The Coordination of Intermediation[*]

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Abstract

We study liquidity provision by decentralized financial intermediaries (i.e., dealers) in a dynamic model of asset markets. When inventory cost is low (high), dealers provide more (less) liquidity by holding more (less) inventory, the market is liquid (illiquid), and interdealer trading is active (inactive). When inventory cost is medium, a dealer provides more liquidity if and only if other dealers do so, leading to strategic coordination motives and multiple equilibria. Switching between equilibria implies the potential for liquidity declines without fundamental shocks. A small trading friction among dealers effectively reduces inventory cost and hence reduces the possibility of multiple equilibria.

KEYWORDS: intermediation, liquidity, inventory, coordination, multiple equilibria, interdealer trading.

JEL: D02, G01, G11, G12, G21, G23

1 Introduction

Many over-the-counter (OTC) asset markets are intermediated in the sense that buyers and sellers cannot trade with each other directly but only through an intermediary (i.e., a dealer). When intermediating an asset, the intermediary buys it from a seller first, temporarily holds it on the balance sheet as an inventory, and sells it to a buyer later. In this sense, the intermediary provides liquidity to the asset market, and market liquidity hinges on the intermediary's willingness to hold inventory. This process of intermediaries using own balance sheet space to hold inventory and provide liquidity is ubiquitous in various economically important markets, such as the corporate bond (Di Maggio, Kermani and Song, 2017), municipal bond (Green, Hollifield and Schurhoff, 2006, Li and Schurhoff, 2018), asset-backed security (Hollifield, Neklyudov and Spatt, 2017), mortgage-backed security (Gao, Schultz and Song, 2017), and credit default swap (Eisfeldt et al., 2018) markets, among others.

Our paper is motivated by the observation that various OTC asset markets have experienced large declines in intermediary liquidity provision yet without a clear negative fundamental shock. For instance, Bessembinder et al. (2018) find a significant decline in dealer balance sheet utilization and inventory holding in the U.S. corporate bond market in the 2010 to 2014 period compared to the pre-crisis period. Choi and Huh (2017) find that among large trades wherein dealers use their balance sheet space to hold inventory, buyers and sellers pay 40 to 80 percent higher spreads than pre-crisis. Given the huge market size of $9.3 trillion, the implied reduction of market liquidity is of unquestionable economic importance. So far, the leading explanation is that the Volcker Rule that prevented dealer proprietary trading had contributed to dealer balance sheet space costs, leading to the observed liquidity declines (e.g., Bao, O'Hara and Zhou, 2018, Bessembinder et al., 2018, Dick-Nielsen and Rossi, 2019).

Nevertheless, counter-arguments prevail that economic forces other than regulation must be at play (see Adrian et al., 2017, for example). The main point is that the Volcker Rule had not yet been implemented until 2015, and dealers did not face realized balance sheet space costs before that.[1] Rather, some industry practitioners and regulators have

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[1]As part of the Dodd-Frank Act, passed in 2010, section 13 (i.e, the Volcker Rule) was added to the Bank Holding Company Act of 1956. The Volcker Rule generally prohibits banking entities, many of which are corporate bond dealers, from using their balance sheet in proprietary trading. Implementation of the Volcker

hypothesized that a dealer may stop providing liquidity simply because it worries about other dealers stopping providing liquidity,[2] which has been echoed by academics in policy discussions (e.g., Greenwood et al., 2017, Duffie, 2018). Thus, it remains a first-order question as to whether and why, theoretically, intermediaries may commit less balance sheet space to holding inventory and reduce liquidity provision *without* a negative fundamental shock.

Our paper answers this question. Our main contribution is to provide a new strategic perspective to explain intermediaries' incentives in using balance sheet space to hold inventory, the associated patterns of decentralized trading, and the implications on market liquidity. We in particular focus on the existence and properties of *multiple* equilibria, the switching between which implies declines in 1) aggregate dealer inventory holding and in 2) market liquidity without a negative fundamental shock. Built upon the modern OTC asset pricing literature, which typically delivers a unique equilibrium, our model shows that multiple equilibria can generically arise due to a coordination failure in intermediaries' liquidity provision decisions. This coordination failure is reminiscent of the classic strategic coordination failure in the bank run literature (e.g., Diamond and Dybvig, 1983): a depositor's run decision depends on other depositors'. Similarly, in our framework, if one dealer is worrying about other dealers not providing liquidity, it may stop providing liquidity. In this sense, our theory provides one complementary explanation for why, for example, dealer banks committed less balance sheet space and provided less liquidity as they worried about each other's liquidity provision decisions in preparing for the to-be-implemented Volcker Rule before 2015.[3]

In this paper, we formulate those ideas in a dynamic model of decentralized asset market à la Rubinstein and Wolinsky (1987) and Duffie, Garleanu and Pedersen (2005). We set up the model in Section 2. There, buyers and sellers, that is, customers, can trade an asset only through a competitive dealer sector with many long-lived dealers. Customers randomly

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Rule, however, was not immediate and followed a laborious process. Final regulations were issued in early 2014 and implemented in mid-2015.

[2]See Bill Gross, "It Never Rains in California," appeared in the *Investment Outlook from Bill Gross*, Janus Capital, July 2015, and former U.S. SEC Commissioner Daniel Gallagher, "Remarks Regarding the Fixed Income Markets at the Conference on Financial Markets Quality," September 2012.

