WORKING paper

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The Circular Relationship Between Productivity Growth and Real Interest Rates[1]

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Abstract

In most advanced economies, both real interest rates and productivity growth have decreased since the early 1990s. In this paper, we explore the mechanism whereby a circular relationship links these two quantities. While productivity is a key driver of potential output which affects the level of interest rates, the level of interest rates is a determinant of the expected return from investment projects, and thus of the productivity level required for investment. In our model, absent of a technology shock, this specific relationship can only converge to an equilibrium where growth and interest rates are both low. We test this using macroeconomic data on 17 OECD countries and simulate the effect of a temporary productivity shock.

Keywords: Productivity,Slowdown, Secular Stagnation, Interest Rates

JEL classification: O43, O47, O57, E43

Non-Technical Summary

Economic growth in all advanced countries has slowed continuously since the 1970s and has fallen to a historical low since the Great Recession. This secular slowdown is mainly the result of weaker growth in total factor productivity whose widespread stagnation is difficult to interpret with a standard growth framework. The picture becomes even more puzzling if we consider the diversity of productivity levels, of new technology diffusion, of average human capital and of openness to trade across all advanced countries, which are all affected by a slowdown. Such a shared TFP trend in a context of significant structural heterogeneity suggests that a common global factor could be at play.

In this paper, we investigate one possible explanation: the decline in long-term real interest rates observed since the early 1990s in all developed countries. Specifically, we discuss and test the existence of a circular relationship between interest rates and productivity growth. It is of course well-known that productivity is a long-term determinant of return on capital and thereby of interest rates, which explains a positive correlation between these two quantities. But we argue that this is only one

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side of the coin as interest rates are also a determinant of the minimum expected return from investment projects, and therefore of the productivity level required for such an investment. The decline in long-term real interest rates, notably due to negative demographic pressures, may have led to a slowdown in productivity by making an increasing number of weakly-productive companies and projects profitable (we refer to this mechanism as the ``cleansing effect'').

If this channel dominates the negative impact of tougher financial constraints on innovation financing, a negative permanent shock on interest rates, for example due to population aging, would indeed lead to a secular fall in productivity growth. This fall would in turn lead to a decline in interest rates and create a circular relationship between these two indicators that ultimately would converge to a steady-state characterized by low growth and low interest rates. When real interest rates are low (as it has been the case for several decades), it is likely that this second channel will dominates the first one. In this case, only a technology shock could disrupt this downward spiral.

In order to test this mechanism, and in particular the existence of a circular relationship between real interest rates and TFP growth, we take a long run view. We first rely on the the Long Term Productivity database built by Bergeaud et al. (2016) which provides comparable cross-country TFP estimates from the end of the 19th century, and on the work of Jorda et al. (2017). We estimate this circular relationship by cross-country panel regressions using annual data on a sample of 17 advanced countries over the period 1950-2017. We jointly estimate the two relationships (from real interest rates to productivity growth and from productivity growth to real interest rates) using different methods and use the point estimates to look at the past and the future.

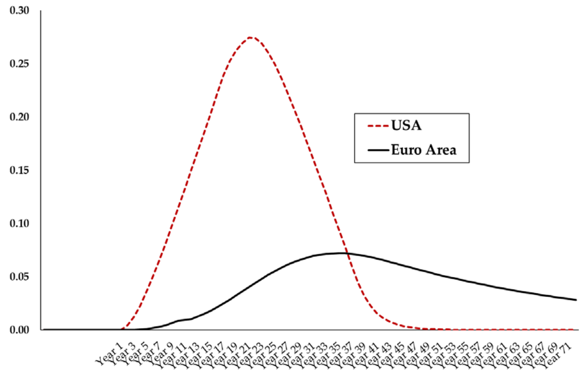
Our results hint at the existence of a circular relationship that results in a secular stagnation equilibrium: a situation where productivity grows slowly and where real interest rates are low. Between the two sub-periods 1984-1995 and 2005-2016, TFP annual growth declined by about 0.66pp in the United States and 1.51pp in the euro area and the contribution of real interest rates that we estimate fell by 0.6pp and 0.56pp respectively. While of course other factors are at play during this period, and in particular, in the case of the euro area, a slowdown in human capital stock, such contributions suggest that real interest rates could account for a significant share of the productivity slowdown.

One way to break out of this circular relationship is via a new technological revolution linked to the digital economy, or, in countries where there is still room for convergence, via structural reforms to improve the diffusion of new technologies. Using our estimate results, we propose some simulations to test the impact of such shocks in the frontier economy. The results from these simulations confirm the intuition. We assess the impact of a negative shock on relative equipment prices with a magnitude that could be comparable to the ``ICT shock'' in the United States between 1985 and 2007 (cf. figures below). This shock would be enough to escape the secular stagnation trap, with TFP growth higher than the baseline rate by 0.6pp at the peak. This technology shock in the United States would spread to other countries through the catching-up process and lead to a slow but lasting acceleration in TFP, as its level converges with that of the United States. In the euro area, TFP growth relative to the baseline reaches a peak of 0.2pp, about ten years after the US peak.

The global economy will face several headwinds in the foreseeable future (see Gordon, 2010). In particular, significant productivity growth would be required to finance the energy transition towards a more sustainable growth, to lead to an ordered decrease in the crisis-inherited high debt level and to face the consequences of an aging population. This technology shock, the impact of which would be maximised by the low interest rate environment, would hence be necessary to be able to face these headwinds with confidence. The debate on its emergence is still highly controversial among economists, but as the after-effects of the crisis on productivity growth vanish, a clearer view of what we can expect in the coming years should be warranted.

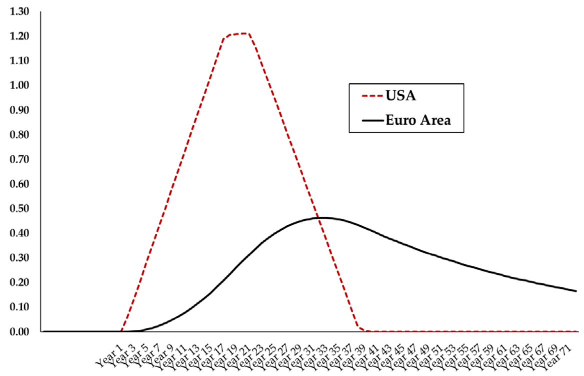
Simulation results in the Euro Area and in the US for a shock of relative investment prices in the US.

Response of the growth rate of TFP



Simulation results in the Euro Area and in the US for a shock of relative investment prices in the US.

Response of the real interest rate



La relation circulaire entre la croissance de la productivité et les taux d'intérêt réels

RÉSUMÉ

Dans la plupart des pays avancés, les taux d'intérêt réels et la croissance de la productivité ont décru depuis le début des années 1990. Dans ce papier, nous explorons le mécanisme par lequel une relation circulaire lie ces deux indicateurs. Alors que la productivité est un déterminant de la croissance potentielle et affecte donc le niveau des taux d'intérêt, le niveau des taux est un déterminant du rendement attendu des projets d'investissement et ainsi du niveau de productivité requis pour de tels investissements. Dans notre modèle, en l'absence de choc technologique, cette relation spécifique ne peut converger que vers un équilibre où taux d'intérêt et croissance sont bas. Nous testons cette relation en utilisant des données macroéconomiques sur 17 pays de l'OCDE et simulons l'effet d'un choc de productivité temporaire.

Mots-clés : Productivité, Ralentissement, Stagnation Séculaire, Taux d'Intérêt

1 Introduction

Economic growth in all advanced countries has slowed continuously since the 1970s and has fallen to a historical low since the Great Recession (at least compared to 20$^{\text{th}}$ century standards, see Bergeaud et al., 2016, 2017). This secular slowdown is mainly the result of weaker growth in total factor productivity whose widespread stagnation is difficult to interpret with a standard growth framework. The picture becomes even more puzzling if we consider the diversity of productivity levels, of new technology diffusion, of average human capital and of openness to trade across all advanced countries which are all affected by a slowdown. Such a shared TFP trend in a context of significant structural heterogeneity suggests common global factors could be at play.

In this paper, we investigate one possible explanation: the decline in long-term real interest rates observed since the early 1990s in all developed countries. Specifically, we discuss and test the existence of a circular relationship between interest rates and productivity growth. It is of course well-known that productivity is a long-term determinant of return on capital and thereby of interest rates, which explains a positive correlation between these two quantities. But we argue that this is only one side of the coin as interest rates are also a determinant of the minimum expected return from investment projects, and therefore of the productivity level required for such an investment. The decline in long-term real interest rates, notably due to negative demographic pressures, may have led to a slowdown in productivity by making an increasing number of weakly-productive companies and projects profitable (we refer to this mechanism as the "cleansing effect").

The first causal relationship, from potential growth to long- term real interest rates, is standard in the literature. Even if other factors have been shown to influence the equilibrium level of long- term interest rates, the decline in productivity gains and hence in potential growth appears to be an important contributing factor behind the fall in real interest rates since the early 1980s (for an empirical analysis of this relationship and a summary of the existing literature, see Teulings and Baldwin, 2014, Bean, 2016, Eggertsson et al., 2016 and Marx et al., 2017).

The reverse causal relationship, from long-term real interest rates to productivity, and consequently GDP growth, has recently received wide attention, with contrasted conclusions. Some authors document a negative impact of higher interest rates on productivity growth, which arises from tougher financial conditions when interest rates are too high and dampen investments in R&D. Different theoretical models show how lower credit constraints can foster innovation-led growth by reducing the costs of screening promising projects (e.g., see King and Levine, 1993 and Aghion et al., 2009). Empirically, this relation has been confirmed by numerous papers, such as Levine (1997), Rajan and Zingales (1998), Aghion et al. (2010) and Aghion et al. (2012). Using individual firm datasets in the context of the financial crisis, recent empirical contributions highlight similar results (see for instance, Duval et al., 2017 and Manaresi and Pierri, 2017 respectively for US and Italian firms) and show that financial constraints have a detrimental impact on productivity growth.[1] But on the other side, the fall in real interest rates from the mid-1980s could have reduced mortality rates for less productive firms (a decline in the "cleansing effect") and could thus have hampered the reallocation of production factors towards firms at the frontier. Lower rates could also have made it possible to finance less efficient projects, and this could in turn have reduced productivity gains. Several studies have provided support for this explanation (see, for example, Reis, 2013, Gopinath et al., 2017, Gorton and Ordonez, 2016 and Cette et al., 2016). [2] [3] In this paper, we focus on testing this latter explanation, which appears to be dominant over the period of estimation due to the overall level of real interest rates.

