Activist Settlements

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Abstract

Recently, activist investors have been reaching settlements with boards more often than they have been challenging boards in a proxy fight. In this paper, I provide a theoretical framework to study the economics of these settlements. The activist can demand that his proposal be implemented right away ("action settlement") or demand a number of board seats ("board settlement"), which also gives the activist access to better information. I find that the incumbent's rejection of board settlement reflects more of its private information than the rejection of action settlement does. Therefore, demanding board settlement increases the activist's credibility to run a proxy fight upon rejection and leads to a higher likelihood of reaching a settlement in the first place. Consistently with the empirical evidence by Bebchuk, Brav, Jiang, and Keusch (forthcoming), the likelihood of board (action) settlement increases (decreases) with information asymmetry. Moreover, while the average ex-post shareholder value upon reaching board settlement is lower than upon reaching action settlement, the ex-ante value created by demanding board settlement can be higher. Finally, even though value-destroying projects are typically not implemented following settlements, the existence of settlements may nevertheless destroy shareholder value due to the free-rider problem. However, strikingly, making activism less costly can actually further exacerbate this problem.

KEYWORDS: Settlements, Shareholder Activism, Boards of Directors, Allocation of Control, Authority.

JEL CLASSIFICATION: C70, D74, D83, G23, G34

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1 Introduction

Shareholder activism is on the rise.[1] To influence the corporate policies of their target firms, activist investors employ a variety of tactics, some more antagonistic than others. For example, under many jurisdictions, one path utilized by activists to exert control on firms is to challenge boards with a contested election ("proxy fight"), which is widely studied in literature.[2] However, there is a second path an activist can pursue to influence control: The activist can negotiate directly with the incumbent board, and if the incumbent agrees to the activist's demands, they reach a settlement, thereby effectively bypassing the shareholders. Interestingly, as documented by Bebchuk et al. (forthcoming), such settlements are common and their number has surpassed the number of proxy fights launched.[3] In spite of the prevalence of activist settlements, they have not received much attention in literature. The objective of this paper is to provide a theoretical framework in order to study the economics behind activist settlements.

Importantly, there are two types of settlements that can be reached between activists and incumbents. In a "board settlement", the activist obtains board seats and joins the decision-making in the boardroom to execute his agenda. For example, in 2008, Carl Icahn first received representation on the board of Motorola with two directors after approaching the firm with the aim of splitting it.[4] Bebchuk et al. (forthcoming) document that 84% of the settlement contracts with activist hedge funds between 2000-2013 resulted in the appointment of new directors to the board. On the other hand, in an "action settlement", the incumbent agrees to

[1] One example of this rise is the tremendous growth in activist hedge funds over the last two decades, recently exceeding \$170 billion in assets under management (see HLS Forum on Corporate Governance and Financial Regulation, "The Activist Investing Annual Review 2017", 02/21/2017.). Moreover, there is empirical evidence suggesting that there are positive returns around activist interventions (see, e.g., Brav et al. (2008), Greenwood and Schor (2009), Bebchuk et al. (2015), and Becht et al. (2017)).

[2] For empirical literature, see, e.g., Dodd and Warner (1983), DeAngelo and DeAngelo (1989), Ikenberry and Lakonishok (1993), Mulherin and Poulsen (1998), Alexander et al. (2010), Buchanan et al. (2012), Helwege et al. (2012), Gantchev (2013), Fos and Tsoutsoura (2014), Appel et al. (2019), Fos and Jiang (2016), Fos (2017). For theoretical literature, see, e.g., Shleifer and Vishny (1986), Harris and Raviv (1988), Bhattacharya (1997), Maug (1999), Yılmaz (1999), Bebchuk and Hart (2001), Gilson and Schwartz (2001), Corum and Levit (2019).

[3] Bebchuk et al. (forthcoming) focus on campaigns by activist hedge funds and report that while 195 settlements were reached from 2009 to 2013, in the same time frame 114 proxy fights were initiated. Moreover, 50 of the latter were settled and therefore did not to go to a shareholder vote. Schoenfeld (2019) documents that in the US the total number of agreements reached between a firm and its shareholder is over 4,400 from 1996 to 2018.

[4] Motorola eventually split into two subsidiaries in 2011, one of which got acquired shortly after by Google for \$12.5bn. See The New York Times, "Motorola and Icahn Reach Compromise on Directors", 4/08/2008; The New York Times, "In Google's Motorola Deal, Icahn Gets His Wish (Again)", 8/15/2011.

implement the activist's proposal right away.[5] For example, in 2012, AOL agreed to sell more than 800 patents for \$1.1bn to Microsoft after pressured by the hedge fund Starboard Value, although Starboard did not have any presence on the board of AOL.[6]

Given that the activist can demand that the firm implement his proposal, why do we see so many board settlements? More generally, what are the trade-offs between board and action settlements, and what determines the likelihood of reaching a settlement? Also, as shareholders in general are left out of the settlement negotiation, do shareholders benefit from these settlements? For example, Blackrock, the world's largest asset manager, has expressed that "there is a real concern among investors that standard negotiated settlements—such as giving board seats to a dissident or announcing a stock buyback—may favor short-term gains at the expense of long-term performance".[7]

I tackle these questions by analyzing a model in which the activist can either settle (i.e., negotiate a compromise) with the incumbent board or run a proxy fight to replace it. The incumbent board is privately informed about the value of the project on the table, and the incumbent enjoys private benefits from keeping the status quo. The activist is uninformed about the project's value but he is aligned with the shareholders, an assumption which I relax later. The novel feature of my model is that the activist can demand a settlement; specifically, he can demand that the incumbent implement the project (action settlement) or give him several board seats (board settlement), the latter of which provides the activist with better information regarding the value of the project and some level of decision authority over the implementation of the project.

A key insight in my findings is that compared to action settlement, the response of the incumbent to the demand of board settlement is more sensitive to the incumbent's private information, because the future decision of the activist in the board depends on the value of the project. If the activist demands action settlement, the incumbent's incentives to reject are stronger for project returns that are smaller. Therefore, although the incumbent rejects some positive NPV projects (due to the private benefits it keeps by doing so), it always rejects when the project NPV is negative, as illustrated in Figure 1. This rejection behavior of the

[5] Moreover, many of these settlements are not explicit, implying that the real number of settlements reached between activists and firms is even larger than those measured by contracts.

[6] The share price of AOL jumped 43% upon the announcement of this sale. See Wall Street Journal, "AOL's Deal Eases Pressure", 9/04/2012.

[7] See HLS Forum on Corporate Governance and Financial Regulation, "Getting Along with BlackRock", 11/06/2017.

incumbent upon action settlement demand leaves the activist in the dark regarding whether the project return is large enough to justify the costs of launching a proxy fight. Moreover, the weak credibility of the activist to run a proxy fight further encourages the incumbent to reject the action settlement in the first place.

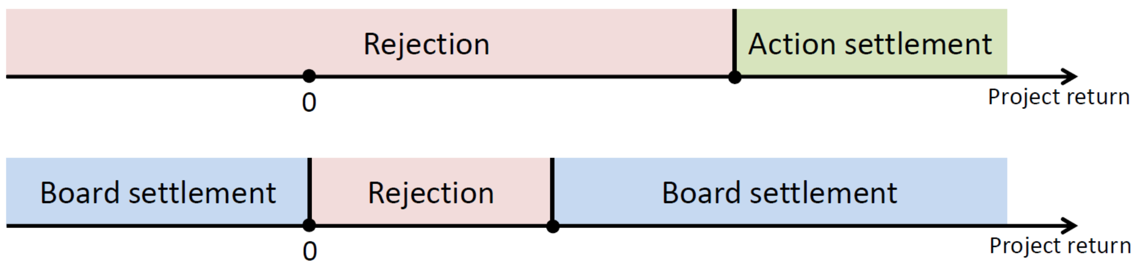

Figure 1 - Illustration of the incumbent's equilibrium behavior under different settlement demands.

In contrast, if the activist demands board settlement, the incumbent's incentive to accept is non-monotonic with respect to the project return. Specifically, the incumbent accepts board settlement when the project is negative NPV because it knows that the activist will not implement the project upon joining the board. Thus, compared to action settlement, there are two advantages in demanding board settlement: First, the activist saves the cost of a proxy fight when the project is negative NPV. Second, upon rejection of his demand the activist perfectly understands that the project NPV is positive. This inference of the activist increases the credibility of his proxy fight threat upon rejection, which in turn pushes the incumbent to accept the settlement with higher likelihood even when the project has a positive NPV. Indeed, consistently with this result, Gantchev (2013) documents that activists succeed in only about 7% of the cases where they aim to reach a settlement over the action they are demanding, in a stark contrast to 39% of the cases where they aim to reach a board settlement.

While the discussion above points out a very important advantage of board settlements, it does not imply that the activist will always demand one. Indeed, a drawback of board settlement is that unlike action settlement, it does not ensure the implementation of the project, because the activist does not obtain full decision authority. Therefore, an important question is what determines the likelihood of board and action settlements. I show that, due to advantage of board settlement that arises from information asymmetry, the likelihood of board (action) settlement increases (decreases) with information asymmetry. This finding is consistent with Bebchuk et al. (forthcoming), who document a negative relation between information asymmetry and the probability that a settlement specifies actions.

The model has several other interesting implications and predictions. First, acceptance of action settlements always leads to higher average shareholder return than acceptance of board settlements. This result is consistent with empirical evidence provided in literature (Bebchuk et al. (forthcoming), Gantchev (2013), Becht et al. (2009)). For example, Bebchuk et al. (forthcoming) find that a settlement that contracts actions leads to an average announcement return of 2.6-8.9%, while a settlement that gives the activist board seats on average yields an announcement return of about 1.2%.[8] Therefore, one may raise the question of whether the ability of activists to demand board seats through settlements decreases shareholder value. However, I find that demanding a high number of board seats in fact increases *ex-ante* shareholder value more than demanding action settlement, because it increases the likelihood of reaching a settlement, as well as the likelihood of a proxy fight upon rejection. Related, given any settlement demand, decreasing the cost of waging a proxy fight reduces the shareholder return conditional on settlement as well as conditional on proxy fight, although the shareholder value conditional on the activist's demand increases.[9] For these reasons, when evaluating the effects of shareholder activism, proxy fights and settlements should be taken into account together. In other words, measuring shareholder value conditional on the demand of the activist rather than conditional on the ex-post response of the incumbent may yield more accurate estimates for the effect of activism on firm value.

Second, the probability that the activist's proposal is implemented conditional on obtaining access to the board is lower if these board seats were obtained through a settlement than through a proxy fight (even if the number of board seats is identical in both cases). This observation follows from the result that the incumbent rejects board settlement only if the project NPV is positive, while it always accepts if the project NPV is negative. Therefore, although some might interpret activists' insistence on their proposal after winning a proxy fight as short-termism or as overconfidence, this result provides another explanation as to why activists might be more aggressive with their agenda in the boardroom after a successful proxy fight. Third, the number of board seats demanded by the activist, the likelihood of a proxy fight, and shareholder value can be non-monotonic with respect to the cost of waging a proxy fight. Therefore, making activist interventions difficult can improve value of the firm even when

[8] Specifically, the average announcement return is 8.1–8.9% if the settlement contracts merging or selling the target firm or selling part of its asset, and 2.6–2.7% if the settlement contracts departure of the CEO.

[9] Gantchev (2013) finds that a campaign ending in a proxy fight has average costs of $10.7 million for the activist and that these costs are equal to the two-thirds of the mean abnormal activist return, pointing to significance of these costs from the perspective of the activist.

the activist's preferences do not conflict with maximizing firm value. This result complements the policy proposals to curb activism that often build on the argument that activists destroy firm value due to their short-term focus.[10]

Finally, in Section 4, I consider a generalization of the model where I relax the assumption that the activist's preferences are aligned with shareholders, allowing for the activist to be willing to undertake a value-destroying project (i.e., to have a "bias" for the project). Specifically, I examine the question of whether activists destroy value through settlements, as some shareholders have expressed concern that settlements may harm shareholder value. Interestingly, I show that whenever a proxy fight occurs with positive probability, the activist never destroys shareholder value ex-post through settlements but only after the activist wins a proxy fight with the support of the shareholders. However, this finding does not imply that allowing for settlements always benefits shareholders. In contrast, I find that even if the activist is unbiased, the average shareholder created might be smaller under the existence of settlements than under the hypothetical scenario where settlements were not an option. Specifically, this is the case if the presence of settlements substantially decreases the incentives of the activist to run a proxy fight. Moreover, although this is essentially a free-rider problem, reducing the cost of launching a proxy fight does not necessarily alleviate this problem.

My paper is related to the literature on corporate governance and shareholder activism. In general, there are two kinds of governance mechanisms: Voice and exit.[11] My paper belongs to the strand of literature that focuses on voice. Typically, this strand does not distinguish between different types of intervention methods and builds on the notion that the activist can force his intervention on the firm without persuading the incumbent or shareholders.[12] On the contrary, my paper includes settlements as a form of voice mechanism, alongside proxy fight. Moreover, the success of the activist's intervention attempts depends on the belief of

[10] One example is the Brokaw Act proposed in 2016 by the US senators Tammy Baldwin and Jeff Merkley. The bill introduces more stringent disclosure rules for activists, aiming to make it more difficult for activists to accumulate shares in firms, which would therefore make intervention more costly per share owned by activists. In the press release of the proposal, it is stated that "Activist hedge funds are leading the short-termism charge in our economy. [...] They often make demands to benefit themselves at the expense of the company's long-term interests." See www.baldwin.senate.gov/press-releases/brokaw-act.

[11] For surveys on voice and exit, see, e.g., Edmans (2014) and Edmans and Holderness (2017). For the literature on exit, see, e.g., Admati and Pfleiderer (2009), Edmans (2009), Goldman and Strobl (2013), and Edmans et al. (2019).

[12] See, e.g., Shleifer and Vishny (1986), Kyle and Vila (1991), Admati et al. (1994), Burkart et al. (1997), Maug (1998), Kahn and Winton (1998), Bolton and von Thadden (1998), Noe (2002), Aghion et al. (2004), Faure-Grimaud and Gromb (2004), Edmans and Manso (2011), and Dasgupta and Piacentino (2015).

the incumbent regarding the activist's threat of running a proxy fight and on the belief of the shareholders regarding the value the activist will create in the event of a proxy fight. The role of proxy fights in exerting control is extensively studied in literature.[13] Distinctively from this literature, however, here I focus on the trade-off between different types of settlements, and their interaction with the activist's decision to run a proxy fight as a result of the activist's inference. My paper is also related to Levit (2019) who studies communication, alongside with voice and exit, as a form of shareholder activism. Although both models share the idea that voice (i.e., a proxy fight) is an outcome of a failure to resolve the conflict by other means, Levit (2019) focuses on persuasion (i.e., communication of private information) by the activist, while my model focuses on settlements as a form of bargaining. I show that a key factor behind the activist's demand is the information content of the incumbent's response, and that this endogeneity results in many novel predictions. Cohn and Rajan (2013) also study the effect of an activist investor on the board's decision-making. However, in their model the role of the activist is to produce information, and the board acts as an unbiased arbitrator between the management and the activist with the aim of maximizing shareholder value. The focus of their analysis is the interaction between the "internal governance" determined by the board and the "external governance" provided by the activist. In contrast, in my model I treat the board and management as a monolithic entity, who is conflicted with maximizing shareholder value, and I study the relation among different kind of intervention methods (i.e., settlements and proxy fight) the activist can utilize to correct this behavior.

To estimate the costs of various stages of activism, Gantchev (2013) builds a sequential decision model where the activist decides at each stage whether to exit or escalate his intervention tactic. However, since he employs structural estimation, other aspects are exogenous in his paper, including the intervention method at each stage. Johnson and Swem (forthcoming) also constuct a structural model, with the goal of quantifying dynamic effects of the activist's reputation over time. Specifically, in their paper the information asymmetry is about the activist's cost of proxy fight, rather than the value of the project. However, they simplify their model along other dimensions, excluding shareholder voting and different type of settlements.[14] In contrast to these papers, a novel feature of my framework is that I explicitly model the critical differences between a proxy fight and settlements, as well as within settlements. I show

[13] See, e.g., the papers listed in footnote 2 on page 2.

[14] Boyson and Pichler (2019) also empirically focus on the resistance of activists' targets and the counter-resistance by activists.

that these differences, combined with the endogeneity of the decisions of the incumbent and shareholders, shape the activist's settlement demand as well as the decision to run a proxy fight.

Finally, my paper is also related to the literature on bargaining under asymmetric information (See Kennan and Wilson (1993) for an early survey). In comparison to this literature, I allow the parties to negotiate on two different dimensions, as opposed to one dimension: actions and board composition. The latter is effectively a bargaining over rights on access to information and decision making authority. However, negotiations on action versus board seats do not lead to the same outcome since negotiations over rights can incorporate private information. In this sense, my paper is related to Eraslan and Yılmaz (2007) who consider bargaining with securities that allow eventual payoffs to depend on privately held information at the time of negotiations. Importantly, while the application chosen in my paper is shareholder activism due to the availability of existing empirical evidence, the theoretical framework applies to many other settings. For instance, in many countries, after a general election a coalition government needs to be formed if the leading party has not gathered sufficient support. When the political parties negotiate how to form such a coalition, they can negotiate directly over implementing certain policies (akin to action settlement in my model) or over allocation of control over these policies (akin to board settlement). Moreover, if this negotiation fails and a coalition is not formed, then the parties go to a re-election (akin to a proxy fight), demonstrating the relevance of the model presented in this paper and its implications for settings outside of shareholder activism.

2 Setup

Consider a model with an activist investor, a publicly traded firm which is initially run by its incumbent board of directors, and passive shareholders of the target. The activist and the incumbent own some shares in the firm as well. There is a project that the firm can implement. I use "project" and "action" interchangeably. Denote by $x = 1$ if the project is implemented, and $x = 0$ otherwise (i.e., status quo). The project (e.g., a spin-off) creates a value of Δ per share for the activist and shareholders, while the incumbent's payoff per share from implementation is $\Delta - b$, where b represents the private benefit that the incumbent loses (per share owned) if project is implemented (i.e., $x = 1$).[15] This conflict of interest of the incumbent board is an important friction of the model, since otherwise the incumbent requires no intervention. For example, if a division of the firm is spun-off, the incumbent board may lose some of its private benefits since it will manage a smaller firm. While the preferences of the activist and shareholders are aligned, I relax this assumption in Section 4. Δ follows a continuous probability density function f (and cumulative distribution function F) with full support on (l, b), which is the activist's and shareholders' prior information about Δ.[16] On the other hand, the incumbent privately knows Δ (e.g., since it has access to confidential information), which is the other key friction in the model.[17] I assume that $l < 0$. The timeline of the game consists of three phases and is as follows.

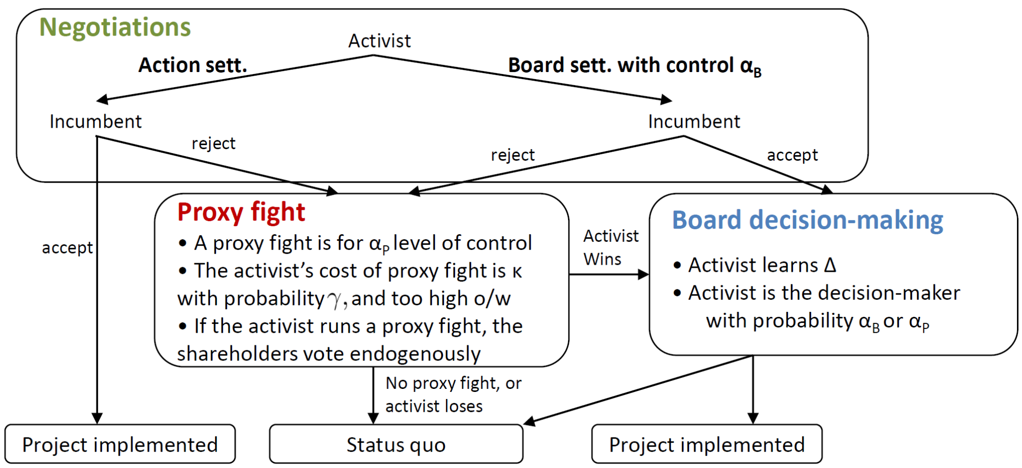

Figure 2 - The timeline

[15]Denoting the incumbent's stake by n_i and absolute private benefits from keeping status quo by B_i, b can be expressed as $b = \frac{B_i}{n_i}$. Therefore, if $n_i\Delta < B_i < \Delta$, then implementing the project is the efficient outcome even when the incumbent's private benefits are considered.

[16]The assumption that $\Delta < b$ is to simplify the analysis and focus on the interesting scenario. Indeed, if $\Delta \geq b$, then it is straightforward to see that the incumbent will implement the project without any pressure since the project's return Δ is so high, and shareholders will have no objection to the implementation. Therefore, I do not consider this case in the model.

[17]I assume that the incumbent cannot verifiably disclose Δ unless the activist joins the board. The rationale behind this assumption is that due to Regulation FD, outside the board, the incumbent has to make public disclosure of any material information disclosed to a shareholder that is likely to trade, such as an activist. However, public disclosure of proprietary information may harm the firm value and therefore may result in the breach of fiduciary duty of the incumbent. Consistent with this, upon joining the board many directors nominated by activists sign confidentiality agreements that restrict their information sharing outside the board. See http://clsbluesky.law.columbia.edu/2016/12/15/sullivan-cromwell-reviews-and-analyzes-2016-u-s-shareholder-activism/

In the first phase (i.e., "negotiations" phase), the activist and the incumbent negotiates. This phase consists of two stages: First, in the "proposal stage", the activist decides whether to demand action settlement or board settlement with some level of control $\alpha_B > 0$. Specifically, the former means that the activist demands that the incumbent implement the project, while the latter means that the activist demands board seat(s) that give him α_B control in the board. A board control of α_B gives the activist decision authority in the implementation stage with probability α_B. Second, in the "response stage", the incumbent decides whether to accept or reject the activist's settlement demand. If the incumbent accepts a settlement, I assume that the activist cannot run a proxy fight, e.g., due to a standstill agreement. If the incumbent accepts action settlement, then the project is implemented, payoffs are realized, and the game ends.

The second phase (i.e., "proxy fight" phase) occurs if the incumbent has rejected the activist's demand. This phase consists of two stages as well: First, in the "proxy fight stage", the activist decides whether to launch a proxy fight by incurring a cost of k per share he owns. $k = \kappa$ with probability γ, and $k = \zeta$ with probability $1 - \gamma$, where $\zeta > \alpha_P b$. The activist learns the value of k at this stage.[18] Let $e = 1$ if a proxy fight is launched, and $e = 0$ otherwise. If the activist runs a proxy fight, the incumbent incurs a cost of $c_{P,1} > 0$. Second, in the "voting stage", shareholders vote if the activist has launched a proxy fight to maximize their own payoff. In other words, the voting of shareholders is endogenous, and the proxy fight succeeds if and only if the shareholders support the activist. Let $\tau = 1$ if the shareholder support the activist, and $\tau = 0$ otherwise. If the activist wins a proxy fight, then he obtains a control of α_P in the board, and the incumbent incurs a cost of $c_{P,2}$, which is in addition to $c_{P,1}$.[19] I let $c_P \equiv c_{P,1} + c_{P,2}$, and assume that the lowerbound l of Δ is sufficiently low compared to c_P, i.e.,

$$l < b - \frac{c_P}{\frac{1}{\gamma} - \alpha_P}. \tag{1}$$

[18] The parameter k can also be interpreted as the opportunity cost of the activist incurs by not liquidating the stake in the target. This opportunity cost can be due to forgoing other new investment opportunities, or alternatively, it can also be due to having to liquidate other existing investments to meet investor outflows (i.e., liquidity shock), if there are any. In either case, if this cost is too high, then the activist will choose not to run a proxy fight and instead will choose to liquidate his stake.

[19] Fos and Tsoutsoura (2014) find that facing a direct threat of removal is associated with \$1.3-2.9 million in foregone income until retirement for the median incumbent director. They also find that after a proxy fight, not only incumbent directors that were up for re-election during the proxy fight lose on average 0.71 on other boards, but also the other incumbent directors (who were not up for re-election) lose on average 0.45 seats on other boards.

If the activist has not launched a proxy fight or loses it, the payoffs are realized, and the game ends.

The third and final phase (i.e., "board decision-making" phase) takes place if the activist has achieved some board control $\alpha > 0$, either through board settlement or proxy fight. Again, this phase consists of two stages: First, in the "learning stage", the activist learns Δ, because he obtains access to confidential information that the incumbent had. Second, in the "implementation stage", the activist obtains decision authority with probability α, and the incumbent obtains decision authority otherwise. Whoever has the decision authority decides whether to implement the action. Payoffs are realized, and the game ends.

2.1 Payoffs

Denoting the payoff of the incumbent, activist, and shareholder by Π_i, Π_a, and Π_s respectively,

$$\begin{aligned} \Pi_i(\Delta, e, \tau) &= x \cdot (\Delta - b) - e \cdot (c_{P,1} + \tau \cdot c_{P,2}), \\ \Pi_a(\Delta, e) &= x \cdot \Delta - e \cdot k, \\ \Pi_s(\Delta, e) &= x \cdot \Delta. \end{aligned}$$

As mentioned earlier, I modify the model in Section 4 such that the activist has a bias as well.

3 Analysis

I solve for the Perfect Bayesian Equilibria of the game, where I allow for mixed strategies. The formal definition of the equilibrium and all proofs not in the main text are in the Appendix. Throughout the analysis, I denote the probability that the activist runs a proxy fight if no settlement is reached by ρ. I start with the following preliminary result.

Lemma 1 (i) *Consider the implementation stage.*

- (a) *If the incumbent board has the decision authority and an action settlement has not been reached, then the incumbent does not implement the project.*
- (b) *If the activist has acquired board seat(s), has the decision authority, and has learned Δ, then the activist implements the project if $\Delta > 0$ and does not implement if $\Delta < 0$.*

(*ii*) *The activist never runs a proxy fight at the proxy fight stage if $k = \zeta$. However, in any equilibrium where the activist runs a proxy fight with positive probability, he always wins.*

At the implementation stage, the incumbent strictly prefers not implementing the project for any Δ since its private benefits b per share from keeping status quo is always strictly larger than the increase Δ in the share value from implementing the project. On the other hand, since the activist does not have private benefits from keeping the status quo, if he learns Δ in the board then he pushes for the project if $\Delta > 0$ and prefers status quo if $\Delta < 0$.

If the activist runs a proxy fight, the shareholders vote according to the shareholder value the activist will create if he wins. For the activist to be willing to incur to cost of a proxy fight, it must be that $\Delta > 0$ with positive probability since the source of the activist's profit is the future increase in the share price. Since the preferences of the shareholders and the activist are aligned, in the event of a proxy fight, the activist always wins.

The cost of proxy fight for the activist is $k \in \{\kappa, \zeta\}$, as described in Section 2. If $k = \zeta$, then this cost is too high, and therefore the activist never runs a proxy fight at the proxy fight stage. Therefore, the activist runs a proxy fight only if his cost of proxy fight is κ. Since the probability of $k = \kappa$ is γ, this is also the highest probability that the activist may run a proxy fight. Throughout the rest of this section, I assume that the activist's cost of proxy fight κ satisfies[20]

$$\kappa \leq \alpha_P E\left[\max\left\{0, \Delta\right\}\right]. \tag{2}$$

Next, I analyze the different subgames where the activist has made a particular settlement demand.

3.1 Action settlement

In this section, I consider the subgame where the activist has demanded action settlement. The proposition below characterizes the equilibrium.