[3]We are neither dismissing the findings in the recent literature such as Bao, O'Hara and Zhou (2018), Bessembinder et al. (2018), Dick-Nielsen and Rossi (2019) nor arguing that the realized balance sheet space costs caused by the Volcker Rule and other regulatory policies were unimportant. Rather, we view our mechanism as a complementary amplification mechanism that helps to rationalize the large drop in intermediary liquidity provision over various OTC markets before a negative shock actually got realized.

arrive in the market, seeking to buy or sell a unit of identical asset with a dealer. Beyond intermediating the asset between buyers and sellers, dealers can also trade with each other bilaterally in an endogenously emerged interdealer market. To model different levels of dealer inventory holding and the rich trading patterns as observed in reality, we purposefully relax the $\{0, 1\}$ asset position restriction commonly used in the literature. Specifically, at any time, each dealer in our model can use its own balance sheet space to hold 1) a high inventory or 2) a low inventory to help provide liquidity, or 3) does not hold any inventory at all, and the inventory cost function is weakly convex. (Looking forward, we will elaborate why modeling three inventory levels is necessary but also the most parsimonious way to capture the economic essence; modeling more inventory levels would not add economic insight given our focus.) When meeting a dealer, a customer negotiates a price with the dealer through bargaining, completes the transaction, consumes, and leaves the market.

Section 3 presents *two* types of trading equilibrium of the model: when trading happens, a dealer may endogenously hold a high (low) level of inventory to provide more (less) liquidity, and an interdealer market where dealers trade with each other endogenously emerges (or not). Consider a dealer who already holds a low level of inventory on its balance sheet. When the inventory cost is sufficiently low (high) relative to the asset fundamental, this dealer is more (less) willing to provide liquidity in the sense of buying the asset from a seller, holding a high level of inventory, and then selling the asset to a buyer later. This implies a higher (lower) aggregate dealer inventory and a larger (smaller) dispersion of inventory distribution among dealers. Since the gains from trade of a potential interdealer trade are large (small) when the dispersion of inventory distribution is sufficiently large (small), interdealer trading is active (inactive), the implied intermediation chain is longer (shorter), and the market is more (less) liquid in equilibrium.

The key idea of our paper is that the two equilibria described above may generically co-exist in the same economy when the inventory cost is medium, which we illustrate in Section 4.1. When equilibrium multiplicity arises, a switch between the two equilibria may result from non-fundamental reasons as those featured in, for example, the bank run literature, leading to declines and volatility in intermediary inventory holding and liquidity provision as observed in reality.

To understand the coordination motives in dealers' liquidity provision decisions, consider the seller's reservation value in a potential trade between a seller and a dealer who already holds a low inventory. Economically, the seller's reservation value represents the lowest possible bid price that a liquidity-providing dealer has to pay to the seller. We show in Section 4.2 that as long as the seller is sufficiently impatient, the seller's reservation value is lower when more dealers buy from sellers and hold a high inventory. This implies that the liquidity-providing dealer in question is able to bid a lower price to the seller when other dealers are willing to provide liquidity. This subsequently implies that the dealer in question is also more able to enjoy a higher profit, rendering its liquidity provision decision more strategically complementary to others'. This contributes to the coordination motives.

Section 4.3 explores another economically intuitive way of seeing the coordination motives, which hinges on how equilibrium market liquidity interacts with dealer liquidity provision. When other dealers all buy from sellers and hold a high inventory, the market becomes more liquid. This implies two competing effects on dealer profitability per unit of time, one on 1) markup per trade and the other on 2) customer-dealer trading volume per unit of time. Markup per trade (i.e., the difference between the buyer-dealer price and the seller-dealer price) decreases due to intensified dealer competition. But trading volume per unit of time increases thanks to a higher aggregate dealer inventory, which implies that more assets are intermediated per unit of time. Thus, if the latter effect dominates, that is, if trading volume per unit of time increases sufficiently fast or markup per trade decreases sufficiently slow, the dealer in question will be willing to join other dealers to provide liquidity, enjoying a higher revenue per unit of time. This contributes to the coordination motives.

Having illustrated the coordination motives in dealers' liquidity provision decisions, we examine the role of interdealer trading in Section 5. It is well-known that trading among intermediaries allows them to share risks more efficiently (e.g., Ho and Stoll, 1983, Viswanathan and Wang, 2004). But how does it affect intermediaries' coordination in liquidity provision? We show that a small trading friction among dealers can mitigate the concern of coordination failures. Without interdealer trading, the only way for a dealer to offload its inventory is to be contacted by and trade with a buyer. In contrast, an active interdealer market allows a dealer that holds a high inventory to trade with another dealer without any inventory,

giving the former an alternative way to offload its costly inventory. This process of inventory sharing is faster when the interdealer trading friction is smaller. Because inventory cost is convex, a smaller interdealer trading friction thus effectively lowers dealer inventory cost in equilibrium, encourages the dealer to buy from sellers, and importantly makes the dealer's liquidity provision decision less strategically complementary to other dealers'. We show that, if the trading friction among dealers is sufficiently small, the endogenously emerged interdealer market with sufficiently fast interdealer trading can eliminate the coordination motives among dealers' liquidity provision decisions. In contrast, as the trading friction among dealers becomes larger it is more likely for the coordination motives to emerge. Together, these findings shed light on the ongoing policy discussion on OTC market reforms and highlight the importance of a well functioning interdealer market.

