Investor Protection, Financial Development, and Corporate Valuation

Preliminary and Incomplete

Yingcong Lan* Xiaoji Lin† Neng Wang‡ Jinqiang Yang§

December 21, 2019

Abstract

Many firms around the world are managed by controlling shareholders who are entrenched with significant wealth exposures to their own illiquid businesses. Lacking investor protection inevitably causes concentrated insiders' equity ownership. However, financial under-development induces significant under-diversification costs for the insiders. We develop a tractable dynamic model to analyze the economic consequences of imperfect investor protection and limited financial development for both the firms' diversified outside investors and their controlling shareholders. The key is the insiders' dynamic non-linear tradeoffs between their private benefits and under-diversification costs. We show that imperfect investor protection and illiquidity/incomplete markets significantly affect firms' investment polices and valuations.

*Cornerstone Research, New York, NY

†University of Minnesota, 321 19th Ave S, Minneapolis, MN

‡Columbia University and NBER, 3022 Broadway, Uris Hall 812, New York, NY 10027

§School of Finance, Shanghai University of Finance and Economics, 200433, China.

In contrast to the common belief that corporations are widely held (Berle and Means (1932)), many corporations around the world, including large publicly traded companies, have controlling shareholders such as founders, founding family members, and sovereign states. La Porta, López-de-Silanes, and Shleifer (1999) document controlling shareholders' concentrated ownership in large firms around the world.[1] With weak investor protection, controlling shareholders, whom we also interchangeably refer to as insiders, become entrenched and pursue private benefits at the expense of outside investors. By "investor protection," we broadly refer to features of institutional, legal, political, regulatory, and market environments as well as corporate governance mechanisms at the firm level, which facilitate financial contracting and contractual enforcement, and protect investors against expropriation by corporate insiders.

Agency problems take a variety of forms including outright stealing from the firm, selling the firm's output to a related party at below market prices, hiring unqualified friends, and self-serving value-destroying investment, just to name a few.[2] It is difficult to verify and contract on decisions such as corporate investment, since they often involve managerial discretion and judgment. Penalizing self-serving insiders based on value-destroying investment is difficult, especially under weak investor protection. We take private benefits and corporate investment as non-contractible in our analysis.

Financial under-development, including imperfect risk sharing, illiquidity, and incomplete markets frictions, usually leads to under-diversification costs for the insiders. We study two major forms of financial under-development. The first is incomplete markets due to unspanned risk for the insider. Namely, the insider's idiosyncratic business risks are only partially spanned by the public financial market. The second is the insider's borrowing constraint, which limits the insider's debt capacity. These two financial frictions imply that the insiders lack of full-diversification, which affects their optimal

[1] Claessens, Djankov, and Lang (2000) and Faccio and Lang (2002) document concentrated ownership for large public firms in East Asian countries and Western European countries, respectively.

[2] For example, see La Porta et al. (2000a) for such a statement in an influential survey on investor protection and corporate governance.

investment policies and firms' valuations.

Our model incorporates the key frictions, imperfect investor protection and the insider's lack of diversification, in an integrated dynamic framework, where the entrenched insider makes *interdependent* business decisions (private benefits, corporate investment, asset sales) and household decisions (consumption-saving and portfolio choice). Using this framework, we address the following questions: What determines corporate investment in firms run by controlling shareholders? How do private benefits of control influence corporate investment and valuation? What determines the insider's private valuation and outside investors' public valuation (Tobin's average q)? What are the effects of imperfect investor protection and limited financial development on the cost of outside equity capital?

Since a key focus of our study is corporate investment, we naturally start with the neoclassical (Tobin's) q theory of investment, and incorporate the key frictions discussed above into the q theoretic framework.[3] Specifically, under the Modigliani-Miller (MM) assumption, our first-best benchmark extends the seminal Hayashi (1982), a widely-used neoclassical q-theoretic model, to a stochastic setting with risk premia by incorporating independently and identically distributed (i.i.d.) capital shocks. In this first-best benchmark, the optimal investment-capital ratio is constant, Tobin's average q equals marginal q, and the capital asset pricing model (CAPM) holds. Because the properties of our first-best benchmark are so strikingly simple, any interesting new dynamics and properties that our model generates are thus attributed to the *interaction* between imperfect investor protection and limited financial development.

In the full model, investor protection and under-diversification frictions have opposing effects on firm investment. On the one hand, the insider under-invests in illiquid (but

[3]Brainard and Tobin (1968) and Tobin (1969) define the ratio between the firms market value to the replacement cost of its capital stock, as incentive to invest in capital. This ratio has become known as Tobin's average q. Hayashi (1982) provides conditions under which average q is equal to marginal q. Abel and Eberly (1994) develop a unified q theory of investment in neoclassic settings. Lucas and Prescott (1971) and Abel (1983) are important early contributors. See Caballero (1999) for a survey on investment.

productive) business in order to lower the idiosyncratic business risk exposure.[4] On the other hand, the insider has incentives to over-invest in business to pursue private benefits. A forward-looking insider thus has a preference to keep less liquidity (bigger firm size) under weaker investor protection, *ceteris paribus*. Both over-investment and under-investment may thus occur. Intuitively, for an insider with a sufficiently small liquid wealth, her concern about under-diversification outweighs incentives to pursue private benefits, leading to under-investment. In contrast, for sufficiently large liquid wealth, the opposite holds and hence the insider over-invests. Notably, our model also generates predictions on time-varying investment dynamics that are purely due to the frictions, rather than changing investment opportunities.

Furthermore, investor protection and financial under-development significantly affect firms' valuations. With imperfect investor protection, public firm value is unambiguously lower than the first-best value. However, improvement of investor protection reduces agency costs and lowers the private benefit of the insider, which in turn increases the firm's public value. With incomplete markets, the insider's liquidity determines her private firm value. In particular, the increase in the insider's liquidity reduces the under-diversification costs, thereby increasing the private firm value for the insider.

Finally, idiosyncratic business risk affects the risk premium of the insider when financial development is limited. We show that with under-developed financial markets, insiders cannot fully diversity their idiosyncratic business risk, hence the risk premium for outside equity depend on the insider's liquidity. However this relation is not monotone due to nonlinear relations between liquidity and the idiosyncratic risk exposure. Interestingly, idiosyncratic business risk affects private benefit, firms' public value and optimal investment ambiguously when financial markets are incomplete. This happens because higher idiosyncratic risk causes higher saving motive and higher liquidity for the insider; however idiosyncratic risk and insiders' liquidity work in the opposite direc-

[4] Panousi and Papanikolaou (2012) find that the firm's investment falls as its idiosyncratic risk rises, and more so when the manager owns a larger fraction of the firm and hence is more exposed to the firm's non-diversifiable idiosyncratic risk.

tions (idiosyncratic risk negatively while liquidity positively) in determining the private benefit, firm value and investment. Taken together, we show that imperfect investor protection and financial under-development are first-order effects in driving corporate investment and valuations both qualitatively and quantitatively.

Related literature. Our paper links to several strands of literature.We are closely related to the large literature on investor protection and financial development. La Porta et al. (2002) provide a static model to explain their empirical findings of lower firm values in countries with weaker investor protection.Stulz (2005) constructs a twin agency model where rulers of sovereign states and corporate insiders pursue their own interests to explain the limit of financial globalization. Beck, Levine and Loayza (2000) empirically show that financial development positively impacts economic growth through increasing productivity growth. More recently, Albuquerque and Schroth (2010) propose a structural estimation of the block pricing model in Burkart, Gromb and Panunzi (2000). Their estimation fixes a few issues in the literature including endogeneity in share price, the downward bias in treating the block discount as the low realization of the block premium, the selection bias which only selects firms that are traded, etc. Albuquerque and Schroth (2015) derive and estimate a search model of block trades that studies the valuation of the illiquid controlling stakes. Unlike these papers, we study how investor protection and financial development affect corporate investment, public firm valuation for diversified investors, private firm valuation for the under-diversified insider, cost of capital for inside and outside equity, as well as the idiosyncratic risk premium for the insider's equity in a unified dynamic incomplete-markets q-theoretic framework.

It is also useful compare our paper to Albuquerque and Wang (2008) (AW hereafter). AW develop an equilibrium model of investment and asset pricing under imperfect investor protection.They show that the firm over-invests, the cost of capital is higher, and Tobin's q is lower when investor protection is weaker. Unlike our model, AW is a general equilibrium model where financial markets are fully developed, whereas we focus on the case of limited financial development. In particular, since the insider does not fully diversify idiosyncratic risk in our model, she may under-invest if under-diversification cost dominates private benefit; in contrast in AW the insider always over-invests to pursue private benefit.

This paper also contributes to the dynamic corporate finance literature including both investment-based and capital-structure-focused models.[5] Almost all models in this literature assume that either the firm is risk neutral or investors price the firm using a stochastic discount factor. Zwiebel (1996), Morellec (2004), and Lambrecht and Myers (2008) develop dynamic capital structure models with managerial entrenchment, building on Jensen (1986) and Stulz (1990).[6] Unlike existing work, we model the *interactive* effects of managerial agency and risk aversion in a dynamic incomplete-markets-based q theory of investment. Our model distinctively allows us to study the impact of frictions on both marginal q and average q for both the insider and outsiders, corporate investment, as well as the cost of capital for inside and outside equity.[7]

Our paper also relates to the literature on ownership dynamics. Admati, Pfleiderer, and Zechner (1994) develop a model where a risk-averse large shareholder trades off the enhanced incentives to monitor the firm's performance against the increased exposure to the firm's idiosyncratic risk. DeMarzo and Urosevic (2006) develop a dynamic model of ownership for the large shareholder in light of the trade-off between monitoring incentives and diversification. We model the insider's tradeoff between private benefits of control and diversification. As in the literature, the insider also faces time inconsistency in our model.[8] Unlike the existing work in the literature, we explicitly incorporate a cost

[5]See Whited (1992), Gomes (2001), Hennessy and Whited (2007), Riddick and Whited (2009), and Bolton, Chen, and Wang (2011), among others, for models with investment and financial frictions. Fischer, Heinkel, and Zechner (1989), Leland (1994), and Goldstein, Ju, and Leland (2001) are examples of contingent-claim-style capital structure models.

[6]Morellec, Nikolov, and Schürhoff (2011) and Nikolov and Whited (2011) estimate dynamic capital structure models with managerial agency.

[7]Lambrecht and Myers (2011) assume risk-averse managers and generate a Lintner-type payout, but do not study investment dynamics, firm valuation, and the cost of capital for outside equity.

[8]Stoughton and Zechner (1998) study time consistency in a two-period model and consider applications to initial public offering (IPO) underpricing.

function for the insider's ownership adjustments, as often done for equity/debt issuance in dynamic capital structure literature.

Lastly our work also contributes to the large and growing literature on misallocation and economic growth [9] by providing a micro-foundation for sources of capital misallocation. In our model, agency frictions generate a wedge between the insider's "effective" productivity and outside investors' perceived productivity, which causes capital to be misallocated.

1 Model

An entrepreneur manages a firm's operations, owns a fraction of the firm's equity, and is fully entrenched. For these reasons, we also refer to the entrepreneur as the insider or controlling shareholder, as is often done in the investor-protection literature.

When investor protection is imperfect, the entrenched insider inevitably has incentives to pursue private benefits. However, as financial development is often limited (e.g., incomplete markets and borrowing constraints), the insider cannot fully insure himself against his exposure to the firm's idiosyncratic risk exposure and hence has to take into account the impact of non-diversifiable risk on his diversion and investment decisions. The interactions among these different margins yield rich implications on corporate investment and how the insider and outside investors value the firm differently.

Capital Accumulation, Liquidation, and Production. The insider uses capital stock, which we denote by K_t, to produce output.[10] The firm's operating revenue (before investment and other decisions) is proportional to its K_t and given by AK_t, where A is

[9] See, e.g., Foster, Haltiwanger and Krizan (2006), Restuccia and Rogerson (2008), and Hsieh and Klenow (2009) on misallocation and productivity growth, Midrigan and Xu (2014) on misallocation and financial frictions, and more recently David, Schmid and Zeke (2018) on misallocation and asset pricing.

[10] We can easily generalize our model to allow for flexible labor demand and constant return to scale production function in capital and labor.

the firm's productivity. (For simplicity, we take as a constant.)

