Efficiency Wages, Unemployment, and Environmental Policy

Abstract

We study the incidence of environmental taxes and their impact on unemployment in an analytical general equilibrium efficiency wage model. We find closed-form solutions for the effect of a pollution tax on unemployment, factor prices, and output prices, and we identify and isolate different channels through which these general equilibrium effects arise. A new effect arises from the efficiency wage specification; this effect depends on the form of the workers' effort function. Numerical simulations further illustrate our results and show that this new efficiency wage effect can fully offset the sources-side incidence results found in models that omit it.

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I. Introduction

The effects that environmental policies may have on labor markets, and in particular whether and to what extent they kill jobs or create jobs, is of utmost importance to policymakers. Much popular aversion to environmental regulation comes from its perceived negative impact on jobs. Additionally, other distributional impacts of policy, like the sources-side and uses-side incidence, can depend on frictions in the labor market that yield unemployment. It is important for policymakers to understand the effect of environmental policies on unemployment and on both factor and output prices.

There are several ways to go about addressing the very general question of how environmental policies affect labor markets and unemployment. Many papers empirically estimate the impact of specific environmental policies on employment, including Greenstone (2002) and Colmer et al. (2018). Other papers use computable general equilibrium (CGE) models to quantify the large-scale effects that policies like an economy-wide carbon tax might have, including Hafstead et al. (2018). A third approach uses analytical general equilibrium modeling, which can shed light on the mechanisms behind the effects that can be quantified through empirical or CGE models. Both Hafstead and Williams (2018) and Aubert and Chiroleu-Assouline (2019) introduce pollution policy and unemployment resulting from labor search frictions into an analytical general equilibrium model.

The purpose of this paper is to study the effect of pollution taxes on unemployment and incidence using an analytical general equilibrium model where unemployment is endogenously generated via efficiency wages. Workers' effort is a function of the real wage and the economy's unemployment level. Pollution is modeled as a production input along with capital and labor, allowing for fully general forms of substitution among these three factors. We solve the model to find closed-form analytic solutions for the general equilibrium responses to a change of pollution tax rate, including expressions for changes in the unemployment rate, factor prices, output prices, and the amount of pollution. The model allows us to clarify the impact of differential factor intensities, substitution effects, and output effects by looking at various special cases. Lastly, we conduct numerical simulations using calibrated parameter values.

Our modeling approach dates back to the canonical tax incidence modeling of Harberger (1962). Like Agell and Lundborg (1992) and Rapanos (2006), our paper adds an efficiency wage theory of unemployment to the model, though those papers do not model pollution. Like Fullerton and Heutel (2007), our paper adds pollution and pollution taxes to the model, though that paper does not model unemployment.[1] We incorporate both efficiency wages and environmental policy into a Harberger-style analytical general equilibrium tax incidence model. Our paper is most similar to Hafstead and Williams (2018) and Aubert and Chiroleu-Assouline (2019), which both also model environmental policy and unemployment in an analytical general equilibrium setting. However, in both of those papers, unemployment arises from Diamond-Mortensen-Pissarides-style search frictions (Pissarides 2000). In our paper, unemployment arises from efficiency wage theory (Akerlof 1982, Shapiro and Stiglitz 1984). Furthermore, Hafstead and Williams (2018) do not provide analytical, closed-form solutions, just numerical simulations, and neither Hafstead and Williams (2018) nor Aubert and Chiroleu-Assouline (2019) include capital in their model.

Our theoretical results add new insights to the tax incidence literature. We identify effects that have been found in previous studies, like the output and substitution effects. These

[1] Other papers that use a similar methodology to incorporate pollution policy into analytical general equilibrium modeling include Fullerton and Heutel (2010), Gonzalez (2012), Fullerton and Monti (2013), Dissou and Siddiqui (2014), and Fullerton and Ta (2019). Fullerton and Muehlegger (2019) review the literature on the incidence of environmental regulations.

effects differ, though, with endogenous unemployment. For example, a substitution effect exists such that the pollution tax increases unemployment more so when capital is a better substitute for pollution than is labor. Along with these standard effects, we identify a new effect that we call the efficiency wage effect. The magnitude and direction of this effect can importantly depend on the form of the workers' effort function. Generally, the less elastic the workers' marginal effort response to real wage is, the less burden labor bears, and the smaller increase in unemployment. When a Cobb-Douglas form is applied to the effort function, we show that the crucial parameters are the elasticities of effort with respect to the real wage and to unemployment. When effort responds more strongly to the real wage, then the magnitude of the efficiency wage effect is higher, but when effort responds more strongly to unemployment, then the magnitude of the efficiency wage is smaller.

The calibrated numerical simulation results provide further insights into these effects. The disproportionate burden of the tax on labor from substitution effects is offset by the disproportionate burden on capital from the efficiency wage effect. Ignoring this effect thus gets the sign of the sources-side incidence wrong. The effect of environmental policy on unemployment is mainly driven by a substitution effect from the larger, untaxed clean sector, rather than substitution within the smaller, taxed dirty sector. Both the analytical and the numerical results highlight the important role of efficiency wage effect and the form of the effort function in the analysis of pollution tax incidence and unemployment effects.

The paper is organized as follows. Section 2 presents the model and derives a system of linearized equations. Section 3 offers a general solution and simplifies it in several special cases to assist in interpreting the results. Section 4 calibrates and numerically simulates the model. The last section concludes.

II. Model

Our model is a simple two-sector, two-factor incidence model, in the spirit of Harberger (1962), with the addition of involuntary unemployment through an efficiency wage as in Agell and Lundborg (1992) and Rapanos (2006), and with the addition of pollution as in Fullerton and Heutel (2007, 2010). We consider a competitive two-sector economy using two factors of production: capital and labor. Both factors are perfectly mobile between sectors. A third variable input is pollution, Z, which is only used in production of one of the goods (the "dirty" good). The constant returns to scale production functions are:

$$X = X(K_X, E_X)$$

$$Y = Y(K_Y, E_Y, Z)$$

where X is the “clean” good, Y is the “dirty” good, K_X and K_Y are the capital used in each sector, and E_X and E_Y are the effective labor, in efficiency units, used in each sector.

The effective labor in each sector is defined as the actual amount of labor L times the effort level e:

$$E_X = e\left(\frac{w}{P}, U\right) \cdot L_X$$

$$E_Y = e\left(\frac{w}{P}, U\right) \cdot L_Y$$

where $e(\frac{w}{P}, U)$, the effort level of a representative worker, depends on the real wage rate $\frac{w}{P}$, and on the level of unemployment U.

This effort function is how the efficiency wage theory of unemployment is incorporated into this model. Unlike in structural models of efficiency wages, in which effort is an endogenously-determined optimal response of workers given the possibility of termination if caught shirking (Shapiro and Stiglitz 1984) or norms of fairness (Akerlof 1982), here the effort function is given as a reduced-form relationship between the wage, unemployment, and the level of effort. Our reduced-form effort function is identical to that of Rapanos (2006). The reduced-form effort function in Agell and Lundborg (1992) is slightly different; effort is a function of the relative wages across industries and the ratio of the wage to capital rental rate.

Structural efficiency wage models predict that effort is positively related to the real wage ($\frac{w}{P}$) and to the economy-wide level of unemployment, so we impose that the first derivatives e_1 and e_2 are positive.[2] The effort level is identical across the two sectors (since neither the real wage nor unemployment are sector-specific). L_X and L_Y are the labor used in each sector in terms of the number of workers. Linearizing the two equations defining effective labor gives us:

$$\widehat{E_X} = \hat{e} + \widehat{L_X} \tag{1}$$

$$\widehat{E_Y} = \hat{e} + \widehat{L_Y} \tag{2}$$

We adopt the "hat" notation where a variable with a hat represents a proportional change in the variable. That is, $\widehat{E_X} \equiv dE_X/E_X$, and likewise for the other variables.