We design our model to drive home the economic insight of strategic coordination of intermediation; thus, our predictions are primarily qualitative but not quantitative. For the same reason, we abstract away from other important aspects of OTC asset markets. First, we follow Duffie, Garleanu and Pedersen (2005) and abstract away from potentially persistent trading relationships. Although trading relationships in OTC markets affect prices both between customers and dealers (Hendershott et al., 2017) and among dealers (Di Maggio, Kermani and Song, 2017), we view on-balance-sheet inventory costs and search and bargaining frictions as first-order despite trading relationships. This view is supported by Friewald and Nagler (2018), who empirically show that inventory, search, and bargaining frictions reduce the unexplained common component in U.S. corporate bond yield spread changes by 20%. Second, we focus on principal intermediated trades that occupy dealer balance sheet space temporarily while ignore agency, riskless principal, or pre-arranged trades in which the dealer effectively serves as a broker. Although agency trades are increasingly popular in some OTC markets (e.g., U.S. corporate bond markets), Schultz (2017) estimates that they account for less than 10% of all trades over 2005-2014. We leave them for future research.

Lastly, although the two-equilibrium structure in our model resembles that in Diamond and Dybvig (1983), we do not interpret the switch between equilibria as sudden "runs" in intermediated asset markets. Precisely, our focus is the coordination in intermediary liquidity provision and its market liquidity implications, which are less explored in the literature but,

as we believe, is first-order important in helping rationalize the large-scale declines in dealer balance sheet utilization and liquidity provision and inform future policy discussions.

Related literature. The main contribution of our paper is to formulate equilibrium multiplicity, its implications on market liquidity and trading patterns, and the underlying coordination motives in an otherwise standard dynamic asset pricing model following Rubinstein and Wolinsky (1987) and Duffie, Garleanu and Pedersen (2005).

This asset pricing literature typically delivers a unique equilibrium,[4] with a few exceptions that feature different economic forces from ours. Vayanos and Weill (2008) show that if buyers and sellers coordinate their trading activities in on-the-run Treasury bonds, on-the-run bonds become more liquid relative to off-the-run bonds. Vayanos and Weill (2008) do not model financial intermediation. Farboodi, Jarosch and Menzio (2018) consider intermediation as a rent extraction activity: they allow traders to acquire a costly commitment technology before trading, which help them gain bargaining power, and show that there may be multiple equilibria in which different fractions of agents acquire the commitment technology.[5] Gu et al. (2018) explore three different ways of modeling financial intermediaries. They show that when intermediaries derive a positive return from holding the asset (as opposed to incurring inventory costs), multiple equilibria may arise because intermediaries may choose to intermediate or just to hoard them.

Several papers in this OTC asset pricing literature consider notions of market fragility, which are related to our focus on equilibrium multiplicity. Weill (2007) and Lagos, Rocheteau, and Weill (2011) consider how dealers provide liquidity during an exogenously specified financial crisis. He and Milbradt (2014) consider the feedback loop between corporate bond defaults and secondary market illiquidity and show that they reinforce each other. These papers do not have multiple equilibria or trading among intermediaries.

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[4]Several other papers in the broader search literature feature multiple equilibria, but get equilibrium multiplicity for quite different reasons. For example, Diamond (1982) get equilibrium multiplicity in a production economy by directly assuming increasing returns to matching, whereas we have constant returns to matching. In Kiyotaki and Wright (1989), multiple equilibria co-exist with different goods acting as money, depending on what the authors call "extrinsic beliefs." Trejos and Wright (2016) show that allowing for non-linear utility functions, as opposed to the linear utility used in the Duffie, Garleanu and Pedersen (2005) paradigm, can also generate multiple equilibria in search models.

[5]As the authors noted, this multiplicity does not come from the effect through changes in the composition of agents, which instead plays a role in our model. Also, after acquiring the commitment technology, their "market equilibrium", which is the counterpart of our trading equilibrium, is always unique.

On modeling innovations, our framework features a fully decentralized, two-tier market structure with separate customer-to-dealer and interdealer markets,[6] and we relax the commonly used $\{0, 1\}$ asset holding restrictions appeared in most papers built upon Duffie, Garleanu and Pedersen (2005).[7] These modeling innovations allow us to parsimoniously capture the economic essence behind different levels of inventory holdings and generate various patterns of decentralized intermediated trading. Thus, our paper also contributes to a burgeoning literature exploring the endogenous emergence of market structures in OTC asset markets. In this literature, a wave of papers focuses on the emergence of the core-periphery trading networks observed in various OTC markets, asking why some traders become customers while others become dealers. These models start either from identical traders (Wang, 2017) or heterogenous traders along the dimension of initial asset positions (Afonso and Lagos, 2015), of asset valuations and trading needs (Chang and Zhang, 2016, Shen, Wei and Yan, 2016), of trading technologies (Neklyudov, 2014, Farboodi, Jarosch and Shimer, 2017), or of both trading needs and technologies (Uslu, 2017). Atkeson, Eisfeldt and Weill (2015), Neklyudov and Sambalaibat (2015) and Colliard, Foucault and Hoffmann (2018) further combine the OTC search and the network literatures to consider how exogenously specified network structures affect dealers' entry, exit and inventory holding decisions. Closer to us, Hugonnier, Lester and Weill (2018) use a fully decentralized two-tier market structure but with ex-ante heterogenous dealers to explore endogenous intermediation chains and quantitative implications. Those papers do not focus on multiple equilibria. Our work complements those papers in that we have a particular focus on the emergence of multiple equilibria and the liquidity implications. In doing so, we use a rich setting where the dealers are ex-ante identical and the trading, pricing, and inventory holding decisions of market participants are all endogenous.