Let I_t denote the firm's investment at t. The firm's accumulates capital is as follows:

$$dK_t = \Phi(I_t, K_t)dt + \sigma_K K_t d\mathcal{B}_t^K - dX_t \,, \tag{1}$$

where $\mathcal{B}^K$ is a standard Brownian motion, σ_K is the volatility for capital shocks, $\Phi(I_t, K_t)$ measures the efficiency of installing investment goods into the firm's capital stock, and $dX_t \geq 0$ is the asset sale over dt.

We model the costly capital adjustment process by assuming that the efficiency of investment, measured by $\Phi(I, K)$, is concave in investment I.[11] Following Hayashi (1982) and Lucas and Prescott (1971), we assume that $\Phi(I, K)$ is homogeneous of degree one in investment I and capital K. That is, we may write

$$\Phi(I_t, K_t) = \phi(i_t)K_t \,, \tag{2}$$

where $i_t = I_t/K_t$ denotes the investment-capital ratio and $\phi(i)$ is increasing and concave. Capital adjustment costs make productive capital illiquid, which is empirically important.[12]

As in Cox, Ingersoll, and Ross (1985), capital stock in our model is subject to the shock, $d\mathcal{B}_t^K$, over dt. One natural interpretation of this shock is stochastic depreciation as emphasized in Barro (2009) and Brunnermeier and Sannikov (2014).

1.1 Investor Protection

When investor protection is imperfect, the insider can be entrenched with control rights, and inevitably pursues private benefits, which may take a variety of forms such as

[11]Our model builds on the q-theory literature which assumes that capital accumulation involves adjustment costs. See Hayashi (1982), Abel (1983), and Abel and Eberly (1994) on the role of adjustment costs on investment and the value of capital. See Caballero (1999) for a survey. Note that here the capital adjustment cost function is concave in I because it appears in capital accumulation equation. We can equivalently incorporate convex capital adjustment costs in the cash flow equation as in Hayashi (1982).

[12]For example, installing new equipment or upgrading capital may disrupt production lines, and require additional time and resources.

excessive salary, transfer pricing, employing unqualified relatives and friends, just to name a few.[13] As a result, firm profits are not shared on a *pro rata* basis between the entrenched insider and outsiders. In practice, the controlling shareholders can achieve full control of the firm with far less than majority cash flow rights via dual class shares, pyramidal structure, a controlled board, and/or other strategies. In our model, the insiders' discretion does not depend on their cash flow rights.[14]

Insider's Diversion Cost and Firm Output. Without loss of generality, let S_t denote the amount of output diverted by the insider away from the firm at time t. Following the investor protection literature (e.g., La Porta et al. (2002), Johnson et al. (2000), Stulz (2005), and Albuquerque and Wang (2008)), we assume that when diverting amount of S_t, the insider incurs a personal diversion cost

$$\Psi_t = \Psi(S_t, K_t)\,. \tag{3}$$

For tractability, we assume that $\Psi(S_t, K_t)$ is homogeneous with degree one in the gross diversion amount S_t and firm size K_t: $\Psi(S_t, K_t) = \psi(s_t)K_t$, where $s_t = S_t/K_t$ is the scaled diversion. We make the standard assumption that $\psi(s)$ is increasing and convex. Intuitively, the marginal cost of diverting resources is increasing.

The insider controls the firm's resource allocation and the payout rate to outside shareholders is then given by

$$Y_t = AK_t - I_t - S_t = (A - i_t - s_t)K_t\,. \tag{4}$$

Insider's Net Flow Payoff. Let α denote the insider's equity ownership of the firm. In addition to the pro rata share of Y_t, (i.e., αY_t), the insider also diverts amount of S_t

[13]See Barclay and Holderness (1989), Dyck and Zingales (2004), and Albuquerque and Schroth (2010) on the empirical evidence in support of private benefits of control.

[14]We can generalize by allowing the insider's control to be a function of his cash-flow rights, i.e., ownership.

away from the firm and incurs a personal cost of diversion Ψ_t. That is, the insider's net cash-flow is equal to αZ_t, where

$$Z_t = \frac{1}{\alpha}\left[\alpha Y_t + (S_t - \Psi_t)\right] = Y_t + \frac{1}{\alpha}\left(s_t - \psi(s_t)\right) K_t\,, \tag{5}$$

where the second term in the second equality in (5) reflects the wedge between the dividend Y_t and Z_t. By scaling Z_t by K_t, we have

$$z_t \quad = \quad (A - i_t) + \frac{(1-\alpha)s_t - \psi(s_t)}{\alpha}\,. \tag{6}$$

The second term in (6) gives the flow transfer (netting out of the diversion cost and benefit) from outside investors to the insider anticipated by outside investors.

Next, we model financial development along two dimensions: 1.) the degree of market incompleteness, which implies that the firm's idiosyncratic risk cannot be spanned by tradable securities and 2.) and the tightness of borrowing constraints.

1.2 Financial Markets and Development

Both outside investors and the insider can invest in the risk-free asset which pays interest at a constant rate, r, and the risky market portfolio. As in Merton (1971), the stock market's rate of return is (*i.i.d.*) with an expected rate of return μ_M and volatility $\sigma_M > 0$. Let $\mathcal{B}^M$ denote the shock to the market portfolio's return. Let η denote this portfolio's Sharpe ratio, which is given by

$$\eta = \frac{\mu_M - r}{\sigma_M}\,. \tag{7}$$

Let ρ denote the correlation coefficient between the firm's capital shock, $d\mathcal{B}_t^K$, and the market-return shock $d\mathcal{B}_t^M$. Whenever $|\rho| < 1$, unlike diversified outside investors, the insider cannot fully diversify his risk exposures.

Incomplete Markets due to Unspanned Risk. In general, the insider's business risks are only partially spanned by the public market. Let ν denote the portion of the capital stock volatility, σ_K, spanned by the public market portfolio. That is, we have

$$\nu = \rho\sigma_K \,. \tag{8}$$

Similarly, let ϵ denote the *unspanned* volatility of the capital shock

$$\epsilon = \sqrt{1-\rho^2}\,\sigma_K \,. \tag{9}$$

By the orthogonality condition, we have $\sigma_K^2 = \nu^2 + \epsilon^2$.

For a fixed level of systematic volatility ν, the higher the unspanned idiosyncratic volatility ϵ, the larger fraction of the firm's total risk that is not traded. A higher value of ϵ^2/σ_K^2 corresponds to a lower degree of financial development, *ceteris paribus.*

Insider's Wealth Dynamics. Let W and C denote the insider's liquid wealth and consumption, respectively. Let Π denote his allocation in the market portfolio implying that $(W-\Pi)$ is the amount invested in the risk-free asset. The insider can also manage his W_t by liquidating the firm's capital stock at a unit price λ.

Therefore, the insider's financial wealth, W, evolves as follows:

$$dW_t = r\left(W_t - \Pi_t\right)dt + \Pi_t\left(\mu_M dt + \sigma_M d\mathcal{B}_t^M\right) - C_t dt + \alpha Z_t dt + \alpha\lambda dX_t \,, \tag{10}$$

where the first term gives the returns from investments in the risk-free asset, the second term describes the allocation to the risky market portfolio, the third term gives the consumption outflow, the fourth term $\alpha Z_t dt$ gives the insider's *pro rata* net flow payoff, and the last term $\alpha\lambda dX_t$ gives the increase in the insider's personal wealth when the firm liquidates its capital at a unit price of λ. Insiders can divert cash flows but cannot steal stock of capital. The insider and outside shareholders collect their *pro rata* share of the firm's piece-wise liquidation.

Insider's Borrowing Constraints and Debt Capacity. The insider can borrow but is subject to the following borrowing constraint:

$$W_t \geq -\alpha L_t \,, \tag{11}$$

where $L_t > 0$ is the insider's borrowing limit, i.e., debt capacity, for each unit of his ownership in the firm. We assume that the insider's debt capacity, L_t, per unit of ownership is proportional to the firm's size:

$$L_t = \ell K_t \,, \tag{12}$$

where ℓ is the insider's scaled debt capacity as a function of his ownership α. Because the firm liquidates capital at a unit price of λ, we require $\ell \leq \lambda$ to ensure that the insider's debt is risk-free.

1.3 Insider's Optimization and Firm's Market Value

Insider's Optimization Problem. The insider's preferences over consumption $\{C_t : t \geq 0\}$ is given by

$$\mathbb{E}\left[\int_0^\infty e^{-\zeta t}\, U(C_t)\, dt\right] \,, \tag{13}$$

where $\zeta > 0$ is his subjective discount rate and $U(C)$ is increasing and concave. We choose the widely-used constant relative risk-averse (CRRA) utility:

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \,, \tag{14}$$

where $\gamma > 0$ is the coefficient of relative risk aversion.[15] As is standard, the case with $\gamma = 1$ corresponds to the logarithmic utility.

The insider chooses consumption C_t, market-portfolio allocation amount Π_t, corporate investment I_t, diversion S_t, and cumulative capital liquidation X_t to maximize (13) given the output process (4), the firm's capital accumulation (1), the insider's diversion cost function (3), his wealth dynamics (10), and his borrowing constraint (11). Let $F(K, W)$ denote the insider's value function, the solution of this optimization problem.

[15] Our model can be generalized to Epstein-Zin utility, which allows the coefficient of relative risk aversion to be different from the inverse of the elasticity of substitution without losing tractability.

Insider's Value, Certainty Equivalent Wealth, and Insider's Average q. We show that the investors' value function, $F(K,W)$, is given by

$$F(K,W)=\frac{(hP(K,W))^{1-\gamma}}{1-\gamma}\,, \tag{15}$$

where h is a constant given by

$$h=\left[r+\frac{\eta^2}{2\gamma}+\gamma^{-1}\left(\zeta-r-\frac{\eta^2}{2\gamma}\right)\right]^{\frac{\gamma}{\gamma-1}}\,, \tag{16}$$

Importantly, $P(K_t,W_t)$ can be interpreted as the insider's certainty equivalent wealth. In words, $P(K_t,W_t)$ is the amount that the insider would demand to permanently give up his control over the firm and retire as a Merton- style consumer who optimally invests in the market portfolio and the risk-free asset.

Define $V_t^{in}=V^{in}(K_t,W_t)$ as the difference between the insider's certainty equivalent wealth and his financial wealth W, scaled by his ownership stake:

$$V^{in}(K_t,W_t)=\frac{1}{\alpha}(P(K,W)-W)\,. \tag{17}$$

We may interpret V_t^{in} as the insider's subjective (private) valuation of the firm. Scaling V_t^{in} by K_t, we obtain

$$q^{in}(w)=\frac{V^{in}(K,W)}{K}=\frac{p(w)-w}{\alpha}\,. \tag{18}$$

Equation (18) captures the effects of imperfect investor protection and limited FD on the insider's welfare. With limited FD, q^{in} depends not only on the firm's production technology also on the insider's preferences and his investment opportunity.

Before providing the solution of our model, it is helpful to first summarize the insider's optimal diversion decision as this result holds regardless of the degree of financial development and investor protection.

Insider's Optimal Diversion and Net Private Benefits. As the insider's net flow payoff, z_t given in (6), only involves a static tradeoff , the optimal diversion, s_t, solves

$$\max_{s_t}\,(1-\alpha)s_t-\psi(s_t)\,. \tag{19}$$

Naturally, the optimal $s^*(\alpha)$ depends on both α and η:

$$1-\alpha = \psi'(s^*(\alpha)) \longrightarrow s^*(\alpha) = (\psi')^{-1}(1-\alpha)\,. \tag{20}$$

Substituting (20) into (4), we obtain $Y_t = y_t K_t$, where

$$y_t = A - s^*(\alpha) - i_t \tag{21}$$

is the scaled payout to an outside investor with one share of equity. By substituting (20) into (6), we obtain the following payout rate to the insider:

$$z_t = (A + b^*(\alpha)) - i_t\,, \tag{22}$$

where $b^*(\alpha)$ is the insider's net private benefit given by

$$b^*(\alpha) = \frac{1-\alpha}{\alpha}\left(s^*(\alpha) - \frac{\psi(s^*(\alpha))}{1-\alpha}\right)\,, \tag{23}$$

and $s^*(\alpha)$ is the insider's diversion rule given in (20). The wedge between z_t given in (22) and y_t given in (21) describes the impact of agency on the flow payoff.

