.40	0.68	9	0.44	0.10	0.42	0.76
	A	68	0.43	0.18	0.42	0.71	28	0.34	0.12	0.33	0.60
	BBB	63	0.50	0.28	0.51	0.70	40	0.33	0.11	0.30	0.61
	HY	0	-	-	-	-	26	0.42	0.23	0.38	0.62
σ^E	AA+	32	0.26	0.15	0.24	0.39	9	0.24	0.16	0.21	0.38
	A	68	0.31	0.18	0.30	0.45	28	0.30	0.17	0.26	0.45
	BBB	63	0.37	0.23	0.36	0.54	40	0.29	0.18	0.26	0.45
	HY	0	-	-	-	-	26	0.37	0.22	0.32	0.54
σ^A	AA+	32	0.16	0.07	0.15	0.28	9	0.15	0.06	0.15	0.28
	A	68	0.18	0.07	0.18	0.27	28	0.20	0.13	0.18	0.29
	BBB	63	0.18	0.11	0.18	0.25	40	0.19	0.12	0.19	0.29
	HY	0	-	-	-	-	26	0.22	0.14	0.22	0.27
δ	AA+	32	0.009	0.000	0.008	0.016	9	0.012	0.000	0.007	0.035
	A	68	0.009	0.000	0.008	0.016	28	0.022	0.004	0.018	0.046
	BBB	63	0.005	0.000	0.004	0.012	40	0.028	0.008	0.022	0.055
	HY	0	-	-	-	-	26	0.030	0.009	0.029	0.052
		UK					*France*				
K/A	AA+	15	0.20	0.07	0.18	0.35	10	0.25	0.06	0.19	0.69
	A	50	0.29	0.11	0.25	0.51	27	0.28	0.07	0.25	0.56
	BBB	49	0.30	0.13	0.28	0.53	42	0.33	0.12	0.32	0.53
	HY	24	0.42	0.13	0.39	0.76	23	0.44	0.18	0.44	0.72
σ^E	AA+	15	0.25	0.16	0.24	0.38	10	0.28	0.17	0.26	0.42
	A	50	0.26	0.14	0.24	0.42	27	0.28	0.17	0.25	0.46
	BBB	49	0.28	0.17	0.24	0.45	42	0.29	0.17	0.26	0.46
	HY	24	0.44	0.23	0.36	0.80	23	0.39	0.22	0.36	0.59
σ^A	AA+	15	0.20	0.16	0.20	0.27	10	0.22	0.08	0.22	0.29
	A	50	0.19	0.12	0.18	0.28	27	0.20	0.13	0.20	0.27
	BBB	49	0.20	0.14	0.19	0.26	42	0.20	0.13	0.19	0.26
	HY	24	0.27	0.18	0.23	0.31	23	0.22	0.15	0.21	0.27
δ	AA+	15	0.011	0.000	0.004	0.038	10	0.019	0.000	0.019	0.041
	A	50	0.016	0.000	0.000	0.051	27	0.019	0.000	0.018	0.042
	BBB	49	0.024	0.000	0.028	0.049	42	0.022	0.002	0.020	0.042
	HY	24	0.034	0.000	0.034	0.074	23	0.023	0.001	0.018	0.050

This table presents summary statistics for non-financial firms matched to all bonds, including callable bonds. We do not use callable bonds in computing credit spreads, but we still use these firms in estimating default boundary. The sample is from 1997 to 2017.