If the second channel (negative reallocation effect) dominates the first (positive relation from real interest rates to productivity growth), a negative permanent shock on interest rates, for example due to population aging, would indeed lead to a secular fall in productivity growth. This fall would in turn lead to a decline in interest rates and create a circular relationship between these two quantities that ultimately would converge to a steady -state characterized by low growth and low interest rates. When real interest rates are low (as it has been the case for several decades), it is likely that this second channel will dominates the first one. A recent paper from Aghion et al. (2019a) proposes a theoretical framework that combines these two channels in an inverted-U relationship between interest rates and productivity, with a positive relationship at low interest rates levels. Using French microdata, the authors confirm that, at least over the last two decades, the second channel has been active and has weighed on productivity. In this case, only a technology shock could disrupt this downward spiral.[4]

[1] In this paper, we study solely interest rates and not a direct measure of financial constraints. High financial constraints may not arise solely in a high interest rates environment, but high interest rates leads to tougher financial constraints, *ceteris paribus*

[2] In a recent contribution using Italian data and a Melitz and Polanec (2015) decomposition, Linarello et al. (2019) show that a negative credit supply shock reduces aggregate productivity by lowering incumbent firms average productivity. However, they also show that these effects are counterbalanced because such negative credit shocks also improve the allocation of resources and push the least productive firms to exit the market.

[3] Low interest rates can also negatively impact productivity growth if they give a comparative advantage to the market leader as emphasised by Liu et al. (2019). This is corroborated by the increasing market concentration recently observed in the United States, as well as the decline in the labour share and business dynamism (see Aghion et al., 2019b). We do not investigate this channel in this paper.

[4] However, in order to reap the full benefits of such a shock, an economy needs to have the right institutions (see e.g. Acemoglu et al., 2006). In a more realistic view, all countries would draw different gains from a technology shock, whereas because of the increasing mobility of capital, all would experience the subsequent equilibrium rise in real interest rates. Countries with poorly adapted institutions would thus be penalised twice: real interest rates would rise, but productivity would not accelerate.

In order to test this mechanism, and in particular the existence of a circular relationship between real interest rates and TFP growth, we take a long run view. We first rely on the Long Term Productivity database built by Bergeaud et al. (2016) which provides comparable cross-country TFP estimates from the end of the 19th century, and on the work of Jordà et al. (2017) and in particular their Macrohistory database which provides yearly average values for long-term interest rates. We estimate this circular relationship by cross-country panel regressions using annual data on a sample of 17 advanced countries over the period 1950-2017. We jointly estimate the two relationships (from real interest rates to productivity growth and from productivity growth to real interest rates) using different methods and use the point estimates to look at the past and the future. To the best of our knowledge, our paper is the first to propose estimates of such circular relationship between real interest rates and TFP growth.

Our results hint at the existence of a circular relationship that results in a secular stagnation equilibrium: a situation where productivity grows slowly and where real interest rates are low. Between the two sub-periods 1984-1995 and 2005-2016, TFP annual growth declined by about 0.66pp in the United States and 1.51pp in the euro area and the contribution of real interest rates that we estimate fell by 0.6pp and 0.56pp respectively. While of course other factors are at play during this period, and in particular in the case of the euro area a slowdown in human capital stock, such contributions suggest that real interest rates could account for a significant share of the productivity slowdown.

One way to break out of this circular relationship is via a new technological revolution linked to the digital economy, or, in countries where there is still room for convergence, via structural reforms to improve the diffusion of new technologies. Using our estimate results, we propose some simulations to test the impact of such shocks in the frontier economy. The results from these simulations confirm the intuition. We assess the impact of a negative shock on relative equipment prices with a magnitude that could be comparable to the "ICT shock" in the United States between 1985 and 2007. This shock would be enough to escape the secular stagnation trap, with TFP growth higher than the baseline rate by 0.6pp at the peak. This technology shock in the United States would spread to other countries through the catching-up process and lead to a slow but lasting acceleration in TFP, as its level converges with that of the United States. In the euro area, TFP growth relative to the baseline reaches a peak of 0.2pp, about ten years after the US peak. The digital revolution and its substantial effect on productivity that some economists have forecasted could correspond to or could even been larger than such a shock over the next decades (see for instance Van Ark, 2016, Brynjolfsson et al. (2017, 2018) or Branstetter and Sichel, 2017).

The remainder of the paper is organised as follows: Section 2 motivates our analysis and describes the data, Section 3 briefly sketches the theory that was developed in Aghion et al. (2019a) to understand how an increase in real interest rates can result in increasing growth. Section 4 details and estimates the empirical model. Finally, Section 5 shows our model's response to a technology shock in the productivity leader.

2 Background and Descriptive Evidence

Before turning to the estimation of the circular relationship between interest rates and TFP growth, we first consider some descriptive evidence. As briefly explained in the introduction, we draw our data from two different sources. First, we rely on the long-term productivity database built by Bergeaud et al. (2016) which provides comparable TFP estimates over a very long time dimension and for a large panel of countries. Second, we complete and backdate long-term real interest rate data provided by the OECD using the work of Jordà et al. (2017). Our final dataset includes 17 countries over a period of 68 years (from 1950 to 2017).[5]

Figures 1 and 2 show the median and confidence intervals, over the period 1950-2017 for our set of 17 developed countries, respectively for TFP growth and for long-term real interest rates. We can make the following observations.

TFP growth trends: We distinguish two sub-periods: i) Up to the first oil shock at the beginning of the 1970s, TFP growth fluctuated around a stable rate of around 3%; ii) From the first oil shock onward, TFP growth declined by steps and its level at the end of the period varied between 0.5% and 1%. Bergeaud et al. (2016, 2017) look more accurately at these data and highlight in fact three sub-periods:

- After WWII, continental European countries and Japan benefited from the big wave of productivity that the United States had experienced decades earlier, and progressively caught up with the technology leader.[6] During this catching-up process, TFP growth declined in the United States.
- From the oil shock to 1995, the post-war convergence process slowly came to an end as many countries caught up with the United States' productivity level. After 1995 and until 2004, productivity growth in the United States overtook that of other countries, benefiting from a new productivity wave, albeit much lower

[5]These 17 advanced countries are: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States.

[6]Throughout, we will refer to the United States as the technology leader (or frontier) even if some countries may have a higher level of TFP over some sub-periods for particular reasons, for instance Norway due to the importance of its oil sector.

than what was observed in the 1930s, 1940s and 1950s. As documented in numerous papers, this productivity growth wave corresponds to the third industrial revolution linked to ICT (see Jorgenson, 2001, Jorgenson et al. 2008, Van Ark et al., 2008, Timmer et al., 2011, Bergeaud et al., 2016 and Cette, 2014 for a survey). Apart from the very short second productivity wave observed mostly in the United States (and to a lesser extent in the United Kingdom), TFP growth continued to decline dramatically in advanced economies, irrespective of the original TFP level.

- From the mid-2000s, before the beginning of the Great Recession, TFP growth decreased in all countries. The current pace of TFP growth appears very low compared to what was observed previously, except during the world wars. This low growth performance for most advanced countries cannot be explained solely by the Great Recession and its consequences (Fernald et al. 2017).

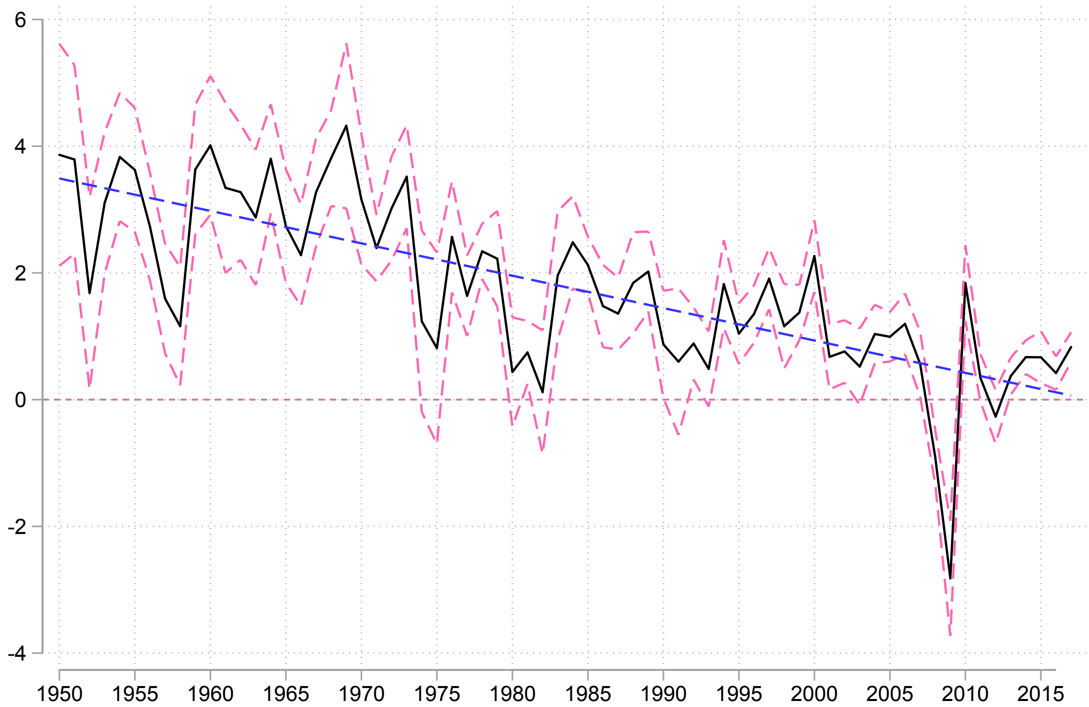


Figure 1: Yearly median growth rate of TFP growth (in %) and its trend (dotted blue line). The dotted red lines around the median are confidence intervals, defined as the median + and - 2 standard errors across the 17 OECD countries of the sample.

Real-interest rates: We mainly distinguish between three sub-periods: i) Until the early 1970s and first oil shock, real interest rates remained stable and close to 2.5%; ii) During the decade covering the two oil shocks, from the early 1970s to the early 1980s, real interest rates posted large fluctuations. First a sharp decrease - fueled by high inflation rates that resulted from the acceleration in oil prices - which even led to negative real interest rates. A sharp increase then followed, explained by a rise in nominal interest rates and a decline in inflation. Real interest rates grew to more than 5.5%; iii) From the mid-1980s to the end of the period, real interest rates declined dramatically by at least 5 pp; this decline is observed simultaneously in all developed countries; their standard deviation across countries decreases, which points to an increasing role of international determinants for real interest rates.

