Entrepreneurial Experimentation and Duration

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Abstract

Entrepreneurial ventures surviving longer tend to earn more. Therefore, censoring on entrepreneurial duration may bias the average earnings, causing discrepancies between observed earnings and its expected long-run value. We propose a continuous-time dynamic model in which a risk-averse entrepreneur learns the unknown venture quality through experimentation over time and chooses the optimal time to exit. We derive in closed form the optimal decision rules and the risk-adjusted expected value of entrepreneurship. In the model, duration captures the optimal amount of experiment, which predicts long-term success and thus the expected earnings. The structural estimation suggests that the bias of entrepreneurial earnings due to right-censoring on duration in our current sample is 4.7% of the annual wage of salaried workers, or $1,652 per year.

JEL Classification: D81, D83, J24, L26, M13.

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1 Introduction

The motivation of entrepreneurship has been puzzling, given the low average realize earnings and return compared to alternatives (Hamilton (2000) and Moskowitz and Vissing-Jorgensen (2002)). In this paper, we focus on the role of entrepreneurial duration and show that incorporating information from duration helps to explain this puzzle.

Why is duration crucial for studying entrepreneurship? First, duration is an optimal choice of the entrepreneur and an observable outcome. It captures the optimal amount of experimentation and information about entrepreneur's subjective valuation of the venture, and such information is not contained in the observed earnings. Second, analyses on duration highlight an important but often overlooked bias of average entrepreneurial earnings, which is due to the prevailing right-censoring on duration. We study the role of duration and quantify such a bias of earnings in this paper.

We propose a tractable model of entrepreneurial experimentation. In our model, entrepreneurial ventures have different unobservable qualities (e.g., the long-run venture earnings). A risk-averse entrepreneur does not know the quality of her venture and can only learn about it by (full-time) working on her venture. However, she has an option to quit her venture and takes a salaried job. Once she quits, she no longer learns about the quality of her venture. We analytically characterize the entrepreneur's optimal experimentation strategy and her private valuation of her venture, taking into account her option to exit and risk aversion.

In our model, the entrepreneur trades off between the current pay cut during experimentation and the potential successful benefits in the future, which arrives randomly in time if the venture quality is high. Therefore, a higher entrepreneurial belief of the venture quality leads to longer duration and higher certainty-equivalent value of entrepreneurship. Furthermore, a longer duration leads to a higher probability to succeed and thus higher long-run earnings because the success arrives independently over time. As a result, the duration, the certainty equivalent value of entrepreneurship, and the observed earnings are all positively correlated.

Because of the positive correlation between duration and earnings, empirical measures of earnings underestimates its long-run mean when the duration is censored on the right in surveys. To quantify such a bias, we structurally estimate our model and conduct a counterfactual experiment. The counterfactual experiment hypothetically extends the censoring time, the time from the venture starting to the survey end, in the current sample to infinite time or the average time where workers typically retire. Results show that due to the right censoring on duration, the average earnings in the current sample underestimates its long-run value by about 4.7% of the wage of salaried workers, or \$1,652 per year. After the correction this new and other biases, the entrepreneurs earn 3.0% average premium or \$1,054 per year relative to salaried workers.

This paper is related to the literature that explains the private equity premium puzzle with real options.[1] Among them, Manso (2016) proposes a two-period dynamic model to study the real option in entrepreneurial experimentation. Dillon and Stanton (2017) and Catherine (2017) analyze dynamic lifetime models with learning to explain the entrepreneurial incentives for entry.[2] We differ from these papers in two important ways. First, we highlight the role of duration in capturing the information of entrepreneurial decision making, in addition to earnings. Second, we point out and quantify a new bias in the literature to bridge the gap between theories and empirical findings.

Our new bias of earnings due to censored duration differs from the two biases Manso (2016) considers. Manso points out that the conventional cross-sectional mean of entrepreneurial earnings neglects the earnings after entrepreneurs abandon the ventures. As a result, the cross-sectional mean underestimates lifetime earnings of exiting entrepreneurs, inducing an experimentation bias, and overweights the earnings of entrepreneurs who survive, causing a survivorship bias. In our paper, the new bias measures the distance of the current mean of earnings with censoring on duration, to its long-run mean without censoring on duration. We compare these biases quantitatively and show that this new bias exists even after taking into account of entrepreneurs' post-exit earnings and thus complements the two biases in Manso (2016).

Our model also contributes to the literature of entrepreneurship as experimentation. (See Kerr, Nanda, and Rhodes-Kropf (2014) for a survey.) In particular, Keller, Rady, and Cripps (2005) studies the strategic experimentation issues. Manso (2011) derives the optimal contract to motivate a risk-neutral manager to venture her innovations, in which an experimentation process with early costs and long-term reward are similar to ours here. Besides investigating different topics, we add to the literature by analyzing the effects of risk aversion on agents' experimentation strategies. We show that entrepreneurs' risk attitudes influence their consumption/saving decisions and career choices because entrepreneurs cannot fully diversify away the ambiguity of the venture quality and the idiosyncratic venture risk. In addition, we show that when entrepreneurial beliefs are not observable, the time

[1] See Brennan and Schwartz (1985), McDonald and Siegel (1986), and Dixit and Pindyck (1994) for the standard real option approach to investment.

[2] Besides, Catherine (2017) suggests that non-pecuniary earnings may explain the inferior private equity return.

to exit is a sufficient statistic for the entrepreneurial behaviors.

Another strand of literature studies the roles of entrepreneurs or venture capitalists in innovation and macroeconomic growth. For example, Opp (2019) develops a general equilibrium model of venture capital intermediation, in which entrepreneurial experimentation is a static problem. We differ from this literature by developing a partial equilibrium model of an individual entrepreneur's dynamic experimentation problem. By doing so, we derive the analytical solutions of entrepreneurial optimal decision rules, which enables us to zoom in on the relationship between entrepreneurial duration and earnings.

Empirically, our model prediction and results are consistent to the empirical evidence showing a positive correlation between entrepreneurial earnings and venture age. Hamilton (2000) shows that entrepreneurial earnings grow faster than salaried workers unconditionally. In a sample of venture capital-backed startups, Ewens, Nanda, and Stanton (2019) document that venture earnings and firm age are positively correlated. Both papers are consistent with our empirical evidence and support our model implications that duration matters for estimated earnings. Additionally, while Ewens, Nanda, and Stanton (2019) find that the earnings of the ventures reaching certain revenue and product milestones grow faster than those fail to reach such milestones, our paper presents evidence conditioning on entrepreneurship survival status.

Finally, Hall and Woodward (2010) study how entrepreneurial duration affects exit value and the certainty equivalent value of entrepreneurship of venture-backed entrepreneurs, among other results on exit value. Our paper complement their results in two aspects. First, we emphasize the effects of duration on venture earnings instead of exit value. Adding exit value to our model strengthens the value of entrepreneurship relative to salaried jobs. Second, we incorporate quality ambiguity and learning in our model. As a result, the certainty-equivalent value in our model decreases over time because of downward updated beliefs, in addition to deteriorating entrepreneurial assets in their model.

The rest of the paper proceeds as follows. Section 2 presents stylized empirical facts about earnings and duration. Section 3 proposes a parsimonious dynamic model of entrepreneurs with learning and option to exit. Section 4 estimates the model and characterizes the model solutions, focusing on duration, beliefs, and the net certainty equivalent value of entrepreneurship. Section 5 characterizes the bias of earnings coming from censored duration. Section 6 concludes.

2 Data and Stylized Facts

In this section, we describe the data and present stylized facts that motivate our model. The key empirical finding is that survived entrepreneurs have longer durations and higher average earnings. This relation suggests the importance of joint consideration of entrepreneurial duration and earnings as the right-censoring bias of duration may cause biases of the cross-sectional average of earnings.

Data. We use the National Longitudinal Survey of Youth 1979 (NLSY79), a longitudinal project that follows a sample of American youth born between 1957-1964. The original data consist of 12,686 individuals surveyed between 1979 and 2014. As in Manso (2016), we drop military members (1,280) and representative minorities (5,295), leaving 6,111 individuals in the final sample. Only variables in even-year surveys are kept, because the survey was initially conducted each year but every two years after the year 1994.

In the data, we classify an individual-year observation as entrepreneurial if its primary worker type is self-employment.[3] The observations are classified as salaried workers if they reported working in government, private companies, or non-profit organizations.[4] We require a respondent to start his career as a salaried worker in the survey. This requirement avoids the left-censoring bias for the venture duration, which occurs when we cannot observe the time when the self-employment starts.[5]

Labor-earnings and wealth variables are in the unit of 2012 dollars using the Consumer Price Index (CPI). We use family net worth in NLSY79 to measure wealth. Because the survey only collects the net worth data in certain years, analysis with net worth has a smaller sample size.[6]

Table 1 presents the summary statistics. Among the self-employment (SE) businesses, we categorize the ones that exist in the last available survey as survived ones and others as abandoned. The first two rows report the numbers of observations. In the first row, salaried workers account for about 90% $(55,865/(55,865+5,836))$ of all individual-year

[3] As in Manso (2016) and other papers, we do not distinguish between self-employment and entrepreneurship. Because our model applies to broadly defined self-employment with learning and risk-bearing.

[4] We exclude observations if the respondents were neither salaried workers nor self-employed, e.g., unemployed or students.

[5] Had we allowed the respondent to start as an entrepreneur, our estimate of his duration would be biased due to left censoring. Below we illustrate this via an example. Consider an entrepreneur, self-employed from 1970 through 1988. Because the sample starts from 1980, the actual duration of venture is 18 years (i.e., 1988-1970), however, the reported duration is only 8 years (i.e., 1988-1980.)

[6] Total net worth amount in NLSY79 was only collected in the following survey years: 1985-1990, 1992-2000, 2004, 2008, and 2012.

Table 1: Summary Statistics

This table presents the summary statistics of the National Longitudinal Survey of Youth-1979 (NLSY79). The sample is from 1980 to 2014 and uses only even-year surveys. Among the self-employment (SE) businesses, we categorize the ones that remain active until the last available survey as survived ones and others as abandoned.

	Workers	SE Businesses		
		All	Abandoned	Survived
Number of individual-year	55,865	5,836	3,383	2,453
Number of businesses		2,309	1,708	601
Average earnings ($)	44,309	36,522	30,424	53,854
Earnings volatility ($)	44,791	50,256	41,158	67,002

observations. Among the SE businesses, the survived ones constitute a 42% (2,453/5,836) of all business-year observations. When we compare the numbers of business in the second row, the survived ones account for 26% of the SE business.

The next two rows report the average earnings and the standard deviation.[7] Self-employers have lower and riskier earnings (with a mean of $36,522 and a standard deviation of $50,256$) than the salaried workers (with a mean of $44,309 and a standard deviation of $44,791$). These results are consistent with the empirical findings, which are referred to as the private equity premium puzzle by Moskowitz and Vissing-Jorgensen (2002).

