Asset Safety versus Asset Liquidity

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ABSTRACT

Recently, a lot of attention has been paid to the role "safe and liquid assets" play in the macroeconomy. Many economists take as given that safer assets will also be more liquid, and some go a step further by practically using the two terms as synonyms. However, they are not synonyms: safety refers to the probability that the (issuer of the) asset will pay the promised cash flow, and liquidity refers to the ease with which an asset can be sold if needed. Mixing up these terms can lead to confusion and wrong policy recommendations. In this paper, we build a multi-asset model in which an asset's safety and liquidity are well-defined and *distinct* from one another. Treating safety as a primitive, we examine the relationship between an asset's safety and liquidity in general equilibrium. We show that the commonly held belief that "safety implies liquidity" is generally justified, but there may be exceptions. We then describe the conditions under which a relatively riskier asset can be more liquid than its safe(r) counterparts. Finally, we use our model to rationalize the puzzling observation that AAA corporate bonds are considered less liquid than (the riskier) AA corporate bonds.

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1 Introduction

Recently, there has been a lot of attention on the role of safe assets and liquid assets in the macroeconomy. Many economists, both academics and practitioners, seem to believe that safe(r) assets are also more liquid, and some go a step further by practically using the two terms as synonyms or by merging them into the single term "safe and liquid assets".[1] However, these terms are not synonyms: Safety refers to the probability that the (issuer of the) asset will pay the promised cash flow, at maturity, and liquidity refers to the ease with which an investor can sell the asset, if needed, before maturity.[2] Mixing up these terms can lead to false conclusions and misguided policy recommendations. For instance, when a credit rating agency characterizes a certain bond as AAA, should investors think of this as an assessment (only) of its safety or also of its liquidity? And, if the answer is affirmative, how can one explain the fact that (the virtually default free) AAA corporate bonds are considered less liquid than their riskier AA counterparts? Finally, policy makers and financial regulators are concerned about liquidity in certain assets markets. If safety implies liquidity, could one just increase safety and let liquidity follow?

These questions reveal that it is essential to carefully study the relationship between asset safety and asset liquidity, rather than just assume that one implies the other. To do so, we build a multi-asset model in which an asset's safety and liquidity are well-defined and *distinct* from one another. Treating safety as a primitive, we examine the relationship between an asset's safety and its liquidity, which in our model is endogenous. We show that the commonly held belief that "safer assets will be more liquid" is generally justified, but there may be exceptions. We then describe the conditions under which a riskier asset can be more liquid than its safe(r) counterparts, and use our model to rationalize the observation that, in the U.S., AAA corporate bonds are less liquid than AA corporate bonds. Finally, we highlight a surprising implication of our model about the effect of an increase

[1] The examples are numerous, so for the sake of brevity we highlight just two. From the IMF's 2012 Global Financial Stability Report: "Safe assets are a desirable part of a portfolio from an investor's perspective, as they [...] are highly liquid, permitting investors to liquidate positions easily." And at the 2017 American Economic Association meeting, one session was titled: "How *safe and liquid assets* impact monetary and financial policy" (emphasis added).

[2] Although there are economists who adopt slightly different definitions for both of these terms. For instance, Gorton and Ordonez (2013) emphasize that an important aspect of *safe* assets is that they are "information insensitive". Also, a large number of papers in the New Monetarist literature, assume that an asset's *liquidity* refers to the ease with which that asset can be used to purchase consumption, e.g., by serving as a means of payment; see Lagos, Rocheteau, and Wright (2017). For a careful comparison of the approaches, see the Literature Review (Section 1.1).

in the supply of safe assets on welfare.

To answer the research question at hand we consider a dynamic, general equilibrium model with two assets, A and B. The concept of asset safety is extremely straightforward in our framework. Throughout the paper we label asset A the "safe asset", which always pays the promised cash flow, unlike asset B which may default with a certain probability, known to everyone.[3] The concept of liquidity is more involved. To capture the idea that an asset's liquidity refers to the ease with which an agent can sell it for cash (if needed), we employ the monetary model of Lagos and Wright (2005), extended to incorporate asset trade in over-the-counter (OTC) secondary asset markets, as in Duffie, Gârleanu, and Pedersen (2005). Another important ingredient we introduce is an entry decision made by the agents. Each asset trades in a distinct OTC market, and agents choose to visit the market where they expect to find the best terms. Thus, in our model, an asset's liquidity depends on the *endogenous* decision of agents to visit the secondary market where that asset trades, not just the exogenous characteristics of that market.

More precisely, after agents make their portfolio decisions, two shocks are realized. The first is an idiosyncratic shock that determines whether an agent will have a consumption opportunity in that period, and the second is an aggregate shock that determines whether asset B will default in that period. Since purchasing the consumption good necessitates the use of a medium of exchange (i.e., money) and carrying money is costly, in equilibrium, agents who receive a consumption opportunity will visit the secondary market to *sell* assets and boost their cash holdings. Hence, assets have indirect liquidity properties (they can be sold for cash, although they do not serve directly as means of payment), and their equilibrium price in the primary market will typically contain a *liquidity premium*, i.e., it will exceed the fundamental value.

The first result of the paper is that, other things equal, the safer asset carries a higher liquidity premium, and that premium is increasing in the default probability of asset B.[4] The intuition behind this result is as follows. An agent who turns out to be an asset seller can only visit one OTC market at a time; since, typically, assets are costly to own due to the liquidity premium, agents choose to 'specialize' ex-ante in asset A or B. Unlike sellers, who are committed to visit the market of the asset in which they chose to specialize, asset

[3] Modeling asset A as a default-free asset is not necessary for the main results; all one needs is that asset A is *safer* than asset B, i.e., that it defaults with a lower probability.

[4] This statement adopts the liquidity premium as the measure of an asset's liquidity. Later in the analysis, we also consider an alternative measure of liquidity, namely, trade volume, and show that the result is still valid. That is, we show that trade volume is higher in the secondary market for the safer asset, and that the difference in trade volumes between markets A and B is increasing in the default probability of asset B.

buyers are free to visit any market they wish, since their money is good to buy any asset. As a result, in the event of default, all the asset buyers (even those who had chosen to specialize in asset B) will rush into the market for asset A. Naturally, the *ex-post* possibility of a market flooded with buyers (in the event of default) is a powerful force attracting agents to specialize in asset A *ex-ante*, as they realize that in this market they will have a high expected trade probability, if they turn out to be sellers. This is crucial because it is the sell-probability that affects an asset's issue price: an agent who buys an asset (in the primary market) is willing to pay a higher price if she expects that it will be easy to sell that asset 'down the road'. Through this channel, even a small default probability for asset B can be *magnified* into a big endogenous liquidity advantage for asset A, even with constant returns to scale (CRS) in the OTC matching technology.

So far we have assumed that all parameters other than the assets' safety are kept equal. Allowing for differences in asset supplies delivers the second important result of the paper.[5] Even with slight increasing returns to scale (IRS) in OTC matching, demand curves can be *upward sloping*, because an asset in large supply is likely to be more liquid. Consequently, asset B can be more liquid than asset A, despite being less safe, as long as the supply of the former is large enough compared to the latter.

The intuition is as follows. As we have seen, our model gives rise to an endogenous channel whereby a safer asset also acquires a liquidity advantage. However, whether this advantage will materialize, also depends on the relative supply of the safe asset. If the supply of asset A is limited, as more agents choose to specialize in that asset each one of them will only hold a small amount, and any bilateral meeting in the market for asset A will generate a small surplus. This effect, which we dub the "*dilution effect*", tends to make an asset in large supply more attractive to agents. Now, with the dilution effect in mind, consider an increase in the supply of asset B. As the supply rises, more agents are willing to trade in the secondary market for asset B because of the increase in the expected trading surplus (conditional on no-default). Crucially, asset buyers are more sensitive to this increase because their entry choice is more 'elastic' due to the lack of precommitment. As a result, the trade probability in market B for sellers increases by far more than that for buyers, and, as we have highlighted, it is the sell-probability that matters most for the

[5] There are two more parameters held equal in the background: the efficiency of matching in each OTC market and the bargaining power of buyers versus sellers in each OTC market, often put together under the umbrella of "OTC market micro-structure". Since our goal is to develop a theory that links asset safety and asset liquidity in an unbiased way, we assume that these parameters are always equal in both OTC markets. This guarantees that any difference in liquidity between the two assets is driven exclusively by differences in safety and not by exogenous market characteristics.

determination of the issue price. If, to the channel described so far, one adds (even slight) IRS in the matching technology, the agents' incentive to coordinate on the market of the asset in high supply becomes so strong that demand curves can slope upwards, and the less safe asset can carry the higher liquidity premium.

An interesting fact that has recently drawn the attention of economists is that, in the U.S., the virtually default-free AAA bonds are less liquid than (the less safe) AA corporate bonds (see Section 4.3 for details and empirical evidence). Our model can shed some light on this puzzling empirical observation. In recent years, regulations introduced to improve the stability and transparency of the financial system (most prominently, the Dodd-Frank Act) have made it especially hard for corporations to attain the AAA score. As a result, the supply of such bonds has fallen dramatically. During the same time, the yield on AA corporate bonds has been *lower* than that on AAA bonds, even without controlling for the risk premium associated with the riskier AA bonds. While it is plausible to attribute this differential to a higher liquidity premium enjoyed by AA corporate bonds—and this is precisely what practitioners have claimed—existing models of asset liquidity cannot capture this stylized fact (for details, see Section 1.1). Our 'indirect liquidity' approach, coupled with endogenous market entry, is key for explaining why an asset in limited supply tends to be illiquid.

The model also delivers a surprising result regarding welfare. A large body of recent literature highlights that the supply of safe assets has been scarce, and that increasing this supply would be beneficial for welfare (see for example Caballero, Farhi, and Gourinchas, 2017). In our model this result is not necessarily true: there exists a region of parameter values for which welfare is decreasing in the supply of the safe asset. The intuition is as follows. In our model agents have the opportunity to acquire additional cash by selling assets in the secondary market. When the safe asset becomes more plentiful, agents expect that it will be easier to acquire extra cash *ex-post* and, thus, choose to hold less money *ex-ante*. This channel depresses the money demand, which, in turn, decreases the value of money and of the trade that the existing money supply can support.

1.1 Literature Review

Our paper is related to the recent "New Monetarist" literature (see Lagos et al., 2017) that has highlighted the importance of asset liquidity for the determination of asset prices. See for example Geromichalos, Licari, and Suárez-Lledó (2007), Lagos (2011), Lester, Postlewaite, and Wright (2012), Nosal and Rocheteau (2012), Andolfatto, Berentsen, and Waller (2013), and Hu and Rocheteau (2015). In these papers the liquidity properties of assets are 'direct', in the sense that assets serve as a media of exchange or collateral, thus, helping to facilitate trade in frictional decentralized markets for *goods*. In this paper asset liquidity is *indirect*, and it stems from the fact that agents can sell assets for money in secondary *asset* markets. This approach to asset liquidity is not only empirically relevant, but also integrates the concepts of liquidity adopted by monetary economics and finance. (For a more detailed argument, see Geromichalos and Herrenbrueck, 2016a). The indirect liquidity approach is employed in a number of recent papers, including Berentsen, Huber, and Marchesiani (2014, 2016), Mattesini and Nosal (2016), Han (2015), Herrenbrueck and Geromichalos (2017), Herrenbrueck (2019), and Madison (2019).

Naturally, our paper is also related to the growing literature that studies the role of safe assets in the macroeconomy. Examples of such papers include Gorton, Lewellen, and Metrick (2012), Piazzesi and Schneider (2016), He, Krishnamurthy, and Milbradt (2019), Caballero et al. (2017), Gorton (2017). None of these papers study explicitly the relationship between asset safety and asset liquidity. Also, to the best of our knowledge, our paper is the only one that highlights the possibility that welfare can be decreasing in the supply of the safe asset.

Our paper is related to Andolfatto and Martin (2013) who consider a model where a physical asset, whose expected short-run return is subject to a news shock, can serve as medium of exchange. The authors show that the non-disclosure of news can enhance the asset's property as an exchange medium. As we have already highlighted, the concept of (indirect) liquidity adopted here is different, and so is the concept of safety.[6] Here, an asset's safety is simply the (ex-ante) probability with which the assets will not pay the promised cash flow, which, in turn, is a function of the issuer's credit worthiness. This probability is public knowledge and can be thought of (or approximated by) a credit rating agency's score. Rocheteau (2011) studies a model where bonds serve as media of exchange alongside with money. The author shows that if the bond holders (and goods buyers) have private information about the bond's return, then money will endogenously arise as more liquid asset (i.e., a better medium of exchange). Our paper studies the link between asset safety and liquidity, assuming that the various assets' safety characteristics are public knowledge, i.e., we do not have a story of private information.

Our paper is also related to Lagos (2010), who considers a model where bonds, whose return is deterministic, and stocks, whose return is stochastic, compete as media of exchange. The author quantitatively demonstrates that the equity premium puzzle can be

[6] In fact, the authors of that paper never use the term "safety". However, the idea that some assets are more "information sensitive" than others is close to the definition of safety adopted by Gorton and Ordonez (2013); see footnote 2.

explained through a liquidity differential between the safe and the risky asset. Jacquet (2015) employs a similar model, but includes a larger variety of asset classes and ex-ante heterogeneous agents. The author shows that the equilibrium displays a “class structure” in the sense that agents with different liquidity needs will only be willing to hold assets of a certain risk structure. Our paper differs from the aforementioned papers, not only because it employs a different model of liquidity, but also because it predicts that an asset in large(r) supply may carry a higher liquidity premium. This result cannot be obtained in Lagos (2010), or other related papers, as in these papers the asset demand curve is typically decreasing. Thus, our model of indirect liquidity and endogenous market entry has the unique ability to rationalize why assets in limited supply can be highly illiquid, even when they enjoy a high credit rating (e.g., AAA corporate bonds in the U.S.).

He and Milbradt (2014) study a one-asset model where defaultable corporate bonds are traded in an OTC secondary market, and show that the bond liquidity, defined as the bid-ask spread in the OTC market, is positively related with credit ratings. We consider a two-asset (easily extended to an N-asset) model, which allows us to study the liquidity of assets with varying degrees of safety, in general equilibrium. Also, He and Milbradt (2014) employ the model of Duffie et al. (2005) where assets are indivisible, i.e., agents can hold either 0 or 1 units of the asset. Our model also incorporates OTC secondary asset trade *á la* Duffie et al. (2005), but does so within the monetary model of Lagos and Wright (2005), which allows us to study perfectly divisible asset supplies, and offers a number of important new insights. Such insights include the possibility of upward-sloping demand curves, the possibility that a riskier asset can be more liquid in general equilibrium, and the fact that welfare can be decreasing in the supply of safe assets.

In related empirical work, Krishnamurthy and Vissing-Jorgensen (2012) clearly distinguish between asset safety and liquidity, and extract safety and liquidity premia from the data to explain why Treasury yields have been decreasing. Their model identifies the safety premium through the spreads between AAA and BAA bonds, assuming that both of these types of bonds are equally illiquid. The present paper demonstrates that certain assets may carry different liquidity premia precisely because they are characterized by different default risks. This, in turn, highlights that we need more theory that studies the relationship between asset safety and liquidity, and we view the present paper as a part of this important agenda.

2 The model

Our model builds on the one developed in Geromichalos and Herrenbrueck (2016b), which, in turn, is a hybrid of Lagos and Wright (2005) (henceforth, LW) and Duffie et al. (2005).[7] Time is discrete and continues forever. Each period is divided in three subperiods, characterized by different types of trade. In the first subperiod, agents trade in OTC secondary asset markets. In the second subperiod, they trade in a decentralized goods market (DM). Finally, in the third subperiod, agents trade in a centralized market (CM). The CM is the typical settlement market of LW, where agents settle their old portfolios and choose new ones. The DM is a decentralized market characterized by anonymity and imperfect commitment, where agents meet bilaterally and trade a special good. These frictions make a medium of exchange necessary, and we assume that only money can serve this role. The OTC markets allow agents with different liquidity needs to rebalance their portfolio by selling assets for money.

Agents live forever and discount future between periods, but not subperiods, at rate $\beta \in (0,1)$. There are two types of agents, consumers and producers, distinguished by their roles in the DM. The measure of each type is normalized to the unit. Consumers consume in the DM and the CM and supply labor in the CM; producers produce in the DM and consume and supply labor in the CM. All agents have access to a technology that transforms one unit of labor in the CM into one unit of the CM good, which is also the numeraire. The preferences of consumers and producers within a period are given by $\mathcal{U}(X,H,q) = X - H + u(q)$ and $\mathcal{V}(X,H,q) = X - H - q$, respectively, where X denotes consumption of CM goods, H is labor supply in the CM, and q stands for DM goods produced and consumed. We assume that u is twice continuously differentiable, with $u' > 0$, $u'(0) = \infty$, $u'(\infty) = 0$, and $u'' < 0$. The term q^* denotes the first-best level of trade in the DM, i.e., it satisfies $u'(q^*) = 1$. All goods are perishable between periods.

Notice that in this model the agents dubbed "producers" will never choose to hold any assets, as long as these assets are priced at a premium for their liquidity. The reason is simple; a producer's identity is permanent, so why would she ever pay this premium when she knows that she will never have a liquidity need (in the DM)? As a result, all the interesting portfolio choices in this model are made by the "consumers". Thus, henceforth, we will refer to the "consumers" simply as "agents". When we use the terms

[7] In Geromichalos and Herrenbrueck (2016b) the goal is to explain liquidity differences between assets that have the same risk characteristics, but trade in OTC markets with different micro-structures. Here, we assume that there are no differences in exogenous market characteristics. Hence, any difference in liquidity between two assets will be the result of differences in safety. See also footnote 5.

"buyer" and "seller", it will be exclusively to characterize the role of these agents in the secondary asset market. We now describe all the assets available in this economy.

There is a perfectly divisible object called fiat money that can be purchased in the CM at the price φ in terms of CM goods. The supply of money is controlled by a monetary authority, and follows the rule $M_{t+1} = (1+\mu)M_t$, with $\mu > \beta - 1$. New money is introduced if $\mu > 0$, or withdrawn if $\mu < 0$, via lump-sum transfers in the CM. Money has no intrinsic value, but it possesses all the properties that make it an acceptable medium of exchange in the DM, e.g., it is portable, storable, and recognizable by everyone in the economy. Using the Fisher equation, we summarize the money growth rate by $i = (1 + \mu + \beta)/\beta$; the rate i will be a useful benchmark as the yield on a completely illiquid asset. (Thus, i should not be thought of as representing the yield on T-bills; see Geromichalos and Herrenbrueck (2017) for a discussion.)

There are also two types of assets, asset A and asset B. These are one-period, nominal bonds with a face value of one dollar; their supply is exogenous and denoted by S_A and S_B, respectively. Asset j can be purchased at price p_j, $j = \{A, B\}$, in the CM, which we think of as the primary market. After leaving the CM agents receive an idiosyncratic consumption shock (discussed below) and may trade these assets (before maturity) in a secondary OTC market. Each asset j trades in a distinct secondary market, which we dub OTC$_j$, $j = \{A, B\}$. To make things tractable, we assume that *agents can only hold either asset A or asset B*, and can visit only one OTC market per period. Thus, we say they "specialize" in holding asset A or B.[8] However, agents are free to choose any quantity of money and the asset of their choice.

The economy is characterized by two shocks, both of which are revealed after the CM closes and before the OTC round of trade opens. The first is an aggregate shock that determines whether asset B will default or not in that period. More precisely, with probability π each unit of asset B pays the promised dollar, but with probability $1 - \pi$, asset B defaults and pays nothing. Throughout the paper we assume that asset A is a perfectly safe and default-free asset.[9] This aggregate default shock is *iid* across time.

[8] This implies some loss of generality but not too much. As shown in Geromichalos and Herrenbrueck (2016b), specialization is actually a *result* when both assets are safe which follows from the fact that agents can only visit one OTC market per period. Here, it can be shown that full specialization would still be the endogenous outcome if the supply of asset A was small enough, and asset B was safe enough. Otherwise, there may be partial specialization where some agents only hold asset A, and other agents hold B but also small amounts of A which they plan to sell in case B defaults.

[9] Our results are robust to different model specifications. For instance, modeling asset A as a default-free asset is done for simplicity and because many real-world assets characterized as AAA are virtually default free. However, all one needs is that asset A defaults with a lower probability than asset B. Similarly, when

The second shock is an idiosyncratic consumption shock that determines whether an agent will have an opportunity/desire to consume in the forthcoming DM. We assume that a fraction $\ell < 1$ of agents will obtain such an opportunity and the rest will not. Thus, a measure ℓ of agents will be of type C ("Consuming"), and a measure $1 - \ell$ of agents will be of type N ("Not consuming"). This shock is *iid* across agents and time. Since the various types are realized after agents have made their portfolio choices in the CM, N-types will typically hold some cash that they do not need in the current period, and C-types may find themselves short of cash, since carrying money is costly. Placing the OTC round of trade after the CM but before the DM allows agents to reallocate money into the hands of the agents who need it most, i.e., the C-types.[10]

As we have discussed, agents can only trade in one OTC market per period, and they will choose to trade in the market where they expect to find the best terms. Suppose that a measure C_j of C-types and a measure N_j of N-types have chosen to trade in the market for asset $j = \{A, B\}$ (of course, these measures will be determined endogenously). Then, the matching technology

$$f(C_j, N_j) = \left(\frac{C_j N_j}{C_j + N_j} \right)^{1-\rho} (C_j N_j)^{\rho}, \quad \rho \in [0, 1],$$

determines the measure of successful matches in OTC$_j$. The suggested matching function satisfies $f(C, N) \leq \min\{C, N\}$, and is useful because it admits both constant and increasing returns to scale (CRS and IRS, respectively) as subcases: when $\rho = 0$, the matching technology features CRS, while $\rho > 0$ implies IRS. Within each successful match the buyer and seller split the available surplus based on proportional bargaining (Kalai, 1977), with $\theta \in (0, 1)$ denoting the seller's (C-type's) bargaining power.[11] Notice that the matching technology and the bargaining protocol are identical in both OTC markets. This guarantees that any differences in liquidity between assets A and B will be driven by differences in safety, and not by exogenous market characteristics (see footnotes 5 and 7).

Since all the action of the model takes place in the CM and, more importantly, the OTC markets, we wish to keep the DM as simple as possible. To that end, we assume that

asset B defaults, it defaults completely. Qualitatively, our results would not change if we assumed that the default is partial; i.e., at default, asset B pays only $x < 100$ cents on the dollar.

[10] The first paper to incorporate this idea into the LW framework is Berentsen, Camera, and Waller (2007), but there the reallocation of money takes place through a competitive banking system.

[11] The proportional bargaining solution of Kalai (1977) has important advantages over Nash bargaining (Nash Jr, 1950). First, it is significantly more tractable. Second, in recent work, Rocheteau, Hu, Lebeau, and In (2018) solve a sophisticated model of bargaining with strategic foundations, and find that, under fairly general conditions, their solution converges to the proportional one.

all C-type consumers match with a producer, and they make a take-it-or-leave-it (TIOLI) offer (i.e., C-type consumers grasp all the surplus in the DM).

Figures 1 and 2 summarize the timing of events and the important economic actions of the model. A few details are worth emphasizing. First notice that agents who turn out to be C-types are *committed* to visit the OTC market of the asset they chose to specialize in. (One cannot sell asset B in OTC$_A$.) However, this is not true for N-types: an agent who turns out to be an N-type can visit either OTC market, because *her money is good* to buy any type of assets. This has an important consequence. In the default state (see Figure 2), OTC$_B$ will shut down so *all* N-types will rush into OTC$_A$. And what about the agents who specialized in asset B and turned out to be C-types? Unfortunately, they must proceed to the DM only with the money that they carried from the CM. But it is important to remember that agents are aware of this possibility and may choose to hold asset B anyway. Part of what makes this choice optimal is that they may pay a low(er) price for asset B and choose to carry more money as a precaution.