Table IA4 – Continued

	Rating	NObs	Mean	10%	50%	90%	NObs	Mean	10%	50%	90%
		Italy					*Australia*				
K/A	AA+	3	0.27	0.12	0.28	0.36	5	0.52	0.02	0.64	0.83
	A	10	0.39	0.20	0.40	0.58	13	0.26	0.10	0.23	0.51
	BBB	18	0.51	0.36	0.50	0.69	22	0.28	0.11	0.25	0.54
	HY	10	0.64	0.33	0.62	0.91	0	-	-	-	-
σ^E	AA+	3	0.21	0.12	0.17	0.43	5	0.31	0.19	0.27	0.45
	A	10	0.24	0.16	0.22	0.37	13	0.22	0.14	0.19	0.32
	BBB	18	0.26	0.18	0.24	0.36	22	0.29	0.18	0.25	0.43
	HY	10	0.39	0.28	0.36	0.53	0	-	-	-	-
σ^A	AA+	3	0.15	0.11	0.13	0.21	5	0.18	0.08	0.09	0.40
	A	10	0.15	0.12	0.14	0.19	13	0.16	0.09	0.15	0.23
	BBB	18	0.14	0.11	0.14	0.18	22	0.20	0.11	0.20	0.26
	HY	10	0.18	0.14	0.17	0.22	0	-	-	-	-
δ	AA+	3	0.049	0.000	0.055	0.062	5	0.006	0.000	0.000	0.021
	A	10	0.037	0.013	0.040	0.059	13	0.024	0.000	0.000	0.065
	BBB	18	0.038	0.011	0.037	0.065	22	0.034	0.000	0.034	0.059
	HY	10	0.037	0.000	0.027	0.087	0	-	-	-	-
		Canada					*U.S.*				
K/A	AA+	3	0.27	0.12	0.21	0.43	137	0.14	0.03	0.11	0.30
	A	30	0.32	0.12	0.33	0.50	501	0.22	0.06	0.19	0.40
	BBB	66	0.31	0.14	0.28	0.53	832	0.29	0.09	0.26	0.54
	HY	29	0.39	0.13	0.32	0.71	2060	0.47	0.15	0.46	0.82
σ^E	AA+	3	0.25	0.11	0.27	0.33	137	0.27	0.19	0.27	0.37
	A	30	0.23	0.14	0.22	0.35	501	0.31	0.21	0.30	0.42
	BBB	66	0.25	0.15	0.22	0.39	832	0.37	0.24	0.35	0.52
	HY	29	0.43	0.20	0.34	0.61	2060	0.56	0.32	0.51	0.86
σ^A	AA+	3	0.16	0.13	0.13	0.20	137	0.24	0.19	0.23	0.28
	A	30	0.17	0.10	0.14	0.28	501	0.25	0.19	0.24	0.33
	BBB	66	0.18	0.10	0.17	0.27	832	0.27	0.20	0.26	0.38
	HY	29	0.25	0.17	0.22	0.32	2060	0.34	0.20	0.31	0.51
δ	AA+	3	0.045	0.021	0.040	0.084	137	0.028	0.002	0.021	0.061
	A	30	0.036	0.012	0.037	0.062	501	0.036	0.003	0.029	0.075
	BBB	66	0.033	0.000	0.031	0.061	832	0.039	0.005	0.032	0.083
	HY	29	0.042	0.013	0.040	0.073	2060	0.042	0.003	0.036	0.087

Table IA5: Excess Returns on Value-Weighted Portfolios of Corporate Bonds: Monthly from January 1997 to December 2017