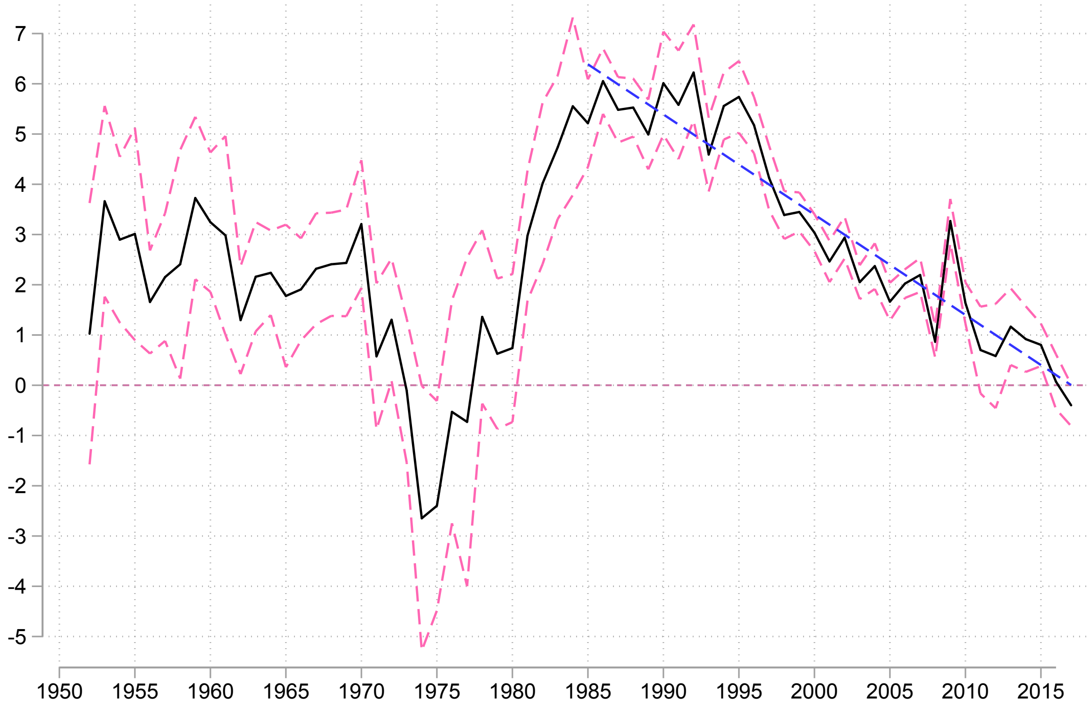


Figure 2: Yearly median growth rate of real interest rate (in %) and its trend since 1985 (dotted blue line). The dotted red lines around the median are confidence intervals, defined as the median + and - 2 standard errors across the 17 OECD countries of the sample.

Comovement: Over the entire period 1950-2017, we observe clear similarities in the dynamics of TFP growth and real interest rates. In particular, the two series are stable from the early 1950s to the early 1970s, and decline in a parallel way from the 1990s. These observations hint at a possible relationship between TFP growth and long-term real interest rates.

As the TFP slowdown from the first oil choc to the mid-1980s is not simultaneous to a parallel decline of the real interest rates, other factors have contributed to explain the TFP slowdown during this sub-period and also afterwards. Numerous possible candidates are mentioned in the literature, and among them the progressive exhaustion of the impact of the second industrial revolution and a slowdown (or even a stabilization in some countries) in the increase of the average duration of education among the working age population (see Bergeaud et al., 2018 for a survey and estimations). Real interest rates dynamics may have had a particularly important contribution from the 1990s onward. The high levels of real interest rates from the mid-1980s to the mid-1990s are unusual and we have to keep in mind that they could at least partly be related to high risk premium. In this case, they could keep a similar impact on TFP growth but would not be themselves explained by TFP growth as in other sub-periods. Nevertheless, a recent paper from Jordà et al. (2019) seems to contradict this interpretation: it shows that the risk premium would have been at its lower post WW2 levels during this sub-period mid-1980s to mid-1990s.

The decline in real interest rate could be a common factor behind the universal decline in TFP growth since the early 1990s in all developed countries. This would be a plausible explanation if the fall in real interest rates from the mid-1980s had slowed default rates in less productive firms (decline in the "cleansing effect"), thereby hampering the reallocation of factors of production to more cutting-edge firms. Lower rates could also have made it easier to finance less efficient projects, and this combination of factors could in turn have reduced productivity gains. As already discussed, several studies have provided empirical support for a decline in the "cleansing effect". It is interesting to note that the majority of these studies, in particular those of Reis (2013) and Gopinath et al. (2017), have focused on southern European countries (notably Spain, Italy and Portugal) and on a relatively recent period (since the 2000s). The literature has not reported such a relationship between financial conditions and productivity in other countries under review (such as Norway, Germany and France, see Cette et al., 2016).

These dynamics are reinforced by the reverse relationship: the decline in productivity gains and hence in potential growth is itself a contributing factor behind the fall in real interest rates (for an empirical analysis of this relationship and a summary of the existing literature, see Teulings and Baldwin, 2014, Bean, 2016, or Marx et al., 2017).

The decrease in real interest rates could thus lead to a fall in productivity, which in turn could lead to a decline in rates, creating a circular relationship between TFP growth and real interest rates. Evidence of such a mechanism can be shown by plotting the filtered TFP growth against the filtered level of real interest rates, as in Figure 3. In this chart, we clearly see a trend since the late 1990s converging to a state with low growth and low interest rates. A similar pattern can be seen before the oil crisis, which was interrupted by an inflation shock that shifted the curve upward, followed by a technology shock that shifted the curve to the right. Without such shocks, the economy

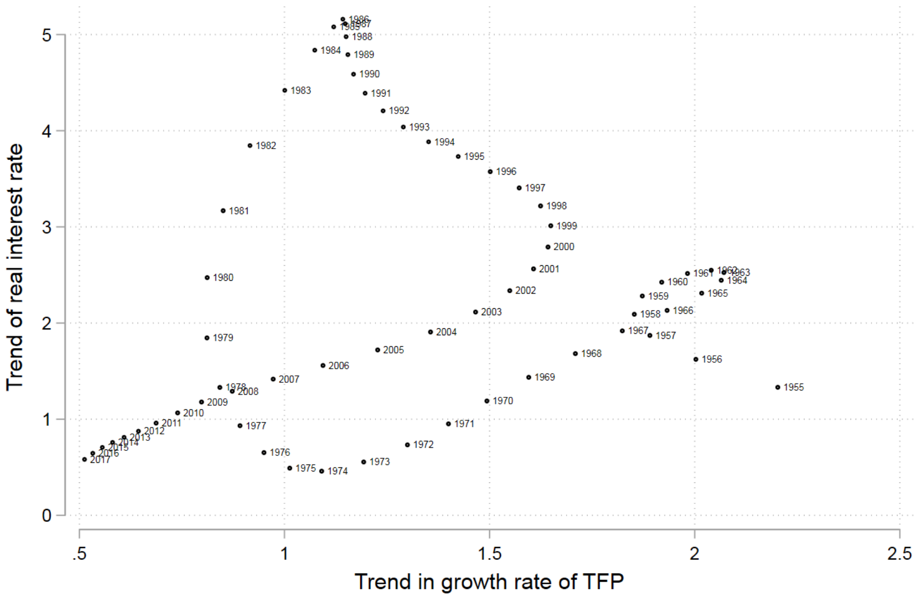


Figure 3: Coevolution of real interest rates and growth rate of TFP in the USA since 1955. Both trends have been obtained using a HP filter with a smoothing parameter set to 100 over the period 1950-2017.

converges towards a low growth and low real interest rate equilibrium.[7]

3 The Theory in a Nutshell

In order to derive a theoretical framework that explains how an increase in interest rates (i.e. larger financial constraints) can affect growth positively, we turn to a Schumpeterian growth model as developed in Aghion and Howitt (1992) with firm dynamics a la Klette and Kortum (2004) and Aghion et al. (2015). It is essentially a summary of the toy model presented in Aghion et al. (2019a) to which we refer the reader for further details.

The model is a standard version of Klette and Kortum (2004) and yields a growth rate g:

$$g = (z_e + z_i)\ln(\gamma),$$

[7] Benigno and Fornaro (2017) built a Keynesian growth framework in which an economy converges towards a permanent state of weak growth through a connection between depressed demand, low interest rates and growth. In their model, weak aggregate demand has a negative impact on firms' investment in innovation (similar to the "investment effect" of Aghion et al., 2019a), which results in a offal in productivity growth. On the other hand, periods of slow growth dampen aggregate demand and push real interest rates to 0.

where γ is a step size of productivity when a firm successfully innovates, which occurs with frequency z_e and z_i for an entrant and an incumbent respectively. As shown in Aghion et al. (2019a), z_e and z_i depend upon γ, the number of scientists ψ, the size of the population L, the Poisson innovation rate of innovation $\frac{1}{\eta}$, a scale parameter ζ and the discount rate ρ:

$$z_e = \frac{\gamma-1}{\gamma}\frac{L}{\psi} - \frac{1}{\eta}\left(\frac{\psi}{\eta\zeta}\right)^{\frac{1}{\eta-1}} - \frac{\rho}{\gamma} \tag{1}$$

$$z_i = \left(\frac{\psi}{\eta\zeta}\right)^{\frac{1}{\eta-1}}. \tag{2}$$

Introducing credit constraints, Aghion et al. (2019a) assume that firms cannot invest more than μ times their current market value. They further assume that this condition is not binding for entrants, for example because they have accumulated enough wealth.[8] This yields:

$$z_e = \frac{\gamma-1}{\gamma}\frac{L}{\psi} - \mu - \frac{\rho}{\gamma} \tag{3}$$

$$z_i = \left(\frac{\mu\psi}{\zeta}\right)^{\frac{1}{\eta}} \tag{4}$$

μ captures the inverse of the tightness of credit constraints in the economy, which in turns is directly linked to interest rates r. The exact relationship between the two is not directly modelled in this framework, but in any case we expect g to vary with r as g varies with $-\mu$.

In the case where μ is large enough, introducing (and raising) credit constraints has a positive effect on growth, because it increases the contribution of entrants relatively more than it reduces the contribution of incumbents to growth. To see this, it is necessary to assume that entrants and incumbents have different step sizes γ_e and γ_i, and we suppose that $\gamma_e > \gamma_i$ (see e.g. Akcigit and Kerr, 2018). In this case:[9]

$$\frac{dg}{d\mu} = -\ln(\gamma_e) + \frac{\psi}{\zeta\eta}\left(\frac{\mu\psi}{\zeta}\right)^{\frac{1}{\eta}-1}\ln(\gamma_i) < 0,$$

if $\mu > \left(\frac{\psi}{\zeta}\right)^{\frac{1}{\eta-1}}\left(\frac{\ln(\gamma_i)}{\ln(\gamma_e)\eta}\right)^{\frac{\eta}{\eta-1}}$.

[8] More specifically, they assume that the channel through which entrants are constrained is not the same as incumbent, and in particular is not affected by μ.

[9] In the case where the two step sizes are the same, then $\frac{dg}{d\mu} < 0$ would violate the fact that innovation by incumbent is limited by the value of μ, see Aghion et al. (2019a).

This relationship shows how decreasing real interest rates can negatively impact growth through a reallocation effect if the level of credit constraints is already low enough. On the contrary, if μ is close to 0, a fall in real interest rates reduces the cost of capital and spurs corporate investment (see e.g. Mazet-Sonilhac and Mésonnier, 2016 and Carluccio et al., 2018) with a positive impact on growth.

The other relationship, from growth to interest rate, is more standard and can be derived from the Euler equation $g = r - \rho$. The intuition behind it is that in the long-run, we assume the ratio of capital to output ratio to be constant, thus a lower (higher) expected growth leads to a lower (higher) demand for investment. Consequently, agents borrow less (more) to finance lower (higher) investment, which decreases (increases) the price of financial capital (i.e. the real interest rate). Marx et al. (2017) propose a literature survey on this relation and a model of overlapping generations to represent it.