3 Analysis of the model

3.1 Summary of value functions and bargaining solutions

In order to streamline the analysis, we relegate the derivations of the value functions and the solutions of the various bargaining problems to Appendix A.1 and A.2. Here is a summary of the main results. As is standard in models that build on LW, all agents have linear value functions in the CM, a result that follows from the (quasi) linear preferences. This makes the bargaining solution in the DM easy to characterize. Consider a DM meeting between a producer and a C-type agent who carries m units of money, and define $m^* \equiv q^*/\varphi$ as the amount of money that (given the price φ) allows the agent to purchase the first-best quantity, q^*. Then, either $m \geq m^*$, and the buyer can purchase q^*, or $m < m^*$, and she spends all of her money to purchase the amount $q = \varphi m < q^*$.

Next, consider a meeting in OTC$_j$, $j = \{A, B\}$, where the N-type brings a quantity $\tilde{m}$ of money, and the C-type brings a portfolio (m, d_j) of money and asset j. Since money is costly to carry, in equilibrium we will have $m < m^*$, and the C-type will want to acquire the amount of money that she is missing in order to reach m^*, namely, $m^* - m$. Whether she will be able to acquire that amount of money depends on her asset holdings. If her asset holdings are enough (of course, how much is "enough" depends on the bargaining power θ), then she will acquire exactly $m^* - m$ units of money. If not, then she will give up all her assets to obtain an amount of money $\xi(m, d_j) < m^* - m$, which is increasing

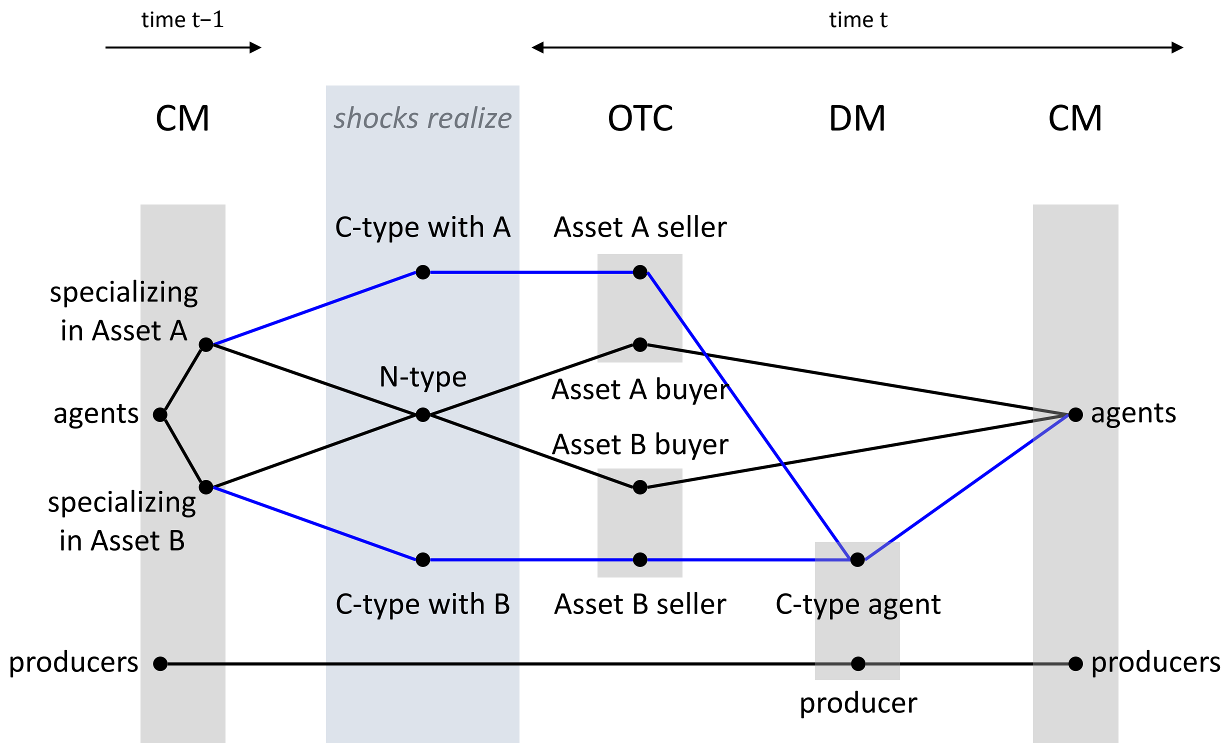


Figure 1. Timeline when both assets pay out

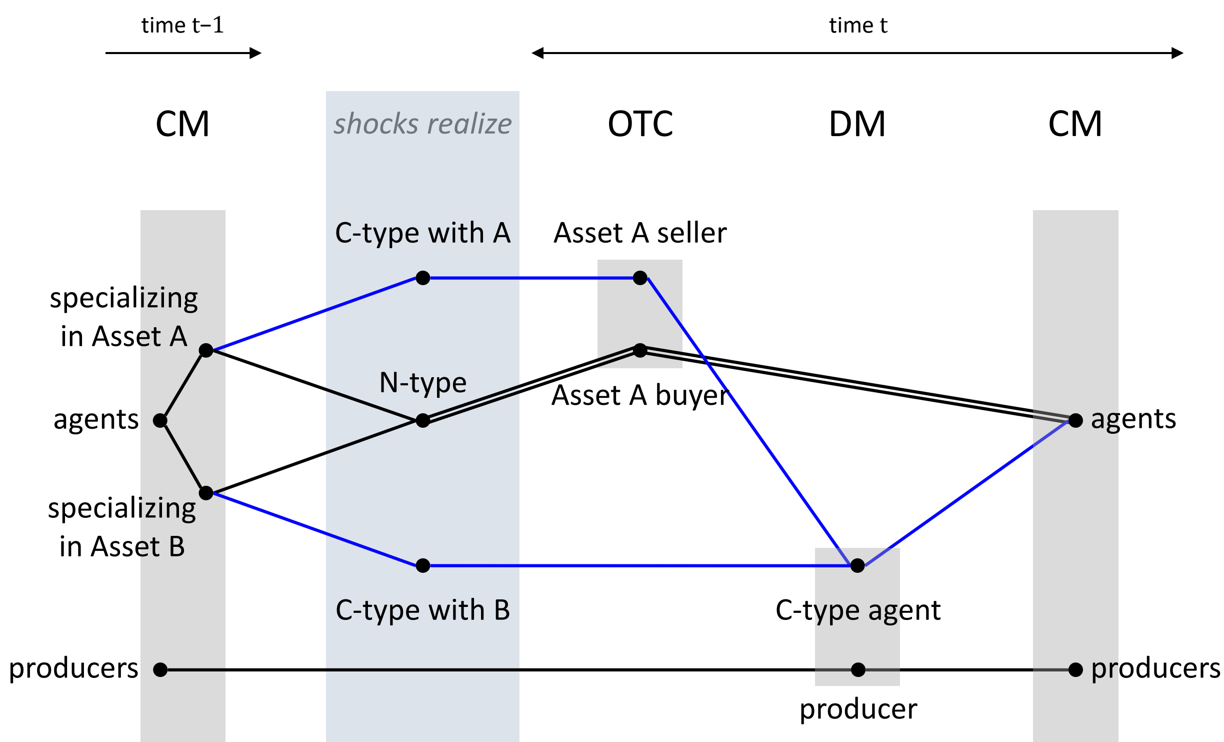


Figure 2. Timeline when asset B defaults

in d_j (the more assets she has, the more money she can acquire) and decreasing in m (the more money she carries, the less she needs to acquire through OTC trade). This last case, where assets are scarce, is especially interesting, because it is precisely then that having a few more assets would have allowed the agent to alleviate the binding cash constraint, which is why an asset price will carry a liquidity premium.[12] A take-away point of this discussion is that the OTC terms of trade depend only on the C-type's portfolio.

3.2 Matching probabilities

Next, consider the matching probabilities in each OTC market. Let $e_C \in [0,1]$ be the fraction of C-type agents who specialize in asset A and are thus committed to trading in OTC$_A$, no matter the eventual aggregate state. And let $e_N^s \in [0,1]$ be the fraction of N-type agents who enter OTC$_A$ in state s, where $s = \{n, d\}$ denotes the aggregate state (n for "normal" and d for "default"). Then, in state s, $e_C\ell$ is the measure of C-types and $e_N^s(1-\ell)$ is the measure of N-types who enter OTC$_A$. Similarly, $(1-e_C)\ell$ is the measure of C-types and $(1-e_N^s)(1-\ell)$ is the measure of N-types who enter OTC$_B$. Letting α_{ij}^s denote the matching probability of an i-type who enters OTC$_j$ in state s, we have:

$$\alpha_{CA}^n \equiv \frac{f[e_C\ell, e_N^n(1-\ell)]}{e_C\ell}, \qquad \alpha_{CB}^n \equiv \frac{f[(1-e_C)\ell, (1-e_N^n)(1-\ell)]}{(1-e_C)\ell},$$
$$\alpha_{NA}^n \equiv \frac{f[e_C\ell, e_N^n(1-\ell)]}{e_N^n(1-\ell)}, \qquad \alpha_{NB}^n \equiv \frac{f[(1-e_C)\ell, (1-e_N^n)(1-\ell)]}{(1-e_N^n)(1-\ell)},$$
$$\alpha_{CA}^d \equiv \frac{f[e_C\ell, e_N^d(1-\ell)]}{e_C\ell}, \qquad \alpha_{CB}^d \equiv \frac{f[(1-e_C)\ell, (1-e_N^d)(1-\ell)]}{(1-e_C)\ell},$$
$$\alpha_{NA}^d \equiv \frac{f[e_C\ell, e_N^d(1-\ell)]}{e_N^d(1-\ell)}, \qquad \alpha_{NB}^d \equiv \frac{f[(1-e_C)\ell, (1-e_N^d)(1-\ell)]}{(1-e_N^d)(1-\ell)}.$$

3.3 Optimal portfolio choice

As is standard in models that build on LW, all agents choose their optimal portfolio in the CM independently of their trading histories in previous markets. In our model, in addition to choosing an optimal portfolio of money and assets, $(\hat{m}, \hat{d}_A, \hat{d}_B)$, agents also choose which OTC market they will enter in order to sell or buy assets, once the shocks have

[12] This discussion assumes that $m + \tilde{m} \geq m^*$, i.e., that the money holdings of the C-type and the N-type pulled together is enough to allow the C-type to purchase the first best quantity q^*. Allowing for $m + \tilde{m} < m^*$ adds many complications to the model without offering any valuable insights (see Geromichalos and Herrenbrueck, 2016a). Therefore, in what follows, we will assume that we are always in the region where $m + \tilde{m} \geq m^*$. This condition will be always satisfied as long as inflation is not too large, so that all agents carry at least *half* of the first-best amount of money.

been realized. The agent's choice can be analyzed with an objective function, $J(\hat{m}, \hat{d}_A, \hat{d}_B)$, which we derive in Appendix A.3 and reproduce here for convenience:

$$\begin{aligned} J(\hat{m}, \hat{d}_A, \hat{d}_B) \equiv & -\varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) + \beta\hat{\varphi}(\hat{m} + \hat{d}_A + \pi\hat{d}_B) \\ & + \beta\ell\Big(u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{m} + \pi\max\{\alpha^n_{CA}\mathcal{S}_{CA}, \alpha^n_{CB}\mathcal{S}_{CB}\} + (1-\pi)\alpha^d_{CA}\mathcal{S}_{CA}\Big), \end{aligned}$$

where $\mathcal{S}_{Cj}$ is the surplus of an agent who turns out to be a C-type and trades in OTC$_j$:

$$\mathcal{S}_{Cj} = u(\hat{\varphi}(\hat{m} + \xi_j(\hat{m}, \hat{d}_j))) - u(\hat{\varphi}\hat{m}) - \hat{\varphi}\chi_j(\hat{m}, \hat{d}_j).$$

In the above expression, ξ_j stands for the amount of money that the agent can acquire by selling assets, and χ_j stands for the amount of assets sold in OTC$_j$, $j = \{A, B\}$.

The interpretation of J is straightforward. The first term is the cost of choosing the portfolio $(\hat{m}, \hat{d}_A, \hat{d}_B)$. This portfolio guarantees the base return $\hat{\varphi}(\hat{m} + \hat{d}_A + \pi\hat{d}_B)$ in next period's CM (the second term of J). This portfolio also offers certain liquidity benefits, but these will only be relevant if the agent turns out to be a C-type; thus, the term in the second line of J is multiplied by ℓ. The C-type can enjoy at least $u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{m}$ just with the money that she brought from the CM. Furthermore, she can enjoy an additional benefit by selling assets for cash in the secondary market. How large this benefit is depends on the market choice of the agent (the term inside the $\max$ operator) and on the realization of the aggregate shock: if asset B defaults, an event that happens with probability $1 - \pi$, a C-type who specialized in that asset has no benefit. A default of asset B is not the only reason why the C-type may not trade in the OTC markets; it may just be that she did not match with a trading partner. This is why the various surplus terms $\mathcal{S}_{Cj}$ are multiplied by the α terms, i.e., the matching probabilities described in the previous section.[13]

3.4 Equilibrium

Before we move on to the characterization of equilibrium, we first need to understand the structure of equilibrium. To that end, notice that we have twelve endogenous variables to be determined in equilibrium (this count does not include the terms of trade in the OTCs):

- equilibrium prices: φ, p_A, p_B
- equilibrium real balances: z_A, z_B

[13] There are two reasons why the objective function does not contain any term that represents the event in which the agent is an N-type. First, and most obviously, N-types are defined as the agents who do not get to consume in the DM. Second, the OTC terms of trade, χ and ξ, depend only on the portfolio of the C-type. An intuitive explanation was presented in Section 3.1. For the details, see Appendix A.2.2.

- equilibrium entry choices: $e_C(\equiv e_C^n = e_C^d)$, e_N^n, e_N^d
- equilibrium DM production: $q_{0A}(\equiv q_{0A}^n = q_{0A}^d)$, $q_{1A}(\equiv q_{1A}^n = q_{1A}^d)$, $q_{0B}(\equiv q_{0B}^n = q_{0B}^d)$, $q_{1B}(\equiv q_{1B}^n)$

In this list of equilibrium variables, the asset prices are obvious, and z_j, $j = \{A, B\}$, is simply the real balances held by the typical agent who chooses to specialize in asset j. The remaining terms deserve some discussion. First, notice that the fraction of C-types who enter OTC$_A$, e_C, does not depend on the aggregate state $s = \{n, d\}$. This is because C-types are committed to visiting the OTC market of the asset they chose to specialize in (and this choice is effectively made before the realization of the shock).

Regarding the DM production terms q_{kj}, $k = \{0, 1\}$ indicates whether the C-type did ($k = 1$) or did not ($k = 0$) trade in the preceding OTC market, and $j = \{A, B\}$ indicates the asset in which she specializes. For example, q_{0A} is the amount of DM good purchased by an agent who specialized in asset A and did not match in OTC$_A$, and so on. These terms do not depend on the aggregate state $s = \{n, d\}$. To see why, notice that q_{0A} depends only on the amount of real balances that the agent carried from the CM (this agent did not trade in the OTC), and that choice was made before s was realized. The same reasoning applies to q_{0B}. How about the term q_{1A}?[14] This term depends on the real balances that the agent carried from the CM (which, we just argued, is independent of the shock realization), and on the amount of assets that this agent carries from the CM (see Section 3.1). How many asset does this agent carry? The answer is S_A/e_C: the exogenous asset supply, S_A, divided by the measure of agents who specialize on asset A. Since S_A is a parameter, and e_C is independent of the state s, the same will be true for the term q_{1A}.

To simplify the exposition of the equilibrium analysis, we now establish that the variables $\{z_A, z_B, \varphi, p_A, p_B\}$ follow immediately from $\{q_{0A}, q_{1A}, q_{0B}, q_{1B}, e_C, e_N^n\}$ (and e_N^d is always equal to 1). First, since the C-types have all the bargaining power in the DM, the equilibrium real balances satisfy

$$z_A = q_{0A},\ z_B = q_{0B}. \tag{1}$$

Second, the equilibrium price of money solves the money market clearing condition:

$$\varphi M = e_C q_{0A} + (1 - e_C) q_{0B}. \tag{2}$$

Third, the equilibrium asset prices solve the following asset demand equations (reproduced from (A.8) in Appendix A.4):

[14] Notice that the term q_{1B}^d is left undefined, since OTC$_B$ shuts down in the default state.

$$p_A = \frac{1}{1+i}\left(1+\ell\frac{\theta}{\omega_\theta(q_{1A})}\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)(u'(q_{1A})-1)\right), \tag{3}$$
$$p_B = \frac{1}{1+i}\left(\pi+\ell\frac{\theta}{\omega_\theta(q_{1B})}\pi\alpha^n_{CB}(u'(q_{1B})-1)\right), \tag{4}$$

where:

$$\omega_\theta(q) \;\equiv\; \theta+(1-\theta)u'(q) \;\geq\; 1.$$

Next, we study the determination of $\{q_{0A}, q_{1A}, q_{0B}, q_{1B}, e_C, e^n_N, e^d_N\}$, keeping in mind that all other variables follow easily once these "core" variables have been determined.

3.4.1 Core variable equilibrium conditions

To determine the six core variables we have six equilibrium conditions. First, we have two money demand equations for agents who specialize in asset A and B:[15]

$$\begin{aligned} i &= \ell\Big(1-\theta\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)\Big)(u'(q_{0A})-1) \\ &\quad+\ell\theta\frac{\omega_\theta(q_{0A})}{\omega_\theta(q_{1A})}\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)(u'(q_{1A})-1), \end{aligned} \tag{5}$$
$$i = \ell(1-\theta\pi\alpha^n_{CB})(u'(q_{0B})-1)+\ell\theta\frac{\omega_\theta(q_{0B})}{\omega_\theta(q_{1B})}\pi\alpha^n_{CB}(u'(q_{1B})-1). \tag{6}$$

Next, we have the trading protocol in OTC$_j$, $j=\{A,B\}$, that links q_{0j} and q_{1j}:

$$q_{1A} = \min\Big\{q^*,\, q_{0A}+\varphi\tilde{\xi}_A\Big\},\quad \varphi d_A = z(\tilde{\xi}_A),$$
$$q_{1B} = \min\Big\{q^*,\, q_{0B}+\varphi\tilde{\xi}_B\Big\},\quad \varphi d_B = z(\tilde{\xi}_B),$$

where

$$z(\tilde{\xi}) \equiv (1-\theta)\Big(u(\varphi(m+\tilde{\xi}))-u(\varphi m)\Big)+\theta\varphi\tilde{\xi},$$
$$d_A = \begin{cases} S_A/e_C, & \text{if } e_C>0,\\ 0, & \text{otherwise,}\end{cases}$$
$$d_B = \begin{cases} S_B/(1-e_C), & \text{if } e_C<1,\\ 0, & \text{otherwise.}\end{cases}$$

[15] The details of this derivation can be found in Appendix A.5. What is important here is to remind the reader that agents who choose to specialize in different assets will typically carry different amounts of money. Not surprisingly, agents who choose to carry the less safe asset B self-insure against the probability of default (and the shutting down of OTC$_B$) by carrying more money.

The equations for d_A, d_B can be interpreted as asset market clearing with 'free disposal': we require agents to choose either asset A or B to specialize in, so it is possible that everyone chooses the same asset. In that case, demand for the other asset is zero. If demand for an asset is positive, the market for that asset clears with equality.

The other equations also have intuitive interpretations. They state that if the agent's asset holdings are large, then $q_{1j} = q^*$, because the agent will be able to acquire (through selling assets) the money necessary to purchase the first-best quantity. On the other hand, if the agent's asset holdings are scarce, she will give up all her assets and purchase an amount of DM good equal to q_{0j} (the amount she could have purchased without any OTC trade) plus $\varphi\tilde{\xi}_j$ (the additional amount she can now afford by selling assets for extra cash). The terms d_j represent the amount of assets held by the typical agent who specializes in asset j. With some additional work, we can re-write the OTC bargaining protocols in a form that involves only core equilibrium variables (and parameters):

$$q_{1A} = \min\left\{ q^*,\, q_{0A} + \frac{\frac{S_A}{M}\frac{e_C q_{0A} + (1-e_C)q_{0B}}{e_C} - (1-\theta)\Big(u(q_{1A}) - u(q_{0A})\Big)}{\theta} \right\}, \tag{7}$$

$$q_{1B} = \min\left\{ q^*,\, q_{0B} + \frac{\frac{S_B}{M}\frac{e_C q_{0A} + (1-e_C)q_{0B}}{1-e_C} - (1-\theta)\Big(u(q_{1B}) - u(q_{0B})\Big)}{\theta} \right\}. \tag{8}$$

Our last two equilibrium conditions come from the optimal OTC market entry decisions of agents. An important remark is that the OTC surplus of N-types does not depend on their portfolios (see Section 3.1 or Appendix A.2.2), whereas the OTC surplus of C-types does depend on their portfolios. Hence, in making their entry decisions, C-types consider not only the expected surplus of entering in either market, as is the case for N-types, but also the cost associated with each entry decision. Another way of stating this is to say that e_C is determined *ex-ante* and represents the decision to specialize in asset A, while e_N^n is determined *ex-post* and represents the fraction of N-types who enter OTC$_A$ in the normal state. Therefore, the optimal entry of C-types is characterized by

$$e_C = \begin{cases} 1, & \tilde{\mathcal{S}}_{CA} > \tilde{\mathcal{S}}_{CB} \\ 0, & \tilde{\mathcal{S}}_{CA} < \tilde{\mathcal{S}}_{CB} \\ \in [0,1], & \tilde{\mathcal{S}}_{CA} = \tilde{\mathcal{S}}_{CB}\ , \end{cases} \tag{9}$$

where

$$
\begin{aligned}
\tilde{\mathcal{S}}_{CA} =& - iq_{0A} - ((1+i)p_A - 1)\Big((1-\theta)(u(q_{1A}) - u(q_{0A})) + \theta(q_{1A} - q_{0A})\Big) \\
&+ \ell(u(q_{0A}) - q_{0A}) + \ell\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)\mathcal{S}_{CA}, \\
\mathcal{S}_{CA} =& \theta\Big(u(q_{1A}) - u(q_{0A}) - q_{1A} + q_{0A}\Big),
\end{aligned}
$$

and

$$
\begin{aligned}
\tilde{\mathcal{S}}_{CB} =& - iq_{0B} - ((1+i)p_B - \pi)\Big((1-\theta)(u(q_{1B}) - u(q_{0B})) + \theta(q_{1B} - q_{0B})\Big) \\
&+ \ell(u(q_{0B}) - q_{0B}) + \ell\pi\alpha^n_{CB}\mathcal{S}_{CB}, \\
\mathcal{S}_{CB} =& \theta\Big(u(q_{1B}) - u(q_{0B}) - q_{1B} + q_{0B}\Big).
\end{aligned}
$$

The optimal entry of N-types is more simple and characterized by

$$
e^n_N = \begin{cases} 1, & \alpha^n_{NA}\mathcal{S}_{NA} > \alpha^n_{NB}\mathcal{S}_{NB} \\ 0, & \alpha^n_{NA}\mathcal{S}_{NA} < \alpha^n_{NB}\mathcal{S}_{NB} \\ \in [0,1], & \alpha^n_{NA}\mathcal{S}_{NA} = \alpha^n_{NB}\mathcal{S}_{NB} \end{cases}, \tag{10}
$$

in the normal state, where

$$
\begin{aligned}
\mathcal{S}_{NA} &= (1-\theta)\Big(u(q_{1A}) - u(q_{0A}) - q_{1A} + q_{0A}\Big), \\
\mathcal{S}_{NB} &= (1-\theta)\Big(u(q_{1B}) - u(q_{0B}) - q_{1B} + q_{0B}\Big),
\end{aligned}
$$

and by

$$
e^d_N = \begin{cases} 1, & e_C > 0 \\ \in [0,1], & e_C = 0 \end{cases}, \tag{11}
$$

in the default state.