	Japan	UK	Germany	France	Italy	Canada	US	Australia
Panel A. Investment-Grade Bonds								
$Avg.R_t^e$	0.015	0.101	0.086	0.076	0.086	0.119	0.067	0.089
α	0.010	0.035	0.033	0.022	0.019	0.058	-0.016	0.073
$t(\alpha)$	(0.74)	(0.41)	(0.81)	(0.43)	(0.25)	(0.85)	(-0.18)	(1.60)
b_1	0.005	0.072	0.070	0.087	0.142	0.079	0.106	0.022
$t(b_1)$	(0.78)	(1.84)	(2.51)	(3.35)	(3.20)	(2.17)	(1.90)	(1.26)
b_2	0.009	0.105	0.083	0.081	0.095	0.096	0.121	0.025
$t(b_2)$	(2.67)	(2.80)	(3.63)	(3.04)	(2.27)	(2.47)	(2.13)	(1.56)
$\bar{R}^2$	0.02	0.11	0.21	0.21	0.17	0.15	0.10	0.04
Panel B. High-Yield Bonds								
$Avg.R_t^e$	-	0.470	0.394	0.405	0.231	0.416	0.891	-
α	-	0.274	0.275	0.249	0.204	0.315	0.435	-
$t(\alpha)$	-	(1.33)	(1.63)	(1.91)	(2.01)	(1.85)	(1.03)	-
b_1	-	0.225	0.523	0.259	0.129	0.220	-0.219	-
$t(b_1)$	-	(2.26)	(2.19)	(3.74)	(2.34)	(1.35)	(-0.40)	-
b_2	-	0.310	0.125	0.233	0.027	0.142	0.760	-
$t(b_2)$	-	(2.72)	(1.38)	(2.53)	(0.74)	(2.97)	(2.21)	-
$\bar{R}^2$	-	0.09	0.14	0.11	0.05	0.08	0.03	-

This table reports the estimates of the time-series regression,

$$R^e_{c,t} = \alpha_{1,c} + \beta_{1,c}FXPC_{1,t} + \beta_{2,c}FXPC_{2,t} + u_{c,t}$$

where $FXPC_t$ is the principal component of currency returns of Lustig et al. (2011). $R^e_{c,t}$ is the value-weighted average returns on corporate bonds in U.S. dollars in excess of the value-weighted average returns on government security in each country in U.S. dollars. $Avg.R_t^e$ is the time-series average of each portfolio, and values in parentheses are t-statistics.

Table IA6: Comparison of Quoted Bid-Ask Spreads with Other Liquidity Measures

Penal A: Descriptive statistics for liquidity measures

	US						International					
	NObs	Mean	Std	10%	50%	90%	NObs	Mean	Std	10%	50%	90%
Roll	20775	0.70	1.06	0.00	0.41	1.57	35839	0.15	0.57	0.00	0.05	0.32
Hasbrouck	21074	0.70	2.40	0.10	0.33	1.34	34748	0.12	0.87	0.02	0.05	0.17
Spread_BGN	21074	0.61	1.33	0.18	0.46	1.13	35957	0.46	0.33	0.18	0.38	0.82
RunLength	20775	1.83	0.70	1.33	1.67	2.38	35839	2.01	0.63	1.46	1.90	2.67
Depth	19408	2.84	2.14	0.76	2.62	4.63	34931	4.96	1.64	2.76	5.05	7.05

Penal B: Average cross-sectional correlations

	US				International			
	Roll	Hasbrouck	Spread_BGN	RunLength	Roll	Hasbrouck	Spread_BGN	RunLength
Hasbrouck	0.559				0.489			
Spread_BGN	0.217	0.207			0.259	0.324		
RunLength	−0.259	−0.107	0.068		−0.216	−0.039	0.030	
Depth	−0.265	−0.278	−0.058	0.030	−0.128	−0.199	−0.056	−0.008

Penal C: Deviation from benchmark measures

	US				International			
Metrics	Mean Bias		RMSE		Mean Bias		RMSE	
Benchmark	Roll	Hasbrouck	Roll	Hasbrouck	Roll	Hasbrouck	Roll	Hasbrouck
	−0.082	−0.083	1.650	2.718	0.312	0.340	0.701	0.970

Panel A presents summary statistics for five monthly liquidity measures: the negative price autocovariance based on the model of Roll (1984) and refined by Hasbrouck (2009) with Gibbs sampling, the quoted bid-ask spreads from Bloomberg BGN, the run length measure by Das and Hanouna (2010), and the market depth defined by Markit. Entries for the first three measures are reported in percentage. Panel B shows the average cross-sectional correlations between these measures. Panel C reports the mean bias and RMSE of the quoted bid-ask spreads against two benchmark measures for transaction costs: the Roll and Hasbrouck measures. The sample is from 2003 to 2017.