Proposition 1 *Suppose that the activist has demanded action settlement. Then, an equilibrium of this subgame exists, is unique, and in equilibrium:*

[20] This assumption ensures that in any equilibrium, the activist runs a proxy fight with positive probability upon the incumbent board's rejection of a settlement. In the Appendix I relax this assumption and show that a type of equilibrium that arises in addition to those described in the main text are the one where the activist never runs a proxy fight upon rejection, and hence the project is never implemented.

(i) The incumbent accepts the action settlement if and only if $\Delta > \Delta_A^*$*, where*

$$\Delta_A^* \equiv \max\left\{\hat{\Delta}_A,\ b - \frac{c_P}{\frac{1}{\gamma} - \alpha_P}\right\} \in (0, b)\,, \tag{3}$$

where $\hat{\Delta}_A$ *is unique and given by the solution of*

$$\kappa = \phi E\left[\max\{0, \Delta\}\,|\Delta \leq \hat{\Delta}_A\right]. \tag{4}$$

(ii) Upon rejection, the activist runs a proxy fight with probability

$$\rho_A^* \equiv \begin{cases} \gamma, & \textit{if } \hat{\Delta}_A \leq b - \frac{c_P}{\frac{1}{\gamma} - \alpha_P} \\ \frac{1}{\alpha_P + \frac{c_P}{b - \hat{\Delta}_A}} \in (0, \gamma), & \textit{otherwise} \end{cases} \tag{5}$$

One of the results of Proposition 1 is that in equilibrium the incumbent follows a threshold strategy, as illustrated in Figure 3. For given project value $\Delta \geq 0$ and the probability ρ that the activist runs a proxy fight upon rejection, accepting action settlement gives the incumbent a payoff of $\Delta - b$, while rejecting gives it an expected payoff of $\rho[-c_P + \alpha_P(\Delta - b)]$ if $\Delta \geq 0$. Intuitively, the incumbent faces a trade-off when deciding whether to reject action settlement: On the one hand, rejection bears the risk of facing a proxy fight (and hence the associated expected cost of ρc_P), while accepting action settlement guarantees that there will be no proxy fight. On the other hand, the probability that the project will be implemented following a rejection, $\rho\alpha_P$, is smaller than one, while under action settlement it is equal to one. Therefore, for Δ values that are sufficiently large, the incumbent is better off by just accepting to implement the project instead facing the risk of proxy fight. In contrast, for smaller Δ, the incumbent is willing to incur the damages of a proxy fight in order to decrease the probability that the project is eventually implemented. Moreover, if $\Delta < 0$, the activist does not implement the project upon winning a proxy fight, and therefore the incumbent's incentive to reject the settlement is larger. Therefore, the incumbent's incentive to accept the settlement is strictly increasing for all Δ, and there is a threshold $\Delta_A(\rho)$ such that the incumbent accepts action settlement if and only if $\Delta > \Delta_A(\rho)$, where

$$\Delta_A(\rho) \equiv b - \frac{c_P}{\frac{1}{\rho} - \phi}, \tag{6}$$

which is positive if $\rho < \frac{1}{\phi + \frac{c_P}{b}}$. Although $\Delta_A(\rho)$ decreases further and becomes nonpositive if ρ is any larger, this never takes place in equilibrium since otherwise the activist would have no incentive to launch a proxy fight upon rejection. Note that $\Delta_A(\rho)$ is decreasing in the probability ρ of a proxy fight upon rejection and in the incumbent's cost c_P of facing a proxy fight.

Figure 3 - Illustration of the incumbent's equilibrium behavior under action settlement demand.

Combined with the threshold strategy of the incumbent, the activist's cost κ of running a proxy fight pins down the equilibrium ρ^*, and Δ_A^* as a result. Due to the discussion above, upon rejection of action settlement the activist infers that the project return Δ is smaller than the incumbent's threshold Δ_A^*. Since the activist is willing to run a proxy fight upon rejection if and only if there is sufficient potential to increase the value of the firm, the activist has his own unique threshold $\hat{\Delta}_A$ given by (4) such that upon rejection he always runs a proxy fight if $\Delta_A^* > \hat{\Delta}_A$, never runs a proxy fight if $\Delta_A^* < \hat{\Delta}_A$, and is indifferent if $\Delta_A^* = \hat{\Delta}_A$, where $\hat{\Delta}_A$ is increasing in κ. Specifically, if the activist's cost of proxy fight is small, i.e., $\hat{\Delta}_A(\kappa) \leq b - \frac{c_P}{\frac{1}{\gamma} - \alpha_P}$, then the incumbent's threshold strategy Δ_A^* is sufficiently large to incentivize the activist to always run a proxy fight upon rejection when $k = \kappa$, resulting in $\rho^* = \gamma$.

However, if the cost of proxy fight is relatively large, i.e., $\hat{\Delta}_A(\kappa) > b - \frac{c_P}{\frac{1}{\gamma} - \alpha_P}$, then the activist cannot sustain the same level of proxy fight threat $\rho = \gamma$, because high cost makes it uncredible. Indeed, if it were credible, then (6) immediately implies that the resulting equilibrium threshold would be too low (i.e., $\Delta_A < \hat{\Delta}_A$) for the activist to maintain this credibility. On the other hand, the activist's threat ρ cannot be too low either, because that would result in too high of a equilibrium threshold (i.e., $\Delta_A > \hat{\Delta}_A$), which would make proxy fight very appealing to the activist. In other words, $\rho^* \in (0, \gamma)$, and ρ^* is sensitive to the incumbent's strategy Δ_A^*, and specifically, $\Delta_A^* = \hat{\Delta}_A$ must be satisfied so that upon the incumbent's rejection, the activist is indifferent between running a proxy fight and not. In turn, since the incumbent's threshold is strictly decreasing with the threat of a proxy fight, the threshold $\hat{\Delta}_A$ uniquely determines ρ^*, as described in (5).

3.2 Board settlement

In this section, I assume that the activist has demanded board settlement that gives him a control of $\alpha_B > 0$. The next proposition characterizes the equilibrium.

Proposition 2 *Suppose that the activist has demanded board settlement that gives him decision authority with probability $\alpha_B \in (0,1]$. Then, an equilibrium of this subgame with on the equilibrium path proxy fight exists, is unique, and in this equilibrium:*[21]

(i) *The incumbent accepts the settlement if and only if $\Delta \in (l,0) \cup (\Delta_B^*(\alpha_B), b)$, where $\Delta_B^* \in (0,b)$ is given by*

$$\Delta_B^*(\alpha_B) \equiv \begin{cases} b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P}, & \text{if } \alpha_B > \hat{\alpha}_L \\ \hat{\Delta}_B, & \text{otherwise} \end{cases}, \tag{7}$$

where

$$\hat{\alpha}_L \equiv \gamma\alpha_P + \gamma \frac{c_P}{b - \hat{\Delta}_B}, \tag{8}$$

and $\hat{\Delta}_B$ is unique and given by the solution of

$$\kappa = \alpha_P E\left[\Delta | 0 \leq \Delta \leq \hat{\Delta}_B\right]. \tag{9}$$

(ii) *Upon rejection the activist runs a proxy fight with probability $\rho_B^*(\alpha_B) \equiv \gamma \min\left\{1, \frac{\alpha_B}{\hat{\alpha}_L}\right\}$.*

To understand the incumbent's strategy, as illustrated in Figure 4, there are two cases to consider. If $\Delta < 0$, then the incumbent does not have anything to fear from having the activist in the board, since the activist will learn that $\Delta < 0$ and hence will not push to implement the project. On the other hand, the incumbent faces the risk of a proxy fight if it

[21] Throughout the analysis, I focus on this equilibrium, where proxy fight is on the equilibrium path. As I show in the Appendix, the only other equilibrium that exists is the one where the incumbent accepts for all Δ, which exists if and only if $\alpha_B \leq \alpha_P(\gamma + \frac{c}{b})$. However, in an unreported analysis I analyze a modified version of the model where activist's cost k of proxy fight is continuously distributed and show that this type of equilibrium is not robust to Grossman and Perry (1986) criterion, while the equilibrium where proxy fight is on the equilibrium path is robust and converges to the equilibrium described in Proposition 2 as the distribution of k converges to the one described in Section 2. Nevertheless, I show in the Online Appendix that the main results continue to hold qualitatively under the alternative selection of equilibrium.

rejects. Therefore, the incumbent strictly prefers to accept board settlement for all $\Delta < 0$.[22] In contrast, if $\Delta \geq 0$, then the incumbent follows a threshold strategy similar to what it followed under action settlement demand in Section 3.1. Specifically, the incumbent knows that if the activist gets decision authority in the board then he will implement the project, and therefore the incumbent accepts board settlement to avoid the risk of a proxy fight if and only if the value of the project is sufficiently high. To be precise, the incumbent accepts board settlement if and only if

$$\alpha_B(\Delta - b) > \rho[-c_P + \alpha_P(\Delta - b)],$$

or, equivalently for all $\Delta > \Delta_B(\alpha_B, \rho)$, where

$$\Delta_B(\alpha_B, \rho) \equiv b - \frac{c_P}{\frac{\alpha_B}{\rho} - \alpha_P}, \tag{10}$$

which is positive if $\rho < \frac{\alpha_B}{\phi + \frac{c_P}{b}}$. Although $\Delta_B^*(\alpha_B, \rho)$ decreases further and becomes nonpositive if ρ is any larger, this would imply that the incumbent accepts for all Δ, which does not take place in the equilibrium where proxy fight is on the equilibrium path. Note that Δ_B is decreasing in the probability ρ of a proxy fight upon rejection and in the incumbent's cost c_P of facing a proxy fight, while it is increasing in the level of control α_B that the activist is demanding.

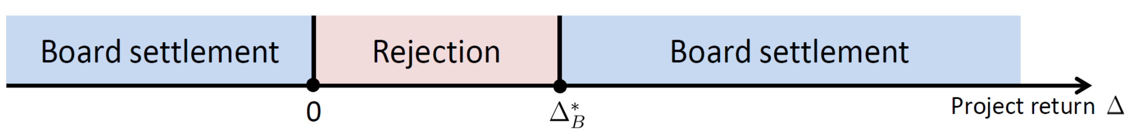

Figure 4 - Illustration of the incumbent's equilibrium behavior under board settlement demand.

In turn, ρ^* is determined in a similar fashion to the case where the activist demands action settlement. Specifically, there is a unique threshold $\hat{\Delta}_B$ such that upon rejection the activist always runs a proxy fight if $\Delta_B^* > \hat{\Delta}_B$, never runs a proxy fight if $\Delta_B^* < \hat{\Delta}_B$, and is indifferent if $\Delta_B^* = \hat{\Delta}_B$, where $\hat{\Delta}_B$ is increasing in κ. If the control demand of the activist is high, i.e., $\alpha_B > \hat{\alpha}_L$, then the incumbent does not have high incentives to accept when $\Delta > 0$. Therefore,

[22] Note that while I assume in the main model that the incumbent does not incur any cost by accepting board settlement if the project will not implemented (i.e., $\Delta < 0$), this may not be the case in reality. However, since the analysis implies that the incumbent *strictly* prefers to accept board settlement for $\Delta < 0$, the equilibrium described in Proposition 2 will continue to hold qualitatively under such a cost as well, as I show in the Appendix.

the threshold that the incumbent follows is always larger than $\hat{\Delta}_B$, and hence the activist always run a proxy fight upon rejection, resulting in $\rho^* = 1$. In contrast, if the control demand of the activist is lower, i.e., $\alpha_B \leq \hat{\alpha}_L$, then the incumbent has higher incentives to accept for a given ρ. However, then $\rho = \gamma$ is not a credible level of threat anymore, since the resulting threshold of the incumbent would be too low to incentivize the activist to launch a proxy fight upon rejection. In other words, ρ^* is sensitive to the incumbent's strategy Δ_B^*, and $\rho^* \in (0, \gamma)$. Specifically, ρ^* must be such that the resulting threshold of the incumbent makes the activist indifferent between running a proxy fight upon rejection and not running (i.e., $\Delta_B^* = \hat{\Delta}_B$), which is uniquely satisfied by $\rho^* = \frac{\alpha_B}{\hat{\alpha}_L}$.

3.3 The type of settlement in equilibrium

In this section, I derive the type of settlement that will be reached in equilibrium. However, first, to better understand the dynamics involved, comparing Proposition 1 and 2 results in a key insight: The incumbent's rejection of board settlement reveals more of its private information than the rejection of action settlement reveals. The following corollary formalizes this insight

Corollary 1 $\Delta_B^*(\alpha_B) \leq \Delta_A^*$ *for all* $\alpha_B \in (0, 1]$. *Moreover, this inequality is strict if* $\rho_A^* < 1$.

To better understand the intuition behind the key insight described above, compare the scenarios where the activist has demanded action settlement to where he has demanded board settlement, as illustrated in Figure 5. First, consider the scenario where the activist has demanded action settlement. Then, the incumbent accepts if and only if the project's return Δ is sufficiently large, because only then it suffers less by accepting compared to by facing the risk of a proxy fight. Therefore, upon the rejection of action settlement the activist infers that the project's return is either mildly positive or negative. For this reason, the activist does not have much incentive to run a proxy fight upon rejection. Since the incumbent is aware of this dynamic, it is not very likely to accept the action settlement in the first place (i.e., large Δ_A^*).

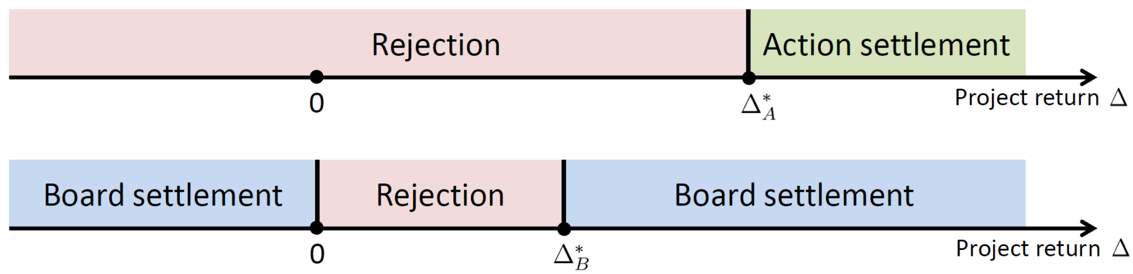

Figure 5 - Illustration of the incumbent's equilibrium behavior under different settlement demands.

Next, consider the scenario where the activist has demanded board settlement. If the project's return Δ is positive, then the equilibrium is similar to the one following action settlement demand: The incumbent accepts if and only if project is highly promising because it does not want to face the risk of a proxy fight. In contrast, if the project's return is negative, then there is a key difference compared to the action settlement. In particular, the incumbent always rejects action settlement because it is too costly for it to implement a negative NPV project on top of losing its private benefits. However, the incumbent accepts board settlement because it knows that the activist will learn that the project's return is negative upon joining the board and will not pursue the project. Therefore, the incumbent rejects board settlement if and only if the project's return is mildly positive.

The above observation is important, because the activist's threat of a proxy fight depends on the information revealed by the activist. If a board settlement is rejected, then the activist and the shareholders infer that the project's return must be positive. This is in contrast to the rejection of action settlement, where the project's return can also be negative. Therefore, demanding board settlement creates a more credible threat of running a proxy fight upon rejection and leads to a higher likelihood of reaching a settlement in the first place (i.e., small Δ_B^*). This observation helps us understand why we see so many board settlements in practice.

However, does this imply that we will observe only board settlement and no action settlement in equilibrium? The answer is no, because it is unlikely for the activist to be able to demand full control of the board via a board settlement. Why would the activist not be able or may not want to get more seats? This might be the case because, for example, the activist might be under too much legal liability if he acquires too many seats. Or, alternatively, valuable information might be lost in the boardroom if too many incumbent directors leave.[23] Indeed, in board settlements, activists do not get the full control of the board, and activists obtain 2 seats on average (see, e.g., Bebchuk et al. (forthcoming)). For these reasons, I restrict the level of control the activist can demand in board settlement to $\alpha_B \in (0, \bar{\alpha}]$, where $\bar{\alpha} \in (0,1)$ is exogenous. Therefore, in contrast to action settlement is, an important disadvantage of board settlement is that conditional on reaching a settlement, the project is not always implemented even if its value is positive, because the activist will not always have the decision authority.

With this trade-off in mind, I next analyze the effect of information asymmetry on the kind of settlement that will be observed in equilibrium. Specifically, in the next Proposition, I show that the likelihood of board (action) settlement increases (decreases) as information asymmetry increases. Importantly, this finding is consistent with Bebchuk et al. (forthcoming), who document a negative relation between information asymmetry and the probability that a settlement contracts over actions.[24]

Proposition 3 *Suppose that Δ is uniformly distributed over (l, b), i.e., $\Delta \sim U(l, b)$, and suppose that $l \in \left(\underline{l}, \bar{l}\right)$, where $\underline{l} \equiv b - \alpha_P \frac{b^2}{2\kappa}$ and $\bar{l} \equiv \max\{0, b - \frac{c_P}{\frac{1}{\gamma} - \alpha_P}\}$.*[25] *Then, as l decreases, conditional on the activist demanding action (board) settlement, the likelihood of reaching settlement decreases (increases). Moreover, these exists a unique $\hat{l} \in (\underline{l}, \bar{l}]$ such that the activist demands board settlement if and only if $l < \hat{l}$.*

When the NPV Δ is distributed uniformly over (l, b), information asymmetry increases as l decreases.[26] Note that I impose the restriction that the upperbound of Δ stays fixed at b as information asymmetry increases, because b represents the private benefits of the incumbent from keeping the status quo, and hence the agency problem is not relevant if $\Delta \geq b$. In other words, the actual range of Δ might expand beyond b, but then the incumbent will implement the project himself anyway, before the activist even shows up. Therefore, when the activist arrives, the distribution of Δ will be bounded from above by b.

[23] Another possible reason is that the activist does not always learn the value of the project upon joining the board, independently from the number of seats he obtains. Indeed, I analyze this modified version of the model in an unreported analysis, and I have shown that the activist might prefer demanding action settlement over board settlement This is because even if $\Delta > 0$ and the activist has the decision authority after a board settlement, he will not implement the project when he cannot learn Δ and his posterior expectation of Δ conditional on reaching a board settlement is negative.

[24] Specifically, they use cash flow volatility as their proxy for information asymmetry.

[25] Note that the parameter restriction $l \in \left(\underline{l}, \bar{l}\right)$ is not a new restriction, but it is rather a result of the restrictions made earlier. Specifically, (2) implies $l > \underline{l}$, and (1) and $l < 0$ implies that $l < \bar{l}$.

[26] In the Appendix, I show that the results of Proposition 3 holds for any distribution of Δ.

Suppose that the activist has demanded action settlement. Then, as information asymmetry increases, upon rejection the project is more likely to be negative NPV, and for this reason the activist's credibility of a proxy fight upon rejection decreases. Therefore, as illustrated in Figure 6, incumbent becomes less likely to accept action settlement (i.e., larger Δ_A^*), and this reduces the activist's profit for two reasons: Even if he runs a proxy fight upon rejection, unlike action settlement this does not guarantee that the project will be implemented, and moreover he ends up incurring the cost of running a proxy fight more often.

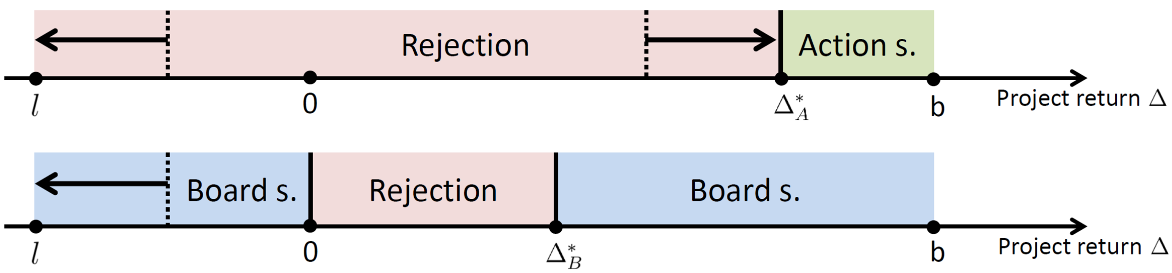

Figure 6 - Illustration of the changes in the settlement regions as information asymmetry increases.

Now, suppose that the activist has demanded board settlement. Then, since board settlement is accepted when NPV is negative, the information revealed by the incumbent's rejection about the NPV does not change (because Δ_B^* does not change). This implies that the activist's credibility of running a proxy fight upon rejection does not decrease as information asymmetry increases. Therefore, in contrast with action settlement, the probability of reaching board settlement increases. An important result of this contrast is that demanding board settlement dampens the negative effect of information asymmetry on activist's profit. This is exactly why as the information asymmetry increases, the activist switches from demanding action settlement to demanding board settlement. These findings not only imply that the *relative* likelihood of board settlement to action settlement increases with information asymmetry, which is consistent with the empirical evidence documented in Bebchuk et al. (forthcoming), but also imply a stronger result: The *absolute* likelihood of observing a board settlement increases and that of observing an action settlement decreases with information asymmetry.[27]

[27] In an unreported analysis, I check and verify the robustness of this result to the modification where the incumbent incurs additional cost by accepting board settlement even if the project will not implemented (i.e., $\Delta < 0$). Specifically, I find that the activist then demands a *menu*, i.e., demands that the incumbent accept either an action settlement or a board settlement. Moreover, there exist $0 < \Delta_B^* \leq \Delta_A^* < b$ such that the incumbent accepts board settlement if $\Delta \in (l, 0) \cup (\Delta_B^*, \Delta_A^*]$ and action settlement if $\Delta > \Delta_A^*$.

3.4 Implications and empirical predictions

Some of the questions that the analysis and discussion up to here have attempted to address are why we see so many board settlements in practice, and when we should expect to observe them, especially in comparison to action settlements. However, how about the effects of settlements on shareholder value? In this section I investigate several implications and empirical predictions of the model, most of which are related to shareholder value. I start with the opposing implications of action and board settlements.

Corollary 2 *(i) Suppose that Δ is distributed as described in Proposition 3. Then,*

(a) The expected shareholder value is strictly larger upon reaching an action settlement than upon reaching a board settlement or upon a successful proxy fight by the activist.

(b) The likelihood of reaching a settlement is strictly higher if the activist demands board settlement than if he demands action settlement.

(ii) There exists $\hat{\alpha} \in (0, 1]$ such that if $\alpha_B \geq \hat{\alpha}$, then the unconditional expected shareholder value is always weakly larger if the activist demands board settlement for a control of $\alpha_B \geq \hat{\alpha}$ than if he demands action settlement, and strictly larger if $\rho^*_A < 1$.

Part (i.a) compares the CAR (cumulative average return) conditional on various outcomes: board settlement, action settlement, and a successful proxy fight. An important difference between action and board settlement is that while action settlement takes place if and only $\Delta \geq \Delta^*_A$, where the project is implemented with probability one, board settlement takes place for all $\Delta \in (l, 0) \cup (\Delta^*_B(\alpha_B), b)$, where $\Delta^*_B(\alpha_B) \leq \Delta^*_A$. In other words, reaching an action settlement bears the positive news that the project NPV is high, while reaching a board settlement bears the negative news that the project might actually have a lower NPV, or even worse, might be worthless. While the inequality $\Delta^*_B \leq \Delta^*_A$ might seem obvious from Corollary 1, recall that Corollary 1 establishes this inequality by comparing two subgames (i.e., where the activist has demanded action settlement vs. board settlement). Therefore, the question that arises here is whether this inequality continues to hold under the *endogeneity* of the settlement type. The answer to this question is yes,[28] and the intuition behind this result can

Therefore, as information asymmetry increases, probability of board settlement increases while probability of action settlement decreases.

[28] Specifically, the question is whether $\Delta^*_B(\alpha_B; l') \leq \Delta^*_A(l'')$ holds for any $\alpha_B \in (0, 1]$ and $l', l'' \in (\underline{l}, \bar{l}]$. I prove in the Appendix that this inequality holds under any distribution of Δ.

again be traced back to the key insight described in the previous section: The incumbent's rejection of board settlement reveals that the project NPV is positive, while its rejection of action settlement reflects that it may also be negative. Therefore, the incumbent is more likely to accept board settlement even if $\Delta > 0$, since otherwise it would have to face a higher risk of a proxy fight. In other words, even though the *ex-ante* information asymmetry in higher when the activist *demands board settlement*, the *ex-post* information asymmetry upon the *rejection of action settlement* is higher.

For the reasons mentioned above, the CAR upon board settlement is strictly smaller than upon action settlement and upon a successful proxy fight. This result is consistent with various empirical evidence documented in literature. Bebchuk et al. (forthcoming) find that board settlements have an average announcement return of about 1.2%, while a settlement that contracts actions (e.g., CEO departure, strategic transactions) leads to an average announcement return of 2.6-8.9%.[29] Gantchev (2013) finds that the mean cumulative abnormal returns ending with the "demand negotiations" stage (where the activist aims to reach a settlement over the action he is demanding) is larger than the returns ending with the "board representation" (where the activist aims to reach a board settlement) as well as than the returns ending with "proxy contest" stage.[30] Becht et al. (2009) document that in their study of activist campaigns, announcement of the implementation of an action demanded by the activist (e.g., restructuring, CEO turnover, payout) results in statistically significant positive abnormal returns, while announcement of director turnover results in negative and insignificant returns.[31]

An immediate implication of the intuition behind part (i.a) of Corollary 2 that was discussed above is given by part (i.b), which states that the conditional probability of reaching a settlement is higher if the type of settlement demanded is board settlement. This result is also consistent with Gantchev (2013), who documents that activists succeed in only about 7% of the cases where they aim to reach a settlement over the action they are demanding, in a stark contrast to 39% of the cases where they aim to reach a board settlement.

Although part (i.a) of Corollary 2 states that the *ex-post* return of board settlements will be smaller compared to action settlements, this does not actually imply that *demanding* board

[29] See table 10.

[30] See table 8.

[31] See table 8. Note that although the UK fund that Becht et al. (2009) study does not typically launch a proxy contest to acquire board seats, the risk of losing director seats still exists for the incumbent board due to the difference of governance rules in the UK compared to the US: As Becht et al. (2009) puts, "At annual general meetings, the statutory rule in the United Kingdom is cumulative majority voting, meaning that each and every director must receive a majority of the 'yes' votes cast to be elected (excluding abstentions)."

settlement produces less value then demanding action settlement. Part (ii) articulates this contrast, and tells that the *ex-ante* shareholders value is in fact higher if the activist demands a sufficiently high number of board seats than if he demands action settlement. This result is intuitive, since if the activist demands action settlement, the likelihood of reaching a settlement is low, and so is the probability of a proxy fight upon incumbent's rejection. In contrast, if the activist demands board settlement, the likelihood of reaching a settlement is higher, and so is the probability of a proxy fight upon incumbent's rejection, given that the activist has demanded a high level of control. This comparison explains how the ex-ante value created by board settlement can be larger. Importantly, this result also points out how careful empirical findings should be interpreted when evaluating the effectiveness of activism. Specifically, measuring the shareholder returns following only settlements or only proxy fights might be misleading, because they are intertwined.

The following corollary focuses on a difference between the ways the activist can acquire access to the board.