Both of the representative firms face the same effort function e, and they set their wages w to minimize the effective wage cost per worker $v \equiv w/e$. Formally, the optimization problem for the representative firm is:

$$\min_{w} v = \frac{w}{e\left(\frac{w}{P}, U\right)}$$

The first-order condition is

$$e - e_1 \frac{w}{P} = 0$$

[2] Empirical support for this reduced-form relationship is found in Raff and Summers (1987) and Cappelli and Chauvin (1991).

where e_1 is the first derivative of the effort function with respect to the real wage. This condition can be written as $\varepsilon_1 \equiv \frac{e_1 w}{ep} = 1$, meaning that the wage is set so that the elasticity of effort with respect to the wage is one. Totally differentiating this first-order condition, and employing the assumption that $e_{12} = 0$, we obtain:

$$e_2 dU = \frac{e_{11} w^2}{P^2}\left(\frac{dw}{w} - \frac{dP}{P}\right)$$

which can be rewritten as

$$\hat{U} = \frac{e_{11} w^2}{e_2 U P^2}\left(\hat{w} - \hat{P}\right) = \frac{\frac{e_{11}}{e_1} \cdot \frac{w}{P}}{\frac{e_2}{e} \cdot U}\left(\hat{w} - \hat{P}\right)$$

$$\hat{U} = \frac{\varepsilon_{11}}{\varepsilon_2}\left(\hat{w} - \hat{P}\right) \qquad (3)$$

where $\varepsilon_{11} \equiv \left(\frac{e_{11}}{e_1}\right)\left(\frac{w}{P}\right)$, and $\varepsilon_2 \equiv \left(\frac{e_2}{e}\right) U$. Since $e_2 > 0$, we also have $\varepsilon_2 > 0$, which is the elasticity of effort with respect to unemployment. We assume concavity of the effort function with respect to the real wage w/P to ensure an interior solution to the minimization problem, so $e_{11} < 0$, which implies that $\varepsilon_{11} < 0$. This parameter, ε_{11}, is important throughout the analysis and arises in the closed-form solutions presented below. It is a measure of the concavity of the effort function with respect to the real wage. If it is close to zero, the effort function is close to linear in the real wage. If it is large in absolute value, then the marginal effort with respect to the real wage (e_1) declines quickly as the wage increases.[3]

Totally differentiating the effort function $e = e(\frac{w}{P}, U)$ obtains

$$\hat{e} = \hat{w} - \hat{P} + \varepsilon_2 \hat{U} \qquad (4)$$

From the definition of effective wage v, we have

[3] Rapanos (2006) describes the parameter ε_{11} as "the rate at which workers get satisfied with real wages." (p. 481).

$$\hat{v} = \widehat{w} - \hat{e} \tag{5}$$

The first five equations of our model are identical to those in the efficiency wage model of Rapanos (2006).

The resource constraints are:

$$K_X + K_Y = \bar{K}$$

$$L_X + L_Y = \bar{L} - U$$

where $\bar{K}$ and $\bar{L}$ are the fixed total amounts of capital and labor in the economy. All capital is fully employed, while labor faces a level of unemployment U. Totally differentiating the resource constraints (noting that $\bar{K}$ and $\bar{L}$ remain fixed) yields

$$\widehat{K_X} \cdot \lambda_{KX} + \widehat{K_Y} \cdot \lambda_{KY} = 0 \tag{6}$$

$$\widehat{L_X} \cdot \lambda_{LX} + \widehat{L_Y} \cdot \lambda_{LY} = -\widehat{U} \cdot \lambda_{LU} \tag{7}$$

where λ_{ij} denotes sector j's share of factor i ($\lambda_{KX} = \frac{K_X}{\bar{K}}$). λ_{LU} denotes the unemployment rate ($\lambda_{LU} = \frac{U}{\bar{L}}$). Pollution Z has no equivalent resource constraint. As in Fullerton and Heutel (2007), we start with a preexisting positive tax τ_Z on pollution.

When modeling producer behavior, we consider the producers responding to the price and quantity of effective labor rather than actual labor. The price of a unit of effective labor is v, and the quantities are E_X and E_Y. Producers of X can substitute between factors in response to changes in the factor prices $p_K \equiv r(1 + \tau_K)$ and $p_E \equiv v(1 + \tau_E)$, where τ_K and τ_E are the *ad valorem* taxes on capital and effective labor. We will only consider a change in the pollution tax, not in any of the other taxes, so, $\widehat{p_K} = \hat{r}$ and $\widehat{p_E} = \hat{v}$. The elasticity of substitution in production σ_X is defined to capture this response to factor price changes:

$$\widehat{K_X} - \widehat{E_X} = \sigma_X(\hat{v} - \hat{r}) \tag{8}$$

where σ_X is defined to be positive.

Producers of Y use three inputs: capital, effective labor, and pollution. Firms face no market price for pollution, just a tax on per unit of pollution, so $p_Z = \tau_z$ and $\widehat{p_Z} = \widehat{\tau_z}$. Firm Y's behavior can be modeled using Allen elasticities of substitution between inputs i and j, e_{ij}. This elasticity is positive for two substitutes and negative for two complements, and the own price Allen elasticity must always be negative. We assume that cross-price Allen elasticities are always positive, so that any two inputs are substitutes for each other. The magnitudes of the Allen elasticities determine which inputs are better substitutes. For example, if $e_{KZ} > e_{EZ}$, then capital is a better substitute for pollution than is labor.

Following Fullerton and Heutel (2007), we arrive at two equations describing the dirty sector's production decisions:

$$\widehat{K_Y} - \hat{Z} = \theta_{YK}(e_{KK} - e_{ZK})\hat{r} + \theta_{YE}(e_{KE} - e_{ZE})\hat{v} + \theta_{YZ}(e_{KZ} - e_{ZZ})\widehat{\tau_Z} \tag{9}$$

$$\widehat{E_Y} - \hat{Z} = \theta_{YK}(e_{EK} - e_{ZK})\hat{r} + \theta_{YE}(e_{EE} - e_{ZE})\hat{v} + \theta_{YZ}(e_{EZ} - e_{ZZ})\widehat{\tau_Z} \tag{10}$$

Here $\theta_{YK} \equiv \frac{r(1+\tau_K)K_Y}{p_Y \cdot Y}$, $\theta_{YE} \equiv \frac{v(1+\tau_E)E_Y}{p_Y \cdot Y}$ and $\theta_{YZ} \equiv \frac{\tau_Z \cdot Z}{p_Y \cdot Y}$ are the share of sales revenue from Y that is paid to capital, to effective labor, and to pollution (through the tax), respectively. Equations (9) and (10) show how a change in any of the input prices affects the relative demand for the three inputs. The change in demand is a function of both initial shares (the θs) and the Allen elasticities of substitution.

Using the assumptions of perfect competition and constant returns to scale, we get

$$\widehat{p_X} + \hat{X} = \theta_{XK}(\hat{r} + \widehat{K_X}) + \theta_{XE}(\hat{v} + \widehat{E_X}) \tag{11}$$

$$\widehat{p_Y} + \hat{Y} = \theta_{YK}\left(\hat{r} + \widehat{K_Y}\right) + \theta_{YE}\left(\hat{v} + \widehat{E_Y}\right) + \theta_{YZ}\left(\hat{Z} + \widehat{\tau_Z}\right) \tag{12}$$

Define θ_{XK} and θ_{XE} similarly to θ_{YK}. (Note that $\theta_{XK} + \theta_{XE} = 1$ and $\theta_{YK} + \theta_{YE} + \theta_{YZ} = 1$.) Totally differentiate each sector's production function and substitute in the conditions from the perfect competition assumption to get

$$\hat{X} = \theta_{XK}\widehat{K_X} + \theta_{XE}\widehat{E_X} \tag{13}$$

$$\hat{Y} = \theta_{YK}\widehat{K_Y} + \theta_{YE}\widehat{E_Y} + \theta_{YZ}\hat{Z} \tag{14}$$

The details of the derivation of equations 9 through 14 can be found in Fullerton and Heutel (2007, Appendix A).

On the consumer side, consumer preferences are modeled using σ_u, the elasticity of substitution between goods X and Y. The definition of this elasticity yields

$$\hat{X} - \hat{Y} = \sigma_u(\widehat{p_Y} - \widehat{p_X}) \tag{15}$$

Lastly, the price index P, which appears in the effort function, is defined to equal a weighted average of the output prices of the two goods, i.e. $P = {p_X}^{\eta} \cdot {p_Y}^{1-\eta}, (\eta < 1)$. Then the change in the price index can be written as

$$\hat{P} = \eta\widehat{p_X} + (1-\eta)\widehat{p_Y} \tag{16}$$

The full model is equations (1) through (16). It contains just one exogenous policy variable ($\hat{\tau}_Z$), and 17 endogenous variables. To solve it, we impose a normalization assumption by assuming that the price index P is the numeraire and unchanged, so that $\hat{P} = 0$. Dropping $\hat{P}$ from the model thus yields 16 equations with 16 unknowns ($\widehat{K_X}, \widehat{K_Y}, \widehat{E_X}, \widehat{E_Y}, \widehat{L_X}, \widehat{L_Y}, \hat{Z}, \hat{U}, \hat{e}, \hat{w}, \hat{v}, \widehat{p_X}, \widehat{p_Y}, \hat{r},\ \hat{X},\ \hat{Y}$). The model is solved with successive substitution, as described in the appendix.