We may interpret $A + b^*(\alpha)$ as the "effective" return on capital for the insider and $A - s^*(\alpha)$ as the "effective" return on capital for outside investors. The return-on-capital wedge between the insider and outside investors, $b^*(\alpha) - s^*(\alpha)$, induced by agency, has first-order effects on investment and different valuations for a share of the firm's equity by the two types of agents.

Naturally, under perfect investor protection because diversion is prohibitively expensive even for any tiny amount, meaning $\psi(s^*) \to \infty$ for any $s^* > 0$, the insider chooses $s^*(\alpha) = 0$ and hence $b^*(\alpha) = 0$.

1.4 Firm's Market Value and Tobin's Average q: Outside Investors' Perspective.

Unlike the insider, well diversified outside investors only demand systematic risk premium. Although they have no control rights, they are rational and take into account the insider's incentives when valuing the firm. In equilibrium, the insider bears the cost of the agency cost.

Technically speaking, outside investors use the stochastic discounted factor (SDF) to value financial claims. Let $\mathbb{M}_t$ denote the SDF. As the only source of the aggregate shock is $\mathcal{B}_t$ and the equilibrium risk-free rate is constant, we may write down the SDF as follows (e.g., Duffie, 2001):

$$\frac{d\mathbb{M}_t}{\mathbb{M}_t} = -r dt - \eta d\mathcal{B}_t^M \,. \tag{24}$$

Here, by no arbitrage, the drift of the SDF is equal to $-r$ and the market price of risk for the SDF (the negative of the SDF volatility) is equal to the market portfolio's Sharpe ratio, η, given by (7).

Let V_t^{out} denote the firm's public equity value at t. Taking the SDF specified in (24) and the insider's optimal decisions as given, outside investors price the firm's equity as follows:

$$V_t^{out} = \mathbb{E}_t \left(\int_t^{\infty} \frac{\mathbb{M}_s}{\mathbb{M}_t} Y_s \, ds \right) \,, \tag{25}$$

where the SDF $\mathbb{M}$ captures both the time value of money and the risk premium. Tobin's average q for the firm from outside investors' perspective is then given by

$$q_t^{out} = \frac{V_t^{out}}{K_t} \,. \tag{26}$$

While we can use the SDF to value publicly traded equity held by outside investors, we cannot use the SDF to assess how much the insider values his equity claim even if we take into account the net private benefits accrued to the insider. This is because the insider is not well diversified and cares about the firm's idiosyncratic risk. For this reason, we have to value inside equity based on his utility optimization.[16]

[16]This result in our model differs from that in Albuquerque and Wang (2008), where both the insider's and the outsiders' equity claims are price with the same unique SDF, a general-equilibrium outcome in their model.

2 Full Financial-Development Benchmark

In this section, we provide the solution for the full-FD case. By full FD, we mean that all agents can trade a complete set of Arrow-Debreu securities at actuarially fair terms.[17] As a result, the insider can fully hedge his idiosyncratic risk exposures and hence is only exposed to the systematic component of the firm's risk.

This insider's welfare in this full-FD benchmark serves as the upper bound for the insider's welfare. Also, with this benchmark, we can decompose the welfare loss into a pure agency component and the remaining part, which is due to limited FD.

Corporate Investment and Insider's Value of Firm Equity. With full FD, the insider can completely hedge out his idiosyncratic risk exposure and hence we can apply Arrow-Debreu complete-markets solution approach.[18] By using this approach, we show that the insider's original joint (consumption, portfolio-choice, firm investment, and diversion) optimization problem summarized in Section 1.3 can be decomposed into two independent problems for the insider: a firm-side problem (involving capital accumulation) and a consumer-side problem (involving consumption and portfolio choice).

As the insider's diversion decision cannot be contracted away (due to incomplete contracting assumption in our model), the equilibrium solution is still inefficient. Indeed, in Section 1.3, we have already summarized the solution for the diversion decision for the general case including this full-FD special case.

As we show, the firm-side problem boils down to the following one where the firm

[17]In our model, it is sufficient to introduce a new financial asset whose return is purely driven by a diffusion shock, which is orthogonal to the shock to the market portfolio $d\mathcal{B}_t^M$. Together with the the market portfolio, this financial asset fully spans, $d\mathcal{B}_t^K$, the shock to the firm's capital stock. As we assume that the only source of the aggregate shock in our model $\mathcal{B}_t^M$, the equilibrium rate of return for this newly introduced financial asset is equal to the risk-free rate, r. See Appendix for details.

[18]For Arrow-Debreu analysis in continuous-time formulations, see Duffie and Huang (1985) and Cox and Huang (1989). See Duffie (2001) for a textbook treatment.

chooses an optimal constant i to maximize:

$$\mathbb{E}_t\left(\int_t^\infty e^{-r^*(s-t)}Z_s ds\right) = \int_t^\infty e^{-r^*(s-t)}z(i)K_t e^{\phi(i)(s-t)}ds\,. \tag{27}$$

The first equality in (27) is the definition and the second equality follows from several key features of the model: (i) the homogeneity properties for the firm's capital accumulation and production technology as in (Hayashi, 1982); (ii) the constant investment opportunity set over time (e.g., constant productivity A); and (iii) full FD. These properties jointly imply that the optimal investment-capital ratio, i_t, is constant over time and hence we turn a stochastic optimization problem into a deterministic one.[19] With full FD, the value function obtained above is equal to V_t^{in} defined in (17).

Let i^* denote the optimal i_t. Integrating (27) yields $V_t^{in} = q_*^{in}K_t$, where

$$q_*^{in} \;=\; \max_i \;\frac{z(i)}{r_V^* - \phi(i)} = \frac{A + b^*(\alpha) - i^*(\alpha)}{r_V^* - \phi(i^*(\alpha))}\,, \tag{28}$$

and the optimal level of i^* is the maximand of the first expression in (28). We refer to q_*^{in} as the insider's average q under full FD. For clarity, we make the dependence of q_*^{in} on the insider's ownership explicit by writing q_*^{in} as $q_*^{in}(\alpha)$.

Equation (28) is the widely-used Gordon growth formula but for the insider: The numerator in (28) is equal to $z(i^*) = A + b^*(\alpha) - i^*$, the scaled flow payoff to the insider per unit of equity ownership, which includes both the cash flow rights and also the private benefits. The denominator in (28) is equal to the difference between the discount rate, r_V^*, and the expected growth rate of Z_t, $\phi(i^*)$.

Because the insider is fully entrenched, the insider's return on capital is $A + b^*(\alpha)$ exceeding A. As a result, he over-invests compared with the first-best outcome. This is an example where capital is mis-allocated between investment and payouts (which shareholders can use either for consumption or other investment activities.)

By using dynamic programming, we can also show that investment satisfies the FOC:

$$q_*^{in}(\alpha) = \frac{1}{\phi'(i^*(\alpha))}\,, \tag{29}$$

[19] A key step is to recognize that $\mathbb{E}_t[K_s] = K_t e^{\phi(i)(s-t)}$.

which equates the insider's marginal benefit of investing (the insider's q) with his marginal cost of investing $1/\phi'(i^*(\alpha))$. Corporate investment is chosen with the insider's future diversion benefits fully taken into account.

Insider's "Total" Wealth and Consumption. As we have discussed earlier, with full FD, the insider's consumption problem can be separated out from the firm's problem, because the insider hedges all his idiosyncratic risk and chooses an efficient mean-variance exposure to the systematic risk for his entire wealth portfolio.[20]

Therefore, the insider's certainty equivalent wealth, which is also equal to his "total" wealth, is given by $P^*(K_t, W_t) = W_t + \alpha V^{in}(K_t) = (w_t + \alpha q_*^{in})\, K_t$, where $w_t = W_t/K_t$. That is, the insider's total wealth is additively separable. Second, the insider's optimal consumption follows the standard Merton's rule in that he consumes a constant fraction of his total wealth, i.e., $C^*(K_t, W_t) = mP^*(K_t, W_t)$ and $c^*(w_t) = C^*(K_t, W_t)/K_t$, where

$$c^*(w_t) = mp^*(w_t) = m\left(w_t + \alpha q_*^{in}\right) . \tag{30}$$

Here, m is the marginal propensity to consume (MPC) under full FD an is given by

$$m = r + \frac{\eta^2}{2\gamma} + \gamma^{-1}\left(\zeta - r - \frac{\eta^2}{2\gamma}\right) , \tag{31}$$

as in Merton (1971). See Appendix for the optimal hedging and portfolio choices supporting the insider's consumption and firm investment decisions.

Outside Investors' Value of Firm Equity. With full FD, the key difference for the two types of agents is that the (scaled) flow payoff: for the insider, it is equal to z given in (22) and for outside investor it is equal to y given in (21). Recall that the cost of capital is the same for the two types.[21]

[20] Technically speaking, the insider only has one budget equation under full FD. We thus can solve the insider's optimization problem by using either the Arrow-Debreu complete-markets approach (with one budget constraint) or the standard dynamic programing method, both of which are summarized in the Appendix.

[21] Our derivation for outside investors' valuation is essentially the same as that for the insider. Hence, we skip the details and summarize our results for the outside investors' valuation.

The firm's total market capitalization is $V_t^{out} = q_*^{out} K_t$, where q_*^{out} is the Tobin's average q for outside investors:[22]

$$q_*^{out}(\alpha) = \frac{A - s^*(\alpha) - i^*(\alpha)}{r_V^* - \phi(i^*(\alpha))} \,. \tag{32}$$

Non-contractible diversion by the insider distorts the firm's investment away from the first-best, therefore, the firm's market value is lower for both direct diversion and also indirect inefficient investment reasons. Outside investors receive their pro rata share $(1-\alpha) q_*^{out} K_t$.

First-Best (FB) Benchmark and Welfare Comparison with Full-FD Case. The FB benchmark is a special case of the full-FD case when investor protection is perfect. For the FB benchmark, there are no frictions at all and therefore the insider diverts nothing: $s^*(\alpha) = 0$. The insider and outside investors value the firm in the same way at $q^{FB} K_t$, where q^{FB} is the Tobin's average q under the FB:

$$q^{FB} = \frac{y^{FB}}{r_V^* - \phi(i^{FB})} = \frac{1}{\phi'(i^{FB})} \,. \tag{33}$$

Here, the FB investment-capital ratio i^{FB} can be solved by using the second equality in (33), $z^{FB} = y^{FB} = A - i^{FB}$. Note that with full FD, the level of investor protection has no impact on the firm's cost of capital.

Lacking investor protection causes the insider to prefer investing over paying out to shareholders as he collects a larger private benefits from a larger firm, causing $i^* > i^{FB}$ and the insider's Tobin's average q to be larger than q^{FB}, i.e., $q_*^{in} > q^{FB}$. Diversion and excessive investment cause an *ex post* wealth transfer from outside investors to the insider, i.e., $q^{FB} > q_*^{out}$ as outside investors rationally price the insider's agency. Therefore, in equilibrium, it is the insider who ultimately *ex ante* bears the cost of lacking strong investor protection. The following inequality holds:

$$q_*^{in} > q^{FB} > q_*^{out} \,, \tag{34}$$

[22] This q is the empirically measured Tobin's average q if our full-FD case were the (true) model generating the data.

where q_*^{in} and q_*^{out} are given by (28) and (32), respectively.

Under full FD, a natural measure of the aggregate welfare is the ownership-weighted average of the insider's Tobin's average q and outside investors' average q, as all risks can be hedged and hence total valuation for the firm is a reasonable measure of welfare. With imperfect investor protection even with full FD, the insider and outside investors cannot credibly contract with each other in the long term to achieve the first-best outcome which involves zero diversion and the first-best investment level, i^{FB}. As a result, there is a net welfare loss, in that

$$\alpha q_*^{in} + (1-\alpha) q_*^{out} < q^{FB} \,. \tag{35}$$

In our full-FD case, the lower the degree of investor protection, the more excessive the firm's investment, the more inefficient the resource allocation and the larger the gap in the preceding inequality.