Corollary 3 *Suppose that the activist has demanded board settlement that gives him decision authority with probability* $\alpha_B = \alpha_P$ *.Then, the probability that the project is implemented is higher when the incumbent rejects and the activist wins a proxy fight, compared to when the incumbent accepts the settlement.*

Corollary 3 compares the likelihoods of project implementation in the cases where the activist obtains the same level of board control through board settlement versus through a proxy fight. Upon any board settlement, it is revealed that the project NPV is negative with some probability. Therefore, whenever this is the case, the activist does not push for the project in the boardroom. Therefore, upon reaching board settlement, the activist sometimes does not implement the project even if he achieves decision authority. On the other hand, upon rejection of settlement the activist perfectly infers that the project NPV is positive. Therefore, if the activist runs a proxy fight upon rejection, then upon winning he will be very aggressive with his agenda in the boardroom, always implementing the project as long as he has the decision authority. For this reason, the dynamics in the boardroom not only depend on the amount of control the activist has achieved, but also heavily depend on the path the activist has achieved that control (i.e., proxy fight vs. board settlement).

In contrast to the implications discussed above, the next corollary lists some of the similarities between action and board settlements regarding their implications on shareholder value.

Corollary 4 *Suppose that the activist has demanded action settlement or board settlement for some control $\alpha_B \in (0,1]$. Then,*

(i) *If $c_P > (\frac{1}{\gamma} - \alpha_P)b$, then the unconditional shareholder value decreases as c_P increases.*

(ii) *As activist's cost of proxy fight κ decreases, the expected shareholder value conditional on settlement as well as conditional on proxy fight decreases, while the unconditional expected shareholder value increases.*

Part (i) states that fixing the activist's demand, making a proxy fight too costly for the incumbent actually harms shareholder value, even if the activist is unbiased. This result might seem counter-intuitive at a first glance, since between the incumbent board and the activist, the incumbent is the one who is not aligned with shareholders in terms of its preferences with respect to the implementation of the project. However, another important misalignment that exists is the one between the shareholders and the activist due to the cost of proxy fight for the activist. Specifically, the activist internalizes the cost of a proxy fight, while the shareholders do not. The role this friction plays in part (i) of Corollary 4 is the following. If the incumbent's cost of proxy fight c_P is high, then the incumbent is highly incentivized to accept the settlement. In particular, Propositions 1 and 2 imply that the incumbent's rejection threshold is at its minimum: $\Delta_A^* = \hat{\Delta}_A$ if the activist has demanded action settlement, and $\Delta_B^* = \hat{\Delta}_B$ if the activist has demanded board settlement. In either case, upon rejection, the activist is indifferent between running a proxy fight and not upon rejection if his cost of proxy fight is $k = \kappa$. Now, suppose that c_P increases. Then, if the activist were to keep running a proxy fight with the same probability upon rejection, the incumbent would be incentivized to accept the settlement even more, which would reduce the rejection thresholds of the incumbent (i.e., Δ_A^* and Δ_B^*). However, then the activist would have no credible threat of a proxy fight to begin with. Therefore, as c_P increases, the probability that the activist runs a proxy fight decreases, so that the incumbent's rejection threshold remain unchanged. However, the effect of this dynamic on shareholder value is negative, since the project is now less likely to be implemented if the project NPV is smaller then the threshold.

In contrast, in part (ii) of Corollary 4, as the activist's cost κ of running a proxy fight increases, the activist's threat ρ^* increases. Therefore, the incumbent is more likely to accept the settlement, resulting in a drop in the incumbent's rejection threshold, which in turn decreases both the expected shareholder value conditional on settlement and the expected shareholder value conditional on proxy fight. On the other hand, the unconditional shareholder value becomes larger for two reasons: The probability of reaching a settlement increases, and the probability ρ^* of a proxy fight upon rejection increases as well. This result once again underscores the importance of considering the impact of a change on all possible outcomes together, rather than only on settlement or on proxy fight alone.

The last result might leave the impression that reducing the activist's cost of proxy fight always benefits shareholders, at least if the activist is unbiased. However, as we will see later, this is actually not true, because Corollary 4 *fixes* the activist's demand. Indeed, once we endogenize in the next section the level of control the activist demands in equilibrium via board settlement, we will see that lowering activist's cost of proxy fight does not always benefit shareholders.

3.5 The number of seats the activist demands in equilibrium

In this section, I endogenize the level of control α_B the activist demands in equilibrium when he is demanding board settlement.

Corollary 5 *Suppose that the activist demands board settlement equilibrium, and for any $\alpha_B > 0$, the equilibrium with on the equilibrium path proxy fight is in play. Then, an equilibrium always exists, and in any equilibrium the activist demands a control of $\alpha_B^* \in \Lambda \equiv \arg\max_{\min\{\bar{\alpha},\hat{\alpha}_L\}\leq\alpha_B\leq\bar{\alpha}} \Pi_a(\alpha_B)$. Moreover, if $\Delta \sim U(l,b)$, then*

(i) *α_B^* is unique and* [32]

$$\alpha_B^* = \begin{cases} \bar{\alpha}, & \text{if } \hat{\alpha}_L \geq \bar{\alpha} \text{ or } \kappa \leq \frac{c_P}{2} \\ \hat{\alpha}_L, & \text{otherwise.} \end{cases}$$

where $\hat{\alpha}_L(\kappa) = \gamma\alpha_P + \gamma\frac{c_P}{b-\frac{2\kappa}{\alpha_P}}$.

(ii) *The unconditional expected shareholder value is strictly increasing in α_B for any κ. Moreover, if $\kappa_H > \frac{c_P}{2}$ and $\bar{\alpha} > \gamma\alpha_P$, then the shareholder value is maximized if and only if $\kappa \in (0, \frac{c_P}{2}] \cup \{\kappa_H\}$, where*

$$\kappa_H = \frac{\alpha_P}{2}\left(b - \frac{c_P}{\frac{\bar{\alpha}}{\gamma} - \alpha_P}\right).$$

[32] If $\kappa = \frac{c_P}{2}$ and $\hat{\alpha}_L < 1$, the activist's profit is maximized at any $\alpha \in [\hat{\alpha}_L, 1]$.

To see how the activist determines the level of control α_B to demand when he decides to demand board settlement, note that the activist's expected profit by demanding board settlement is given by

$$\Pi_a(\alpha_B) = \int_0^{\Delta_B^*(\alpha_B)} \gamma(\alpha_P\Delta - \kappa)\, dF(\Delta) + \alpha_B \int_{\Delta_B^*(\alpha_B)}^{b} \Delta dF(\Delta), \tag{11}$$

where the first and second terms represent the activist's expected payoff upon rejection of board settlement and upon reaching settlement, respectively. If $\alpha_B < \hat{\alpha}_L$, then $\Delta_B^*(\alpha_B) = \hat{\Delta}_B$ does not change with α_B, and hence $\Pi_A(\alpha_B)$ is strictly increasing in α_B. This is because for the activist to have credibility against the incumbent, $\Delta_B^*(\alpha_B)$ has to be sufficiently large, i.e., $\Delta_B^*(\alpha_B) \geq \hat{\Delta}_B$. When $\alpha_B < \hat{\alpha}_L$, this constraint binds, resulting in $\Delta_B^*(\alpha_B) = \hat{\Delta}_B$. Since the activist enjoys no benefit from demanding a lower α_B, he demands at least $\alpha_B = \hat{\alpha}_L$ level of control (that is, if the upperbound of what he can demand allows it, i.e., if $\hat{\alpha}_L \leq \bar{\alpha}$). On the other hand, if $\alpha_B > \hat{\alpha}_L$, then

$$\Pi_a'(\alpha_B) = \int_{b-\frac{c_P}{\frac{\alpha_B}{\gamma}-\alpha_P}}^{b} \Delta dF(\Delta) - \frac{1}{\gamma}\frac{c_P}{\left(\frac{\alpha_B}{\gamma} - \alpha_P\right)^2} f\left(b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P}\right)\left((\alpha_B - \gamma\alpha_P)\left(b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P}\right) + \gamma\kappa\right). \tag{12}$$

In this case, demanding higher α_B has three distinct effects: While it gives the activist higher control conditional on board settlement, as represented by the first term in (12), it also increases the likelihood of rejection, as represented by the second term. In the latter case, not only the activist has to incur the cost of a proxy fight to have the project implemented, but also the probability that it will be eventually implemented drops from α_B to $\gamma\alpha_P$ even though the activist runs a proxy fight when his cost of proxy fight is $k = \kappa$. These effects determine in equilibrium the level of α_B the activist demands. In particular, if κ is large, then the downsides of demanding high α_B will dominate, and the activist will demand exactly $\alpha_B = \hat{\alpha}_L$ level of control.

Note that while the activist internalizes the cost of proxy fight, shareholders do not. Therefore, the shareholders do not mind the rejection risk of the incumbent as much as the activist does, and therefore they may end up wishing that the activist were more aggressive in his demand and would demand higher α_B. In other words, the number of board seats the activist demand might be too low to maximize shareholder value. To understand the implications of this disparity between the activist and shareholders more concretely with an example, let us consider the uniform distribution for Δ. As implied by Corollary 5, then the shareholder value is maximized if the activist demands as many board seats as possible, i.e., $\alpha_B = \bar{\alpha}$. However, does the activist actually end up demanding this many seats? If the activist's cost of proxy fight is low (i.e., $\kappa < \frac{c_P}{2}$), he indeed demands $\alpha_B^* = \bar{\alpha}$, because he does not fear rejection and does not minimize the incidence of proxy fights. In contrast, if κ is moderate (i.e., $\kappa \in (\frac{c_P}{2}, \kappa_H)$), then the activist demands a lower number of seats (i.e., $\alpha_B^* = \hat{\alpha}_L < \bar{\alpha}$), because doing so reduces the probability of rejection and therefore the need for a proxy fight. Therefore, the resulting shareholder value is lower.

However, if κ is moderate and increases further, then the activist starts demanding higher number of seats, increasing the shareholder value back up as a result. Why is this the case? If the activist demands a lower number of seats, the incumbent's rejection threshold Δ_B^* does not decrease anymore (if it were to decrease the activist would never run a proxy fight upon rejection, because the expected NPV would be low compared to the cost of proxy fight). However, then the activist does not benefit anymore from demanding a low number of seats, so he demands a higher number of seats instead. In other words, activist's incentive constraint for having sufficient credible threat binds, and this is reflected by an increase in $\hat{\alpha}_L(\kappa)$ as κ increases, resulting in a higher α_B^* since $\alpha_B^* = \hat{\alpha}_L$. Moreover, if κ increases to $\kappa = \kappa_H$, then the activist completely reverts back to demanding highest control possible, i.e., $\alpha_B^* = \bar{\alpha}$, because $\hat{\alpha}_L(\kappa)$ reaches $\bar{\alpha}$.[33]

To summarize, an important takeaway from Corollary 5 is that making activism costlier can actually increase shareholder value, even if the activist unbiased. First of all, at a practical level, activists obtain about two seats on average in board settlements (Bebchuk et al., (forthcoming)), which suggests that the reality might be often falling under the case of moderate proxy fight cost described above. Second, a theoretically interesting aspect is that this result stems from the free-rider problem itself, although the result itself is stark contrast to the vast majority of literature on free-rider problem.

[33]More generally, I show in an unreported analysis that under any distribution, as long as for some κ the activist does not demand the highest level of control he can demand, then expected shareholder value is nonmonotonic with respect to κ and is maximized if and only if $\kappa \in [0, \kappa_L] \cup \{\kappa_H\}$ for some $0 < \kappa_L < \kappa_H$.

4 Biased activist

A common concern expressed by institutional investors is that settlements may harm shareholders, since activists effectively bypass other shareholders in settlements. In this section, I generalize the setup by allowing for a bias of $b_A \in (-\infty, \infty)$ in the activist's payoff, and I show that whenever a proxy fight occurs with positive probability, the activist never destroys shareholder value through settlements but only after the activist wins a proxy fight with the support of the shareholders.

Specifically, I assume that the activist's payoff when the project is implemented is $\Delta - b_A$, and hence with this modification the activist's payoff is given by

$$\Pi_a(\Delta, x, e) = x \cdot (\Delta - b_A) - e \cdot k,$$

where $e = 1$ if the activist runs a proxy fight and $x = 1$ if the project is implemented. Therefore, compared to the shareholders, $b_A > 0$ ($b_A < 0$) means that the activist has a bias against (in favor) of implementing the project.

Part (i) of the following Proposition states that the main results from the baseline model continue to hold, while part (ii) focuses on the implications of the activist's bias for shareholder value.

Proposition 4 *(i) Suppose that the activist has a bias of $b_A \in (-\infty, b)$.*[34] *Then, Proposition 3 and Corollaries 2, 3, and 4 continue to hold, where $\underline{l}$ and $\bar{l}$ are replaced with $\underline{l} \equiv b - \alpha_P \frac{(b-b_A)^2}{2\kappa}$ and $\bar{l} \equiv \max\{b_A, b - \frac{c_P}{\frac{1}{\gamma} - \alpha_P}\}$.*

(ii) Suppose that the activist has a bias of $b_A \in (-\infty, \infty)$, and the activist has demanded a settlement. Then, an equilibrium always exist. Moreover,

(a) Suppose $b_A < 0$. Then, there always exists an equilibrium where the project is never implemented upon settlement if $\Delta < 0$. Moreover, in any equilibrium where proxy fight is on the equilibrium path, for any $\Delta < 0$ the project is never implemented following a settlement, while for all $b_A < \Delta < 0$ the activist runs and wins the proxy fight and implements the project with positive probability.

[34] Also, suppose that $l < b_A$ and $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$, where the former is the modified version of the parameter restriction $l < 0$ from the main model, while the latter is the modified version of the restriction (2).

(*b*) *Suppose* $b_A \geq 0$. *Then, in any equilibrium, the project is never implemented if* $\Delta < 0$.

If the activist has a bias against the project (i.e., $b_A > 0$), then a shareholder value destroying project is never implemented since both the activist and the incumbent are against such a project. Therefore, if a value destroying project is ever implemented in equilibrium, it must be that the activist has a bias in favor of the project, i.e., he must be profiting from the implementation of the project if $b_A < \Delta < 0$ for some $b_A < 0$. Therefore, a question that arises is that, since shareholders are not at the negotiation table when settlements are reached, could the activist be destroying shareholder value through settlements?

However, even if $b_A < 0$, Proposition 4 tells that in any equilibrium where proxy fight is on the equilibrium path,[35] the project is never implemented upon settlement if the project destroys shareholder value. On the other hand, in any such equilibrium, negative NPV projects do get implemented, and interestingly this takes place exclusively after the activist runs and wins a proxy fight! To see the intuition behind this result, note that if proxy fight is on the equilibrium path, it must be that the shareholders elect the activist with positive probability. Consider any settlement demand that can be made by the activist. If the activist has demanded action settlement, then similar to the intuition in Proposition 1, the incumbent follows a threshold strategy Δ_A^* such that it accepts if and only if $\Delta > \Delta_A^*$. Therefore, for any value destruction to take place through settlement, it must be $\Delta_A^* < 0$. However, then upon rejection and the activist's proxy fight, shareholder perfectly understand the activist will destroy value if he gets seats in the board, and therefore they never elect him. For this reason, value destroying projects are implemented only after the activist runs a proxy fight. The case where the activist demands board settlement is similar. In this case, the incumbent accepts board settlement for all $\Delta < b_A$, because it knows that the activist will not push for the project once he learns Δ. Similar to the case with the unbiased activist in Proposition 2, there is a second threshold Δ_B^* such that the incumbent rejects if and only if $\Delta \in [b_A, \Delta_B^*]$. However, it cannot be that $\Delta_B^* < 0$, because then the shareholders never elect the activist to the board in the case of a proxy fight. Therefore, again, no value destroying project is implemented upon board settlement, but such projects do get implemented with positive probability after the activist runs a proxy fight.

[35] As I did for the main model, in an unreported analysis I analyze a modified version of the model where activist's cost k of proxy fight is continuously distributed and show that any equilibrium where proxy fight is not on the equilibrium path is not robust to Grossman and Perry (1986) criterion, while the equilibrium where proxy fight is on the equilibrium path is robust and converges to the equilibrium of the model in this section.

4.1 Does the existence of settlements always benefit shareholders?

Proposition 4 and the discussion following it describes how settlements do not *ex-post* destroy value. However, this does not mean that the existence of settlements is always good for shareholders. In fact, I find that even if the activist unbiased (i.e., $b_A = 0$ as in the main model), allowing for settlements can decrease *ex-ante* shareholder value. Specifically, this is the case if settlements substantially decrease the incentives of the activist to run a proxy fight. The next corollary formalizes this result.

Corollary 6 *Suppose that $b_A = 0$, and the activist has demanded any settlement. Then, given the settlement η that the activist has demanded, there exists $\bar{\rho}(\eta)$ such that the expected shareholder value is smaller than in the equilibrium where settlements are not allowed if and only if $\rho^*(\eta) < \bar{\rho}(\eta)$.*

To understand this result, recall that the main wedge between the activist and shareholders is that the shareholders do not internalize the cost of running a proxy fight but activist does. In contrast with the comparisons we have made up to this point in the paper, rather than comparing the activist's incentives to run a proxy fight upon demanding different kind of settlements, we will rather focus on the comparison of these incentives if the activist demands a settlement versus if a settlement is not an option. If hypothetically settlement were not an option, then the activist would always run a proxy fight since the expected value of the project would be high and there would be no additional information that the activist would learn before deciding to launch a proxy fight. In contrast, when the activist demands a settlement, he makes an inference from the rejection, and sometimes it signals that the NPV is lower. In those cases the activist will often choose not to launch a proxy fight even though it is better to do so for the shareholders. If this decrease in the likelihood of proxy fight is large, then the ex-ante shareholder value in this case is smaller compared to the first case where the activist did not demand any settlement. Consistent with the concern that the activists may not be launching proxy fight as often as they should from the perspective of the shareholders, Bebchuk et al. (forthcoming) document that, over the last five years their data sample (i.e., between 2009-2013), the number of activist campaigns ending with a settlement is about fives times of the number of campaigns ending with a voted proxy fight.

Since the disparity between the interests of the activist and the shareholders stems from the activist's cost of proxy fight, a question that might arise is whether the shareholder value can be increased by decreasing cost of proxy fight. By Corollary 4, this is indeed the case if the activist is demanding action settlement in equilibrium. However, by Proposition 4 and the discussion follows it, this may not hold if the activist is demanding board settlement. In particular, while reducing the activist's cost of proxy fight all the way to zero would increase shareholder value, reducing the cost by less actually harms shareholders when the activist demands fewer board seats as a result. Therefore, reducing activist's cost of proxy fight does not provide a guarantee that the shareholders will benefit from the existence of settlements more.

5 Conclusion

In this paper, I study the economics of settlements between activist investors and incumbent boards. The activist can demand an action to be implemented right away ("action settlement") or demand a number of board seats ("board settlement") which gives the activist partial decision authority and access to better information about the prospects of the proposal.

I find that the incumbent's rejection of board settlement reflects more of its private information than the rejection of action settlement does. Therefore, demanding board settlement increases the activist's credibility to run a proxy fight upon rejection and leads to a higher likelihood of reaching a settlement in the first place. Due to this informational advantage of board settlement over action settlement, the likelihood of board (action) settlement increases (decreases) with information asymmetry, consistently with the empirical evidence by Bebchuk et al. (forthcoming). Moreover, while the average ex-post shareholder value upon reaching board settlement is lower than upon reaching action settlement, the ex-ante value created by demanding board settlement can be higher. Therefore, when trying to identify the effects of activism on firm value, measuring shareholder value at the demand level of the activist might produce more accurate results. On the other hand, to minimize the risk of rejection, activists may demand too few seats and not maximize shareholder value. Surprisingly, increasing the cost of a proxy fight can alleviate this conflict and make the activist more aggressive. Finally, even though value-destroying projects are typically not implemented following settlements, the existence of settlements may nevertheless destroy shareholder value due to the free-rider problem. However, strikingly, making activism less costly can actually further exacerbate this problem.

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A Proofs of main results

In many of the proofs below, I prove the results with generalizing the model to activist's bias $b_A \in (-\infty, b)$. To see the details of the biased activist modification, see the beginning of Section 4. Whenever I do such a generalization, I assume $\zeta > \alpha_P(b - b_A)$, which reduces to $\zeta > \alpha_P b$ in Section 2 since the activist does not have any bias in the main model, i.e., $b_A = 0$. The generalizations also often include additional cost that the incumbent board incurs solely from the activist gaining board seats. Specifically, I denote the cost that the incumbent incurs when the activist obtains α level of decision authority by αc_b. This cost is in addition to any other cost that the incumbent board may incur as described in Section 2 (i.e., in addition to the cost of proxy fight as well as the disutility the incumbent board incurs if the project is implemented), and hence this cost is independent of whether the activist has obtained board seats through proxy fight or through board settlement. Whenever I do such a generalization, I assume $l < b - \frac{c_P + \alpha_P c_b}{\frac{1}{\gamma} - \alpha_p}$, which reduces to $l < b - \frac{c_P}{\frac{1}{\gamma} - \alpha_p}$ in Section 2 since $c_b = 0$ in the main model.

Definition. Denote the activist's demand as $\eta = A$ if he has demanded action settlement, and as $\eta = \alpha_B$ if he has demanded board settlement that gives him $\alpha_B \in [0, 1]$ control in the board. Then, given the activist's demand $\eta \in \{A\} \cup \{\alpha_B\}_{\alpha_B \in [0,1]}$, a Perfect Bayesian Equilibrium (PBE) of this subgame is $\left(\{\iota^*(\Delta)\}_{\Delta \in (l,b)}, \{\rho^*(k)\}_{k \in \{\kappa, \zeta\}}, \sigma^*, \{x_i^*(\Delta)\}_{\Delta \in (l,b)}, \{x_a^*(\Delta)\}_{\Delta \in (l,b)}\right)$ such that there exist cumulative distribution functions (F_a, F_s) with full support on (l, b) such that:[36]

(i) At the implementation stage, for any $\Delta \in (l, b)$, the incumbent (the activist) implements the project with probability $x_i^*(\Delta)$ ($x_i^*(\Delta)$) such that it maximizes its (his) payoff, i.e.,[37]

$$
\begin{aligned}
x_i^*(\Delta) &\in X_i(\Delta) \equiv \arg\max_{x \in [0,1]} x\Delta - b \\
x_a^*(\Delta) &\in X_a(\Delta) \equiv \arg\max_{x \in [0,1]} x\Delta - b_A
\end{aligned}
$$

(ii) At the voting stage, the activist wins the proxy fight with probability σ^* such that it maximizes shareholders' payoff given their belief $F_s(\cdot)$ and the rest of the strategy profile,

[36]Here, F_a (F_s) is the activist's (shareholders') belief about Δ upon incumbent's rejection.

[37]Note that there is no belief involved at this stage, since both the activist and the incumbent know Δ at this stage.

i.e., $\sigma^* \in BR_s(F_s(\cdot), \{x_i^*(\Delta)\}_{\Delta\in(l,b)}, \{x_a^*(\Delta)\}_{\Delta\in(l,b)})$, where

$$BR_s(F_s(\cdot), \{x_i(\Delta)\}_{\Delta\in(l,b)}, \{x_a(\Delta)\}_{\Delta\in(l,b)}) \equiv \arg\max_{\sigma'\in[0,1]} \sigma' \int_l^b [(1-\alpha_P)x_i\Delta + \alpha_P x_a \Delta]\, dF_s(\Delta)$$

(*iii*) At the proxy fight stage, the activist runs proxy fight with probability $\rho^*(k)$ such that it maximizes his payoff given his belief $F_a(\cdot)$, cost of proxy fight k, and the rest of the strategy profile, i.e., $\rho^*(k) \in BR_a(k, F_a(\cdot), \sigma^*, \{x_i^*(\Delta)\}_{\Delta\in(l,b)}, \{x_a^*(\Delta)\}_{\Delta\in(l,b)})$, where

$$\begin{aligned} & BR_a(k, F_a(\cdot), \sigma, \{x_i(\Delta)\}_{\Delta\in(l,b)}, \{x_a(\Delta)\}_{\Delta\in(l,b)}) \\ \equiv\ & \arg\max_{\rho'} \rho' \left[-k + \sigma \int_l^b [(1-\alpha_P)x_i\Delta + \alpha_P x_a\Delta]\, dF_a(\Delta)\right] \end{aligned}$$

(*iv*) At the response stage, for any $\Delta \in (l,b)$, the incumbent accepts with probability $\iota^*(\Delta)$ such that it maximizes its payoff given the rest of the strategy profile, i.e., $\iota^*(\Delta) \in BR_i(\Delta, \eta, E[\rho^*], \sigma^*, \{x_i^*(\Delta)\}_{\Delta\in(l,b)}, \{x_a^*(\Delta)\}_{\Delta\in(l,b)})$, where

$$\begin{aligned} & BR_i(\Delta, \eta, \rho, \sigma, \{x_i(\Delta)\}_{\Delta\in(l,b)}, \{x_a(\Delta)\}_{\Delta\in(l,b)}) \\ \equiv\ & \arg\max_{\iota'} \left[\begin{array}{l} \iota'[1_{\{\eta=A\}} \cdot (\Delta - b) + 1_{\{\eta\neq A\}} \cdot [(1-\eta)x_i\Delta + \eta x_a\Delta]] \\ +(1-\iota')\rho\,[-c_{p,1} + \sigma\,[-c_{p,2} + (1-\alpha_P)x_i\,(\Delta-b) + \alpha_P x_a\,(\Delta - b)]] \end{array} \right] \end{aligned}$$

(*v*) If rejection is on the equilibrium path in any equilibrium (i.e., $\iota^*(\Delta) < 1$ for any $\Delta \in (l,b)$), then the beliefs must satisfy Bayes' rule, i.e., for any $\Delta \in (l,b)$,

$$F_s^*(\Delta) = F_a^*(\Delta) = \begin{cases} \frac{\int_l^{\Delta}[1-\iota^*(\Delta')]dF(\Delta')}{\int_l^b[1-\iota^*(\Delta')]dF(\Delta')}, & \text{if } \int_l^b [1-\iota^*(\Delta')]\, dF(\Delta') > 0 \\ \frac{\sum_{\Delta'\in(l,\Delta] \text{ s.t. } \iota^*(\Delta')\neq 1}[1-\iota^*(\Delta)]f(\Delta)}{\sum_{\Delta'\in(l,b) \text{ s.t. } \iota^*(\Delta')\neq 1}[1-\iota^*(\Delta')]f(\Delta')}, & \text{otherwise.} \end{cases}$$

Remark. Note that I do not put any restriction for the off-equilibrium-path beliefs. To simplify the exposition, from this point onwards I express $E[\rho]$ simply as ρ.

I prove Lemma 1 with the following generalized lemma, where $b_A \in (-\infty,\infty)$, $c_b \in [0,\infty)$, $\kappa \in (0,\infty)$.

Lemma 2 *Suppose that* $b_A \in (-\infty,\infty)$, $c_b \in [0,\infty)$, $\kappa \in (0,\infty)$. *In any subgame,*

(*i*) *At the implementation stage:*

(*a*) *If the incumbent board has the decision authority, then the incumbent does not implement the project.*

(*b*) *If the activist has acquired board seat(s), has the decision authority, and has learned* Δ*, then the activist implements the project if* $\Delta > b_A$ *and does not implement if* $\Delta < b_A$.

(*ii*) *At the proxy fight stage, the activist never runs a proxy fight if* $k = \zeta$.

Proof. Proof of part (i) is the same with the proof of part (i) Lemma 1, which is provided right after Lemma 1.

Consider part (ii). At the proxy fight stage, for any belief the activist has about Δ and for any probability of winning a proxy fight upon launching it, the activist's expected payoff from running a proxy fight is strictly less than $\alpha_P (b - b_A) - k$. Since $\zeta > \alpha_P (b - b_A)$, this implies that the activist never runs a proxy fight if $k = \zeta$. ■

The next lemma characterizes the best responses of the activist, the incumbent board, and the shareholders as a function of their beliefs, given the activist has demanded action settlement.