III. Solution

Our focus is on incidence and unemployment effects, so we are most interested in solutions for changes in factor prices ($\hat{w}$ and $\hat{r}$), output prices ($\widehat{p_X}$ and $\widehat{p_Y}$), and unemployment $\hat{U}$. The appendix describes the solution method.

We present three closed-form solutions. The first is $\widehat{U}$, the change in unemployment or the change in the unemployment rate.[4] The second is $\widehat{w} - \hat{r}$, which represents the sources-side incidence, i.e. the relative burden on labor versus capital. If $\widehat{w} - \hat{r}$ is positive, then the wage increases more than the rental rate does (or decreases less), so the burden of the tax falls relatively more on capital than on labor. The third is $\widehat{p_Y} - \widehat{p_X}$, which represents the uses-side incidence, i.e. the relative burden on consumers of the dirty good versus consumers of the clean good. If $\widehat{p_Y} - \widehat{p_X}$ is positive, then the difference in these prices increases, so the burden of the tax falls relatively more on consumers of the dirty good versus consumers of the clean good. These solutions are:

$$\widehat{U} = \frac{\varepsilon_{11}}{\varepsilon_2}\frac{\theta_{YZ}}{D}\left\{\begin{array}{c} A[\theta_{YK}(1-\eta)(e_{KK}-e_{ZK}) - \eta_K(e_{KZ}-e_{ZZ})] \\ +B[-\theta_{YK}(1-\eta)(e_{KE}-e_{ZK}) + \eta_K(e_{EZ}-e_{ZZ})] \\ -\sigma_u\theta_{XK}(\gamma_L-\gamma_K) - C\sigma_X(1-\eta) \end{array}\right\}\widehat{\tau_Z} \qquad (17)$$

$$\widehat{w} - \hat{r} =$$
$$\frac{\theta_{YZ}}{D}\left\{\begin{array}{c} A[\theta_{YK}(1-\eta)(e_{KK}-e_{ZK}) - (\eta_K - \varepsilon_{11}\eta_E)(e_{KZ}-e_{ZZ}) - \theta_{YE}(1-\eta)\varepsilon_{11}(e_{KE}-e_{ZE})] \\ +B[-\theta_{YK}(1-\eta)(e_{KE}-e_{ZK}) + (\eta_K-\varepsilon_{11}\eta_E)(e_{EZ}-e_{ZZ}) + \theta_{YE}(1-\eta)\varepsilon_{11}(e_{EE}-e_{ZE})] \\ +\sigma_u(\gamma_L-\gamma_K)(-\theta_{XK}+\varepsilon_{11}\theta_{XE}) - C\sigma_X(1-\eta)(1+\varepsilon_{11}) + (1-\eta)M \end{array}\right\}\widehat{\tau_Z} \qquad (18)$$

$$\widehat{p_Y} - \widehat{p_X}$$

$$= \frac{\varepsilon_{11}\theta_{YZ}}{D}\left\{\begin{array}{c} \theta_{XE}\theta_{YK}[A(e_{KK}-e_{ZK}+e_{ZZ}-e_{KZ}) + B(e_{ZK}-e_{KE}+e_{EZ}-e_{ZZ})] \\ +\theta_{XK}\theta_{YE}[A(e_{ZE}-e_{KE}+e_{KZ}-e_{ZZ}) + B(e_{EE}-e_{ZE}+e_{ZZ}-e_{EZ})] \\ -C\sigma_X + M\dfrac{\theta_{XK}}{\varepsilon_{11}} \end{array}\right\}\widehat{\tau_Z} \qquad (19)$$

These solutions use the following definitions and simplifications: $\gamma_L \equiv \frac{\lambda_{LY}}{\lambda_{LX}}, \gamma_K \equiv \frac{\lambda_{KY}}{\lambda_{KX}}$, $\mathrm{A} \equiv \gamma_L\beta_K + \gamma_K(\beta_L + \theta_{YZ})$, $\mathrm{B} \equiv \gamma_K\beta_L + \gamma_L(\beta_K + \theta_{YZ})$, $C \equiv \beta_K + \beta_L + \theta_{YZ}$, where $\beta_K \equiv \theta_{XK}\gamma_K + \theta_{YK}$ and $\beta_L \equiv \theta_{XE}\gamma_L + \theta_{YE}$, $\eta_K \equiv \theta_{XK}\eta + \theta_{YK}(1-\eta)$, $\eta_E \equiv \theta_{XE}\eta + \theta_{YE}(1-\eta)$, and $D \equiv$

[4] U is the level of unemployment, so $\widehat{U}$ is defined as the percentage change in the level of unemployment. But since the total labor force is fixed, $\widehat{U}$ is also the percent change in the unemployment rate. It is not a percentage point change. For example, if the baseline unemployment rate is 4%, then $\widehat{U} = 0.1$ is a ten-percent increase in that baseline rate, to 4.4%.

$\varepsilon_{11}\Big\{A[\theta_{YK}(e_{KK}-e_{ZK})\eta_E-\theta_{YE}(e_{KE}-e_{ZE})\eta_K]+B[-\theta_{YK}(e_{KE}-e_{ZK})\eta_E+\theta_{YE}(e_{EE}-$

$e_{ZE})\eta_K]-\sigma_u(\gamma_L-\gamma_K)(\theta_{XK}\theta_{YE}-\theta_{XE}\theta_{YK})-M\frac{\eta_K}{\varepsilon_{11}}-C\sigma_X(\eta_K+\eta_E)\Big\}$, where $M\equiv$

$(1+\gamma_K)\left[(1+\gamma_L)(1+\varepsilon_{11})-\frac{\lambda_{LU}}{\lambda_{LX}}\frac{\varepsilon_{11}}{\varepsilon_2}\right]$.

All three expressions linear functions of the change of the pollution tax $\widehat{\tau_Z}$, since the model is linearized and $\widehat{\tau_Z}$ is the only exogenous policy variable. These expressions are long and difficult to interpret, but nonetheless they can be decomposed into several effects that can be separately analyzed.[5] In the following subsections we decompose each expression into terms representing several intuitive effects, in the spirit of Mieszkowski (1967): an output effect, two substitution effects (one from the clean sector and one from the dirty sector), and a new effect that we call the efficiency wage effect. The output effect is represented by the terms that include the elasticity of substitution in utility, σ_u. The clean sector substitution effect is represented by the terms that include the elasticity of substitution in production in the clean sector, σ_X. The dirty sector substitution effect is represented by all of the terms that include the Allen elasticities of substitution in the dirty sector, e_{ij}. Finally, the efficiency wage effect is represented by the terms that includes M.[6]

III.A. Efficiency wage effect

We begin by interpreting a new effect that we identify, which we call the efficiency wage effect. This of course is absent in previous models without an efficiency wage or endogenous

[5] Throughout the analysis below, we assume that the denominator D is positive, which is true under the following conditions: $e_{KK}<e_{ZK}<e_{EK}$, $e_{EE}<e_{ZE}<e_{KE}$, $\varepsilon_{11}>-1$ and $(\gamma_L-\gamma_K)(\theta_{XK}\theta_{YE}-\theta_{XE}\theta_{YK})>0$.
[6] Since these results are so complicated, we also consider a simpler model that does not include capital. This model is presented in Appendix A.II. While the results are simpler than those from the main model, it cannot be used to analyze sources-side incidence or to see how substitution between labor and capital affects unemployment.

unemployment.[7] This efficiency wage effect is absent from the equation for unemployment, equation (17) (there is no term with M in that equation). That may seem counterintuitive, since of course the efficiency wage component of the model must affect unemployment. However, the coefficient $\frac{\varepsilon_{11}}{\varepsilon_2}$ in front of equation (17) captures this relationship. All of the previously identified effects are scaled by this coefficient, which shows how the form of the effort function translates these effects into unemployment.