Table IA7: CDS Absolute Pricing Errors of the Black-Cox Model

Panel A: Mean Pricing Error

	AA+	A	BAA	HY	AA+	A	BAA	HY
	Japan				Italy			
d^{FS}	22	37	51	244	24	60	82	1100
d^{BGY}	22	36	51	244	26	69	93	1332
d^{HNS}	15	29	53	348	15	47	58	608
	UK				Canada			
d^{FS}	21	48	81	217	-	64	61	183
d^{BGY}	21	49	76	214	-	66	61	179
d^{HNS}	19	26	57	265	-	42	55	213
	Germany				Australia			
d^{FS}	31	66	110	159	28	75	115	-
d^{BGY}	29	60	92	165	28	75	116	-
d^{HNS}	20	31	50	238	22	54	70	-
	France				US			
d^{FS}	31	120	126	325	36	48	89	269
d^{BGY}	31	123	130	347	31	44	86	266
d^{HNS}	21	29	58	286	21	27	64	251

Panel B: Mean Percentage Pricing Error

	AA+	A	BAA	HY	AA+	A	BAA	HY
	Japan				Italy			
d^{FS}	22	32	40	72	24	47	60	69
d^{BGY}	22	32	40	71	26	53	65	94
d^{HNS}	15	24	39	251	15	33	34	67
	UK				Canada			
d^{FS}	21	45	57	91	-	46	47	64
d^{BGY}	21	46	56	87	-	47	48	64
d^{HNS}	19	24	39	156	-	27	32	108
	Germany				Australia			
d^{FS}	29	58	88	62	27	57	73	-
d^{BGY}	28	53	73	63	27	58	74	-
d^{HNS}	17	25	33	132	21	38	36	-
	France				US			
d^{FS}	31	111	86	108	32	44	60	76
d^{BGY}	31	113	89	114	29	40	57	66
d^{HNS}	20	27	40	160	18	22	35	91

The table summarizes the absolute pricing errors of CDS spreads under the Black-Cox model. Pricing errors are reported as the absolute differences, $|s_{k,t}^{BC} - s_{k,t}|$, or as the absolute percentage differences, $\frac{|s_{k,t}^{BC} - s_{k,t}|}{s_{k,t}}$, between the model implied and observed spreads. CDS observations are grouped into rating categories: $AAA\&AA$, A, BBB and high-yield (HY).

Figure IA1: Median CDS Spreads and the Black-Cox Model

The figures compare the median CDS spreads for different tenor in each country. For each tenor and country, we compute median value for IG firms using the CDS spreads in the data and the Black-Cox model with the FS-method of d. The sample is monthly from January 2001 to May 2015.