Lemma 3 *Suppose that* $b_A \in (-\infty, \infty)$, $c_b \in [0, \infty)$, $\kappa \in (0, \infty)$, *and the activist has demanded action settlement. Moreover:*

(*i*) *Suppose that the activist runs a proxy fight with probability* ρ *upon rejection and the shareholders support the activist with probability* σ *upon a proxy fight. Then, the incumbent board follows a threshold strategy, i.e., it accepts action settlement if* $\Delta > \Delta_A(\rho, \sigma)$ *and rejects if* $\Delta < \Delta_A(\rho, \sigma)$*, where*

$$\Delta_A(\rho, \sigma) \equiv \begin{cases} b - \frac{\rho c_{p,1} + \rho\sigma(c_{p,2} + \alpha_p c_b)}{1 - \rho\sigma\alpha_p}, & \textit{if } b_A \leq b - \frac{\rho c_{p,1} + \rho\sigma(c_{p,2} + \alpha_p c_b)}{1 - \rho\sigma\alpha_p} \\ \min\left\{b_A, b - \rho c_{p,1} - \rho\sigma\left(c_{p,2} + \alpha_p c_b\right)\right\}, & \textit{otherwise} \end{cases}$$

(*ii*) *Suppose that the incumbent board accepts action settlement if* $\Delta > \Delta_A(\rho, \sigma)$ *and rejects if* $\Delta < \Delta_A(\rho, \sigma)$ *for some* $\Delta_A > l$*, and upon launching a proxy fight, the activist wins with probability* $\sigma \in [0, 1]$*. Then, upon rejection of action settlement, the activist's best* *response is to run a proxy fight with probability* $\rho(\Delta_A, \sigma) \in BR_a(\Delta_A, \sigma; \hat{\Delta}_A(\sigma))$, *where*

$$BR_a(\Delta_A, \sigma; \hat{\Delta}_A(\sigma)) \equiv \begin{cases} \{\gamma\}, & \textit{if } \Delta_A > \hat{\Delta}_A(\sigma), \\ [0, \gamma] & \textit{if } \Delta_A = \hat{\Delta}_A(\sigma), \\ \{0\}, & \textit{if } \Delta_A < \hat{\Delta}_A(\sigma), \end{cases} \tag{13}$$

and $\hat{\Delta}_A(\sigma)$ *is given by*

$$\hat{\Delta}_A(\sigma) \equiv \begin{cases} l, & \textit{if } \kappa \leq \sigma\alpha_P (l - b_A), \\ \begin{array}{l} x > \max\{l, b_A\} \textit{ such that} \\ \kappa = \sigma\alpha_P \frac{1}{F(x)} \int_{b_A}^{x} (\Delta - b_A)\, dF(\Delta), \end{array} & \textit{if } \sigma\alpha_P (l - b_A) < \kappa \leq \sigma\alpha_P \int_{\min\{b, b_A\}}^{b} (\Delta - b_A)\, dF(\Delta \\ \infty, & \textit{otherwise.} \end{cases} \tag{14}$$

(*iii*) *Suppose that the incumbent board ccepts action settlement if* $\Delta > \Delta_A(\rho, \sigma)$ *and rejects if* $\Delta < \Delta_A(\rho, \sigma)$ *for some* $\Delta_A > l$. *Then, if the activist launches a proxy fight, the shareholders' best response is to support him with probability* $\sigma(\Delta_A) \in BR_s(\Delta_A)$, *where*

$$BR_s(\Delta_A) \equiv \begin{cases} \{1\}, & \textit{if } \Delta_A > \check{\Delta}, \\ [0, 1] & \textit{if } \Delta_A \in (l, b_A] \cup \{\check{\Delta}\}, \\ \{0\}, & \textit{if } \Delta_A \in (b_A, \check{\Delta}), \end{cases} \tag{15}$$

where $\check{\Delta}$ *is given by*

$$\check{\Delta} \equiv \begin{cases} b_A, & \textit{if } b_A \geq 0 \\ x > \max\{l, b_A\} \;\; \textit{s.t. } 0 = \int_{b_A}^{x} \Delta dF(\Delta), & \textit{if } b_A < 0 \textit{ and } \int_{b_A}^{b} \Delta dF(\Delta) \geq 0 \\ \infty, & \textit{otherwise.} \end{cases} \tag{16}$$

Proof. Note that $\Delta \in (l, b)$ by assumption. Consider part (i). If the incumbent board accepts action settlement, then its payoff is $\Delta - b$. If the incumbent board rejects it, then by Lemma 2, its payoff is

$$\pi_i = \begin{cases} -\rho c_{p,1} + \rho\sigma\left[-c_{p,2} + \alpha_p(-c_b + \Delta - b)\right], & \text{if } \Delta > b_A, \\ \begin{array}{l} \in \left[-\rho c_{p,1} + \rho\sigma\left[-c_{p,2} + \alpha_p(-c_b + \Delta - b)\right],\right. \\ \left. -\rho c_{p,1} + \rho\sigma\left[-c_{p,2} + \alpha_p(-c_b)\right]\right], \end{array} & \text{if } \Delta = b_A, \\ -\rho c_{p,1} + \rho\sigma\left[-c_{p,2} + \alpha_p(-c_b)\right], & \text{if } \Delta < b_A, \end{cases} \tag{17}$$

where the value of π_i within the specified interval when $\Delta = b_A$ depends on the probability that the activist implements the action when he is indifferent between implementing and not implementing. If $\Delta > b_A$, then the incumbent board rejects (accepts) action settlement if

$$\Delta - b < (>) - \rho c_{p,1} + \rho\sigma \left[-c_{p,2} + \alpha_p(-c_b + \Delta - b)\right],$$

or, equivalently, rejects if

$$\Delta < b - \frac{\rho c_{p,1} + \rho\sigma\left(c_{p,2} + \alpha_p c_b\right)}{1 - \rho\sigma\alpha_p} \tag{18}$$

and accepts if

$$\Delta > b - \frac{\rho c_{p,1} + \rho\sigma\left(c_{p,2} + \alpha_p c_b\right)}{1 - \rho\sigma\alpha_p}. \tag{19}$$

If $\Delta < b_A$, the incumbent board rejects (accepts) settlement if

$$\Delta - b < (>) - \rho c_{p,1} + \rho\sigma \left[-c_{p,2} + \alpha_p(-c_b)\right],$$

or, equivalently, rejects if

$$\Delta < b - \rho c_{p,1} - \rho\sigma\left(c_{p,2} + \alpha_p c_b\right) \tag{20}$$

and accepts if

$$\Delta > b - \rho c_{p,1} - \rho\sigma\left(c_{p,2} + \alpha_p c_b\right). \tag{21}$$

Finally, if $\Delta = b_A$, the incumbent board rejects if (18) holds and accepts if (21) holds.

Note that if (18) is satisfied for some Δ, then (20) is satisfied for all $\Delta' \leq \Delta$. Therefore, if $b_A < b - \frac{\rho c_{p,1} + \rho\sigma(c_{p,2} + \alpha_p c_b)}{1 - \rho\sigma\alpha_p}$, then the incumbent rejects action settlement for all $\Delta \leq b_A$. This implies that for any Δ, the incumbent rejects if (18) holds and accepts if (19) holds. If $b_A = b - \frac{\rho c_{p,1} + \rho\sigma(c_{p,2} + \alpha_p c_b)}{1 - \rho\sigma\alpha_p}$, then this implies that (20) holds (and hence the incumbent rejects) for all $\Delta < b_A$ and (19) holds (and hence the incumbent accepts) for all $\Delta > b_A$. Finally, if $b_A > b - \frac{\rho c_{p,1} + \rho\sigma(c_{p,2} + \alpha_p c_b)}{1 - \rho\sigma\alpha_p}$, then (19) is satisfied for all $\Delta > b_A$ (and hence the incumbent accepts action settlement for no $\Delta > b_A$), and what remains to be shown is that if $\Delta \leq b_A$, then the incumbent rejects for $\Delta < \min\left\{b_A, b - \rho c_{p,1} - \rho\sigma\left(c_{p,2} + \alpha_p c_b\right)\right\}$ and accepts for $\Delta > \min\left\{b_A, b - \rho c_{p,1} - \rho\sigma\left(c_{p,2} + \alpha_p c_b\right)\right\}$. To see this, consider two subcases: First, suppose that $b_A \leq b - \rho c_{p,1} - \rho\sigma\left(c_{p,2} + \alpha_p c_b\right)$. Then, (20) holds for all $\Delta < b_A$, and hence the incumbent rejects for all $\Delta < b_A$. Second, suppose that $b_A > b - \rho c_{p,1} - \rho\sigma\left(c_{p,2} + \alpha_p c_b\right)$. Then, (21) holds (and hence the incumbent accepts) for all $\Delta \in \left(b - \rho c_{p,1} - \rho\sigma\left(c_{p,2} + \alpha_p c_b\right), b_A\right]$, and (20) holds (and hence the incumbent rejects) for all $\Delta < \left(b - \rho c_{p,1} - \rho\sigma\left(c_{p,2} + \alpha_p c_b\right)\right.$. This concludes the proof of part (i).

Next, consider part (ii). Note that if the incumbent rejects, then with probability γ the activist's cost of proxy fight is ζ and hence he does not run a proxy fight by Lemma 2. With probability $1-\gamma$, the activist's cost of proxy fight is κ, and his expected payoff from not running a proxy fight is zero and from running a proxy fight is

$$\pi_a = -\kappa + \sigma\alpha_P \frac{1}{F(\Delta_A)} \int_{\min\{b_A,\Delta_A\}}^{\Delta_A} (\Delta - b_A)\, dF(\Delta),$$

which is strictly positive if $\Delta_A > \hat{\Delta}_A(\sigma)$, zero if $\Delta_A = \hat{\Delta}_A(\sigma)$, and strictly negative if $\Delta_A < \hat{\Delta}_A(\sigma)$. This concludes this part.

Finally, consider part (iii). If the activist loses the proxy fight, then by Lemma 2 the shareholder value will be zero. On the other hand, if the activist wins the proxy fight, then under the specified beliefs the expected shareholder value will be

$$\pi_s = \alpha_P \frac{1}{F(\Delta_A)} \int_{\min\{b_A,\Delta_A\}}^{\Delta_A} \Delta dF(\Delta).$$

Therefore, if $b_A \geq 0$, then $\pi_s > 0$ when $\Delta_A > b_A$ and $\pi_s = 0$ when $\Delta_A \leq b_A$. On the other hand, if $b_A < 0$, then $\pi_s > 0$ when $\Delta_A > \check{\Delta}$ and $\pi_s = 0$ when $\Delta_A \in (l, b_A] \cup \check{\Delta}$, and $\pi_s < 0$ if $\Delta_A \in (b_A, \check{\Delta})$. ■

Lemma 4 *Suppose that $b_A \in (-\infty,\infty)$, $c_b \in [0,\infty)$, $\kappa \in (0,\infty)$, and the activist has demanded action settlement. Then, in any equilibrium of this subgame:*

(i) $l < \Delta_A^*$.

(ii) If $b_A < b$, then $b_A < \Delta_A^* = b - \frac{\rho^* c_{p,1} + \rho^*\sigma^*(c_{p,2}+\alpha_p c_b)}{1-\rho^*\sigma^*\alpha_p}$.

Proof. First, I prove part (i). There are two cases to consider. First, suppose that $b_A \leq l$. Then, since $b_A \leq l < b - \frac{c_p + \alpha_p c_b}{\frac{1}{\gamma} - \alpha_p} \leq b - \frac{c_{p,1}+\sigma(c_{p,2}+\alpha_p c_b)}{\frac{1}{\rho} - \sigma\alpha_p}$ for any $\rho \in [0,\gamma]$ and $\sigma \in [0,1]$, any equilibrium satisfies $\Delta_A^* = b - \frac{c_{p,1}+\sigma^*(c_{p,2}+\alpha_p c_b)}{\frac{1}{\rho^*} - \sigma^*\alpha_p} > l$ by Lemma 3. Second, suppose that $l < b_A$. Then, by Lemma 3, any equilibrium satisfies either $\Delta_A^* = b - \frac{c_{p,1}+\sigma^*(c_{p,2}+\alpha_p c_b)}{\frac{1}{\rho^*} - \sigma^*\alpha_p}$ or $\Delta_A^* = b -$

$\rho^* c_{p,1} - \rho^* \sigma^* (c_{p,2} + \alpha_p c_b)$. Since $l < b - \frac{c_p + \alpha_p c_b}{\frac{1}{\gamma} - \alpha_p} \leq b - \frac{c_{p,1} + \sigma(c_{p,2} + \alpha_p c_b)}{\frac{1}{\rho_i} - \sigma \alpha_p} \leq b - \rho c_{p,1} - \rho\sigma (c_{p,2} + \alpha_p c_b)$ for any $\rho \in [0, \gamma]$ and $\sigma \in [0, 1]$, this implies that $l < \Delta_A^*$ in any equilibrium.

Second, I prove part (ii). Suppose that $b_A < b$, but there is an equilibrium such that $\Delta_A^* \leq b_A$. However, since $l < \Delta_A^*$ by part (i), then by Lemma 3 this implies that it must be $\rho^* = 0$ in this equilibrium. However, then $b_A \leq b - \frac{\rho^* c_{p,1} + \rho^* \sigma^* (c_{p,2} + \alpha_p c_b)}{1 - \rho^* \sigma^* \alpha_p} = b$, and hence by Lemma 3, $\Delta_A^* = b > b_A$, yielding a contradiction. Therefore, in any equilibrium, it must be that $b_A < \Delta_A^*$. Then, again by Lemma 3, it must be that $\Delta_A^* = b - \frac{\rho^* c_{p,1} + \rho^* \sigma^* (c_{p,2} + \alpha_p c_b)}{1 - \rho^* \sigma^* \alpha_p}$. ■

I prove Proposition 1 with the following generalized proposition, where $b_A \in (-\infty, b)$ and $c_b \in [0, \infty)$. Note that $b_A < b$ is implied by $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$ since $0 < \kappa$.

Proposition 5 *Suppose that $c_b \in [0, \infty)$, $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$, $E[\Delta | \Delta \geq b_A] > 0$, and the activist has demanded action settlement. Then, an equilibrium of this subgame exists, the equilibrium is unique, and in equilibrium:*

(i) The incumbent board accepts the action settlement if $\Delta > \Delta_A^*$ and rejects if $\Delta < \Delta_A^*$, where[38]

$$\Delta_A^* = \max\left\{\hat{\Delta}_A(1),\ b - \frac{c_p + \alpha_p c_b}{\frac{1}{\gamma} - \alpha_p}, \check{\Delta}\right\} \in (0, b), \tag{22}$$

where $\hat{\Delta}_A(\cdot)$ and $\check{\Delta}$ are given by (14) and (16) in Lemma 3, respectively.

(ii) Upon rejection, the activist runs a proxy fight with probability $\rho^* = \rho_A$, where

$$\rho_A^* \equiv \begin{cases} \gamma, & \text{if } \max\left\{\hat{\Delta}_A(1), \check{\Delta}\right\} \leq b - \frac{c_p + \alpha_p c_b}{\frac{1}{\gamma} - \alpha_p}, \\ \frac{b - \hat{\Delta}_A(1)}{\alpha_p \left(b - \hat{\Delta}_A(1)\right) + c_p + \alpha_p c_b} \in (0, \gamma), & \text{if } b - \frac{c_p + \alpha_p c_b}{\frac{1}{\gamma} - \alpha_p} < \hat{\Delta}_A(1) \text{ and } \check{\Delta} \leq \hat{\Delta}_A(1) \\ \frac{b - \check{\Delta}}{\tilde{\sigma}_A \alpha_p \left(b - \check{\Delta}\right) + c_{p,1} + \tilde{\sigma}_A (c_{p,2} + \alpha_p c_b)} \in (0, \gamma), & \text{if } \max\left\{\hat{\Delta}_A(1),\ b - \frac{c_p + \alpha_p c_b}{\frac{1}{\gamma} - \alpha_p}\right\} < \check{\Delta}, \text{ and } \\ & \quad c_{p,1} \geq \frac{1}{\gamma}\left(b - \check{\Delta}\right) \text{ or } \check{\Delta} < \hat{\Delta}_A(\tilde{\sigma}_I) \\ \gamma, & \text{otherwise,} \end{cases} \tag{23}$$

[38] Recall that $c_p = c_{p,1} + c_{p,2}$.

where $\tilde{\sigma}_A$ and $\tilde{\sigma}_I$ are unique and given by

$$\begin{aligned}\tilde{\sigma}_A &\equiv \frac{\kappa}{\alpha_P \frac{1}{F(\check{\Delta})} \int_{b_A}^{\check{\Delta}} (\Delta - b_A)\, dF(\Delta)} \\ \tilde{\sigma}_I &\equiv \frac{\frac{1}{\gamma}\left(b - \check{\Delta}\right) - c_{p,1}}{\alpha_p \left(b - \check{\Delta}\right) + c_{p,2} + \alpha_p c_b}\end{aligned} \tag{24}$$

(*iii*) *Upon launching a proxy fight, the activist wins the proxy fight with probability $\sigma^* = \sigma_A^*$, where*

$$\sigma_A^* = \begin{cases} 1, & \text{if } 0 \le b_A \text{ or } \check{\Delta} \le \max\left\{\hat{\Delta}_A(1), b - \frac{c_p + \alpha_p c_b}{\frac{1}{\gamma} - \alpha_p}\right\}, \\ \max\{\tilde{\sigma}_I, \tilde{\sigma}_A\} \in (0,1), & \text{if } b_A < 0 \text{ and } \max\left\{\hat{\Delta}_A(1), b - \frac{c_p + \alpha_p c_b}{\frac{1}{\gamma} - \alpha_p}\right\} < \check{\Delta}, \end{cases} \tag{25}$$

Proof. Suppose that $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$, $E[\Delta | \Delta \ge b_A] > 0$, and the activist has demanded action settlement. Note that by Lemma 3, in any equilibrium of this subgame, the incumbent follows a threshold strategy. That is, there exists a Δ_A^* such that the incumbent rejects action settlement for all $\Delta < \Delta_A^*$ and accepts for all $\Delta > \Delta_A^*$. Moreover, note that $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$ implies that $b_A < b$. Hence, in any equilibrium $\Delta_A^* < \check{\Delta}$ implies that $\sigma^* = 0$, since $\Delta_A^* > b_A$ in any equilibrium by Lemma 4, and $\sigma = 0$ is the shareholders' unique best response to $\Delta_A \in \left(b_A, \check{\Delta}\right)$ by Lemma 3.

First, I prove part (i). Note that $b_A < b$, $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$ and $E[\Delta | \Delta \ge b_A] > 0$ imply that $\max\left\{\hat{\Delta}_A(1), \check{\Delta}\right\} \in (0, b)$, and hence Δ_A^* given by (22) satisfies $\Delta_A^* \in (0, b)$. Note that in any equilibrium, Lemma 4 implies that $b - \frac{c_p + \alpha_p c_b}{\frac{1}{\gamma} - \alpha_p} \le b - \frac{\rho^* c_{p,1} + \rho^* \sigma^* (c_{p,2} + \alpha_p c_b)}{1 - \rho^* \sigma^* \alpha_p} = \Delta_A^*$, and Lemma 3 implies that $\hat{\Delta}_A(1) \le \Delta_A^*$ (because if $\Delta_A^* < \hat{\Delta}_A(1)$ then $\rho^* = 0$, which contradicts with $\Delta_A^* < b$) and $\check{\Delta} \le \Delta_A^*$ (because if $\Delta_A^* < \check{\Delta}$ then $\sigma^* = 0$ and hence $\rho^* = 0$, which again contradicts with $\Delta_A^* < b$). Therefore, in any equilibrium

$$\Delta_A^* \ge \max\left\{\hat{\Delta}_A(1),\ b - \frac{c_p + \alpha_p c_b}{\frac{1}{\gamma} - \alpha_p}, \check{\Delta}\right\}.$$

Therefore, it remains to show that there cannot be an equilibrium where Δ_A^* is strictly larger than (22). Suppose there is. Then, by Lemma 3, it must be that $\sigma^* = 1$ since $\check{\Delta} < \Delta_A^*$, and $\rho^* = \gamma$ since $\hat{\Delta}_A(1) < \Delta_A^*$. However, then $\Delta_A^* = b - \frac{c_p + \alpha_p c_b}{\frac{1}{\gamma} - \alpha_p}$ again by Lemma 3, yielding a contradiction with $\Delta_A^* > b - \frac{c_p+\alpha_p c_b}{\frac{1}{\gamma}-\alpha_p}$.

Next, I prove parts (ii) and (iii). There are five cases to consider. First, suppose that $\max\left\{\hat{\Delta}_A(1), \check{\Delta}\right\} \leq b - \frac{c_p+\alpha_p c_b}{\frac{1}{\gamma}-\alpha_p}$. Then, in any equilibrium, $\Delta_A^* = b - \frac{c_p+\alpha_p c_b}{\frac{1}{\gamma}-\alpha_p} = b - \frac{\rho^* c_{p,1}+\rho^*\sigma^*(c_{p,2}+\alpha_p c_b)}{1-\rho^*\sigma^*\alpha_p}$ by part (i) and Lemma 4. Moreover, since $b - \frac{c_p+\alpha_p c_b}{\frac{1}{\gamma}-\alpha_p} < b - \frac{\rho c_{p,1}+\rho\sigma(c_{p,2}+\alpha_p c_b)}{1-\rho\sigma\alpha_p}$ for any $\sigma \in [0,1]$ and $\rho \in [0,\gamma]$ other than $\sigma = 1$ and $\rho = \gamma$, Lemma 3 implies it must be that $\sigma^* = 1$ and $\rho^* = \gamma$. Indeed, each of $\Delta_A = b - \frac{c_p+\alpha_p c_b}{\frac{1}{\gamma}-\alpha_p}$, $\sigma = 1$, and $\rho = \gamma$ is a best response given others' strategies, making this strategy profile the unique equilibrium.

Second, suppose that $\max\{b - \frac{c_p+\alpha_p c_b}{\frac{1}{\gamma}-\alpha_p}, \check{\Delta}\} < \hat{\Delta}_A(1)$, or $b - \frac{c_p+\alpha_p c_b}{\frac{1}{\gamma}-\alpha_p} < \hat{\Delta}_A(1) = \check{\Delta}$. Then, in any equilibrium, $\Delta_A^* = \hat{\Delta}_A(1) = b - \frac{\rho^* c_{p,1}+\rho^*\sigma^*(c_{p,2}+\alpha_p c_b)}{1-\rho^*\sigma^*\alpha_p}$ by part (i) and Lemma 4. Then, it must be that $\sigma^* = 1$, because otherwise $\Delta_A^* < \hat{\Delta}_A(\sigma^*)$ and hence $\rho^* = 0$ by Lemma 3, which yields a contradiction with $\Delta_A^* < b$. Moreover, $\sigma^* = 1$ in turn implies that it must be $\hat{\Delta}_A(1) = b - \frac{c_{p,1}+(c_{p,2}+\alpha_p c_b)}{\frac{1}{\rho^*}-\alpha_p}$, or equivalently,

$$\rho^* = \frac{b - \hat{\Delta}_A(1)}{\alpha_p\left(b - \hat{\Delta}_A(1)\right) + c_{p,1} + (c_{p,2} + \alpha_p c_b)}.$$

Note that $\rho^* \in (0,\gamma)$ since $\hat{\Delta}_A(1) < b$ and $b - \frac{c_p+\alpha_p c_b}{\frac{1}{\gamma}-\alpha_p} < \hat{\Delta}_A(1)$. Indeed, each of $\Delta_A = \hat{\Delta}_A(1)$, $\sigma = 1$, and $\rho = \rho^*$ is a best response given others' strategies, making this strategy profile the unique equilibrium.

Before proving the remaining cases, I show that any equilibrium satisfies

$$\Delta_A^* = \max\left\{\hat{\Delta}_A(\sigma^*), b - \frac{c_{p,1} + \sigma^*\left(c_{p,2} + \alpha_p c_b\right)}{\frac{1}{\gamma} - \sigma^*\alpha_p}\right\}. \tag{26}$$

Suppose there exists an equilibrium such that Δ_A^* is strictly larger than (26). However, by Lemma 3, then it must be that $\rho^* = \gamma$ and hence $\Delta_A^* = b - \frac{c_{p,1}+\sigma^*(c_{p,2}+\alpha_p c_b)}{\frac{1}{\gamma}-\sigma^*\alpha_p}$, which yields a contradiction. It remains to show that there cannot be an equilibrium such that Δ_A^* is strictly smaller than (26). Suppose there is. However, then again by Lemma 3, it must be that $\rho^* = 0$ and hence $\Delta_A^* = b$, yielding contradiction.

Third, suppose that $\max\{\hat{\Delta}_A(1),\ b - \frac{c_p+\alpha_p c_b}{\frac{1}{\gamma}-\alpha_p}\} < \check{\Delta}$ and $c_{p,1} \geq \frac{1}{\gamma}\left(b - \check{\Delta}\right)$. Then, in any equilibrium, $\Delta_A^* = \check{\Delta}$ by part (i). Since $\hat{\Delta}_A(0) = \infty$ and $b - \frac{c_{p,1}+\sigma(c_{p,2}+\alpha_p c_b)}{\frac{1}{\gamma}-\sigma\alpha_p} < \check{\Delta}$ for all $\sigma \in (0,1]$, by (26) it must be that $\Delta_A^* = \check{\Delta} = \hat{\Delta}_A(\sigma^*)$. The only σ^* that satisfies the latter equality is $\sigma^* = \tilde{\sigma}_A$, and therefore it must be that $\sigma^* = \tilde{\sigma}_A$. Furthermore, $\Delta_A^* = b - \frac{c_{p,1}+\tilde{\sigma}_A(c_{p,2}+\alpha_p c_b)}{\frac{1}{\rho^*}-\tilde{\sigma}_A\alpha_p}$ by

Lemma 3, hence together with $\Delta_A^* = \check{\Delta}$, this implies that

$$\rho^* = \frac{b - \check{\Delta}}{\tilde{\sigma}_A \alpha_p \left(b - \check{\Delta}\right) + c_{p,1} + \tilde{\sigma}_A \left(c_{p,2} + \alpha_p c_b\right)}. \tag{27}$$

Moreover, $\hat{\Delta}_A(1) < \check{\Delta} = \hat{\Delta}_A(\tilde{\sigma}_A)$ implies that $\tilde{\sigma}_A \in (0,1)$, and hence

$$b - \frac{c_{p,1} + \tilde{\sigma}_A \left(c_{p,2} + \alpha_p c_b\right)}{\frac{1}{\gamma} - \tilde{\sigma}_A \alpha_p} < \check{\Delta} = b - \frac{c_{p,1} + \tilde{\sigma}_A \left(c_{p,2} + \alpha_p c_b\right)}{\frac{1}{\rho^*} - \tilde{\sigma}_A \alpha_p} < b \tag{28}$$

implies that $\rho^* \in (0, \gamma)$, where the first inequality follows from $c_{p,1} \geq \frac{1}{\gamma}\left(b - \check{\Delta}\right)$. Moreover, note that $c_{p,1} \geq \frac{1}{\gamma}\left(b - \check{\Delta}\right)$ implies that $\tilde{\sigma}_I \leq 0$, hence $\max\{\tilde{\sigma}_I, \tilde{\sigma}_A\} = \tilde{\sigma}_A$. Indeed, each of $\Delta_A = \check{\Delta}$, $\sigma = \tilde{\sigma}_A$, and $\rho = \rho^*$ is a best response given others' strategies, making this strategy profile the unique equilibrium.