The efficiency wage effect is its own term in the expressions for the sources-side and uses-side incidence; it is represented by the terms with M in it. As defined earlier, $M \equiv (1+\gamma_K)\left[(1+\gamma_L)(1+\varepsilon_{11}) - \frac{\lambda_{LU}}{\lambda_{LX}}\frac{\varepsilon_{11}}{\varepsilon_2}\right]$, which is strictly positive if $\varepsilon_{11} > -1$. The efficiency wage effect in the expression for sources-side incidence (equation 18) is $\frac{\theta_{YZ}}{D}(1-\eta)M$ and so is the same sign of M. If workers' marginal effort with respect to the real wage does not decline too fast as the wage increases (i.e. $\varepsilon_{11} > -1$), then the efficiency wage effect on $\widehat{w} - \hat{r}$ is strictly positive, meaning that a pollution tax disproportionately burdens capital. The uses-side efficiency wage effect from equation (19) is $\frac{\theta_{YZ}\theta_{XK}}{D}M$. Under the same assumption that $\varepsilon_{11} > -1$, this effect is strictly positive, which means the dirty good price increases more than the clean good price, and the uses-side incidence falls more on consumers of the dirty good.

To further interpret this effect, we can make an additional assumption: that the workers' effort function is a Cobb-Douglas function of real wage and unemployment:

$$e\left(\frac{W}{P}, U\right) = \left(\frac{W}{P}\right)^{\alpha} U^{\beta}$$

[7] A corresponding term is found in Rapanos's (2007) results, which he calls an "unemployment effect."

where $\alpha > 0$ is the elasticity of effort with respect to the real wage, and $\beta > 0$ is the elasticity of effort with respect to unemployment. Then we have $\varepsilon_{11} = \alpha - 1$ and $\varepsilon_2 = \beta$. The concavity of the effort function with respect to the real wage implies that $\varepsilon_{11} = \alpha - 1 < 0$, and thus $\alpha < 1$. The functional form assumption also implies that $\varepsilon_{11} > -1$. With Cobb-Douglas effort, $M = \frac{1}{\lambda_{LX}\lambda_{KX}}\left[(1-\lambda_{LU})\alpha + \lambda_{LU}\left(\frac{1-\alpha}{\beta}\right)\right]$. This expression is strictly positive, and its magnitude depends on the unemployment rate λ_{LU} and both elasticities of the effort function α and β.

We explore how the magnitude of M, and thus of the source-side and uses-side efficiency wage effects, depends on these parameters. First, the elasticity of the effort function with respect to the real wage affects the efficiency wage effect thusly: $\frac{\partial M}{\partial \alpha} = \frac{1}{\lambda_{LX}\lambda_{KX}}\left[1 - \lambda_{LU}\left(1 + \frac{1}{\beta}\right)\right]$. This derivative is positive as long as $\beta > \frac{\lambda_{LU}}{1-\lambda_{LU}}$, which is likely true given that the unemployment rate (λ_{LU}) is likely small. Given this condition, the efficiency wage effect becomes larger in magnitude (M increases) as workers' effort becomes more responsive to the real wage. Second, the elasticity of effort with respect to unemployment affects the efficiency wage effect thusly: $\frac{\partial M}{\partial \beta} = -\frac{\lambda_{LU}}{\lambda_{LX}\lambda_{KX}}\frac{1-\alpha}{\beta^2}$. This derivative is strictly negative, which means the efficiency wage effect is smaller (M decreases) as workers' effort becomes more responsive to unemployment. These two derivatives demonstrate how the source of the efficiency wage matters greatly to incidence effects. A high α means effort is very responsive to the real wage, which is likely to be true in a gift exchange or fair wage efficiency wage model like Akerlof (1982). A high β means effort is very responsive to unemployment, which is likely to be true in a shirking and firing model like Shapiro and Stiglitz (1984). If a fair wage model is more accurate, then the efficiency wage incidence effect is large, whereas if a shirking and firing model is more accurate, then the efficiency wage incidence effect is smaller.

Third, the unemployment rate affects the efficiency wage effect thusly: $\frac{\partial M}{\partial \lambda_{LU}} = \frac{1}{\lambda_{LX}\lambda_{KX}} \cdot \frac{1-\alpha-\alpha\beta}{\beta}$. This is positive if and only if $\beta < \frac{1-\alpha}{\alpha}$. All else equal one might predict that a larger baseline unemployment rate will increase the magnitude of the efficiency wage effect. But if the effort function is very responsive to unemployment (β is large) then this might not be the case.

III.B. Output Effect

The remaining effects in equations (17), (18), and (19) are the output effect and two substitution effects. These are standard effects found in the tax incidence literature dating back to Harberger (1962) and Mieszkowski (1967). Here, we focus on how the inclusion of pollution and unemployment modifies these effects.

In both equations (17) and (18), the terms that includes σ_u, the substitution elasticity of demand between the two goods X and Y, represent an output effect (this effect is also identified in Mieszkowski (1967)). What we mean by "output effect" is that the pollution tax disproportionately affects the dirty sector — merely because the dirty sector is the only sector that uses pollution as an input — and reduces its output in a way that depends on consumer preferences via σ_u. Less output means less demand for all inputs, but particularly the input used intensively in that sector.

The output effect caused by one-unit change in the pollution tax ($\widehat{\tau_Z}$) on unemployment $\widehat{U}$ is $\frac{\varepsilon_{11}}{\varepsilon_2}\frac{\theta_{YZ}}{D}\{-\sigma_u\theta_{XK}(\gamma_L - \gamma_K)\}$. This term is negative whenever $\gamma_K > \gamma_L$, which holds whenever the dirty sector Y is relatively capital-intensive.[8] The dirty sector being capital-intensive means

[8] $\gamma_K > \gamma_L$ implies $\frac{\lambda_{KY}}{\lambda_{KX}} > \frac{\lambda_{LY}}{\lambda_{LX}}$, which implies $\frac{K_Y}{K_X} > \frac{L_Y}{L_X}$.

that the pollution tax will impose a larger burden on capital than on labor, which translates to a decrease in unemployment, captured in this term.

In the expression for sources-side incidence $\hat{w} - \hat{r}$, equation (18), the output effect $\frac{\theta_{YZ}}{D}\{\sigma_u(\gamma_L - \gamma_K)(-\theta_{XK} + \varepsilon_{11}\theta_{XE})\}$ is positive whenever $\gamma_K > \gamma_L$. If the dirty sector is relatively capital-intensive ($\gamma_K > \gamma_L$), then this output effect will decrease the price of capital relative to the wage ($\hat{w} - \hat{r} > 0$). The magnitude of this effect is proportional to the substitution elasticity of demand between the two goods, σ_u.

There is no output effect on the uses-side incidence $\widehat{p_Y} - \widehat{p_X}$; the relative factor intensities do not affect uses-side incidence, only sources-side incidence.

The analysis of the output effect is similar to the corresponding effect in Fullerton and Heutel (2007), though the expressions are somewhat different due to the presence of efficiency wages and endogenous unemployment. Likewise, a similar term is found in Rapanos (2006), though that model omits pollution.

III.C. Clean Sector Substitution Effect

Next we identify two kinds of substitution effects. In equations (17), (18), and (19), the terms that include σ_X, the substitution elasticity of input demand between capital and labor for the clean (X) sector, are what we call the clean sector substitution effect. This captures the change through the response of the clean sector towards the change of relative input prices. Because the model is general equilibrium and total factor quantities (capital and labor) across sectors are fixed, the effect of substitutability within the clean industry impacts the incidence of a tax levied only on the dirty industry.

In the expression for change in unemployment $\hat{U}$ (equation 17), the clean sector substitution effect is $\frac{\varepsilon_{11}}{\varepsilon_2}\frac{\theta_{YZ}}{D}\{-C\sigma_X(1-\eta)\}$. This term is unambiguously positive, given the negative ε_{11} out front. An increase in the pollution tax unambiguously increases unemployment through the clear sector substitution effect.

Similarly, for the sources-side incidence (equation 18), the clean sector substitution effect is $\frac{\theta_{YZ}}{D}\{-C\sigma_X(1-\eta)(1+\varepsilon_{11})\}$. This term is unambiguously negative, so when the pollution tax increases, this effect decreases $\hat{w}-\hat{r}$ and places more burden of the tax on labor.

The clean sector substitution effect's impact on both unemployment $\hat{U}$ and sources-side incidence $\hat{w}-\hat{r}$ arises from the same intuition. The tax increase is an overall distortion to the economy. While the total amount of capital employed is fixed, the total amount of labor employed varies because of endogenous unemployment. The overall distortion from the pollution tax thus exacerbates the tax wedge affecting unemployment, increasing overall unemployment and disproportionately burdening labor income (due to the link between unemployment and labor income from the effort function).