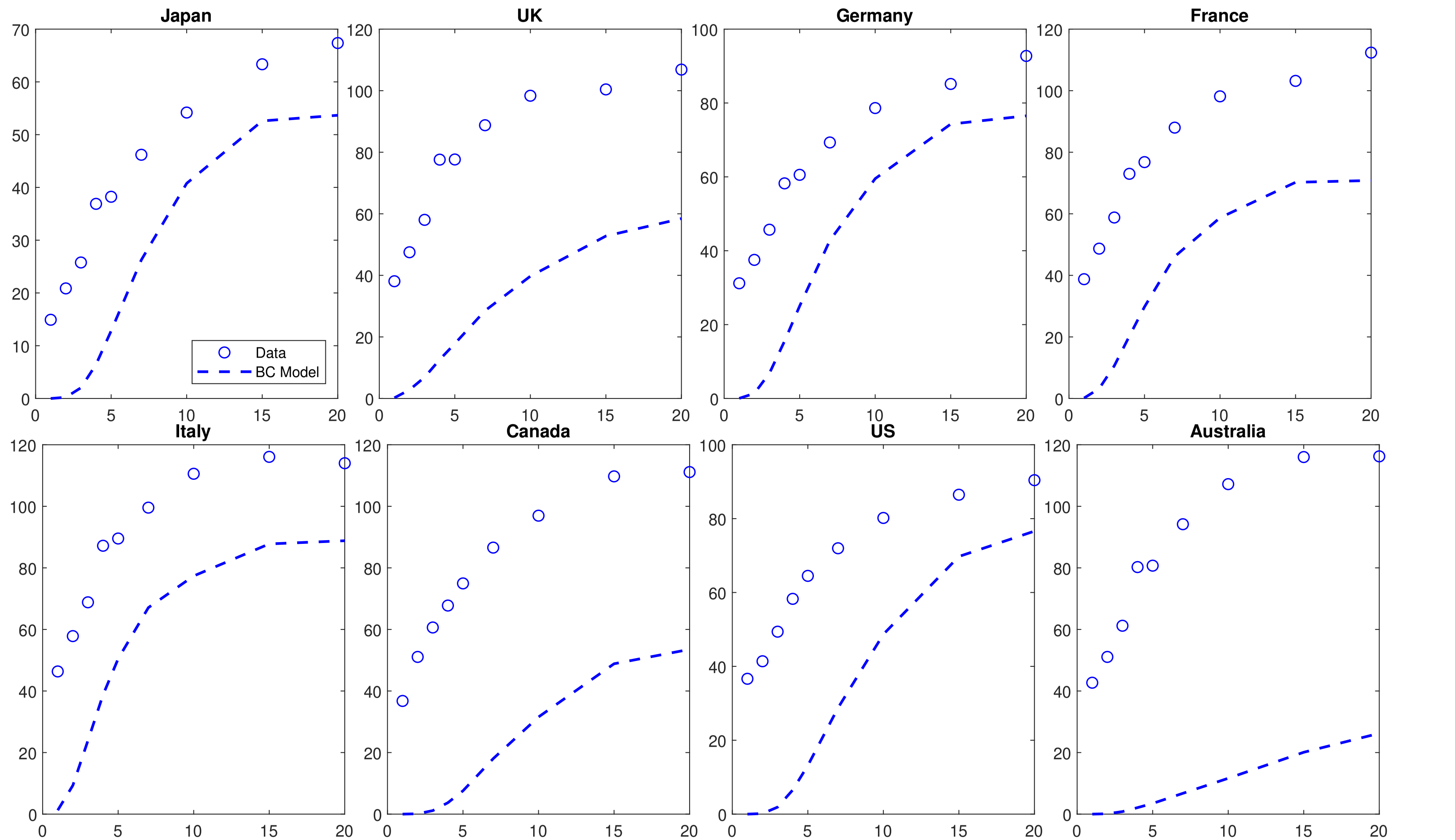

Figure IA2: Swap Rates as Risk-Free Rates

The figures on the left column shows the mean (dot) and median (star) credit spreads in the data and in the Black-Cox model, in which default boundary is estimated using Feldhütter and Schaefer (2018) approach. The figures on the right compares the median credit spreads using two approaches to estimate d: i) Bai et al. (diamond) and ii) matching individual bond's P-default probability (circle).