While the abandoned businesses ($30,424) have significantly lower earnings than salaried workers, the survived ones earn more ($53,854) than workers. We confirm this result using additional econometric methods, including fixed effects regressions and propensity-score matching. In the propensity-score matching results, while abandoned businesses earn $4,958 less than the salaried workers, survived businesses outperform the abandoned ones by $8,270. (See Table A1 in Appendix for more details.)

These stylized facts motivate us to develop a dynamic model of entrepreneurship with experimentation. As in Manso (2016), experimentation naturally implies that entrepreneurs are willing to take an earnings cut early on in career with the hope of making a substantially higher earnings in the future.

Finally, we obtain individual characteristics including age, gender, race, working experiences, years of schooling, and cognitive traits. For details of these variables, we refer readers to Table IA.1 in the Internet Appendix.

[7] For SE businesses, reported mean of corresponding variables are averaged first across years within the same businesses and then across different businesses.

Entrepreneurial duration and right censoring. Figure 1 plots the empirical distribution of observed duration for both abandoned and survived businesses.[8] Panel A shows that for abandoned businesses, more than half entrepreneurs abandon within two years since the start of their businesses and three quarters of them quit within four years since the start. The average duration for the sample of all abandoned businesses is about four years. For this group, the business exit rate declines over time at an increasing rate, which is consistent with Evans and Leighton (1989). This observation motivates us to use Gamma distribution to fit the duration in our structural estimation later.

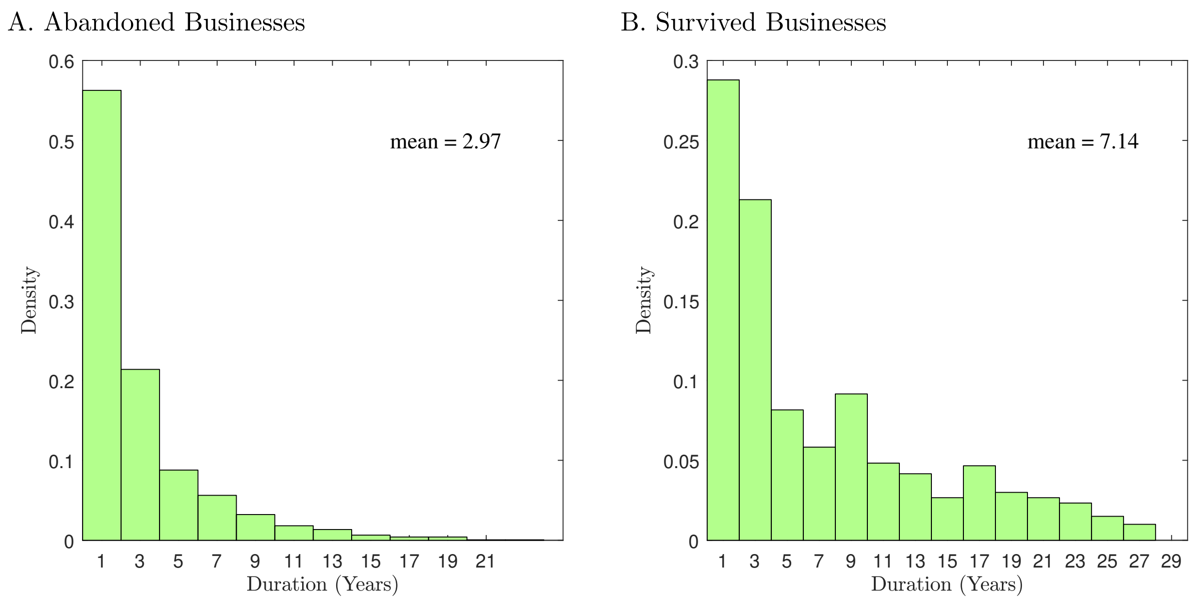

Figure 1: Empirical density of duration for abandoned and survived SE businesses. For an abandoned SE business, the duration is from the starting of the business to the switching to the salaried job. For a survived business, the duration is from the starting of the business to the last reported date of the survey.

Panel B shows that the average duration for the survived businesses is about eight years, more than doubling that for the abandoned ones. This statistic even under-states the duration differences between the two groups, because the observed duration is inevitably truncated by the end year of the survey. This empirically observed duration is thus lower than the actual duration of the survived businesses. The difference between these two is known as the right-censoring bias of duration in the survival analysis.[9] In this paper, we

[8]Because the time interval is every two years, we take the last observed year minus one as the business duration. For example, if the business is discontinued in the survey in its eighth year, it actual duration is between six and eight years. We report seven year as this example's duration.

[9]See Chapter 22 in Wooldridge (2010) on the right-censoring bias and the survival analysis.

show that this right censoring on duration not only biases the empirical measure of duration but also observed entrepreneurial earnings.

3 Model and Solution

In this section, we introduce our model and present its solution.

3.1 Setup

Time is continuous. At any time t, an infinitely-lived agent chooses to work as either a self-employed entrepreneur or a salaried worker. These two options are mutually exclusive. Being an entrepreneur means receiving a lower compensation in the near term but may potentially yield much higher payoffs than being a salaried worker.

We use s_t to denote the agent's status at t. If the agent is a salaried worker at t, then $s_t = 0$. If the agent is an entrepreneur at t, there are only two possible stages. If she is experimenting with her entrepreneurial idea, then $s_t = 1$. Otherwise, she is already revealed to be a successful entrepreneur by t, then $s_t = 2$.

The agent's earnings over the interval $(t, t + dt)$, dY_t, follows

$$dY_t = \mu(s_t)dt + \sigma(s_t)dB_t \, , \tag{1}$$

where B_t is a standard Brownian motion. That is, $\{Y_t : t \geq 0\}$ denote the agent's cumulative (un-discounted) earnings. This type of stochastic process is widely used in dynamic contracting and corporate finance models.[10]

Importantly, the drift $\mu(\,\cdot\,)$ and the volatility $\sigma(\,\cdot\,)$ are functions of status s_t. Conditioning on s_t, the agent's earnings is independently and identically distributed (*i.i.d.*). That is, $\mu(s_t)$ and $\sigma(s_t)$ are constant given s_t. We denote these constants to be (μ_W, σ_W) for salaried workers ($s_t = 1$), (μ_L, σ_L) for experimenting entrepreneurs ($s_t = 1$), and (μ_H, σ_H) for successful entrepreneurs ($s_t = 2$).

To generate an economically interesting trade-off, and to be consistent with the empirical evidence, we impose the key assumption that an experimenting entrepreneur makes less than a salaried worker and the agent prefers being a successful entrepreneur than a

[10]DeMarzo and Sannikov (2006) and Decamps, Mariotti, Rochet, and Villeneuve (2011) use a simpler formulation of the earnings process (1) to model the firm's earnings. In their models, both m_t and σ_t are constant and hence the firm's earnings are i.i.d.. DeMarzo and Fishman (2007) use the same i.i.d. process in a discrete-time setting.

salaried worker.[11] We assume the expected earnings satisfy $\mu_L < \mu_W < \mu_H$.

How do we model the agent's dynamic experimentation decision and when does the agent know his experimentation is successful? The quality of the agent's entrepreneurial idea quality is either low (L) or high (H). For both types, no one knows the true quality, as it can only be learned via the entrepreneur's active experimentation. Therefore, every entrepreneur starts from the experimentation stage, i.e., $s_t = 1$.

If her venture idea is L, she will never succeed. In this case, until the moment that she quits and becomes a salaried worker at T, she always makes losses compared with her outside option, being a salaried worker, i.e., $\{s_t = 1 : t \leq T\}$.

If her venture idea is H, she will succeed with a constant probability λdt over a small time interval $(t, t + dt)$, provided that she is actively experimenting on the venture. In this case, if the project is revealed to be of high quality at τ when she is experimenting, we have $\{s_t = 1 : t < \tau\}$ and $\{s_t = 2 : t \geq \tau\}$. The entrepreneur's learning is achieved via active experimentation on the project. Once the project is revealed to be high quality, everyone becomes informed.

Entrepreneurs are often not well diversified and have limited access to capital markets. We thus assume that the agent cannot insure against her earnings risk. We assume that the agent can borrow and save at the constant risk-free rate, r. Let $\{x_t; t \geq 0\}$ and $\{c_t; t \geq 0\}$ denote the agent's wealth and consumption processes, respectively. Her wealth evolves as

$$dx_t = (rx_t - c_t)\, dt + dY_t\,. \tag{2}$$

The agent chooses consumption and stage $\{c_t,\, s_t\,;\, t \geq 0\}$ to maximize her utility

$$\mathbb{E}\left(\int_0^\infty e^{-rt} u\,(c_t)\, dt\right)\,. \tag{3}$$

For tractability, we assume $u(c) = -e^{-\gamma c}/\gamma$, which is the constant absolute risk-aversion (CARA) utility and $\gamma > 0$ is the coefficient of absolute risk aversion. We can generalize our model to allow the agent to partially hedge her labor earnings shocks.

3.2 Solution

Learning. Let p_t denote the agent's time-t posterior belief that her venture quality is high given that $s_t = 1$. Let τ denote the time at which the entrepreneurial venture succeeds and is revealed to be H. For the L-type venture, $\tau = \infty$. In Appendix B, we show that

[11] We also include the volatility effect in our analysis.

Bayesian learning implies for $t < \tau$,

$$dp_t = -\lambda p_t \left(1 - p_t\right) dt \,. \tag{4}$$

The belief p_t is decreasing in t. Intuitively, the longer the entrepreneur experiments without success the less promising the entrepreneurial idea. That is, no news is bad news.

Entrepreneur's Problem. We solve the agent's optimization problem in three steps. First, a successful entrepreneur solves a standard savings problem without experimentation, as $s_t = 2$ is an absorbing state. Second, once the agent stops experimenting with her venture and takes a salaried job, it is suboptimal to go back to the venture. The intuition is that her belief on the current business quality is low enough to continue experimenting and will not increase after she quits. Therefore, $s_t = 0$ is also an absorbing state. A salaried worker's and a successful entrepreneur's optimization problems are essentially the same other than different parameter values. For these absorbing states, given the current wealth level x, a constant expected earnings $\mu(s)$, and a constant volatility $\sigma(s)$, we denote the agent's value function being $\overline{V}\left(x; \mu(s), \sigma(s)\right)$.

The last step is to solve the agent's optimization problem when $s_t = 1$. Denote the value function of an experimenting entrepreneur to be $V\left(x, p\right)$. We conjecture that the value function takes the following form:

$$V(x,p) = -\frac{1}{\gamma r} e^{-\gamma r \left(x + \frac{1}{r}\mu_W - \frac{1}{2}\gamma\sigma_W^2 + m(p)\right)} \,. \tag{5}$$

Here, $m(p)$ is the net certainty-equivalent value of entrepreneurship. It is the lowest certainty equivalent wealth given to the entrepreneur so that she is indifferent between taking the salaried job and experimenting the business, i.e., $\overline{V}\left(x + m\left(p\right); \mu_W, \sigma_W\right) = V\left(x, p\right)$. Note that $m(p)$ is a net value in excess of $x + \frac{1}{r}\mu_W - \frac{1}{2}\gamma\sigma_W^2$, where $\frac{1}{r}\mu_W - \frac{1}{2}\gamma\sigma_W^2$ is the certainty-equivalent value of a salaried job.