The clean sector substitutions effect on the uses-side incidence $\widehat{p_Y}-\widehat{p_X}$ is $\frac{\varepsilon_{11}\theta_{YZ}}{D}\{-C\sigma_X\}$ which is always positive. An increase in the pollution tax burdens consumers of the dirty good less than it burdens consumers of the clean good through this effect.

The clean sector substitution effect's magnitude on all three outcomes is scaled by the magnitude of σ_X. The easier it is for the clean sector to substitute between capital and labor (larger σ_X), the larger is the size of each of the effects described above. A very similar effect is found in Rapanos (2006).[9]

III.D. Dirty Sector Substitution Effect

Finally, the other substitution effect comes from substitutability among inputs in the dirty sector. These are the longest and most complicated parts of equations (17), (18), and (19), which are all of the terms that contain the Allen elasticities for the dirty sector, e_{ij}, along with the constants that are functions of factor shares and output shares. These are difficult to interpret in the general case; this is a byproduct of the very flexible form of substitution that we allow between the three inputs in the dirty sector. It can be simplified under either one of two assumptions.

Thus, here we impose a simplifying assumption to aid in interpreting this effect: that the two sectors have equal factor intensities; that is, $\gamma_K = \gamma_L \equiv \gamma$. Then we have $A = B = (1+\gamma)\gamma$ and $C = \gamma + 1$. This eliminates the output effect described earlier. It also greatly simplifies the complicated dirty sector substitution effect. The solutions under this assumption are:

$$\widehat{U} = \frac{\varepsilon_{11}}{\varepsilon_2}\frac{\theta_{YZ}}{D}(1+\gamma)\{\gamma[\theta_{YK}(1-\eta)(e_{KK}-e_{KE}) + \eta_K(e_{EZ}-e_{KZ})] - \sigma_X(1-\eta)\}\,\widehat{\tau_Z} \qquad (20)$$

$$\widehat{w} - \widehat{r}$$
$$= \frac{\theta_{YZ}}{D}\begin{Bmatrix} \gamma(1+\gamma)\big[(1-\eta)\big(-2e_{KE}(1-\theta_{YZ})\big) - \theta_{YZ}(e_{KZ}+e_{EZ}) + (\eta_K - \varepsilon_{11}\eta_E)(e_{EZ}-e_{KZ})\big] \\ -(1+\gamma)\sigma_X(1-\eta)(1+\varepsilon_{11}) + (1-\eta)M \end{Bmatrix}\widehat{\tau_Z} \; (21)$$

$$\widehat{p_Y} - \widehat{p_X} = \frac{\varepsilon_{11}\theta_{YZ}}{D}\begin{Bmatrix} \gamma(1+\gamma)[\theta_{XE}\theta_{YK}(e_{KK}-e_{KE}+e_{KZ}+e_{EE})] \\ -(1+\gamma)\sigma_X + M\dfrac{\theta_{XK}}{\varepsilon_{11}} \end{Bmatrix}\widehat{\tau_Z} \qquad (22)$$

[9] For example, the first term in equation 34 in Rapanos (2006) is the clean sector substitution effect on sources-side incidence, and it also is scaled by the substitution elasticity in consumption between the two goods (denoted by σ_D in his model).

The dirty sector substitution effect on unemployment $\widehat{U}$ is $\frac{\varepsilon_{11}}{\varepsilon_2}\frac{\theta_{YZ}}{D}(1+\gamma)\gamma[\theta_{YK}(1-\eta)(e_{KK}-e_{KE})+\eta_K(e_{EZ}-e_{KZ})]$. We can easily sign the following parts: $\frac{\varepsilon_{11}}{\varepsilon_2}\frac{\theta_{YZ}}{D}(1+\gamma)\gamma<0$ and $\theta_{YK}(1-\eta)(e_{KK}-e_{KE})<0$. Therefore, as long as $e_{EZ}-e_{KZ}<0$, this effect is positive. If capital is a better substitute for pollution than is labor ($e_{EZ}-e_{KZ}<0$), then an increase in the pollution tax increases unemployment through this effect. However, if labor is a better substitute for pollution than is capital ($e_{EZ}-e_{KZ}>0$), we cannot say with certainty whether it increases or decreases the unemployment through this effect.

The dirty sector substitution effect on $\widehat{w}-\widehat{r}$ is $\frac{\theta_{YZ}}{D}\gamma(1+\gamma)\big[(1-\eta)\big(-2e_{KE}(1-\theta_{YZ})\big)-\theta_{YZ}(e_{KZ}+e_{EZ})+(\eta_K-\varepsilon_{11}\eta_E)(e_{EZ}-e_{KZ})\big]$. If capital is a better substitute for pollution than is labor ($e_{EZ}-e_{KZ}<0$), then this effect is strictly negative, so the pollution tax imposes more burden on labor.

When it comes to then uses-side incidence, the dirty sector substitution effect is $\frac{\varepsilon_{11}\theta_{YZ}}{D}\gamma(1+\gamma)[\theta_{XE}\theta_{YK}(e_{KK}-e_{KE}+e_{KZ}+e_{EE})]$. The sign of this term is determined by $e_{KK}-e_{KE}+e_{KZ}+e_{EE}$. One simple case is that if capital and labor are better substitutes than are capital and pollution ($e_{KE}>e_{KZ}$), then the dirty sector substitution effect on $\widehat{p_Y}-\widehat{p_X}$ is positive, which means the price of dirty good increases more than clean good through this effect.

IV. Numerical Analysis

Here we numerically simulate the model by assigning parameter values calibrated from data and taken from the previous literature. Ours is a simple two-sector, two-input model, not a CGE model, so the purpose of these simulations is not to pin down plausible quantitative values for the magnitudes of these effects. Rather, the purpose is to explore the size of the effects discussed in the previous section and how they relate to various parameter values. We begin with presenting base case simulations decomposed into the effects from the analytical model. Then we vary parameter values, including the effort function elasticities.

We use the 2017 Integrated Industry-Level Production Account (KLEMS) data provided by the U.S. Bureau of Economic Analysis for the calibration of the factor share and factor intensity parameters.[10] This data set traces the sources of growth in GDP and output from the industry origins by examining changes in capital, labor, intermediate purchases of energy, materials, and services.

First, we use the energy inputs (in millions of dollars) as a measurement of the input Z in our dirty sector. The KLEMS data contains 64 major industries. We rank them based on their ratios of energy inputs to the gross outputs of the industry and assign the top 16 energy-demanding industries as the dirty sector, and the remaining industries as the clean sector. The dirty sector includes utilities (with energy inputs at 17.63% of output), rail transportation (10.08%), truck transportation (9.39%), to the 16th dirtiest industry, primary metals (2.81%). The 47 clean industries include from accommodation (2.63%) to insurance carriers and related activities (0.06%), which we assume use a negligible portion of energy compared to the industry outputs. This assignment implies that dirty sector makes up about 30 percent of gross outputs. We let the weight of price of X on price index P, η = 0.7, mirroring the fact that the clean sector is 70% of income.

Second, the shares of each sector's revenue paid to labor, capital, and energy are roughly measured using the ratios of compensation to labor, capital, and energy to the outputs of each sector. We find that the clean sector is more labor-intensive, with about 61% of the revenue paid

[10] U.S. Bureau of Economic Analysis, "Production Account Tables, 1998-2017," https://www.bea.gov/data/special-topics/integrated-industry-level-production-account-klems (accessed December 12, 2019).

to labor, so $\theta_{XK} = 0.39$ and $\theta_{XE} = 0.61$. The dirty sector is more capital-intensive and pays about 7% of its revenue to energy inputs, so we have $\theta_{YZ} = 0.07$, $\theta_{YK} = 0.56$ and $\theta_{YE} = 0.37$.

Third, we use the different factor intensities of the two sectors and their share of the gross output to calculate each sector's share of capital and labor. Sector X's share of capital $\lambda_{KX} = 0.62$ and $\lambda_{KY} = 0.38$, showing that even if the dirty sector Y is capital-intensive, it still uses a smaller share of the capital because the dirty sector only accounts for 30% of the economy. We set the unemployment rate λ_{LU} to be 0.04 to roughly coincide with the average U.S. monthly unemployment rate (4.35%) in 2017. Thus, we get $\lambda_{LX} = 0.76$, $\lambda_{LY} = 0.20$. These imply that $\gamma_L = 0.26$ and $\gamma_K = 0.61$, which means that the clean and dirty sectors have different capital or labor intensities.