4 Estimations

4.1 Econometric Model

We consider the following system of simultaneous equations where countries are indexed by $i \in \{1 \dots N\}$ and year by $t \in \{1 \dots T\}$:

$$\begin{cases} g_{i,t} & = & a g_{i,t-1} + b r_{i,t-1} + C' X_{i,t} + \varepsilon_{i,t} \\ r_{i,t} & = & \alpha g_{i,t} + \beta r_{i,t-1} + \Gamma' Z_{i,t} + \eta_{i,t} \end{cases} \tag{5}$$

where g is the growth rate of total factor productivity and r the level of interest rate. X and Z are two vectors of time varying and time unvarying exogenous covariates and ε and η are error terms. We first assume that these error terms are iid and uncorrelated with each other and run separate estimations. In both equations, we add an autoregressive term that captures the persistence of both productivity growth and interest rates. In terms of timing, we assume that there is no direct contemporaneous effects of r on g in contrast with the effects of g on r.[10]

We are interested in the values of α and b and their long-term counterparts $\alpha/(1-\beta)$ and $b/(1-a)$. α corresponds to the marginal effect of a change in interest rates on the contemporaneous growth rate of TFP. In the other equation, we are mostly interested in the value of b, which corresponds to the marginal effect of a change in interest

[10] As changes in factor allocation takes time, the impact of interest rates on TFP growth is delayed, which is not the case for the impact of TFP growth on interest rates, changes in financial capital allocation being possibly fast.

rates on the lagged productivity growth rate. In line with the model and our discussion, we expect both α and b to be positive.

In theory, it is possible to estimate model (5) equation by equation. This makes the rather strong assumption that errors are not serially correlated (otherwise we would have for example $\mathbb{E}[\varepsilon_{i,t} r_{i,t-1}] \neq 0$, violating the identification assumptions of the first equation). For illustrative purposes, we therefore first report results obtained by separately estimating the equations using an OLS estimator. We shall however keep in mind that coefficients can only capture correlations between the different quantities involved and that their magnitude might be biased. Finally, since we have a very long time window when compared to the number of countries ($N/T \approx 0.25$), we first abstract from the Nickell (1981)'s bias and use a standard within estimator before turning to a GMM estimator to correct for endogeneity.

We then turn to an estimation of the full model, taking into account the complete data generating process as described in (5). More specifically, we relax the assumption implicitly made in the previous estimation procedures that errors ε and η are fully independent. We consider that the system of equation (5) displays contemporaneous cross-equation error correlation and is therefore a seemingly unrelated regression system (see Zellner, 1962). We estimate this system using an iterative GLS method.

We estimate equation (5) for our set of 17 OECD countries and over the period 1950-2016. Although we have information on interest rates and productivity before 1950, we prefer not to consider pre-WW2 data because during this period interest rates cover a very different reality across countries and time (see Levy-Garboua and Monnet, 2016 in the case of France) and because the war periods yield fragile data.

4.2 Choice of exogenous variables

In selecting covariates to include in vector C and Γ, we need to bear in mind various criteria. First, we want these variables to explain part of the dynamics of growth or interest rates; and second, we want these variables to be as exogenous as possible; and third, we need these variables to be available for all our 17 countries and to be consistently measured since 1950.

As regards the first point, we know from the vast growth literature that the two main contributors to long-term TFP growth are improvement in human capital and technological progress. For most countries, the latter factor evolves following a catching-up process with the frontier economy (since World War II, this would be the United States, see Bergeaud et al., 2016, 2018 for a review). We shall therefore control for the variation in human capital, measured both as the change in the average duration of education among the population and as life expectancy. Both the average duration of education and life expectancy are slowly varying series whose dynamics are mostly driven by historical policy decisions, demographics and long-term technological change in the health system, and we consider that they are essentially unaffected by the contemporaneous growth rate of TFP.

For worldwide technological progress, we use the relative price of investment in the United States (i.e. the ratio of the price of investment over the price of GDP)[11] and we model its diffusion using the relative distance to the TFP level of the United States. All these variables are included in vector C and are available from 1950 for all our 17 countries. Data source, description and measurement are detailed in Appendix A.

Constant-quality investment price indexes attempt to take productive performance improvements in investment into account. For a stable value of investment spending over two years, an embodied productive performance improvement would correspond to an increase in the investment volume and to a decrease in the investment price index. The embodied technical change is, from this point of view, a determinant of the price of investment. Nevertheless, the measurement of investment price indexes takes only partly into account the improvements in investment productive performance for several reasons, and at least for the two following ones: (i) these improvements are taken into account only for some products, mainly automobiles and, within ICT, hardware, prepackaged and partly custom software, and some communication equipment. For other investment products, there is almost no impact of an investment quality change on the measurement of investment prices. This partial approach is explained by the cost of the methods (hedonic or matching approaches, mainly) used to take into account changes in quality in investment price indexes; (ii) whatever the efforts of national accountants and their degree of sophistication, these methods remain imperfect and take only partially into account the embodied technical progress in investment price indexes. For these reasons, an unknown part of the embodied technical progress is not included in the increase in investment volume and a decrease in investment prices, but as we do not have other indexes more appropriate, we introduce them in our estimates as a proxy of technical progress.[12]

For the second equation, we control for the volatility of inflation in the past three years by the age structure of the population, more precisely by the ratio of dependent population (i.e. below 16 or above 65, see Ang and Madsen, 2016) to total population, and by a measure of the stability of economic policy, i.e. the number of changes in finance ministers in the past three years. We expect inflation volatility to capture the risk premium that stems from changes in expected inflation, the age dependency ratio

[11]Quality improvement in investment leading to increases in productive performances is partly incorporated into national accounts through investment prices, especially for ICTs. This incorporation is deeper in the US national accounts than in those of other countries (for a summary on these aspects, see Byrne et al., 2013 and Cette, 2014)

[12]See Bergeaud et al. (2018) for more details.

to proxy the supply of savings, which weighs on real interest rates, and changes in finance ministers to capture economic political uncertainty.

4.3 Results

As discussed above, we start to show results when each equation of system (5) is estimated separately, using a panel fixed -effect estimator, presented in columns 1, 3 and 5 of Table 1. We then replicate the same specifications using GMM. Indeed, as explained above, in the likely case that errors are serially correlated in each equation of (5), we can have $\mathbb{E}[\varepsilon_{i,t} r_{i,t-1}] \neq 0$, violating the identification assumptions of the first equation. We therefore instrument $r_{i,t-1}$ by its past value in nominal terms so as to also correct for inflation shocks. Results are shown in columns 2, 4 and 6 of Table 1.

From real -interest rates to growth: Results obtained from the estimation of the first equation of (5) are shown in Table 1. The columns differ by the measure of the relative price of investment. Columns 1 and 2 use the variation in the price of equipment divided by the variation in the price of GDP for the United States applied to all countries. Indeed, as mentioned above, US national accounts are the most advanced in incorporating quality adjustment resulting from technological progress into investment prices. As this technological progress should be common to all countries, we use this measure as a proxy for the pace of global innovation. Columns 3 and 4 use the same measure but only set to 0 for countries that are farther than 1% from the productivity level of the frontier.[13] The underlying idea is that only countries that are close enough to the technological frontier directly benefit from an innovation shock, while other countries indirectly benefit from the shock through the catching-up dynamics. Finally, columns 5 and 6 use the same measure as columns 3 and 4, but consider the price of all investment assets instead of focusing solely on equipment.[14]

In all instances, we see that the marginal effect of $r_{i,t-1}$ on $g_{i,t}$ is positive, significant and of similar magnitude. It is higher in GMM estimates but often less precisely estimated. There is therefore a positive correlation between the previous year's level of interest rates and current productivity growth. Education has the expected magnitude:

[13] Formally the variable is set to 0 for observation (i, t) such that:

$$\frac{1}{5}\sum_{k=1}^{5}\left(\mathrm{tfp}_{i,t-k} - \mathrm{tfp}_{US,t-k}\right) > 0.01,$$

where tfp is the logarithm of the level of TFP.

[14] We extend our investment price measure as some technological assets might be included in structure investment series, although these are likely to be limited. The US national accounts also reports price indices for specific IT assets such as computers. However, such series may suffer from imprecise price measurements; despite the efforts of the BEA and BLS, they remain imperfectly captured, as underlined by Byrne et al., 2013.

a one-year increase in the average level of education of the population raises productivity by around 7-8 percentage points, in line with the literature (see Psacharopoulos and Patrinos, 2004 and Bergeaud et al., 2018 for a review). Life expectancy, which proxies the general health of the labour force, is also positively correlated (although not significantly) with growth, one additional year of life expectancy increasing TFP by about 0.3%. The catching-up coefficient implies that countries that are far from the productivity frontier tend to grow faster. The coefficient suggests that the speed of convergence is about 5% per year. Finally, relative investment price coefficients are negative and precisely estimated. Our preferred estimate is column 2, which takes into account potential endogeneity of nominal interest rates and allows all countries to benefit directly from technological progress. In this estimate, a 1% decrease in the investment price increases TFP by about 0.13%.

Implied magnitudes: A simple back of the envelop calculation gives an idea of the overall effect of the decline in real interest rates on TFP growth. As shown in Figures 1 and 2 the median real interest rate in our sample declined from 5.2% in 1985 to 0.5% at the end of the period. Over the same period, median TFP growth declined from 2.5% to 0.5%. According to our preferred estimate (column 2 of Table 1), 0.7 percentage point of the decline in TFP growth or 35% of the slowdown could be attributed to the decrease in interest rates *ceteris paribus*.

From growth to real-interest rate: As regards the second equation of (5), results are shown in Table 2. Column 1 simply estimates this equation using a panel fixed effect within estimator. To control for endogeneity, we also estimate this equation using GMM in columns 2-5. We instrument the TFP growth rate using one-year lagged values of the intensive margin of two technologies: first, information technologies which are measured by the ratio of ICT capital stock to GDP, in value; and second, electricity is measured by the logarithm of electricity consumption per capita (both variables are taken from Bergeaud et al., 2018 where more details on these two measures and their impact on TFP are given). Results are shown in columns 2 and 3 of Table 2. Column 2 only uses the ICT instrument, while column 3 uses both instruments. In columns 4 and 5, we further address potential endogeneity. We perform the same exercise as in columns 2 and 3, but consider the 5-year lagged value of ICT and electricity consumption per capita in the United States as an instrument for the growth rate of TFP (and exclude the United States from the estimation sample).

The correlation between TFP growth and real interest rates is positive and usually significant. Its magnitude is higher when using our 5-year lagged value of technology proxies as instruments, although the differences between the coefficients are not statistically significant. Given a 0.7 autoregressive coefficient and a 0.1 TFP coefficient, the long run impact of a 1 pp increase in TFP on the level of interest rates is about 0.3

pp. We may note that, according to our model and to Aghion et al. (2019a), this positive correlation is only valid in a low interest rates environment. If interest rates were significantly higher, the economy would stand on the other side of the "inverted-U" and an increase in TFP could in fact lead to a reduction of real interest rates. Empirical results from Aghion et al. (2019a) shows that at least in the recent period this is not the case.[15]

A higher age dependency ratio, which could lead to an increased supply of savings, weighs on interest rates, with a negative significant coefficient of similar amplitude across estimates: a 1pp increase in the age dependency decreases TFP by about 0.07pp in the long run. This relationship is important for the future: demography may exert a continuous downward pressure on long-term real interest rates as this ratio is expected to increase in the next decades (see Basso and Jimeno, 2018). Inflation volatility may increase the risk premium on interest rates and has a positive coefficient, albeit not always significant. Policy instability, as proxied by changes in finance ministers, pushes up interest rates as expected although the standard errors remain too large to conclude precisely.