Our paper is also related to the literature on OTC intermediation chains. Glode and

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[6]In the OTC asset pricing literature, some papers feature an explicit but reduced-form (usually modeled as centralized) interdealer market to facilitate the analysis of other aspects. For example, Lagos and Rocheteau (2009) and subsequent papers built on them consider unlimited asset holding positions by investors. Babus and Parlatore (2018) consider how strategic traders choose a dealer to trade with.

[7]Lagos and Rocheteau (2009) are among the first to allow agents to have unrestricted asset holdings positions but require centralized interdealer trading. Afonso and Lagos (2015) and Uslu (2017) consider decentralized trading and allow for a finite N and unrestricted asset holding positions, respectively. But they do not specifically distinguish between customer-to-dealer and interdealer trading as we do, and their focuses are also different from ours.

Opp (2016) show that longer intermediation chains help mitigate adverse selection and lead to more efficient trading. Babus and Hu (2017) show that intermediation chains can help incentivize better monitoring. On the other hand, a longer intermediation chain can lead to higher intermediation costs (Gofman, 2014) and real inefficiencies (Philippon, 2015). Ours differs by offering a balanced message: longer intermediation chains are desirable in term of liquidity provision, but the intermediation chain itself may be fragile due to the switch between multiple equilibria.

2 The model

Timeline, asset, and preferences. Time is continuous and runs infinitely: $t \in [0, \infty)$. Consider a market for a single indivisible asset. Agents are of three types: sellers, dealers, and buyers. All agents are risk-neutral with time preferences determined by a constant discount rate $r > 0$. The monetary value of a unit of the asset is $\theta > 0$ for a buyer, and is normalized to 0 for a seller and for a dealer. We call θ the fundamental of the asset. We also call buyers and sellers jointly as customers.

Buyers and sellers (customers). Buyers and sellers constantly arrive in and leave the market. The arrival rate of new buyers and of new sellers is n.[8] When arriving, a seller brings one unit of asset to the market while the buyer has nothing. We focus on intermediated trading in the sense that buyers and sellers cannot trade with each other directly but only through dealers. At any time t, an endogenous distribution of b_t active buyers and s_t active sellers are present in the market searching for dealers. We denote by $z_t = b_t + s_t$ the total mass of searching customers. A seller or a buyer who has successfully met and transacted with an dealer leaves the market and consumes. We also assume that, according to an independent Poisson process with rate $\eta > 0$, a searching customer defaults and departs the market without a successful transaction. This default shock captures that a buyer or seller may lose its trading opportunity if cannot find an intermediary in a timely manner.[9] Hence, the customers are effectively more impatient if η is larger. We assume that

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[8]The model can be extended to handle different arrival rates of new buyers and sellers without changing the economic mechanism, but doing so significantly complicates the calculation.

[9]For example, a junk bond mutual fund running out of cash rushes to find a dealer to sell its illiquid bonds, but it defaults and departs the market if cannot find a dealer timely.

dealers cannot short as typically observed in reality; thus, any asset will flow from a seller to a dealer's balance sheet as an inventory first, and then to a buyer later.

Dealers. Dealers are ex-ante homogeneous and long-lived in the market. We normalize the mass of dealers to 1. At any time t, each dealer can hold 0, 1, or 2 units of asset on its balance sheet. As standard in the literature (e.g., Amihud and Mendelson, 1980), we assume a weakly convex inventory cost per unit of time: $C(0) = 0, C(1) = c > 0$ where c captures the level of inventory costs, and $C(2) = \rho c$ where $\rho \geqslant 2$ captures the the convexity of the cost structure. We denote the endogenous distribution of dealers who hold 0, 1, and 2 units of the asset by d_{0t}, d_{1t}, and d_{2t}, respectively, where $d_{0t} + d_{1t} + d_{2t} = 1$. We call them type-0, type-1, and type-2 dealers, respectively.

It is worth noting that we purposefully relax the common assumption in the literature that limits the agents to hold either 0 or 1 unit of asset. By allowing a dealer to hold 0, 1, or 2 units of the asset, we parsimoniously capture that at any time a dealer may hold no inventory at all (0), or a low level of inventory (1), or a high level of inventory (2) on its balance sheet as observed in reality. As we show later, this deviation from the literature is also economically essential in generating gains from trade between intermediaries, and thereby driving our results regarding trading patterns. It is possible to extend the model to allow for more than three asset holding positions, but doing so will significantly complicate the analysis without adding economic insight regarding the coordination among intermediaries.[10]

Matching protocols. To make the model flexible and realistic, we assume a fully decentralized two-tier market structure.

First, on the customer-to-dealer market, a customer contacts a random dealer according to an independent Poisson process with rate $\alpha > 0$. Economically, the inverse of α captures the friction of the customer-to-dealer market; α can be also interpreted as a customer search technology: a larger α implies a better technology held by customers. This setting immediately implies that a dealer is contacted by a random customer at endogenous Poisson rate $\beta_t = \alpha(b_t + s_t)$, which depends on $b_t + s_t$, the mass of searching customers. Intuitively, a dealer will be contacted faster when more customers are actively searching.

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[10]This treatment is common in the broader literature of coordination games. Specifically, for any N-strategy supermodular game, the underlying economic insight can be appropriately captured by a two-strategy coordination game. See Vives (1990) for a more detailed discussion.

Second, in the interdealer market, a dealer contacts another random dealer according to an independent Poisson process with rate $\lambda \geqslant 0$, which is also independent to the dealer's inventory holdings. Similarly, the inverse of λ captures the trading friction of the interdealer market: a larger λ suggests that dealers are easier to meet each other. As discussed in the introduction, we do not explicitly model persistent trading relationships, but stronger trading relationships among dealers can be captured by a larger λ in our framework.