Fourth, suppose that $\max\{\hat{\Delta}_A(1), b - \frac{c_p + \alpha_p c_b}{\frac{1}{\gamma} - \alpha_p}\} < \check{\Delta}$, $c_{p,1} < \frac{1}{\gamma}\left(b - \check{\Delta}\right)$, and $\hat{\Delta}_A(\tilde{\sigma}_I) \leq \check{\Delta}$. Then, in any equilibrium, $\Delta_A^* = \check{\Delta}$ by part (i). Note that $\tilde{\sigma}_I$ satisfies $b - \frac{c_{p,1} + \tilde{\sigma}_I (c_{p,2} + \alpha_p c_b)}{\frac{1}{\gamma} - \tilde{\sigma}_I \alpha_p} = \check{\Delta}$ and $\hat{\Delta}_A(\tilde{\sigma}_I) \leq \check{\Delta}$. Therefore, the only σ^* that satisfies (26) is $\sigma^* = \tilde{\sigma}_I$, and therefore it must be that $\sigma^* = \tilde{\sigma}_I$. Furthermore, $\Delta_A^* = b - \frac{c_{p,1} + \sigma^* (c_{p,2} + \alpha_p c_b)}{\frac{1}{\rho^*} - \sigma^* \alpha_p}$ by Lemma 3, which implies that $\rho^* = \gamma$. Indeed, each of $\Delta_A = \check{\Delta}$, $\sigma = \tilde{\sigma}_I$, and $\rho = \gamma$ is a best response given others' strategies, making this strategy profile the unique equilibrium. Moreover, $\hat{\Delta}_A(\tilde{\sigma}_A) = \check{\Delta}$, and hence $\hat{\Delta}_A(\tilde{\sigma}_I) \leq \check{\Delta}$ implies that $\tilde{\sigma}_I \geq \tilde{\sigma}_A$. Also note that $b - \frac{c_p + \alpha_p c_b}{\frac{1}{\gamma} - \alpha_p} < \check{\Delta} = b - \frac{c_{p,1} + \tilde{\sigma}_I (c_{p,2} + \alpha_p c_b)}{\frac{1}{\gamma} - \tilde{\sigma}_I \alpha_p}$ implies that $\tilde{\sigma}_I < 1$, and $c_{p,1} < \frac{1}{\gamma}\left(b - \check{\Delta}\right)$ implies that $\tilde{\sigma}_I > 0$.

Fifth, suppose that $\max\{\hat{\Delta}_A(1), b - \frac{c_p + \alpha_p c_b}{\frac{1}{\gamma} - \alpha_p}\} < \check{\Delta}$, $c_{p,1} < \frac{1}{\gamma}\left(b - \check{\Delta}\right)$, and $\check{\Delta} < \hat{\Delta}_A(\tilde{\sigma}_I)$. Then, in any equilibrium, $\Delta_A^* = \check{\Delta}$ by part (i). Note that $\tilde{\sigma}_I$ and $\tilde{\sigma}_A$ satisfy $\hat{\Delta}_A(\tilde{\sigma}_A) = \check{\Delta} < \hat{\Delta}_A(\tilde{\sigma}_I)$. Therefore, $\tilde{\sigma}_A > \tilde{\sigma}_I$. Since $\tilde{\sigma}_I$ satisfies $b - \frac{c_{p,1} + \tilde{\sigma}_I (c_{p,2} + \alpha_p c_b)}{\frac{1}{\gamma} - \tilde{\sigma}_I \alpha_p} = \check{\Delta}$, this implies that $b - \frac{c_{p,1} + \tilde{\sigma}_A (c_{p,2} + \alpha_p c_b)}{\frac{1}{\gamma} - \tilde{\sigma}_A \alpha_p} < \check{\Delta}$. Therefore, the only σ^* that satisfies (26) is $\sigma^* = \tilde{\sigma}_A$, and therefore it must be that $\sigma^* = \tilde{\sigma}_A$. Furthermore, $\Delta_A^* = b - \frac{c_{p,1} + \tilde{\sigma}_A (c_{p,2} + \alpha_p c_b)}{\frac{1}{\rho^*} - \tilde{\sigma}_A \alpha_p}$ by Lemma 3, hence together with $\Delta_A^* = \check{\Delta}$, this implies (27). Moreover, $\hat{\Delta}_A(1) < \check{\Delta} = \hat{\Delta}_A(\tilde{\sigma}_A)$ implies that $\tilde{\sigma}_A \in (0,1)$, and since also (28) holds, it must be that $\rho^* \in (0, \gamma)$ (note that the first inequality in (28) holds because $\tilde{\sigma}_A > \tilde{\sigma}_I$). Indeed, each of $\Delta_A = \check{\Delta}$, $\sigma = \tilde{\sigma}_A$, and $\rho = \rho^*$ is a best response given others' strategies, making this strategy profile the unique equilibrium. ■

Proposition 6 *Suppose that $c_b \in [0, \infty)$ and the activist has demanded action settlement. Moreover, suppose that $b_A \in (-\infty, \infty)$ and $\kappa \in (0, \infty)$, or $b_A < b$ and at least one of $\kappa \geq \alpha_P E[\max\{0, \Delta - b_A\}]$ and $E[\Delta|\Delta \geq b_A] \leq 0$ holds. Then, an equilibrium of this subgame exists, the equilibrium is unique, and in equilibrium the incumbent board always rejects the settlement, and the activist never runs a proxy fight.*

Proof. Suppose that $c_b \in [0, \infty)$, $b_A \geq b$, $\kappa \in (0, \infty)$, and the activist has demanded action settlement. Note that $\Delta \in (l, b)$ by assumption. By Lemma 2, the project is never implemented in any subgame, regardless of who has decision authority. Therefore, the activist's payoff from running a proxy fight at any subgame is always $-k < 0$. Since his payoff from not running a proxy fight at any subgame is zero, $\rho = 0$ is the activist's unique best response in any subgame. Given $\rho = 0$, the incumbent's expected payoff from rejecting the settlement is zero, while its payoff from accepting is strictly smaller than zero. Therefore, the incumbent rejects for all Δ.

Suppose that $c_b \in [0, \infty)$, $b_A < b$, at least one of $\kappa \geq \alpha_p E[\max\{0, \Delta - b_A\}]$ or $E[\Delta|\Delta \geq b_A] \leq 0$ holds, and the activist has demanded action settlement. Note that $\Delta \in (l, b)$ by assumption. Also note that by Lemma 3, in any equilibrium of this subgame, the incumbent follows a threshold strategy. That is, there exists a Δ_A^* such that the incumbent rejects action settlement for all $\Delta < \Delta_A^*$ and accepts for all $\Delta > \Delta_A^*$. I prove in four steps.

First, suppose that $\kappa > \alpha_P E[\max\{0, \Delta - b_A\}]$. Then $\hat{\Delta}_A(\sigma) = \infty$ for all $\sigma \in [0, 1]$, and hence by Lemma 3, the unique best response of the activist is $\rho^* = 0$. Therefore, again by Lemma 3, the unique best response of the incumbent is $\Delta_A^* = b$, that is, the incumbent rejects the action settlement for all $\Delta < b$.

Second, suppose that $\kappa = \alpha_P E[\max\{0, \Delta - b_A\}]$. Note that then $\hat{\Delta}_A(\sigma) \geq b$ for any $\sigma \in [0, 1]$ by (14). There are two types of equilibrium candidates to consider. First, consider $\rho^* = 0$. Indeed, by Lemma 3, $\rho = 0$ is a best response of the activist for any given $\sigma \in [0, 1]$ and $\Delta_A > l$. Moreover, $\Delta_A = b$ is the unique best response of the incumbent given $\rho = 0$. Therefore, $\rho^* = 0$ and $\Delta_A^* = b$ constitute an equilibrium. Second, consider any $\rho^* > 0$. Then by Lemma 3, $\Delta_A^* < b$ in any equilibrium. However, then by the same lemma the unique best response of the activist given $\Delta_A = \Delta_A^*$ and any σ is $\rho = 0$, since $\hat{\Delta}_A(\sigma) \geq b > \Delta_A^*$ for any $\sigma \in [0, 1]$. This yields a contradiction with $\rho^* > 0$, eliminating any such equilibrium candidate.

Third, suppose that $E[\Delta|\Delta \geq b_A] < 0$. Note that this implies that $b_A < 0$. Then, $\check{\Delta} = \infty$ by (16), and therefore the shareholders' unique best response is $\sigma^* = 0$ by Lemma 3. Therefore, the activist's unique best response is $\rho^* = 0$, and in turn, the incumbent's unique best response is $\Delta_A^* = b$.

Fourth, suppose that $E[\Delta|\Delta \geq b_A] = 0$. Again, note that this implies that $b_A < 0$. Note that then $\check{\Delta} \geq b$, and therefore $\sigma = 0$ is a best response of the shareholders for any Δ_A by Lemma 3. There are two types of equilibrium candidates to consider. First, consider $\rho^* = 0$. Indeed, by Lemma 3, $\rho = 0$ is a best response of the activist given $\sigma = 0$ and any $\Delta_A > l$. Moreover, $\Delta_A = b$ is the unique best response for the incumbent given $\rho = 0$. Therefore, $\Delta_A^* = b$, $\rho^* = 0$, and $\sigma^* = 0$ constitute an equilibrium. Second, consider any $\rho^* > 0$. Then by Lemma 3, $\Delta_A^* < b$ in any equilibrium. However, then by the same lemma the unique best response of the shareholders given $\Delta_A = \Delta_A^*$ is $\sigma = 0$, since $\check{\Delta} \geq b > \Delta_A^*$. However, if $\sigma^* = 0$, then the unique best response of the activist is $\rho = 0$, yielding a contradiction with $\rho^* > 0$, and hence eliminating any equilibrium candidate with $\rho^* > 0$. ■

The next lemma characterizes the best responses of the activist, the incumbent board, and the shareholders as a function of their beliefs, given the activist has demanded board settlement.

Lemma 5 *Suppose that $b_A \in (-\infty, b)$, $c_b \in [0, \infty)$, $\kappa \in (0, \infty)$, and the activist has demanded board settlement that gives him decision authority with probability $\alpha_B \in (0, 1]$. Moreover:*

(i) Suppose that the activist runs a proxy fight with probability ρ upon rejection and the shareholders support the activist with probability σ upon a proxy fight. Then,

(a) If $\rho\,[c_{p,1} + \sigma\,(c_{p,2} + \alpha_p c_b)] < \alpha_B c_b$, then the incumbent board rejects board settlement for all $\Delta \in (l, b_A) \cup (b_A, b)$.

(b) If $\rho\,[c_{p,1} + \sigma\,(c_{p,2} + \alpha_p c_b)] = \alpha_B c_b$, then the incumbent board rejects board settlement if $\Delta > b_A$, and is indifferent between accepting and rejecting if $\Delta < b_A$.

(c) If $\rho\,[c_{p,1} + \sigma\,(c_{p,2} + \alpha_p c_b)] > \alpha_B c_b$, then the incumbent board accepts board settlement if $\Delta \in (l, b_A) \cup (\Delta_B, b)$ and rejects if $\Delta \in (b_A, \Delta_B)$, where $\Delta_B < b$, and Δ_B is given by

$$\Delta_B(\alpha_B, \rho, \sigma) \equiv \begin{cases} \max\left\{b_A, b + c_b - \frac{c_{P,1} + \sigma c_{P,2}}{\frac{\alpha_B}{\rho} - \sigma\alpha_P}\right\}, & \textit{if } \alpha_B > \rho\sigma\alpha_P, \\ b_A, & \textit{otherwise.} \end{cases} \tag{29}$$

(ii) Suppose that the incumbent board accepts board settlement if $\Delta \in (l, b_A) \cup (\Delta_B, b)$ and rejects if $\Delta \in (b_A, \Delta_B)$ for some $\Delta_B > \max\{l, b_A\}$, and upon launching a proxy fight, the

activist wins with probability $\sigma \in [0,1]$. Then, upon rejection of board settlement, the activist's best response is to run a proxy fight with probability $\rho(\Delta_B, \sigma) \in BR_a(\Delta_B, \sigma; \hat{\Delta}_B(\sigma))$, where BR_a is given by (13) and $\hat{\Delta}_B(\sigma)$ is given by

$$\hat{\Delta}_B(\sigma) \equiv \begin{cases} l, & \textit{if } \kappa \leq \sigma\alpha_P \left(l - b_A\right), \\ x > \max\{l, b_A\} \ \textit{ such that} \\ \kappa = \sigma\alpha_P \frac{1}{F(x)-F(b_A)} \int_{b_A}^{x} \left(\Delta - b_A\right) dF(\Delta), & \textit{if } \sigma\alpha_P \left(l - b_A\right) < \kappa \leq \sigma\alpha_P \frac{1}{1-F(b_A)} \int_{\min\{b,b_A\}}^{b} (\Delta \\ \infty, & \textit{otherwise.} \end{cases} \tag{30}$$

(*iii*) *Suppose that the incumbent board accepts board settlement if $\Delta \in (l, b_A) \cup (\Delta_B, b)$ and rejects if $\Delta \in (b_A, \Delta_B)$ for some $\Delta_B > \max\{l, b_A\}$. Then, if the activist launches a proxy fight, the shareholders' best response is to support him with probability $\sigma(\Delta_B) \in BR_s(\Delta_B)$, where BR_s is given by (15).*

Proof. Note that $\Delta \in (l, b)$ by assumption. Consider part (i). If the incumbent board accepts board settlement, then its payoff is $\alpha_B(\Delta - b) - \alpha_B c_b$. If the incumbent board rejects it, then by Lemma 2, its payoff is given by (17), where the value of π_i within the specified interval when $\Delta = b_A$ again depends on the probability that the activist implements the action when he is indifferent between implementing and not implementing. If $\Delta > b_A$, then the incumbent board rejects (accepts) action settlement if

$$\alpha_B\left(\Delta - b\right) - \alpha_B c_b < (>) - \rho c_{p,1} + \rho\sigma\left[-c_{p,2} + \alpha_p(-c_b + \Delta - b)\right],$$

or, equivalently, rejects if

$$\left(\Delta - b\right)\left(\alpha_B - \rho\sigma\alpha_p\right) < \alpha_B c_b - \rho\left[c_{p,1} + \sigma(c_{p,2} + \alpha_p c_b)\right] \tag{31}$$

and accepts if

$$\left(\Delta - b\right)\left(\alpha_B - \rho\sigma\alpha_p\right) > \alpha_B c_b - \rho\left[c_{p,1} + \sigma(c_{p,2} + \alpha_p c_b)\right]. \tag{32}$$

If $\Delta < b_A$, the incumbent board rejects (accepts) settlement if

$$-\alpha_B c_b < (>) - \rho c_{p,1} + \rho\sigma\left[-c_{p,2} + \alpha_p(-c_b)\right],$$

or, equivalently, rejects if

$$0 < \alpha_B c_b - \rho\left[c_{p,1} + \sigma(c_{p,2} + \alpha_p c_b)\right] \tag{33}$$

and accepts if

$$0 > \alpha_B c_b - \rho\left[c_{p,1} + \sigma(c_{p,2} + \alpha_p c_b)\right]. \tag{34}$$

Consider part (i.a), i.e., suppose that $\rho\left[c_{p,1} + \sigma\left(c_{p,2} + \alpha_p c_b\right)\right] < \alpha_B c_b$. Note that this implies that $\rho\sigma\alpha_p < \alpha_B$. Then, (31) and (33) hold for all $\Delta < b$. Therefore, the incumbent rejects for all $\Delta \in (l, b_A) \cup (b_A, b)$.

Next, consider part (i.b), i.e., suppose that $\rho\left[c_{p,1} + \sigma\left(c_{p,2} + \alpha_p c_b\right)\right] = \alpha_B c_b$. Since $\alpha_B > 0$, this implies that $\rho\sigma\alpha_p < \alpha_B$ (because if $\rho\sigma\alpha_p \geq \alpha_B$, then $\rho > 0$ and $\sigma > 0$, and hence $\rho\left[c_{p,1} + \sigma\left(c_{p,2} + \alpha_p c_b\right)\right] > \alpha_B c_b$, yielding a contradiction). Therefore, for all $\Delta > b_A$ (31) holds, i.e., the incumbent rejects. However, for any $\Delta < b_A$, the incumbent is indifferent between accepting and not, because

$$-\alpha_B c_b = -\rho c_{p,1} + \rho\sigma\left[-c_{p,2} + \alpha_p(-c_b)\right],$$

where LHS is the incumbent's payoff from accepting and the RHS is its payoff from rejecting.

Next, consider part (i.c), i.e., suppose $\rho\left[c_{p,1} + \sigma\left(c_{p,2} + \alpha_p c_b\right)\right] > \alpha_B c_b$. Then, (34) holds, and therefore the incumbent accepts if $\Delta < b_A$. If $\Delta > b_A$, then there are two cases to consider. If $\alpha_B \leq \rho\sigma\alpha_p$, then (32) holds for all $\Delta > b_A$, and hence the incumbent accepts if $\Delta > b_A$. However, if $\alpha_B > \rho\sigma\alpha_p$, then (32) holds (and hence the incumbent accepts) for all $\Delta \in (\max\{b_A, b + c_b - \frac{c_{P,1}+\sigma c_{P,2}}{\frac{\alpha_B}{\rho} - \sigma\alpha_P}\}, b)$, and (31) holds (and hence the incumbent rejects) for all $\Delta \in (b_A, b + c_b - \frac{c_{P,1}+\sigma c_{P,2}}{\frac{\alpha_B}{\rho} - \sigma\alpha_P})$. Moreover, note that $\rho\left[c_{p,1} + \sigma\left(c_{p,2} + \alpha_p c_b\right)\right] > \alpha_B c_b$ and $b_A < b$ implies that $\Delta_B < b$. This concludes the proof of part (i).

Next, consider part (ii). Note that if the incumbent rejects, then with probability γ the activist's cost of proxy fight is ζ and hence he does not run a proxy fight by Lemma 2. With probability $1 - \gamma$, the activist's cost of proxy fight is κ, and his expected payoff from not running a proxy fight is zero and from running a proxy fight is

$$\pi_a = -\kappa + \sigma\alpha_P \frac{1}{F(\Delta_B) - F(b_A)} \int_{b_A}^{\Delta_B} (\Delta - b_A)\, dF(\Delta),$$

which is strictly positive if $\Delta_B > \hat{\Delta}_B(\sigma)$, zero if $\Delta_B = \hat{\Delta}_B(\sigma)$, and strictly negative if $\Delta_B < \hat{\Delta}_B(\sigma)$. This concludes this part.

Finally, consider part (iii). If the activist loses the proxy fight, then by Lemma 2 the shareholder value will be zero. On the other hand, if the activist wins the proxy fight, then under the specified beliefs the expected shareholder value will be

$$\pi_s = \alpha_P \frac{1}{F(\Delta_B) - F(b_A)} \int_{b_A}^{\Delta_B} \Delta dF(\Delta).$$

Therefore, if $b_A \geq 0$, then $\pi_s > 0$ since $\Delta_B > b_A$. On the other hand, if $b_A < 0$, then $\pi_s > 0$ when $\Delta_B > \check{\Delta}$ and $\pi_s = 0$ when $\Delta_B = \check{\Delta}$, and $\pi_s < 0$ if $\Delta_B \in (b_A, \check{\Delta})$. ■

I prove Proposition 2 with the following generalization that $b_A \in (-\infty, b)$ and $c_b \in [0, \frac{c_P}{\frac{1}{\gamma} - \alpha_p})$. Note that $b_A < b$ is implied by $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$ since $0 < \kappa$.

Proposition 7 *Suppose that* $c_b \in [0, \frac{c_P}{\frac{1}{\gamma} - \alpha_p})$, $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$, $E[\Delta | \Delta \geq b_A] > 0$, *and the activist has demanded board settlement that gives him decision authority with probability* $\alpha_B \in (0, 1]$. *Then, an equilibrium of this subgame always exists, and* $\rho^* > 0$ *in any equilibrium. Moreover, an equilibrium where the incumbent board rejects board settlement with positive probability for some* Δ *always exists, and in any such equilibrium,*

(*i*) *The incumbent accepts the board settlement if* $\Delta \in (l, b_A) \cup (\Delta_B^*, b)$ *and rejects if* $\Delta \in (b_A, \Delta_B^*)$, *where* $\Delta_B^* \in (0, b)$ *is given by*

$$\Delta_B^*(\alpha_B) \equiv \begin{cases} b + c_b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P}, & \text{if } \alpha_B \geq \alpha_L, \\ \max\left\{\hat{\Delta}_B(1), \check{\Delta}\right\}, & \text{otherwise,} \end{cases} \tag{35}$$

where $\hat{\Delta}_B(\cdot), \check{\Delta}$, *and* α_L *are respectively given by (14), (16), and*

$$\alpha_L \equiv \gamma\alpha_P + \gamma \frac{c_P}{b + c_b - \max\left\{\hat{\Delta}_B(1), \check{\Delta}\right\}}. \tag{36}$$

(*ii*) *Upon rejection the activist runs a proxy fight with probability* $\rho^* = \rho_B^*(\alpha_B)$, *where*

$$\rho_B^*(\alpha_B) \equiv \begin{cases} \gamma, & \text{if } \alpha_B \geq \alpha_L, \\ \gamma\frac{\alpha_B}{\alpha_L} \in (0, \gamma), & \text{if } \alpha_B < \alpha_L \text{ and } \check{\Delta} \leq \hat{\Delta}_B(1), \\ \alpha_B \frac{c_b + b - \check{\Delta}}{\tilde{\sigma}_A \alpha_P \left(b - \check{\Delta}\right) + c_{P,1} + \tilde{\sigma}_A \left(c_{P,2} + \alpha_P c_b\right)} \in (0, \gamma), & \text{if } \begin{array}{l} \alpha_B < \alpha_L \text{ and } \hat{\Delta}_B(1) < \check{\Delta}, \text{ and} \\ \alpha_B \leq \gamma\frac{c_{P,1}}{c_b + b - \check{\Delta}} \text{ or } \check{\Delta} < \hat{\Delta}_B(\tilde{\sigma}_I(\alpha_B)) \end{array} \\ \gamma, & \text{otherwise,} \end{cases} \tag{37}$$

where $\hat{\sigma}_A$ *and* $\hat{\sigma}_I$ *are unique and given by*

$$\begin{aligned} \hat{\sigma}_A &\equiv \frac{\kappa}{\alpha_P \frac{1}{F(\check{\Delta})-F(b_A)} \int_{b_A}^{\check{\Delta}} (\Delta - b_A)\, dF(\Delta)} \\ \hat{\sigma}_I(\alpha_B) &\equiv \frac{\frac{\alpha_B}{\gamma}\left(c_b + b - \check{\Delta}\right) - c_{P,1}}{\alpha_P \left(c_b + b - \check{\Delta}\right) + c_{P,2}} \end{aligned} \tag{38}$$

(iii) Upon proxy fight, shareholders support the activist with probability $\sigma^* = \sigma_B^*(\alpha_B)$, *where*

$$\sigma_B^*(\alpha_B) = \begin{cases} 1, & \text{if } 0 \le b_A, \alpha_B \ge \alpha_L, \text{ or } \check{\Delta} \le \hat{\Delta}_B(1), \\ \max\{\tilde{\sigma}_I, \tilde{\sigma}_A\} \in (0,1), & \text{if } b_A < 0, \alpha_B < \alpha_L, \text{ and } \hat{\Delta}_B(1) < \check{\Delta}, \end{cases} \tag{39}$$

Proof. Suppose that $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$, $E[\Delta|\Delta \ge b_A] > 0$, and the activist has demanded board settlement that gives him decision authority with probability $\alpha_B \in (0,1]$. Note that $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$ implies that $b_A < b$.

First, to prove that there does not exist any equilibrium such that $\rho^* = 0$, I prove a broader statement: There is not any equilibrium where $\rho^*[c_{P,1} + \sigma^*(c_{P,2} + \alpha_P c_b)] \le \alpha_B c_b$. Suppose there is. Then, by Lemma 5, the incumbent rejects board settlement for all $\Delta > b_A$. By Lemma 2, upon the incumbent's rejection of settlement, the expected payoff of shareholders is zero if the activist does not run or win a proxy fight, and is

$$\pi_s = \frac{1}{\int_l^b [1 - \iota^*(\Delta)]\, dF(\Delta)} \alpha_P \int_{b_A}^b \Delta [1 - \iota^*(\Delta)]\, dF(\Delta)$$

if the activist runs and wins a proxy fight, where $1 - \iota^*(\Delta)$ is the incumbent's probability of rejection board settlement given Δ, and hence $\int_l^b [1 - \iota^*(\Delta)]\, dF(\Delta)$ is the unconditional probability of rejection by the incumbent. Note that $\iota^*(\Delta) = 0$ for all $\Delta > b_A$, hence π_s can be simplified as

$$\pi_s = \frac{1}{\int_l^{\max\{l,b_A\}} [1 - \iota^*(\Delta)]\, dF(\Delta) + \int_{\max\{l,b_A\}}^b dF(\Delta)} \alpha_P \int_{b_A}^b \Delta dF(\Delta),$$

which is strictly greater than zero since $E[\Delta|\Delta \ge b_A] > 0$. Therefore, it must be that $\sigma^* = 1$. Then, again by Lemma 2, upon the incumbent's rejection, the activist's expected payoff from not running a proxy fight is zero and is

$$\pi_a = -\kappa + \frac{1}{\int_l^b \left[1 - \iota^*(\Delta)\right] dF\left(\Delta\right)} \alpha_P \int_{b_A}^{b} \left(\Delta - b_A\right) \left[1 - \iota^*(\Delta)\right] dF\left(\Delta\right)$$

from running a proxy fight when the activist's cost of proxy fight $k = \kappa$. Since $\iota^*(\Delta) = 0$ for all $\Delta > b_A$ and $\int_l^b \left[1 - \iota^*(\Delta)\right] dF\left(\Delta\right) \leq 1$, $\pi_a \geq -\kappa + \alpha_P E\left[\max\left\{0, \Delta - b_A\right\}\right] > 0$. Then, the activist always runs a proxy fight when $\kappa = \kappa$. Combining with Lemma 2, then the activist runs a proxy fight with expected probability of $\rho^* = \gamma$. However, this implies that

$$\rho^* \left[c_{P,1} + \sigma^* \left(c_{P,2} + \alpha_P c_b\right)\right] = \gamma \left(c_P + \alpha_P c_b\right) > c_b \geq \alpha_B c_b,$$

where the equality follows from $c_P = c_{P,1} + c_{P,2}$, and the first inequality follows from $c_b < \frac{c_P}{\frac{1}{\gamma} - \alpha_P}$. Hence, this creates a contradiction with $\rho^* \left[c_{P,1} + \sigma^* \left(c_{P,2} + \alpha_P c_b\right)\right] \leq \alpha_B c_b$.

Second, note that by the first step above,

$$\rho^* \left[c_{P,1} + \sigma^* \left(c_{P,2} + \alpha_P c_b\right)\right] > \alpha_B c_b \tag{40}$$

in any equilibrium. Hence, by Lemma 5, in any equilibrium the incumbent accepts the settlement if $\Delta \in (l, b_A) \cup (\Delta_B^*, b)$ and rejects if $\Delta \in (b_A, \Delta_B^*)$, where $\Delta_B^* < b$ and Δ_B^* is given by (29). Therefore, in any equilibrium where the incumbent rejects the settlement with positive probability (i.e., $\iota^*(\Delta) < 1$) for some Δ, it must be that

$$\Delta_B^* > l \text{ and } \Delta_B^* > b_A. \tag{41}$$

Here, the latter inequality holds because otherwise Lemma 5 implies that the incumbent rejects only when $\Delta = b_A$, resulting in $\rho^* = 0$ since $b_A < \hat{\Delta}_B(\sigma)$ for all $\sigma \in [0, 1]$, which yields a contradiction with (40). Note that (41) combined with Lemma 5 imply that in any equilibrium with $\iota^*(\Delta) < 1$ for some Δ, it must be that

$$\alpha_B > \rho^* \sigma^* \alpha_P \tag{42}$$

and

$$\Delta_B^* = b + c_b - \frac{c_{P,1} + \sigma^* c_{P,2}}{\frac{\alpha_B}{\rho^*} - \sigma^* \alpha_P} \in \left(\max\left\{b_A, l\right\}, b\right). \tag{43}$$

In addition, in any such equilibrium with $\iota^*(\Delta) < 1$ for some Δ, it must be that

$$\sigma^* > 0, \tag{44}$$

because if $\sigma^* = 0$ then $\hat{\Delta}_B(\sigma^*) = \infty$, and hence $\rho^* = 0$ by Lemma 5, yielding a contradiction with (40).