4.4 Simultaneous estimations

We now turn to an estimation of the full system (5) assuming that it is a seemingly unrelated regression system, which supposes that error terms across equations are related. Results are presented in Table 3. Columns differ as in Table 1 by relative investment prices (US equipment prices in column 1; US equipment prices for countries at 1% maximum from the frontier in column 2 and total US equipment prices in column 3). These results are qualitatively similar to the ones obtained using separate within estimations of the system, i.e. the coefficient of interest rates is positive and significant, estimated at around 0.04, while the auto-regressive coefficient of TFP growth is estimated at around 0.23, suggesting a 0.045 pp long run impact on TFP growth of a 1 pp increase in interest rates. From these results, 0.3 point of the decline in TFP growth or 15% of the slowdown could be attributed to the decrease in interest rates *ceteris paribus*. Changes in education, life expectancy and distance to the productivity frontier have the expected signs, although the life expectancy coefficient is not significant. Turning to interest rate estimates in panel B, results are qualitatively similar to the separate estimates, with a lower magnitude for the TFP coefficient. A one point increase in the TFP growth rate leads to a long-term impact of 0.2 point on the level of

[15]While we are not directly testing the cleansing effect as a channel through which credit and productivity could be positively correlated, we refer to Aghion et al. (2019a) for formal evidence that this channel is at play (in particular, they look at the exit rate of low productivity firms following a credit shock).

Table 1: Separate estimation, baseline results

Dependent variable: $g_{i,t}$						
Estimator	OLS	GMM	OLS	GMM	OLS	GMM
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
$g_{i,t-1}$	0.214***	0.204***	0.210***	0.202***	0.214***	0.206***
	(0.036)	(0.036)	(0.035)	(0.036)	(0.036)	(0.037)
$r_{i,t-1}$	0.042*	0.123**	0.056**	0.118*	0.054**	0.113*
	(0.023)	(0.061)	(0.024)	(0.062)	(0.025)	(0.066)
Catch-up	-6.026***	-5.945***	-5.428***	-5.446***	-5.366***	-5.395***
	(0.595)	(0.601)	(0.559)	(0.578)	(0.543)	(0.563)
Variation in Relat. Price	-0.174***	-0.134**	-0.088**	-0.059	-0.197**	-0.128
	(0.049)	(0.056)	(0.036)	(0.044)	(0.092)	(0.118)
Variation in educ.	7.953***	8.013***	7.335***	7.432***	7.163***	7.292***
	(1.339)	(1.364)	(1.296)	(1.300)	(1.270)	(1.276)
Variation in Life Exp.	0.308	0.337	0.291	0.321	0.304	0.328
	(0.289)	(0.291)	(0.292)	(0.292)	(0.291)	(0.291)
R^2	0.273	0.263	0.264	0.258	0.264	0.259
Observations	1122	1122	1122	1122	1122	1122
KP LM stat. p-val.		0.000		0.000		0.000
KP Wald F-stat.		50.191		45.114		39.999

Notes: This Table reports regression results from a separate estimation of the first equation of (5). Columns 1 and 2 measure relative investment price of investment using its current value for equipment in the US for all countries, columns 3 and 4 use this value for countries with a TFP level at least 1% below the US and columns 5 and 6 do the same as 3 and 4 but consider relative investment price for total investment (instead of only focusing on equipment). Coefficients and standard errors (in parentheses) are obtained using a panel-fixed effect estimators (columns 1, 3 and 5) or the GMM (columns 2, 4 and 6) using past realization of the nominal interest rate as an instrument. Autocorrelation and heteroskedasticity robust standard errors have been estimated using the Newey-West variance estimator with a bandwidth of 5 years. KP LM stat. p-val. stands for the p-value of the Kleibergen-Paap statistic of underidentification. KP Wald F-stat. stands for the F-statistics of the Kleibergen-Paap Wald test of weak instruments.

Table 2: Separate estimation, baseline results

Dependent variable: $r_{i,t}$					
Estimator	OLS	GMM	GMM	GMM	GMM
	(1)	**(2)**	**(3)**	**(4)**	**(5)**
$r_{i,t-1}$	0.705***	0.692***	0.691***	0.690***	0.697***
	(0.040)	(0.040)	(0.039)	(0.041)	(0.040)
$g_{i,t}$	0.103***	0.112	0.121*	0.218***	0.208***
	(0.032)	(0.088)	(0.073)	(0.070)	(0.068)
Age Dep Ratio	-0.048***	-0.048***	-0.050***	-0.064***	-0.060***
	(0.014)	(0.017)	(0.014)	(0.016)	(0.015)
Inflation Volat.	0.125*	0.106	0.105	0.109	0.120
	(0.074)	(0.076)	(0.076)	(0.076)	(0.074)
Policy Instability	0.071	0.084	0.083	0.089	0.095
	(0.065)	(0.067)	(0.067)	(0.068)	(0.067)
R^2	0.540	0.527	0.527	0.530	0.531
Observations	1122	1088	1088	1056	1056
KP LM stat. p-val.		0.000	0.000	0.000	0.000
KP Wald F-stat.		46.588	47.917	125.583	63.217
Hansen-J p-val.			0.842		0.478

Notes: This Table reports regression results from a separate estimation of the second equation of model (5). Coefficients and standard errors are obtained using a panel-fixed effect estimators (in column 1) or the GMM (colums 2 to 5). Column 2 uses the ratio of ICT capital stock over GDP in value taken at $t-1$ as an instrument for $g_{i,t}$. Column 3 adds another instrument to column 2's specification: the logarithm of electricity consumption divided by population in $t-1$. Column 4 uses the ratio of ICT capital stock over GDP in value taken at $t-5$ in the US and column 5 adds the logarithm of electricity consumption per capita in the US at $t-5$. In columns 4 and 5, the US is excluded from the sample. These variables are taken from Bergeaud et al. (2018). Autocorrelation and heteroskedasticity robust standard errors (in parentheses) have been estimated using the Newey-West variance estimator with a bandwidth of 5 years. KP LM stat. p-val. stands for the p-value of the Kleibergen-Paap statistic of underidentification. KP Wald F-stat. stands for the F-statistics of the Kleibergen-Paap Wald test of weak instruments. Hansen-J p-val. is the p-value of the Hansen J-statistic of over identification.

interest rates, with a 0.06 short-term coefficient of TFP growth and a 0.7 autoregressive term. The age dependency ratio, inflation volatility and political instability have the expected signs and remain close to the within estimates.

5 Simulations and discussion

In this section, we use our estimation results and conduct two exercises. First, we use the first line of the equation (5) to decompose the impact of r on g over different sub-periods, and similarly we use the second line of the same system to decompose the impact of g on r. These two decompositions will indicate the contribution of the decline in real interest rates to the decrease in TFP growth and, similarly, the contribution of the decline in TFP growth to the decrease in real interest rates. Second, we simulate the full model and look at the impact on TFP growth and on real interest rates of a negative shock in the US relative investment price considered as resulting from a technology shock.

5.1 Long run evolution breakdown

Table 4 uses point estimates of column 2 of Table 1 to estimate the long run effects of interest rates on the evolution of TFP over different sub-periods. We then use the point estimates of column 3 of Table 2 to look at the average contribution of TFP growth to real interest rates. Results are shown in Table 5. We select five sub-periods that correspond to the overall evolution of real interest rates as documented in Section 2, 1950-1973, 1973-1984, 1984-1995, 1995-2005 and 2005-2016, and compare the United States with the euro area.

The contribution of real interest rates to TFP growth reaches a maximum in the sub-period 1984-1995 both in the United States and in the euro area. Between the two sub-periods 1984-1995 and 2005-2016, the decline in TFP annual growth was about 0.66pp in the United States and 1.51pp in the euro area. The contribution of the decrease in interest rates to the decline in TFP growth in the euro area is similar to that in the United States, but TFP decelerates less in the United States, as other factors have contributed more to the decline in the latter. Of these factors, the slowdown in the average level of education plays the biggest role: the level of education in the euro area was still in a rapid catching-up process with the United States during the 1984-1995 sub-period; this was no longer the case during the 2005-2016 sub-period, when convergence was almost achieved. In the future, because of the negative contribution of demography to real interest rates, the contribution of this variable to TFP growth could remain low, without a favourable technology shock.

The contribution of TFP growth to real interest rates reaches a maximum in the

Table 3: Simultaneous estimation, baseline results

Estimator	SURE	SURE	SURE
	(1)	**(2)**	**(3)**
	Panel A: dependent variable: $g_{i,t}$		
$g_{i,t-1}$	0.228***	0.223***	0.226***
	(0.028)	(0.028)	(0.028)
$r_{i,t-1}$	0.035*	0.051**	0.050**
	(0.021)	(0.021)	(0.021)
Catch-up	-5.657***	-5.025***	-4.961***
	(0.476)	(0.453)	(0.447)
Variation in Relat. Price	-0.188***	-0.087*	-0.187*
	(0.041)	(0.046)	(0.103)
Variation in educ	8.147***	7.430***	7.252***
	(1.296)	(1.299)	(1.287)
Variation in Life Exp.	0.184	0.181	0.196
	(0.181)	(0.183)	(0.183)
	Panel B: dependent variable: $r_{i,t}$		
$r_{i,t-1}$	0.705***	0.704***	0.704***
	(0.021)	(0.021)	(0.021)
$g_{i,t}$	0.055**	0.064**	0.068**
	(0.027)	(0.027)	(0.027)
Age Dep Ratio	-0.043***	-0.043***	-0.043***
	(0.011)	(0.011)	(0.011)
Inflation Volat.	0.022***	0.022***	0.022***
	(0.005)	(0.005)	(0.005)
Policy Instability	0.076	0.077	0.077
	(0.052)	(0.052)	(0.052)
R^2	0.255	0.245	0.245
Observations	1105	1105	1105

Notes: This table presents simultaneous estimation results of system (5). All columns consider the model as SURE (seemingly unrelated regression). Columns differ by the way relative price of investment is measured which is the same as in Table 1 (respectively columns 1, 3 and 5 for this Table's columns 1, 2 and 3). Heteroskedasticity robust standard errors are reported in parenthesis.

sub-period 1995-2005 in the United States and in the sub-period 1984-1995 in the euro area. Between these sub-periods and the final sub-period 2005-2016, the decline in real interest rates was about 2.08pp in the United States and 3.75pp in the euro area. The decrease in TFP growth explains 0.44pp (21%) of this decline in the United States and 0.59pp (16%) in the euro area. This small contribution of the decrease in TFP growth to the decline in real interest rates, both for the United States and the euro area, is due to the fact that many other factors have contributed to the dynamics of interest rates. Of these factors, the increase in age dependency plays an important role in the two areas. As age dependency is expected to continue to increase in the future, TFP growth should continue to decrease without a positive technology shock.