Bilateral trading. Trading prices are determined through generalized Nash bargaining. As a benchmark, we assume that all agents, when meet bilaterally, have equal bargaining powers.[11] Hence, trading happens when the gains from trade are positive, and the two meeting agents equally share the gains from trade.

3 Equilibrium analysis

As standard in the literature, we focus on steady-state (i.e., stationary) trading equilibria, that is, equilibria in which the mass of each type of agents is constant, and trading happens. Thus, we suppress the time argument t in equilibrium outcome variables, and we call a steady-state trading equilibrium simply an equilibrium below.

We first note that in any equilibrium, the inflow-outflow balance of the dealer sector implies $n - \eta b = n - \eta s$, which further implies that the mass of searching buyers must equal that of searching sellers, that is, $b = s$. We highlight that this is not an assumption but instead an equilibrium outcome from the inflow-outflow balance.

Given our focus on whether a dealer is willing to hold a high level of inventory to provide liquidity, that is, whether there are a positive mass of type-2 dealers in equilibrium, we give a sufficient condition for the mass of type-2 dealers to be positive.

LEMMA **1.** *In any trading equilibrium, $d_2 > 0$ if interdealer trading happens.*

Intuitively, if there were only type-0 and type-1 dealers in the market, interdealer trading would not happen because of the lack of gains from trade. Instead, a type-2 dealer may potentially trade with a type-0 dealer and both become type-1 dealers because the gains

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[11]The predictions of our model are robust to more sophisticated bargaining protocols that may give rise to unequal relative bargaining powers between the two bargaining parties.

from trade could be positive. Looking forward, we will further show that interdealer trading must happen when a type-2 dealer meets a type-0 dealer, that is, the interdealer market being active is also a necessary condition for $d_2 > 0$. These observations provide direct guidance for us to characterize the trading patterns of two possible trading equilibria, as we do below.

3.1 Trading equilibrium with high inventory: $d_2 > 0$

We first characterize a type of equilibrium where $d_2 > 0$. In this type of equilibrium, a dealer is willing to hold a high level of inventory. Figure 1 below illustrates the trading pattern of this type of equilibrium, where solid arrows indicate the flows of assets and dashed arrows indicate changes in the distribution of agents, that is, the flows of customers in and out of the market as well as the changes in dealers' inventory-holding status. As Figure 1 clearly shows, to sustain such an equilibrium requires five relevant trades to happen: a seller sells to a type-0 dealer or to a type-1 dealer, a buyer buys from a type-1 dealer or from a type-2 dealer, and a type-0 dealer buys from a type-2 dealer as an interdealer trade.
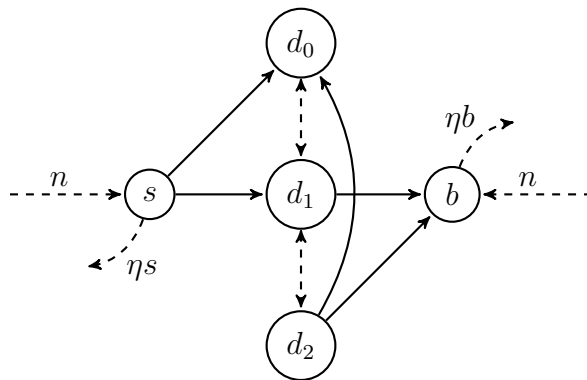
Figure 1: Trading equilibrium with the interdealer market

Solid arrows indicate the flows of assets. Dashed arrows indicate changes in the distribution of agents, that is, the flows of customers in and out of the market as well as changes in dealers' inventory-holding status.

Under this equilibrium, the asset inflows to the buyers must equal the asset outflows from type-1 and type-2 dealers:

$$n - \eta b = \alpha b(d_1 + d_2) , \tag{3.1}$$

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and similarly, the asset outflows from the sellers must equal the asset inflows taken by type-0 and type-1 dealers:

$$n - \eta s = \alpha s (d_0 + d_1) \,. \tag{3.2}$$

Since $b = s$ holds in any equilibrium, these two accounting identities (3.1) and (3.2) imply that

$$d_0 = d_2 \,, \tag{3.3}$$

that is, the mass of type-0 dealers must equal that of type-2 dealers. In what follows, we define $d_I = d_0 = d_2$ when refer to the equilibrium with $d_2 > 0$.

Denote by $V_b^1, V_s^1, V_0^1, V_1^1$, and V_2^1 the equilibrium value functions of an active buyer, seller, type-0 dealer, type-1 dealer, and type-2 dealer, respectively, where the superscript 1 indicates that these value functions are evaluated under the distribution of agents in an equilibrium with $d_2 > 0$. We derive the agents' equilibrium Hamilton-Jacobi-Bellman (HJB) equations, taking into account the endogenous matching and bargaining outcomes:

$$(\eta + r)V_b^1 = \alpha \left(d_2 \frac{1}{2}(\theta + V_1^1 - V_2^1 - V_b^1)^+ + d_1 \frac{1}{2}(\theta + V_0^1 - V_1^1 - V_b^1)^+ \right) , \tag{3.4}$$

$$(\eta + r)V_s^1 = \alpha \left(d_0 \frac{1}{2}(V_1^1 - V_0^1 - V_s^1)^+ + d_1 \frac{1}{2}(V_2^1 - V_1^1 - V_s^1)^+ \right) , \tag{3.5}$$

$$rV_0^1 = \beta \frac{s}{b+s} \frac{1}{2}(V_1^1 - V_0^1 - V_s^1)^+ + \lambda d_2 \frac{1}{2}(2V_1^1 - V_0^1 - V_2^1)^+ , \tag{3.6}$$

$$rV_1^1 = \beta \left(\frac{b}{b+s} \frac{1}{2}(\theta + V_0^1 - V_1^1 - V_b^1)^+ + \frac{s}{b+s} \frac{1}{2}(V_2^1 - V_1^1 - V_s^1)^+ \right) - c \,, \tag{3.7}$$

$$rV_2^1 = \beta \frac{b}{b+s} \frac{1}{2}(\theta + V_1^1 - V_2^1 - V_b^1)^+ + \lambda d_0 \frac{1}{2}(2V_1^1 - V_0^1 - V_2^1)^+ - \rho c \,. \tag{3.8}$$