We can now define the steady state equilibrium of the model.

Definition 1. For given asset supplies $\{A, B\}$, the steady state equilibrium for the core variables of the model is the equilibrium quantities and entry choices, $\{q_{0A}, q_{1A}, q_{0B}, q_{1B}, e_C, e^n_N\}$, such that (5), (6), (7), (8), (9) and (10) hold. The equilibrium real balances, $\{z_A, z_B\}$, satisfy (1), the equilibrium price of money, φ, solves (2), and the equilibrium asset prices, $\{p_A, p_B\}$, solve (3) and (4).

3.5 Equilibrium market entry

In this section we analyze the optimal entry decision of agents, which is a key channel in our model. More precisely, we want to study the best response of the representative C-type, who takes as given e_C (the proportion of other C-types who enter market A), and the associated optimal entry decision of N-types (in the normal state), $e_N^n(e_C)$. This task becomes easier by recognizing that there are three opposing forces at work. We dub them the congestion effect, the coordination effect, and the dilution effect.

The congestion effect means that a high e_C will *discourage* the representative C-type from going to market A because it implies a low matching probability. However, a high e_C also means that many N-types are attracted to market A, i.e., a high e_N^n, and a high e_N^n is a force that *encourages* the representative C-type to visit OTC$_A$. This is the coordination effect, which may completely or more than completely offset congestion. A more subtle force is the dilution effect. When e_C is high, many agents specialize in asset A, and each one of them carries a small fraction of the (fixed) asset supply. As a result, the surplus generated in a meeting in OTC$_A$ will be small. This is yet another force that *discourages* the representative C-type from entering market A when e_C is high, because that agent forecasts that few N-types will be attracted to that market.

Moving to the formal analysis, we construct equilibria as fixed points of e_C. To be specific: first, we fix a level of e_C; then we solve for the optimal portfolio choices through Equations (5)-(8) and (10); and finally, we define the C-types' *best response function*:

$$G(e_C) \equiv \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB},$$

where the surplus terms have the optimal choices substituted. This function measures the relative benefit to an *individual* C-type from specializing in asset A over asset B, assuming a proportion e_C of all *other* C-type agents specialize in A, and all other decisions (portfolios and entry of N-types) are conditionally optimal. We say that a value of e_C is part of an "interior" equilibrium if $e_C \in (0,1)$ and $G(e_C) = 0$, or a "corner" equilibrium if $e_C = 0$ and $G(0) \leq 0$ or $e_C = 1$ and $G(1) \geq 0$.

Proposition 1. *The following types of equilibria exist, and have these properties:*

(a) There exists a corner equilibrium where $e_C = 0$, $e_N^n = e_N^d = 0$.

(b) There exists a corner equilibrium where $e_C = 1$, $e_N^n = e_N^d = 1$.

(c) Assume $\rho = 0$ *(CRS) and asset supplies are low enough so that assets are scarce in OTC trade. Then,* $\lim_{e_C \to 0+} G(e_C) > 0 > G(0)$*; the equilibrium at the* B*-corner is not robust to small trembles.*

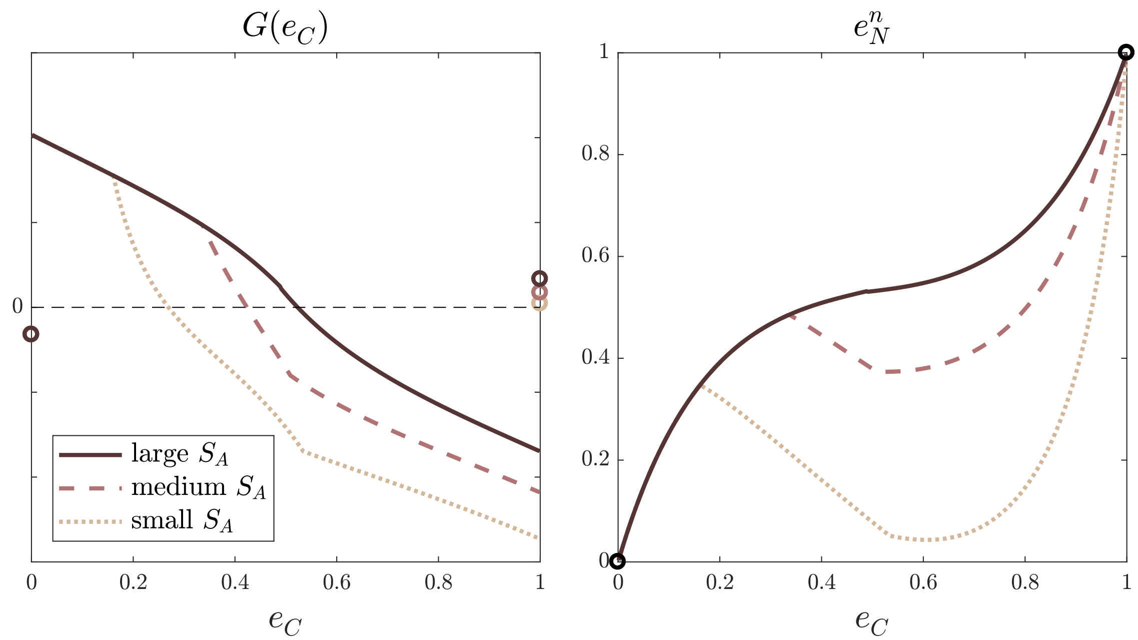


Figure 3. C-types' incentive to deviate and N-types' optimal entry choice, given e_C, for the case of CRS

Notes: The figure depicts the function $G(e_C) \equiv \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB}$ (left panel) and the optimal response of N-types, e_N^n (right panel), as functions of aggregate e_C, assuming CRS in matching ($\rho = 0$). Equilibrium entry is illustrated for three levels of asset supply S_A, keeping the supply of asset B constant. Here, $\pi = 0.95$.

(d) *Assume $\rho = 0$ (CRS) and asset supplies are low enough so that assets are scarce in OTC trade. Then, $\lim_{e_C \to 1} G(e_C) < G(1)$. If $\pi \to 1$, then the limit is negative, and the equilibrium at the A-corner is not robust, either.*

(e) *Assume $\rho = 0$ (CRS), $\pi \to 1$, and asset supplies are low enough so that assets are scarce in OTC trade. Then, there exists at least one interior equilibrium which is robust.*

(f) *Given $\rho > 0$ (IRS), $\lim_{e_C \to 0+} G(e_C) \neq G(0)$.*

(g) *Given $\rho > 0$ (IRS), $\lim_{e_C \to 1-} G(e_C) = G(1) > 0$; the equilibrium at the A-corner is robust.*

Proof. See Appendix A.7.1. □

Figures 3 and 4 illustrate these results; the former for the case of CRS, and the latter for the case of IRS. The left panel of each figure depicts the individual C-type's best response function, $G(e_C)$. Since this function depends not only on the behavior of fellow C-types, but also on that of N-types, on the right panel of each figure we show the optimal entry choice of N-types, $e_N^n(e_C)$, as a function of e_C. The figures also illustrate how equilibrium entry is affected by changes in the supply of asset A, keeping the supply of asset B constant.

As indicated in the right panel of each figure, we have $e_N^n(0) = 0$ and $e_N^n(1) = 1$: when all C-types are concentrated in one market, the N-types will follow. Generally, the higher e_C is, the more N-types would like to go to market A: this is just the coordination effect and it tends to make $e_N^n(e_C)$ increasing. Whether it will be strictly increasing or not, ultimately depends on the strength of the dilution effect relative to the coordination effect. This is why in both figures, $e_N^n(e_C)$ is increasing when A is large: it is a large asset supply that weakens the dilution effect.[16]

Next, we have $G(0) < 0$ and $G(1) > 0$, while $e_N^n(0) = 0$ and $e_N^n(1) = 1$. This illustrates parts (a) and (b) of Proposition 1; the corners are always equilibria (marked with circles on the left panel of the figures). However, with CRS these equilibria are not robust (unless π is so small that the entire G-function is positive, in which case the A-corner is the only equilibrium); this illustrates parts (c) and (d) of the proposition.[17] Also, with CRS the congestion effect is so dominant that the G-function is globally decreasing in the interior, as shown in part (e) of the proposition and illustrated in Figure 3. Therefore, there exists a robust interior equilibrium where the representative C-type is indifferent between entering market A or B; i.e., $G(e_C) = 0$. As the supply of asset A increases, so does the equilibrium value of e_C, because a larger asset supply weakens the dilution effect and increases the incentives of agents to concentrate on market A.

Figure 4 illustrates equilibrium entry under various values of S_A for the case of IRS. Naturally, the two corner solutions are still equilibria, and since IRS strengthen the coordination effect, the equilibrium where all agents go to OTC$_A$ ($e_N^n = e_C = 1$) is now robust (part (g) of the proposition). This may or may not be true for the equilibrium with $e_N^n = e_C = 0$, depending on the values of π and ρ.[18] Figure 4 demonstrates the case of

[16] There is also a difference between the two figures. In Figure 3 (CRS case), $e_N^n(e_C)$ is strictly increasing in its entire domain. However, in Figure 4 (IRS case), and for the case of large S_A, $e_N^n(e_C)$ reaches 1 for a rather small value of e_C and becomes flat afterwards. This is because with IRS, the desire of N-types to go to the market with many C-types, i.e., the coordination effect, is supercharged.

[17] More precisely, they are not "trembling hand perfect" Nash equilibria. Consider for example the equilibrium with $e_N^n = e_C = 1$ (a similar argument applies to the one with $e_N^n = e_C = 0$). Since all N-types visit market A, the representative C-type also wishes to visit that market. (Why try to trade in a ghost town, such as market B?) However, if an arbitrarily small measure ε of N-types visited market B by error, the representative C-type would have an incentive to unilaterally deviate to market B, where her chance of matching is now extremely high (since $e_C = 1$, she would be the only C-type in that market).

[18] Consider first the equilibrium with $e_N^n = e_C = 1$. With IRS the desire to go to OTC$_A$ (where all agents are concentrated) is so strong that, even if some N-types visit OTC$_B$ by error, the representative C-type no longer has an incentive to deviate to that market (unlike the CRS case; see footnote 17). But the channel described so far is relevant for both corners. So why is the equilibrium where all agents go to OTC$_B$ not always robust as well? Because OTC$_B$ is the market of the asset that may default. When that happens

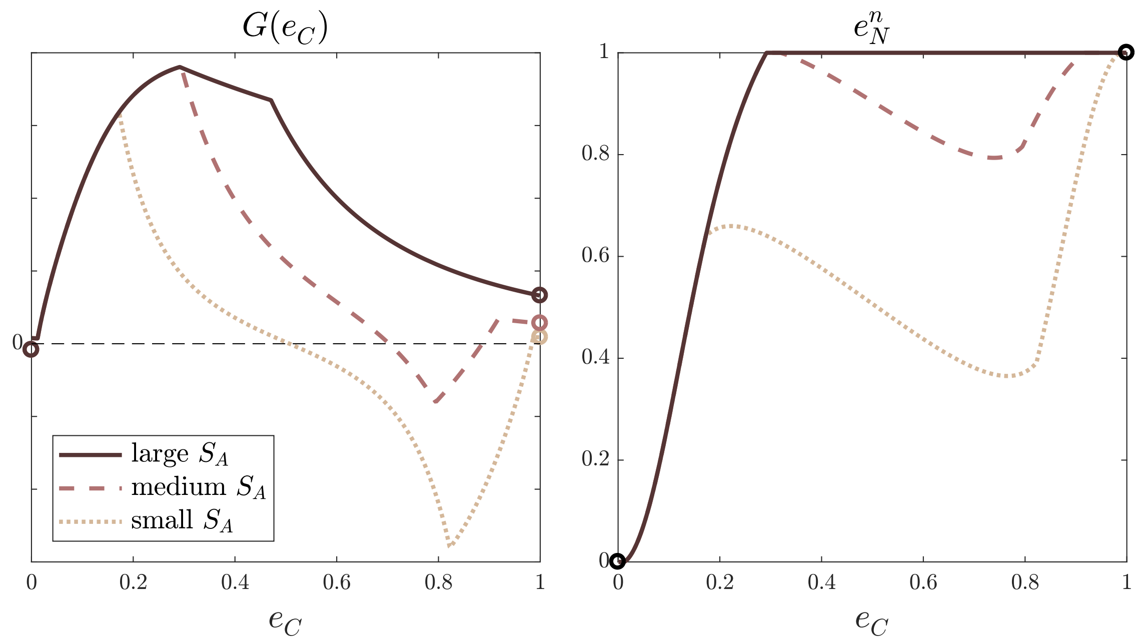


Figure 4. C-types' incentive to deviate and N-types' optimal entry choice, given e_C, for the case of IRS

Notes: The figure depicts the function $G(e_C) \equiv \tilde{S}_{CA} - \tilde{S}_{CB}$ (left panel) and the optimal response of N-types, e^n_N (right panel), as functions of aggregate e_C, assuming IRS in matching. Equilibrium entry is illustrated for three levels of asset supply S_A, keeping the supply of asset B constant. Here, $\pi = 0.95$ and $\rho = 0.3$.

non-robustness; as shown in part (f) of the proposition, the best response function is discontinuous at the B-corner, though it is continuous at the A-corner. With the coordination effect amplified, multiple interior equilibria are typical (as in the case of "small S_A" and "medium S_A"). However, the only interior equilibrium that is robust is the one where G has a negative slope. A rise in S_A will lead to an increase in the (interior and robust) equilibrium value of e_C. But with IRS, another interesting possibility arises: if S_A is large enough, the desire of agents to coordinate on OTC$_A$ is so strong that interior equilibria cease to exist. This is depicted in the "large S_A" case in the figure, where one can see that the A-corner (with $e^n_N = e_C = 1$) is the unique robust equilibrium entry outcome.

(ex-post), all N-types will rush to market A, i.e., $e^d_N = 1$, and this creates an additional incentive for the representative C-type to deviate to market A (a decision made ex-ante). This additional incentive will be relatively large, when π is low (high default probability) and ρ is low (weak coordination effect). Therefore, the equilibrium with $e^n_N = e_C = 0$ is likely to be non-robust for relatively low values of π and ρ.

3.6 Liquidity premium

Most of our main results will be about the *liquidity premia* assets A and B may carry in equilibrium. As we have seen in the asset pricing equations (3) and (4), asset prices consist of the fundamental value multiplied by a premium that reflects the possibility to sell the asset in OTC markets. We define the fundamental value of an asset as the equilibrium price that would emerge if this possibility was eliminated. In that case, agents would value the assets only for their cash flows, and the equilibrium prices would be given by $1/(1+i)$, for asset A, and $\pi/(1+i)$, for asset B.

The liquidity premium of asset j, denoted by L_j, is therefore defined as the percentage difference between an asset's price and its fundamental value:

$$p_A = \frac{1}{1+i}(1+L_A), \qquad\qquad p_B = \frac{\pi}{1+i}(1+L_B), \tag{12}$$

where:

$$\begin{aligned} L_A &= \ell \cdot \left(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\right) \cdot \frac{\theta}{\omega_\theta(q_{1A})} \cdot (u'(q_{1A}) - 1), \\ L_B &= \ell \cdot \alpha^n_{CB} \cdot \frac{\theta}{\omega_\theta(q_{1B})} \cdot (u'(q_{1B}) - 1). \end{aligned} \tag{13}$$

Each liquidity premium is the product of four terms. First, the probability that a consumer turns out to be a C-type and thus needs liquidity at all (ℓ). Second, given that the consumer is a C-type, the expected probability of matching in the respective OTC market (conditional on the market being open). Third, the share of the marginal surplus captured by the C-type (θ/ω_θ), which is endogenous but constrained to the interval $(0, \theta]$. And fourth, the marginal surplus of the match: the utility gained by a consumer who brings one more unit of real balances into the DM, net of the production cost ($u'(q_{1j}) - 1$).

Thus, there are two ways a liquidity premium can be zero: either the relevant OTC market is closed ($\alpha_{Cj} = 0$), or assets are so plentiful that selling an extra asset in the OTC does not create additional surplus in the DM ($q_{1j} = q^*$, thus $u'(q_{1j}) = 1$). In the latter case, the asset is still "liquid", but its liquidity is *inframarginal* so it does not affect the price.

4 Main Results

4.1 Result 1: Safe and liquid

The first result of the paper is that, other things equal, the safer asset (asset A) tends to be more liquid. We demonstrate this result employing two measures of liquidity, the liquidity premium and the volume of trade in each OTC market. Throughout Section 4.1 we assume that the supplies of the two assets are equal ($S_A = S_B$), in order to focus on liquidity differences purely due to the assets' safety differential. Because of the complexity of our model, a full analytical characterization is impossible and we break our analysis into two stages.[19] First, we take a local approximation of our model around $\pi = 1$, assuming constant returns to scale ($\rho = 0$). In this case, a symmetric interior equilibrium exists where the two assets are perfect substitutes and their equilibrium prices (and liquidity premia) are equal, and the perturbation of this equilibrium with small changes in π can be solved in closed form (see Appendix A.7). Second, in order to obtain global results away from $\pi \to 1$, and to conduct comparative statics with respect to ρ, we solve the model numerically.

Proposition 2. *Assume that asset supplies S_A and S_B are equal and are low enough so that assets are scarce in OTC trade. Then:*

(a) At $\pi = 1$, there exists a symmetric equilibrium where $e_C = e_N = 0.5$, $q_{0A} = q_{0B}$, $q_{1A} = q_{1B}$, and $L_A = L_B$.

(b) Assume $\rho = 0$ (CRS) and $(1-\ell)\theta$ is sufficiently large. Then, locally, $\pi < 1$ implies $L_A > L_B$: the safer asset is more liquid.

Proof. See Appendix A.7.2. □

Naturally, when $\pi = 1$, the two assets are perfect substitutes and their equilibrium prices (and liquidity premia) will be equal. However, as π falls below 1, the liquidity premium of asset A generally exceeds that of asset B. Near the symmetric equilibrium, the derivative of the difference between the liquidity premia with respect to π is:

$$\begin{aligned}\left.\frac{d(L_A - L_B)}{d\pi}\right|_{\pi\to 1} = \;& \ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)}(\alpha^n_{CA} - \alpha^d_{CA}) \\ & + \ell\theta\frac{u''(q_1)}{w_\theta(q_1)^2} \times \frac{d(q_{1A} - q_{1B})}{d\pi} + \ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)} \times \frac{d(\alpha^n_{CA} - \alpha^n_{CB})}{d\pi}\end{aligned}$$

The first term on the right-hand side represents the obvious negative *direct effect*: the probability of meeting a buyer for asset A is always lower in the normal state than in the state where B defaults ($\alpha^n_{CA} < \alpha^d_{CA}$), therefore the liquidity advantage of asset A increases as

[19] Our model has six 'core' equilibrium variables, most of which show up in multiple equations; these equations are non-linear and include kinks, due to the various branches of the bargaining solutions and the agents' market entry decisions. Simply put, every time a parameter value changes, all six endogenous variables are affected by simultaneous and, typically, opposing forces. For more detail, one can inspect matrix equation (A.18) in Appendix A.7, which describes the effect of changes in π on the core variables in general equilibrium, keeping in mind that this matrix is evaluated at the limit as $\pi \to 1$.

B becomes less safe ($\pi \downarrow$). But this liquidity advantage is magnified by the endogenous responses of agents to *perceived* default risk, which affect what happens even in the state where asset B pays out. Consider the second term in the equation. An agent who specializes in asset B despite the default risk will self-insure by carrying more money, which translates (after OTC trade) to a higher q_{1B}, resulting in a lower trading surplus (indicated by multiplication with $u''(q_1) < 0$) and thus a lower liquidity premium for asset B; thus, this *intensive margin effect* always reinforces the direct effect.

Finally, there is the the third term in the equation, representing an *extensive margin effect*: generally, when $\pi < 1$, N-types respond more strongly to the lower trading surplus in the B-market, thus the matching probability for C-types is higher in OTC$_A$. If so, then all three effects point in the same direction and thus the overall sign of the equation is negative, as per part (b) of Proposition 2. Analytically, we can show that this is indeed the case when $(1 - \ell)\theta$ is sufficiently large; numerically, we can find counterexamples, but the overall negative sign is still the predominant result.[20]

Figure 5 illustrates our result for a range of π, and for both CRS and an intermediate degree of IRS. In each of these cases, the difference between L_A and L_B is strictly decreasing in π. It is important to remind the reader that this differential is purely due to liquidity; it is not a risk premium. Indeed, decreasing π makes agents less willing to hold asset B because that asset is now at higher risk of default, but that effect is already included in the fundamental value of the assets (see Equations 12). The new result here is that as asset B becomes less safe it also enjoys a smaller liquidity premium, precisely because it becomes less safe.

The intuition behind Result 1 is as follows. Unlike C-types, who are committed to visit the market of the asset in which they chose to specialize, N-types are free to visit any market they wish, since their money is good to buy any asset. Consequently, in the event of default, all the N-types (even those who had chosen to specialize in asset B) will rush into OTC$_A$. Of course, agents who are currently making their portfolio and market entry decisions in the CM correctly anticipate this possibility. Thus, the chance of a market flooded with buyers *ex-post* (i.e., OTC$_A$ in the event of default) serves as a powerful incentive attracting agents to specialize in asset A *ex-ante*, as they forecast that

[20] To be precise, we checked the sign for all combinations of θ and ℓ in $\{0.1, 0.5, 0.9\}$, and asset supplies of $S_A = S_B \in \{.02, .05, .10, .15\}$, with $\rho = 0$, $i = .1$, and $M = 1$ maintained. Out of these 36 parameter combinations, in four of them the assets are so plentiful that both liquidity premia are zero for any π; in three of them, all with maximal ℓ and minimal asset supplies, the sign is reversed so that $L_A < L_B$ when $\pi < 1$, i.e., the safer asset is less liquid; in the remaining 29 cases, we have the 'normal' result where the safer asset is more liquid. For more detail, see Appendix A.7.2.

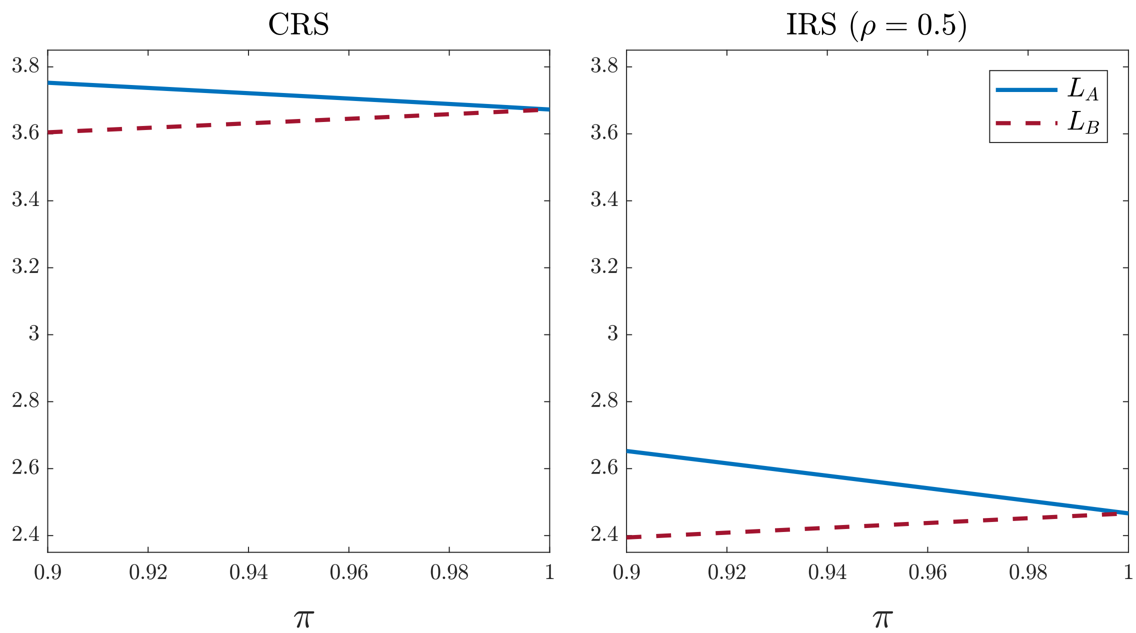


Figure 5. Liquidity premia as functions of π

Notes: The figure depicts the liquidity premia of assets A, B as functions of π, assuming symmetric asset supplies. The left panel illustrates the case of a CRS matching technology, and the right panel represents the the case of IRS ($\rho = 0.5$).

this market will offer a high matching probability, if they turn out to be C-types.