For convenience, we define δ_L as the risk-adjusted pay cut when experimenting and δ_H as the successful entrepreneur's risk-adjusted pay premium over salaried workers:

$$\delta_L \equiv \left(\mu_W - \mu_L\right) - \frac{1}{2} r\gamma \left(\sigma_W^2 - \sigma_L^2\right) \tag{6}$$

$$\delta_H \equiv \left(\mu_H - \mu_W\right) - \frac{1}{2} r\gamma \left(\sigma_H^2 - \sigma_W^2\right) \tag{7}$$

The optimal strategy of entrepreneurs is presented in the following proposition:

Proposition 1. *If* $0 < r\delta_L < \frac{\lambda}{\gamma}\left(1 - e^{-\gamma\delta_H}\right)$, *then the value function* $V(x,p)$ *is given by (5) and the optimal consumption rule is given by*

$$c(x,p) = r\left(x + \frac{\mu_W}{r} - \frac{1}{2}\gamma\sigma_W^2 + m(p)\right), \tag{8}$$

Here, for $p \geq p^*$, $m(p)$ *solves the following differential equation*

$$rm(p) \quad = \quad -\delta_L + \frac{\lambda p}{\gamma r}\left(1 - e^{-\gamma(\delta_H - rm(p))}\right) - \lambda p(1-p)\,m'(p), \tag{9}$$

subject to boundary conditions $m(p^*) = 0$ *and* $m'(p^*) = 0$*. For* $p < p^*$, $m(p) = 0$.

The belief threshold $p^*(x)$ *is independent of the wealth level* x *and given by*

$$p^*(x) = p^* \equiv \frac{\gamma r\delta_L}{\lambda\left(1 - e^{-\gamma\delta_H}\right)}. \tag{10}$$

Additionally, p^* *is increasing with* λ *if* $\sigma_H \geq \sigma_W$ *and* $\sigma_L \geq \sigma_W$.

The time to exit $z(p;p^*)$ *is the length of time for the belief updating from* p *to* p^*,

$$z(p;p^*) = \frac{1}{\lambda}\left(\ln\frac{p}{1-p} - \ln\frac{p^*}{1-p^*}\right). \tag{11}$$

The entrepreneurial duration is $T(p_0) \equiv z(p_0;p^*)$.

See Appendix B for the proof.

Private value of entrepreneurship. Equation (9) presents the solution of the private value of entrepreneurship $m(p)$. The left side is the required return of the entrepreneurial business. The right side has three terms. The first term $-\delta_L$ is the risk-adjusted earnings loss while experimenting. The second term captures the potential benefits of the business succeeding in the next moment, where the value of the business changes from $m(p)$ to δ_H/r, the net certainty-equivalent value of a successful business in excess of salaried jobs. The last term on the right, $-\lambda p(1-p)\,m'(p)$, reflects the marginal return due to belief updating on the value, which is the instantaneous change in belief multiplying the marginal effects of belief changes on values.

Belief threshold The belief threshold p^* is derived from two boundary conditions. $m(p) = 0$ states that the agent must be indifferent between the salaried job and staying in business at the threshold. $m'(p) = 0$ is the smooth-pasting condition, because net marginal benefits of experimenting must be zero at the threshold p^*.

The explicit formula of the belief threshold (10) gives intuitive comparative statics. A higher interest rate r or a higher risk-adjusted earnings loss during experimentation δ_L increases p^*. Because higher values of these two increase the costs of staying in the business, the entrepreneur exits sooner. Additionally, a higher arrival rate of success λ or a higher risk-adjusted premium of success δ_H decreases p^* because the expected benefits of success are higher.

Proposition 1 also shows that the belief threshold increases with the risk aversion γ. A higher risk aversion lowers the entrepreneur's valuation of private equity, which in turn reduces an entrepreneur's willingness to stay in the business and encourages exit.

Entrepreneurial Duration. It is evident that the time to exit $z\,(p;p^*)$ is an increasing function of the entrepreneur's current belief p in (11). Intuitively, the experimental duration in (11) captures the optimal amount of experimentation, which is longer when the current belief p is higher. This result is similar to the Lemma 3.1 in Keller, Rady, and Cripps (2005).

In the model, the time to exit z is a key state variable of the entrepreneur's problem as the belief. In fact, inverting the function, $z\,(p;p^*)$ with respect to p in (11), the current belief p becomes a function of time to exit z:

$$p(z) = \left[1 + \left(\frac{1}{p^*} - 1\right) e^{-\lambda z}\right]^{-1}. \tag{12}$$

Therefore, the private value of entrepreneurship and the expected earnings can be expressed as functions of the duration. Missing data on z or T can lead to biases of observed entrepreneurial performance.

4 Estimation and Solution

In this section, we first estimate and then solve the model.

4.1 Calibration

We first choose parameters that are directly observable in data. Table 2 reports them. The annual risk-free rate r is 4%. We target the entrepreneur's relative risk aversion to 2 (as in Hall and Woodward (2010)) by setting the coefficient of risk aversion γ to

$2/68594 = 2.92 \times 10^{-5}$, as the entrepreneur's median starting wealth is $\$68,594$ in our sample.[12]

The expected annual wage of salaried workers is $\mu_W = \$35,147$, which is the pre-entry average earnings of entrepreneurs.[13] (See Table IA.2 in Internet Appendix.) By using Table 1, we set the annual earnings volatility of salaried workers σ_W at $44,791 and the entrepreneur's annual earnings volatilities before and after success, σ_L and σ_H, at $50,256.

Table 2: Calibrated Parameter Values

This table summarizes the calibrated parameter values. Whenever applicable, parameter values are annualized.

Parameter	Definition	Value
r	Risk-free rate	4%
γ	Absolute risk aversion (ARA)	2.92×10^{-5}
μ_W	Expected wage of salaried workers	$35,147
σ_W	Earnings volatility of salary workers	$44,791
$\sigma_L = \sigma_H$	Entrepreneur's earnings volatility	$50,256
α_L	Pay cut when experimenting $\mu_W - \mu_L$	$4,958

Let α_L denote the annual (expected) pay cut when experimenting and α_H denote annual earnings jump over salaried workers after successful experimentation:

$$\alpha_L \equiv \mu_W - \mu_L \quad \text{and} \quad \alpha_H \equiv \mu_H - \mu_W\,. \tag{13}$$

The calibrated value of α_L is $4,958, based on the earnings difference between the abandoned SE businesses and the propensity-score matched salaried workers in Table A1. Therefore, the abandoned businesses on average make $\mu_L = \mu_W - \alpha_L = 35,147 - 4,958 = \$30,189$ per year.

[12] The estimated relative risk aversion ranges from below 1 in Hansen and Singleton (1982) to around 10 in Mehra and Prescott (1995). All else equal, a lower value of risk aversion yields higher estimate of α_H and implies a larger indirect censoring bias.

[13] Since we do not have information of the post-exit earnings of survived businesses, we use the pre-entry earnings to approximate the post-exit earnings for all businesses, i.e., the earnings after exercising the outside option. Table IA.2 in Internet Appendix shows that the pre-entry and post-exit earnings are very similar when the latter is available for the abandoned businesses. Hamilton (2000) and Manso (2016) find that self-employment experiences boost earnings in the salaried jobs. We also try this specification, which might change the quantitative magnitudes of some results but not the main implications.

4.2 Estimation

We estimate the remaining parameters using the duration data. Our entrepreneurial survival problem differs from conventional survival analyses because it involves entrepreneurial learning and exit decisions. We derive the likelihood function of our model and take into account that some duration observations are right-censored.

Estimation method. Motivated by the downward-sloping pattern of histogram in Figure 1, we assume that the entrepreneurial duration T follows a Gamma distribution whose probability density function is given by $\phi(s;k,\theta) = \frac{\theta^k}{\Gamma(k)} s^{k-1} e^{-\theta s}$, where k and $\theta > 0$ are the two parameters to be estimated. Let $\Phi(s;k,\theta)$ denote the corresponding cumulative distribution function.

Let $P_A(t)$ denote the probability that a business is abandoned before time t, where the subscript (A) means "abandoned". The probability that a business is abandoned before time t is given by

$$P_A(t) \equiv (1-\pi)\,\mathbb{P}(T \leq t|L) + \pi\mathbb{P}(T \leq t \wedge \tau|H)\ , \tag{14}$$

where π is the probability that a business is of High quality in the sample and τ is the stochastic arrival time that the High-quality project is revealed.[14] The first term in (14) gives the probability that a Low-quality project is abandoned and the second term is the probability that a High-quality project is abandoned. In Appendix C, we the explicit formula for $P_A(t)$.

We next derive the likelihood function, taking into account the right censoring on duration. Let η_i denote the censoring time for Business i, the time lapsed from the beginning of the business i to the survey end.[15] For a survived business, which has not been abandoned by the end of the survey, its true duration is either infinity if it succeeds or a finite number greater than η_i if is abandoned after the survey end. In either case, using η_i as the duration of this survived business leads to a downward bias of duration, which is defined as the right-censoring bias. Obviously, such a bias only applies to survived businesses but not the abandoned ones.

The business survives in the sample with probability $1 - P_A(\eta_i)$. If we observe that Business i is abandoned t_i years from its entry, then the duration T is between year $t_i - 2$

[14] In (14), $\mathbb{P}(\cdot)$ is the probability operator and "$\wedge$" is the minimum operator, i.e., $x_1 \wedge x_2 = \min\{x_1, x_2\}$.

[15] In the sample, different businesses have different censoring time for different businesses mainly because the beginning time of individual businesses is different.

and t_i. The probability for this outcome is $P_A(t_i) - P_A(t_i - 2)$. The log-likelihood function for the duration of each business i is

$$LL(t_i, \eta_i; \pi, \lambda, k, \theta) = \mathbb{I}_{\{t_i<\eta_i\}} \log\left[P_A(t_i) - P_A(t_i - 2)\right] + \mathbb{I}_{\{t_i=\eta_i\}} \log\left(1 - P_A(\eta_i)\right). \quad (15)$$

Here, $\mathbb{I}_{\{\cdot\}}$ is an indicator function, which is equal to one if the event in the subscript is realized and zero otherwise. The second term on the right accounts for the right-censoring bias of duration.