These HJB equations are intuitive. First, since all agents have equal bargaining power, they equally share the potential gains from trade, if positive. The bargaining process also determines the trading price in every bilateral trading. These are reflected in the right hand sides of the HJBs.

Second, a buyer may buy a unit of asset from either a type-2 or type-1 dealer, while a seller may sell to either a type-0 or type-1 dealer. This is reflected in the buyer's and seller's HJB equations (3.4) and (3.5).

Finally, for the three types of dealers, they all have their unique pattern of trading,

reflected by (3.6), (3.7), and (3.8). A type-0 dealer can either buy a unit of asset from a seller or from a type-2 dealer, a type-1 dealer can either buy from a seller or sell to a buyer, while a type-2 dealer can either sell to a buyer or to a type-0 dealer. Consistent with our earlier argument, only the type-0 and type-2 dealers are potential candidates to participate in an interdealer trade.

In view of Lemma 1, we now use the HJBs to show that interdealer trading must happen if a type-0 dealer meets a type-2 dealer.

LEMMA 2. *In any equilibrium with $d_2 > 0$, $2V_1^1 - V_0^1 - V_2^1 > 0$ holds. That is, the gains from trade between a type-0 and a type-2 dealer are always strictly positive and thus interdealer trading happens.*

In the proof for Lemma 2, we show that the gains from a potential trade between a type-0 and a type-2 dealer can be expressed as

$$2V_1^1 - V_0^1 - V_2^1 = \frac{(\rho - 2)c + \kappa_1 \beta}{\kappa_2} \, ,$$

where $\kappa_1 > 0$ and $\kappa_2 > 0$ are two strictly positive constants that are determined in equilibrium. The numerator of the right hand side shows that the potential gains from trade are captured by two independent terms. The first term captures an instantaneous effect: it shows that an interdealer trade allows the two dealers to jointly save their inventory costs, consistent with the classical view of inventory sharing. The second term captures a continuation effect: it implies that an interdealer trade allows the two dealers, who subsequently become two type-1 dealers, to jointly intermediate more assets between buyers and sellers in a given time period. Since β is always strictly positive in any equilibrium, an interdealer trade may happen even if the inventory cost is not convex.

So far, Lemmas 1 and 2 show that the interdealer market being active is both a sufficient and a necessary condition for $d_2 > 0$. This verifies our equilibrium categorization: the equilibrium features an active interdealer market when $d_2 > 0$ while the interdealer market is inactive when $d_2 = 0$.

A natural and important question is when a trading equilibrium with $d_2 > 0$ exists. In principle, the existence of such an equilibrium requires all the five relevant trades, illustrated

by the five solid arrows in Figure 1, to happen. That is, the gains from trade as shown in the right hand sides of value functions (3.4), (3.5), (3.6), (3.7), and (3.8) must be weakly positive. It is not surprising, however, that many of these constraints will be slack in equilibrium. The following proposition shows that, intuitively, when the trade between a type-1 dealer and a seller happens, the equilibrium exists:

PROPOSITION 1. *When*

$$V_2^1 - V_1^1 - V_s^1 \geqslant 0 \,, \tag{3.9}$$

a trading equilibrium with $d_2 > 0$ exists, where V_2^1, V_1^1 and V_s^1 satisfy the value functions (3.4), (3.5), (3.6), (3.7), and (3.8).

Proposition 1 formally indicates that the sufficient criterion for the existence of the equilibrium with $d_2 > 0$ is that a type-1 dealer is willing to buy from a seller, given the corresponding distribution of agents. Other relevant trades must happen as a consequence.

The intuition is as follows. First, holding a higher inventory is more costly to a dealer due to the weakly convex inventory cost structure. Thus, that a type-1 dealer, who already holds a low level of inventory, finds it profitable to increase its inventory implies that a type-0 dealer must also find it profitable to increase its inventory. This implies that the trade between a type-0 dealer and a seller must happen.

Second, dealers do not have any ultimate interest in holding the asset. Thus, that a dealer finds it profitable to buy from a seller implies that the dealer must also find it profitable to sell to a buyer later, regardless of its type. This implies that the trade between a type-1 dealer and a buyer and that between a type-2 dealer and a buyer must happen.

Finally, notice that Lemma 2 already guarantees that the trade between a type-0 and a type-2 dealer, that is, interdealer trading, must happen when the two dealers meet.

Hence, given the economic importance of whether a type-1 dealer is willing to buy from a seller, as indicated by Proposition 1, in what follows we formally call this decision a type-1 dealer's *liquidity provision decision*, or equivalently, whether a type-1 dealer is willing to *provide liquidity* to a seller.

Following Proposition 1, we may further formulate the equilibrium existence with respect to the asset fundamental θ or inventory cost c. Since the HJBs are linear in θ, we have the

following straightforward corollary. It suggests that a type-1 dealer is willing to provide liquidity when the asset fundamental is high enough.