The discussion following Equations (13) reveals why this is important for liquidity: an agent who buys an asset today (in the primary market) is willing to pay a higher price if she expects that it will be easy to sell that asset 'down the road', and, importantly, it is the C-types who sell assets down the road. Through this channel, any positive default probability for asset B translates into a matching advantage for C-types in OTC$_A$. This, in turn, translates into a higher liquidity premium for asset A, because that premium depends on the (anticipated) ease with which the agent can sell the asset if she turns out to be a C-type. Naturally, this channel, and the liquidity differential between the two assets, will be magnified if matching is characterized by IRS.

This last point can be seen more clearly in Figure 6. Instead of liquidity premia, we plot the percentage difference between the two asset prices, $(p_A - p_B)/p_A$, for various values of ρ (and as functions of π), and we contrast them to the difference between the fundamental values.[21] Thus, any difference between the curve indexed as "Fundamental" and the curves representing the various ρ's is a *pure* liquidity difference. The bottom panel of this figure performs the same exercise, but for high values of ρ (including $\rho = 1$,

[21] Clearly, the percentage difference between the fundamental values of assets A and B is $1 - \pi$.

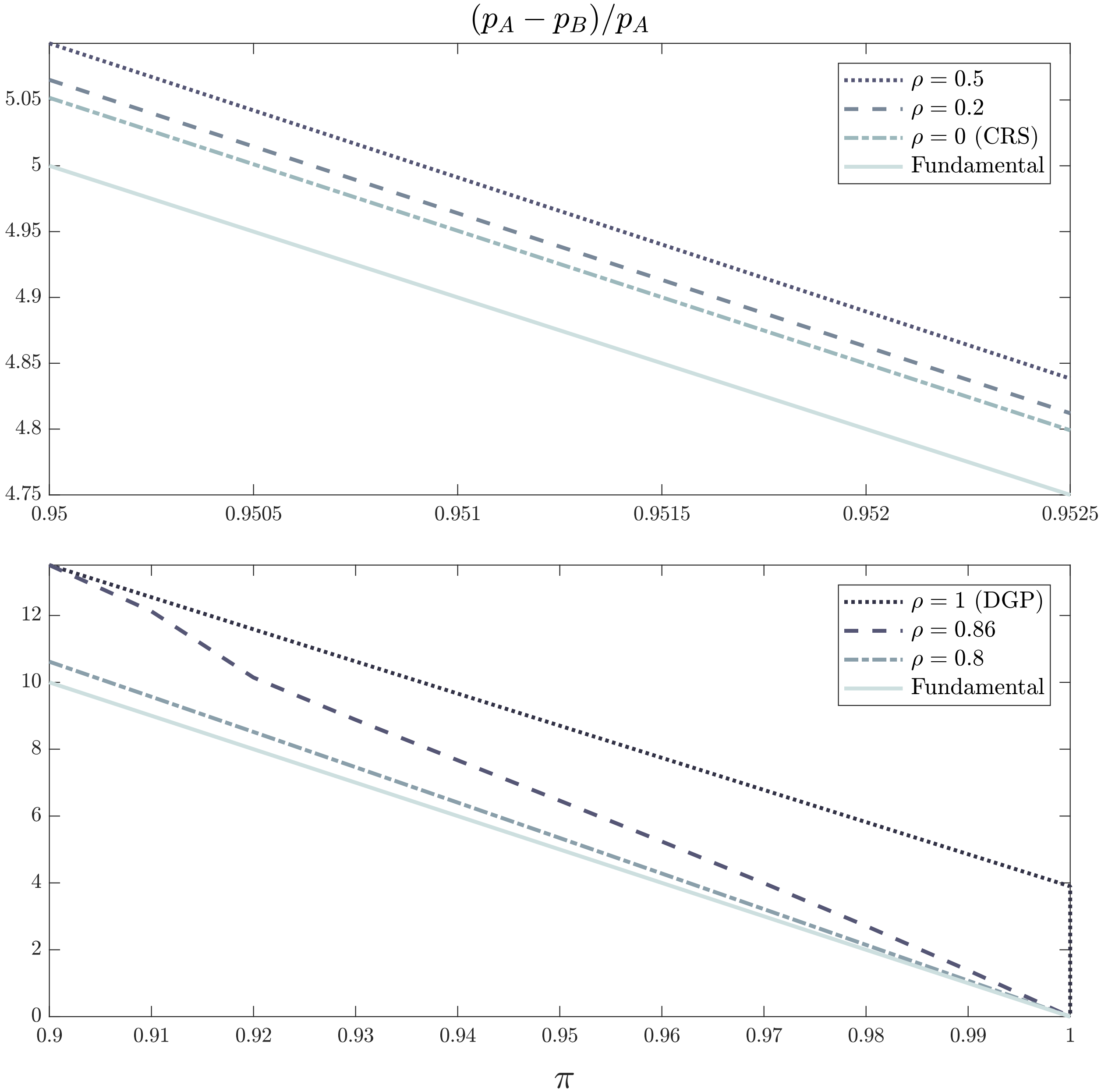


Figure 6. Price differentials as functions of π

Notes: The top panel depicts the price differential $(p_A - p_B)/p_A$ as a function of π, for various values of ρ, assuming symmetric asset supplies. The curve dubbed "Fundamental" represents the percentage difference between the price of assets A and B, if the liquidity channel was shut down, namely, the term $1 - \pi$. The difference between the "Fundamental" curve and the curves corresponding to the various ρ's represent a pure liquidity difference between the two assets. The bottom panel repeats the same exercise for high values of ρ, including the "congestion-free" case where $\rho = 1$. For high enough ρ and for values of π arbitrarily close to 1, the price differential $(p_A - p_B)/p_A$ jumps discontinuously, representing a large liquidity advantage of asset A versus asset B.

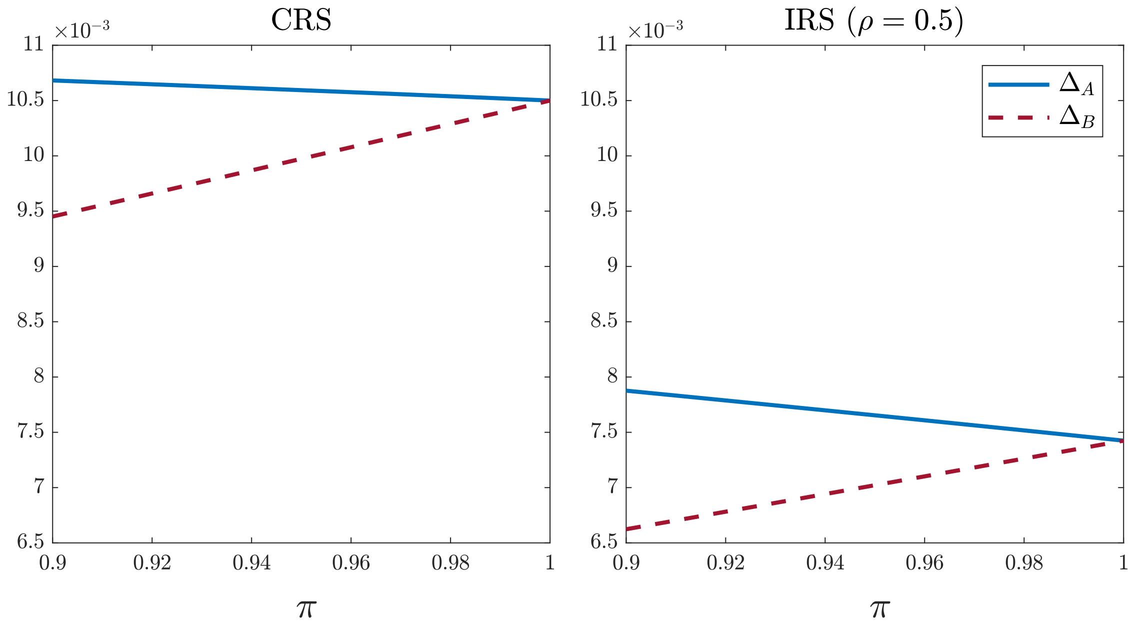


Figure 7. OTC trade volumes as functions of π

Notes: The figure depicts the trade volumes in OTC$_A$ and OTC$_B$, Δ_A and Δ_B as defined in Appendix A.6, as functions of π, for $S_A = S_B$. In the left panel we assume the matching technology is CRS ($\rho = 0$), and in the right panel we have $\rho = 0.5$.

i.e., the congestion-free matching function adopted by Duffie et al. (2005) and most of the papers that build on their framework). This figure highlights that with strong IRS ($\rho \to 1$), even a tiny default probability for asset B can be magnified into an enormous liquidity advantage for asset A. This is visualized by the function $(p_A - p_B)/p_A$, which has a jump at $\pi \to 1$ as long as $\rho \geq .87$ in the specific example.

The description of the mechanism behind Result 1 also highlights that as π decreases, more agents will choose to coordinate in the market for asset A. Thus, it is not only the liquidity premium of asset A that increases in the default probability, but also the volume of trade in that market. This is highlighted in Figure 7, which illustrates the trade volumes in the two OTC markets as functions of π for the cases of CRS and IRS. The details of the derivation of OTC trade volume are relegated to Appendix A.6. As seen in the figure, the trade volume is higher in the secondary market for the safer asset, and the difference in trade volumes between OTC$_A$ and OTC$_B$ is decreasing in π. Since secondary market trade volume is often adopted in the finance literature as a measure of an asset's liquidity, we view this result as an alternative way of establishing that a safer asset will also be more liquid – *other things being equal*.

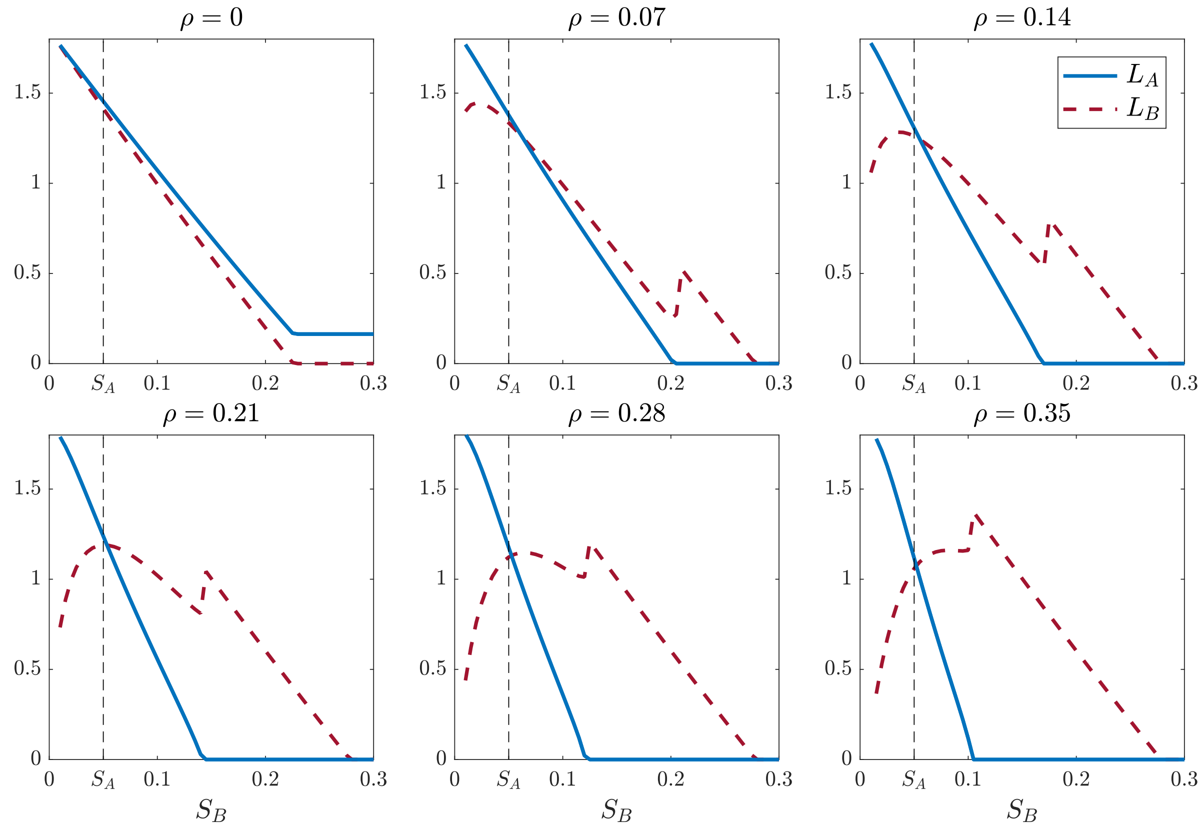


Figure 8. Liquidity premia with varying degrees of IRS

Notes: The figure depicts the liquidity premia of assets A and B as functions of S_B, for a constant S_A, and for varying degrees of IRS. The dashed vertical line indicates the (fixed) supply of asset A; π is set to 0.95.

4.2 Result 2: Safer yet less liquid

Of course, other things are not always equal, and we are particularly interested in asset supplies.[22] Allowing for different asset supplies delivers the second important result of the paper: even with slight IRS in OTC matching, the coordination channel becomes so strong that asset demand curves can be upward sloping. Consequently, asset B can carry a higher liquidity premium than the safe asset A, as long as the supply of the former is sufficiently larger than that of the latter.

Figure 8 depicts the liquidity premia for assets A and B as functions of the supply S_B, keeping S_A fixed, and for various degrees of IRS in matching. First, notice that the liquidity premium on asset A is always decreasing in S_B. This is also true for the liquidity premium on asset B, under CRS (top-left panel), as is standard in existing models of asset liquidity. However, with even a small degree of IRS, the coordination channel becomes so

[22] Recall that the matching efficiency and the bargaining protocol are assumed to be identical in both markets, because we do not want to give one of the assets an exogenous liquidity advantage.

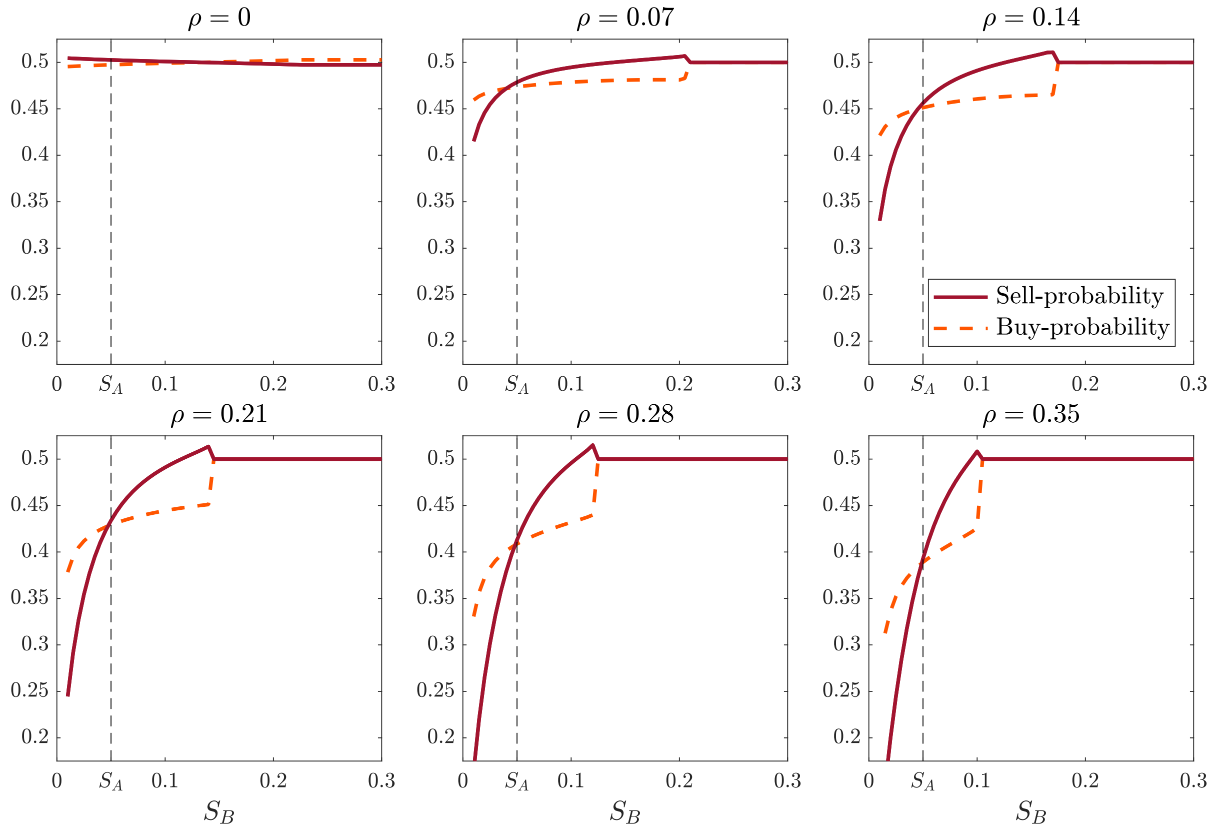


Figure 9. Sell- and buy-probabilities in the OTC_B

Notes: The figure depicts the sell-probability, α^n_{CB}, and the buy-probability, α^n_{NB}, in the secondary market for asset B, in the normal state, as a function of S_B (and for varying degrees of IRS). The dashed vertical line indicates the (fixed) supply of asset A; for the default probability, we set $\pi = 0.95$.

strong that asset demands can slope upwards. And if S_B is significantly larger than S_A, we observe $L_B > L_A$, i.e., the less safe asset can emerge as more liquid.

The intuition for this result is as follows. As we have seen, our model has a channel whereby a safer asset also enjoys an endogenous liquidity advantage. However, whether this advantage will materialize depends on the relative strength of the dilution effect: If the supply of asset A is limited, as more agents choose to specialize in that asset, each one of them will only hold a small amount, and any bilateral meeting in OTC_A will generate a small surplus. Keeping this effect in mind, consider an increase in the supply of asset B. As a result, more agents are willing to trade in OTC_B because of the increase in the expected trading surplus in that market (conditional on no-default). Crucially, N-types respond more elastically to this increase because their market entry choice is not governed by their asset specialization choice (which is already sunk). Consequently, the trade probability in market B for C-types increases by far more than that for N-types, as illustrated

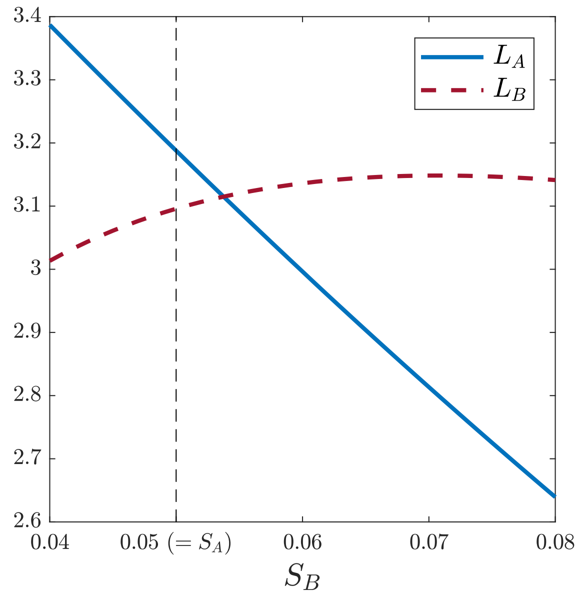


Figure 10. Liquidity premia

Notes: The figure depicts the liquidity premia of assets A and B as functions of S_B, for a constant S_A, and for $\rho = 0.2$. The dashed vertical line indicates the (fixed) supply of asset A; π is set to 0.95.

in Figure 9. Why this is important for liquidity should now be completely transparent: the agent will be willing to pay a high liquidity premium for an asset if she expects a high probability of selling that asset (conditional on needing to sell, i.e., being a C-type). And with some IRS in matching, the aforementioned channel becomes so strong that the premium an agent is willing to pay for an asset is increasing in that asset's supply.

Figure 10 summarizes Results 1 and 2. It depicts the liquidity premia of assets A and B as functions of S_B, keeping S_A fixed, with a slight degree of IRS, $\rho = 0.2$. When the supplies of the two assets are equal ($S_B = S_A$), asset A carries a higher liquidity premium (Result 1). However, as S_B increases further, we enter the region where the demand for asset B becomes upward sloping, until eventually L_B surpasses L_A (Result 2).

4.3 Result 3: Rationalizing the illiquidity of AAA corporate bonds

An interesting fact that has recently drawn the attention of practitioners (but not so much that of academic researchers yet) is that, in the U.S., the virtually default-free AAA bonds are less liquid than (the riskier) AA corporate bonds. Figure 11 plots the time-series yields of AAA versus AA corporate bonds (as well as their difference) on the top panel, and, as a reference point, it does the same for the AAA versus AA municipal bond yields in the bottom panel.[23] The bottom panel is consistent with what one would expect to see: the riskier AA municipal bonds command a higher yield than the one on AAA municipal bonds, because investors who choose to hold the former want to be compensated for their higher default probability.

Interestingly, this logical pattern is reversed in the case of corporate bonds. Indeed, on the top panel of the figure, we see that in the past 5 years, the yield on AA corporate bonds has been consistently lower than that on AAA bonds. Why do investors command a higher yield in order to hold (the virtually default-free) AAA corporate bonds? Many practitioners have claimed that this is so because the secondary market for AAA corporate bonds is extremely illiquid.[24] This narrative is consistent with the observations depicted in Figure 11, and it is supported by further evidence. For instance, He and Milbradt (2014) document that the bid-ask spread in the market for AAA corporate bonds is higher than the one in the market for AA corporate bonds. Additionally, in recent years Bloomberg has ceased constructing its price index for AAA-rated corporate bonds, due to the dearth of outstanding bonds and the lack of secondary market trading. Of course, a high bid-ask spread and a low trade volume are both strong indicators of an illiquid market.

Our model could shed some light on this empirical observation, if it was the case that AAA corporate bonds have a scarce supply relative to AA corporate bonds. This turns out to be overwhelmingly true. In the years following the financial crisis, regulations introduced to improve the stability and transparency of the financial system (such as the Dodd-Frank Act) have made it especially hard for corporations to attain the AAA score. This resulted in a large decrease in the outstanding supply of this class of bonds.[25] As a benchmark of comparison, in June 2018, the outstanding supply of AAA over AA corporate bonds was 1/10, while the same statistic for municipal bonds was 1/3.

While it is plausible to attribute the irregularity observed on the top panel of Figure 11 to 'some liquidity story', existing models of liquidity cannot help us understand this puzzling observation (see a review of the literature in Section 1.1). In these papers, the asset demand curves are decreasing, hence, an asset in large (small) supply will tend

[23] The data on municipal bonds comes from Standard & Poor's, and the data on corporate bonds comes from Federal Reserve Economic Data (FRED). The original data is on a daily base, but, to make the graphs more legible, it is converted to a monthly base. The graphs show the historical yields for the past 10 years.