To be consistent with the model, we require that the average prior of entrepreneurs to be the same as the average quality of businesses in the data:

$$\mathbb{E}_0(p_0) = \pi. \quad (16)$$

This condition rules out entrepreneurial overconfidence (Cooper, Woo, and Dunkelberg (1988); Arabsheibani, de Meza, Maloney, and Pearson (2000); and Bernardo and Welch (2001)).[16]

We infer the distribution of p_0 as a function of (p^*, T) from (11):

$$p_0 = \left[1 + \left(\frac{1}{p^*} - 1\right) e^{-\lambda T}\right]^{-1}. \quad (17)$$

Therefore, we can rewrite (16) explicitly as:

$$\pi = \mathbb{E}_0(p_0) = \int_0^\infty \left[1 + \left(\frac{1}{p^*} - 1\right) e^{-\lambda s}\right]^{-1} \phi(s; k, \theta)\, ds. \quad (18)$$

Equations (17) and (18) depend on the belief threshold p^*, which in turn depends on the earnings premium of successful businesses α_H as in (10). Because α_H is not directly observed in sample, we infer it from the in-sample average of the survivors' earnings premium, α_S, which is a biased estimate of α_H. First, we use propensity-score matching to calibrate the value of α_S. With this calibrated value of $\alpha_S = \$3,312$ in specification (4) of Table A1, we then it link back to the unobserved α_H by using the following equation:

$$\alpha_S = -\alpha_L + (\alpha_H + \alpha_L)\left[\frac{\sum_{i=1}^{N_S} \eta_i \varphi_H(\eta_i)}{\sum_{i=1}^{N_S} \eta_i}\right], \quad (19)$$

where $\varphi_H(\eta_i)$ is the number of years Business i receives successful earnings (i.e., successful

[16] This condition also rules out other behavioral distortions. See Appendix C.

years) divided by η_i, and N_S is the total number of surviving businesses. (See Appendix C for its derivation.)

In summary, our estimation problem is to choose $\left(\widehat{\pi}, \widehat{\lambda}, \widehat{k}, \widehat{\theta}, \widehat{\alpha}_H\right)$ to solve the following optimization problem:

$$\max \sum_{i=1}^{N} LL\left(t_i, \eta_i; \pi, \lambda, k, \theta\right) , \tag{20}$$

subject to equations (18) and (19).

Estimation Results Table 3 reports the estimation results. The estimated probability that a business in the sample is of high-quality, π, is 32% . The estimated value of λ is 0.136, which implies that a high-quality business takes on average 7 years to succeed.

A successful entrepreneur's earnings premium, $\alpha_H = \mu_H - \mu_W$, is \$9,316 per year, translating into 26.5% of the average salaried worker's wage μ_W. Thus, entrepreneurs expect an increase of expected earnings from $\mu_L = \$30,189$ to $\mu_H = \alpha_H + \mu_W = \$43,569$ once they succeed, which is about 41% of μ_W.

Note that $\alpha_H = \$9,316$ is much higher than the average survivor premium, $\alpha_S = \$3,312$ because surviving is not the same as succeeding. As duration is right-censored, the survived businesses include those with low expected earnings μ_L but are still experimenting, additional to successful businesses. As a result, the average survivor premium α_S is lower than the success premium α_H.

Table 3: Estimation Results

S.D. denotes the standard deviation of the estimators.

Parameter	Definition	Value	S.D.
π	Probability of high-quality businesses	0.320	0.026
λ	Intensity that a high-quality business is revealed	0.136	0.040
α_H	Successful entrepreneurs' pay premium $\mu_H - \mu_W$	9,316	365
k	Scale of the Gamma distribution of duration T	0.680	0.042
θ	Rate of the Gamma distribution of duration T	0.149	0.015

Distribution of duration. The last two estimators in Table 3, k and θ, describe the distribution of duration T. In Panel A of Figure 2, we use these parameter values to plot the distribution of duration T, which follows a Gamma distribution with a fat right tail (with a kurtosis 11.8). The expected duration T is 4.58 years, longer than the empirical conditional duration of abandoned businesses, which is 2.97 years. (See Figure 1.) This suggests that right-censoring induces substantial downward bias of observed duration.

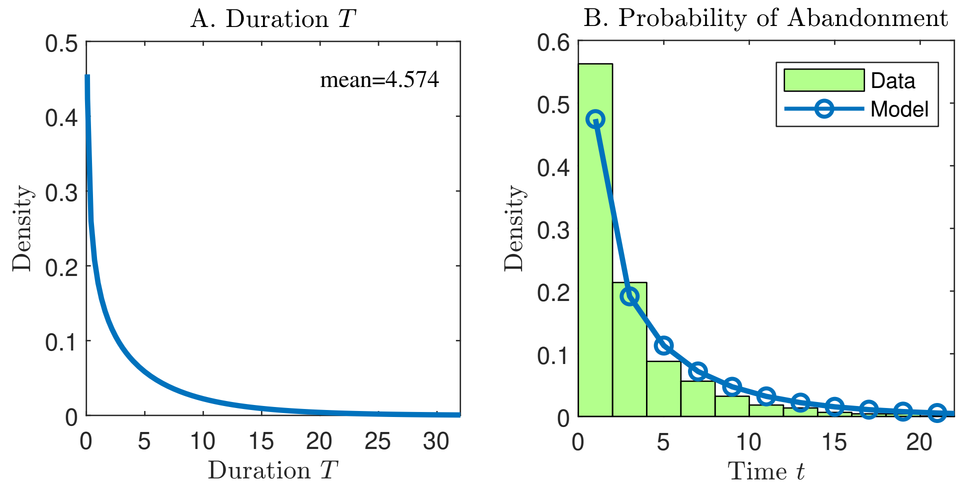

Figure 2: Distribution of duration T and probability of abandonment. In Panel A, the density function for duration T is a Gamma distribution $\phi(T; k, \theta)$ with $k = 0.680$ and $\theta = 0.149$. In Panel B, the probability of abandonment in data is the empirical density of abandoned businesses. The estimated density is $P_A(t) - P_A(t-2)$, which is the probability that a business is abandoned between time $t-2$ and t. (P_A is defined in (14), which gives the probability that a business is abandoned before time t.)

We validate our model's fitness of data by comparing the probability of abandonment observed in data with our estimated distribution $P_A(t) - P_A(t-2)$ in Panel B. Overall, the estimated model fits the data well.

Distribution of Prior. Figure 3 plots the distribution of prior p_0, which is indirectly inferred from the distribution of duration T using (17). The belief threshold p^* is 0.481, derived from Proposition 1 with our estimated parameter values. This threshold is the lower bound of the prior as we only observe entrepreneurs who enter into self-employment with priors greater than the threshold p^*. Similar to the duration T, the prior's distribution exhibits a fat tail with a kurtosis of 6.5.

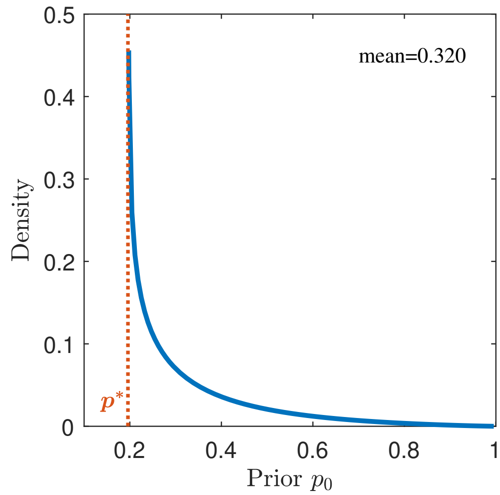

Figure 3: Probability density function of prior p_0. We indirectly infer the distribution of p_0 from the distribution of duration T by using (17). The belief threshold p^* is 0.196.

4.3 Net Certainty-Equivalent Value of Entrepreneurship

In Section 3, we have defined the net certainty-equivalent value of entrepreneurship as the net gain to be an entrepreneur measured in her certainty equivalent wealth, which is optimization-based and hence explicitly takes into account effects of risks, belief updating, and the opportunity cost of being a salaried worker.

In Figure 4, we plot the net certainty-equivalent value $m(p)$ with our estimated parameter values. Panel A shows that $m(p)$ is increasing and convex in belief p in $[p^*, 1]$. The higher the perceived quality of the business (i.e., a higher belief p), the higher $m(p)$. As the belief approaches one, the entrepreneur rationally never quits and her net certainty equivalent value, $m(1)$, is equal to \$140,927.[17] The convexity of $m(p)$ in belief p is due to the option value of experimentation.[18]

At the belief threshold p^*, both $m(p)$ and its slope are zero, because the entrepreneur optimally chooses to stop experimenting at p^*. These results correspond to the boundary conditions in Proposition 1.

Panel B of Figure 4 plots the net certainty equivalent value of entrepreneurship as a function of the time to exit z, where we define $\widehat{m}(z) \equiv m(p(z))$. Here, $\widehat{m}(z)$ is strictly increasing over time to exit, z. The function is initially convex when z is small and then

[17]Note that $m(1)$ is not the same as δ_H/r, as a high quality business takes time to become successful. See Appendix B for a detailed derivation of $m(1)$.

[18]However, in general, because the entrepreneur is risk-averse and markets are incomplete, her certainty-equivalent wealth may be concave in p. This could be the case when the entrepreneur is sufficiently risk averse, for example.

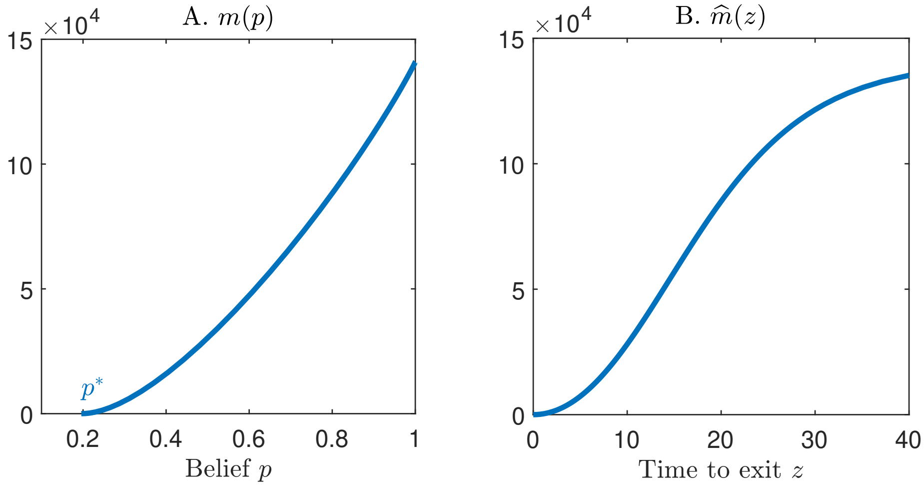

Figure 4: Net certainty-equivalent value of entrepreneurship. Panel A plots $m(p)$ and Panel B plots $\widehat{m}(z) = m(p(z))$ as a function of time to exit $z = T - t$.

concave when z is large. This is because the private value m is a convex function of the current belief p but the convexity of $p(z)$ depends on the value of z.[19]

Panel B provides an important insight about the relation between duration and entrepreneurial performance. In the model, conditioning on eventual exit, the longer the duration is, the higher the prior the entrepreneur had when she entered into the business, and the higher the starting private value of entrepreneurship $m(p_0)$. This relation emphasizes the role of duration analysis in entrepreneurship. It also suggests why neglecting the right censoring on duration can cause biases in the value of entrepreneurship and in the cross-sectional average earnings. We will study these points in more details in next section.