Additionally, we use unity for the elasticity of substitution between capital and labor in the clean sector ($\sigma_X = 1$) and the elasticity of substitution in consumption between the clean and dirty goods ($\sigma_u = 1$), following Fullerton and Heutel (2007). We use the calibration of Allen cross-price elasticities e_{KE}, e_{KZ}, and e_{EZ} from Fullerton and Heutel (2010), which sets $e_{KE} = 0.5$, $e_{KZ} = 0.5$, and $e_{EZ} = 0.3$.[11] This indicates that capital is a slightly better substitute for pollution than is labor ($e_{KZ} > e_{EZ}$).

Finally, we found no source for the parameter values related to the effort function, ε_{11} and ε_2. When the effort function is Cobb-Douglas, these parameters are $\varepsilon_{11} = \alpha - 1$ and $\varepsilon_2 = \beta$. So, we arbitraitily assume a Cobb-Douglas effort function where each elasticity α and β is set to 0.5, implying that $\varepsilon_{11} = -0.5$ and $\varepsilon_2 = 0.5$. Table 1 summarizes the base-case parameter values.

[11] In Fullerton and Heutel (2010), there is no effective labor E, just labor L, so we additionally assume that their e_{KL} is equal to our e_{KE}, etc.

Table 1 – Base Case Parameter Values

Parameter	Value	Parameter	Value
θ_{XK}	0.39	λ_{KX}	0.62
θ_{XE}	0.61	λ_{KY}	0.38
θ_{YK}	0.56	λ_{LX}	0.76
θ_{YE}	0.37	λ_{LY}	0.20
θ_{YZ}	0.07	λ_{LU}	0.04
γ_K	0.61	γ_L	0.26
η	0.7	σ_X	1
ε_{11}	-0.5	σ_u	1
ε_2	0.5	e_{KE}	0.5
e_{KZ}	0.5	e_{EZ}	0.3

Note: These values are calibrated based on data and on the previous literature as described in the text.

The exogenous policy choice variable is the change in the pollution tax $\widehat{\tau_Z}$. We consider the change in the price of energy under a carbon tax that accounts for the social cost of carbon. Interagency Working Group on Social Cost of Greenhouse Gases established by the US Government provides an updated estimation of SCC based on new versions of three IAM models (DICE, PAGE, and FUND) in 2016. We adopt an estimate of $40 per metric ton of CO_2 based on the report (Interagency Working Group on Social Cost of Greenhouse Gases, 2016). Then we calculate the weighted average energy price with and without a carbon tax at $40 per metric ton CO_2. The calculation is based on the fuel price calculator provided by Hafstead and Picciano (2017), and we use the 2015 energy price and industrial sector energy usage data provided by U.S. Energy Information Administration.[12] In 2015, the energy used from generated from coal, petroleum, and natural gas is 1.38, 8.25, and 9.43 quadrillion British thermal units (BTU), respectively. Here we leave out the electricity for now, because the price effects will vary across

[12] U.S. Energy Information Administration, "U.S. industrial sector energy use by source, 1950-2018," https://www.eia.gov/energyexplained/use-of-energy/industry.php (accessed December 12, 2019).

regions due to differences in market structure and the initial mix of generation technologies. The average percentage increase of prices for coal (all types), petroleum product and natural gas is 264%, 25% and 50%, respectively. Weighted by the energy usage amount, we get that the energy price increases 35% on average after imposing the $40 carbon tax. Therefore, we present simulation results with $\widehat{\tau_Z} = 0.35$.

We begin by presenting the results under the base-case parameterization. In Table 2, and all of the numerical simulation tables, we presents the effects of a 35% increase in the pollution tax on unemployment ($\widehat{U}$), the sources-side incidence ($\hat{w} - \hat{r}$), and the uses-side incidence ($\widehat{p_Y} - \widehat{p_X}$). The last row of Table 2 (row 5) presents the net effect of the tax, and rows 1 through 4 decompose this net effect into the four effects discussed earlier.

Table 2 – Base Case Simulation Results

Row		$\boldsymbol{\widehat{U}}$	$\boldsymbol{\hat{w} - \hat{r}}$	$\boldsymbol{\widehat{p_Y} - \widehat{p_X}}$
1	Output Effect	-0.25%	0.44%	–
2	Clean Sector Substitution Effect	0.77%	-0.38%	1.28%
3	Dirty Sector Substitution Effect	0.37%	-0.42%	0.30%
4	Efficiency Wage Effect	–	0.60%	0.78%
5	Net Effect	0.88%	0.24%	2.36%

Note: This table presents the simulated effects on unemployment, sources-side incidence, and uses-side incidence of a $40 per metric ton carbon tax (a 35% increase in the pollution tax) under the base case parameter values (listed in Table 1).

From the theoretical results, there is no efficiency wage effect present in the expression for unemployment, and there is no output effect present in the expression for uses-side incidence (so these entries in Table 2 are blank).

The net effect of a 35% increase in the pollution tax on unemployment is a 0.884% increase in unemployment. This is small, because the dirty (taxed) sector is just 30% of the overall economy, and pollution is just 7% of the value of its inputs, and the tax rate increase is just 35%. The increase in unemployment is mainly driven by the clean sector substitution effect (0.765% increase) versus the dirty sector substitution effect (0.367% increase). Even though the dirty sector is the taxed sector, substitution among inputs in the clean sector has a larger effect on unemployment. This is because the clean sector is the larger sector (70%), and in general equilibrium its substitution possibility is more important for employment than is the dirty sector's substitution. The output effect is negative, since the dirty (taxed) sector is capital-intensive.

For the sources-side incidence, the efficiency wage effect plays a significant role. Both dirty and clean sector substitution effects serve to increase the relative burden on labor ($\hat{w} - \hat{r} < 0$). From these two effects alone, the wage relative to the capital rental rate decreases by about 0.8%. The output effect offsets these effects somewhat, again since the dirty sector is capital-intensive. But, the efficiency wage effect reverses the sign and completely offsets the substitution effects and decreases the relative burden on labor. The sources-side incidence goes from favoring capital to favoring labor.

For the uses-side incidence (the relative burden on output prices), we see a positive sign from all three effects; each puts more of the burden on consumers of the clean good than on consumers of the dirty good. Here, ignoring the efficiency wage effect would miss about 30% of the net effect.

The base case simulation results in Table 2 demonstrate the importance of the new efficiency wage effect, especially its effect on the sources-side incidence. Ignoring this effect would yield a prediction of the wrong sign. The base case results depend on the base case parameters, so we next consider several simulations that test for the differences in outcomes as functions of parameter values. First, we vary the effort function elasticity parameters ε_{11} and ε_2. These results are presented in Table 3, which presents the outcomes when all of the parameters are at the base case, except for these two parameters. In Table 3 and the remaining tables, we also present the resulting change in pollution, $\hat{Z}$. We do not have theoretical results related to the change in pollution since it is not the focus of this paper, but we explore it here numerically.

Table 3 – Sensitivity Analysis – Varying Effort Function Elasticities

Row	ε_{11}	ε_2	$\hat{U}$	$\hat{w} - \hat{r}$	$\widehat{p_Y} - \widehat{p_X}$	$\hat{Z}$
1	-0.1	0.1	1.17%	0.36%	2.22%	-16.62%
2	-0.1	0.5	0.24%	0.32%	2.22%	-16.61%
3	-0.1	0.9	0.13%	0.31%	2.22%	-16.61%
4	-0.5	0.1	3.98%	0.39%	2.34%	-16.35%
5	-0.5	0.5	0.88%	0.24%	2.36%	-16.30%
6	-0.5	0.9	0.50%	0.23%	2.37%	-16.30%
7	-0.9	0.1	5.43%	0.40%	2.41%	-16.21%
8	-0.9	0.5	1.26%	0.20%	2.45%	-16.12%
9	-0.9	0.9	0.71%	0.17%	2.45%	-16.11%

Note: This table presents the simulated effects on unemployment, sources-side incidence, uses-side incidence, and pollution of a \$40 per metric ton carbon tax (a 35% increase in the pollution tax) for different values of the effort function elasticities. Their base-case values are used in row 5. All of the other parameters are kept at their base case values (listed in Table 1).

The unemployment increases the least and capital bears more relative burden ($\hat{w} - \hat{r}$ with larger values) when the elasticity of marginal effort with respect to wage (ε_{11} in absolute value) is small and the elasticity of effort with respect to unemployment (ε_2) is large. The potential explanation is that if ε_{11} is large in absolute value, workers' marginal reduced effort increases quickly as the wage drops. This will restrain the magnitude of the wage dropping relative to capital price, because reduced wage will cause extra loss of productivity due to quickly decreased effort level. If ε_2 is large, workers are more sensitive to unemployment and work much harder, then their extra productivity will offset the rising cost of energy and there will be less increase in unemployment.