COROLLARY **1.** *If a trading equilibrium with $d_2 > 0$ exists for an asset fundamental θ, then a trading equilibrium with $d_2 > 0$ exists for any asset fundamental $\theta' \geqslant \theta$, other model parameters fixed.*

Corollary 1 can be also stated with respect to the inventory cost c because the HJBs are also linear in c. A type-1 dealer is willing to provide liquidity when the inventory cost is low enough. That is:

COROLLARY **1′.** *If a trading equilibrium with $d_2 > 0$ exists for an inventory cost c, then a trading equilibrium with $d_2 > 0$ exists for any inventory cost $c' \leqslant c$, other model parameters fixed.*

3.2 Trading equilibrium with low inventory: $d_2 = 0$

Next, we characterize the other type of trading equilibrium where $d_2 = 0$, that is, no dealers are willing to hold a high level of inventory. In view of Lemmas 1 and 2, interdealer trading being inactive is a sufficient and necessary condition for $d_2 = 0$. Figure 2 below illustrates the trading pattern of this type of equilibrium. It shows that to sustain such an equilibrium requires two relevant trades to happen: a seller sells to a type-0 dealer, and a buyer buys from a type-1 dealer.

Before proceeding, we highlight that the inactiveness of interdealer trading in the equilibrium with $d_2 = 0$ should not be literally interpreted as a complete shutdown of interdealer trading in reality. Rather, it parsimoniously captures the observed less intensive interdealer trading when intermediary liquidity provision to customers is low. This mapping is consistent with the empirical evidence in Bessembinder et al. (2018) that interdealer trading as a proportion of total trading volume in the U.S. corporate bond market had declined from 25% right after the financial crisis to 16% in 2013 as intermediary liquidity provision declined.

In this equilibrium with $d_2 = 0$, the inflow-outflow balance implies

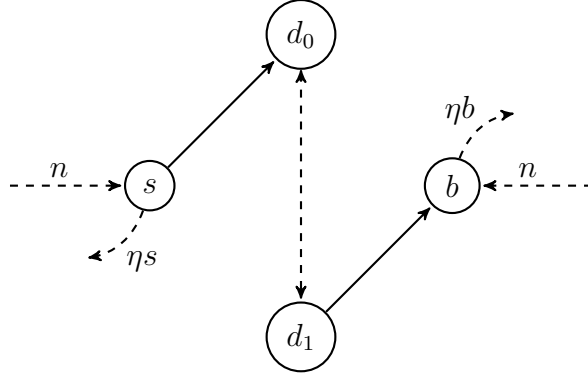$$n - \eta b = \alpha b d_1 \,, \tag{3.10}$$

Figure 2: Equilibrium without the interdealer market

Solid arrows indicate the flows of assets. Dashed arrows indicate changes in the distribution of agents, that is, the flows of customers in and out of the market as well as changes in dealers' inventory-holding status.

and

$$n - \eta s = \alpha s d_0 \,. \tag{3.11}$$

Since $b = s$ in any equilibrium, (3.10) and (3.11) together imply

$$d_0 = d_1 = \frac{1}{2} \,, \tag{3.12}$$

that is, the mass of type-0 dealers must equal that of type-1 dealers in the equilibrium with $d_2 = 0$.

Denote by $V_b^0, V_s^0, V_0^0,$ and V_1^0 the equilibrium value functions of an active buyer, seller, type-0 dealer, and type-1 dealer, respectively, where the superscript 0 indicates that these value functions are evaluated under the distribution of agents in an equilibrium with $d_2 = 0$. We can similarly derive the agents' equilibrium HJB equations:

$$(\eta + r)V_b^0 = \alpha d_1 \frac{1}{2}(\theta + V_0^0 - V_1^0 - V_b^0)^+ \,, \tag{3.13}$$

$$(\eta + r)V_s^0 = \alpha d_0 \frac{1}{2}(V_1^0 - V_0^0 - V_s^0)^+ \,, \tag{3.14}$$

$$rV_0^0 = \beta \frac{s}{b+s} \frac{1}{2}(V_1^0 - V_0^0 - V_s^0)^+ \,, \tag{3.15}$$

$$rV_1^0 = \beta \frac{b}{b+s} \frac{1}{2}(\theta + V_0^0 - V_1^0 - V_b^0)^+ - c \,. \tag{3.16}$$

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Similar to Proposition 1, the following proposition characterizes the conditions under which a trading equilibrium with $d_2 = 0$ exists:

PROPOSITION 2. *When $V_0^0 \geqslant 0$ and*

$$V_2^0 - V_1^0 - V_s^0 \leqslant 0 \,, \tag{3.17}$$

a trading equilibrium with $d_2 = 0$ exists, where the hypothetical value function V_2^0 is given by

$$rV_2^0 = \beta \frac{b}{b+s} \frac{1}{2}(\theta + V_1^0 - V_2^0 - V_b^0)^+ + \lambda d_0 \frac{1}{2}(2V_1^0 - V_0^0 - V_2^0)^+ - \rho c \,, \tag{3.18}$$

and V_b^0, V_s^0, V_0^0 and V_1^0 satisfy the value functions (3.13), (3.14), (3.15), and (3.16).