5 Downward Biases of Entrepreneurial Earnings

In this section, we show that the downward bias of average entrepreneurial earnings caused by right censoring on duration is quantitatively significant by conducting counterfactual experiments.

Observed earnings as functions of censoring time. We first express the cross-sectional average of entrepreneurial earnings as functions of the censoring time η. Following Manso (2016), we consider both the cross-sectional and the lifetime mean of the self-employment earnings. The former measures only the entrepreneur's earnings during

[19]As $p''(z) = \lambda^2 p(1-p)(1-2p)$, $p(z)$ is convex when $p < 1/2$ and concave when $p > 1/2$.

her self-employment spell, and the latter includes both her earnings during self-employment and the wages earned from her salaried job after she quits self-employment.

We define the excess earnings as the average entrepreneurial earnings minus the expected salaried job wage μ_W. We use $\Psi_j^k(\eta)$ to denote this excess earnings scaled by μ_W, where the superscript refers to the cross-sectional and the lifetime mean of earnings, $k \in \{\mathrm{CS}, \mathrm{LT}\}$, and the subscript refers to abandoned and survived businesses, $j \in \{A, S\}$.

Let $\Psi_S(\eta)$ denote the scaled excess earnings for survived businesses with both cross-sectional and lifetime mean as their self-employment spell and the censoring time are the same. Let $\varphi_H(\eta)$ denote the fraction of time in the business that an entrepreneur's business earns high-quality income. A survived business earns in excess to salaried job wage α_H with $\varphi_H(\eta)$ fraction of time or $-\alpha_L$ with the remaining $1-\varphi_H(\eta)$ fraction of time in expectation. Therefore, we have

$$\Psi_S(\eta) \equiv \varphi_H(\eta) \frac{\alpha_H}{\mu_W} - (1 - \varphi_H) \frac{\alpha_L}{\mu_W} . \tag{21}$$

Similarly, for abandoned businesses, the cross-sectional and the lifetime mean of the scaled excess earnings are:

$$\Psi_A^{\mathrm{CS}}(\eta) = -\frac{\alpha_L}{\mu_W} , \tag{22}$$

$$\Psi_A^{\mathrm{LT}}(\eta) = -\varphi_L(\eta) \frac{\alpha_L}{\mu_W} . \tag{23}$$

Here, $\varphi_L(\eta) \equiv \frac{1}{\eta} \mathbb{E}(T | T < \eta \wedge \tau)$ denotes the fraction of lifetime an entrepreneur receiving the low expected experimenting earnings, conditioning on her exit before η.

The lifetime mean of the scaled unconditional excess earnings is

$$\Psi^{\mathrm{LT}}(\eta) = P_A(\eta) \Psi_A^{\mathrm{LT}}(\eta) + (1 - P_A(\eta)) \Psi_S(\eta) , \tag{24}$$

where $P_A(\eta)$ is the probability that they are abandoned before time η, given in (14). The corresponding cross-sectional mean is

$$\Psi^{\mathrm{CS}}(\eta) = \frac{P_A(\eta) \varphi_L(\eta) \Psi_A^{\mathrm{CS}}(\eta) + (1 - P_A(\eta)) \Psi_S(\eta)}{P_A(\eta) \varphi_L(\eta) + (1 - P_A(\eta))} \tag{25}$$

$$= \frac{\Psi^{\mathrm{LT}}(\eta)}{1 - [1 - \varphi_L(\eta)] P_A(\eta)} . \tag{26}$$

(See Appendix C for full explicit representation of all these scaled excess earnings.)

Bias of earnings. As η goes to the infinity, right censoring on duration disappears and thus $\lim_{x\to\infty}\Psi_j^k(x)$ is the scaled excess earnings with uncensored duration. The bias of scaled earnings due to the right-censoring on duration is the difference between the average scaled excess earnings given the current censoring time η, $\Psi_j^k(\eta)$, and its corresponding limit:

$$\mathrm{Bias}_j^k(\eta) \equiv \Psi_j^k(\eta) - \lim_{x\to\infty}\Psi_j^k(x)\ . \tag{27}$$

Panels A and B of Figure 5 plot various $\Psi_j^k(\eta)$ (conditional on status j) and the unconditional $\Psi^k(\eta)$ as functions of the censoring time η, respectively. The most important message in Figure 5 is that average earnings of entrepreneurship are strictly increasing with the censoring time η due to positive correlation between earnings and duration.[20] It also shows that when the censoring time η is long enough, observed average of the excess earnings of entrepreneurship becomes positive.

This monotonicity of these earnings suggests that $\mathrm{Bias}_j^k(\eta) < 0$, i.e., right-censoring on duration leads to a downward bias of average earnings. Such a result is intuitive: right-censoring on duration means that the observations with older business age are dropped, which happen to be the ones with better entrepreneurial performance. It further implies that the shorter the censoring time η, the more severe the downward bias of average earnings, as confirmed by the monotonicity of the scaled excess earnings.

Comparing with biases in Manso (2016). The bias of average earnings due to right-censoring on duration complements the experimentation and the survivorship biases in Manso (2016).

We first represent the two biases in Manso (2016) in our model. The experimentation bias arises because the cross-sectional mean of earnings neglects the salaried job earnings after an entrepreneur quits her experimentation. It corresponds to $\Psi_A^{\mathrm{CS}} - \Psi_A^{\mathrm{LT}} = (1-\varphi_L)\Psi_A^{\mathrm{CS}} < 0$, with $\varphi_L(\eta) \in (0,1]$ being the fraction of lifetime an entrepreneur spending in experimentation and $\Psi_A^{\mathrm{CS}} < 0$. (See (23).) As in Panel A of Figure 5, the cross-sectional mean of scaled excess earnings of the abandoned ones (Ψ_A^{CS}) biases downward the lifetime mean (Ψ_A^{LT}).

The survivorship bias states the earnings of survived businesses is over-weighted in the cross-sectional mean ($\frac{1-P_A(\eta)}{P_A(\eta)\varphi_L(\eta)+1-P_A(\eta)}$ in (25)) than in the lifetime mean ($1-P_A$ in (24)) of unconditional earnings. This arises again because the cross-sectional mean leaves out the post-exit earnings of failed entrepreneurship, i.e., $\varphi_L(\eta) \le 1$. As the survived businesses

[20] The exception is the cross-sectional mean for abandoned businesses Ψ_A^{CS}, which is a constant.

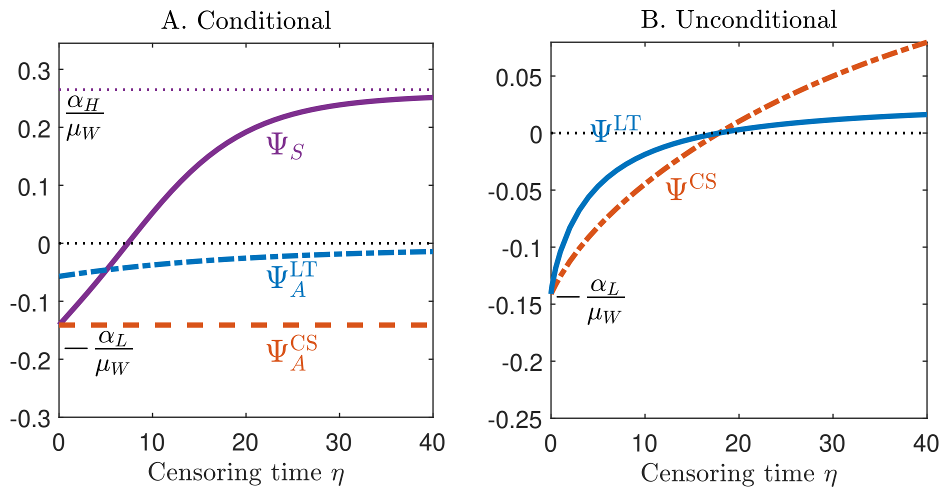

Figure 5: Scaled excess earnings as functions of censoring time η. Panel A plots the scaled excess earnings conditional on survival status, Ψ_j^k, for the cross-sectional ($k = \mathrm{CS}$) or the lifetime ($k = \mathrm{LT}$) mean of self-employment earnings and for the abandoned ($j = A$) and the survived ($j = S$) businesses. Panel B plots the cases for unconditional scaled excess earnings for all businesses, which have no subscript. See Appendix D for detailed values of the limits of scaled excess earnings.

earn more than the abandoned ones in data and in the model, the survivorship bias can lead to overestimation in the cross-sectional mean of excess earnings relative to the lifetime mean.

The two biases together capture the difference between the cross-sectional and the lifetime mean of unconditional entrepreneurial earnings, which is measured by $\Psi^{\mathrm{CS}} - \Psi^{\mathrm{LT}}$. By (26), Ψ^{CS} is Ψ^{LT} divided by a function strictly between zero and one. Therefore, $\left|\Psi^{\mathrm{CS}}\right| \geq \left|\Psi^{\mathrm{LT}}\right|$ and they have the same positive and negative signs, as in Panel B of Figure 5. This result is consistent with Proposition 1 in Manso (2016).

Additional to these two biases, we show that the new bias defined in (27) also contributes to the estimated average earnings when duration is right-censored. For example, the unbiased estimate of the average entrepreneurial earnings is $\lim_{\eta\to\infty} \Psi^{\mathrm{LT}}(\eta)$. The two biases in Manso (2016) capture the different between the cross-sectional mean $\Psi^{\mathrm{CS}}(\eta)$ and the lifetime mean $\Psi^{\mathrm{LT}}(\eta)$, which is the difference between the red dot-dash and the blue solid curves in Panel B of Figure 5. Our new bias further states that with limited censoring time η, the current lifetime mean $\Psi^{\mathrm{LT}}(\eta)$ can still be significantly different from its long-run average, which is uncensored on duration. In Figure 5, the new bias is the distance of the blue solid curve to it limiting value (0.03).

Empirical earnings growth. Table 4 reports empirical evidence that the earnings of survived businesses outgrows that of their propensity-scored matched workers, confirming the positive correlation between duration and earnings in survived businesses. These results suffice to confirm the existence of downward bias of earnings due to the right-censoring of duration because duration of the abandoned businesses is fully observed and not grows when we extend the censoring time. (For additional evidence on earnings growth, please see Table A2 in Appendix A.)

Table 4: Earnings growth.