Third, note that since $b_A < b$, in any equilibrium where $\iota^*(\Delta) < 1$ for some Δ, $\Delta_B^* < \check{\Delta}$ implies that $\sigma^* = 0$. This is because $\Delta_B^* > b_A$ in any such equilibrium by the previous step, and $\sigma = 0$ is the shareholders' unique best response to $\Delta_B \in \left(b_A, \check{\Delta}\right)$ by Lemma 5.

Fourth, I prove that part (i) holds for any equilibrium where the incumbent rejects the settlement with positive probability (i.e, $\iota^*(\Delta) < 1$) for some Δ. Note that due to second step above, it remains to show that in equilibrium Δ_B^* is given by (35) and satisfies $\Delta_B^* \in (0, b)$. Note that $b_A < b$, $\kappa < \alpha_P E\left[\max\left\{0, \Delta - b_A\right\}\right]$ and $E\left[\Delta | \Delta \geq b_A\right] > 0$ imply that

$$\max\left\{\hat{\Delta}_B(1), \check{\Delta}\right\} \in (\max\left\{0, b_A, l\right\}, b). \tag{45}$$

Moreover, $b + c_b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P} \in [\max\left\{\hat{\Delta}_B(1), \check{\Delta}\right\}, b)$ for all $\alpha_B \in [\alpha_L, 1)$ due to (36) and the assumption that $c_b < \frac{c_P}{\frac{1}{\gamma} - \alpha_P}$. Therefore, Δ_B^* given by (35) satisfies $\Delta_B^* \in (0, b)$. Next, I show that Δ_B^* is given by (35). Consider two possible cases. First, suppose that $\alpha_B \geq \alpha_L$. Then,

$$\begin{aligned} b_A &< \max\left\{\hat{\Delta}_B(1), \check{\Delta}\right\} = b + c_b - \frac{c_P}{\frac{\alpha_L}{\gamma} - \alpha_P} \leq b + c_b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P} \\ &\leq b + c_b - \frac{c_{P,1} + \sigma^* c_{P,2}}{\frac{\alpha_B}{\rho^*} - \sigma^* \alpha_P} = \Delta_B^*, \end{aligned} \tag{46}$$

where the last inequality follows from $\rho^* \in [0, \gamma]$ and $\sigma^* \in [0, 1]$. Therefore, $b + c_b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P} \leq \Delta_B^*$, and it remains to show that there cannot be an equilibrium where $\Delta_B^* > b + c_b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P}$. Suppose there is. Then, by (46) $\Delta_B^* > \max\left\{\hat{\Delta}_B(1), \check{\Delta}\right\}$, and hence by Lemma 5 it must be that $\rho^* = \gamma$ and $\sigma^* = 1$. However, then $\Delta_B^* = b + c_b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P}$, yielding a contradiction. Next, suppose that $\alpha_B < \alpha_L$. Note that Lemma 5 implies that $\hat{\Delta}_B(1) \leq \Delta_B^*$ (because due to (41) if $\Delta_B^* < \hat{\Delta}_B(1)$ then $\rho^* = 0$, which contradicts with (40)) and $\check{\Delta} \leq \Delta_B^*$ (because if $\Delta_B^* < \check{\Delta}$ then $\sigma^* = 0$ and hence $\rho^* = 0$, which again contradicts with (40)). Therefore, $\Delta_B^* \geq \max\left\{\hat{\Delta}_B(1), \check{\Delta}\right\}$, and it remains to show that there cannot be an equilibrium where $\Delta_B^* > \max\left\{\hat{\Delta}_B(1), \check{\Delta}\right\}$. Suppose there is. Then, by (41) and Lemma 5 it must be that $\rho^* = \gamma$ and $\sigma^* = 1$. However, then again by (41) and the same lemma it must be that $\Delta_B^* = b + c_b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P} < b$. However, then

$$\Delta_B^* = b + c_b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P} < b + c_b - \frac{c_P}{\frac{\alpha_L}{\gamma} - \alpha_P} = \max\left\{\hat{\Delta}_B(1), \check{\Delta}\right\},$$

where the first inequality follows from $\alpha_B < \alpha_L$ and the last equality follows from the definition of α_L, yielding a contradiction with $\Delta_B^* > \max\left\{\hat{\Delta}_B(1), \check{\Delta}\right\}$. This concludes the proof of part (i).

Next, I prove that an equilibrium where the incumbent board rejects board settlement with positive probability (i.e., $\iota^*(\Delta) < 1$) for some Δ always exists, along with parts (ii) and (iii). There are five cases to consider. First, suppose that $\alpha_B \geq \alpha_L$. Then, in any such equilibrium $\Delta_B^* = b + c_b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P} = b + c_b - \frac{c_{P,1} + \sigma^* c_{P,2}}{\frac{\alpha_B}{\rho^*} - \sigma^* \alpha_P}$ by part (i) and (46). Moreover, since $b + c_b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P} < b + c_b - \frac{c_{P,1} + \sigma c_{P,2}}{\frac{\alpha_B}{\rho} - \sigma \alpha_P}$ for any $\sigma \in [0,1]$ and $\rho \in [0,\gamma]$ other than $\sigma = 1$ and $\rho = \gamma$, Lemma 5 implies it must be that $\sigma^* = 1$ and $\rho^* = \gamma$. It remains to show that $\Delta_B^* = b + c_b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P}$, $\sigma^* = 1$, and $\rho^* = \gamma$ is indeed an equilibrium. Note that $\rho^* \left[c_{P,1} + \sigma^* \left(c_{P,2} + \alpha_P c_b\right)\right] > \alpha_B c_b$ and $\alpha_B > \rho^* \sigma^* \alpha_P$ is satisfied by $c_b < \frac{c_P}{\frac{1}{\gamma} - \alpha_p}$, $\alpha_B \geq \alpha_L$, and $\max\left\{\hat{\Delta}_B(1), \check{\Delta}\right\} < b$, where the last inequality follows from (45). Therefore, the best response of the incumbent given others' strategies is described by part (i.c) of Lemma 5, and specifically by

$$\Delta_B = \max\{b_A, b + c_b - \frac{c_{P,1} + \sigma c_{P,2}}{\frac{\alpha_B}{\rho} - \sigma \alpha_P}\}. \tag{47}$$

Note that given $\sigma = 1$ and $\rho = \gamma$, (47) reduces to $\Delta_B = b + c_b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P}$ because $b + c_b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P} \geq \max\left\{\hat{\Delta}_B(1), \check{\Delta}\right\} > \max\left\{b_A, l\right\}$, where the first inequality follows from $\alpha_B \geq \alpha_L$ and the second one follows from (45). By Lemma 5, these two inequalities also imply that $\sigma = 1$ is a best response of the incumbent given $\Delta_B = b + c_b - \frac{c_P}{\frac{\alpha_B}{\gamma} - \alpha_P}$, and that $\rho = \gamma$ is a best response of the activist given $\sigma = 1$ and the same Δ_B. Therefore, the described strategy profile is indeed an equilibrium.

Second, suppose that $\alpha_B < \alpha_L$ and $\check{\Delta} \leq \hat{\Delta}_B(1)$. Then, in any equilibrium with $\iota^*(\Delta) < 1$ for some Δ, it must be that $\Delta_B^* = \hat{\Delta}_B(1) \in (0, b)$ by part (i) and hence $\Delta_B^* = b + c_b - \frac{c_{P,1} + \sigma^* c_{P,2}}{\frac{\alpha_B}{\rho^*} - \sigma^* \alpha_P}$ by (41) and Lemma 5. Then it must be that $\sigma^* = 1$, because otherwise $\Delta_B^* < \hat{\Delta}_B(\sigma^*)$ and hence $\rho^* = 0$ by (41) and Lemma 5, which yields a contradiction with (40). Moreover, $\sigma^* = 1$ in turn implies that it must be $\hat{\Delta}_B(1) = b + c_b - \frac{c_P}{\frac{\alpha_B}{\rho^*} - \alpha_P}$, or equivalently, $\rho^* = \gamma \frac{\alpha_B}{\alpha_L}$. Note that $\gamma \frac{\alpha_B}{\alpha_L} \in (0, \gamma)$ since $\alpha_B < \alpha_L$. It remains to show that $\Delta_B^* = \hat{\Delta}_B(1)$, $\sigma^* = 1$, and $\rho^* = \gamma \frac{\alpha_B}{\alpha_L}$

is indeed an equilibrium. Note that $\rho^*\left[c_{P,1}+\sigma^*\left(c_{P,2}+\alpha_P c_b\right)\right] > \alpha_B c_b$ and $\alpha_B > \rho^*\sigma^*\alpha_P$ are satisfied since $\hat{\Delta}_B\left(1\right) < b$ by (which follows from (45)) and $\hat{\Delta}_B(1) = b + c_b - \frac{c_P}{\frac{\alpha_B}{\rho^*}-\alpha_P}$. Therefore, the best response of the incumbent given others' strategies is described by part (i.c) of Lemma 5, and specifically by (47), which reduces to $\Delta_B = \hat{\Delta}_B(1)$ given $\sigma = 1$ and $\rho = \gamma\frac{\alpha_B}{\alpha_L}$, since $\check{\Delta} \leq \hat{\Delta}_B(1)$ and $b_A < \hat{\Delta}_B(1)$ (note that the last inequality follows from $\check{\Delta} \leq \hat{\Delta}_B(1)$ and (45)). Similarly, given $\Delta_B = \hat{\Delta}_B(1)$, $\sigma = 1$ is a best response of the incumbent given Lemma 5 since $\check{\Delta} \leq \hat{\Delta}_B(1)$ and hence $\max\{l, b_A\} < \hat{\Delta}_B(1)$ by (45). The last inequality also implies that given $\Delta_B = \hat{\Delta}_B(1)$ and $\sigma = 1$, $\rho = \gamma\frac{\alpha_B}{\alpha_L}$ is a best response of the activist given Lemma 5 since $\gamma\frac{\alpha_B}{\alpha_L} \in [0, \gamma]$. Therefore, the described strategy profile is indeed an equilibrium.

Before considering the remaining cases, I show that any equilibrium where $\iota^*(\Delta) < 1$ for some Δ must satisfy

$$\Delta_B^* = \begin{cases} b + c_b - \frac{c_{P,1}+\sigma^* c_{P,2}}{\frac{\alpha_B}{\gamma}-\sigma^*\alpha_P}, & \text{if } \alpha_B \geq \hat{\alpha}_B(\sigma^*) \equiv \gamma\sigma^*\alpha_P + \gamma\frac{c_{P,1}+\sigma^* c_{P,2}}{b+c_b-\hat{\Delta}_B(\sigma^*)}, \\ \hat{\Delta}_B(\sigma^*), & \text{otherwise}, \end{cases} \tag{48}$$

Moreover, note that $\hat{\Delta}_B(\sigma^*) < b$ since otherwise $\Pr(\Delta \geq b) = 0$ and Lemma 5 implies $\hat{\Delta}_B(\sigma^*) = \infty$ and hence $\rho^* = 0$, which is a contradiction with (40). Therefore, $b + c_b - \frac{c_{P,1}+\sigma^* c_{P,2}}{\frac{\alpha_B}{\gamma}-\sigma^*\alpha_P} \geq \hat{\Delta}_B(\sigma^*)$ if $\alpha_B \geq \hat{\alpha}_B(\sigma^*)$. Suppose there exists an equilibrium such that Δ_B^* is strictly larger than (48). However, this implies that $b > \Delta_B^* > \hat{\Delta}_B(\sigma^*)$, where the first inequality follows from part (i), and hence Lemma 5 implies that $\rho^* = \gamma$ and hence $\Delta_B^* = b + c_b - \frac{c_{P,1}+\sigma^* c_{P,2}}{\frac{\alpha_B}{\gamma}-\sigma^*\alpha_P}$. However, since $\Delta_B^* \in (0, b)$ by part (i) and Δ_B^* is not equal to (48), this means that it must be $\alpha_B < \hat{\alpha}_B(\sigma^*)$. However, then $b + c_b - \frac{c_{P,1}+\sigma^* c_{P,2}}{\frac{\alpha_B}{\gamma}-\sigma^*\alpha_P} < \hat{\Delta}_B(\sigma^*)$ (since $\hat{\Delta}_B(\sigma^*) < b$), which yields a contradiction with $\Delta_B^* = b + c_b - \frac{c_{P,1}+\sigma^* c_{P,2}}{\frac{\alpha_B}{\gamma}-\sigma^*\alpha_P} > \hat{\Delta}_B(\sigma^*)$. It remains to show that there cannot be an equilibrium such that Δ_B^* is strictly smaller than (26). Suppose there is. There are two cases to consider. First, suppose that $\alpha_B \geq \hat{\alpha}_B(\sigma^*)$ and $\Delta_B^* < b + c_b - \frac{c_{P,1}+\sigma^* c_{P,2}}{\frac{\alpha_B}{\gamma}-\sigma^*\alpha_P}$. However, this yields contradiction with (43) for any $\rho^* \in [0, \gamma]$, which must be satisfied due to Lemma 5. Second, suppose that $\alpha_B < \hat{\alpha}_B(\sigma^*)$ and $\Delta_B^* < \hat{\Delta}_B(\sigma^*)$. However, then (41) and Lemma 5 imply that $\rho^* = 0$, yielding a contradiction with (40).

As the third case, suppose that $\alpha_B \leq \gamma\frac{c_{P,1}}{c_b+b-\check{\Delta}}$ and $\hat{\Delta}_B(1) < \check{\Delta}$. Then, since $\alpha_B \leq \gamma\frac{c_{P,1}}{c_b+b-\check{\Delta}}$ implies that $\alpha_B < \alpha_L$, in any equilibrium with $\iota^*(\Delta) < 1$ for some Δ, it must be that $\Delta_B^* = \check{\Delta} \in (0, b)$ by part (i) and hence $\Delta_B^* = b + c_b - \frac{c_{P,1}+\sigma^* c_{P,2}}{\frac{\alpha_B}{\rho^*}-\sigma^*\alpha_P}$ by (43). Since $b + c_b - \frac{c_{P,1}+\sigma c_{P,2}}{\frac{\alpha_B}{\gamma}-\sigma\alpha_P} < \check{\Delta}$ for all $\sigma \in (0, 1]$, by (44) and (48) it must be that $\Delta_B^* = \hat{\Delta}_B(\sigma^*)$. Because $\Delta_B^* \in (0, b)$ by part (i), the only σ^* that satisfies $\check{\Delta} = \hat{\Delta}_B(\sigma^*)$ is $\sigma^* = \hat{\sigma}_A$, and therefore it must be that $\sigma^* = \hat{\sigma}_A$.

Furthermore, $\Delta_B^* = b + c_b - \frac{c_{P,1}+\hat{\sigma}_A c_{P,2}}{\frac{\alpha_B}{\rho^*}-\hat{\sigma}_A\alpha_P}$ by (43), hence together with $\Delta_B^* = \check{\Delta}$, this implies that

$$\rho^* = \alpha_B \frac{c_b + b - \check{\Delta}}{\hat{\sigma}_A\alpha_P(c_b + b - \check{\Delta}) + c_{P,1} + \hat{\sigma}_A c_{P,2}}, \tag{49}$$

Moreover, $\hat{\Delta}_B(1) < \check{\Delta} = \hat{\Delta}_B(\hat{\sigma}_A)$ implies that $\hat{\sigma}_A \in (0,1)$, and hence (49) together with $\alpha_B \leq \gamma\frac{c_{P,1}}{c_b+b-\check{\Delta}}$, $\check{\Delta} < b$, and $\hat{\sigma}_A \in (0,1)$ imply that $\rho^* \in (0,\gamma)$. Note that $\alpha_B \leq \gamma\frac{c_{P,1}}{c_b+b-\check{\Delta}}$ also implies that $\hat{\sigma}_I \leq 0$, hence $\max\{\hat{\sigma}_I, \hat{\sigma}_A\} = \hat{\sigma}_A$.

It remains to show that $\Delta_B^* = \check{\Delta}$, $\sigma^* = \hat{\sigma}_A$, and $\rho = \rho^*$ given by (49) is indeed an equilibrium. Note that $\rho^*\left[c_{P,1} + \sigma^*\left(c_{P,2} + \alpha_P c_b\right)\right] > \alpha_B c_b$ and $\alpha_B > \rho^*\sigma^*\alpha_P$ are satisfied since $\check{\Delta} < b$ (which follows from (45)) and $\check{\Delta} = b + c_b - \frac{c_{P,1}+\hat{\sigma}_A c_{P,2}}{\frac{\alpha_B}{\rho^*}-\hat{\sigma}_A\alpha_P}$. Therefore, the best response of the incumbent given others' strategies is described by part (i.c) of Lemma 5, and specifically by (47), which reduces to $\Delta_B = \check{\Delta}$ given $\sigma = \hat{\sigma}_A$ and $\rho = \rho^*$, since $b_A < \check{\Delta}$ (note that this inequality follows from $\hat{\Delta}_B(1) < \check{\Delta}$ and (45)). Similarly, given $\Delta_B = \check{\Delta}$, $\sigma = \hat{\sigma}_A \in (0,1)$ is a best response of the incumbent given Lemma 5 since $\hat{\Delta}_B(1) < \check{\Delta}$ and hence $\max\{l, b_A\} < \check{\Delta}$ by (45). The last inequality also implies that given $\Delta_B = \check{\Delta}$ and $\sigma = \hat{\sigma}_A$, $\rho = \rho^*$ is a best response of the activist given Lemma 5 since $\check{\Delta} = \hat{\Delta}_B(\hat{\sigma}_A)$ and $\rho^* \in [0,\gamma]$. Therefore, the described strategy profile is indeed an equilibrium.

Fourth, suppose that $\gamma\frac{c_{P,1}}{c_b+b-\check{\Delta}} < \alpha_B < \alpha_L$, $\hat{\Delta}_B(1) < \check{\Delta}$, and $\hat{\Delta}_B(\hat{\sigma}_I(\alpha_B)) \leq \check{\Delta}$. Then, since $\alpha_B < \alpha_L$, in any equilibrium with $\iota^*(\Delta) < 1$ for some Δ, it must be that $\Delta_B^* = \check{\Delta} \in (0,b)$ by part (i) and hence $\Delta_B^* = b + c_b - \frac{c_{P,1}+\sigma^* c_{P,2}}{\frac{\alpha_B}{\rho^*}-\sigma^*\alpha_P}$ by (43).Note that $\hat{\sigma}_I(\alpha_B)$ and $\hat{\sigma}_A$ satisfy $\hat{\Delta}_B(\hat{\sigma}_I(\alpha_B)) \leq \check{\Delta} = \hat{\Delta}_B(\hat{\sigma}_A)$. Therefore, $\hat{\sigma}_I(\alpha_B) \geq \hat{\sigma}_A$. Also, note that $\hat{\sigma}_I(\alpha_B)$ satisfies $b + c_b - \frac{c_{P,1}+\hat{\sigma}_I c_{P,2}}{\frac{\alpha_B}{\gamma}-\hat{\sigma}_I\alpha_P} = \check{\Delta}$. Next I prove that $\sigma^* = \hat{\sigma}_I(\alpha_B)$. Suppose $\sigma^* \neq \hat{\sigma}_I(\alpha_B)$. Then, (48) and $\Delta_B^* = \check{\Delta}$ imply that it must be $\alpha_B < \hat{\alpha}_B(\sigma^*)$ and $\Delta_B^* = \hat{\Delta}_B(\sigma^*)$. The latter implies that $\sigma^* = \hat{\sigma}_A$, and hence $\alpha_B < \hat{\alpha}_B(\hat{\sigma}_A)$. Note that $b + c_b - \frac{c_{P,1}+\hat{\sigma}_I c_{P,2}}{\frac{\alpha_B}{\gamma}-\hat{\sigma}_I\alpha_P} = \check{\Delta} \in (0,b)$ and $\hat{\sigma}_I(\alpha_B) \geq \hat{\sigma}_A$ imply that $\alpha_B - \gamma\hat{\sigma}_A\alpha_P > 0$. Combined with $\hat{\Delta}_B(\hat{\sigma}_A) = \check{\Delta} < b$ and $\alpha_B < \hat{\alpha}_B(\hat{\sigma}_A)$, this implies that $b + c_b - \frac{c_{P,1}+\hat{\sigma}_A c_{P,2}}{\frac{\alpha_B}{\gamma}-\hat{\sigma}_A\alpha_P} < \hat{\Delta}_B(\hat{\sigma}_A) = \check{\Delta}$, which yields a contradiction with $b + c_b - \frac{c_{P,1}+\hat{\sigma}_I c_{P,2}}{\frac{\alpha_B}{\gamma}-\hat{\sigma}_I\alpha_P} = \check{\Delta}$ and $\hat{\sigma}_I(\alpha_B) > \hat{\sigma}_A > 0$ (note that $\hat{\sigma}_I(\alpha_B) > \hat{\sigma}_A$ follows from $\hat{\sigma}_I(\alpha_B) \geq \hat{\sigma}_A$, $\sigma^* \neq \hat{\sigma}_I(\alpha_B)$, and $\sigma^* = \hat{\sigma}_A$). Hence, it must be that $\sigma^* = \hat{\sigma}_I(\alpha_B)$. Furthermore, $\Delta_B^* = b + c_b - \frac{c_{P,1}+\hat{\sigma}_I c_{P,2}}{\frac{\alpha_B}{\rho^*}-\hat{\sigma}_I\alpha_P}$ by (43), hence together with $\Delta_B^* = \check{\Delta} \in (0,b)$ and $b + c_b - \frac{c_{P,1}+\hat{\sigma}_I c_{P,2}}{\frac{\alpha_B}{\gamma}-\hat{\sigma}_I\alpha_P} = \check{\Delta}$, this implies that $\rho^* = \gamma$. Moreover, $\gamma\frac{c_{P,1}}{c_b+b-\check{\Delta}} < \alpha_B$ and $\check{\Delta} < b$ imply that $\hat{\sigma}_I > 0$, and $\hat{\Delta}_B(1) < \check{\Delta} < b$ and $\alpha_B < \alpha_L$ imply that $\hat{\sigma}_I < 1$.

It remains to show that $\Delta_B^* = \check{\Delta}$, $\sigma^* = \hat{\sigma}_I(\alpha_B)$, and $\rho^* = \gamma$ is indeed an equilibrium.

Note that $\rho^* \left[c_{P,1} + \sigma^* \left(c_{P,2} + \alpha_P c_b\right)\right] > \alpha_B c_b$ and $\alpha_B > \rho^* \sigma^* \alpha_P$ are satisfied since $\check{\Delta} < b$ (which follows from (45)) and $\check{\Delta} = b + c_b - \frac{c_{P,1} + \hat{\sigma}_I c_{P,2}}{\frac{\alpha_B}{\gamma} - \hat{\sigma}_I \alpha_P}$. Therefore, the best response of the incumbent given others' strategies is described by part (i.c) of Lemma 5, and specifically by (47), which reduces to $\Delta_B = \check{\Delta}$ given $\sigma = \hat{\sigma}_I(\alpha_B)$ and $\rho = \gamma$, since $b_A < \check{\Delta}$ (note that this inequality follows from $\hat{\Delta}_B(1) < \check{\Delta}$ and (45)). Similarly, given $\Delta_B = \check{\Delta}$, $\sigma = \hat{\sigma}_I(\alpha_B) \in (0,1)$ is a best response of the incumbent given Lemma 5 since $\hat{\Delta}_B(1) < \check{\Delta}$ and hence $\max\{l, b_A\} < \check{\Delta}$ by (45). The last inequality also implies that given $\Delta_B = \check{\Delta}$ and $\sigma = \hat{\sigma}_I(\alpha_B)$, $\rho = \gamma$ is a best response of the activist given Lemma 5 since $\hat{\Delta}_B(\hat{\sigma}_I(\alpha_B)) \leq \check{\Delta}$. Therefore, the described strategy profile is indeed an equilibrium.

Fifth, suppose that $\gamma \frac{c_{P,1}}{c_b + b - \check{\Delta}} < \alpha_B < \alpha_L$, $\hat{\Delta}_B(1) < \check{\Delta}$, and $\check{\Delta} < \hat{\Delta}_B(\hat{\sigma}_I(\alpha_B))$. Then, since $\alpha_B < \alpha_L$, in any equilibrium with $\iota^*(\Delta) < 1$ for some Δ, it must be that $\Delta_B^* = \check{\Delta} \in (0,b)$ by part (i) and hence $\Delta_B^* = b + c_b - \frac{c_{P,1} + \sigma^* c_{P,2}}{\frac{\alpha_B}{\rho^*} - \sigma^* \alpha_P}$ by (43). Note that $\hat{\sigma}_I(\alpha_B)$ and $\hat{\sigma}_A$ satisfy $\hat{\Delta}_B(\hat{\sigma}_A) = \check{\Delta} < \hat{\Delta}_B(\hat{\sigma}_I(\alpha_B))$. Therefore, $\hat{\sigma}_A > \hat{\sigma}_I(\alpha_B)$. Also, note that $\hat{\sigma}_I(\alpha_B)$ satisfies $b + c_b - \frac{c_{P,1} + \hat{\sigma}_I c_{P,2}}{\frac{\alpha_B}{\gamma} - \hat{\sigma}_I \alpha_P} = \check{\Delta}$. Note that it must be $\sigma^* \neq \hat{\sigma}_I(\alpha_B)$, because if $\sigma^* = \hat{\sigma}_I(\alpha_B)$ then $\Delta_B^* = \check{\Delta} < \hat{\Delta}_B(\hat{\sigma}_I(\alpha_B))$ and Lemma 5 imply that $\rho^* = 0$, yielding a contradiction with (40). However, since $\check{\Delta} \in (0,b)$ implies that $b + c_b - \frac{c_{P,1} + \sigma^* c_{P,2}}{\frac{\alpha_B}{\gamma} - \sigma^* \alpha_P} \neq \check{\Delta}$ for any $\sigma^* \neq \hat{\sigma}_I(\alpha_B)$, it must be that $\Delta_B^* = \hat{\Delta}_B(\sigma^*)$ by (48). Since $\Delta_B^* = \check{\Delta}$, this implies that it must be that $\sigma^* = \hat{\sigma}_A$. Furthermore, $\Delta_B^* = b + c_b - \frac{c_{P,1} + \hat{\sigma}_A c_{P,2}}{\frac{\alpha_B}{\rho^*} - \hat{\sigma}_A \alpha_P}$ by (43), hence together with $\Delta_B^* = \check{\Delta}$, this implies that ρ^* is given by (49). Moreover, $\hat{\Delta}_B(1) < \check{\Delta} = \hat{\Delta}_B(\hat{\sigma}_A)$ implies that $\hat{\sigma}_A \in (0,1)$, and hence (49) together with $b + c_b - \frac{c_{P,1} + \hat{\sigma}_I c_{P,2}}{\frac{\alpha_B}{\gamma} - \hat{\sigma}_I \alpha_P} = \check{\Delta}$, $\hat{\sigma}_A > \hat{\sigma}_I(\alpha_B)$, $\check{\Delta} < b$, and $\hat{\sigma}_A \in (0,1)$ imply that $\rho^* \in (0,\gamma)$. It remains to show that $\Delta_B^* = \check{\Delta}$, $\sigma^* = \hat{\sigma}_A$, and $\rho = \rho^*$ given by (49) is indeed an equilibrium. The proof of this identical to that in the third case above (i.e., when $\alpha_B \leq \gamma \frac{c_{P,1}}{c_b + b - \check{\Delta}}$ and $\hat{\Delta}_B(1) < \check{\Delta}$), and therefore is not repeated here. ■

Proposition 8 *Suppose that $c_b \in [0,\infty)$, $\kappa < \alpha_P E\left[\max\{0, \Delta - b_A\}\right]$, $E\left[\Delta | \Delta \geq b_A\right] > 0$, and the activist has demanded board settlement that gives him decision authority with probability $\alpha_B \in (0,1]$. Then, an equilibrium of this subgame where the incumbent accepts board settlement for all Δ exists if and only if*

$$\alpha_B \leq \gamma \alpha_P + \gamma \frac{c_P}{c_b + b - \max\{b_A, l\}}. \tag{50}$$

Moreover, whenever such an equilibrium exists, it also exists with $\rho^* = \gamma$ (i.e., if the incumbent *rejects the settlement, the activist runs a proxy fight with probability γ) and $\sigma^* = 1$ (i.e., if the activist runs a proxy fight he always wins).*

Proof. Suppose that $c_b \in [0, \infty)$, $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$, $E[\Delta | \Delta \geq b_A] > 0$, and the activist has demanded board settlement that gives him decision authority with probability $\alpha_B \in (0, 1]$. Note that $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$ implies that $b_A < b$. Note that $\Delta \in (l, b)$ by assumption.