Our model shows that another channel through which resources are mis-allocated in the economy is that the outside investors perceive the firm's productivity (netting out of agency) as $A - s^*$, but the insider, who makes the decision, effectively treats the firm's productivity (including his private benefits) as $A + b^*$. The wedge between the insider's "effective" productivity and outside investors' perceived productivity (netting out of agency) causes resource to be misallocated.

As Albuquerque and Wang (2008), our full-FD case allows us to isolate the sole effect of imperfect investor protection on the insider's and outsiders' valuation of the firm's equity. As a result, this full-FD case helps us to better understand the mechanism and decompose the effects of investor protection and financial development when both frictions are at play. In our full-FD case the insider's w_t is stochastic, while $w_t = 0$ in Albuquerque and Wang (2008) due to the general-equilibrium no-trade result between the two types of agents. As one can see, our full-FD case and Albuquerque and Wang (2008) focus on different economic dynamics and variables.

We show that the cost of capital for the insider is equal to that for outside investors, despite the fact that the former has control rights while the latter do not.

CAPM, Cost of Capital for Insider and Outside Investors, and SDF. Let r_V^* denote the same cost of capital for both the insider and outside investors with full FD. We use $*$ as the superscript to denote the solution associated with the full-FD case. In Appendix, we show that CAPM gives the correct cost of capital for the firm's equity for both insider and outside investors:

$$r_V^* = r + \beta_V^* \left(\mu_M - r\right) \,, \tag{36}$$

where the firm's beta under full FD, β_V^*, is given by

$$\beta_V^* = \frac{\rho\sigma_K}{\sigma_M} = \frac{\nu}{\sigma_M} \,. \tag{37}$$

This is intuitive. If the aggregate component of the firm's volatility is less than the market portfolio's volatility, the firm's beta under full FD is less than one, $\beta_V^* < 1$.

In general, agency costs have a negative effect on firm value due to diversion and investment distortions. However, the effect of agency on the expected return and beta for the firm is not obvious at all. Indeed, in our full-FD case, agency has no impact at all on β, as seen from (36) and (37). Therefore, the insider and outside investors use the same cost of capital to value the firm. Relatedly, the insider prices the firm's cash flows using the same SDF as outside investors do. The reason that they value the same firm differently is because they receive different net flow payoffs.

Next, we analyze the general case with imperfect investor protection and limited financial development.

3 Solution: Limited Financial Development

With limited FD, the insider can no longer completely hedge his exposure to the firm's idiosyncratic risk. Lacking of diversification significantly alters not just the insider's portfolio choice but also his consumption and corporate investment decisions.

Portfolio Choice. The scaled investment in the market portfolio is given by

$$\pi(w) = \frac{\eta}{\sigma_M}\frac{p(w)}{\gamma^{in}(w)} - \frac{\nu}{\sigma_M}\left(\frac{\gamma p(w)}{\gamma^{in}(w)} - w\right) = \frac{\eta}{\sigma_M}\frac{p(w)}{\gamma^{in}(w)} - \beta_V^*\left(\frac{\gamma p(w)}{\gamma^{in}(w)} - w\right), \quad (38)$$

where β_V^* is the firm's beta under FD given in (36) and

$$\gamma^{in}(w) \equiv -\frac{F_{WW}}{F_W} \times P(K,W) = \gamma p'(w) - \frac{p(w)p''(w)}{p'(w)}. \quad (39)$$

We interpret γ^{in} as a measure of the insider's relative risk aversion.[23]

The first term in (38) gives the mean-variance demand where γ^{in} plays the role of the CRRA coefficient as in Merton (1971). Because of his concentrated ownership in the firm and the firm's β, the insider inevitably has an excessive exposure to the aggregate risk if he were to simply follow Merton's mean-variance asset-allocation rule. In order to bring his total risk exposure to the aggregate market risk in line with his effective risk aversion, the insider reduces his allocation to the stock market portfolio by the second term in (38), which is the insider's hedging demand due to his concentrated equity ownership in the firm.

Insider's Consumption and Corporate Investment. The optimal consumption rule is given by

$$c(w) = mp(w)(p'(w))^{-1/\gamma}, \quad (40)$$

where m is the MPC under full FD and is given by (31). In general, $c(w_t)$ is nonlinear in w_t and significantly deviates from the full-FD linear solution, especially when the insider's q is low.

[23]First, $-F_{WW}/F_W$ measures the insider's level of absolute risk aversion by using his value function. By multiplying $-F_{WW}/F_W$ with an appropriate measure of the insider's "net worth" measure, we obtain a measure of the insider's relative risk aversion. With incomplete markets, there is no well-defined market measure for the insider's "total" wealth. The insider's certainty equivalent wealth $P(K,W)$ seems a natural measure of the insider's welfare in the unit of "total" wealth. This motivates our definition of γ^{in} in (39). Because of limited FD and borrowing constraints, the insider's relative risk aversion, γ^{in}, is generally greater than γ. Indeed, $\gamma^{in} = \gamma$ only when $w \to \infty$ or FD is full.

The optimality condition for investment equates $F_K(K,W)\Phi_I(I,K)$, which is the insider's marginal (utility) value of investing, with his marginal (utility) cost of investing, $\alpha F_W(K,W)$, where α reflects his ownership of the firm. That is, $F_K(K,W)\Phi_I(I,K) = \alpha F_W(K,W)$. Using the homogeneity property to simplify the investment FOC gives

$$\phi'(i(w)) \cdot (p(w) - wp'(w)) = \alpha p'(w)\,. \tag{41}$$

The left side of (41) is the product of marginal efficiency of investing $\phi'(i)$ and the insider's marginal certainty equivalent value of capital $P_K(K,W) = p(w) - wp'(w)$. The right side is the insider's marginal certainty equivalent wealth of liquidity $\alpha P_W = \alpha p'(w)$. With full FD, $p'(w) = 1$ as the insider's marginal cost of investing is only his equity share α. Next, we report the dynamics for the key state variable, w, the ratio between the insider's wealth W and the firm's capital stock K.

Dynamics of w_t. By using Ito's formula, we obtain the following dynamics of w_t:

$$dw_t = \mu_w(w_t)dt + \pi_t\sigma_M d\mathcal{B}_t^M - w_t\sigma_K d\mathcal{B}_t^K + (w_t + \alpha\lambda)dx_t\,, \tag{42}$$

where $dx_t = dX_t/K_t$ and $\mu_w(w_t)$ is given by

$$\mu_w(w_t) = \left[r - \phi(i_t) + \sigma_K^2\right] w_t + \alpha z_t + \pi_t(\mu_M - r - \rho\sigma_K\sigma_M) - c_t\,. \tag{43}$$

Next, we characterize the insider's welfare, measured by the insider's (scaled) certainty-equivalent wealth.

3.1 Scaled Certainty Equivalent Wealth $p(w)$.

The insider's scaled certainty equivalent wealth $p(w)$ satisfies the following ordinary differential equation (ODE):

$$0 = \left[\frac{\gamma m(p'(w))^{1-\frac{1}{\gamma}} - \zeta}{1-\gamma} + \phi(i(w)) - \frac{\gamma\sigma_K^2}{2}\right] p(w) + \alpha[A + b^*(\alpha) - i(w)]p'(w)$$

$$+ \left(r_V^* - \phi(i(w)) + \gamma\epsilon^2\right) wp'(w) - \gamma^{in}(w)\frac{\epsilon^2 w^2}{2}\frac{p'(w)}{p(w)} + \frac{(\eta - \gamma\nu)^2 p'(w)p(w)}{2\gamma^{in}(w)}\,, \tag{44}$$

where $\gamma^{in}(w)$ measures the insider's endogenous risk aversion, given by (39), and $i(w)$ satisfies (41).

The Full-FD Solution as the Boundary Condition as $w \to \infty$. When the insider has infinity wealth relative to the firm's size K, i.e., when $w \to \infty$, the insider can perfectly insure against his exposure to the firm's idiosyncratic risk via his savings. Therefore, as $w \to \infty$, we have the following limiting solution:

$$\lim_{w \to \infty} p(w) = p^*(w) = w + \alpha\, q_*^{in}(\alpha)\,, \tag{45}$$

where $q_*^{in}(\alpha)$ is given by (28). That is, his scaled certainty equivalent wealth $p(w)$ is equal to $p^*(w)$ under full FD, which is additively separable in w and his personal value of his equity stake in the firm, $\alpha\, q_*^{in}(\alpha)$. Note that in this case, there is no value discount due for the insider as there is no cost for him to bear idiosyncratic risk. Hence, his subjective valuation of the firm is $\alpha\, q_*^{in}(\alpha)$, where $q_*^{in}(\alpha)$ is his valuation under full FD.

The Binding Borrowing Constraint as the Left-end Boundary Condition. When exhausting his debt capacity, i.e., when $W_t = -\alpha L_t$, the insider honors his liability by selling capital. By selling a unit of capital, the firm receives λ unit in cash and thus the insider receives his pro rata share, $\alpha\lambda$. The continuity of the insider's value function before and after his piece-wise liquidation of capital requires

$$F(K, -\alpha L - \alpha\lambda\Delta K) = F(K - \Delta K, -\alpha L)\,. \tag{46}$$

And then by taking $\Delta K \to 0$, we have $\alpha\lambda F_W(K, -\alpha L) = F_K(K, -\alpha L)$, which implies that $\alpha\lambda P_W(K, -\alpha L) = P_K(K, -\alpha L)$. By using the homogeneity property, we know that $P_W = p'(-\alpha\ell(\alpha))$ and $P_K = p(-\alpha\ell(\alpha)) + \alpha\ell(\alpha)p'(-\alpha\ell(\alpha))$. Therefore, using scaled variables, we have

$$p(-\alpha\ell(\alpha)) = \alpha(\lambda - \ell(\alpha))p'(-\alpha\ell(\alpha))\,. \tag{47}$$

We summarize the solution as follows. The optimal diversion $s^*(\alpha)$ is given by (20), the optimal corporate investment $i(w)$ satisfies (41), the insider's optimal asset allocation rule $\pi(w)$ is given by (38), and his optimal consumption rule $c(w)$ solves (40). Finally, the scaled certainty equivalent wealth $p(w)$ solves the ODE given by (44) subject to the boundary conditions (45) and (47).

3.2 Firm Value (by Outside Investors) and Tobin's Average q

Next, we calculate the firm's total market capitalization, which we denote by $V(K,W)$, from the perspective of well diversified outside shareholders. To calculate firm value for outside investors, we need to take into account not only the insider's corporate investment and diversion decisions, but also his personal portfolio considerations.

Next, we summarize the mode's implications for the firm's market value. The scaled firm value, $q^{out}(w) = V^{out}(W,K)/K$, for outside investors solves the ODE,

$$\begin{aligned}(r_V^* - \phi(i(w)))q^{out}(w) \quad &= \quad y(w) + [(r_V^* - \phi(i(w)))w - c(w) + \alpha z(w)]q_w^{out}(w) \\ &\quad + \frac{\sigma_M^2 \pi(w)^2 + \sigma_K^2 w^2 - 2\nu\sigma_M \pi(w) w}{2} q_{ww}^{out}(w)\,, \qquad (48)\end{aligned}$$

subject to the following boundary conditions:

$$q^{out}(-\alpha\ell(\alpha)) \quad = \quad \alpha(\lambda - \ell(\alpha))q_w^{out}(-\alpha\ell(\alpha)) + \lambda\,, \qquad (49)$$

$$\lim_{w\to\infty} q^{out}(w) \quad = \quad q_*^{out}\,. \qquad (50)$$

The ODE (48) is the valuation equation for the firm's market value from outside investors' perspective. The scaled payout is y_t, which is a function of w_t.

4 Quantitative Analysis

In this section, we explore the quantitative implications of imperfect investor protection and limited financial development for corporate valuation and investment.

4.1 Data

The data are from Thomson Reuters Worldscope database. A company is identified by Worldscope permanent ID (ITEM6105). Country is identified by nation (ITEM6026) in which the company is domiciled and matched to the country-level index described below.[24] We exclude financial firms (SIC code from 6000 to 6999) and regulated firms (SIC code from 3730 to 3743 and from 4900 to 4999). We further exclude firm-year observations with fiscal year-end price less than 0.01 in native currency or percentage of closely held shares greater than 100. A firm needs to have more than one observation in the period to be included in the sample. We winsorize variables at 2% level at both ends for each country. For the sample period of 1989 to 2016, we have 467802 observations.