[24] In recent work, Christensen and Mirkov (2019) highlight yet another class of bonds – Swiss Confederation Bonds – that are considered extremely safe, yet not particularly liquid.

[25] The number of AAA-rated corporations in the U.S., never high, decreased to four – Automatic Data Processing, Exxon Mobil, Johnson & Johnson, and Microsoft – in 2011. Automatic Data Processing got downgraded in 2014, and Exxon Mobil in 2016. Today, there are only two AAA-rated companies.

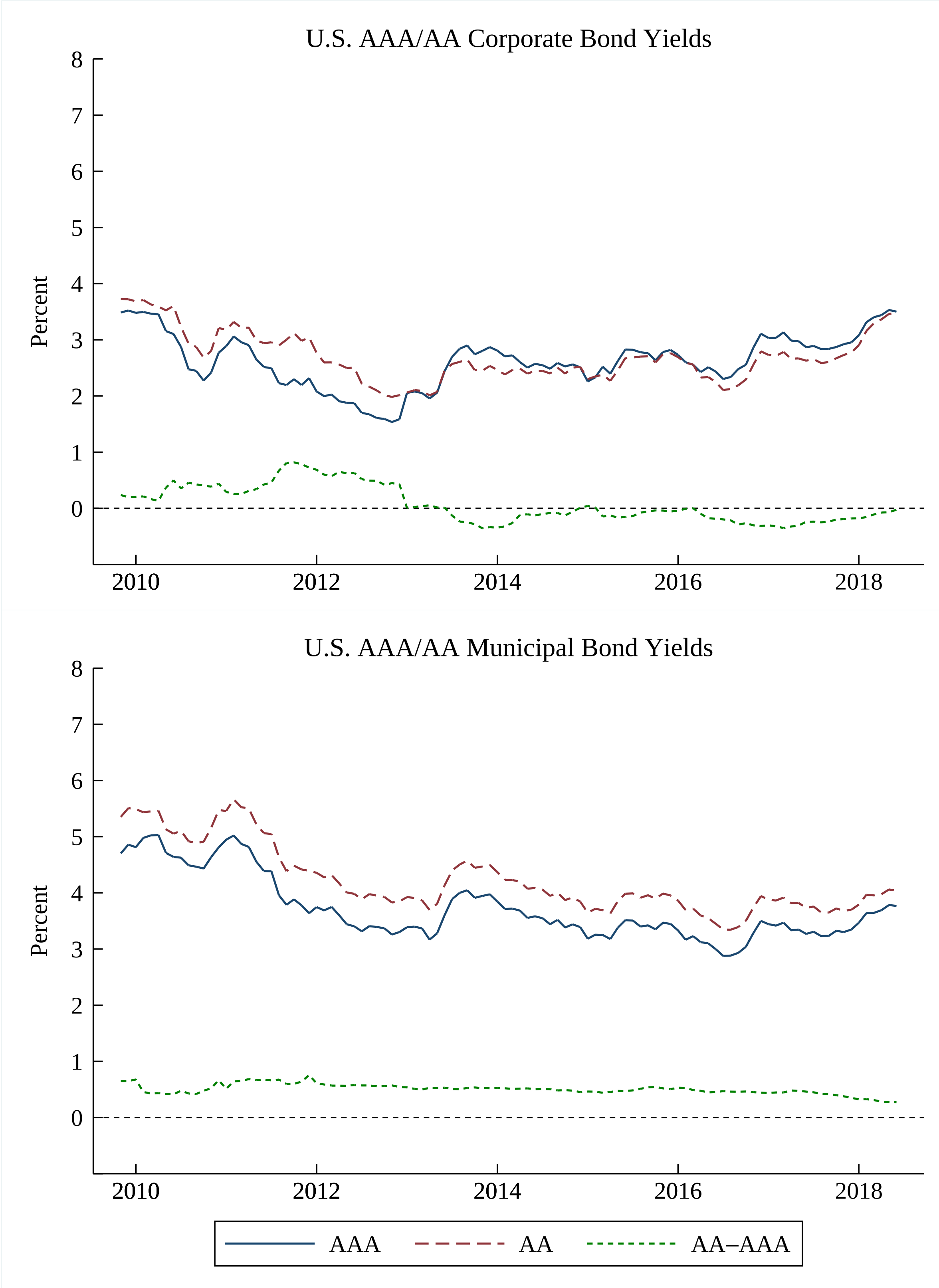


Figure 11. Historical yields of AAA/AA corporate/municipal bonds

Sources: FRED; Standard & Poor's.

to have a low (high) liquidity premium. Our model formalizes the idea that an asset in very scarce supply will be illiquid, even if it maintains an excellent credit rating. And our 'indirect liquidity' approach, coupled with endogenous market entry, is key for delivering this empirically relevant result.

4.4 Result 4: Safe asset supply and welfare

In our final result, we highlight an important implication of our model about the effect of an increase in the supply of safe assets on welfare. A large body of recent literature highlights that the supply of safe assets has been scarce, and that increasing this supply would be beneficial for welfare (see, for example, Caballero et al., 2017). In our model this result is not necessarily true. In particular, welfare may not be monotonic in S_A.

First, let us define the welfare function of this economy, which is the C-type agent's surplus in the DM, averaged between agents who had the opportunity to rebalance their portfolios in the OTC round of trade, and those who did not.[26] Clearly, one also needs to remember that here we have agents who chose to specialize in different assets, and two possible aggregate states (default and no-default). In the normal state, welfare is:

$$\begin{aligned}
\mathcal{W}^n &= \big(e_C\ell - f(e_C\ell, e_N^n(1-\ell))\big)\cdot\big(u(q_{0A})-q_{0A}\big) + f(e_C\ell, e_N^n(1-\ell))\cdot\big(u(q_{1A})-q_{1A}\big) \\
&\quad + \big((1-e_C)\ell - f((1-e_C)\ell,(1-e_N^n)(1-\ell))\big)\cdot\big(u(q_{0B})-q_{0B}\big) \\
&\quad + f((1-e_C)\ell,(1-e_N^n)(1-\ell))\cdot\big(u(q_{1B})-q_{1B}\big) \\
&= e_C\ell\cdot\big[(1-\alpha_{CA}^n)\big(u(q_{0A})-q_{0A}\big)+\alpha_{CA}^n\big(u(q_{1A})-q_{1A}\big)\big] \\
&\quad + (1-e_C)\ell\cdot\big[(1-\alpha_{CB}^n)\big(u(q_{0B})-q_{0B}\big)+\alpha_{CB}^n\big(u(q_{1B})-q_{1B}\big)\big],
\end{aligned}$$

and in the default state, it is:

$$\begin{aligned}
\mathcal{W}^d &= \big(e_C\ell - f(e_C\ell, 1-\ell)\big)\cdot\big(u(q_{0A})-q_{0A}\big) + f(e_C\ell,1-\ell)\cdot\big(u(q_{1A})-q_{1A}\big) + (1-e_C)\ell\cdot\big(u(q_{0B})-q_{0B}\big) \\
&= e_C\ell\cdot\big[(1-\alpha_{CA}^d)\big(u(q_{0A})-q_{0A}\big)+\alpha_{CA}^d\big(u(q_{1A})-q_{1A}\big)\big] + (1-e_C)\ell\cdot\big(u(q_{0B})-q_{0B}\big).
\end{aligned}$$

We define aggregate welfare as:

$$\mathcal{W} = \pi\mathcal{W}^n + (1-\pi)\mathcal{W}^d. \tag{14}$$

Figure 12 plots equilibrium welfare as a function of the supply of the safe asset, and highlights the case in which welfare is non-monotonic in S_A. This result may seem surprising at first. A higher supply of asset A enhances the liquidity role of that asset (or,

[26] In models that build on LW, steady-state welfare depends only on the volume of DM trade. Hence, a sufficient statistic for welfare is how close the average DM production is to the first-best quantity, q^*.

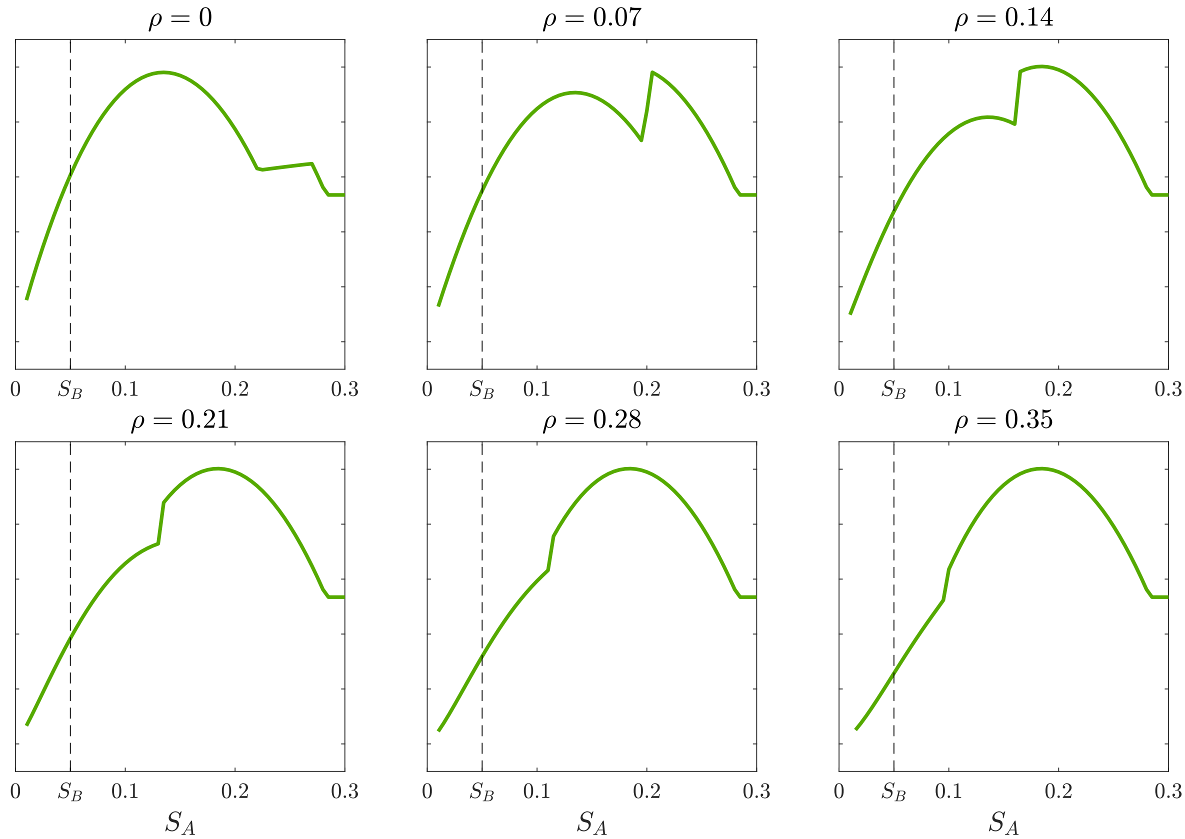


Figure 12. Safe asset supply and welfare

Notes: The figure depicts equilibrium welfare as a function of S_A, for various values of ρ, including $\rho = 0$ (CRS). The dashed vertical line indicates the (fixed) supply of asset B; π is set to 0.95.

equivalently, allows for more secondary market asset trade), which, in turn, should allow agents to purchase more goods in the DM. While not wrong, this intuition is incomplete. What is missing is that when the safe asset becomes more plentiful, agents expect that it will be easier to acquire extra cash ex-post and, thus, they choose to hold less of it ex-ante. In other words, our model is characterized by an externality: agents prefer to carry assets rather than money, and they wish to acquire money in the secondary market(s) only after they have learned that they really need it (i.e., only if they have turned out to be a C-type). But someone has to bring the money, and that someone will not be adequately compensated. This channel depresses the demand for money, which, in turn, decreases the value of money and the volume of trade that the existing money supply can support.

An interesting detail seen in Figure 12 is that welfare always decreases when S_A is large enough. This feature of equilibrium can be explained a follows. As S_A increases, the amount of DM goods purchased by an agent who traded in OTC$_A$, q_{1A}, also increases, because that agent was able to sell more assets and boost her money holdings. On the other hand, as S_A increases, the amount of DM goods purchased by an agent who did not trade in OTC$_A$, q_{0A}, decreases, because the higher asset supply induced that agent to carry fewer money balances *ex-ante* (see previous paragraph). Hence, an increase in S_A generates two opposing effects on welfare: the surplus term $u(q_{1A}) - q_{1A}$ (involving agents who traded in OTC$_A$) increases, but the surplus term $u(q_{0A}) - q_{0A}$ (involving agents who did not trade in OTC$_A$) decreases.[27] While it is hard to know which effect prevails for any value of S_A, what is certain is that if S_A keeps rising, there will come a point where the *marginal* liquidity benefit of more A-assets will be zero (because $q_{1A} \to q^*$ implies $u'(q_{1A}) \to 1$). Near that point, an increase in S_A still hurts welfare by depressing $u(q_{0A}) - q_{0A}$ (because $u'(q_{0A}) \gg 1$), but now it generates no countervailing benefit.

5 Conclusion

We argue that understanding the link between an asset's safety and its liquidity is crucial. To this end, we present a general equilibrium model where asset safety and asset liquidity are well-defined and *distinct* from one another. Treating safety as a primitive, we examine the relationship between an asset's safety and liquidity. We show that the commonly held belief that "safety implies liquidity" is generally justified, but there may be exceptions. In particular, we highlight that a safe asset in scarce supply may be less liquid than a less-safe asset in large supply. Thus, our model can rationalize the puzzling observation that AAA corporate bonds in the U.S. are less liquid than (the riskier) AA corporate bonds. Contrary to a recent literature on the role of safe assets, we show that in our model increasing the supply of the safe asset is not always beneficial for welfare.

[27] Of course, this is a general equilibrium model where any change in S_A affects not only the terms q_{0A}, q_{1A}, but also the terms q_{0B}, q_{1B}. However, the latter is a secondary effect which turns out to be quantitatively not too important.

Appendix

A.1 Value functions

A.1.1 Value functions in the CM

Consider first an agent who enters the CM with m units of money and d_j units of asset j, $j = \{A, B\}$ in state $s = \{n, d\}$. The value function of the agent is given by

$$W_s(m, d_A, d_B) = \max_{\substack{X,H,\\ \hat{m},\hat{d}_A,\hat{d}_B}} \left\{ X - H + \beta\, \mathbb{E}_{s,i}\Big[\max\Big\{\Omega^s_{iA}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big), \Omega^s_{iB}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\Big\}\Big] \right\}$$

$$\begin{aligned}
\text{s.t.}\quad & X + \varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) = H + \varphi(m + \mu M + d_A + d_B), && \text{if } s = n \quad \text{(normal)},\\
& X + \varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) = H + \varphi(m + \mu M + d_A), && \text{if } s = d \quad \text{(default)},
\end{aligned}$$

where variables with hats denote portfolio choices for the next period, and $\mathbb{E}$ is the expectation operator over states and types of consumers. Ω^s_{ij} denotes a value function of an i-type agent, $i = \{C, N\}$, who enters the OTC market for asset j in state s, and it is described in the next section. Replacing $X - H$ from the budget constraint yields

$$\begin{aligned}
W_s(m, d_A, d_B) = {} & \varphi(m + \mu M + d_A + d_B \cdot \mathbb{I}\{s = n\}) \\
& + \max_{\hat{m},\hat{d}_A,\hat{d}_B} \Big\{ -\varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) \\
& \qquad + \beta\pi\ell \max\Big\{\Omega^n_{CA}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big), \Omega^n_{CB}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\Big\} \\
& \qquad + \beta\pi(1-\ell) \max\Big\{\Omega^n_{NA}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big), \Omega^n_{NB}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\Big\} \\
& \qquad + \beta(1-\pi)\ell \max\Big\{\Omega^d_{CA}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big), \Omega^d_{CB}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\Big\} \\
& \qquad + \beta(1-\pi)(1-\ell) \max\Big\{\Omega^d_{NA}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big), \Omega^d_{NB}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\Big\} \Big\},
\end{aligned} \tag{A.1}$$

where $\mathbb{I}$ is the indicator function, and we have used the fact that asset B defaults with probability $1 - \pi$ and that an agent becomes C-type with probability ℓ. This simply can be written as follows:

$$\begin{aligned}
W_n(m, d_A, d_B) &= \varphi(m + d_A + d_B) + \Lambda,\\
W_d(m, d_A, d_B) &= \varphi(m + d_A) + \Lambda,
\end{aligned} \tag{A.2}$$

where Λ collects the remaining terms that do not depend on the current states.

The value function for a producer is much simpler. Note that producers will not want to leave the CM with a positive amount of money or assets, as long as the assets are priced at a liquidity premium. The reason is that a producer's identity is permanent; so, there is no reason for her to bring money or buy assets with paying a liquidity premium when she knows that she will never have a liquidity need in the DM. Therefore, when entering the CM, a producer will only hold money that she received as payment in the preceding DM. Thus, the value function for a producer is given by

$$W^P(m) = \max_{X,H} \left\{ X - H + \beta\, \mathbb{E}_s \big[V_s^P \big] \right\}$$
$$\text{s.t.} \;\; X = H + \varphi m,$$

where V_s^S denotes a value function of a producer in the DM in state s that will be described later. Notice that the value function does not depend on states of the economy. Using the budget constraint, we can re-write the value function as follows:

$$W^P(m) = \varphi m + \beta \left(\pi V_n^P + (1-\pi) V_d^P \right) \equiv \varphi m + \Lambda^P.$$

Note that all agents have linear value functions in the CM. This is standard in models that build on LW, a result that follows from the (quasi) linear preferences, and it makes the bargaining solution in the DM easy to characterize.

A.1.2 Value functions in the OTC markets

In the OTC markets, C-type agents are selling assets, and N-type agents are buying assets. Let $\Omega_{ij}^s(m, d_A, d_B)$ denote a value function of an agent of type i who decides to enter OTC$_j$ in state s. ξ_j is the amount of money that gets transferred to a C-type, and χ_j the amount of asset j that gets transferred to an N-type in a typical match in OTC$_j$. These terms of trade are described in the next section. The value functions are given by

$$\begin{aligned}
\Omega_{CA}^n(m, d_A, d_B) &= \alpha_{CA}^n V_n(m + \xi_A, d_A - \chi_A, d_B) + (1 - \alpha_{CA}^n) V_n(m, d_A, d_B), \\
\Omega_{CA}^d(m, d_A, d_B) &= \alpha_{CA}^d V_d(m + \xi_A, d_A - \chi_A, d_B) + (1 - \alpha_{CA}^d) V_d(m, d_A, d_B), \\
\Omega_{CB}^n(m, d_A, d_B) &= \alpha_{CB}^n V_n(m + \xi_B, d_A, d_B - \chi_B) + (1 - \alpha_{CB}^n) V_n(m, d_A, d_B), \\
\Omega_{CB}^d(m, d_A, d_B) &= V_d(m, d_A, d_B), \\
\Omega_{NA}^n(m, d_A, d_B) &= \alpha_{NA}^n W_n(m - \xi_A, d_A + \chi_A, d_B) + (1 - \alpha_{NA}^n) W_n(m, d_A, d_B), \\
\Omega_{NA}^d(m, d_A, d_B) &= \alpha_{NA}^d W_d(m - \xi_A, d_A + \chi_A, d_B) + (1 - \alpha_{NA}^d) W_d(m, d_A, d_B), \\
\Omega_{NB}^n(m, d_A, d_B) &= \alpha_{NB}^n W_n(m - \xi_B, d_A, d_B + \chi_B) + (1 - \alpha_{NB}^n) W_n(m, d_A, d_B), \\
\Omega_{NB}^d(m, d_A, d_B) &= W_d(m, d_A, d_B),
\end{aligned} \tag{A.3}$$

where V_s denotes a C-type agent's value function in the DM in state s. Note that OTC$_B$ shuts down when asset B defaults, and thus N-type agents proceed directly to the CM, whereas C-type agents move on to the DM.

A.1.3 Value functions in the DM

In the DM, C-type agents meet producers. Let q denote the quantity of DM goods traded and τ the total payment in units of money. These terms of trade are described in the next section. The value function of an agent who enters the DM with a portfolio (m, d_A, d_B) in state s is given by

$$V_s(m, d_A, d_B) = u(q) + W_s(m - \tau, d_A, d_B), \tag{A.4}$$

The value function of a producer, who enters with no money or assets, is given by

$$V^P = -q + W^P(\tau),$$

which does not depend on states of the economy.

A.2 Terms of trade

A.2.1 Terms of trade in the DM

Consider a meeting between a producer and a C-type agent with a portfolio (m, d_A, d_B). The two parties bargain over a quantity q to be produced by the producer and a cash payment τ to be made by the agent. The agent makes a TIOLI offer maximizing her surplus subject to the producer's participation condition and the cash constraint. The bargaining problem is described as

$$\max_{\tau,\, q} \Big\{ u(q) + W_s(m - \tau, d_A, d_B) - W_s(m, d_A, d_B) \Big\}$$
$$\text{s.t.} \quad -q + W^P(\tau) - W^P(0) = 0, \quad \tau \leq m.$$

Using the linearity of the CM value functions, the C-type agent's surplus becomes $u(q) - \varphi\tau$ and the producer's surplus $-q + \varphi\tau$. This implies that the bargaining solution must satisfy $q(m) = \varphi\tau(m)$—that is, the producer will require $\tau(m)$ units of money for producing $q(m)$ of goods. When the agent has enough money to have the optimal level produced, that is, when $\varphi m \geq q^*$, q^* will be produced. Otherwise, φm will be produced. Define $m^* \equiv q^*/\varphi$ as the amount of money that allows an agent to purchase the first-best quantity, q^*. Then, the solution can be expressed in a concise way:

$$\begin{aligned} q(m) &= \min\{q^*, \varphi m\} \ \big(= \varphi\tau(m)\big), \\ \tau(m) &= \min\{m^*, m\}, \quad m^* \equiv q^*/\varphi. \end{aligned} \tag{A.5}$$

Since an agent will never choose to hold $m > m^*$ due to the cost of carrying money, we will focus on the binding branch of the bargaining solution, $q(m) = \varphi m$ and $\tau(m) = m$.