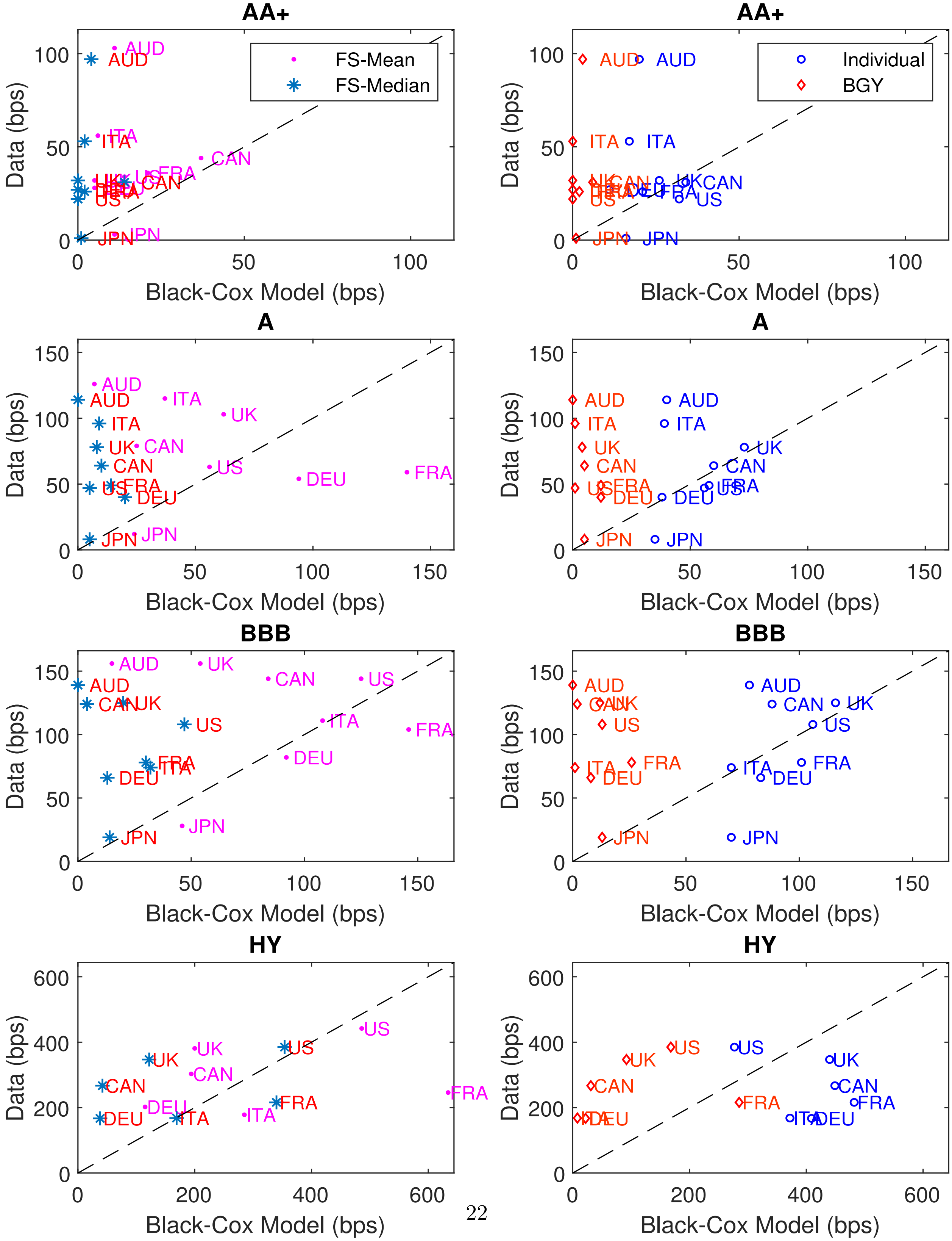

Figure IA3: Debt Securities Domestic Ownership in 2017

The figure presents the share of corporate debt securities held by different class of domestic investors as of 2017. The data source is the flow of funds in each country. The type of security included is debt securities issued by domestic nonfinancial private corporations in countries other than U.K. and Canada. In U.K. and Canada, such data is not available, and thus we use debt securities issued by UK monetary, financial institutions and other U.K. residents in U.K., and other Canadian bonds (other than government debt securities) in Canada.

In the European countries, the breakdown between insurance and pension funds is not available, and thus pension fund's holding is included in "Insurance". Furthermore, in the European countries, other financial institutions are included in "Mutual Funds".