Note that by Lemma 5, any equilibrium where the incumbent accepts board settlement for all Δ has to satisfy

$$\rho^* [c_{p,1} + \sigma^* (c_{p,2} + \alpha_p c_b)] > \alpha_B c_b \text{ and } \Delta_B^* < b_A,$$

where Δ_B is given by (29). I start by showing that an equilibrium where the incumbent accepts board settlement for all Δ does not exist if (50) does not hold. Note that if (50) does not hold, then $\alpha_B > \gamma \alpha_P$. Since $\rho^* \leq \gamma$ by Lemma 2, this implies that $\alpha_B > \rho^* \sigma^* \alpha_P$, and hence Δ_B^* is given by

$$\Delta_B^* (\alpha_B, \rho^*, \sigma^*) = \max \left\{ b_A, b - \frac{\rho^* [c_{P,1} + \sigma^* (c_{P,2} + \alpha_P c_b)] - \alpha_B c_b}{\alpha_B - \rho^* \sigma^* \alpha_P} \right\} \tag{51}$$

The incumbent does not accept for all Δ if $\Delta_B^* > \max\{b_A, l\}$, or equivalently,

$$\alpha_B > \rho^* \sigma^* \alpha_P + \rho^* \frac{c_{P,1} + \sigma^* c_{P,2}}{c_b + b - \max\{b_A, l\}},$$

which holds since (50) is violated and $c_P = c_{P,1} + c_{P,2}$.

Next, I show that if (50) holds, then there exists an equilibrium where $\rho^* = \gamma$, $\sigma^* = 1$, and the incumbent accepts board settlement for all Δ. Suppose that upon rejection of settlement, the off-equilibrium path beliefs of the activist and the shareholders are $\Delta \in (\max\{0, b_A + \frac{\kappa}{\alpha_P}\}, b)$. Note that this belief is non-empty since $b_A + \frac{\kappa}{\alpha_P} < b$, or equivalently $\kappa < \alpha_P (b - b_A)$, which follows from $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$. Then, given these beliefs and $\rho^* = \gamma$, by Lemma 2 upon the incumbent's rejection the payoff of shareholders is zero if the activist does not run or win a proxy fight, and is strictly larger than zero if the activist runs and wins a proxy fight, making $\sigma = 1$ a best response of the shareholders. Similarly, given the same beliefs and $\sigma^* = 1$, by Lemma 2 upon the incumbent's rejection the payoff of the activist is zero if he does not run a proxy fight, and is

$$-k + \sigma^* \alpha_P E[\max\{0, \Delta - b_A\} \mid \text{rejection}] > -k + \sigma^* \alpha_P (b_A + \frac{\kappa}{\alpha_P} - b_A) = 0$$

if he runs a proxy fight (where the RHS is zero if $k = \kappa$) , making running a proxy fight if $k = \kappa$ a best response of the activist in turn. Note that by Lemma 2, this implies that $\rho = \gamma$. Therefore, it remains to show that given $\rho^* = \gamma$ and $\sigma^* = 1$, accepting board settlement for all Δ is a best response of the incumbent if (50) holds. Note that $\rho^* \left[c_{p,1} + \sigma^* \left(c_{p,2} + \alpha_P c_b\right)\right] > \alpha_B c_b$ is satisfied since (50) implies that $c_P + \gamma\alpha_P c_b \geq \alpha_B c_b$, where $c_P = c_{P,1} + c_{P,2}$. Therefore, by Lemma 5, the incumbent accepts board settlement for all $\Delta \in (l, b_A) \cup (\Delta_B^*, b)$. Note that $\Delta_B^* = \max\{b_A, l\}$ since

$$b - \frac{\rho^* \left[c_{P,1} + \sigma^* \left(c_{P,2} + \alpha_P c_b\right)\right] - \alpha_B c_b}{\alpha_B - \rho^*\sigma^*\alpha_P} \leq \max\{b_A, l\},$$

which holds since $\rho^* = \gamma$, $\sigma^* = 1$, and (50) holds. This implies that the incumbent accepts board settlement for all $\Delta \in (l, b_A) \cup (b_A, b)$, and it remains to show that it is also a best response of the incumbent to accepts when $\Delta = b_A$ if $b_A > l$. To see this, recall that the probability that the activist implements the project when he has the decision authority and $\Delta = b_A$ is given by $x_a^*(b_A)$. Then, the incumbent's expected payoff is $\alpha_B x_a^*(b_A - b) - \alpha_B c_b$ from accepting the settlement and is

$$\rho^* \left[-c_{P,1} + \sigma^* \left(-c_{P,2} + \alpha_P \left(-c_b + x_a^*(b_A - b)\right)\right)\right]$$

from rejecting. Here, the former is weakly larger since $b_A > l, x_a^* \in [0,1], \rho^* = \gamma, \sigma^* = 1, c_P = c_{P,1} + c_{P,2}$, and (50) holds. Therefore, accepting the settlement is a best response of the incumbent as well if $\Delta = b_A$ and $b_A > l$, concluding the proof. ■

Proposition 9 *Suppose that $c_b \in [0, \infty)$ and the activist has demanded a board settlement that gives him decision authority with probability $\alpha_B \in (0, 1]$. Moreover, suppose that $b_A \in (-\infty, \infty)$ and $\kappa \in (0, \infty)$, or $b_A < b$ and at least one of $\kappa \geq \alpha_P E\left[\max\{0, \Delta - b_A\}\right]$ and $E\left[\Delta | \Delta \geq b_A\right] \leq 0$ holds. Then, an equilibrium of this subgame where $\rho^* = 0$ (i.e., the activist never runs a proxy fight upon rejection of the settlement) always exists. Moreover, in any such equilibrium, the project is never implemented for any Δ, and the incumbent rejects board settlement for all Δ if $c_b > 0$ and rejects for all $\Delta > b_A$ if $c_b = 0$. Finally, whenever such an equilibrium exists, there exists an equilibrium where $\rho^* = 0$ and the incumbent rejects board settlement for all Δ.*

Proof. Suppose that $c_b \in [0, \infty)$, $b_A \geq b$, $\kappa \in (0, \infty)$, and the activist has demanded a board settlement that gives him decision authority with probability $\alpha_B \in (0, 1]$. Note that $\Delta \in (l, b)$

by assumption. By Lemma 2, the project is never implemented in any subgame, regardless of who has decision authority. Therefore, the activist's payoff from running a proxy fight at any subgame is always $-k < 0$. Since his payoff from not running a proxy fight at any subgame is zero, $\rho = 0$ is the activist's unique best response in any subgame. Given $\rho = 0$, the incumbent's expected payoff from rejecting the settlement is zero, while its payoff from accepting is $-\alpha_B c_b$ as well. Therefore, the incumbent rejects for all Δ if $c_b > 0$, and is indifferent between rejecting and not for all Δ if $c_b = 0$.

Suppose that $c_b \in [0, \infty)$, $b_A < b$, at least one of $\kappa \geq \alpha_p E\left[\max\{0, \Delta - b_A\}\right]$ or $E\left[\Delta | \Delta \geq b_A\right] \leq 0$ holds, and the activist has demanded a board settlement that gives him decision authority with probability $\alpha_B \in (0, 1]$. Note that $\Delta \in (l, b)$ by assumption. First, I prove that in any equilibrium with $\rho^* = 0$, the project is never implemented for any Δ, and the incumbent rejects board settlement for all Δ if $c_b > 0$ and rejects for all $\Delta > b_A$ if $c_b = 0$. Note that by Lemma 2, the incumbent never implements the project itself if it has the decision authority at the implementation stage. Therefore, if a board settlement is reached, the incumbent's payoff is $\alpha_B x_a^*(\Delta)(\Delta - b) - \alpha_B c_b$, where $x_a^*(b_A)$ is the probability that the activist implements the project for a given Δ if he has the decision authority. Since the incumbent's payoff is zero from rejecting the settlement, and $x_a^*(\Delta) = 1$ for all $\Delta > b_A$ by Lemma 2, the incumbent rejects board settlement for all Δ if $c_b > 0$ and rejects for all $\Delta > b_A$ if $c_b = 0$. Moreover, if $x_a^*(b_A) > 0$, then the incumbent rejects board settlement for $\Delta = b_A$ as well. Combining with $x_a^*(\Delta) = 0$ for all $\Delta < b_A$ by Lemma 2, these imply that the project is never implemented for any Δ.

Next, I start by proving that there exists an equilibrium where $\rho^* = 0$ and the incumbent rejects board settlement for all Δ. I prove this statement in two steps. First, suppose that $\kappa \geq \alpha_P E\left[\max\{0, \Delta - b_A\}\right]$. Then, by Lemma 2, upon the incumbent's rejection, in this equilibrium candidate the activist's expected payoff from not running a proxy fight is zero and is $-k + \sigma^* \alpha_P E\left[\max\{0, \Delta - b_A\}\right]$ from running a proxy fight. Since the latter is weakly smaller than zero for all $\sigma^* \in [0, 1]$ and $k \in \{\zeta, \kappa\}$, $\rho = 0$ is a best response of the activist given other players' strategies. If $\Delta = b_A$, then by Lemma 2 the incumbent's payoff is zero from rejecting and is weakly smaller than zero from accepting, since in the latter case it is given by $\alpha_B x_a^*(b_A)(\Delta - b) - \alpha_B c_b$. Combining with Lemma 5, rejecting is a best response of the incumbent for all Δ given other players' strategies, confirming the existence of this equilibrium.

Second, suppose that $E\left[\Delta | \Delta \geq b_A\right] \leq 0$. Note that this implies that $b_A < b$. Then, by Lemma 2, if the activist runs a proxy fight, in this equilibrium candidate the shareholders' payoff is zero if the activist loses and is $\alpha_P \Pr(\Delta \geq b_A) E\left[\Delta | \Delta \geq b_A\right]$ if the activist wins. Therefore, $\sigma = 0$ is a best response of the shareholders given that incumbent rejects for all Δ.

In turn, $\rho = 0$ is a best response of the activist, since upon rejection his payoff is zero from not running a proxy fight and is $-k$ from running one. If $\Delta = b_A$, then again by Lemma 2 the incumbent's payoff is zero from rejecting and is weakly smaller than zero from accepting. Combining with Lemma 5, rejecting is a best response of the incumbent for all Δ given other players' strategies, confirming the existence of this equilibrium. ■

I prove Proposition 3 with the following generalization that $b_A \in (l, b)$. Note that $b_A < b$ is implied by $\kappa < \alpha_P E\left[\max\left\{0, \Delta - b_A\right\}\right]$ since $0 < \kappa$.

Proposition 10 *Suppose that* $c_b = 0$, $b_A \in (l, b)$, $\kappa < \alpha_P E\left[\max\left\{0, \Delta - b_A\right\}\right]$, *and* $E\left[\Delta | \Delta \geq b_A\right] > 0$. *Moreover, suppose that* Δ *follows the cumulative distribution function* $F^{\lambda}(\cdot)$, *which is continuously differentiable with full support on* $(\hat{l}(\lambda), b)$ *such that* $\hat{l}(\lambda)$ *is a weakly decreasing function of* λ *that satisfies* $\hat{l}(1) = l$, *and* $f^{\lambda}(\Delta) = \frac{1}{\lambda} f(\Delta)$ *for* $\Delta \in [b_A, b)$, *where* f^{λ} *is the probability density function of* F^{λ}. *Let*

$$\hat{\lambda} \equiv \sup\left\{\lambda \geq 1 : \kappa < \alpha_P E^{\lambda}\left[\max\left\{0, \Delta - b_A\right\}\right]\right\}, \tag{52}$$

where $E^{\lambda}[\cdot]$ *denotes the expectation under the distribution* $F^{\lambda}(\cdot)$. *Then,*

- *(i) There exists* $\bar{\lambda} \in (0, \hat{\lambda})$ *such that for any* $\lambda \in [1, \hat{\lambda})$, *the activist demands action settlement if* $\lambda < \bar{\lambda}$ *and board settlement if* $\lambda > \bar{\lambda}$.
- *(ii) Suppose that the activist demands action settlement for some* $\lambda \in [1, \hat{\lambda})$. *Then, the average shareholder return of board settlement is always strictly smaller than the average return of an action settlement.*

Proof. Note that action settlement is denoted with $\eta = A$, while board settlement that gives the activist decision authority with probability α_B with $\eta = \alpha_B$. Note that for all $\lambda \geq 1$, $\hat{l}(\lambda) < b_A$ is satisfied and hence $E^{\lambda}\left[\Delta | \Delta \geq b_A\right] > 0$. Moreover, $\kappa < \alpha_P E^{\lambda}\left[\max\left\{0, \Delta - b_A\right\}\right]$ for $\lambda = 1$, $\alpha_P E^{\lambda}\left[\max\left\{0, \Delta - b_A\right\}\right]$ is a strictly decreasing function of λ, and $\lim_{\lambda \to \infty} \alpha_P E^{\lambda}\left[\max\left\{0, \Delta - b_A\right\}\right] = 0$. Hence $\hat{\lambda}$ exists, satisfies $\hat{\lambda} < \infty$, and is unique. Moreover, this also implies that

$$\kappa < \alpha_P E^{\lambda}\left[\max\left\{0, \Delta - b_A\right\}\right] \text{ and } E^{\lambda}\left[\Delta | \Delta \geq b_A\right] > 0 \text{ for all } \lambda \in [1, \hat{\lambda}). \tag{53}$$

Denote the expected payoff of the activist from demanding settlement η by $\Pi_a(\eta | \lambda)$.

Consider part (i). Pick any $\lambda_1 \in [1, \hat{\lambda})$, and suppose that $\Pi_a(\alpha_B|\lambda_1) \geq \Pi_a(A|\lambda_1)$ for any $\alpha_B \in (0, 1]$. I show that $\Pi_a(\alpha_B|\lambda_2) \geq \Pi_a(A|\lambda_2)$ for all $\lambda_2 \in (\lambda_1, \hat{\lambda})$.[39] Consider any $\lambda_2 \in (\lambda_1, \bar{\lambda})$. I prove in two steps. First, I show that

$$\Pi_a(\alpha_B|\lambda_2) = \frac{\lambda_1}{\lambda_2}\Pi_a(\alpha_B|\lambda_1) \tag{54}$$

Consider any $\lambda \in [1, \hat{\lambda})$, and suppose that the activist has demanded $\eta = \alpha_B$. Then, due to (53), by Proposition 7 $\rho^* = \rho_B(\alpha_B|\lambda) > 0$ in any equilibrium, an equilibrium where the incumbent board rejects the settlement with positive probability for some $\Delta \in (\hat{l}(\lambda), b)$ always exists, and when combined with Lemma 2, the activist's expected payoff in this equilibrium is given by

$$\Pi_a(\alpha_B|\lambda) = \int_{b_A}^{\Delta_B^*(\alpha_B|\lambda)} \rho_B^*(\alpha_B|\lambda)\left[\sigma_B^*(\alpha_B|\lambda)\alpha_P\left(\Delta - b_A\right) - \kappa\right] f^\lambda(\Delta)d\Delta + \int_{\Delta_B^*(\alpha_B|\lambda)}^{b} \alpha_B\left(\Delta - b_A\right) f^\lambda(\Delta)d\Delta, \tag{55}$$

where $\Delta_B^*(\alpha_B|\lambda)$, $\rho_B^*(\alpha_B|\lambda)$, and $\sigma_B^*(\alpha_B|\lambda)$ are given by (35), (37), and (39), respectively (Note that λ is not explicit in these expressions, because it enters into them via the cumulative distribution function (i.e., by replacing $F(\cdot)$ with $F^\lambda(\cdot)$). Also, note that $\Delta_B^*(\alpha_B|\lambda) \in (\max\{0, b_A\}, b)$ again due to (53) and Proposition 7. Moreover, $\hat{l}(\lambda_1) < b_A$ and $\hat{l}(\lambda_2) < b_A$ imply that $\hat{\Delta}_B(1|\lambda_1) = \hat{\Delta}_B(1|\lambda_2)$ and $\check{\Delta}(\lambda_1) = \check{\Delta}(\lambda_2)$, where $\hat{\Delta}_B(\cdot|\lambda)$ and $\check{\Delta}(\lambda)$ are given by (30) and (16), respectively. Therefore, $\Delta_B^*(\alpha_B|\lambda_1) = \Delta_B^*(\alpha_B|\lambda_2)$, $\rho_B^*(\alpha_B|\lambda_1) = \rho_B^*(\alpha_B|\lambda_2)$, and $\sigma_B^*(\alpha_B|\lambda_1) = \sigma_B^*(\alpha_B|\lambda_2)$. Hence, $f^{\lambda_2}(\Delta) = \frac{\lambda_1}{\lambda_2} f^{\lambda_1}(\Delta)$ implies that (54) holds. Propositions 7 and 8 imply that the only other equilibrium that can exist is the one where the incumbent accepts the settlement for all $\Delta \in (\hat{l}(\lambda), b)$. Moreover, Proposition 8 further implies that if this equilibrium exists for $\lambda = \lambda_1$ then it also exists for any $\lambda \in [1, \hat{\lambda})$ (which includes λ_2), due to $\hat{l}(\lambda) < b_A$ and (53). When this equilibrium indeed exists, for any given $\lambda \in [1, \hat{\lambda})$ the activist's expected payoff in this equilibrium is given by

$$\Pi_a(\alpha_B|\lambda) = \int_{b_A}^{b} \alpha_B\left(\Delta - b_A\right) f^\lambda(\Delta)d\Delta. \tag{56}$$

Hence, $f^{\lambda_2}(\Delta) = \frac{\lambda_1}{\lambda_2} f^{\lambda_1}(\Delta)$ again implies that (54) holds.[40]

[39] I assume that if the activist is indifferent between demanding board settlement and action settlement for both $\lambda = \lambda_1$ and $\lambda = \lambda_2$, the activist demands the same type of settlement in both cases.

[40] A simple assumption I make is that if for any $\lambda \in \{\lambda_1, \lambda_2\}$ the equilibrium described in Proposition 8 is selected in the subgame that the activist demands board settlement that gives him decision authority with

Second, I show that

$$\Pi_a(A|\lambda_2) \leq \frac{\lambda_1}{\lambda_2}\Pi_a(A|\lambda_1) \tag{57}$$

Consider any $\lambda \in [1, \hat{\lambda})$, and suppose that the activist has demanded $\eta = A$. Then, due to (53), by Lemma 2 and Proposition 5 an equilibrium always exists and the activist's expected payoff in this equilibrium is given by

$$\Pi_a(A|\lambda) = F^{\lambda}(b_A)\rho_A^*(\lambda)\left[-\kappa\right]+\int_{b_A}^{\Delta_A^*(\lambda)} \rho_A^*(\lambda)\left[\sigma_A^*(\lambda)\alpha_P\left(\Delta - b_A\right) - \kappa\right] f^{\lambda}(\Delta)d\Delta+\int_{\Delta_A^*(\lambda)}^{b} \left(\Delta - b_A\right) f^{\lambda}(\Delta)d\Delta, \tag{58}$$

where $\Delta_A^*(\lambda)$, $\rho_A^*(\lambda)$, and $\sigma_A^*(\lambda)$ are given by (22), (23), and (25), respectively, and $\Delta_A^*(\lambda) \in (\max\{0, b_A\}, b)$ by Lemma 4 and Proposition 5. Note that (16) implies that $\check{\Delta}(\lambda_1) = \check{\Delta}(\lambda_2)$. Moreover, for any $x > \max\{\hat{l}(\lambda_1), b_A\}$, $F^{\lambda_2}(x) \geq F^{\lambda_1}(x)$ holds (since $F^{\lambda_2}(b) = F^{\lambda_1}(b) = 1$ and $f^{\lambda}(\Delta) = \frac{1}{\lambda}f(\Delta)$ for $\Delta \in [b_A, b)$), and hence

$$\frac{1}{F^{\lambda_2}(x)}\int_{b_A}^{x} \left(\Delta - b_A\right) dF^{\lambda_2}(\Delta) \leq \frac{1}{F^{\lambda_1}(x)}\int_{b_A}^{x} \left(\Delta - b_A\right) dF^{\lambda_1}(\Delta). \tag{59}$$

This combined with $\hat{l}(\lambda_2) \leq \hat{l}(\lambda_1) < b_A$ and (14) together imply that $\hat{\Delta}_A(1|\lambda_1) \leq \hat{\Delta}_A(1|\lambda_2)$. Importantly, $\hat{\Delta}_A(1|\lambda_1) \leq \hat{\Delta}_A(1|\lambda_2)$, $\check{\Delta}(\lambda_1) = \check{\Delta}(\lambda_2)$, and (22) imply that $\Delta_A^*(\lambda_1) \leq \Delta_A^*(\lambda_2)$. Since $\rho_A^*(\lambda_2) \in (0, \gamma]$ by Proposition 5, to prove (57), there are two cases to consider. First, suppose that $\rho_A^*(\lambda_2) \in (0, \gamma)$. This means that if $\lambda = \lambda_2$ and $k = \kappa$, then upon the incumbent's rejection, the activist is indifferent between running a proxy fight and not running, where his payoff from the latter is zero. This implies that if $\lambda = \lambda_2$ then the sum of the first two terms in (58) is equal to zero, and hence

$$\Pi_a(A|\lambda_2) = \int_{\Delta_A^*(\lambda_2)}^{b} \left(\Delta - b_A\right) f^{\lambda_2}(\Delta)d\Delta \leq \frac{\lambda_1}{\lambda_2}\int_{\Delta_A^*(\lambda_1)}^{b} \left(\Delta - b_A\right) f^{\lambda_1}(\Delta)d\Delta, \tag{60}$$

where the inequality follows from $\Delta_A^*(\lambda_1) \leq \Delta_A^*(\lambda_2)$. Moreover, Proposition 5 implies that $\rho_A^*(\lambda_1) > 0$. This means that if $\lambda = \lambda_1$, then upon the incumbent's rejection, the activist weakly prefers running a proxy fight over not running when $k = \kappa$. Therefore, if $\lambda = \lambda_1$ then the sum of the first two terms in (58) is weakly greater than zero, in other words, $\int_{\Delta_A^*(\lambda_1)}^{b} \left(\Delta - b_A\right) f^{\lambda_1}(\Delta)d\Delta \leq \Pi_a(A|\lambda_1)$. When combined with (60), this implies that (57)

probability $\alpha_B = \tilde{\alpha}_B$, then the same type of equilibrium is selected in this subgame for all $\lambda \in \{\lambda_1, \lambda_2\}$ if it exists.

indeed holds, which is what we wanted to show.

Second, suppose that $\rho_A^*(\lambda_2) = \gamma$. I start by showing that $\rho_A^*(\lambda_1) = \gamma$ and $\sigma_A^*(\lambda_1) = \sigma_A^*(\lambda_2)$. To see this, Proposition 5 and $\rho_A^*(\lambda_2) = \gamma$ implies that there are two possible cases for $\lambda = \lambda_2$: Either

$$\max\left\{\hat{\Delta}_A(1|\lambda), \check{\Delta}(\lambda)\right\} \leq b - \frac{c_p + \alpha_p c_b}{\frac{1}{\gamma} - \alpha_p}, \tag{61}$$

or

$$\max\{\hat{\Delta}_A(1|\lambda), b - \frac{c_p + \alpha_p c_b}{\frac{1}{\gamma} - \alpha_p}\} < \check{\Delta},\ c_{p,1} < \frac{1}{\gamma}\left(b - \check{\Delta}\right) \text{ and } \hat{\Delta}_A(\tilde{\sigma}_I|\lambda) \leq \check{\Delta}(\lambda), \tag{62}$$

where $\tilde{\sigma}_I$ is given by (24). Suppose that (61) holds for $\lambda = \lambda_2$. Note that then $\sigma_A^*(\lambda_2) = 1$. Moreover, $\hat{\Delta}_A(1|\lambda_1) \leq \hat{\Delta}_A(1|\lambda_2)$ and $\check{\Delta}(\lambda_1) = \check{\Delta}(\lambda_2)$ imply that (61) also holds for $\lambda = \lambda_1$, and therefore by Proposition 5 it must be that $\rho_A^*(\lambda_1) = \gamma$ and $\sigma_A^*(\lambda_1) = \sigma_A^*(\lambda_2) = 1$. Now, suppose that (62) holds for $\lambda = \lambda_2$ instead. Note that then $\hat{\Delta}_A(\tilde{\sigma}_I(\lambda_2)|\lambda_2) \leq \hat{\Delta}_A(\tilde{\sigma}_A(\lambda_2)|\lambda_2)$ implies that $\tilde{\sigma}_I(\lambda_2) \geq \tilde{\sigma}_A(\lambda_2)$, and hence $\sigma_A^*(\lambda_2) = \tilde{\sigma}_I(\lambda_2)$. Moreover, $\hat{\Delta}_A(1|\lambda_1) \leq \hat{\Delta}_A(1|\lambda_2)$ and $\check{\Delta}(\lambda_1) = \check{\Delta}(\lambda_2)$ again imply that (62) also holds for $\lambda = \lambda_1$, and specifically $\tilde{\sigma}_I(\lambda_1) = \tilde{\sigma}_I(\lambda_2)$ and $\tilde{\sigma}_A(\lambda_1) \leq \tilde{\sigma}_A(\lambda_2)$, where the last inequality follows from (59). Combined with $\tilde{\sigma}_A(\lambda_2) \leq \tilde{\sigma}_I(\lambda_2)$, by Proposition 5 it must be that $\rho_A^*(\lambda_1) = \gamma$ and $\sigma_A^*(\lambda_1) = \sigma_A^*(\lambda_2) = \tilde{\sigma}_I(\lambda_2)$. Also note that $\rho_A^*(\lambda_1) = \rho_A^*(\lambda_2) = \gamma$ implies that if $\lambda \in \{\lambda_1, \lambda_2\}$, then upon the incumbent's rejection, the activist weakly prefers running a proxy fight over not running when $k = \kappa$. Therefore, if $\lambda \in \{\lambda_1, \lambda_2\}$ then the second term in (58) is weakly greater than zero. Combined with $\sigma_A^*(\lambda_1) = \sigma_A^*(\lambda_2)$ and (58), this implies that

$$\begin{aligned}
\Pi_a(A|\lambda_2) &= F^{\lambda_2}(b_A)\gamma\left[-\kappa\right] + \int_{b_A}^{\Delta_A^*(\lambda_2)} \gamma\left[\sigma_A^*(\lambda_2)\alpha_P\left(\Delta - b_A\right) - \kappa\right] f^{\lambda_2}(\Delta)d\Delta + \int_{\Delta_A^*(\lambda_2)}^{b} \left(\Delta - b_A\right) f^{\lambda_2}(\Delta)d\Delta \\
&< \frac{\lambda_1}{\lambda_2}[F^{\lambda_1}(b_A)\gamma\left[-\kappa\right] + \int_{b_A}^{\Delta_A^*(\lambda_2)} \gamma\left[\sigma_A^*(\lambda_2)\alpha_P\left(\Delta - b_A\right) - \kappa\right] f^{\lambda_1}(\Delta)d\Delta + \int_{\Delta_A^*(\lambda_2)}^{b} \left(\Delta - b_A\right) f^{\lambda_1}d\Delta] \\
&\leq \frac{\lambda_1}{\lambda_2}[F^{\lambda_1}(b_A)\gamma\left[-\kappa\right] + \int_{b_A}^{\Delta_A^*(\lambda_1)} \gamma\left[\sigma_A^*(\lambda_1)\alpha_P\left(\Delta - b_A\right) - \kappa\right] f^{\lambda_1}(\Delta)d\Delta + \int_{\Delta_A^*(\lambda_1)}^{b} \left(\Delta - b_A\right) f^{\lambda_1}d\Delta] \\
&= \frac{\lambda_1}{\lambda_2}\Pi_a(A|\lambda_1),
\end{aligned}$$

where the first inequality utilizes $f^{\lambda}(\Delta) = \frac{1}{\lambda}f(\Delta)$ for $\Delta \in [b_A, b)$, and $F^{\lambda_2}(b_A) > F^{\lambda_1}(b_A)$ (since $F^{\lambda_2}(b) = F^{\lambda_1}(b) = 1$), and the second inequality utilizes $\Delta_A^*(\lambda_1) \leq \Delta_A^*(\lambda_2)$ and $\sigma_A^*(\lambda_1) = \sigma_A^*(\lambda_2)$. Hence, (57) again holds, which is again what we wanted to show.