This table reports the earnings growth of survived self-employment (SE) businesses relative to that of salaried workers. Survived businesses are SE that remains active until the last available survey. The test sample consists only the survived entrepreneurial businesses and their propensity-scored matched workers in corresponding years. Because we only use even-year surveys, the earnings growth is bi-annual growth. Earnings growth is defined as $\text{earnings}_t - \text{earnings}_{t-2}$ and log growth is defined as $\log(1 + \text{earnings}_t) - \log(1 + \text{earnings}_{t-2})$. Control variables include age, race, sex, years of schooling, Rosenberg scales, AFQT scores, Rotter scores, year fixed effects, and business inception year fixed effects. Standard errors are clustered at the SE-matched-worker pairs. *, **, and *** denote 10%, 5%, and 1% statistical significance, respectively.

	(1) Earnings growth	(2) log Earnings growth
Self-employment:	2,259*	0.151*
	(1.90)	(1.67)
Sample size	2,373	2,373

A counterfactual experiment To quantify these biases in the current sample[21], we propose a counterfactual experiment. We ask the question what would happen if more surveys are available in the future years. The experiment results are reported in Table 5.

Panel A of Table 5 reports the moments in the original sample, while Panel B reports the limiting case without right-censoring on duration. Because the average duration in Panel B is significantly longer than that in Panel A, the current sample is subject to right censoring on duration.

Within the same panel, the differences between the cross-sectional and the lifetime mean account for the joint effects of the experimentation and the survivorship biases. In Panel A,

[21]Results in Figure 5 are not sufficient to quantify the biases in current sample because SE businesses in the sample start from different time and have different censoring time. Results in Table 4 only suggest the existence of downward bias on earnings but fails to quantify it because of the nonlinear relationship between earnings and duration.

Table 5: A counterfactual experiment extending the censoring time.

This experiment hypothetically extends the censoring time in the current sample of entrepreneurial businesses. $\triangle\eta$ denotes the number of years extended: Panel A with $\triangle\eta = 0$ represents the current sample statistics; Panel B with $\triangle\eta = +\infty$ is the case without censoring (as in our model); Panel C with $\triangle\eta = 16$ extends the censoring time by 16 years. Scaled excess earnings refers to $\Psi_j^k(\eta)$. Scaled annuity of net certainty-equivalent value is the annuity of the certainty-equivalent value, $rm(z)$, scaled by μ_W. The limited values are derived in Appendix D.

A. $\triangle\eta = 0$:	All SE	Survived	Abandoned
Percentage of businesses	100%	26.03%	73.97%
Average duration (years)	4.055	7.136	2.971
Scaled excess earnings (cross-sectional)	-0.043	0.094	-0.141
Scaled excess earnings (lifetime)	-0.017	0.094	-0.041
Scaled annuity of net certainty-equivalent value	0.028	0.116	0

B. $\triangle\eta = +\infty$:	All SE	Survived	Abandoned
Percentage of businesses	100%	11.39%	88.61%
Average duration (years)	$+\infty$	$+\infty$	4.574
Scaled excess earnings (cross-sectional)	0.265	0.265	-0.141
Scaled excess earnings (lifetime)	0.030	0.265	0
Scaled annuity of net certainty-equivalent value	0.029	0.256	0

C. $\triangle\eta = 16$:	All SE	Survived	Abandoned
Percentage of businesses	100%	12.19%	87.81%
Average duration (years)	6.820	27.070	4.008
Scaled excess earnings (cross-sectional)	0.025	0.238	-0.141
Scaled excess earnings (lifetime)	0.007	0.238	-0.025
Scaled annuity of net certainty-equivalent value	0.029	0.241	0

the joint effects account for a downward bias of 2.6% of μ_W per year. $(-4.3\% - (-1.7\%) = -2.6\%.)$

Besides the above two biases, we have the new bias due to right-censoring on duration when comparing Panel A and B. For example, the lifetime mean of the unconditional scaled excess earnings is significantly lower than the uncensored values in Panel B by 4.7% of μ_W, or \$1,652 per year. $(-1.7\% - 3.0\% = -4.7\%.)$ Analyses conditional on survival status reveal that such a bias mainly comes from underestimating the earnings of the survived businesses, the ones subject to right censoring on duration.

Combining all biases together, we find that the total downward bias accounts for 7.3% of μ_W, or \$2,567 per year. This total bias is economically substantial and pivotal, as it turns the average entrepreneurial earnings in excess to salaried workers from negative (-4.3% of μ_W) to positive (3.0% of μ_W).

The last row quantifies the scaled annuity of the net certainty-equivalent value of entrepreneurship. This value differs from the average excess earnings in the same Panel, especially in the current sample in Panel A. This is because the empirically observed earnings fails to take into account the option to abandon, the belief updating, the discount over time, and the risk-aversion.[22] All these suggest that empirical earnings may not reflect the rationally expected performance, especially when the duration of business is censored.

The average of such value for all businesses (survived businesses) in Panel A, 2.8% (11.6%), is lower than that in Panel B, 2.9% (25.6%) . This is because the incomplete information on business quality and uncertain arrival of earnings jump resolves gradually over time. The 2.9% of scaled net certainty equivalent value of entrepreneurship is an economically significant gain as it is the risk-adjusted net gain from entrepreneurial experimentation.

One may argue that such a large bias is due to the infinite time horizon. In response to that, we extend the censoring time by 16 years in Panel C, which virtually extends the average age in the last year survey from 51 to 67 (the social security retirement age).[23] Most of the previous results hold qualitatively. The unconditional scaled excess earnings is 2.5% during self-employment and 0.7% during lifetime, both of which are significantly higher than the original sample. Similarly, the conditional scaled excess earnings for lifetime income in Panel C is much higher than in the original sample, as they are subject to less downward bias due to the longer censoring time. Finally, the net certainty equivalent value of entrepreneurship is also higher than in Panel A.

Overall, our counterfactual experiment results suggest that extending the censoring time would raise the observed earnings. In particular, with sufficiently long censoring time, the unconditional excess earnings can become positive, which partially explains why people choose entrepreneurship despite of the observed negative excess earnings. [24]

[22]In the long run, the lifetime mean of scaled excess earnings converge to a value higher than the corresponding certainty-equivalent value. This is because the option to exit and ambiguous business quality resolves in the long run, but the earnings volatilities remain.

[23]See Internet Appendix Table IA.2 for the respondents' age in the last year of the self-employment business.

[24]Note that our counterfactual experiment extends the censoring time of SE businesses that are already in the sample without introducing new entries of SE businesses. Newly-entered businesses after the survey end will be likely to have the low experimentation earnings and simultaneously survive Therefore, they will be subject to the most severe right-censoring bias if included in the extended sample, which bias the mean earnings downward. This idea is similar to Manso (2016), which rules out new entries after the formation of the cohort of the SE businesses in 2002 and is thus immune from the new-entry issues.

Further discussion The wealth effect on entrepreneurial duration is not a focus of our paper but we can extend our framework to accommodate it.[25] In the Internet Appendix, we construct a dynamic model using our current theoretical framework but assuming the agents have a constant relative risk aversion utility, in which we remove the Brownian earnings volatility for tractability. We show that entrepreneurs starting with higher wealth tend to have longer duration higher average earnings and the positive correlation between duration and earnings persists in this model.

Risk aversion in our calibration is to match a relative risk aversion of two for entrepreneurs with median starting wealth. To match the duration moments in data, a higher risk aversion requires a higher successful premium α_H. This implies an even larger bias of earnings due to right-censoring on duration.

In our model, we assume that once the entrepreneur succeeds, she will be successful forever and the high expected earnings persists. Such an assumption on the persistence of the expected earnings is for tractability reason. In fact, as long as the expected earnings are persistent enough and rational entrepreneurs have options to quit, one will observe the positive correlation between earnings and duration, and most of our results will go through. Furthermore, if the successful high earnings lasts for a finite period, given the observed earnings profiles in data, a rational entrepreneur will demand an even higher successful pay premium to start her business. This implies an even larger gap between the observed average earnings and the successful pay premium, and thus a larger bias.[26]

6 Conclusions

Entrepreneurial duration and earnings are endogenously positively correlated. Such a correlation implies that right-censoring on duration, which is typical in survey data of entrepreneurship, biases the average earnings downward. While the bias of duration is taken into account in standard duration analyses, its effects on average earnings are often neglected. With quantitative analyses and counterfactual experiments, we show that such a bias in average earnings can be economically substantial, with which the motivation of entrepreneurship can be justified in a rational model despite the low observed entrepreneurial

[25] A literature on entrepreneurial survival has shown that relaxing financial constraints can affect entrepreneurs' performance and surviving probabilities. This literature includes, but is not limited to, Holtz-Eakin, Joulfaian, and Rosen (1994), Hurst and Lusardi (2004), Andersen and Nielsen (2012), Adelino, Schoar, and Severino (2015), Corradin and Popov (2015), and Schmalz, Sraer, and Thesmar (2017).

[26] Consider another model, after succeeding, the business can revert back to low income with some random arrival time. This in effect increases the discount rate after success and therefore the entrepreneur must require a higher successful pay premium to compensate this higher effective discount rate.

earnings.

For tractability reason, we assume that entrepreneurs observe the expected earnings instead of the realized earnings at the current stage. We also developed a more realistic model that the entrepreneurs learn from realized earnings. While the estimation procedure and quantitative results can be different, the positive relation between earnings and duration persists. Therefore, most of our qualitative results will survive. (Internet Appendix provides some details of this model.)

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Appendices

A Additional Empirical Results

Conditional earnings differences. We confirm with different regression approaches that survived businesses earn much higher earnings than the abandoned ones and may even outperform the salaried workers. Table A1 reports the results, with fixed effects and propensity score matching approaches, and on annual earnings and the logarithm of it. (See the Internet Appendix for detailed procedure of the propensity-score matching.) We use Specification (2) coefficients to calibrate α_L and α_S in the estimation section.

Table A1: Conditional earnings differences.

This table compares the average earnings of self-employment (SE) to that of salaried workers. We categorize the SE businesses that remain active until the last available survey as survived ones and others as abandoned. Annual earnings is in 2012 dollars. The fixed effects method (FE) are in (1) and (3), and the propensity-score matching method (PSM) are in (2) and (4), in which the coefficients give the entrepreneurial earnings relative to earnings of the matched control group. Control variables include age, race, sex, years of schooling, Rosenberg scales, AFQT scores, Rotter scores, year fixed effects, and business inception year fixed effects. Standard errors are clustered at individual level for FE and at SE-matched-worker pair level for PSM. *, **, and *** denote 10%, 5%, and 1% statistical significance, respectively.

	Earnings		Log Earnings	
	(1) FE	(2) PSM	(3) FE	(4) PSM
Abandoned	-4,775***	-4,958***	-1.467***	-1.566***
	(-4.50)	(-3.62)	(-16.02)	(-16.84)
Surv.-Aban.	6,489**	8,270**	0.296**	0.439***
	(2.42)	(2.01)	(2.07)	(3.01)
Sample size	55,642	8,319	55,642	8,319

Additional evidence on earnings growth. Table A2 reports additional empirical evidence on earnings growth. Panel A focuses on earnings growth and log earnings growth, where Table 4 uses results on survived businesses. Panel B studies how earnings and log earnings change when business age increases. The results on earnings in Panel B are consistent with Hamilton (2000) except that we further condition our results on survival status. Finally, we do not directly test on the relationship between earnings and duration because duration is biased.