From these decompositions, we observe that without any positive technology shock, a secular stagnation situation seems to be the most realistic scenario for the future of the United States and the euro area. These two areas could suffer from a continuous decline in real interest rates and TFP growth. The next section is dedicated to the simulation of the possible impact of a favourable technology shock, on TFP growth and real interest rates, in the United States and the euro area.

Table 4: Decomposition of TFP growth

USA	1950-1973	1973-1984	1984-1995	1995-2005	2005-2016
TFP growth	2.00	0.79	1.29	1.75	0.63
Contribution of Real Interest Rates	0.28	0.22	0.78	0.50	0.18

Euro Area	1950-1973	1973-1984	1984-1995	1995-2005	2005-2016
TFP growth	3.92	1.60	1.84	0.80	0.33
Contribution of Real Interest Rates	0.44	0.22	0.84	0.58	0.26

Notes: This Table shows the average value of TFP growth and the long-run contribution of interest rates using estimates from column (2) in Table 1 from the first equation of model (5). Long-run contributions are defined as $\hat{x}/(1-\alpha)\bar{X}$ where $\hat{x}$ is the estimated coefficient associated with variable X and $\bar{X}$ its average value over the subperiod (α is the auto-regressive coefficient, see (5)).

5.2 Simulations

We now simulate the model (5) using estimates presented in Table 3. The dynamics of this model allows us to consider the long-term impact of a shock in any covariate or in the error terms. To see this formally, it is useful at this stage to rewrite the model (5) as a moving average representation. This can be done easily and results in the following equivalent equation system:

Table 5: Decomposition of interest rates

USA	1950-1973	1973-1984	1984-1995	1995-2005	2005-2016
Interest Rates	1.84	1.42	5.07	3.25	1.17
Contribution of TFP growth	0.78	0.31	0.51	0.69	0.25

Euro Area	1950-1973	1973-1984	1984-1995	1995-2005	2005-2016
Interest Rates	2.85	1.45	5.45	3.78	1.70
Contribution of TFP growth	1.53	0.63	0.72	0.31	0.13

Notes: This Table shows the average value of real interest rates and the long-run contribution of TFP growth using estimates from column (3) in Table 2 from the second equation of model (5). Long-run contributions are defined as $\hat{x}/(1-a)\bar{X}$ where $\hat{x}$ is the estimated coefficient associated with variable X and $\bar{X}$ its average value over the subperiod (a is the autoregressive coefficient, see (5)).

$$\begin{cases} g_{i,t} &= a g_{i,t-1} + b r_{i,t-1} + C' X_{i,t} + \varepsilon_{i,t} \\ r_{i,t} &= \alpha a g_{i,t-1} + (\alpha b + \beta) r_{i,t-1} + (\alpha C' X_{i,t} + \Gamma' Z_{i,t}) + (\alpha \varepsilon_{i,t} + \eta_{i,t}) \end{cases} \tag{6}$$

Or with matrices: $Y_{i,t} = AY_{i,t-1} + \mathcal{C}\mathcal{X}_{i,t} + \mathcal{E}_{i,t}$, where: $Y_{i,t} = (g_{i,t}, r_{i,t})'$, $\mathcal{E}_{i,t} = (\varepsilon_{i,t}, \alpha\varepsilon_{i,t} + \eta_{i,t})'$, $\mathcal{X}_{i,t} = (X_{i,t}, Z_{i,t})$. We also have defined:

$$A = \begin{bmatrix} a & b \\ \alpha a & \alpha b + \beta \end{bmatrix} \text{ and } \mathcal{C} = \begin{bmatrix} C' & 0 \\ \alpha C' & \Gamma' \end{bmatrix}$$

Since A is invertible ($\det(A) = \alpha\beta$), then assuming that we know future realizations of $\mathcal{X}_{i,t}$, we can estimate the response of Y to any shock. More specifically, the recursive model yields for each $t > t_0$

$$Y_{i,t} = A^{t-t_0} Y_{i,t_0} + \sum_{k=0}^{t-t_0} A^k \left(\mathcal{C}\mathcal{X}_{i,t_0+k} + \mathcal{E}_{i,t_0+k} \right)$$

We can then simulate a shock in relative investment prices, which will impact $Y_{i,t}$ through its effect on $\mathcal{X}$.

Implementation: We use the coefficients as estimated in column 2 of Table 3 and consider a negative shock in relative investment prices of equipment that has the same magnitude and duration as the one corresponding to the "ICT shock" in the United

States between 1985 and 2007 (see Appendix B, Figure B1).[16] We then plot the evolution of the TFP growth rate and of real interest rates over time, measuring the difference between a simulation with this technology shock and a simulation without any shock.

The shock in relative investment prices directly impacts TFP growth rate in the United States and therefore its contemporaneous level of real interest rates, which in turns will impact the TFP growth rate in the next period and so on. For other countries, the impact of the shock depends on their relative TFP level compared to the United States: countries that are close or above the TFP level of the United States also directly benefit from the shock, while other countries are only indirectly affected through the catching-up process. This is the case of the euro area which, taken as a whole, is too far below the US TFP level at the time of the shock to be directly impacted.[17]

For the United States, most of the resulting effect is essentially homothetic to the shock as real interest rates are too low to play a significant role at the time of the shock. Consequently, when the technology shock is over and relative investment prices do not change anymore, we expect the effect on TFP growth to quickly vanish. For the euro area, on the contrary, most of the effect stems from the catching-up with the frontier, as differences in TFP levels relative to that of the United States increase after the shock kicks in. We thus expect the effect to last longer (as long as it takes for the euro area to converge towards the US productivity level), but to be lower in magnitude. This is indeed what we see in Figure 4. A similar pattern emerges when looking at the evolution of real interest rates in Figure 5, whose dynamics are dictated by the growth rate of TFP.

Note that had we used the specification in column 1 of Table 3 where every country's TFP growth rate is directly impacted by relative investment prices, the difference between the simulated curve for the United States and that of the euro area would be less significant (see Appendix B, Figures B2 and B3). In fact, in such a case, the only difference would stem from the catching-up terms and from the heterogeneous level of interest rates. However, these two contributions are minor at the time of the shock and the difference in growth rates between the United States and the euro area is negligible. The magnitude of the shock is also different in this case, because the marginal effect of relative investment prices is larger. It therefore matters which specification we choose in the simulation.

[16]One advantage of using the relative investment price as a reduced form measure of technological progress is that we do not have to explicitly model and identify this change on the economy which proved to be empirically challenging, see e.g. Malgouyres et al. (2019)

[17]As in Bergeaud et al. (2017), we aggregate data from eight euro area countries, which we consider as the whole euro area. More specifically, in the years following 2016, we measure TFP growth and real interest rates as the sum of each of the eight countries, weighted by their average population between 2000 and 2015. See Appendix A.

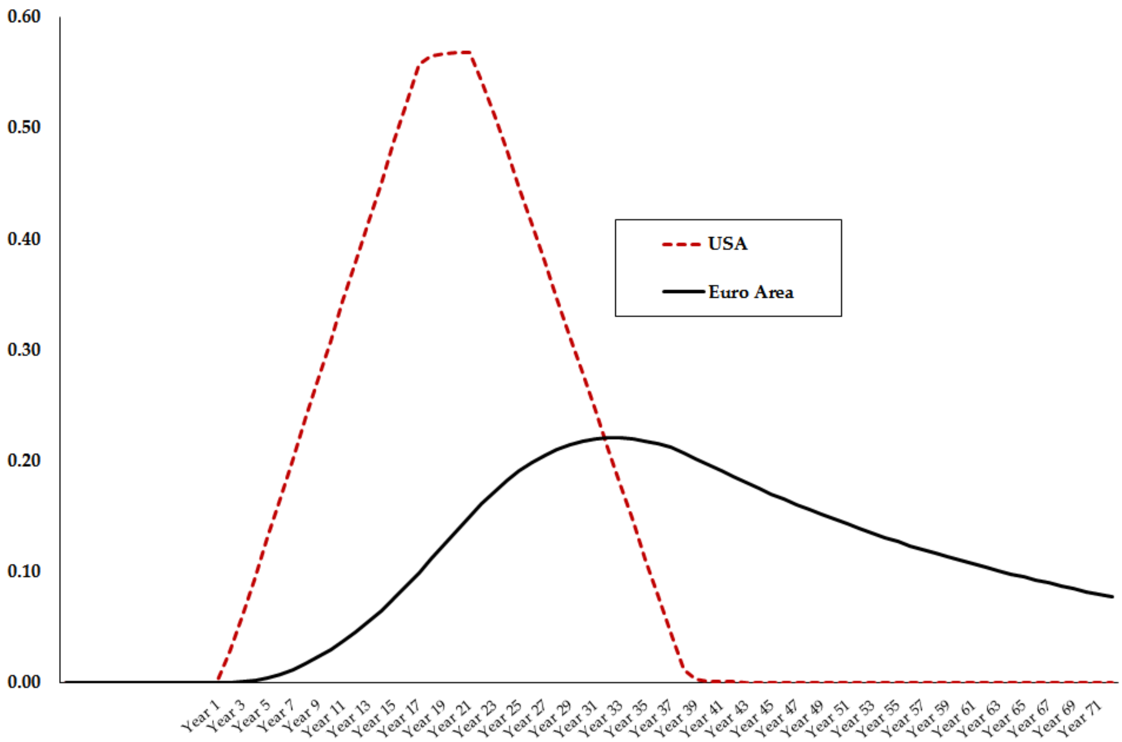


Figure 4: Simulation results in the Euro Area and in the US for a shock in the US. Response of the growth rate of TFP g

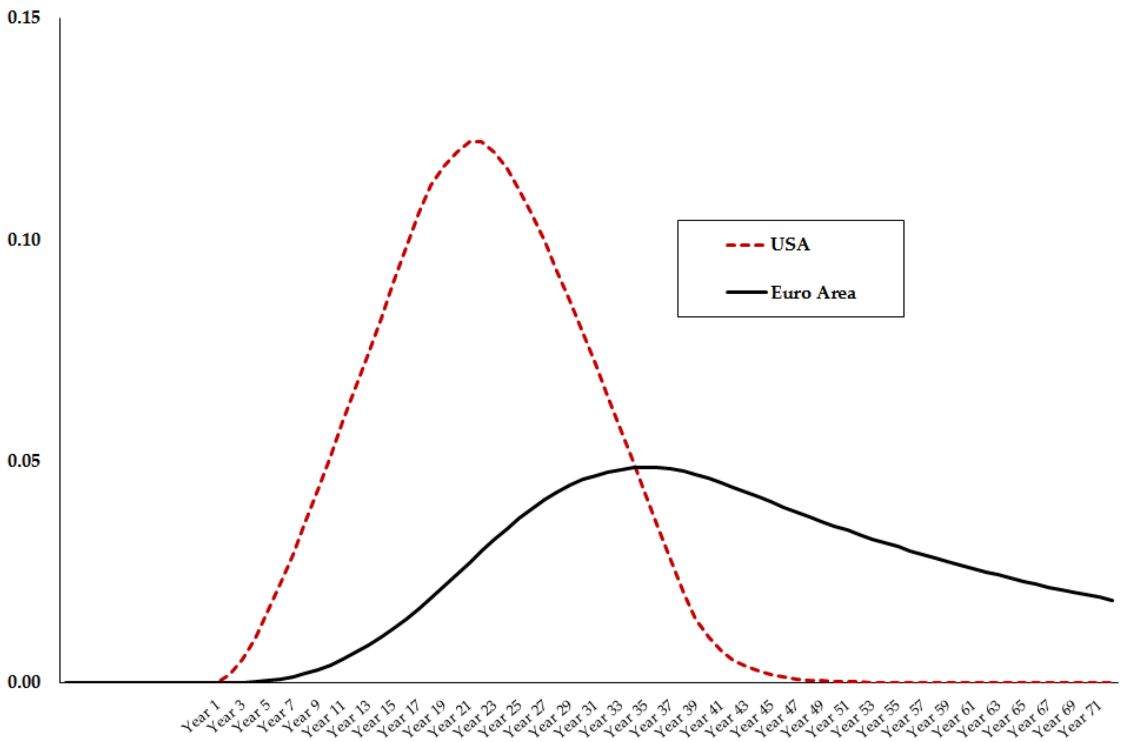


Figure 5: Simulation results in the Euro Area and in the US for a shock in the US. Response of the real interest rate r