The uses-side incidence always falls disproportionately on consumers of the dirty good and is not much affected by the effort function elasticities. Likewise, the fall in pollution is largely unaffected by these elasticities: a 35% increase in the tax rate yields a pollution reduction of about 16%.

Next, in Table 4, we vary the Allen cross-price elasticities of substitution in production of the dirty good. We keep the elasticity between labor and capital, e_{KE}, equal to its base-case value of 0.5, and we vary the other two cross-price elasticities e_{KZ} and e_{EZ} to vary among 0, 0.5 and 1. All of the other parameters are kept at their base case values, except that the own-price elasticities e_{KK}, e_{EE}, and e_{ZZ} must also vary with the cross-price elasticities. To demonstrate, we also include in the third column of Table 4 the resulting value of the own-price elasticity e_{ZZ}.

Table 4 – Sensitivity Analysis – Varying Dirty Sector Substitution Elasticities

Row	e_{KZ}	e_{EZ}	e_{ZZ}	$\widehat{U}$	$\hat{w} - \hat{r}$	$\widehat{p_Y} - \widehat{p_X}$	$\widehat{Z}$
1	0.5	0.3	-5.59	0.88%	0.24%	2.36%	-16.30%
2	0	0	0	0.66%	0.60%	2.32%	-1.85%
3	0	0.5	-2.64	0.49%	0.88%	2.28%	-8.43%
4	0	1	-5.29	0.32%	1.15%	2.24%	-14.99%
5	0.5	0	-4	0.99%	0.08%	2.39%	-12.31%
6	0.5	0.5	-6.64	0.81%	0.35%	2.35%	-18.96%
7	0.5	1	-9.29	0.64%	0.63%	2.31%	-25.57%
8	1	0	-8	1.31%	-0.44%	2.46%	-22.66%
9	1	0.5	-10.64	1.13%	-0.16%	2.42%	-29.37%
10	1	1	-13.29	0.96%	0.12%	2.38%	-36.04%

Note: This table presents the simulated effects on unemployment, sources-side incidence, uses-side incidence, and pollution of a $40 per metric ton carbon tax (a 35% increase in the pollution tax) for different values of the substitution elasticities e_{KZ} and e_{EZ}. Their base-case values are used in row 1. All of the other parameters are kept at their base case values (listed in Table 1).

In Table 4, the change of unemployment $\widehat{U}$ is always positive. Unemployment always increases with a 35% increase in carbon tax, and it increases the most when capital is a better substitute for pollution relative to labor ($e_{KZ} > e_{EZ}$). The value of $\hat{w} - \hat{r}$ varies across different parameter values, and it is small or even negative when $e_{KZ} > e_{EZ}$. The change of pollution $\hat{Z}$ is always negative. The pollution tax is drastically more effective in reducing pollution when other inputs are strong substitutes. This can be seen by noting that when e_{ZZ} is large in absolute value (as in the last row), then the change in pollution is large in absolute value.

Lastly, in Table 5 we hold the factor substitution elasticities to be fixed and consider the impact of changes in factor intensities. We assume $e_{KE} = 0.5$, $e_{KZ} = 0.5$, and $e_{EZ} = 0.3$ as in base case, while varying the value of $\gamma_K - \gamma_L$ from –0.05 to 0.5. We maintain the assumption that clean sector is 70% of income and we set the ratio of total capital to labor in economy to be 0.45/0.55 that is roughly consistent with our first few simulations. $\gamma_K - \gamma_L$ is positive if the dirty sector is more capital intensive than the clean sector.

Table 5 – Sensitivity Analysis – Varying Factor Intensities

Row	$\boldsymbol{\gamma_K - \gamma_L}$	$\boldsymbol{\hat{U}}$	$\boldsymbol{\hat{w} - \hat{r}}$	$\boldsymbol{\widehat{p_Y} - \widehat{p_X}}$	$\boldsymbol{\hat{Z}}$
1	-0.35	1.23%	-0.31%	2.56%	-14.44%
2	-0.25	1.18%	-0.23%	2.55%	-14.71%
3	0	1.05%	-0.03%	2.50%	-15.35%
4	0.25	0.94%	0.16%	2.42%	-15.96%
5	0.35	0.89%	0.23%	2.38%	-16.18%
6	0.55	0.82%	0.36%	2.29%	-16.61%

Note: This table presents the simulated effects on unemployment, sources-side incidence, uses-side incidence, and pollution of a $40 per metric ton carbon tax (a 35% increase in the pollution tax) for different values of relative factor intensities. All of the other parameters are kept at their base case values (listed in Table 1). Their base-case values (rounded to the nearest hundredth) are used in row 5.

As the dirty sector becomes more capital-intensive (as $\gamma_K - \gamma_L$ increases), the increase in unemployment declines and capital bears an increasing share of the burden ($\hat{w} - \hat{r}$ increases). Varying capital intensities yields only minor variation in the relative change in output prices and the change in pollution.

V. Conclusion

We use an analytical general equilibrium model with unemployment generated through efficiency wages to analyze the effect of a pollution tax on unemployment and on sources-side and uses-side incidence. Worker effort depends on unemployment and the real wage. Pollution is modeled as an input to production with a general form of substitutability between the other inputs. We decompose the general equilibrium impact of the tax on unemployment and incidence into several effects, including an output effect, substitution effects, and a new effect that we call the efficiency wage effect. The magnitude of this efficiency wage effect depends crucially on how workers' effort responds to both the real wage and to unemployment. When workers are more responsive to the real wage, the efficiency wage effect is larger, and when workers are more responsive to unemployment, the efficiency wage effect is smaller.

We further illustrate our results through calibrated numerical simulations. At the base case parameterization, the new efficiency wage effect offsets the substitution and output effects on the sources-side incidence. Ignoring the efficiency wage effect, the burden of a pollution tax increase falls mostly on labor, while including it, the burden falls mostly on capital. On unemployment, the output effect reduces unemployment since the dirty sector is capital-intensive, but it is dominated by substitution effects that increase unemployment. The magnitudes of the effects on unemployment and on sources-side incidence depend greatly on the structure of the effort function, though the magnitude of the uses-side incidence is largely independent of that. The uses-side incidence always falls disproportionately on consumers of the dirty good.

We employ a parsimonious model to be able to interpret the intuition behind our results, so there are many ways in which the model could be extended by relaxing various assumptions. For example, further work could consider other effort functions, including one that depends on the wage to rental rate ratio (Agell and Lundborg 1992), or could include heterogeneity among workers (Fullerton and Monti 2013). We do not consider the benefit of pollution reduction and its incidence or effect on unemployment.

Nevertheless, our results provide theoretical insights into the impact of environmental policy on labor markets that could inform policymakers. A key takeaway is that the effect of policy on unemployment depends on *how* unemployment is generated in the economy. Our model is an efficiency wage model, rather than a search-and-matching model (Hafstead and Williams 2018, Aubert and Chiroleu-Assouline 2019) or another model of unemployment. But our model nests several different structural causes of efficiency wages. Under a fair wage model, worker effort may respond greatly to the real wage, while under a shirking and firing model, worker effort may respond greatly to unemployment. We show that how effort responds is a crucial determinant of how overall unemployment will be affected by a pollution tax, as well as its incidence.