Note that (54) and (57) imply that $\Pi_a(\alpha_B|\lambda_2) \geq \Pi_a(A|\lambda_2)$ for all $\lambda_2 \in (\lambda_1, \hat{\lambda})$. Therefore, to complete the proof of part (i), it remains to show that $\bar{\lambda} < \hat{\lambda}$. For this, it is sufficient to show that for any $\alpha_B \in (0,1]$, there exists $\lambda'(\alpha_B) \in (1,\hat{\lambda})$ such that $\Pi_a(\alpha_B|\lambda') > \Pi_a(A|\lambda')$. Note that by Proposition 5, $\lim_{\lambda\uparrow\hat{\lambda}} \hat{\Delta}_A(1|\lambda) = b$. Moreover, $b_A < b$ and (53) imply that $\check{\Delta}(\lambda)$ is the same and $\check{\Delta}(\lambda) < b$ for all $\lambda \in [1,\hat{\lambda})$, where $\check{\Delta}(\lambda)$ is given by (16). Therefore, there exists $\lambda'' < \hat{\lambda}$ such that $\max\{b - \frac{c_p+\alpha_p c_b}{\frac{1}{\gamma}-\alpha_p}, \check{\Delta}(\lambda)\} < \hat{\Delta}_A(1|\lambda) < b$ for all $\lambda \in (\lambda'',\hat{\lambda})$. Then, again by Proposition 5, $\Delta_A^*(\lambda) = \hat{\Delta}_A(1|\lambda)$ and $\rho_A^* \in (0,\gamma)$ for all $\lambda \in (\lambda'',\hat{\lambda})$. This means that if $\lambda \in (\lambda'',\hat{\lambda})$ and $k = \kappa$, then upon the incumbent's rejection, the activist is indifferent between running a proxy fight and not running, where his payoff from the latter is zero. Therefore, if $\lambda \in (\lambda'',\hat{\lambda})$ then the sum of the first two terms in (58) is equal to zero, and hence

$$\Pi_a(A|\lambda) = \int_{\Delta_A^*(\lambda)}^{b} (\Delta - b_A)\,\frac{1}{\lambda} f(\Delta)d\Delta < \frac{1}{\lambda}\left[F(b) - F(\Delta_A^*(\lambda))\right](b - b_A). \tag{63}$$

Note that $\Delta_A^*(\lambda) \in (\max\{0,b_A\},b)$ by (53), Lemma 4 and Proposition 5. Moreover, $\Delta_A^*(\lambda) = \hat{\Delta}_A(1|\lambda)$ for all $\lambda \in (\lambda'',\hat{\lambda})$ also implies that $\lim_{\lambda\uparrow\hat{\lambda}} \Delta_A^*(\lambda) = \lim_{\lambda\uparrow\hat{\lambda}} \hat{\Delta}_A(1|\lambda) = b$.

With regard to $\Pi_a(\alpha_B|\lambda)$, there are two cases to consider depending on what equilibrium is chosen for the subgame $\eta = \alpha_B$. First, consider the equilibrium where the incumbent rejects the settlement $\eta = \alpha_B$ with some probability for some $\Delta \in (\hat{l}(\lambda), b)$. Recall that by (53) and Proposition 7, this equilibrium exists for all $\lambda \in [1,\hat{\lambda})$, and in this equilibrium $\Pi_a(\alpha_B|\lambda)$ is given by (55). This combined with (58) imply that for any $\lambda \in (\lambda'',\hat{\lambda})$,

$$\Pi_a(\alpha_B|\lambda) = \frac{1}{\lambda}\Pi_a(\alpha_B|1) \geq \frac{1}{\lambda}\int_{\Delta_B^*(\alpha_B|1)}^{b} \alpha_B\,(\Delta - b_A)\, f(\Delta)d\Delta \geq \frac{1}{\lambda}\left[1 - F(\Delta_B^*(\alpha_B|1))\right]\alpha_B\left[\Delta_B^*(\alpha_B|1) - b_A\right], \tag{64}$$

where $\Delta_B^*(\alpha_B|1) \in (\max\{0,b_A\},b)$ again due to (53) and Proposition 7. Note that the first inequality in (64) follows from the fact that the first term in (55) is nonnegative (since otherwise the activist would deviate to $\rho = 0$, yielding a contradiction). Then, (63), (64), and $\lim_{\lambda\uparrow\hat{\lambda}} \Delta_A^*(\lambda) = b$ imply that for any given α_B, there exists $\lambda' \in (\lambda'',\hat{\lambda})$ such that $\Pi_a(\alpha_B|\lambda') > \Pi_a(A|\lambda')$, which is what we wanted to show.

Second, by Proposition 8, the only other equilibrium that can exist in the subgame $\eta = \alpha_B$ is the one where the incumbent accept the settlement $\eta = \alpha_B$ for all $\Delta \in (\hat{l}(\lambda), b)$. Recall that Proposition 8 implies that if this equilibrium exists for any $\lambda \in [1,\hat{\lambda})$ then it also exists for any $\lambda \in [1,\hat{\lambda})$, due to $\hat{l}(\lambda) < b_A$ and (53). Suppose this equilibrium is in play when $\eta = \alpha_B$. Also recall that in this equilibrium, $\Pi_a(\alpha_B|\lambda)$ is given by (56), which implies that for any $\lambda \in (\lambda'',\hat{\lambda})$, $\Pi_a(\alpha_B|\lambda) = \frac{1}{\lambda}\int_{b_A}^{b} \alpha_B\,(\Delta - b_A)\, f(\Delta)d\Delta > 0$. When combined with (64) and $\lim_{\lambda\uparrow\hat{\lambda}} \Delta_A^*(\lambda) = b$, this implies that for any given α_B, there exists $\lambda' \in (\lambda'', \hat{\lambda})$ such that $\Pi_a(\alpha_B|\lambda') > \Pi_a(A|\lambda')$, which is what we wanted to show.

Consider part (ii). Suppose that the activist demands action settlement for some $\tilde{\lambda} \in [1, \hat{\lambda})$. By part (i), there exists $\bar{\lambda} \in (0, \hat{\lambda})$ such that for any $\lambda \in [1, \hat{\lambda})$, the activist demands action settlement if $\lambda < \bar{\lambda}$ and board settlement if $\lambda > \bar{\lambda}$. Denote the expected shareholder value conditional on reaching settlement η by $\pi_{sh}(\eta|\lambda)$. Pick any $\alpha_B \in (0, 1]$ and $\lambda, \lambda' \in [1, \hat{\lambda})$. Then, it is sufficient to show that $\pi_{sh}(A|\lambda) > \pi_{sh}(\alpha_B|\lambda')$. Note that (53), Lemma 2, and Proposition 5 imply that

$$\pi_{sh}(A|\lambda) = \frac{1}{1 - F^{\lambda}(\Delta_A^*(\lambda))} \int_{\Delta_A^*(\lambda)}^{b} (\Delta - b_A)\, f^{\lambda}(\Delta) d\Delta = \frac{1}{1 - F(\Delta_A^*(\lambda))} \int_{\Delta_A^*(\lambda)}^{b} (\Delta - b_A)\, f(\Delta) d\Delta \tag{65}$$

where $\Delta_A^*(\lambda)$ is given by (22), and $\Delta_A^*(\lambda) \in (\max\{0, b_A\}, b)$ by Lemma 4 and Proposition 5. Note that the last equality in (65) follows from $F^{\lambda}(b) = 1$, $f^{\lambda}(\Delta) = \frac{1}{\lambda} f(\Delta)$ for $\Delta \in [b_A, b)$, and, which in turn imply $1 - F^{\lambda}(\Delta) = \frac{1}{\lambda}[1 - F(\Delta)]$ for all $\Delta \in [b_A, b)$.

With regard to $\pi_{sh}(\alpha_B|\lambda')$, just as in the proof of part (i), there are again two cases to consider depending on what equilibrium is chosen for the subgame $\eta = \alpha_B$. First, consider the equilibrium where the incumbent rejects the settlement $\eta = \alpha_B$ with some probability for some $\Delta \in (\hat{l}(\lambda'), b)$. If this equilibrium is in play, Proposition 7 implies that

$$\begin{aligned}
\pi_{sh}(\alpha_B|\lambda') &= \frac{1}{1 - F^{\lambda'}(\Delta_B^*(\alpha_B|\lambda')) + F^{\lambda'}(b_A)} \int_{\Delta_B^*(\alpha_B|\lambda')}^{b} \alpha_B\, (\Delta - b_A)\, f^{\lambda'}(\Delta) d\Delta \qquad (66)\\
&= \frac{1}{\frac{1}{\lambda'}[1 - F(\Delta_B^*(\alpha_B|\lambda'))] + F^{\lambda'}(b_A)} \int_{\Delta_B^*(\alpha_B|\lambda')}^{b} \alpha_B\, (\Delta - b_A)\, \frac{1}{\lambda'} f(\Delta) d\Delta
\end{aligned}$$

where $\Delta_B^*(\alpha_B|\lambda')$ is given by (35), and $\Delta_B^*(\lambda') \in (\max\{0, b_A\}, b)$ by Proposition 7. Note that $\Delta_B^*(\alpha_B|\lambda') = \Delta_B^*(\alpha_B|\lambda)$ by the discussion following (55) in the proof of part (i). Moreover, for all $\lambda'' \in [1, \hat{\lambda})$, $\hat{\Delta}_A(1|\lambda'') \geq \hat{\Delta}_B(1|\lambda'')$, and hence $\Delta_A^*(\lambda) \geq \Delta_B^*(\alpha_B|\lambda)$ by Proposition 5 since $c_b = 0$. Therefore, it must be that $\Delta_A^*(\lambda) \geq \Delta_B^*(\alpha_B|\lambda) = \Delta_B^*(\alpha_B|\lambda')$. When combined with (65), (66), and $F^{\lambda'}(b_A) > 0$ (since $\hat{l}(\lambda) < b_A$), this implies that $\pi_{sh}(A|\lambda) > \pi_{sh}(\alpha_B|\lambda)$, which is what we wanted to show.

Second, by Proposition 8, the only other equilibrium that can exist in the subgame $\eta = \alpha_B$ is the one where the incumbent accepts the settlement $\eta = \alpha_B$ for all $\Delta \in (\hat{l}(\lambda'), b)$. Suppose this equilibrium is in play when $\eta = \alpha_B$. Then, Proposition 8 implies that

$$\pi_{sh}(\alpha_B|\lambda') = \frac{1}{1 - F^{\lambda'}(b_A)} \int_{b_A}^{b} \alpha_B (\Delta - b_A) f^{\lambda'}(\Delta) d\Delta = \frac{1}{1 - F(b_A)} \int_{b_A}^{b} \alpha_B (\Delta - b_A) f(\Delta) d\Delta, \tag{67}$$

where the last equality follows from $F^{\lambda'}(b) = 1$, $f^{\lambda'}(\Delta) = \frac{1}{\lambda'} f(\Delta)$ for $\Delta \in [b_A, b)$, and, which in turn imply $1 - F^{\lambda'}(\Delta) = \frac{1}{\lambda'} [1 - F(\Delta)]$ for all $\Delta \in [b_A, b)$. Since $\Delta_A^*(\lambda) \in (\max\{0, b_A\}, b)$, (65), (67), and $F(b_A) > 0$ (since $l < b_A$) imply that $\pi_{sh}(A|\lambda) > \pi_{sh}(\alpha_B|\lambda)$, which is again what we wanted to show. ■

Proof of Proposition 4. Proposition 4 follows from Propositions 5, 7, and 10. ■

Corollary 7 *Suppose that $c_b = 0$, $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$, $E[\Delta|\Delta \geq b_A] > 0$, and the activist has demanded board settlement that gives him decision authority with probability α_B.*

(i) There exists $\hat{\alpha} \in (0, 1]$ such that if $\alpha_B \geq \hat{\alpha}$, then the expected shareholder value is always weakly larger than in the equilibrium where he the activist demanded action settlement, and strictly larger if $l < b_A$, $\rho_A^* < 1$, and $\sigma_B^*(1) = 1$.

(ii) If the incumbent accepts the settlement and $b_A > l$, the activist does not always implement the project even if he obtains decision authority. However, if the incumbent rejects the settlement, then the activist always implements the project if he obtains decision authority upon launching and winning a proxy fight.

Proof. Consider part (i). Note that action settlement is denoted with $\eta = A$, while board settlement that gives the activist decision authority with probability $\alpha_B = 1$ with $\eta = 1$. Denote the expected shareholder value by $\Pi_{sh}(\eta)$ if the activist has demanded η. Suppose that $c_b = 0$, $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$, $E[\Delta|\Delta \geq b_A] > 0$. Then, by Proposition 5,

$$\Pi_{sh}(A) = \int_{b_A}^{\Delta_A^*} \rho_A^* \sigma_A^* \alpha_P \Delta dF(\Delta) + \int_{\Delta_A^*}^{b} \Delta dF(\Delta), \tag{68}$$

where $\Delta_A^* \in (\max\{0, b_A\}, b)$ by Lemma 4 and Proposition 5. With regard to $\Pi_{sh}(1)$, there are two cases to consider depending on what equilibrium is chosen for the subgame $\eta = 1$.

First, consider the equilibrium where the incumbent rejects the settlement $\eta = 1$ with some probability for some $\Delta \in (\hat{l}(\lambda'), b)$. If this equilibrium is in play, Proposition 7 implies that

$$\Pi_{sh}(1) = \int_{b_A}^{\Delta_B^*(1)} \rho_B^* \sigma_B^*(1) \alpha_P \Delta dF(\Delta) + \int_{\Delta_B^*(1)}^{b} \Delta dF(\Delta), \tag{69}$$

where $\Delta_B^*(1) \in (\max\{0, b_A\}, b)$ by Proposition 7. Moreover, Propositions 5 and 7, $\hat{\Delta}_B(1) \leq \hat{\Delta}_A(1)$, and $c_b = 0$ imply that $\Delta_B^*(1) \leq \Delta_A^*$, $\rho_B^*(1) \geq \rho_A^*$, and $\sigma_B^*(1) \leq \sigma_A^*$. By (16), $\int_{b_A}^{\check{\Delta}} \Delta dF(\Delta) = 0$ and hence $b_A \leq \check{\Delta} \leq \Delta_B^*(1) \leq \Delta_A^* < b$ implies that (69) reduces to

$$\Pi_{sh}(1) = \int_{\check{\Delta}}^{\Delta_B^*(1)} \rho_B^* \sigma_B^*(1) \alpha_P \Delta dF(\Delta) + \int_{\Delta_B^*(1)}^{b} \Delta dF(\Delta), \tag{70}$$

while (68) reduces to

$$\Pi_{sh}(A) = \int_{\check{\Delta}}^{\Delta_A^*} \rho_A^* \sigma_A^* \alpha_P \Delta dF(\Delta) + \int_{\Delta_A^*}^{b} \Delta dF(\Delta), \tag{71}$$

Since $\sigma_B^*(1) \in (0, 1]$, there are two cases to consider. First, suppose that $\sigma_B^*(1) = 1$. Then, $\sigma_B^*(1) \leq \sigma_A^*$ implies that $\sigma_A^* = 1$, and together with $\rho_B^*(1) \geq \rho_A^*$, $0 < \Delta_B^*(1) \leq \Delta_A^*$, (70), and (71) imply that $\Pi_{sh}(1) \geq \Pi_{sh}(A)$. Moreover, if $l < b_A$ and $\rho_A^* < 1$, then $\hat{\Delta}_B(1) < \hat{\Delta}_A(1) = \Delta_A^*(1)$. Therefore, if in addition $\sigma_B^*(1) = 1$, then Propositions 5 and 7, $c_b = 0$, and $\sigma_B^*(1) \leq \sigma_A^*$ imply that $\sigma_A^* = 1$ and $0 < \check{\Delta} \leq \Delta_B^*(1) < \Delta_A^*$. Hence, (70), (71), and $\check{\Delta} \geq 0$ imply that $\Pi_{sh}(1) > \Pi_{sh}(A)$. Second, suppose that $\sigma_B^*(1) \in (0, 1)$. Then, Proposition 7 implies that $\Delta_B^*(1) = \check{\Delta}$. Therefore, again $0 < \Delta_B^*(1) \leq \Delta_A^*$, (70), and (71) imply that $\Pi_{sh}(1) \geq \Pi_{sh}(A)$.

Second, by Proposition 8, the only other equilibrium that can exist in the subgame where $\eta = 1$ is the one where the incumbent accepts the settlement $\eta = 1$ for all $\Delta \in (\hat{l}(\lambda'), b)$. Suppose this equilibrium is in play when $\eta = 1$. Then, Proposition 8 implies that $\Pi_{sh}(1) = \int_{b_A}^{b} \Delta dF(\Delta)$. Again, $\int_{b_A}^{\check{\Delta}} \Delta dF(\Delta) = 0$ and $b_A \leq \check{\Delta} < b$ imply that $\Pi_{sh}(1) = \int_{\check{\Delta}}^{b} \Delta dF(\Delta)$. This combined with (71) $\Pi_{sh}(1) \geq \Pi_{sh}(A)$. Moreover, if $l < b_A$, $\rho_A^* < 1$, and $\sigma_B^*(1) = 1$, then $\check{\Delta} < \Delta_A^*$ by the previous step (i.e., as discussed after (71)) and hence $\Pi_{sh}(1) > \Pi_{sh}(A)$. ■

Corollary 8 *Suppose that* $c_b \in [0, \frac{c_P}{\frac{1}{\gamma} - \alpha_p})$, $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$, $E[\Delta | \Delta \geq b_A] > 0$. *Then, fixing the settlement demanded by the activist, expected shareholder value weakly in-*

creases as κ decreases.

Proof. Note that action settlement is denoted with $\eta = A$, while board settlement that gives the activist decision authority with probability α_B with $\eta = \alpha_B$. Denote the expected shareholder value by $\Pi_{sh}(\eta)$ if the activist has demanded η. Suppose that $c_b \in [0, \frac{c_P}{\frac{1}{\gamma} - \alpha_p})$, $\kappa < \alpha_P E\left[\max\{0, \Delta - b_A\}\right]$, $E\left[\Delta | \Delta \geq b_A\right] > 0$. Then, by Proposition 5,

$$\Pi_{sh}(A|\kappa) = \int_{b_A}^{\Delta_A^*(\kappa)} \rho_A^*(\kappa)\sigma_A^*(\kappa)\alpha_P \Delta dF(\Delta) + \int_{\Delta_A^*(\kappa)}^{b} \Delta dF(\Delta), \tag{72}$$

where $\Delta_A^*(\kappa) \in (\max\{0, b_A\}, b)$ by Lemma 4 and Proposition 5. There are two cases to consider. First, suppose that $\check{\Delta} = \Delta_A^*(\kappa)$. Then, since $\int_{b_A}^{\check{\Delta}} \Delta dF(\Delta) = 0$ by (16), (72) reduces to $\Pi_{sh}(A|\kappa) = \int_{\check{\Delta}}^{b} \Delta dF(\Delta)$. Moreover, (14) implies that $\hat{\Delta}_A(\kappa') < \hat{\Delta}_A(\kappa)$ for all $\kappa' < \kappa$, and hence Proposition 5 implies that $\check{\Delta} = \Delta_A^*(\kappa)$ for all $\kappa' < \kappa$. Therefore, $\Pi_{sh}(A|\kappa) = \Pi_{sh}(A|\kappa')$ for all $\kappa' < \kappa$.

Second, suppose that $\check{\Delta} < \Delta_A^*(\kappa)$. Then, by Proposition 5, there exists $\kappa'' \in [0, \kappa)$ such that $\check{\Delta} < \Delta_A^*(\kappa') \leq \Delta_A^*(\kappa)$ for all $\kappa' \in (\kappa'', \kappa)$, and $\check{\Delta} = \Delta_A^*(\kappa')$ for all $\kappa' \in (0, \kappa'']$. Note that by the previous step, $\Pi_{sh}(A|\kappa') = \Pi_{sh}(A|\kappa'')$ for all $\kappa' \in (0, \kappa'']$. Since Proposition 5 implies that $\Pi_{sh}(A|\kappa)$ is continuous with respect to κ for all $\kappa < \alpha_P E\left[\max\{0, \Delta - b_A\}\right]$, it remains to show that $\Pi_{sh}(A|\kappa') \geq \Pi_{sh}(A|\kappa)$ for all $\kappa' \in (\kappa'', \kappa)$. Pick any $\kappa' \in (\kappa'', \kappa)$. Note that $\check{\Delta} < \Delta_A^*(\kappa') \leq \Delta_A^*(\kappa)$ and Proposition 5 together imply that $\sigma_A^*(\kappa) = \sigma_A^*(\kappa') = 1$. Moreover, Proposition 5 implies that $\rho_A^*(\kappa') > \rho_A^*(\kappa)$. Therefore, (72) implies that $\Pi_{sh}(A|\kappa') \geq \Pi_{sh}(A|\kappa)$, which is what we wanted to show.

Next, consider $\eta = \alpha_B$ for any $\alpha_B \in (0, 1]$. There are two cases to consider depending on what equilibrium is chosen for the subgame $\eta = \alpha_B$. First, consider the equilibrium where the incumbent accepts the settlement for all $\Delta \in (l, b)$. Note that by Proposition 8, if this equilibrium exists for κ, then it exists for all $\kappa' \in (0, \kappa)$, and $\Pi_{sh}(\alpha_B|\kappa) = \alpha_B \int_{b_A}^{b} (\Delta - b_A) f(\Delta) d\Delta$, which does not change w.r.t κ.[41] Second, by Proposition 7, the only the other equilibrium to consider is the one where the incumbent rejects the settlement with positive probability for some $\Delta \in (l, b)$, which always exists. The proof for this case is very similar to proof when

[41] I assume that if for any $\kappa \in \{\kappa_1, \kappa_2\}$ the equilibrium described in Proposition 8 is selected in the subgame that the activist demands board settlement that gives him decision authority with probability α_B, then the same type of equilibrium is selected in this subgame for all $\kappa \in \{\kappa_1, \kappa_2\}$ if it exists.

$\eta = A$. Specifically, by Proposition 7

$$\Pi_{sh}(\alpha_B|\kappa) = \int_{b_A}^{\Delta_B^*(\alpha_B|\kappa)} \rho_B^*(\kappa)\sigma_B^*(\alpha_B|\kappa)\alpha_P \Delta dF(\Delta) + \alpha_B \int_{\Delta_B^*(\alpha_B|\kappa)}^{b} \Delta dF(\Delta), \tag{73}$$

where $\Delta_B^*(\alpha_B|\kappa) \in (\max\{0, b_A\}, b)$ by Proposition 7. There are two cases to consider. First, suppose that $\check{\Delta} = \Delta_B^*(\alpha_B|\kappa)$. Then, since $\int_{b_A}^{\check{\Delta}} \Delta dF(\Delta) = 0$ by (16), (73) reduces to $\Pi_{sh}(\alpha_B|\kappa) = \alpha_B \int_{\check{\Delta}}^{b} \Delta dF(\Delta)$. Moreover, (30) implies that $\hat{\Delta}_B(\alpha_B|\kappa') < \hat{\Delta}_B(\kappa)$ for all $\kappa' < \kappa$, and hence Proposition 7 implies that $\check{\Delta} = \Delta_B^*(\alpha_B|\kappa)$ for all $\kappa' < \kappa$. Therefore, $\Pi_{sh}(\alpha_B|\kappa) = \Pi_{sh}(A|\kappa')$ for all $\kappa' < \kappa$.

Second, suppose that $\check{\Delta} < \Delta_B^*(\alpha_B|\kappa)$. Then, by Proposition 7, there exists $\kappa'' \in [0, \kappa)$ such that $\check{\Delta} < \Delta_B^*(\alpha_B|\kappa') \leq \Delta_B^*(\alpha_B|\kappa)$ for all $\kappa' \in (\kappa'', \kappa)$, and $\check{\Delta} = \Delta_B^*(\alpha_B|\kappa')$ for all $\kappa' \in (0, \kappa'']$. Note that by the previous step, $\Pi_{sh}(\alpha_B|\kappa') = \Pi_{sh}(\alpha_B|\kappa'')$ for all $\kappa' \in (0, \kappa'']$. Since Proposition 7 implies that $\Pi_{sh}(\alpha_B|\kappa)$ is continuous with respect to κ for all $\kappa < \alpha_P E[\max\{0, \Delta - b_A\}]$, it remains to show that $\Pi_{sh}(\alpha_B|\kappa') \geq \Pi_{sh}(\alpha_B|\kappa)$ for all $\kappa' \in (\kappa'', \kappa)$. Pick any $\kappa' \in (\kappa'', \kappa)$. Note that $\check{\Delta} < \Delta_B^*(\kappa') \leq \Delta_B^*(\kappa)$ and Proposition 7 together imply that $\sigma_B^*(\alpha_B|\kappa) = \sigma_B^*(\alpha_B|\kappa') = 1$. Moreover, Proposition 7 implies that $\rho_B^*(\alpha_B|\kappa') > \rho_B^*(\alpha_B|\kappa)$. Therefore, (73) implies that $\Pi_{sh}(\alpha_B|\kappa') \geq \Pi_{sh}(\alpha_B|\kappa)$, which is what we wanted to show. ■