Table A2: Earnings growth.

This table reports the earnings growth of self-employment (SE) businesses relative to that of salaried workers. We categorize the SE businesses that remain active until the last available survey as survived ones and others as abandoned ones. The samples consist only the entrepreneurial businesses and their propensity-scored matched workers in corresponding years. Earnings growth is defined as $\text{earnings}_t - \text{earnings}_{t-2}$ and log growth is defined as $\log(1 + \text{earnings}_t) - \log(1 + \text{earnings}_{t-2})$. Control variables include age, race, sex, years of schooling, Rosenberg scales, AFQT scores, Rotter scores, year fixed effects, and business inception year fixed effects. Standard errors are clustered at the SE-matched-worker pairs. *, **, and *** denote 10%, 5%, and 1% statistical significance, respectively.

A.	Earnings Growth			Log Earnings		
	Abandoned	Survived	All SE	Abandoned	Survived	All SE
Self-employment	-4,269***	2,259*	-1,092	-0.445	0.151*	-0.154**
	(-3.51)	(1.90)	(-1.28)	(-3.93)	(1.67)	(-2.12)
Sample size	2,526	2,373	4,899	2,526	2,373	4,899

B.	Earnings			Log Earnings Growth		
	Abandoned	Survived	All SE	Abandoned	Survived	All SE
Self-employment: a	-7,598***	-4,945	-7,899***	-1.550***	-1.339***	-1.571***
	(-3.44)	(-1.11)	(-3.66)	(-11.59)	(-7.08)	(-15.21)
Years since inception: b	648	-296	69	0.004	0.002	-0.002
	(1.20)	(-0.80)	(0.23)	(0.28)	(0.12)	(-0.20)
$a \times b$	459	1,298*	1,141**	-0.002	0.028	0.033**
	(0.71)	(1.88)	(2.17)	(-0.08)	(1.63)	(2.49)
Sample size	5,003	3,316	8,319	5,003	3,316	8,319

B Proofs in the Model

Derivation of Bayes rule, equation (4): Over a small time interval $\triangle$, the Bayes rule implies that the posterior probability $p_{t+\triangle}$ is

$$\begin{aligned} p_{t+\triangle} &\equiv \mathbb{P}\left[H \middle| \mu_{t+\Delta} = \mu_L\right] = \frac{\mathbb{P}\left[H \text{ and } \mu_{t+\Delta} = \mu_L\right]}{\mathbb{P}\left[\mu_{t+\Delta} = \mu_L\right]} \\ &= \frac{\mathbb{P}\left[\mu_{t+\Delta} = \mu_L \middle| H\right] p_t}{\mathbb{P}\left[\mu_{t+\Delta} = \mu_L | H\right] p_t + \mathbb{P}\left[\mu_{t+\Delta} = \mu_L | L\right](1 - p_t)} \\ &= \frac{(1 - \lambda\Delta) \times p_t}{(1 - \lambda\Delta) \times p_t + 1 \times (1 - p_t)} = \frac{(1 - \lambda\triangle)\, p_t}{1 - \lambda\triangle p_t}, \end{aligned}$$

where the last but one equation uses $\mathbb{P}_t\left[\mu_{t+\Delta}=\mu_L \middle| H\right]=1-\lambda\Delta$ and $\mathbb{P}_t\left[\mu_{t+\Delta}=\mu_L \middle| L\right]=1$.

Taking the limit as $\triangle \to 0$, we obtain the equation (4). Q.E.D.

Derivation of $\overline{V}$ in absorbing states $s_t \in \{0,2\}$. $\overline{V}(x;\mu,\sigma)$ is the value function in the absorbing states given the current wealth level x, an earnings rate μ, and an earnings volatility σ. Here, μ and σ are constant parameters. The individual solves the following problem,

$$\overline{V}(x;\mu,\sigma) \equiv \max_c \mathbb{E}\left[\int_0^\infty e^{-rt} u(c_t)\, dt\right] \tag{B.1}$$

given the wealth evolution in Equation (2). The Hamilton-Jacobi-Bellman (HJB) equation is thus

$$r\overline{V} = \max_c \left\{u(c) + (rx+\mu-c)\,\overline{V}_w + \frac{1}{2}\sigma^2 \overline{V}_{ww}\right\} \tag{B.2}$$

We have the first-order condition for consumption

$$u'(c) = \overline{V}_x\,, \tag{B.3}$$

and the transversality condition

$$\lim_{t\to\infty} e^{-rt} u'(c_t)\, x_t = 0\,. \tag{B.4}$$

Guess the solution is

$$\overline{V}(x;\mu,\sigma) = -\frac{1}{\gamma r} \exp\left[-\gamma r\left(x + \frac{\mu}{r} - \frac{1}{2}\gamma\sigma^2\right)\right]\,. \tag{B.5}$$

One can verify that the above equation is the solution of (B.2) and (B.4). Q.E.D.

Details about the value function V The HJB equation of the entrepreneur is

$$\begin{aligned} rV(x,p) = \max_{c,T}\ & u(c) + (rx_t+\mu_L-c)\,V_x(x,p) + \frac{1}{2}\sigma_L^2 V_{xx}(x,p) \\ & + \lambda p\left[\overline{V}(x;\mu_H,\sigma_H) - V(x,p)\right] - \lambda p(1-p) V_p(x,p)\,. \end{aligned} \tag{B.6}$$

The left side is the total return of her value function $rV(x,p)$. The right side is the sum of the instantaneous utility payoff $u(c)$ and the instantaneous expected changes of her value function. The second and the third terms are the standard savings and Brownian risk effects due to changes in wealth w. The fourth term gives the expected increase in the value function if the business succeeds, which is the probability λp of succeeding per unit of time multiplying the value increase $\overline{V}(x;\mu_H,\sigma_H) - V(x,p)$. The last term measures the impact of learning on the entrepreneur's value function.

In addition to choosing consumption, the agent also chooses the optimal timing T to terminate her experimentation. There is an optimal belief boundary $p^*(x)$ such that the agent exits when her posterior is lower than $p^*(x)$. At the belief boundary, the agent must be indifferent between the salaried job and staying in business, i.e.,

$$V(x, p^*(x)) = \overline{V}(w;\mu_W,\sigma_W). \tag{B.7}$$

For the belief threshold $p^*(w)$ to be optimal, the following smooth-pasting conditions must also be satisfied (see, for example, Krylov (1980) and Dumas (1991)):

$$\begin{aligned} V_x(x, p^*(x)) &= \overline{V}_x(x;\mu_W,\sigma_W), &\text{(B.8)}\\ V_p(x, p^*(x)) &= 0. &\text{(B.9)}\end{aligned}$$

Equation (B.8) states that the marginal values of wealth are the same right before and after exit. Additionally, the value function must have zero sensitivity to the change of the boundary $p^*(w)$, as in equation (B.9). Otherwise, by slightly changing the belief threshold $p^*(x)$, the agent is better off.

Proof of Propositions 1: By the first-order condition $u'(c) = V_x(x,p)$ and the CARA utility specification, we have

$$u(c) = -\frac{1}{\gamma}V_x(x,p) \tag{B.10}$$

at the optimum.

Using the conjectured functional form given in (5) yields

$$\begin{aligned} c &= r\left(x + \frac{\mu_W}{r} - \frac{1}{2}\gamma\sigma_W^2 + m(p)\right) &\text{(B.11)}\\ V_x(x,p) &= \exp(-\gamma c) = -\gamma r V(x,p), &\text{(B.12)}\\ V_{xx}(x,p) &= -\gamma r \exp(-\gamma c) = -\gamma r V_x(x,p) &\text{(B.13)}\\ V_p(x,p) &= m'(p)\exp(-\gamma c) = m'(p)V_x(x,p) &\text{(B.14)}\end{aligned}$$

Using equations (B.5) and (5) yields

$$\overline{V}\left(x;\mu_H,\sigma_H\right)=-\frac{V_x\left(x,p\right)}{\gamma r}\exp\left[-\gamma\left(\delta_H-rm(p)\right)\right]. \tag{B.15}$$

Plugging the above equations (B.10)-(B.15) into (B.6) gives

$$\begin{aligned}\frac{-V_x}{\gamma} &= \frac{-V_x}{\gamma}-\left[rm\left(p\right)+\mu_W-\mu_L-\frac{1}{2}r\gamma\sigma_W^2\right]V_x-\frac{1}{2}r\gamma\sigma_L^2V_x-\lambda p(1-p)m'\left(p\right)V_x \\ &\quad +\lambda p\frac{V_x}{\gamma r}\left[1-e^{-\gamma(\delta_H-rm(p))}\right]. \end{aligned}\tag{B.16}$$

Canceling V_x gives the equation (9).

Given the conjectured form of the value function (5), the value-matching condition (B.7) and the smooth pasting-conditions (B.8)-(B.9) become

$$m(p^*) = 0, \tag{B.17}$$

$$m'(p^*) = 0. \tag{B.18}$$

Plugging (B.17)-(B.18) into (9) give

$$0=\frac{\delta_L}{\lambda p^*}-\frac{1}{\gamma r}\left(1-e^{-\gamma\delta_H}\right). \tag{B.19}$$

Re-arranging yields (10). To ensure that the posterior belief threshold lies within the interesting region $(0,1)$, we assume $0<r\delta_L<\frac{\lambda}{\gamma}\left(1-e^{-\gamma\delta_H}\right)$. Q.E.D.

Comparative statics of p^* The comparative statics of p^* over other parameters is obvious except γ. To investigate that, we first consider special cases where $\sigma_L=\sigma_H=\sigma_W$. In this case, γ does not affect the risk-adjusted earnings differentials as the incremental variances are zero for both experimentation and success stage. We have

$$\begin{aligned}\frac{\partial p^*}{\partial\gamma} &= \frac{r\delta_L}{\lambda\left(1-e^{-\gamma\delta_H}\right)}-\frac{\gamma r\delta_L\delta_H}{\lambda\left(1-e^{-\gamma\delta_H}\right)^2}e^{-\gamma\delta_H} \\ &= -\frac{r\delta_L}{\lambda\left(1-e^{-\gamma\delta_H}\right)^2}\left(\gamma\delta_He^{-\gamma\delta_H}+e^{-\gamma\delta_H}-1\right). \end{aligned}\tag{B.20}$$

To show $\partial p^*/\partial\gamma>0$, we only need to show the function

$$h(y)\equiv ye^{-y}+e^{-y}-1<0 \text{ for all } y>0. \tag{B.21}$$

This is true since $h(0) = 0$ and $h'(y) = -ye^{-y} < 0$ for $y > 0$.

Then we allow $\sigma_L \geq \sigma_W$ and $\sigma_H \geq \sigma_W$. In this case a higher γ increases δ_L and decreases δ_H, which further increases p^*. Therefore, p^* is increasing with γ.