5.3 Discussion

Without any positive technology or education shocks, the equation system converges towards a low TFP growth/low real interest rate equilibrium, which corresponds to the description of secular stagnation, although the mechanism at play differs partly from that of Hansen in 1939 or more recently Summers (2014, 2015), which relies mainly on a low demand environment. Here, a low cleansing intensity leads to the survival of low productivity firms thanks to low real interest rates. Real interest rates are maintained at a low and even declining level by the projected increase in the age dependency ratio. Indeed, according to United Nations forecasts, the age dependency ratio would increase from around 50% in 2016 to 62% in 2050. According to our estimates, this corresponds to a 0.5pp decline in real interest rates, and twice as much in a country such as Japan where the age dependency ratio is expected to reach 90% in 2050.

An increase in average years of education of the working age population can be foreseen only for a limited number of countries, as leeway may be exhausted in many advanced countries such as the United States, where the expansion of tertiary education has been largely achieved (in the United States, the average number of years of schooling was over 13 toward the end of the period). Yet, improvement in the quality of education or on-the-job training could significantly increase the contribution of human capital to TFP.

Hence, we simulate here a negative shock on relative equipment prices of the magnitude of the "ICT shock" in the United States between 1985 and 2007. This is a small shock in terms of amplitude, as this technology shock had a limited impact on TFP growth compared to other shocks in the 20th century. This shock could stem from a second wave linked to ICT, which could be due to the contribution of AI or robots to production processes. Yet, this shock would be enough to escape the secular stagnation trap, with TFP growth higher than the very low baseline rate by 0.6pp at peak. This technology shock in the United States would spread to other countries through the catching-up process and lead to a slow but lasting acceleration in TFP, as its level converges with that of the United States. In the euro area, TFP growth relative to the baseline reaches a peak of 0.2pp, about ten years after the peak in the United States (see Figure 4). All other things being equal, the gain in terms of TFP level is, at the end of the process, about 25% in the two economic areas. In terms of GDP, the gain would be larger, from the capital deepening channel activated by the investment price decrease (see Cette et al., 2005 for a presentation of these different types of channels). As TFP growth and hence real returns are higher than in the baseline, real interest rates increase by 0.12pp in the United States and 0.05pp in the euro area (See Figure 5). This increase in interest rates would trigger a mutually reinforcing mechanism with TFP growth due to the circular relationship framework. Simulations in case of permanent technology shocks are presented in Appendix B (Figures B4 and B5).

As shown in Aghion et al. (2019a), the relationship between real interest rates and

TFP growth is positive in low interest rate environments when the cleansing effect dominates the negative impact of financial constraints on innovation, but negative in high interest rate environments. Hence, this simulation is valid for a limited technology shock at the current low interest rate juncture. If the shock were stronger, its impact on TFP growth would be dampened by the negative feedback impact of high interest rates on TFP. This negative feedback is not estimated here as our estimation period covers mostly a low real interest rate period.

6 Conclusion

The circular relationship between TFP growth and real interest rates contributes to the understanding of the slowdown in productivity since the 1980s. It contributes to the current secular stagnation debate, and provides an alternative secular stagnation explanation from Hansen (1939), and more recently for instance from Summers (2014, 2015) which are mainly based on demand dynamics. Indeed, a combined low interest rate/low productivity growth environment can be explained by a weak cleansing mechanism, whereby low interest rates support the survival of weakly profitable firms and investment projects. The decrease in real interest rates since the early 1990s can hence help to explain the slowdown in productivity over that period.

We provide here both a rapid theoretical framework for this explanation and estimates over a panel of advanced countries of the circular relationship between interest rates and TFP growth, taking into account endogeneity issues and simultaneous estimates of the TFP and interest rate equations. We show that the decrease in real interest rates since the early 1990s, which relies both on an increase in the age dependency ratio and lower inflation volatility, explains a large part of the slowdown in TFP. Looking forward, simulations show that a new technology shock would be necessary to escape the current secular stagnation situation of low interest rates/low TFP growth, which could be entrenched in the foreseeable future by the negative impact of the increase in the age dependency ratio on interest rates. A technology shock, even modest in amplitude, would push up TFP growth and, through higher expected returns on investments, real interest rates. In turn, high interest rates would free resources from weakly productive firms and foster TFP growth, leading to mutually reinforcing dynamics between interest rates and productivity.[18]

The global economy will face several headwinds in the foreseeable future (see Gordon 2010). In particular, significant productivity growth would be required to finance

[18] Our mechanism also highlights that without a large technological shock, suitable competition policies aimed at favoring the cleansing mechanism, i.e. the reallocation of workers toward more productive firms, could help circumvent, at least temporarily, the convergence toward a secular stagnation equilibrium when credit constraints are too low.

the energy transition towards a more sustainable growth, to lead to an ordered decrease in the crisis-inherited high debt level and to face the consequences of an aging population. This technology shock, the impact of which would be maximised by the low interest rate environment, would hence be necessary to be able to face these headwinds with confidence. The debate on its emergence is still highly controversial among economists, but as the after-effects of the crisis on productivity growth vanish, a clearer view of what we can expect in the coming years should be warranted.

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A Data appendix

TFP growth: Our TFP data comes from the Long Term Productivity database version 2.0 and are freely available for download. The detailed construction of these series are described in Bergeaud et al. (2016) and relies on different assumptions: Cobb-Douglas production function with capital and labor, a stock of capital obtained through a permanent inventory method with constant depreciation rates for five each of the different assets (building, machinery, software, computers and telecommunication equipment) and a constant elasticity of substitution between capital and labor. The average value of TFP growth since 1950 is 1.76, with a standard deviation of 2.13 with almost all the variance being within country.

Real-interest rates: Real interest rates are computed using data on nominal long-term interest rates and inflation, both averaged by year for each country. The primary source of data is the OECD *Main Economic Indicators* (OECD, 2018) which we backdate using the Macrohistory database constructed by Jordà et al. (2017). The average value of real interest rate since 1950 is 2.17, with a standard deviation of 3.69 with almost all the variance being within country.

Covariates in TFP regression: In the model where the dependent variable is the growth rate of TFP, we consider the following control variables:

- Stock of education as measured by the average number of years spent in education after 6 years old in the total population over 15. This measure is originally taken from Van Leeuwen and Li (2014) and updated by Bergeaud et al. (2018) using information on enrollment rates at different education level.
- Life Expectancy is measured at the age of 10 and is drawn from taken from Madsen (2012) and based on the The Human Mortality Database.
- Catch-up is defined as the difference in the log level of TFP with the USA, and lagged by one year. If for a given observation, the level is larger than in the USA, we set the catch-up variable to 0 for this specific year and country.
- Relative Price of investment is measured as the difference between the price index of investment goods and the price index of GDP in the USA taken from the BEA and retrieved via FRED.

Covariates in real interest rate regression: In the model where the dependent variable is the level of the real interest rate, we consider the following control variables:

- Age Dependency Ratio is measured as the share of the population of age larger than 64 or lower than 16 divided by the share of the population between 16 and 64. These data have been drawn from Ang and Madsen (2016).

- Inflation volatility is measured as the standard deviation in inflation from year $t-3$ to year $t-1$, divided by the average value of inflation over the same period. Inflation is taken from the same source as real-interest rates.

- Policy instability is measured by the number of changes in finance ministers in the past 3 years. This information has been hand-collected from official sources.

Euro Area: In Section 4, we present a decomposition of TFP growth and real interest rate for the Euro Area, including in periods when this area did not formally exist. What we call Euro Area is in fact an aggregate of 8 countries: Germany, France, Italy, Spain, Netherlands, Belgium, Portugal and Finland which represents more than 85% of the total Euro Area GDP on average (see Bergeaud et al., 2018). In order to measure TFP growth and real interest rates for this area, we take the average of all 8 countries which we weight using the share of population over the period 2000-2015. Weights are presented in Table A1.

Table A1: Population based weights used in the aggregation of Euro Area's TFP growth and real-interest rates.

Countries	BEL	DEU	ESP	FIN	FRA	ITA	NLD	PRT
	0.04	0.30	0.12	0.02	0.23	0.19	0.07	0.02

B Additional Figures

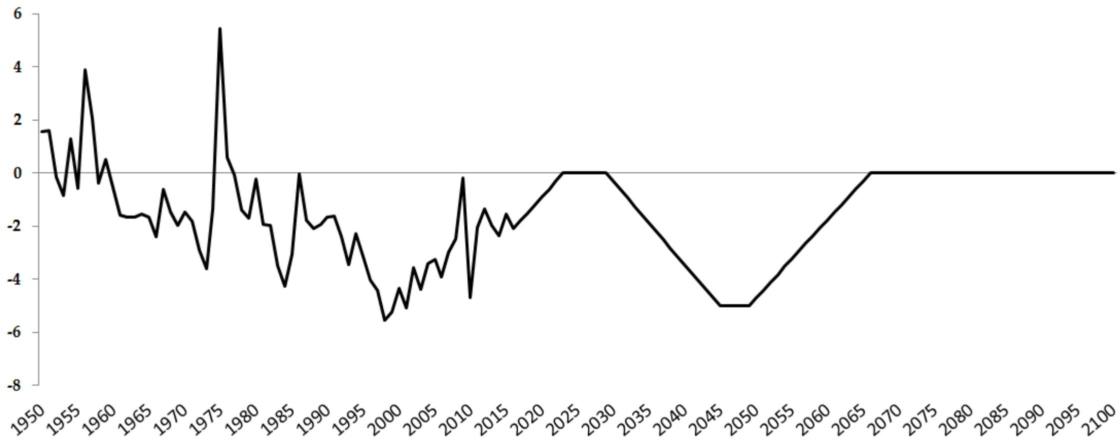

Figure B1: Evolution of relative price of investment in equipment in the US and future projected shock.