A.2.2 Terms of trade in the OTC markets

Consider a meeting in OTC$_j$ between a C-type agent with a portfolio (m, d_A, d_B) who wants to sell assets and an N-type agent with $(\tilde{m}, \tilde{d}_A, \tilde{d}_B)$ who wants to buy assets. Let χ_j be the amount of asset j will be traded for ξ_j amount of money as a result of bargaining. The Kalai bargaining applies with the asset seller's bargaining power denoted by θ. Then, the bargaining surplus of an i-type consumer from an OTC$_j$ trading in state s, $\mathcal{S}^s_{ij}$, are given by

$$\begin{aligned}
\mathcal{S}^n_{CA} &= V_n(m+\xi_A, d_A-\chi_A, d_B) - V_n(m, d_A, d_B) = u(\varphi(m+\xi_A)) - u(\varphi m) - \varphi\chi_A,\\
\mathcal{S}^n_{NA} &= W_n(\tilde{m}-\xi_A, \tilde{d}_A+\chi_A, \tilde{d}_B) - W_n(\tilde{m}, \tilde{d}_A, \tilde{d}_B) = -\varphi\xi_A + \varphi\chi_A,\\
\mathcal{S}^d_{CA} &= V_d(m+\xi_A, d_A-\chi_A, d_B) - V_d(m, d_A, d_B) = u(\varphi(m+\xi_A)) - u(\varphi m) - \varphi\chi_A,\\
\mathcal{S}^d_{NA} &= W_d(\tilde{m}-\xi_A, \tilde{d}_A+\chi_A, \tilde{d}_B) - W_d(\tilde{m}, \tilde{d}_A, \tilde{d}_B) = -\varphi\xi_A + \varphi\chi_A,\\
\mathcal{S}^n_{CB} &= V_n(m+\xi_B, d_A, d_B-\chi_B) - V_n(m, d_A, d_B) = u(\varphi(m+\xi_B)) - u(\varphi m) - \varphi\chi_B,\\
\mathcal{S}^n_{NB} &= W_n(\tilde{m}-\xi_B, \tilde{d}_A, \tilde{d}_B+\chi_B) - W_n(\tilde{m}, \tilde{d}_A, \tilde{d}_B) = -\varphi\xi_B + \varphi\chi_B.
\end{aligned}$$

Notice that $\mathcal{S}^n_{CA} = \mathcal{S}^d_{CA}$ and $\mathcal{S}^n_{NA} = \mathcal{S}^d_{NA}$; thus, the solutions will not depend on states of the economy. $\mathcal{S}^d_{CB}$ and $\mathcal{S}^d_{CB}$ are not defined since OTC$_B$ shuts down when asset B defaults. Thus, we will simply write as follows: $\mathcal{S}_{CA}(\equiv \mathcal{S}^n_{CA} = \mathcal{S}^d_{CA})$, $\mathcal{S}_{NA}(\equiv \mathcal{S}^n_{NA} = \mathcal{S}^d_{NA})$, $\mathcal{S}_{CB}(\equiv \mathcal{S}^n_{CB})$, and $\mathcal{S}_{NB}(\equiv \mathcal{S}^n_{NB})$. The expressions for the surpluses can be simplified as follows:

$$\begin{aligned}
\mathcal{S}_{Cj} &= u(\varphi(m+\xi_j)) - u(\varphi m) - \varphi\chi_j,\\
\mathcal{S}_{Nj} &= -\varphi\xi_j + \varphi\chi_j.
\end{aligned}$$

Since money is costly to carry, in equilibrium, C-type agents will bring $m < m^*$ and want to acquire the amount of money that she is missing in order to reach m^*, namely, $m^* - m$. Whether she will be able to acquire that amount of money depends on her asset holdings. If her asset holdings are enough, then she will be able to acquire $m^* - m$ units of money. If not, she will give up all her assets to obtain as much money as possible.

An assumption behind this discussion is that N-type's money holdings never limit the trade. That is, we assume that $m + \tilde{m} \geq m^*$, i.e., that the money holdings of the C-type and the N-type pulled together is enough to allow the C-type to purchase the first best quantity q^*, hence ignoring the constraint $\xi_j \leq \tilde{m}$ in the bargaining problem. This will be true in equilibrium as long as inflation is not too large so that all agents carry at least *half* of the first-best amount of money (see also footnote 12).

Thus, the bargaining problem is described by

$$\max_{\xi_j, \chi_j} \mathcal{S}_{Cj} \quad \text{s.t.} \quad \mathcal{S}_{Cj} = \frac{\theta}{1-\theta}\mathcal{S}_{Nj}, \quad \chi_j \leq d_j.$$

From the Kalai constraint, we get

$$\varphi\chi_j = z(\xi_j) \equiv (1-\theta)\Big(u(\varphi(m+\xi_j)) - u(\varphi m)\Big) + \theta\varphi\xi_j,$$

which says that the asset seller has to give up $z(\xi_j)/\varphi$ amount of asset j to acquire ξ_j amount of money. Note that $z'(\xi_j) > 0$, and recall that the optimal amount of money that the asset seller wants to achieve is $m^* - m$. When the asset seller has enough assets to compensate $m^* - m$, that is, when $\varphi d_j \geq z(m^* - m)$, $m^* - m$ will be traded. Otherwise, d_j will be traded. The solution can be expressed in a concise way:

$$\begin{aligned} &\chi_j(m, d_j) = \min\{d_j^*, d_j\} \big(= z(\xi_j(m, d_j))/\varphi\big), \quad d_j^* \equiv \frac{z(m^* - m)}{\varphi}, \\ &\xi_j(m, d_j) = \min\{m^* - m, \tilde{\xi}_j(m, d_j)\}, \quad \varphi d_j = z(\tilde{\xi}_j). \end{aligned} \tag{A.6}$$

With the discussion above in mind, note that the solution does not depend on the N-type consumer's portfolio, but only on the C-type's. Also, note that $\xi_j(m, d_j)$ is increasing in d_j (the more assets a C-type has, the more money she can acquire) and decreasing in m (the more money a C-type carries, the less she needs to acquire through OTC trade).

A.3 Objective function

As is standard in models that build on LW, all agents choose their optimal portfolio in the CM independently of their trading histories in previous markets. In our model, in addition to choosing an optimal portfolio of money and assets, $(\hat{m}, \hat{d}_A, \hat{d}_B)$, agents also choose which OTC market they will enter in order to sell or buy assets, once the shocks have been realized. To analyze the agent's choice, we substitute the agent's value functions in the OTC markets and the DM (equations (A.3) and (A.4)) into the maximization operator of the CM value function (A.1) and use the linearity of the CM value functions (equation (A.2)), dropping the terms that do not depend on the choice variables, to obtain

$$\begin{aligned} -\varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) &+ \beta\pi\ell\Big(\hat{\varphi}(\hat{d}_A + \hat{d}_B) + u(\hat{\varphi}\hat{m}) + \max\{\alpha^n_{CA}\mathcal{S}_{CA}, \alpha^n_{CB}\mathcal{S}_{CB}\}\Big) \\ &+ \beta\pi(1-\ell)\hat{\varphi}(\hat{m} + \hat{d}_A + \hat{d}_B) \\ &+ \beta(1-\pi)\ell\Big(\hat{\varphi}\hat{d}_A + u(\hat{\varphi}\hat{m}) + \alpha^d_{CA}\mathcal{S}_{CA}\Big) \\ &+ \beta(1-\pi)(1-\ell)\hat{\varphi}(\hat{m} + \hat{d}_A), \end{aligned}$$

from which we finally get the objective function:

$$
\begin{aligned}
J(\hat{m}, \hat{d}_A, \hat{d}_B) \equiv & -\varphi(\hat{m} + p_A \hat{d}_A + p_B \hat{d}_B) + \beta\hat{\varphi}(\hat{m} + \hat{d}_A + \pi \hat{d}_B) \\
& + \beta\ell\Big(u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{m} + \pi \max\{\alpha^n_{CA}\mathcal{S}_{CA}, \alpha^n_{CB}\mathcal{S}_{CB}\} + (1-\pi)\alpha^d_{CA}\mathcal{S}_{CA}\Big) \\
= & -\beta\hat{\varphi} i \hat{m} - \beta\hat{\varphi}(1+i)\left(p_A - \frac{1}{1+i}\right)\hat{d}_A - \beta\hat{\varphi}(1+i)\left(p_B - \frac{\pi}{1+i}\right)\hat{d}_B \\
& + \beta\ell\Big(u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{m} + \pi \max\{\alpha^n_{CA}\mathcal{S}_{CA}, \alpha^n_{CB}\mathcal{S}_{CB}\} + (1-\pi)\alpha^d_{CA}\mathcal{S}_{CA}\Big),
\end{aligned}
\tag{A.7}
$$

with $i \equiv (1+\mu)/\beta - 1$, where

$$
\mathcal{S}_{Cj} = \theta\left[u(\hat{\varphi}(\hat{m} + \xi_j(\hat{m}, \hat{d}_j))) - u(\hat{\varphi}\hat{m}) - \hat{\varphi}\xi_j(\hat{m}, \hat{d}_j)\right].
$$

A.4 Asset demand

Asset demand equations are derived from the first-order conditions of the objective function (A.7) with respect to $\hat{d}_A$ and $\hat{d}_B$:

$$
\begin{aligned}
&\{\hat{d}_A\} \quad (1+i)p_A - 1 = \ell\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)\frac{1}{\hat{\varphi}}\frac{\partial \mathcal{S}_{CA}}{\partial \hat{d}_A}, \\
&\{\hat{d}_B\} \quad (1+i)p_B - \pi = \ell\pi\alpha^n_{CB}\frac{1}{\hat{\varphi}}\frac{\partial \mathcal{S}_{CB}}{\partial \hat{d}_B},
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{\partial \mathcal{S}_{Cj}}{\partial \hat{d}_j} = \frac{\partial \mathcal{S}_{Cj}}{\partial \tilde{\xi}_j}\frac{\partial \tilde{\xi}_j}{\partial \hat{d}_j} &= \theta\left(u'(\hat{\varphi}(\hat{m} + \tilde{\xi}_j)) - 1\right)\hat{\varphi}\frac{\partial \tilde{\xi}_j}{\partial \hat{d}_j}, \\
\hat{\varphi} = z'(\tilde{\xi}_j)\frac{\partial \tilde{\xi}_j}{\partial \hat{d}_j} &= \hat{\varphi}\left(\theta + (1-\theta)u'(\hat{\varphi}(\hat{m} + \tilde{\xi}_j))\right)\frac{\partial \tilde{\xi}_j}{\partial \hat{d}_j},
\end{aligned}
$$

where the second equation is from total differentiation of $\hat{\varphi}\hat{d}_j = z(\tilde{\xi}_j(\hat{m}, \hat{d}_j))$.

From above, we can get the asset demand equations (3) and (4) by expressing in terms of the equilibrium quantities:

$$
\begin{aligned}
(1+i)p_A - 1 &= \ell\theta\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)(u'(q_{1A}) - 1)\frac{1}{\theta + (1-\theta)u'(q_{1A})}, \\
(1+i)p_B - \pi &= \ell\theta\pi\alpha^n_{CB}(u'(q_{1B}) - 1)\frac{1}{\theta + (1-\theta)u'(q_{1B})}.
\end{aligned}
\tag{A.8}
$$

A.5 Money demand

Money demand equations are derived from the first-order conditions of the objective function (A.7) with respect to $\hat{m}$:

$$\{\hat{m}\} \quad i = \ell\left((u'(\hat{\varphi}\hat{m}) - 1) + \left(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\right)\frac{1}{\hat{\varphi}}\frac{\partial \mathcal{S}_{CA}}{\partial \hat{m}}\right),$$
$$i = \ell\left((u'(\hat{\varphi}\hat{m}) - 1) + \pi\alpha^n_{CB}\frac{1}{\hat{\varphi}}\frac{\partial \mathcal{S}_{CB}}{\partial \hat{m}}\right),$$

where

$$\frac{\partial \mathcal{S}_{Cj}}{\partial \hat{m}} = \theta\left(u'(\hat{\varphi}(\hat{m}+\tilde{\xi}_j)) - u'(\hat{\varphi}\hat{m})\right) + \theta\left(u'(\hat{\varphi}(\hat{m}+\tilde{\xi}_j)) - 1\right)\hat{\varphi}\frac{\partial \tilde{\xi}_j}{\partial \hat{m}},$$
$$0 = (1-\theta)\left(u'(\hat{\varphi}(\hat{m}+\tilde{\xi}_j)) - u'(\hat{\varphi}\hat{m})\right) + (1-\theta)u'(\hat{\varphi}(\hat{m}+\tilde{\xi}_j))\frac{\partial \tilde{\xi}_j}{\partial \hat{m}} + \theta\frac{\partial \tilde{\xi}_j}{\partial \hat{m}},$$

where the second equation is from total differentiation of $\hat{\varphi}\hat{d}_j = z(\tilde{\xi}_j(\hat{m}, \hat{d}_j))$.

From above, we can get the money demand equations (5) and (6) by expressing in terms of the equilibrium quantities:

$$\begin{aligned} i &= \ell\Big(1 - \theta\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)\Big)(u'(q_{0A}) - 1) \\ &\quad + \ell\theta\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)(u'(q_{1A}) - 1)\left(1 - \frac{(1-\theta)(u'(q_{1A}) - u'(q_{0A}))}{\theta + (1-\theta)u'(q_{1A})}\right), \\ i &= \ell(1 - \theta\pi\alpha^n_{CB})(u'(q_{0B}) - 1) + \ell\theta\pi\alpha^n_{CB}(u'(q_{1B}) - 1)\left(1 - \frac{(1-\theta)(u'(q_{1B}) - u'(q_{0B}))}{\theta + (1-\theta)u'(q_{1B})}\right). \end{aligned} \tag{A.9}$$

A.6 OTC trade volumes

The OTC trade volumes in the normal state are defined by

$$\Delta^n_A \equiv f(e_C\ell, e^n_N(1-\ell))\cdot\chi_A(m, d_A),$$
$$\Delta^n_B \equiv f((1-e_C)\ell, (1-e^n_N)(1-\ell))\cdot\chi_B(m, d_B),$$

where

$$\chi_j(m, d_j) = \min\{d^*_j, d_j\}, \quad d^*_j \equiv \frac{z(m^* - m)}{\varphi},$$
$$z(\xi) \equiv (1-\theta)\Big(u(\varphi(m+\xi)) - u(\varphi m)\Big) + \theta\varphi\xi,$$
$$d_A = \frac{S_A}{e_C}, \quad d_B = \frac{S_B}{1-e_C}.$$

These reduce as below:

$$\Delta_A^n = e_C \ell\, \alpha_{CA}^n \cdot \min\left\{ \frac{M\Big((1-\theta)(u(q^*) - u(q_{0A})) + \theta(q^* - q_{0A})\Big)}{e_C q_{0A} + (1-e_C) q_{0B}}, \frac{S_A}{e_C} \right\},$$

$$\Delta_B^n = (1-e_C)\ell\, \alpha_{CB}^n \cdot \min\left\{ \frac{M\Big((1-\theta)(u(q^*) - u(q_{0B})) + \theta(q^* - q_{0B})\Big)}{e_C q_{0A} + (1-e_C) q_{0B}}, \frac{S_B}{1-e_C} \right\}.$$

The OTC trade volume of market A in the default state is defined by[28]

$$\begin{aligned}
\Delta_A^d &\equiv f(e_C \ell, 1-\ell) \cdot \chi_A(m, d_A) \\
&= e_C \ell\, \alpha_{CA}^d \cdot \min\left\{ \frac{M\Big((1-\theta)(u(q^*) - u(q_{0A})) + \theta(q^* - q_{0A})\Big)}{e_C q_{0A} + (1-e_C) q_{0B}}, \frac{S_A}{e_C} \right\}.
\end{aligned}$$

Then, the OTC trade volumes, averaged across the normal and default states, are

$$\begin{aligned}
\Delta_A &\equiv \pi \Delta_A^n + (1-\pi)\Delta_A^d, \\
\Delta_B &\equiv \pi \Delta_B^n.
\end{aligned} \tag{A.10}$$

A.7 Proofs

A.7.1 Classification of equilibria

Proof of Proposition 1. (a) Assume $e_C = 0$. Then, $e_N^n = e_N^d = 0$ is clearly the best response of N-types. We claim that there is no profitable deviation of a C-type, i.e., $G(e_C) \equiv \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB} < 0$ when $e_C = 0$, $e_N^n = e_N^d = 0$. First, notice that when $e_C = 0$, that is, when nobody is holding asset A, $q_{0A} = q_{1A}\, (\equiv \bar{q})$ and $\tilde{\mathcal{S}}_{CA} = -i\bar{q} + \ell(u(\bar{q}) - \bar{q})$. If it were the case that nobody was purchasing asset B either, then $q_{0B} = q_{1B}\, (= \bar{q})$, $\tilde{\mathcal{S}}_{CB} = -i\bar{q} + \ell(u(\bar{q}) - \bar{q})$ and $G(e_C) = 0$. Since q_{1B} increases and q_{0B} decreases as agents hold asset B, it remains to show that $d\tilde{\mathcal{S}}_{CB}/dq_{1B} > 0$, keeping in mind that q_{0B} also changes as q_{1B} changes.

When $e_C = 0$ and $e_N^n = e_N^d = 0$, $\alpha_{CB}^n = 1 - \ell$:

$$\begin{aligned}
\tilde{\mathcal{S}}_{CB} = &-iq_{0B} - \left(\ell \frac{\theta}{\omega_\theta(q_{1B})} \pi(1-\ell)(u'(q_{1B}) - 1)\right)\Big((1-\theta)(u(q_{1B}) - u(q_{0B})) + \theta(q_{1B} - q_{0B})\Big) \\
&+ \ell(u(q_{0B}) - q_{0B}) + \ell\pi(1-\ell)\theta\Big(u(q_{1B}) - u(q_{0B}) - q_{1B} + q_{0B}\Big).
\end{aligned}$$

Then,

$$\begin{aligned}
\frac{d\tilde{\mathcal{S}}_{CB}}{dq_{1B}} = &-\frac{dq_{0B}}{dq_{1B}} \left(i - \frac{\ell((1-(1-\ell)\pi)\theta + (1-\theta)u'(q_{1B}))u'(q_{0B})}{\theta + (1-\theta)u'(q_{1B})} + \frac{\ell(\theta + (1-(1+(1-\ell)\pi)\theta)u'(q_{1B}))}{\theta + (1-\theta)u'(q_{1B})} \right) \\
&- \frac{1}{(\theta + (1-\theta)u'(q_{1B}))^2}(1-\ell)\ell\pi\theta\Big((1-\theta)(u(q_{1B}) - u(q_{0B})) + \theta(q_{1B} - q_{0B})\Big)u''(q_{1B}).
\end{aligned}$$

[28] The OTC trade volume of market B in the default state is 0, since OTC_B shuts down.

The coefficient of dq_{0B}/dq_{1B} in the first term is

$$\begin{aligned}
& i - \frac{\ell((1-(1-\ell)\pi)\theta + (1-\theta)u'(q_{1B}))u'(q_{0B})}{\theta + (1-\theta)u'(q_{1B})} + \frac{\ell(\theta + (1-(1+(1-\ell)\pi)\theta)u'(q_{1B}))}{\theta + (1-\theta)u'(q_{1B})} \\
&= i - \left(\frac{\ell((1-(1-\ell)\pi)\theta + (1-\theta)u'(q_{1B}))u'(q_{0B})}{\theta + (1-\theta)u'(q_{1B})} - \frac{\ell((1-(1-\ell)\pi)\theta + (1-\theta)u'(q_{1B}))}{\theta + (1-\theta)u'(q_{1B})} \right) \\
&\quad - \left(\frac{\ell((1-(1-\ell)\pi)\theta + (1-\theta)u'(q_{1B}))}{\theta + (1-\theta)u'(q_{1B})} - \frac{\ell(\theta + (1-(1+(1-\ell)\pi)\theta)u'(q_{1B}))}{\theta + (1-\theta)u'(q_{1B})} \right) \\
&= i - \ell \left(1 - \frac{\theta}{\theta + (1-\theta)u'(q_{1B})}\pi(1-\ell) \right) (u'(q_{0B}) - 1) - \ell \frac{\theta}{\theta + (1-\theta)u'(q_{1B})}\pi(1-\ell)(u'(q_{1B}) - 1),
\end{aligned}$$

which is 0 since it is equivalent to the money demand equation (6). Thus,

$$\frac{d\tilde{\mathcal{S}}_{CB}}{dq_{1B}} = -\frac{1}{(\theta + (1-\theta)u'(q_{1B}))^2}(1-\ell)\ell\pi\theta\Big((1-\theta)(u(q_{1B}) - u(q_{0B})) + \theta(q_{1B} - q_{0B})\Big)u''(q_{1B}) > 0. \tag{A.11}$$

Therefore, $G(e_C) < 0$ when $e_C = 0$, $e_N^n = e_N^d = 0$. □

(b) Assume $e_C = 1$. Then, $e_N^n = e_N^d = 1$ is clearly the best response of N-types. We claim that there is no profitable deviation of a C-type, i.e., $G(e_C) \equiv \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB} > 0$ when $e_C = 1$, $e_N^n = e_N^d = 1$. First, notice that when $e_C = 1$, that is, when nobody is holding asset B, $q_{0B} = q_{1B}$ ($\equiv \bar{q}$) and $\tilde{\mathcal{S}}_{CB} = -i\bar{q} + \ell(u(\bar{q}) - \bar{q})$. If it were the case that nobody was purchasing asset A either, then $q_{0A} = q_{1A}$ ($= \bar{q}$), $\tilde{\mathcal{S}}_{CA} = -i\bar{q} + \ell(u(\bar{q}) - \bar{q})$ and $G(e_C) = 0$. Since q_{1A} increases and q_{0A} decreases as agents hold asset A, it remains to show that $d\tilde{\mathcal{S}}_{CA}/dq_{1A} > 0$, keeping in mind that q_{0A} also changes as q_{1A} changes.

When $e_C = 1$ and $e_N^n = e_N^d = 1$, $\alpha_{CA}^n = \alpha_{CA}^d = 1 - \ell$:

$$\begin{aligned}
\tilde{\mathcal{S}}_{CA} = & -iq_{0A} - \left(\ell \frac{\theta}{\omega_\theta(q_{1A})}(1-\ell)(u'(q_{1A}) - 1) \right)\Big((1-\theta)(u(q_{1A}) - u(q_{0A})) + \theta(q_{1A} - q_{0A})\Big) \\
& + \ell(u(q_{0A}) - q_{0A}) + \ell(1-\ell)\theta\Big(u(q_{1A}) - u(q_{0A}) - q_{1A} + q_{0A}\Big).
\end{aligned}$$

Then,

$$\begin{aligned}
\frac{d\tilde{\mathcal{S}}_{CA}}{dq_{1A}} = & -\frac{dq_{0A}}{dq_{1A}} \left(i - \frac{\ell(\ell\theta + (1-\theta)u'(q_{1A}))u'(q_{0A})}{\theta + (1-\theta)u'(q_{1A})} + \frac{\ell(\theta + (1-(2-\ell)\theta)u'(q_{1A}))}{\theta + (1-\theta)u'(q_{1A})} \right) \\
& - \frac{1}{(\theta + (1-\theta)u'(q_{1A}))^2}(1-\ell)\ell\theta\Big((1-\theta)(u(q_{1A}) - u(q_{0A})) + \theta(q_{1A} - q_{0A})\Big)u''(q_{1A}).
\end{aligned}$$

The coefficient of dq_{0A}/dq_{1A} in the first term is

$$
\begin{aligned}
& i - \frac{\ell(\ell\theta + (1-\theta)u'(q_{1A}))u'(q_{0A})}{\theta + (1-\theta)u'(q_{1A})} + \frac{\ell(\theta + (1-(2-\ell)\theta)u'(q_{1A}))}{\theta + (1-\theta)u'(q_{1A})} \\
&= i - \left(\frac{\ell(\ell\theta + (1-\theta)u'(q_{1A}))u'(q_{0A})}{\theta + (1-\theta)u'(q_{1A})} - \frac{\ell(\ell\theta + (1-\theta)u'(q_{1A}))}{\theta + (1-\theta)u'(q_{1A})}\right) \\
&\quad - \left(\frac{\ell(\ell\theta + (1-\theta)u'(q_{1A}))}{\theta + (1-\theta)u'(q_{1A})} - \frac{\ell(\theta + (1-(2-\ell)\theta)u'(q_{1A}))}{\theta + (1-\theta)u'(q_{1A})}\right) \\
&= i - \ell\left(1 - \frac{\theta}{\theta + (1-\theta)u'(q_{1A})}(1-\ell)\right)(u'(q_{0A}) - 1) - \ell\frac{\theta}{\theta + (1-\theta)u'(q_{1A})}(1-\ell)(u'(q_{1A}) - 1),
\end{aligned}
$$

which is 0 since it is equivalent to the money demand equation (5). Thus,

$$
\frac{d\tilde{\mathcal{S}}_{CA}}{dq_{1A}} = -\frac{1}{(\theta + (1-\theta)u'(q_{1A}))^2}(1-\ell)\ell\theta\Big((1-\theta)(u(q_{1A}) - u(q_{0A})) + \theta(q_{1A} - q_{0A})\Big)u''(q_{1A}) > 0. \tag{A.12}
$$

Therefore, $G(e_C) > 0$ when $e_C = 1$, $e_N^n = e_N^d = 1$. □

Proof. (c) First observe the value of α_{CA}^n, α_{CA}^d and α_{CB}^n as $e_C \to 0+$. While $\alpha_{CA}^n = \alpha_{CA}^d = 0$ at $e_C = 0$, this is not the case when $e_C \to 0+$. From the optimal entry decision by N-types (10),

$$
e_N^n = \frac{e_C(1 - e_C\ell - \ell S_{NB}/S_{NA} + e_C\ell S_{NB}/S_{NA}}{-(1-\ell)(-e_C - S_{NB}/S_{NA} + e_C S_{NB}/S_{NA})}.
$$

Using this, as $e_C \to 0+$,

$$
\begin{aligned}
\alpha_{CA}^n &\to \lim_{e_C \to 0+} \frac{(1-\ell)e_N^n}{\ell e_C + (1-\ell)e_N^n} = \lim_{e_C \to 0+} \frac{(1-\ell)\frac{e_C(1-e_C\ell-\ell S_{NB}/S_{NA}+e_C\ell S_{NB}/S_{NA}}{-(1-\ell)(-e_C-S_{NB}/S_{NA}+e_C S_{NB}/S_{NA})}}{\ell e_C + (1-\ell)\frac{e_C(1-e_C\ell-\ell S_{NB}/S_{NA}+e_C\ell S_{NB}/S_{NA}}{-(1-\ell)(-e_C-S_{NB}/S_{NA}+e_C S_{NB}/S_{NA})}} \\
&= \lim_{e_C \to 0+} 1 + e_C\ell(-1 + \frac{S_{NB}}{S_{NA}}) - \ell\frac{S_{NB}}{S_{NA}} = 1 - \ell\frac{S_{NB}}{S_{NA}} \; (> 1-\ell), \\
\alpha_{CA}^d &\to \lim_{e_C \to 0+} \frac{1-\ell}{\ell e_C + (1-\ell)} = 1.
\end{aligned}
$$

On the other hand, α_{CB}^n continuously converges to its value, $1-\ell$, at $e_C = 0$ as $e_C \to 0+$.