The inside ownership is proxied by the percentage of closely held shares (ITEM8021). We supplement the inside owner with number of closely held shares (ITEM5475) divided by common shares outstanding multiplied by 100 whenever the percentage is missing. We calculate Tobin's average q as firm's market value divided by 0.9 times firm's book value plus 0.1 times firm's market value, in which market value is market value of equity plus book value of total debt (ITEM3255), and book value is firm's total asset (ITEM2999).[25] Firm's market value of equity is calculated using the fiscal year end price (ITEM5085) multiplied by fiscal year end common shares outstanding (ITEM5301). Returns are calculated as fiscal year end price (ITEM5085) plus dividend per share (ITEM5110) divided by last period fiscal year end price. We estimate the country idiosyncratic volatility as the average firm idiosyncratic stock return volatility which is estimated as the CAPM residual by using the US market portfolio as the market factor. The total volatility is the average firm stock return volatility in each country.

As in Wurgler (2000), we calculate the composite investor protection index by using the legal rights of external investors from LLSV (1998). Specifically, we multiply the sum of anti-director index (0-6) and creditors rights index (0-4) by the LLSV measure of

[24] If we identify country by nation code (ITEM6027) instead, the results do not change.
[25] This approach is used by Baker, Stein, and Wurgler (2003).

the domestic rule of law (0-1). We estimate the dollar value of private benefits (divided by firm value) for countries by using the estimates from Dyck and Zingales (2004).

4.2 Parameter Choices and Calibration

Our calibration strategy explores the cross-country variation in investor protection, as in Wurgler (2000), Stulz (2005), and Pinkowitz, Stulz and Williamson (2006). We divide countries into two groups by sorting them on their composite investor-protection indices: weak and strong investor-protection groups. If a country's index is below the median value of the indices for all countries in Worldscope, it belongs to the weak investor-protection group. Otherwise, it belongs to the strong group.

Next, we specify the functional form of the insider's diversion cost $\psi(s)$. Following LLSV (2002), Shleifer and Wolfenzon (2002), and Albuquerque and Wang (2008), we assume that the insider's diversion cost $\psi(s)$ is quadratic in s,

$$\psi(s) = \frac{\theta_s}{2} s^2 , \tag{51}$$

where θ_s, the key parameter in our analysis, measures the exogenous variation of investor protection in the model. A higher value of θ_s means stronger investor protection.

As in the q-theory of investment literature, we specify the capital adjustment cost function $\phi(i)$ as follows:

$$\phi(i) = \frac{n_i}{1-\theta_i^{-1}} i^{1-\theta_i^{-1}} , \tag{52}$$

where $n_i > 0$ and $\theta_i > 0$.

To identify the two capital adjustment-cost parameters, n_i and θ_i, common to all countries, and the two different values for the investor-protection parameter θ_s for the weak and strong investor-protection groups, we target four key moments on valuation and agency costs in the data. These four moments are Tobin's average q, (q^{out}), for outside investors and the ratio of the dollar value of net private benefits to firm value ($\frac{q^{in}-q^{out}}{q^{out}}$) of weaker and stronger IP countries, respectively. We obtain $n_i = 0.054$,

$\theta_i = 1.4$, $\theta_s = 17.7$ for weaker investor-protection countries and $\theta_s = 25.3$ for stronger IP countries.

To highlight the effect of investor protection, we calibrate all the other parameters to the same value for all countries. We calibrate θ_s, together with the elasticity of the investment-capital ratio and the capital adjustment cost parameters which do not vary with investor protection, to match key investor-protection moments on valuations and agency costs of these two groups. All other parameters are chosen based on either the estimates from the related literatures or the calibration of prior studies.

Table 1 summarizes all variables and baseline parameter values used in the paper. We set the risk-free rate at $r = 4\%$ close to Boldrin, Christiano and Fisher (2001), and the insider's subjective discount rate ζ to 8%, so that the insider's wealth distribution is within a reasonable range. We set the insider's coefficient of relative risk aversion at $\gamma = 2$, a widely used value in the literature, e.g., Hansen and Singleton (1982) and Mehra and Prescott (1985).

For the market-portfolio return, we follow the standard practice by setting the equity risk premium at $\mu_M - r = 6\%$ and the annual return volatility at $\sigma_M = 20\%$, which implies the Sharpe ratio of $\eta = (\mu_M - r)/\sigma_M = 30\%$, consistent with the estimates in Campbell and Cochrane (1999) and Bansal and Yaron (2004). Using the sample of large firms in Compustat from 1981 to 2003, Eberly, Rebelo, and Vincent (2009) provide empirical evidence in support of Hayashi (1982). Using their work as a guideline, we set the expected productivity $A = 12.3\%$.

We set the liquidation value λ per unit of capital at 0.7 and the insider's debt capacity for each unit of capital, ℓ, to 0.3, close to Riddick and Whited (2009) and Bolton, Chen, and Wang (2011). We set the correlation coefficient between the capital shock and the market portfolio to $\rho = 0.8$, consistent with the estimated average ratio of idiosyncratic volatility to total volatility across countries in Worldscope. The insider's equity ownership is set at $\alpha = 33\%$, which is the average of the insider's equity ownership

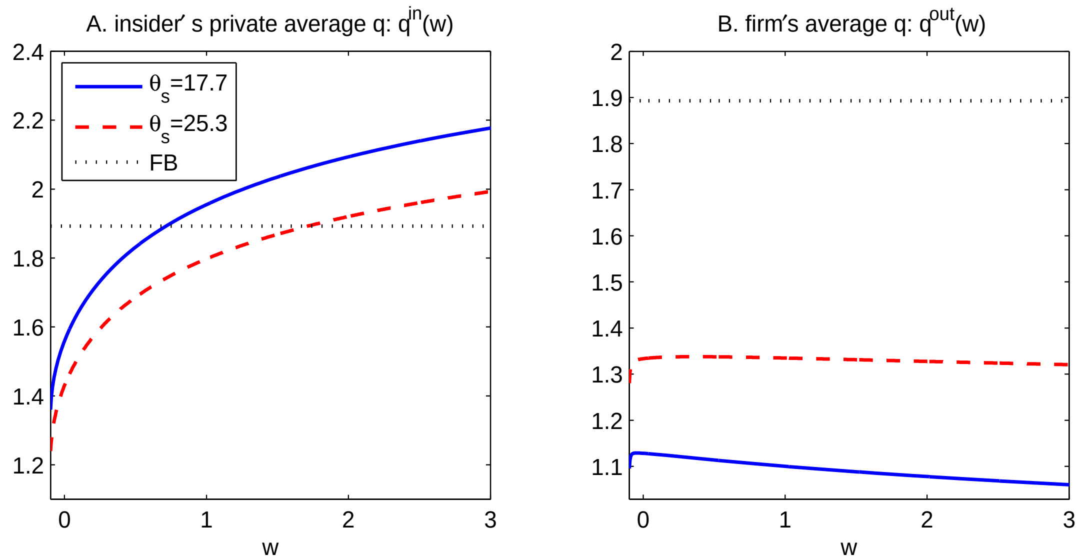

Figure 1: Insider's private average $q^{in}(w)$ and public firm value $q^{out}(w)$. The parameter values are: $A = 0.123$, $r = 4\%$, $\zeta = 8\%$ $\gamma = 2$, $\mu_M - r = 6\%$, $\sigma_M = 20\%$, $\alpha = 0.33$, $\sigma_K = 33\%$, $\lambda = 0.7$, $\ell = 0.3$, $\rho = 0.8$, $n_i = 0.054, \theta_i = 1.4$.

in the firm they control for all countries in the data.[26] We set the total volatility of capital shock, σ_K, to 33% to match the average sales growth volatility of 33% across countries.

4.3 Value Functions and Policy Rules

We now explore the model implications of imperfect investor protection and limited financial development on firm valuations, insiders' wealth, and firm investment.

4.3.1 Insider's Valuation

We study the insider's valuation, i.e., the private average q, $q^{in}(w)$, in Panel A of Figure 1. First, as the insider's scaled liquid wealth w increases, his private average q, $q^{in}(w)$, increases. This is because the insider's idiosyncratic business risk exposure decreases with w, which also causes the insider's under-diversification cost to decrease, leading

[26]The average ownership of equity is 32% in low investor protection countries and 34% in high investor protection countries in the data.

Table 1: Summary of Key Variables and Parameters

This table summarizes the symbols for the key variables used in the model and the parameter values.

Variable	Symbol	Parameter	Symbol	Value
Capital stock	K	Productivity	A	0.123
Business investment	I	Volatility of capital stock	σ_K	33%
Capital adjustment cost	Φ	Capital adjustment cost	n_i	0.054
Diversion cash	S	Elasticity of investment-capital ratio	θ_i	1.4
Controlling shareholder's diversion cost	Ψ	Investor protection parameter	θ_s	$17.7, 25.3$
Controlling shareholder's private value	P	Risk-free rate	r	4%
Controlling shareholder's value function	F	Equity risk premium	$\mu_M - r$	6%
Cash flow to controlling shareholder	Z	Market portfolio volatility	σ_M	20%
Market portfolio	S	Market portfolio Sharpe ratio	η	30%
Controlling shareholder's liquid wealth	W	Ownership of equity	α	33%
Firm value	V^{out}	Correlation(capital/market)	ρ	80%
Cumulative piece-wise liquidation amount	X	Piece-wise liquidation price	λ	0.7
Utility function	U	Scaled debt capacity	ℓ	0.3
Consumption	C	Risk aversion	γ	2
Portfolio allocation	Π	Subject discount rate	ζ	4%
Public Tobin's average q	q^{out}			
Insider's private average q	q^{in}			

to an increase in $q^{in}(w)$. Thus idiosyncratic business risk negatively while the private benefit driven by diversion positively affects $q^{in}(w)$. Specifically, we see that when the insider's liquidity is sufficiently low, e.g. $w < 0.721$ for $\theta_s = 17.7$, the effects of idiosyncratic business risk dominates, thus the insider's private average q is lower than that under First-best case, $q^{FB} = 1.893$. In contrast, when $w > 0.721$, the effects of private benefit dominates causing the insider's private q higher than that under First-best case. Moreover, for sufficiently high liquid wealth ($w \to \infty$), the idiosyncratic risk is negligible for the insider. Hence, $\lim_{w\to\infty} q^{in}(w) = q^{in}_*(\alpha)$, where $q^{in}_*(\alpha)$ is the full FD solution of Section 2.

Second, the higher investor protection, i.e., the bigger the value of θ_s, the lower insider's private average q, i.e., $q^{in}(w)$. This is intuitive: higher IP results in lower net private benefit $b^*(\alpha)$ for the insider due to lower insider's diversion, $s^*(\alpha)$.

4.3.2 Public Firm Value and Average q

Panel B of Figure 1 plots public average q, $q^{out}(w)$. We see that $q^{out}(w)$, in both weaker IP ($\theta_s = 17.7$) and strong IP ($\theta_s = 25.3$) cases, is lower than the first-best benchmark value $q^{FB} = 1.893$ due to agency costs. In the limit as $w \to \infty$, the insider faces no idiosyncratic business risk, the average q approaches the full FD value $q^{out}(w)$, $\lim_{w\to\infty} q^{out}(w) = q^{out}_*$ (Not plotted out).

Furthermore, we find that average q, $q^{out}(w)$, first increases and then decreases in firms' liquid wealth w. This is in sharp contrast to the private average q, $q^{in}(w)$, which monotonically increases in w. When liquid wealth is large, in particular, for $w > 2.60$, as w increases, the idiosyncratic business risk decreases causing the insider to overinvest, which hurts the value for outsiders due to the increase in agency costs led by the over-investment. On the other hand, when liquid wealth is small, e.g., for $w < 2.60$, public average q, $q^{out}(w)$, increases in w as the insider's over-investment motive is mitigated by the under-diversification costs.

Lastly, as expected, the higher IP (higher θ_s), the higher firm's average q, $q^{out}(w)$, due to the larger "effective" return on capital, $A - s^*(\alpha)$, for outside investors. Notably, the effects of IP on the firm's average q are first-order and quantitatively large. For example, when θ_s increases from 17.7 to 25.3, the firm's average q, $q^{out}(w)$, increases by about 20% on average.