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Appendix

A.I. Solution Method

We begin by eliminating through successive substitution several of the endogenous variables from the system of equations. Output quantities $\hat{X}$ and $\hat{Y}$ can be eliminated with equations (13) and (14); effort and the effective wage $\hat{e}$ and $\hat{v}$ can be eliminated with equations (4) and (5); and the effective labor levels $\widehat{E_X}$ and $\widehat{E_Y}$ can be eliminated with equations (1) and (2). Then, capital and labor used in each sector $(\widehat{K_X}, \widehat{K_Y}, \widehat{L_X}, \widehat{L_Y})$ can be eliminated with equations (6), (7), (9), and (10), after substitution in for the variables that had already been eliminated. That leaves six remaining endogenous variables – $\hat{Z}$, $\hat{U}$, $\hat{w}$, $\hat{r}$, $\widehat{p_X}$, and $\widehat{p_y}$ – and the following six equations:

$$\hat{U} = \frac{\varepsilon_{11}}{\varepsilon_2}\hat{w} \tag{A1}$$

$$\begin{gathered}(\gamma_L - \gamma_K)\hat{Z} - (1+\gamma_L)\big(\hat{w} + \varepsilon_2\hat{U}\big) + \theta_{YK}[\gamma_L(e_{KE} - e_{ZK}) - \gamma_K(e_{KK} - e_{ZK})]\hat{r} \\ +\theta_{YE}\varepsilon_{11}[\gamma_K(e_{KE} - e_{ZE}) - \gamma_L(e_{EE} - e_{ZE})]\hat{w} + \theta_{YZ}[\gamma_L(e_{EZ} - e_{ZZ}) - \gamma_K(e_{KZ} - e_{ZZ})]\widehat{\tau_Z} \\ +\frac{\lambda_{LU}}{\lambda_{LX}}\hat{U} = \sigma_X(-\varepsilon_{11}\hat{w} - \hat{r})\end{gathered} \tag{A2}$$

$$\widehat{p_X} = \theta_{XK}\hat{r} - \theta_{XE}\varepsilon_{11}\hat{w} \tag{A3}$$

$$\widehat{p_Y} = \theta_{YK}\hat{r} - \theta_{YE}\varepsilon_{11}\hat{w} + \theta_{YZ}\widehat{\tau_Z} \tag{A4}$$

$$0 = \eta\widehat{p_X} + (1-\eta)\widehat{p_Y} \tag{A5}$$

$$\begin{aligned}\sigma_u(\widehat{p_Y} - \widehat{p_X}) = {} & -(\beta_K + \beta_L + \theta_{YZ})\hat{Z} + \theta_{YK}[-\beta_L(e_{KE} - e_{ZK}) - \beta_K(e_{KK} - e_{ZK})]\hat{r} \\ & + \theta_{YE}\varepsilon_{11}[\beta_K(e_{KE} - e_{ZE}) + \beta_L(e_{EE} - e_{ZE})]\hat{w} \\ & + \theta_{YZ}[-\beta_L(e_{EZ} - e_{ZZ}) - \beta_K(e_{KZ} - e_{ZZ})]\widehat{\tau_Z} \\ & + \theta_{XE}\left[(1+\gamma_L)\big(\hat{w} + \varepsilon_2\hat{U}\big) - \frac{\lambda_{LU}}{\lambda_{LX}}\hat{U}\right]\end{aligned} \tag{A6}$$

where $\gamma_L \equiv \frac{\lambda_{LY}}{\lambda_{LX}}$, $\gamma_K \equiv \frac{\lambda_{KY}}{\lambda_{KX}}$, $\beta_K \equiv \theta_{XK}\gamma_K + \theta_{YK}$ and $\beta_L \equiv \theta_{XE}\gamma_L + \theta_{YE}$.

We then successively solve for the remaining variables.

A.II. Model without Capital

We consider a competitive two-sector economy using only one factor of production: labor, which is perfectly mobile between sectors. The second variable input, pollution, is only used in production of the dirty goods (sector Y). The constant-returns-to-scale production functions become:

$$X = X(E_X)$$

$$Y = Y(E_Y, Z)$$

The labor market equations are the same as equations (1) – (5) in the original model. There is now only one resource constraint, on labor, which is the same as equation (7). Producers of Y face the substitution between labor and pollution. The elasticity of substitution in production σ_Y is defined to capture this response to factor price changes:

$$\hat{Z} - \widehat{E_Y} = \sigma_Y(\hat{v} - \widehat{\tau_z}) \qquad (A7)$$

where σ_Y is defined to be positive.

Using the assumptions of perfect competition and constant returns to scale, we get

$$\widehat{p_X} + \hat{X} = \hat{v} + \widehat{E_X} \qquad (A8)$$

$$\widehat{p_Y} + \hat{Y} = \theta_{YE}\left(\hat{v} + \widehat{E_Y}\right) + \theta_{YZ}\left(\hat{Z} + \widehat{\tau_Z}\right) \qquad (A9)$$

Totally differentiate each sector's production function and substitute in the conditions from the perfect competition assumption to get

$$\hat{X} = \widehat{E_X} \qquad (A10)$$

$$\hat{Y} = \theta_{YE}\widehat{E_Y} + \theta_{YZ}\hat{Z} \qquad (A11)$$

The consumer side is the same as in our original model, represented by equations (15) and (16). We also normalize the overall price level so that $\hat{P} = 0$ and we can drop that variable out of the system.

The full model is equations (1) – (5), (7), (15), (16), and (A7) through (A11). It contains one exogenous policy variable (τ_Z), 13 equations and 13 endogenous variables ($\widehat{E_X}, \widehat{E_Y}, \widehat{L_X}, \widehat{L_Y}, \hat{Z}, \hat{U}, \hat{e}, \hat{w}, \hat{v}, \widehat{p_X}, \widehat{p_Y}, \hat{X}, \hat{Y}$). The model is solved with successive substitution similar to the method in section A.I.

The results are as follows.

$$\hat{U} = \frac{(1-\eta)\theta_{YZ}\widehat{\tau_Z}}{\varepsilon_2((1-\eta)\theta_{YE} + \eta)} \qquad (A12)$$

$$\widehat{w} = \frac{(1-\eta)\theta_{YZ}\widehat{\tau_Z}}{\varepsilon_{11}((1-\eta)\theta_{YE}+\eta)} \qquad (A13)$$

$$\widehat{p_Y} - \widehat{p_X} = \frac{\theta_{YZ}\widehat{\tau_Z}}{(1-\eta)\theta_{YE}+\eta} \qquad (A14)$$

Note that there is no σ_X, σ_Y, or σ_u in the expressions, which means there is no clean sector substitution effect, dirty sector substitution effect or output effect in an economy with no capital. Therefore, $(A12)$ to $(A13)$ fully capture the efficiency wage effect of pollution tax on the change of unemployment, wage and relative output prices. To interpret the results, we need to take a closer look at the term $\frac{(1-\eta)\theta_{YZ}}{(1-\eta)\theta_{YE}+\eta}$. Note that η represents the weight of clean product price in affecting the overall price level or say the share of the clean sector in economy. Therefore, if the overall revenue of the economy is 1 unit, then the compensation to effective labor is $(1-\eta)\theta_{YE} + \eta\theta_{XE} = (1-\eta)\theta_{YE} + \eta$, since all the revenue of clean sector is paid to labor ($\theta_{XE} = 1$). The compensation to pollution or energy is $(1-\eta)\theta_{YZ}$. Then we have

$$\widehat{U} = \frac{revenue\ paid\ to\ energy}{revenue\ paid\ to\ labor}\frac{\widehat{\tau_Z}}{\varepsilon_2} > 0$$

which is very straightforward. The effect of an increase in carbon tax on unemployment is determined by the share of revenue paid to energy compared to labor in the economy, but its effect will be restrained by the elasticity of workers' effort with respect to unemployment (ε_2). The more energy-intensive the economy is, additional carbon tax will hit the employment harder. However, if workers are more sensitive to unemployment and work much harder, then their extra productivity will offset the rising cost of energy and there will be less increase in unemployment.

Similarly,

$$\widehat{w} = \frac{revenue\ paid\ to\ energy}{revenue\ paid\ to\ labor}\frac{\widehat{\tau_Z}}{\varepsilon_{11}} < 0$$

The effect on wage is determined by the share of revenue paid to energy compared to labor, restrained by the rate at which workers get satisfied with wage (ε_{11}). The more energy intensive the economy is, additional carbon tax will lead to lower wages to compensate the rising costs on energy. If ε_{11} is large in absolute value, workers' marginal effort declines quickly as the wage increases, or equivalently, as the wage decreases the marginal reduced effort increases quickly. This will restrain the magnitude of the wage dropping, because reduced wage will cause extra loss of productivity due to quickly decreased effort level.

Lastly,

$$\widehat{p_Y} - \widehat{p_X} = \frac{\theta_{YZ}\widehat{\tau_Z}}{(1-\eta)\theta_{YE} + \eta} = \frac{revenue\ paid\ to\ energy}{revenue\ paid\ to\ labor}\frac{\widehat{\tau_Z}}{1-\eta} > 0$$

The increase of the dirty good price relative to the clean good price is proportional to the ratio of revenue paid to energy compared to labor, whose effect will be restrained by the share of dirty sector in economy $(1-\eta)$.

These results help us tease out the meaning of efficiency wage effect: the weight of energy in economy adjusted by workers' response to changing real wage and unemployment rate due to the tax. However, this model cannot be used to analyze sources-side incidence or to see how substitution between labor and capital affects unemployment, which is why the more complicated model with capital is the main focus of this paper.