If we do not have $\sigma_L \geq \sigma_W$ and $\sigma_H \geq \sigma_W$, the effects of γ on p^* can be ambiguous.

Q.E.D.

Derivation of equation (11): Equation (4) can be rewritten as

$$\frac{dt}{dp_t} = -\frac{1}{\lambda p_t (1 - p_t)} = -\frac{1}{\lambda}\left(\frac{1}{p_t} + \frac{1}{1 - p_t}\right) .$$

It has the solution,

$$t - t_0 = -\frac{1}{\lambda}\left[\ln p_t - \ln (1 - p_t)\right] ,$$

where t_0 is some constant. Let $p_t = p^*$, we get the full duration T:

$$T - t_0 = -\frac{1}{\lambda}\left[\ln p^* - \ln (1 - p^*)\right] ,$$

Subtracting the previous two equations and note that $f = T - t$, we have equation (11). Q.E.D.

C Derivations in Estimation

The cumulative abandoning premium $P_A(t)$ in (14) The probability that a business is abandoned before time t, $P_A(t)$ has two components. When the underlying quality of the business is low, we have

$$\mathbb{P}(T \leq t | L) = \Phi(t; k, \theta) ,$$

where $\Phi(t; k, \theta)$ is the cumulative distribution function of T. When the underlying quality of the business is high, the business is abandoned when $T \leq t \wedge \tau$. In the Internet Appendix, we derive that this case has probability:

$$\mathbb{P}(T \leq t \wedge \tau | H) = \left(\frac{\theta}{\theta + \lambda}\right)^k \Phi(t; k, \theta + \lambda) .$$

Therefore, the complete representation of (14) is

$$P_A(t) = (1 - \pi)\Phi(t; k, \theta) + \pi\left(\frac{\theta}{\theta + \lambda}\right)^k \Phi(t; k, \theta + \lambda) .$$

The model-implied survivor premiums $\alpha_S(\cdot)$ in (19) We here derive the average earnings of survivors. First of all, the probability of businesses surviving by time η is $1 - P_A(\eta)$, where P_A is defined in (C). These survivors may receive different expected earnings. If the businesses are of high quality and succeed before the time η and the exit time T, they receive mean earnings premium α_H after success. These successful businesses account for $\mathbb{P}(\tau < \eta \wedge T, H) / (1 - P_A)$ of the survivors.

However, the successful businesses do not receive the successful earnings from the beginning. Before the random arrival time τ, they receive the experimentation earnings. The expected length of time receiving low experimentation earnings for these businesses is $\mathbb{E}(\tau|\tau < \eta \wedge T, H)$. Therefore, the average time these businesses receiving expected earnings μ_H before time η is $\eta - \mathbb{E}(\tau|\tau < \eta \wedge T, H)$.

Putting these two parts together, we obtain the expected proportion of business-year observations that the survived entrepreneurs receive successful earnings,

$$\varphi_H(\eta) \equiv \frac{\mathbb{P}(\tau < \eta \wedge T, H)}{1 - P_A(\eta)} \left[1 - \frac{1}{\eta}\mathbb{E}(\tau|\tau < \eta \wedge T, H)\right]. \tag{C.1}$$

So the expected excess earnings of a business surviving by the censoring time η is

$$\alpha_S(\eta) = -\alpha_L(1 - \varphi_H(\eta)) + \alpha_H \varphi_H(\eta)\ . \tag{C.2}$$

Taking average across all surviving individual Business i with different censoring time η_i, we get (19).

In Section IA.3 of the Internet Appendix, we derive that

$$\begin{aligned}&\mathbb{P}(\tau < \eta \wedge T, H) = \pi\mathbb{P}(\tau < \eta \wedge T|H)\\ =&\pi\left[1 - \left(\frac{\theta}{\theta+\lambda}\right)^k \Phi(\eta; k, \theta+\lambda) - e^{-\lambda\eta}(1 - \Phi(\eta; k, \theta))\right].\end{aligned}$$

And

$$\begin{aligned}&\mathbb{E}(\tau|\tau < \eta \wedge T|H)\\ =&\frac{1}{\mathbb{P}(\tau < \eta \wedge T, H)}\left[\frac{1}{\lambda} - \left(\eta + \frac{1}{\lambda}\right)e^{-\lambda\eta}(1 - \Phi(\eta; k, \theta))\right.\\ &\left.-\frac{1}{\lambda}\left(\frac{\theta}{\theta+\lambda}\right)^k \Phi(\eta; k, \theta+\lambda) - \frac{k}{\theta+\lambda}\left(\frac{\theta}{\theta+\lambda}\right)^k \Phi(\eta; k+1, \theta+\lambda)\right].\end{aligned}$$

Plugging them in, we get the explicit expression of $\varphi_H(\eta)$ and hence the in-sample survivor earnings premium α_S.

Details about the constraint (16)**.** We want to show that the constraint (16) rules out not only overconfidence but also a few other behavioral distortions.

Consider equations (10) and (11), which show how different behavioral distortions can affect the duration of businesses. For example, consider overconfidence, non-pecuniary benefits, and risk tolerance.[27] Overconfidence can be viewed as a distribution of priors with a mean higher than the average quality in the data. Everything equal, this will lead to prolonged duration according to (11). The unobserved non-pecuniary benefits can be modeled as decreasing the experimentation costs α_L and increasing the success premium α_H by the same amount. The risk tolerance is a reduction in the risk aversion γ, which can even be negative. These two reduce the belief threshold p^* as in (10), inducing a shrinkage in duration. Therefore, despite originating from different sources of behavioral distortions, lower overconfidence, higher non-pecuniary benefits, and larger risk tolerance lead to an equivalent reduction in duration.

The net certainty equivalent value $\boldsymbol{m(1)}$. When $p = 1$, the business quality is high for sure. However, there is still uncertainty about the random arrival time of success. We want to derive $m(1)$ such that $V(x,1) = \overline{V}(x + m(1); \mu_W, \sigma_W)$.

Using the same technique as in the proof of Proposition 1, we have

$$0 = \gamma r \left[\delta_L + rm(1)\right] V(x,1) + \lambda \left[\overline{V}(x; \mu_H, \sigma_H) - V(x,1)\right]$$

Plug in $\overline{V}(x; \mu_H, \sigma_H)$ and $V(x,1) = \overline{V}(x + m(1); \mu_W, \sigma_W)$, we have

$$1 - \frac{\gamma r}{\lambda}\left(\delta_L + rm(1)\right) = \frac{\overline{V}(x; \mu_H, \sigma_H)}{\overline{V}(x + m(1); \mu_W, \sigma_W)} = \exp\left[\gamma\left(rm(1) - \delta_H\right)\right] .$$

Then $m(1)$ is the solution of the above equation.

[27]For example, Cooper, Woo, and Dunkelberg (1988), Arabsheibani, de Meza, Maloney, and Pearson (2000), and Bernardo and Welch (2001) explore the role of overconfidence; Blanchflower and Oswald (1992) and Catherine (2017) study non-pecuniary benefits; Moskowitz and Vissing-Jorgensen (2002) discusses briefly risk tolerance, along with other possible behavioral distortions.

D Derivation in Section 5

Deriving the explicit expressions of Ψ_j^k. For the scaled excess earnings of the survived businesses Ψ_S, we only need to derive the explicit expression of $\varphi_H(\eta)$, which is given in (C.1).

For the cross-sectional mean of the scaled excess earnings, it is simply $\Psi_S^{\mathrm{CS}} = -\alpha_L$. For its lifetime mean, we need to derive φ_L. A business is abandoned in two possible cases, depending on its business quality, low or high. Therefore,

$$\mathbb{E}\left(T|T<\eta\wedge\tau\right) = \frac{\mathbb{P}\left(T<\eta,L\right)\mathbb{E}\left(T|T<\eta,L\right)+\mathbb{P}\left(T<\eta\wedge\tau,H\right)\mathbb{E}\left(T|T<\eta\wedge\tau,H\right)}{\mathbb{P}\left(T<\eta,L\right)+\mathbb{P}\left(T<\eta\wedge\tau,H\right)}\,.$$

The denominator is simply $P_A\left(\eta\right)$. The numerator is derived in Section IA.3 in the Internet Appendix. We have,

$$\mathbb{E}\left(T|T<\eta\wedge\tau\right) = \frac{k}{\theta}\frac{\left(1-\pi\right)\Phi\left(\eta;k+1,\theta\right)+\pi\left(\frac{\theta}{\theta+\lambda}\right)^{k+1}\Phi\left(\eta;k+1,\theta+\lambda\right)}{\left(1-\pi\right)\Phi\left(\eta;k,\theta\right)+\pi\left(\frac{\theta}{\theta+\lambda}\right)^{k}\Phi\left(\eta;k,\theta+\lambda\right)}$$

Plugging this back into $\varphi_L\left(\eta\right) = \frac{1}{\eta}\mathbb{E}\left(T|T<\eta\wedge\tau\right)$, we have the explicit representation of φ_L and hence the lifetime mean of the scaled excess earnings for those who quit, Ψ_A^{LT} in (23).

Furthermore, given all the conditional mean, we have the explicit expressions for the unconditional scaled excess earnings (24) and (26).

Deriving limited values. The limits of scaled excess earnings are:

$$\begin{aligned}
\Psi_S(0) = \Psi_A^{\mathrm{CS}}(0) = \Psi^{\mathrm{CS}}(0) = \Psi^{\mathrm{LT}}(0) &= -\frac{\alpha_L}{\mu_W} = -0.14 \\
\Psi_A^{\mathrm{LT}}(0) &= -\frac{k}{k+1}\frac{\alpha_L}{\mu_W} = -0.06 \\
\lim_{\eta\to\infty}\Psi_S(\eta) = \lim_{\eta\to\infty}\Psi^{\mathrm{CS}}(\eta) &= \frac{\alpha_H}{\mu_W} = 0.27 \\
\lim_{\eta\to\infty}\Psi^{\mathrm{LT}}(\eta) &= \pi\left[1-\left(\frac{\theta}{\theta+\lambda}\right)^k\right]\frac{\alpha_H}{\mu_W} = 0.03\,.
\end{aligned}$$

(See Section IA.2 of Internet Appendix for derivations of the limited values.)

The eventual success probability of entrepreneurship is

$$\lim_{\eta \to \infty} (1 - P_A(\eta)) = \pi \left[1 - \left(\frac{\theta}{\theta + \lambda} \right)^k \right] = 0.114 \,.$$

When time goes to infinity, all survived businesses earn successful earnings, the certainty equivalent value of these survived businesses is therefore $\frac{1}{r}\delta_H = \$225,292$, or 0.256 of μ_W/r. Then the unconditional certainty equivalent value is the above value multiplying the eventual success probability, which is

$$\pi \left[1 - \left(\frac{\theta}{\theta + \lambda} \right)^k \right] \frac{\delta_H}{r} = \$25,650 \,,$$

or 0.029 of μ_W/r.