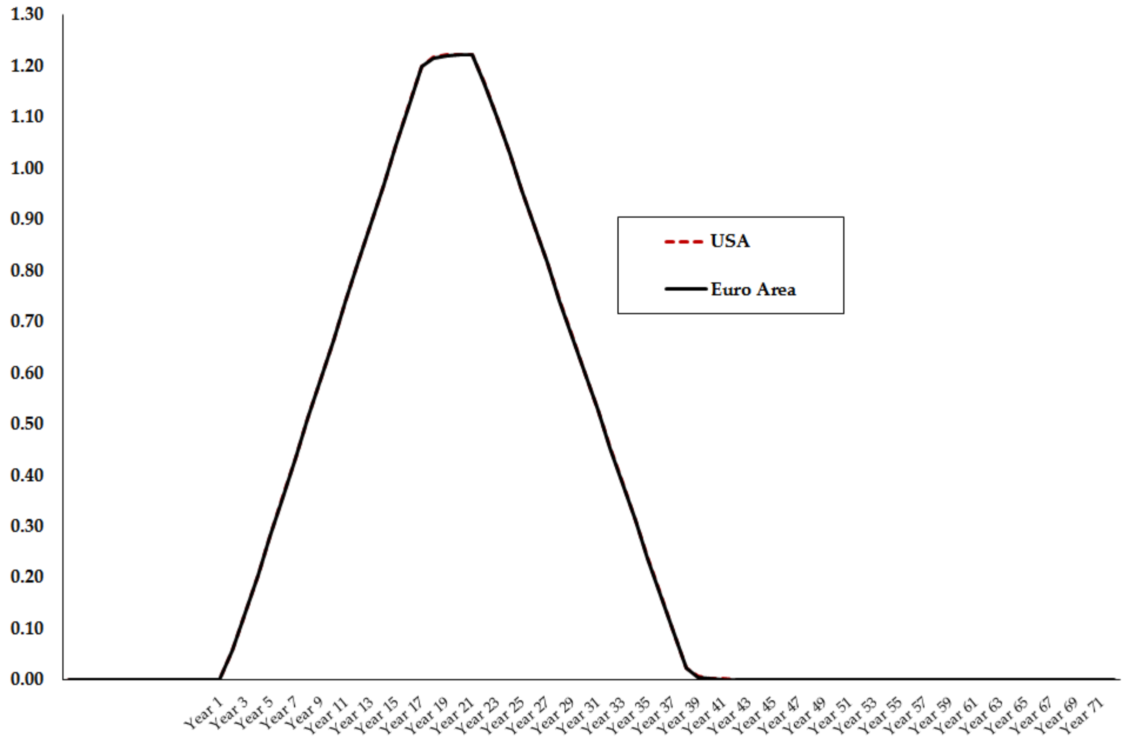

Figure B2: Simulation results in the Euro Area and in the US for a shock in the US in the case of the specification of column 1 of Table 3. Response of the growth rate of TFP g

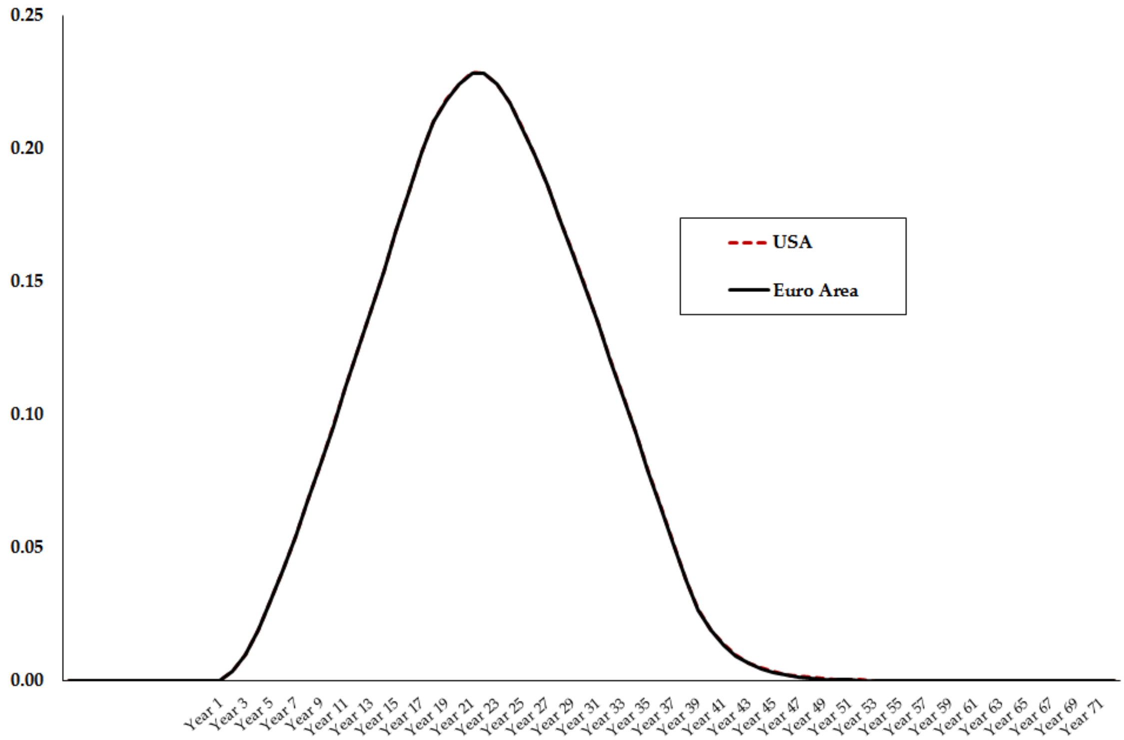

Figure B3: Simulation results in the Euro Area and in the US for a shock in the US in the case of the specification of column 1 of Table 3. Response of the growth rate of real interest rates r

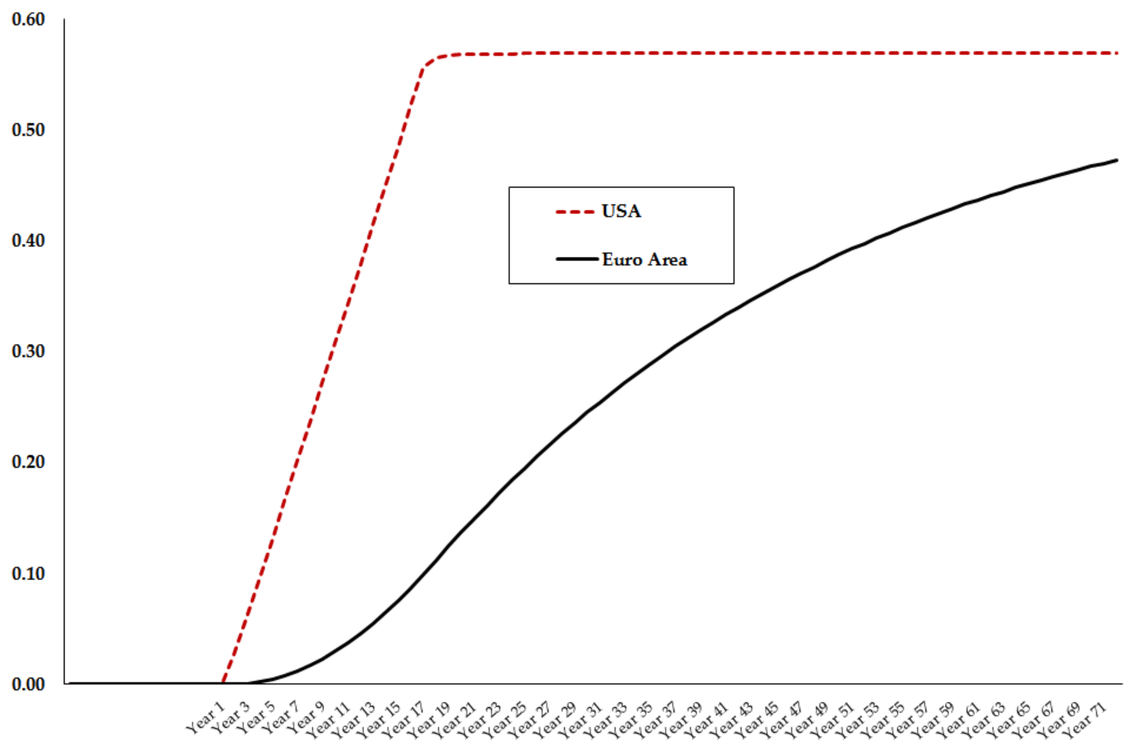

Figure B4: Simulation results in the Euro Area and in the US for a permanent shock in the US. Response of the growth rate of TFP g

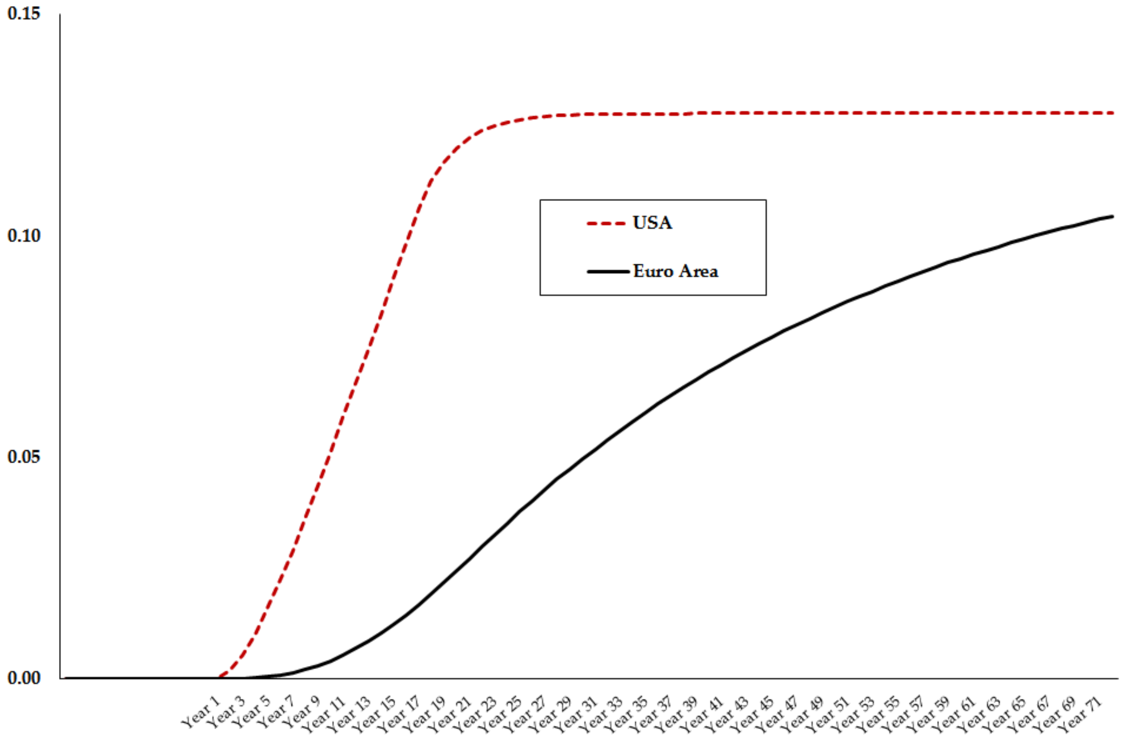

Figure B5: Simulation results in the Euro Area and in the US for a permanent shock in the US. Response of the growth rate of real interest rates r