4.3.3 Firm Investment and the Insider's Consumption

Turning to investment policies, Panel A of Figure 2 plots investment-capital ratio, $i(w)$. We first note that in our model, under-diversification cost together with the flexible piece-wise liquidation option and investor protection jointly generate significant variation in investment-capital ratio $i(w)$ when insider's liquid wealth changes. This happens despite of a constant-returns-to-scale production technology as in Hayashi (1982), which usually implies constant investment-capital ratio.

Furthermore, compared with the First-best level, both over- and under-investment can occur. Intuitively, private benefits of control lead to over-investment whereas under-diversification discourages investment. When the private benefits is stronger than the under-diversification cost ($2.60 < w$ for $\theta_s = 17.7$, or $5.06 < w$ for $\theta_s = 25.3$), the firm over-invests. Otherwise, it's optimal to under-invest. In addition, lower IP may also mitigates the firm's under-investment problem especially when the insider's liquidity is low.

Panel B of Figure 2 shows that the insider's consumption behaves similarly to her private average q. In particular, the insider under-consumes when the firm's liquid wealth is low, i.e., idiosyncratic business risk exposure is high. She slightly over-consumes when the liquidity is sufficiently high, since the private benefit motivation dominates in this case. Overall, the distortion on the insider's consumption is not quantitatively as large as the distortion on investment since the effects of IP and financial under-development on consumption is second order.

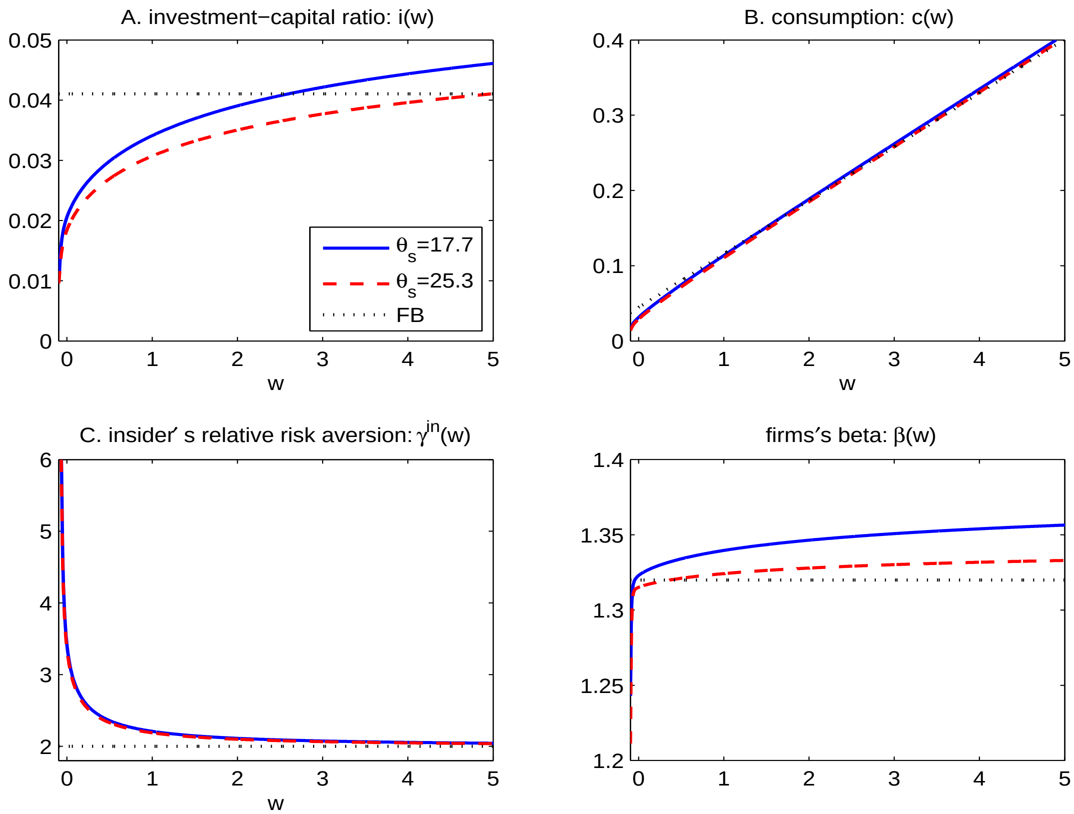

Figure 2: Investment $i(w)$, consumption $c(w)$, insider's relative risk aversion, $\gamma^{in}(w)$, and $\beta(w)$. The parameter values are: $A = 0.123$, $r = 4\%$, $\zeta = 8\%$ $\gamma = 2$, $\mu_M - r = 6\%$, $\sigma_M = 20\%$, $\alpha = 0.33$, $\sigma_K = 33\%$, $\lambda = 0.7$, $\ell = 0.3$, $\rho = 0.8$, $n_i = 0.054, \theta_i = 1.4$.

Panel C of Figure 2 shows that the insider's relative risk aversion is higher than the First-Best level, especially for small values of the insider's liquidity. Moreover, the effects of IP (variations in θ_s) on the insider's relative risk aversion is small, which implies that the high relative risk aversion of insider is mostly generated by the idiosyncratic business risk.

4.4 Mechanism and Results

The model calibration implies three main quantitative results as shown in Table 2.

First, the agency cost measured by private benefit of insiders is quantitatively large.

Our calibration implies that for weaker IP countries (top panel), the firm's average q, q^{out}, is equal to 1.11 and the net private benefits (divided by firm value), $\frac{q^{in}-q^{out}}{q^{out}}$, is equal to 0.63, implying significant loss of outside investors due to agency costs. For stronger IP countries, q^{out} is equal to 1.32 and the net private benefits, $\frac{q^{in}-q^{out}}{q^{out}}$ is 0.24.[27] Even though the private benefit of the insider in stronger IP countries is less than that in weaker IP countries (0.24 vs. 0.63), yet our result also implies that insiders still expropriate outside investors significantly (a significant 24% wedge between outsider's q and insider q) despite that investor protection is stronger. This happens because higher diversion cost in stronger IP countries cannot completely eliminate the over-investment problem caused by agency issues.

Second, the distortion to investment depends on the tradeoff between investor protection and financial development. Our calibration implies the First-best investment i^{FB} is 0.041. Under full FD but imperfect investor protection, investment is above the First-best ($i^* > i^{FB}$) in both weaker and stronger IP countries. Intuitively, because idiosyncratic risk is completely diversified away with complete markets, insiders pursue private benefit by over-investing in capital. In contrast, under limited FD, insiders under-invest causing investment below the First-best ($i^* < i^{FB}$). This is because insiders now cannot fully diversify the idiosyncratic risk due to incomplete markets, thus they optimally lower investment below the First-best to reduce the idiosyncratic risk exposure.

Third, perhaps strikingly, outside investors do not necessarily benefit from full financial development as long as imperfect investor protection is first-order in determining investment. Specifically, the valuation of outside equity (q^{out}_*) under full FD but imperfect investor protection is much lower than the baseline calibration under limited FD and the same degree of investor protection (ave q^{out}). This happens in both stronger IP and weaker IP groups. Furthermore, we see a similar pattern for the private benefit: the

[27]Note that all moments for both weaker and stronger IP countries are exactly matched to the respective empirical moments in the data.

Table 2: **Parameter values:** $A = 0.123$, $r = 4\%$, $\zeta = 8\%$ $\gamma = 2$, $\mu_M - r = 6\%$, $\sigma_M = 20\%$, $\alpha = 0.33$, $\sigma_K = 33\%$, $\lambda = 0.7$, $\ell = 0.3$, $\rho = 0.8$, $n_i = 0.054, \theta_i = 1.4$. That implies $\nu = 26.4\%$, $\epsilon = 19.8\%$, $q^{FB} = 1.893$ and $i^{FB} = 0.041$.

	Data	Data	Model - Limited FD							
	priv. ben	ave. q^{out}	θ_s	ave. w	ave. s	ave. b	ave. q^{out}	ave. i	ave. q^{in}	ave. $\frac{q^{in}-q^{out}}{q^{out}}$
Weaker IP	0.63	1.11	17.7	1.14	0.038	0.038	1.11	0.029	1.80	0.63
Stronger IP	0.24	1.32	25.3	1.38	0.026	0.027	1.32	0.026	1.66	0.24
					Model - Full FD					
				w^*	s^*	b^*	q_*^{out}	i^*	q_*^{in}	$\frac{q_*^{in}-q_*^{out}}{q_*^{out}}$
Weaker IP	0.63	1.11	17.7	-2.977	0.038	0.038	0.276	0.077	2.977	9.78
Stronger IP	0.24	1.32	25.3	-2.594	0.026	0.027	0.994	0.064	2.594	1.61

net private benefit of insiders is higher in full FD than in limited FD. Specifically, for weaker IP countries, with full FD but imperfect investor protection, the private benefit is $\frac{q_*^{in}-q_*^{out}}{q_*^{out}} = 9.78$, which is about fifteen times the private benefit in the baseline calibration with limited FD (0.63) . Similarly, for stronger IP countries, under full FD but imperfect investor protection, the private benefit is $\frac{q_*^{in}-q_*^{out}}{q_*^{out}} = 1.61$, about seven times of the baseline calibration with limited FD (0.24).

The economic mechanism behind this is that with full FD, the insider can fully hedge his idiosyncratic risk exposure, leading him to over-invest to pursue more private benefit, which in turn increases the agency costs. Therefore, with investor protection being imperfect, financial development does not necessarily benefit outsider investors, who may be expropriated even more by insiders.

5 Risk Premium

We further explore the model implications of imperfect investor protection and limited financial development on risk and risk premium

5.1 Beta and the Cost of Outside Equity Capital

We study the asset pricing implications for outsiders. Using Ito's formula, we show that the incremental return dR_t^{out} for outside equity is given by the sum of dividend yield dY_t/V_t^{out} and capital gains dV_t^{out}/V_t^{out},

$$\begin{aligned} dR_t^{out} &\equiv \frac{dY_t + dV_t^{out}}{V_t^{out}} \\ &= \mu^{out}(w_t)dt + \frac{q^{out}(w_t) - w_t q_w^{out}(w_t)}{q^{out}(w_t)}\sigma_K d\mathcal{B}_t^K + \frac{\pi(w_t)q_w^{out}(w_t)}{q^{out}(w_t)}\sigma_M d\mathcal{B}_t^M \,, \end{aligned} \tag{53}$$

where the expected return, $\mu^{out}(w)$, is also referred to as the cost of capital for outside equity. The following proposition summarizes the asset pricing implications for investors.

Proposition 1 *The conditional CAPM holds for outside equity and $\mu^{out}(w)$ satisfies*

$$\mu^{out}(w) = r + \beta(w)(\mu_M - r)\,, \tag{54}$$

$$= r + \eta\left[\nu + \frac{\eta - \gamma\nu}{\gamma^{in}(w)}\frac{p(w)q_w^{out}(w)}{q^{out}(w)}\right]\,, \tag{55}$$

where the conditional beta for outside equity, $\beta(w)$, is given by

$$\beta(w) = \beta_K(w) + \beta_M(w)\,, \tag{56}$$

where

$$\beta_K(w) = \frac{\nu}{\sigma_M}\frac{q^{out}(w) - wq_w^{out}(w)}{q^{out}(w)}\,, \tag{57}$$

$$\beta_M(w) = \frac{\pi(w)q_w^{out}(w)}{q^{out}(w)}\,, \tag{58}$$

Our model implies that the conditional beta has two components: the capital shock beta, $\beta_K(w)$, and the market portfolio allocation beta, $\beta_M(w)$, which we discuss in detail below.

The capital shock beta, $\beta_K(w)$ given in (57), depends on the ratio between the firm's marginal q, $q^{out}(w) - wq_w^{out}(w)$, and average q, $q^{out}(w)$, which can be equivalently expressed as the elasticity of firm value $V^{out}(W,K)$ with respect to capital K, since $d\ln V^{out}(W,K)/d\ln K = (q^{out}(w) - wq_w^{out}(w))/q^{out}(w)$. Intuitively, capital growth is stochastic and co-varies with the aggregate risk, which induces a risk premium described by capital shock beta, $\beta_K(w)$. The market portfolio allocation beta, $\beta_M(w)$ given in (58),

Table 3: The effects of the insider's idiosyncratic business risk. The parameter values for the baseline model are: $A = 0.123$, $r = 4\%$, $\zeta = 8\%$ $\gamma = 2$, $\mu_M - r = 6\%$, $\sigma_M = 20\%$, $\alpha = 0.33$, $\lambda = 0.7$, $\ell = 0.3$, $\nu = 26.4\%$, $n_i = 0.054, \theta_i = 1.4, \theta_s = 17.7$. That implies $q^{FB} = 1.893$ and $i^{FB} = 0.041$.