Hence, as $e_C \to 0+$, $\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA} > \pi\alpha^n_{CB}$. Therefore,

$$\begin{aligned}
\tilde{\mathcal{S}}_{CA} =& -iq_{0A} - \Big(\ell\frac{\theta}{\omega_\theta(q_{1A})}\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)(u'(q_{1A})-1)\Big)\Big((1-\theta)(u(q_{1A})-u(q_{0A})) + \theta(q_{1A}-q_{0A})\Big) \\
&+ \ell(u(q_{0A})-q_{0A}) + \ell\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)\theta\Big(u(q_{1A})-u(q_{0A})-q_{1A}+q_{0A}\Big) \\
>& -iq_{0A} - \Big(\ell\frac{\theta}{\omega_\theta(q_{1A})}\pi\alpha^n_{CB}(u'(q_{1A})-1)\Big)\Big((1-\theta)(u(q_{1A})-u(q_{0A})) + \theta(q_{1A}-q_{0A})\Big) \\
&+ \ell(u(q_{0A})-q_{0A}) + \ell\pi\alpha^n_{CB}\theta\Big(u(q_{1A})-u(q_{0A})-q_{1A}+q_{0A}\Big) \\
\geq& -iq_{0B} - \Big(\ell\frac{\theta}{\omega_\theta(q_{1B})}\pi\alpha^n_{CB}(u'(q_{1B})-1)\Big)\Big((1-\theta)(u(q_{1B})-u(q_{0B})) + \theta(q_{1B}-q_{0B})\Big) \\
&+ \ell(u(q_{0B})-q_{0B}) + \ell\pi\alpha^n_{CB}\theta\Big(u(q_{1B})-u(q_{0B})-q_{1B}+q_{0B}\Big) = \tilde{\mathcal{S}}_{CB},
\end{aligned}$$

where the first inequality comes from

$$\begin{aligned}
&\Big[-iq_{0A} - \Big(\ell\frac{\theta}{\omega_\theta(q_{1A})}\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)(u'(q_{1A})-1)\Big)\Big((1-\theta)(u(q_{1A})-u(q_{0A})) + \theta(q_{1A}-q_{0A})\Big) \\
&\quad + \ell(u(q_{0A})-q_{0A}) + \ell\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)\theta\Big(u(q_{1A})-u(q_{0A})-q_{1A}+q_{0A}\Big)\Big] \\
&-\Big[-iq_{0A} - \Big(\ell\frac{\theta}{\omega_\theta(q_{1A})}\pi\alpha^n_{CB}(u'(q_{1A})-1)\Big)\Big((1-\theta)(u(q_{1A})-u(q_{0A})) + \theta(q_{1A}-q_{0A})\Big) \\
&\quad + \ell(u(q_{0A})-q_{0A}) + \ell\pi\alpha^n_{CB}\theta\Big(u(q_{1A})-u(q_{0A})-q_{1A}+q_{0A}\Big)\Big] \\
=\;& \frac{\ell[(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}) - \pi\alpha^n_{CB}]\theta(q_{1A}-q_{0A})}{\theta + (1-\theta)u'(q_{1A})}\Big(\frac{u(q_{1A})-u(q_{0A})}{q_{1A}-q_{0A}} - u'(q_{1A})\Big) > 0,
\end{aligned}$$

in which we used $\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA} > \pi\alpha^n_{CB}$ and the strict concavity of u; and the second inequality comes from that $\lim_{e_C\to 0+} q_{1B} \leq \lim_{e_C\to 0+} q_{1A} = q^*$ and (A.11). Therefore, $G(e_C) > 0$ as $e_C \to 0+$. □

(d) All we need to show is that one of α^n_{CA}, α^d_{CA} and α^n_{CB} is discontinuous at $e_C = 1$. Here, the discontinuity arises in α^n_{CB}. From the optimal entry decision by N-types (10),

$$e^n_N = \frac{e_C(1 - e_C\ell - \ell S_{NB}/S_{NA} + e_C\ell S_{NB}/S_{NA}}{-(1-\ell)(-e_C - S_{NB}/S_{NA} + e_C S_{NB}/S_{NA})}.$$

Using this, as $e_C \to 1$,

$$\begin{aligned}
\alpha^n_{CB} \to& \lim_{e_C\to 1}\frac{(1-\ell)(1-e^n_N)}{\ell(1-e_C) + (1-\ell)(1-e^n_N)} = \lim_{e_C\to 1}\frac{(1-\ell)(1 - \frac{e_C(1-e_C\ell-\ell S_{NB}/S_{NA}+e_C\ell S_{NB}/S_{NA}}{-(1-\ell)(-e_C-S_{NB}/S_{NA}+e_C S_{NB}/S_{NA})})}{\ell(1-e_C) + (1-\ell)(1 - \frac{e_C(1-e_C\ell-\ell S_{NB}/S_{NA}+e_C\ell S_{NB}/S_{NA}}{-(1-\ell)(-e_C-S_{NB}/S_{NA}+e_C S_{NB}/S_{NA})})} \\
=& \lim_{e_C\to 1} 1 + \ell(-1 + e_C - e_C\frac{S_{NA}}{S_{NB}}) = 1 - \ell\frac{S_{NA}}{S_{NB}}\;(> 1-\ell).
\end{aligned}$$

On the other hand, $\alpha^n_{CB} = 0$ at $e_C = 1$.

Now assume $\pi \to 1$. Unlike α^n_{CB}, α^n_{CA} and α^d_{CA} continuously converge to their values at $e_C = 1$, which are both $1-\ell$. Hence, as $e_C \to 1$ and $\pi \to 1$, $\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA} < \pi\alpha^n_{CB}$. Therefore,

$$\begin{aligned}
\tilde{\mathcal{S}}_{CB} = &- iq_{0B} - \left(\ell\frac{\theta}{\omega_\theta(q_{1B})}\pi\alpha^n_{CB}(u'(q_{1B})-1)\right)\Big((1-\theta)(u(q_{1B})-u(q_{0B}))+\theta(q_{1B}-q_{0B})\Big)\\
&+\ell(u(q_{0B})-q_{0B}) + \ell\pi\alpha^n_{CB}\theta\Big(u(q_{1B})-u(q_{0B})-q_{1B}+q_{0B}\Big)\\
> &- iq_{0B} - \left(\ell\frac{\theta}{\omega_\theta(q_{1B})}\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)(u'(q_{1B})-1)\right)\Big((1-\theta)(u(q_{1B})-u(q_{0B}))+\theta(q_{1B}-q_{0B})\Big)\\
&+\ell(u(q_{0B})-q_{0B}) + \ell\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)\theta\Big(u(q_{1B})-u(q_{0B})-q_{1B}+q_{0B}\Big)\\
\geq &- iq_{0A} - \left(\ell\frac{\theta}{\omega_\theta(q_{1A})}\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)(u'(q_{1A})-1)\right)\Big((1-\theta)(u(q_{1A})-u(q_{0A}))+\theta(q_{1A}-q_{0A})\Big)\\
&+\ell(u(q_{0A})-q_{0A}) + \ell\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)\theta\Big(u(q_{1A})-u(q_{0A})-q_{1A}+q_{0A}\Big) = \tilde{\mathcal{S}}_{CA},
\end{aligned}$$

where the first inequality comes from

$$\begin{aligned}
&\Big[-iq_{0B} - \left(\ell\frac{\theta}{\omega_\theta(q_{1B})}\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)(u'(q_{1B})-1)\right)\Big((1-\theta)(u(q_{1B})-u(q_{0B}))+\theta(q_{1B}-q_{0B})\Big)\\
&\quad+\ell(u(q_{0B})-q_{0B}) + \ell\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)\theta\Big(u(q_{1B})-u(q_{0B})-q_{1B}+q_{0B}\Big)\Big]\\
&-\Big[-iq_{0B} - \left(\ell\frac{\theta}{\omega_\theta(q_{1B})}\pi\alpha^n_{CB}(u'(q_{1B})-1)\right)\Big((1-\theta)(u(q_{1B})-u(q_{0B}))+\theta(q_{1B}-q_{0B})\Big)\\
&\quad+\ell(u(q_{0B})-q_{0B}) + \ell\pi\alpha^n_{CB}\theta\Big(u(q_{1B})-u(q_{0B})-q_{1B}+q_{0B}\Big)\Big]\\
= &\;\frac{\ell[(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA})-\pi\alpha^n_{CB}]\theta(q_{1B}-q_{0B})}{\theta+(1-\theta)u'(q_{1B})}\left(\frac{u(q_{1B})-u(q_{0B})}{q_{1B}-q_{0B}} - u'(q_{1B})\right) < 0,
\end{aligned}$$

in which we used $\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA} < \pi\alpha^n_{CB}$ and the strict concavity of u; and the second inequality comes from that $\lim_{e_C\to 1} q_{1A} \leq \lim_{e_C \to 1} q_{1B} = q^*$ and (A.12). Therefore, $G(e_C) < 0$ as $e_C \to 1$. □

(e) From (c) and (d), we have $\lim_{e_C\to 0+} G(e_C) > 0 > \lim_{e_C\to 1} G(e_C)$ when $\pi \to 1$. The continuity of G immediately implies that there exists at least one robust interior equilibrium. □

(f) All we need to show is that one of α^n_{CA}, α^d_{CA} and α^n_{CB} is discontinuous at $e_C = 1$. Here, the discontinuity arises in α^d_{CA}. As $e_C \to 0+$,

$$\alpha^d_{CA} \to \lim_{e_C \to 0+} \frac{1-\ell}{(\ell e_C + (1-\ell))^{1-\rho}} = (1-\ell)^\rho.$$

On the other hand, $\alpha^d_{CA} = 0$ at $e_C = 0$. $\square$

(g) All we need to show is that all α^n_{CA}, α^d_{CA} and α^n_{CB} continuously converge to their values at $e_C = 1$ as $e_C \to 1$. From the optimal entry decision by N-types (10),

$$e^n_N = \frac{-1 + e_C\ell + e_C\ell\left(\frac{(1-e_C)S_{NB}/S_{NA}}{e_C}\right)^{\frac{1}{1-\rho}}}{-(1-\ell)\left(1 + \left(\frac{(1-e_C)S_{NB}/S_{NA}}{e_C}\right)^{\frac{1}{1-\rho}}\right)}.$$

Then, as $e_C \to 1$,

$$\begin{aligned}
\alpha^n_{CA} &= \frac{(1-\ell)e^n_N}{(\ell e_C + (1-\ell)e^n_N)^{1-\rho}} \\
&= \left(\frac{1}{1 + \left(\left(-1+\frac{1}{e_C}\right)\frac{S_{NB}}{S_{NA}}\right)^{\frac{1}{1-\rho}}}\right)^{\rho}\left(1 - e_C\ell\left(1 + \left(\left(-1+\frac{1}{e_C}\right)\frac{S_{NB}}{S_{NA}}\right)^{\frac{1}{1-\rho}}\right)\right) \to 1-\ell \\
\alpha^d_{CA} &= \frac{1-\ell}{(\ell e_C + (1-\ell))^{1-\rho}} \to 1-\ell \\
\alpha^n_{CB} &= \frac{(1-\ell)(1-e^n_N)}{(\ell(1-e_C) + (1-\ell)(1-e^n_N))^{1-\rho}} \\
&= \left(\frac{1}{1 + \left(\left(-1+\frac{1}{e_C}\right)\frac{S_{NB}}{S_{NA}}\right)^{-\frac{1}{1-\rho}}}\right)^{\rho}\left(1 - (1-e_C)\ell\left(1 + \left(\left(-1+\frac{1}{e_C}\right)\frac{S_{NB}}{S_{NA}}\right)^{-\frac{1}{1-\rho}}\right)\right) \to 0,
\end{aligned}$$

and, at $e_C = e^n_N = 1$,

$$\begin{aligned}
\alpha^n_{CA} &= \frac{(1-\ell)e^n_N}{(\ell e_C + (1-\ell)e^n_N)^{1-\rho}} = 1-\ell \\
\alpha^d_{CA} &= \frac{1-\ell}{(\ell e_C + (1-\ell))^{1-\rho}} = 1-\ell \\
\alpha^n_{CB} &= \frac{(1-\ell)(1-e^n_N)}{(\ell(1-e_C) + (1-\ell)(1-e^n_N))^{1-\rho}} = 0.
\end{aligned}$$

Therefore, α^n_{CA}, α^d_{CA} and α^n_{CB} continuously converge to their values at $e_C = 1$ as $e_C \to 1$, and $G(e_C)$ also continuously converges to its value at $e_C = 1$, which is greater than 0, as $e_C \to 1$. $\square$

A.7.2 Safer implies more liquid

Proof of Proposition 2.

(a) Guess-and-verify: at $\pi = 1$, all the equilibrium equations are symmetric between the

A and B markets, so $q_{0A} = q_{0B}$, $q_{1A} = q_{1B}$, and $L_A = L_B$ are satisfied. And $e_C = e_N = 0.5$ implies $\alpha_{CA} = \alpha_{CB}$ as well as $\alpha_{NA} = \alpha_{NB}$, so symmetry is complete.

(b) In any interior equilibrium where $e_C \in (0,1)$ and both assets are scarce and valued for liquidity so that $q_{1A} < q^*$, $q_{1B} < q^*$, we can totally differentiate the equilibrium equations around the scarce-interior equilibrium:

Post-trade quantities (Equations 7–8)

$$q_{1A} = \min\left\{ q^*,\, q_{0A} + \frac{\frac{S_A}{M}\frac{e_C q_{0A} + (1-e_C)q_{0B}}{e_C} - (1-\theta)\Big(u(q_{1A}) - u(q_{0A})\Big)}{\theta} \right\}$$

$$q_{1B} = \min\left\{ q^*,\, q_{0B} + \frac{\frac{S_B}{M}\frac{e_C q_{0A} + (1-e_C)q_{0B}}{1-e_C} - (1-\theta)\Big(u(q_{1B}) - u(q_{0B})\Big)}{\theta} \right\}$$

Focusing on the scarce branch, total differentiate yields

$$\frac{w_\theta(q_{1A})}{\theta}\, dq_{1A} = \frac{S_A/M + w_\theta(q_{0A})}{\theta}\, dq_{0A} + \frac{S_A}{M}\frac{1-e_C}{e_C\theta}\, dq_{0B} - \frac{S_A}{M}\frac{q_{0B}}{e_C^2\theta}\, de_C \tag{A.13}$$

$$\frac{w_\theta(q_{1B})}{\theta}\, dq_{1B} = \frac{S_B/M + w_\theta(q_{0B})}{\theta}\, dq_{0B} + \frac{S_B}{M}\frac{e_C}{(1-e_C)\theta}\, dq_{0A} + \frac{S_B}{M}\frac{q_{0A}}{(1-e_C)^2\theta}\, de_C \tag{A.14}$$

Money demand (Equations 5–6)

$$i = \ell(1-\theta\bar\alpha_{Cj})(u'(q_{0j}) - 1) + \ell\theta\frac{w_\theta(q_{0j})}{w_\theta(q_{1j})}\bar\alpha_{Cj}(u'(q_{1j}) - 1),$$

which is equivalent to

$$i = \ell\left(1 - \frac{\theta}{w_\theta(q_{1j})}\bar\alpha_{Cj}\right)(u'(q_{0j}) - 1) + \ell\frac{\theta}{w_\theta(q_{1j})}\bar\alpha_{Cj}(u'(q_{1j}) - 1), \quad j = A, B$$

where

$$\begin{aligned} w_\theta(q) &\equiv \theta + (1-\theta)u'(q) \\ \bar\alpha_{CA} &\equiv \pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA} \\ \bar\alpha_{CB} &\equiv \pi\alpha^n_{CB}. \end{aligned}$$

Total differentiation yields

$$\begin{aligned} 0 = {}& \ell\left(1 - \frac{\theta}{w_\theta(q_{1j})}\bar\alpha_{Cj}\right)u''(q_{0j})\, dq_{0j} + \ell\frac{\theta}{w_\theta(q_{1j})^2}\bar\alpha_{Cj}\Big(u''(q_{1j})w_\theta(q_{1j}) - (u'(q_{1j}) - u'(q_{0j}))w'_\theta(q_{1j})\Big)dq_{1j} \\ & + \ell\frac{\theta}{w_\theta(q_{1j})}(u'(q_{1j}) - u'(q_{0j}))\, d\bar\alpha_{Cj} \end{aligned} \tag{A.15}$$

Liquidity premium

Define a new variable:

$$\bar{L}_j \equiv \ell \frac{\theta}{w_\theta(q_{1j})} \bar{\alpha}_{Cj}(u'(q_{1j}) - 1), \quad j = A, B$$

where $\bar{L}_A = L_A = (1+i)p_A - 1$ and $\bar{L}_B = \pi L_B = (1+i)p_B - \pi$. Total differentiation yields

$$d\bar{L}_j = \ell \frac{\theta}{w_\theta(q_{1j})^2} \bar{\alpha}_{Cj}\Big(u''(q_{1j})w_\theta(q_{1j}) - (u'(q_{1j}) - 1)w'_\theta(q_{1j})\Big) dq_{1j} + \ell \frac{\theta}{w_\theta(q_{1j})}(u'(q_{1j}) - 1)\, d\bar{\alpha}_{Cj}, \quad j = A, B$$

C's entry choice (Equations following Equation 9)

$$\begin{aligned}
\mathcal{S}_{Cj} &= \theta\Big(u(q_{1j}) - u(q_{0j}) - q_{1j} + q_{0j}\Big) \\
\tilde{\mathcal{S}}_{Cj} &= -iq_{0j} - \bar{L}_j\Big((1-\theta)(u(q_{1j}) - u(q_{0j})) + \theta(q_{1j} - q_{0j})\Big) + \ell(u(q_{0j}) - q_{0j}) + \ell\bar{\alpha}_{Cj}\mathcal{S}_{Cj}
\end{aligned}$$

Total differentiation yields

$$\begin{aligned}
d\mathcal{S}_{Cj} &= \theta(u'(q_{1j}) - 1)\, dq_{1j} - \theta(u'(q_{0j}) - 1)\, dq_{0j} \\
d\tilde{\mathcal{S}}_{Cj} &= -\Big((1-\theta)(u(q_{1j}) - u(q_{0j})) + \theta(q_{1j} - q_{0j})\Big)d\bar{L}_j + \ell\mathcal{S}_{Cj}\, d\bar{\alpha}_{Cj} \\
&\quad + \Big(-i + \bar{L}_j w_\theta(q_{0j}) + \ell(u'(q_{0j}) - 1)\Big)dq_{0j} - \bar{L}_j w_\theta(q_{1j})\, dq_{1j} + \ell\bar{\alpha}_{Cj}\, d\mathcal{S}_{Cj}
\end{aligned}$$

where

$$\begin{aligned}
&\Big(-i + \bar{L}_j w_\theta(q_{0j}) + \ell(u'(q_{0j}) - 1)\Big)dq_{0j} - \bar{L}_j w_\theta(q_{1j})\, dq_{1j} + \ell\bar{\alpha}_{Cj}\, d\mathcal{S}_{Cj} \\
&= \Big(-i + \bar{L}_j w_\theta(q_{0j}) + \ell(u'(q_{0j}) - 1) - \ell\bar{\alpha}_{Cj}\theta(u'(q_{0j}) - 1)\Big)dq_{0j} + \Big(-\bar{L}_j w_\theta(q_{1j}) + \ell\bar{\alpha}_{Cj}\theta(u'(q_{1j}) - 1)\Big)dq_{1j} \\
&= 0
\end{aligned}$$

since the coefficient of dq_{0j} is equivalent to the first-version money demand. Thus,

$$d\tilde{\mathcal{S}}_{Cj} = -\Big((1-\theta)(u(q_{1j}) - u(q_{0j})) + \theta(q_{1j} - q_{0j})\Big)d\bar{L}_j + \ell\mathcal{S}_{Cj}\, d\bar{\alpha}_{Cj}$$

Therefore, we have

$$G(e_C) = \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB}$$

and total differentiation yields

$$\begin{aligned}
dG = d\tilde{\mathcal{S}}_{CA} - d\tilde{\mathcal{S}}_{CB} &= -\Big((1-\theta)(u(q_{1A}) - u(q_{0A})) + \theta(q_{1A} - q_{0A})\Big)d\bar{L}_A + \ell\mathcal{S}_{CA}\, d\bar{\alpha}_{CA} \\
&\quad + \Big((1-\theta)(u(q_{1B}) - u(q_{0B})) + \theta(q_{1B} - q_{0B})\Big)d\bar{L}_B - \ell\mathcal{S}_{CB}\, d\bar{\alpha}_{CB}
\end{aligned} \tag{A.16}$$

N's entry choice (Equations following Equation 10)

$$\alpha^n_{NA}\mathcal{S}_{NA} = \alpha^n_{NB}\mathcal{S}_{NB}$$
$$\mathcal{S}_{Nj} = (1-\theta)\Big(u(q_{1j}) - u(q_{0j}) - q_{1j} + q_{0j}\Big)$$

Total differentiation yields

$$\mathcal{S}_{NA}\,d\alpha^n_{NA} + \alpha^n_{NA}\,d\mathcal{S}_{NA} = \mathcal{S}_{NB}\,d\alpha^n_{NB} + \alpha^n_{NB}\,d\mathcal{S}_{NB}$$
$$d\mathcal{S}_{Nj} = (1-\theta)(u'(q_{1j})-1)\,dq_{1j} - (1-\theta)(u'(q_{0j})-1)\,dq_{0j}$$

Thus,

$$\begin{aligned} &\mathcal{S}_{NA}\,d\alpha^n_{NA} + \alpha^n_{NA}(1-\theta)(u'(q_{1A})-1)\,dq_{1A} - \alpha^n_{NA}(1-\theta)(u'(q_{0A})-1)\,dq_{0A} \\ &= \mathcal{S}_{NB}\,d\alpha^n_{NB} + \alpha^n_{NB}(1-\theta)(u'(q_{1B})-1)\,dq_{1B} - \alpha^n_{NB}(1-\theta)(u'(q_{0B})-1)\,dq_{0B} \end{aligned} \tag{A.17}$$

Matching probabilities (Section 3.2)

$$\begin{aligned} \alpha^n_{CA} &= e^n_N(1-\ell)\,[e^n_N(1-\ell)+e_C\ell]^{\rho-1} \\ \alpha^n_{CB} &= (1-e^n_N)(1-\ell)\,[(1-e^n_N)(1-\ell)+(1-e_C)\ell]^{\rho-1} \\ \alpha^n_{NA} &= e_C\ell\,[e^n_N(1-\ell)+e_C\ell]^{\rho-1} \\ \alpha^n_{NB} &= (1-e_C)\ell\,[(1-e^n_N)(1-\ell)+(1-e_C)\ell]^{\rho-1} \\ \alpha^d_{CA} &= (1-\ell)\,[(1-\ell)+e_C\ell]^{\rho-1} \\ \alpha^d_{NA} &= e_C\ell\,[(1-\ell)+e_C\ell]^{\rho-1} \\ \alpha^d_{CB} &= \alpha^d_{NB} = 0 \end{aligned}$$

Total differentiation yields

$$\begin{aligned} d\alpha^n_{CA} =& -(1-\rho)\ell(1-\ell)e^n_N\,[e^n_N(1-\ell)+e_C\ell]^{\rho-2}\,de_C \\ &+\left[\frac{\alpha^n_{CA}}{e_N} - (1-\rho)(1-\ell)^2 e^n_N\,[e^n_N(1-\ell)+e_C\ell]^{\rho-2}\right] de^n_N \\ d\alpha^n_{CB} =& (1-\rho)\ell(1-\ell)(1-e^n_N)\,[(1-e^n_N)(1-\ell)+(1-e_C)\ell]^{\rho-2}\,de_C \\ &-\left[\frac{\alpha^n_{CB}}{1-e^n_N} - (1-\rho)(1-\ell)^2(1-e^n_N)\,[(1-e^n_N)(1-\ell)+(1-e_C)\ell]^{\rho-2}\right] de^n_N \\ d\alpha^n_{NA} =& \left[\frac{\alpha^n_{NA}}{e_C} - (1-\rho)\ell^2 e_C\,[e^n_N(1-\ell)+e_C\ell]^{\rho-2}\right] de_C \\ &-(1-\rho)(1-\ell)\ell e_C\,[e^n_N(1-\ell)+e_C\ell]^{\rho-2}\,de^n_N \\ d\alpha^n_{NB} =& -\left[\frac{\alpha^n_{NB}}{1-e_C} - (1-\rho)\ell^2(1-e_C)\,[(1-e^n_N)(1-\ell)+(1-e_C)\ell]^{\rho-2}\right] de_C \end{aligned}$$

$$
\begin{aligned}
&\quad + (1-\rho)(1-\ell)\ell(1-e_C)\left[(1-e_N^n)(1-\ell) + (1-e_C)\ell\right]^{\rho-2} de_N^n \\
d\alpha_{CA}^d &= -(1-\rho)\ell(1-\ell)\left[(1-\ell) + e_C\ell\right]^{\rho-2} de_C \\
d\alpha_{NA}^d &= \left[\frac{\alpha_{NA}^d}{e_C} - (1-\rho)\ell^2 e_C\left[(1-\ell) + e_C\ell\right]^{\rho-2}\right] de_C \\
d\alpha_{CB}^d &= d\alpha_{NB}^d = 0
\end{aligned}
$$

Therefore, we have

$$
\begin{aligned}
d\bar{\alpha}_{CA} &= \pi\, d\alpha_{CA}^n + (1-\pi)\, d\alpha_{CA}^d + (\alpha_{CA}^n - \alpha_{CA}^d)\, d\pi \\
d\bar{\alpha}_{CB} &= \pi\, d\alpha_{CB}^n + \alpha_{CB}^n\, d\pi
\end{aligned}
$$

Now restrict attention to the symmetric equilibrium with CRS matching. If $S_A = S_B \equiv S$ and $\pi \to 1$, then a symmetric equilibrium exists where $e_C = e_N^n = 1/2$. When $\rho = 0$, the matching probabilities becomes

$$
\begin{aligned}
\bar{\alpha}_{CA} &= \bar{\alpha}_{CB} = \alpha_{CA}^n = \alpha_{CB}^n = 1-\ell \\
\alpha_{NA}^n &= \alpha_{NB}^n = \ell \\
\alpha_{CA}^d &= \frac{2(1-\ell)}{2-\ell} \\
\alpha_{NA}^d &= \frac{\ell}{2-\ell} \\
\alpha_{CB}^d &= \alpha_{NB}^d = 0\,,
\end{aligned}
$$

which in turn implies $q_{0A} = q_{0B} \equiv q_0$ and $q_{1A} = q_{1B} \equiv q_1$. Total differentiation yields

$$
\begin{aligned}
d\alpha_{CA}^n &= -d\alpha_{CB}^n = -d\alpha_{NA}^n = d\alpha_{NB}^n = -\,2\ell(1-\ell)\, de_C + 2\ell(1-\ell)\, de_N^n \\
d\alpha_{CA}^d &= -d\alpha_{NA}^d = -(1-\ell)\ell\left(\frac{2-\ell}{2}\right)^{-2} de_C \\
d\alpha_{CB}^d &= d\alpha_{NB}^d = 0
\end{aligned}
$$

Assuming CRS matching ($\rho = 0$), put together (A.13), (A.14), (A.15), (A.16), (A.17) in a matrix form:

$$
\mathbf{A}\boldsymbol{u} = \boldsymbol{b}\, d\pi, \tag{A.18}
$$

where:

$$\mathbf{A} = \begin{bmatrix} \mathfrak{a} & -\mathfrak{a} & -\mathfrak{b} & 0 & -\mathfrak{c} & 0 \\ -\mathfrak{a} & \mathfrak{a} & 0 & -\mathfrak{b} & 0 & -\mathfrak{c} \\ -\mathfrak{d} & 0 & -\mathfrak{e} & 0 & \mathfrak{f} & \mathfrak{g} \\ \mathfrak{d} & 0 & 0 & -\mathfrak{e} & \mathfrak{g} & \mathfrak{f} \\ \mathfrak{h} & -\mathfrak{h} & \mathfrak{j} & -\mathfrak{j} & -\mathfrak{k} & \mathfrak{k} \\ -\mathfrak{m} & \mathfrak{m} & \mathfrak{n} & -\mathfrak{n} & 0 & 0 \end{bmatrix}, \quad \boldsymbol{u} = \begin{bmatrix} de_C \\ de_N^n \\ dq_{1A} \\ dq_{1B} \\ dq_{0A} \\ dq_{0B} \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} -\frac{\mathfrak{a}}{2(2-\ell)} \\ \frac{\mathfrak{a}}{2\ell} \\ 0 \\ 0 \\ 0 \\ \frac{\mathfrak{m}}{2\ell(2-\ell)} \end{bmatrix},$$

and:

$$\mathfrak{a} = \frac{2(1-\ell)\ell\theta[u'(q_0) - u'(q_1)]}{w_\theta(q_1)},$$

$$\mathfrak{b} = -\frac{(1-\ell)\theta w_\theta(q_0)}{w_\theta(q_1)^2} u''(q_1),$$

$$\mathfrak{c} = -\frac{\ell\theta + (1-\ell)u'(q_1)}{w_\theta(q_1)} u''(q_0),$$

$$\mathfrak{d} = \frac{4q_0 S/M}{\theta},$$

$$\mathfrak{e} = \frac{w_\theta(q_1)}{\theta},$$

$$\mathfrak{f} = \frac{S/M + w_\theta(q_0)}{\theta},$$

$$\mathfrak{g} = \frac{S/M}{\theta},$$

$$\mathfrak{h} = 4(1-\ell),$$

$$\mathfrak{j} = \frac{u'(q_1) - 1}{u(q_1) - u(q_0) - q_1 + q_0},$$

$$\mathfrak{k} = \frac{u'(q_0) - 1}{u(q_1) - u(q_0) - q_1 + q_0},$$

$$\mathfrak{m} = \frac{4(1-\ell)\ell^2\theta[u(q_1) - u(q_0) - (q_1 - q_0)u'(q_1)]}{w_\theta(q_1)},$$

$$\mathfrak{n} = -\frac{(1-\ell)\ell\theta[(1-\theta)(u(q_1) - u(q_0)) + \theta(q_1 - q_0)]}{w_\theta(q_1)^2} u''(q_1).$$

Note that $\mathfrak{a}$ to $\mathfrak{n}$ are all positive. The solution is given by:

$$
\boldsymbol{u} = \begin{bmatrix}
-\dfrac{-\mathfrak{cehm}-\mathfrak{bfhm}+\mathfrak{bghm}+2\mathfrak{afhn}-2\mathfrak{aghn}}{4\mathfrak{d}(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})} \\[2ex]
\dfrac{\mathfrak{cehm}+\mathfrak{bfhm}-\mathfrak{bghm}-2\mathfrak{cdjm}-2\mathfrak{bdkm}-2\mathfrak{afhn}+2\mathfrak{aghn}+4\mathfrak{adkn}}{4\mathfrak{d}(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})} \\[2ex]
-\dfrac{\begin{pmatrix} \mathfrak{c}^2\mathfrak{ehm}+\mathfrak{bcfhm}+\mathfrak{bcghm}-2\mathfrak{acfjm}-2\mathfrak{acgjm}-2\mathfrak{abfkm}-2\mathfrak{abgkm} \\ +2\mathfrak{acfj}\ell\mathfrak{m}+2\mathfrak{acgj}\ell\mathfrak{m}+2\mathfrak{abfk}\ell\mathfrak{m}+2\mathfrak{abgk}\ell\mathfrak{m}-2\mathfrak{acfhn}-2\mathfrak{acghn}+4\mathfrak{a}^2\mathfrak{fkn} \\ +4\mathfrak{a}^2\mathfrak{gkn}+2\mathfrak{acfh}\ell\mathfrak{n}+2\mathfrak{acgh}\ell\mathfrak{n}-4\mathfrak{a}^2\mathfrak{fk}\ell\mathfrak{n}-4\mathfrak{a}^2\mathfrak{gk}\ell\mathfrak{n} \end{pmatrix}}{4(\mathfrak{ce}+\mathfrak{bf}+\mathfrak{bg})(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})} \\[2ex]
-\dfrac{\begin{pmatrix} -\mathfrak{c}^2\mathfrak{ehm}-\mathfrak{bcfhm}-\mathfrak{bcghm}-2\mathfrak{acfjm}-2\mathfrak{acgjm}-2\mathfrak{abfkm}-2\mathfrak{abgkm} \\ +2\mathfrak{acfj}\ell\mathfrak{m}+2\mathfrak{acgj}\ell\mathfrak{m}+2\mathfrak{abfk}\ell\mathfrak{m}+2\mathfrak{abgk}\ell\mathfrak{m}-2\mathfrak{acfhn}-2\mathfrak{acghn}+4\mathfrak{a}^2\mathfrak{fkn} \\ +4\mathfrak{a}^2\mathfrak{gkn}+2\mathfrak{acfh}\ell\mathfrak{n}+2\mathfrak{acgh}\ell\mathfrak{n}-4\mathfrak{a}^2\mathfrak{fk}\ell\mathfrak{n}-4\mathfrak{a}^2\mathfrak{gk}\ell\mathfrak{n} \end{pmatrix}}{4(\mathfrak{ce}+\mathfrak{bf}+\mathfrak{bg})(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})} \\[2ex]
-\dfrac{\begin{pmatrix} -\mathfrak{bcehm}-\mathfrak{b}^2\mathfrak{fhm}-\mathfrak{b}^2\mathfrak{ghm}-2\mathfrak{acejm}-2\mathfrak{abekm}+2\mathfrak{acej}\ell\mathfrak{m} \\ +2\mathfrak{abek}\ell\mathfrak{m}+2\mathfrak{abfhn}+2\mathfrak{abghn}+4\mathfrak{a}^2\mathfrak{ekn}+2\mathfrak{aceh}\ell\mathfrak{n}-4\mathfrak{a}^2\mathfrak{ek}\ell\mathfrak{n} \end{pmatrix}}{4(\mathfrak{ce}+\mathfrak{bf}+\mathfrak{bg})(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})} \\[2ex]
\dfrac{\begin{pmatrix} -\mathfrak{bcehm}-\mathfrak{b}^2\mathfrak{fhm}-\mathfrak{b}^2\mathfrak{ghm}+2\mathfrak{acejm}+2\mathfrak{abekm}-2\mathfrak{acej}\ell\mathfrak{m}-2\mathfrak{abek}\ell\mathfrak{m} \\ +4\mathfrak{acehn}+2\mathfrak{abfhn}+2\mathfrak{abghn}-4\mathfrak{a}^2\mathfrak{ekn}-2\mathfrak{aceh}\ell\mathfrak{n}+4\mathfrak{a}^2\mathfrak{ek}\ell\mathfrak{n} \end{pmatrix}}{4(\mathfrak{ce}+\mathfrak{bf}+\mathfrak{bg})(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})}
\end{bmatrix} d\pi \quad \text{(A.19)}
$$

Now look at the liquidity premium:

$$
\begin{aligned}
L_A &= \ell \frac{\theta}{w_\theta(q_{1A})}(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA})(u'(q_{1A})-1) \\
L_B &= \ell \frac{\theta}{w_\theta(q_{1B})}\alpha^n_{CB}(u'(q_{1B})-1).
\end{aligned}
$$

Total differentiation, when $\pi \to 1$ in the symmetric equilibrium, yields

$$
\begin{aligned}
dL_A &= \ell\theta\frac{u''(q_1)}{w_\theta(q_1)^2}\alpha^n_{CA}dq_{1A} + \ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)}(\alpha^n_{CA}-\alpha^d_{CA})d\pi + \ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)}d\alpha^n_{CA} \\
dL_B &= \ell\theta\frac{u''(q_1)}{w_\theta(q_1)^2}\alpha^n_{CB}dq_{1B} + \ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)}d\alpha^n_{CB}.
\end{aligned}
$$

Therefore,

$$
dL_A - dL_B = \ell\theta\frac{u''(q_1)}{w_\theta(q_1)^2}(dq_{1A}-dq_{1B}) + \ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)}(\alpha^n_{CA}-\alpha^d_{CA})d\pi + \ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)}(d\alpha^n_{CA}-d\alpha^n_{CB}).
$$

Since:

$$
\alpha^n_{CA} = 1-\ell, \quad \alpha^d_{CA} = \frac{2(1-\ell)}{2-\ell}, \quad \text{and} \quad d\alpha^n_{CA} = -d\alpha^n_{CB} = -2\ell(1-\ell)(de_C - de^n_N),
$$

we get:

$$dL_A - dL_B = \ell\theta\frac{u''(q_1)}{w_\theta(q_1)^2}(dq_{1A} - dq_{1B}) - \ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)}\frac{\ell(1-\ell)}{2-\ell}d\pi - 4\ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)}\ell(1-\ell)(de_C - de_N^n).$$

In order to have $dL_A - dL_B < 0$, we want each term in $dL_A - dL_B$ to be negative. The second term is obviously negative. To determine the sign of the first term, look at $dq_{1A} - dq_{1B}$. From (A.19),

$$dq_{1A} - dq_{1B} = \frac{\mathfrak{chm}}{2(2-\ell)\ell(\mathfrak{cjm} + \mathfrak{bkm} + \mathfrak{chn} - 2\mathfrak{akn})}.$$

The sign of $dq_{1A} - dq_{1B}$ depends on that of $\mathfrak{cjm} + \mathfrak{bkm} + \mathfrak{chn} - 2\mathfrak{akn}$ in the denominator. We define:

$$\begin{aligned}\mathfrak{D} &\equiv \mathfrak{cjm} + \mathfrak{bkm} + \mathfrak{chn} - 2\mathfrak{akn} \\ &= \left[4\ell(1-\ell)\frac{\theta}{w_\theta(q_1)}\right]\left[-u''(q_0)\left(1 - \frac{(1-\ell)\theta}{w_\theta(q_1)}\right)\ell(u'(q_1)-1)\frac{S^1}{S}\dots\right. \\ &\qquad \left. -u''(q_0)\left(1 - \frac{(1-\ell)\theta}{w_\theta(q_1)}\right)(1-\ell)\frac{-u''(q_1)}{w_\theta(q_1)}S^\theta - u''(q_1)\frac{(1-\ell)\theta}{w_\theta(q_1)}\ell(u'(q_0)-1)\frac{S^0}{S}\right],\end{aligned}$$

where

$$\begin{aligned} S &= u(q_1) - u(q_0) - q_1 + q_0 > 0 \\ S^\theta &= (1-\theta)(u(q_1) - u(q_0)) + \theta(q_1 - q_0) > 0 \\ S^1 &= u(q_1) - u(q_0) - u'(q_1)(q_1 - q_0) > 0 \\ S^0 &= u(q_1) - u(q_0) - u'(q_0)(q_1 - q_0) < 0. \end{aligned}$$

$S^1 > 0$ and $S^0 < 0$ due to the strict concavity of u. For the first term in $dL_A - dL_B$ to be negative, we want $\mathfrak{D} > 0$ so that $dq_{1A} - dq_{1B} > 0$. The first and the second terms in the second bracket in $\mathfrak{D}$ are positive, whereas the third term is negative. If $\theta \to 0$ or $\ell(1-\ell) \to 0$, then $\mathfrak{D} > 0$. In case of quadratic utility, $u(q) \equiv (1+\gamma)q - q^2/2$ with $q^* = \gamma$, we can show that $\mathfrak{D} > 0$ is always the case for all (ℓ, θ). First, observe the following from the sum of the second and the third terms in the second bracket in $\mathfrak{D}$:

$$\begin{aligned} &- u''(q_0)\left(1 - \frac{(1-\ell)\theta}{w_\theta(q_1)}\right)(1-\ell)\frac{-u''(q_1)}{w_\theta(q_1)}S^\theta - u''(q_1)\frac{(1-\ell)\theta}{w_\theta(q_1)}\ell(u'(q_0)-1)\frac{S^0}{S} \\ >& - u''(q_0)\ell(1-\ell)\frac{-u''(q_1)}{w_\theta(q_1)}S^\theta - u''(q_1)\frac{(1-\ell)\theta}{w_\theta(q_1)}\ell(u'(q_0)-1)\frac{S^0}{S} \\ =& \frac{-u''(q_1)}{w_\theta(q_1)}\ell(1-\ell)\frac{1}{S}\left[-u''(q_0)S^\theta S + (u'(q_0)-1)\theta S^0\right], \end{aligned}$$

where the first inequality comes from $u'(q_1) = 1 + q^* - q_1 \geq 1 > \ell$. Denote $\Upsilon(\theta) \equiv -u''(q_0)S^\theta S + (u'(q_0) - 1)\theta S^0$. Observe that $\Upsilon(\theta = 0) = -u''(q_0)(u(q_1) - u(q_0))S > 0$; $\Upsilon(\theta = 1) = (q_1 - q_0)^2(q^* - q_1)/2 > 0$; and $d\Upsilon/d\theta = (u'(q_0) - 1)S^0 + S^2 u''(q_0) < 0$. Therefore, $\Upsilon > 0$ and $\mathfrak{D} > 0$. For other cases, including log utility, we verified numerically and could not find any case where $\mathfrak{D} > 0$ is not satisfied. $\mathfrak{D} > 0$ implies that $dq_{1A} - dq_{1B} > 0$ and that the first term in $dL_A - dL_B$ is negative.

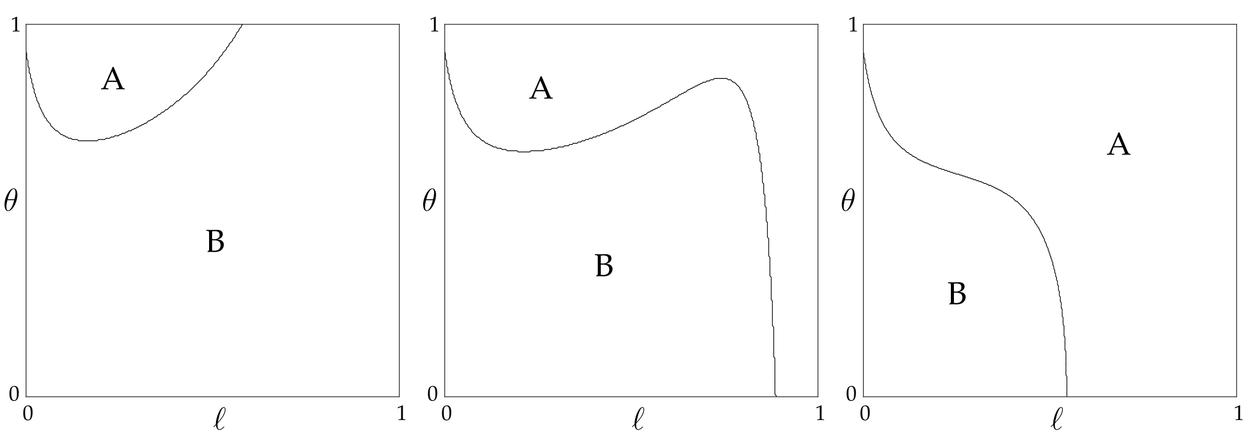
To determine the sign of the last term in $L_A - dL_B$, look at $de_C - de_N^n$. From (A.19),

$$de_C - de_N^n = -\frac{\mathfrak{cjm} + \mathfrak{bkm} - 2\mathfrak{akn}}{2(2-\ell)\ell\, \mathfrak{D}}.$$

Since $\mathfrak{D} > 0$, the sign of $de_C - de_N^n$ depends on that of $\mathfrak{cjm} + \mathfrak{bkm} - 2\mathfrak{akn}$ in the numerator:

$$\mathfrak{cjm} + \mathfrak{bkm} - 2\mathfrak{akn} = \left[4\ell^2(1-\ell)\frac{\theta}{w_\theta(q_1)}\right]\dots$$
$$\times \left[-u''(q_0)\left(1 - \frac{(1-\ell)\theta}{w_\theta(q_1)}\right)(u'(q_1) - 1)\frac{S^1}{S} - u''(q_1)\frac{(1-\ell)\theta}{w_\theta(q_1)}(u'(q_0) - 1)\frac{S^0}{S}\right].$$

For the third term in $dL_A - dL_B$ to be negative, we want $\mathfrak{cjm} + \mathfrak{bkm} - 2\mathfrak{akn} < 0$ so that $de_C - de_N^n > 0$. The first term in the second bracket is positive, whereas the second term is negative. From the equation, we can see that if $(1-\ell)\theta$ is sufficiently large, $\mathfrak{cjm}+\mathfrak{bkm}-2\mathfrak{akn}$ becomes negative, $de_C - de_N^n$ becomes positive, and the third term in $dL_A - dL_B$ becomes negative. Below is the figure that numerically shows in the (ℓ, θ) plane the parameter space where the third term in $dL_A - dL_B$ is negative (A) and where it is not (B):



Small asset supplies | Medium asset supplies | Large asset supplies

In region A, the third term in $dL_A - dL_B$ is negative, so all the components of $dL_A - dL_B$ are negative, while in region B the third term is positive. Under the sufficient condition that $(1-\ell)\theta$ is large enough, we will be in region A so that all the components of $dL_A - dL_B$ become negative and $dL_A - dL_B < 0$, which in turn implies that near $\pi = 1$ we have $L_A > L_B$. □

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