ϵ	ave. w	ave. q^{out}	ave. i	ave. q^{in}	ave. $\frac{q^{in}-q^{out}}{q^{out}}$
5%	-0.01	1.03	0.036	2.26	1.18
10%	0.04	1.09	0.032	2.07	0.90
19.8%	**1.14**	**1.11**	**0.029**	**1.80**	**0.63**
30%	9.55	1.10	0.028	1.73	0.59
40%	21.79	1.09	0.026	1.64	0.52

depends on the ratio between the firm's marginal value of liquidity, $q_w^{out}(w)$, and average q, $q^{out}(w)$.

Panel D of Figure 2 plots the firm's total beta $\beta(w)$. We see that it is monotonically increasing in the insider's liquid wealth w, and is lower (higher) than the First-Best level when liquidity is low (high). This happens because $\beta(w)$ directly depends on the marginal value of average q, i.e. $q_w^{out}(w)$, which is negative when liquidity is low and positive when liquidity is high since $q^{out}(w)$ is decreasing firstly and then increasing in liquidity w as shown in Panel A of Figure 2. Recall that $\beta(w) = \beta_K(w) + \beta_M(w) = \frac{\nu}{\sigma_M} + \frac{\eta-\gamma\nu}{\sigma_M\gamma^{in}(w)}\frac{p(w)q_w^{out}(w)}{q^{out}(w)}$, and hence it's not surprising to see that $\beta(w)$ is lower than the First-Best level when $w < -0.04$ for the weaker IP case with $\theta_s = 17.7$, since $q_w^{out}(-0.04) = 0$; otherwise $\beta(w)$ is higher than the First-Best level when $w > -0.04$. Similarly, for stronger IP case with $\theta_s = 25.3$, $\beta(w)$ is lower than the First-Best level when $w < 0.37$ and higher than First-Best level when $w > 0.37$, where $q_w^{out}(-0.04) = 0$.

5.2 The Effects of the Insider's Idiosyncratic Business Risk

We now examine the effects of insider's idiosyncratic business risk ϵ on the average insider's liquidity, firm's q, investment, insider's private q, and the insider's private benefit. Intuitively, higher idiosyncratic risk exposure will induce higher saving motive for the insider and higher stationary liquidity against under-diversification. Hence, the average w increases in idiosyncratic risk, ϵ.

Interestingly, the model implies non-monotonic relations between the idiosyncratic risk ϵ and the variables studied here. Table 3 reports the result with the systematical risk fixed at $\nu = 26.4\%$. We see all the average valuations and investment are non-monotonic in ϵ. The average investment, insider's private q, and the insider's private benefit are decreasing first and then increasing in ϵ. While the average of firm's q is increasing first and then decreasing in ϵ.

The economic mechanism is as follows. The increase of idiosyncratic business risk has negative effects on the insider's private average q and investment, whereas the increase of insider's liquidity has positive effects on the insider's private q and investment. The first channel, i.e., the negative effects from the increase of idiosyncratic risk, dominates when ϵ is low, whereas the second channel, the positive effects from the increase of liquidity, dominates when ϵ is sufficiently high. Hence the average of insider's private q and investment is non-monotonic in ϵ. The similar intuition applies to the effects of ϵ on the firm's average q and insider's private benefit.

6 Conclusion

Many firms including large publicly traded ones around the world are run by entrenched controlling shareholders, who extract private benefits and choose non-value maximizing investment decisions. We study the economics of lacking investor protection and financial under-development by developing a tractable dynamic q-theory of investment framework where the insider makes *interdependent* consumption-saving, portfolio choice between a risky asset and a risk-free asset, private benefits, and corporate investment decisions. Our model extends the modern q-theory of investment, e.g. Hayashi (1982), along two important dimensions, incomplete markets and imperfect investor protection.

Two opposing forces work in our model. On the one hand, the weaker investor protection, the more private benefits to collect and the stronger the incentives to over-invest as private benefits decrease with insider's liquidity. On the other hand, incomplete markets discourage insiders from investing in their firms as under-diversification costs decrease with liquidity. The insider optimally over-invests when the liquidity is high and under-invests when the liquidity becomes sufficiently small. Investor protection and incomplete markets jointly generate a non-monotoni investment-capital ratio even with a constant-returns-to-scale production technology as in Hayashi (1982).

Our model allows us to (1) solve for the insider's private valuation and diversified outsiders' public valuation, both marginal and average; (2) calculate the cost of capital, i.e. risk/return implications for investors; and (3) characterize corporate investment decisions for a firm run by an under-diversified entrenched insider and link to the insider's private marginal valuation;

We further derive valuation and asset pricing implications for outside equity and inside equity. While clearly reducing firm value for outside investors, agency costs have ambiguous effects on the cost of capital. The cost of outside equity varies with liquidity and can be either higher or lower than the first-best benchmark value. Our model thus provides empirically testable predictions on corporate governance and cross-sectional returns. Moreover, our model generates rich predictions on time-varying investment dynamics purely driven by imperfect investor protection and incomplete markets, rather than changing investment opportunities.

For simplicity, we have taken investor protection as exogenously given. However, insiders may choose governance and investor protection so as to maximize their values. A critical issue in firms run by controlling shareholders is the succession of power.[28] We plan to incorporate these important issues in our future work.

References

Abel, A. B. (1983). Optimal investment under uncertainty. *American Economic Review 73*(1), 228–233.

Abel, A. B. and J. C. Eberly (1994). A unified model of investment under uncertainty. *American Economic Review 84*(5), 1369–1384.

Admati, A., P. Pfleiderer, and J. Zechner (1994). Large shareholder activism, risk sharing, and financial market equilibrium. *Journal of Political Economy 102*(6), 1097–1130.

Albuquerque, R. and E. Schroth (2010). Quantifying private benefits of control from a structural model of block trades. *Journal of Financial Economics 96*, 33–35.

Albuquerque, R. and N. Wang (2008). Agency conflicts, investment, and asset pricing. *Journal of Finance 63*(1), 1–40.

[28]Burkart, Panunzi, and Shleifer (2003) develop a model of family firms.

Barclay, M. and C. Holderness (1989). Private benefits from control of public corporations. *Journal of Financial Economics 25*, 371–395.

Berle, A. and G. Means (1932). *The Modern Corporation and Private Property*. New York: McMillan.

Bolton, P., H. Chen, and N. Wang (2011). A unified theory of Tobin's q, corporate investment, financing, and risk management. *Journal of Finance 66*(5), 1545–1578.

Brainard, W. C. and J. Tobin (1968). Pitfalls in financial model-building. *American Economic Review 58*, 99–122.

Burkart, M., F. Panunzi, and A. Shleifer (2003). Family firms. *Journal of Finance 58*(5), 2167–2202.

Caballero, R. J. (1999). Aggregate investment. In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Volume 1B, pp. 813–862. Amsterdam: North-Holland.

Claessens, S., S. Djankov, and L. Lang (2000). The separation of ownership and control in East Asian corporations. *Journal of Financial Economics 58*, 81–112.

DeMarzo, P. M. and B. Urosevic (2006). Ownership dynamics and asset pricing with a large shareholder. *Journal of Political Economy 114*(4), 774–815.

Dyck, A. and L. Zingales (2004). Private benefits of control: An international comparison. *Journal of Finance 59*, 537–600.

Eberly, J. C., S. Rebelo, and N. Vincent (2009). Investment and value: A neoclassical benchmark. Working Paper, Northwestern University.

Faccio, M. and L. H. P. Lang (2002). The ultimate ownership of Western European corporations. *Journal of Financial Economics 65*, 365–395.

Fischer, E., R. Heinkel, and J. Zechner (1989). Dynamic capital structure choice: Theory and tests. *Journal of Finance 44*, 19–40.

Goldstein, R., N. Ju, and H. Leland (2001). An EBIT-based model of dynamic capital structure. *Journal of Business 74*(4), 483–512.

Gomes, J. F. (2001). Financing investment. *American Economic Review 91*, 1263–85.

Hayashi, F. (1982). Tobin's marginal q and average q: A neoclassical interpretation. *Econometrica 50*, 213–224.

Hennessy, C. A. and T. M. Whited (2007). How costly is external financing? Evidence from a structural estimation. *Journal of Finance 62*, 1705–1745.

Jensen, M. C. (1986). Agency cost of free cash flow, corporate finance, and takeovers. *American Economic Review 76*, 323–329.

Johnson, S., P. Boone, A. Breach, and E. Friedman (2000). Corporate governance in the Asian financial crisis. *Journal of Financial Economics 58*, 141–186.

La Porta, R., F. López-de-Silanes, and A. Shleifer (1999). Corporate ownership around the world. *Journal of Finance 54*(2), 471–517.

La Porta, R., F. López-de-Silanes, A. Shleifer, and R. W. Vishny (2002). Investor protection and corporate valuation. *Journal of Finance 57*(3), 1147–1170.

Lambrecht, B. M. and S. C. Myers (2008). Debt and managerial rents in a real-options model of the firm. *Journal of Financial Economics 89*, 209–231.

Lambrecht, B. M. and S. C. Myers (2011). A Lintner model of payout and managerial rents. *Journal of Finance*. Forthcoming.

Leland, H. (1994). Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance 49*, 1213–1252.

Lucas, R. E. and E. C. Prescott (1971). Investment under uncertainty. *Econometrica 39*(5), 659–681.

Merton, R. C. (1971). Optimum consumption and portfolio rules in a continuous-time model. *Journal of Economic Theory 3*, 373–413.

Morellec, E. (2004). Can managerial discretion explain observed leverage ratios? *Review of Financial Studies 17*(1), 257–294.

Morellec, E., B. Nikolov, and N. Schürhoff (2011). Corporate governance and capital structure dynamics. *Journal of Finance*, Forthcoming.

Nikolov, B. and T. W. Whited (2011). Agency conflicts and cash: Estimates from a structural model. Working Paper, University of Rochester.

Panousi, V. and D. Papanikolaou (2012). Investment, idiosyncratic risk, and ownership. *Journal of Finance 67*(3), 1113–1148.

Riddick, L. A. and T. M. Whited (2009). The corporate propensity to save. *Journal of Finance 64*, 1729–1766.

Stoughton, N. M. and J. Zechner (1998). IPO-mechanisms, monitoring and ownership structure. *Journal of Financial Economics 49*, 45–77.

Stulz, R. M. (1990). Managerial discretion and optimal financing policies. *Journal of Financial Economics 26*, 3–27.

Stulz, R. M. (2005). The limits of financial globalization. *Journal of Finance 60*(4), 1595–1638.

Tobin, J. (1969). A general equilibrium approach to monetary theory. *Journal of Money, Credit and Banking 1*, 15–29.

Whited, T. (1992). Debt, liquidity constraints, and corporate investment: Evidence from panel data. *Journal of Finance 47*, 1425–1460.

Zwiebel, J. (1996). Dynamic capital structure under managerial entrenchment. *American Economic Review 86*, 1197–1215.

Appendices

We first provide details for the full financial development (FD) case and then for the general case with imperfect investor protection and limited FD.

A Derivations and Proofs

A.1 Full Financial Development (FD)

Before presenting proof for the solution under the general case in Section A.2, we first augment the model presented in the preceding section by introducing an additional risky financial asset so that markets are dynamically complete. We refer to this augmented model as the full Financial Development (FD) case, which we analyze in Section 2. The full FD case serves a natural benchmark for us to under the effects of investor protection and FD on investment and value.

Next, let $F^*(K,W)$ denotes the insider's value function under full FD, which is given by

$$F^*(K,W) = \frac{(hP^*(K,W))^{1-\gamma}}{1-\gamma}\,, \tag{A.1}$$

where h is given by

$$h = \left[r + \frac{\eta^2}{2\gamma} + \gamma^{-1}\left(\zeta - r - \frac{\eta^2}{2\gamma}\right)\right]^{\frac{\gamma}{\gamma-1}}\,, \tag{A.2}$$

and the "total" wealth $P^*(K,W) = W + \alpha q_*^{in}(\alpha)K$ where $q_*^{in}(\alpha)$ is given by (28).

We use the optimal portfolio choice approach by introducing new risky asset which capture idiosyncratic risk to give the proofs for the model solutions under full FD.

Without loss of generality, we decompose the capital shock $\mathcal{B}^K$ as follows

$$d\mathcal{B}_t^K = \rho d\mathcal{B}_t^M + \sqrt{1-\rho^2}d\widehat{\mathcal{B}}_t^M\,, \tag{A.3}$$

where $\widehat{\mathcal{B}}^M$ is a standard Brownian motion representing the idiosyncratic risk orthogonal to the systematic risk captured by $\mathcal{B}^M$. Let $\widehat{M}_t$ denote the market value of this tradable asset with no risk premium,

$$\frac{d\widehat{M}_t}{\widehat{M}_t} = rdt + \sigma_{\widehat{M}}d\widehat{\mathcal{B}}_t^M\,. \tag{A.4}$$

Note that the expected growth rate of $\left\{\widehat{M}_t : t \geq 0\right\}$ is the risk-free rate under the physical measure as this asset carries no risk premium. And then, we rewrite the dynamics for K_t as

$$dK_t = \Phi(I_t, K_t)dt + \sigma_K K_t(\rho d\mathcal{B}_t^M + \sqrt{1-\rho^2} d\widehat{\mathcal{B}}_t^M)\,. \tag{A.5}$$

Consider a optimal portfolio choice by dynamically trading in the public equity and the newly introduced public asset with the value process $\widehat{M}_t$ given in (A.4), and the risk-free asset. Let Π_t and $\widehat{\Pi}_t$ denote the number of shares for the publicly traded assets with systematical risk and idiosyncratic risk given in (A.4), respectively. So we have the controlling shareholder's wealth W evolves as follows

$$dW_t = rW_t dt + \Pi_t \sigma_M \left(\eta dt + d\mathcal{B}_t^M\right) + \widehat{\Pi}_t \sigma_{\widehat{M}} d\widehat{\mathcal{B}}_t^M - C_t dt + \alpha Z_t dt + \alpha\lambda dX_t\,.$$

And then the standard dynamic programming argument implies that the value function $F(K,W)$ satisfies the following HJB equation

$$\begin{aligned} \zeta F = \max_{C,\Pi,\widehat{\Pi},I,S} \; & U(C) + \Phi(I,K)F_K + \frac{(\sigma_K K)^2}{2} F_{KK} + (\nu\sigma_M \Pi + \epsilon\sigma_{\widehat{M}}\widehat{\Pi})KF_{KW} \\ & + [rW + \Pi(\mu_M - r) - C + \alpha(AK - I) + (1-\alpha)S - \Psi(S,K)]F_W \\ & + \frac{\sigma_M^2 \Pi^2 + \sigma_{\widehat{M}}^2 \widehat{\Pi}^2}{2} F_{WW}\,. \end{aligned} \tag{A.6}$$

Using the FOC for diversion S, corporate investment I, the insider's consumption C, and investment in market portfolio Π and $\widehat{\Pi}$, respectively, we obtain

$$\Psi_S(S,K) = (1-\alpha)K\,, \tag{A.7}$$

$$\alpha F_W(K,W) = \Phi_I(I,K)F_K(K,W)\,, \tag{A.8}$$

$$U'(C) = F_W(K,W)\,, \tag{A.9}$$

$$\Pi = -\frac{\eta}{\sigma_M}\frac{F_W(K,W)}{F_{WW}(K,W)} - \frac{\nu}{\sigma_M}\frac{KF_{KW}(K,W)}{F_{WW}(K,W)}\,, \tag{A.10}$$

$$\widehat{\Pi} = -\frac{\epsilon}{\sigma_{\widehat{M}}}\frac{KF_{KW}(K,W)}{F_{WW}(K,W)}\,. \tag{A.11}$$

Substituting the controlling shareholder's value function (A.1) with $P^*(K,W) = W + \alpha q_*^{in}(\alpha)K = (w + \alpha q_*^{in}(\alpha))K$ into the FOCs for the real investment (A.8) and the consumption (A.9) respectively, we have $q_*^{in}(\alpha)\phi'(i^*(\alpha)) = 1$ and (30). And then substituting the controlling shareholder's value function given by (A.1) with $P^*(K,W) =$

$W + \alpha q_*^{in}(\alpha)K = (w + \alpha q^{in}(\alpha))K$, and the above consumption strategy and investment in market into the HJB equation (A.6) and simplifying, we have

$$
\begin{aligned}
0 &= \left(\frac{\gamma h^{1-1/\gamma} - \zeta}{1-\gamma} + \frac{\eta^2}{2\gamma}\right)(w + \alpha q_*^{in}(\alpha)) + rw + \alpha((\phi(i^*) - \nu\eta)q_*^{in}(\alpha) + A + b^*(\alpha) - i^*) \\
&= \left(\frac{\gamma h^{1-1/\gamma} - \zeta}{1-\gamma} + \frac{\eta^2}{2\gamma} + r\right)(w + \alpha q_*^{in}(\alpha)) + \alpha(\phi(i^*)q_*^{in}(\alpha) - r_V^* q_*^{in}(\alpha) + A + b^*(\alpha) - i^*) \,.
\end{aligned}
\tag{A.12}
$$

As (A.12) must hold for all w, we obtain $\frac{\gamma(h/\zeta)^{1-1/\gamma} - \zeta}{1-\gamma} + \frac{\eta^2}{2\gamma} + r = 0$ which implies (A.2). For given (A.2), immediately we have

$$
\phi(i^*)q_*^{in}(\alpha) - r_V^* q_*^{in}(\alpha) + A + b^*(\alpha) - i^* = 0 \,. \tag{A.13}
$$

And combining the FOCs for investment, we have (28).

A.2 Sketch of the Model Solution for the General Case

The standard dynamic programming argument implies that $F(K, W)$ satisfies the following HJB equation

$$
\begin{aligned}
\zeta F \;=\; & \max_{C,\Pi,I,S} U(C) + \Phi(I, K)F_K + \frac{(\sigma_K K)^2}{2}F_{KK} + \nu\sigma_M K\Pi F_{KW} + \frac{\sigma_M^2\Pi^2}{2}F_{WW} \\
& + [rW + \Pi(\mu_M - r) - C + \alpha(AK - I) + (1-\alpha)S - \Psi(S, K)]F_W \,.
\end{aligned}
\tag{A.14}
$$

Using the FOC for diversion S, corporate investment I, the insider's consumption C, and market portfolio allocation Π respectively, we obtain

$$
\Psi_S(S, K) = (1-\alpha)K \,, \tag{A.15}
$$

$$
\alpha F_W(K, W) = \Phi_I(I, K)F_K(K, W) \,, \tag{A.16}
$$

$$
U'(C) = F_W(K, W) \,, \tag{A.17}
$$

$$
\Pi = -\frac{\eta}{\sigma_M}\frac{F_W(K, W)}{F_{WW}(K, W)} - \frac{\nu}{\sigma_M}\frac{KF_{KW}(K, W)}{F_{WW}(K, W)} \,. \tag{A.18}
$$

And then substituting (23) and (A.18) into (A.14), we have the HJB equation could be rewritten as

$$
\begin{aligned}
\zeta F = & \max_{C,I} U(C) + \Phi(I, K)F_K + [rW - C + \alpha((A + b^*(\alpha))K - I)]F_W \\
& + \frac{(\sigma_K K)^2}{2}F_{KK} - \frac{(\sigma_M \Pi)^2}{2}F_{WW} \,.
\end{aligned}
\tag{A.19}
$$

And we then conjecture that the controlling shareholder's value function is given by (15), and substituting the value function (15) into the FOCs (A.16) and (A.17) for investment and consumption respectively, and combining the homogeneity property we obtain the firm's investment strategy is given by (41), consumption rule is given by (40), and portfolio allocation rule is given by (38). And then substituting the consumption rule (40), portfolio allocation rule (38), and value function (15) in the HJB equation (A.14) and simplifying, we have ODE(44). Moreover, for the specified $\phi(\cdot)$ as given in (52), we have corresponding ODE for $p(w)$ can be rewritten as

$$
\begin{aligned}
0 \quad = \quad & \left[\frac{\gamma m(p')^{1-\frac{1}{\gamma}}-\zeta}{1-\gamma}-\frac{\gamma\sigma_K^2}{2}\right]p+\alpha[A+b^*(\alpha)]p'+(r_V^*+\gamma\epsilon^2)wp' \\
& +\frac{(\alpha p'(w))^{1-\theta_i}(n_i(p-wp'))^{\theta_i}}{\theta_i-1}-\gamma^{in}(w)\frac{\epsilon^2w^2}{2}\frac{p'(w)}{p(w)}+\frac{(\eta-\gamma\nu)^2p'p}{2\gamma^{in}(w)}\,, \quad \text{(A.20)}
\end{aligned}
$$

and the corresponding investment rule is given by

$$i(w)=\left(\frac{n_i(p(w)-wp'(w))}{\alpha p'(w)}\right)^{\theta_i}. \quad \text{(A.21)}$$

We now turn to analyzing the boundary conditions. When the debt constraint is binding, we have the following value matching condition before and after piece-wise liquidation

$$F(K,-\alpha L-\Delta W)=F(K-\Delta K,-\alpha L)\,, \quad \text{(A.22)}$$

where $\Delta W=\alpha\lambda\Delta K$, and then by taking $\Delta W\to 0$, we have

$$F_W(K,-\alpha L)=F_K(K,-\alpha L)/(\alpha\lambda)\,, \quad \text{(A.23)}$$

which implies (47).

Next, we derive the model solutions for the valuation of outsider investors.

Firstly, by using the Girsanov theorem, we may write the firm's capital stock accumulation under the risk-neutral measure, denoted by $\widetilde{\mathbb{P}}$, as:

$$dK_t=(\Phi(I_t,K_t)-\nu\eta K_t)\,dt+\sigma_K K_t d\widetilde{\mathcal{B}}_t^K\,, \quad \text{(A.24)}$$

where $\widetilde{\mathcal{B}}_t^K=\mathcal{B}_t^K+\rho\eta t$ is standard Brownian motion under risk-neutral measure. And the controlling shareholder's wealth evolves as follows,

$$dW_t=r\,(W_t-\Pi_t)\,dt+\left(rdt+\sigma_M d\widetilde{\mathcal{B}}_t^M\right)\Pi_t-C_tdt+\alpha Z_tdt+\alpha\lambda dX_t\,, \quad \text{(A.25)}$$

where $\widetilde{\mathcal{B}}_t^M = \mathcal{B}_t^M + \eta t$ is standard Brownian motion under risk-neutral measure. And then the standard dynamic programming argument implies that $V^{out}(W, K)$ satisfies the following HJB equation

$$rV^{out} = (A - s(\alpha) - i(w))K + (\phi(i(w)) - \nu\eta)KV_K^{out} + \frac{(\sigma_K K)^2}{2}V_{KK}^{out} + \nu\sigma_M\pi(w)K^2V_{KW}^{out}$$
$$+ [rw - c(w) + \alpha(A - i(w)) + (1-\alpha)s - \psi(s)]KV_W^{out} + \frac{\sigma_M^2\pi(w)^2}{2}K^2V_{WW}^{out}\,. \tag{A.26}$$

And then by using the homogeneity property as $q^{out}(w) = \frac{V^{out}(W,K)}{K}$, immediately we have (48).

With the similarly analysis about the value matching condition before and after piece-wise liquidation for insiders, we have the following condition for firm value for outside investors

$$V^{out}(K, -\alpha L - \Delta W) = V^{out}(K - \Delta K, -\alpha L) + \lambda\Delta K\,, \tag{A.27}$$

and then by taking $\Delta W \to 0$, we have

$$V_W^{out}(K, -\alpha L) = V_K^{out}(K, -\alpha L)/(\alpha\lambda) - 1/\alpha\,, \tag{A.28}$$

which implies (49).