Proposition 2 first requires the value function of a type-0 dealer under the corresponding distribution of agents, V_0^0, to be weakly positive. To see the intuition, recall that we need to verify the gains from trade for all the relevant trades to be positive. When $V_0^0 \geqslant 0$, a type-0 dealer is willing to buy from a seller since this is the only trade that a type-0 dealer can conduct. Then, because dealers do not have any ultimate interest in holding the asset, that a type-0 dealer finds it profitable to buy from a seller implies that the dealer must also find it profitable to sell to a buyer later (i.e., when the type-0 dealer becomes a type-1 dealer). This implies that the trade between a type-1 dealer and a buyer must happen, too.

Provided that $V_0^0 \geqslant 0$, Proposition 2 further indicates that the sufficient criterion for the existence of the trading equilibrium with $d_2 = 0$ is that a type-1 dealer would not find it profitable if it were to buy from a seller, that is, a type-1 dealer is not willing to provide liquidity to a seller. This is captured by the equilibrium criterion (3.17), where the hypothetical value functional for a type-2 dealer is instead evaluated under the distribution with $d_2 = 0$.

Provided that $V_0^0 \geqslant 0$, we can also formulate the equilibrium existence with respect to the asset fundamental or inventory cost. A type-1 dealer is not willing to provide liquidity when the asset fundamental is low enough, or when the inventory cost is high enough.

COROLLARY 2. *If a trading equilibrium with $d_2 = 0$ exists for an asset fundamental θ, then a trading equilibrium with $d_2 = 0$ exists for any asset fundamental $\theta' \leqslant \theta$, other model*

parameters fixed.

COROLLARY **2′**. *If a trading equilibrium with $d_2 = 0$ exists for an inventory cost c, then a trading equilibrium with $d_2 = 0$ exists for any inventory cost $c' \geqslant c$, other model parameters fixed.*

We note that conditions (3.9) and (3.17) regarding the trade between a type-1 dealer and a seller are not mutually exclusive because these value functions are evaluated under different distributions of agents. Conditions (3.9) and (3.17) may both hold. This observation has direct implication on equilibrium multiplicity, which we elaborate below.

4 The coordination of intermediation

Having characterized the two types of trading equilibrium, we consider the potential for equilibrium multiplicity and examine the underlying strategic interactions. We also derive implications on market liquidity, which are empirically relevant but also useful in illustrating the source of the coordination motives. Given our focus on intermediated trading and potential multiple equilibria, in the rest of the paper we focus on the scenarios where trading always happens, that is, when at least one type of trading equilibrium exists.

4.1 Equilibrium multiplicity

The analysis of equilibrium multiplicity hinges on two observations from Section 3. First, Propositions 1 and 2 suggest that whether a type-1 dealer is willing to provide liquidity, that is, to buy from a seller, is the sole criterion to determine the equilibrium type given the distribution of other agents. Second, in turn, the distribution of other agents is solely determined by whether other type-1 dealers buy from sellers and effectively increase their inventory. Since we focus on steady-state equilibria, those observations allow us to use a two-strategy game representation among type-1 dealers to represent the strategic interaction embedded in the dynamic model.

PROPOSITION **3.** *For given model parameters, if*

$$\begin{cases} V_2^1 - V_1^1 \geqslant V_s^1 & (4.1) \\ V_2^0 - V_1^0 \leqslant V_s^0 \,, & (4.2) \end{cases}$$

then an equilibrium with $d_2 > 0$ and an equilibrium with $d_2 = 0$ co-exist.

Proposition 3 follows directly from Propositions 1 and 2. When (4.1) and (4.2) are satisfied simultaneously, those two propositions suggest that both equilibria co-exist. In view of Corollaries 1 and 2, Proposition 3 further suggests that the two equilibria may co-exist for medium values of asset fundamental θ or inventory cost c.

Proposition 3 resembles the classic notion of coordination such as that in the bank run models (e.g., Diamond and Dybvig, 1983). There, whether a depositor runs the bank depends on its belief about whether other depositor run. Here, whether a type-1 dealer buys from a seller and becomes a type-2 dealer, that is, provide liquidity, depends on its belief about whether other type-1 dealers provide liquidity. Thus, we can focus on this binary strategy of type-1 dealers providing liquidity or not to analyze equilibrium multiplicity.[12]

We provide a numerical example in Figure 3 to illustrate Proposition 3. In this example, we choose parameters $n = 1$, $\eta = 1$, $r = 1$, $c = 0.2$, $\rho = 2.5$, $\alpha = 5$, and $\lambda = 0.01$.

The top panel of Figure 3 plots $V_2^1 - V_1^1 - V_s^1$ and $V_2^0 - V_1^0 - V_s^0$, twice of the payoff gains of a type-1 dealer buying from a seller given other type-1 dealers' strategy (recall that other type-1 dealers' strategy determines the distribution of agents), against different values of asset fundamental θ. It shows that in the given economy, type-1 dealers' liquidity provision decisions exhibit coordination motives for medium values of θ. In particular, for medium values of θ, $V_2^1 - V_1^1 - V_s^1$ is positive while $V_2^0 - V_1^0 - V_s^0$ is negative. Therefore, according to Proposition 3, equilibrium multiplicity happens within that medium range of θ. This is confirmed by the bottom panel of Figure 3, which plots the probability ϕ of type-1 dealers providing liquidity against different values of θ. Intuitively, when θ is sufficiently large (small), type-1 dealers provide (do not provide) liquidity. When θ takes medium values, whether a type-1 dealer provides liquidity depends on other dealers' liquidity

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[12]As standard in the game theory literature, we also allow the agents to play mixed strategies to ensure that the strategy space is compact and that a stationary equilibrium always exists. The discussion of mixed strategies in our framework is primarily for technical completeness and not essential to the economic insight. Thus, we provide the details in the online appendix.
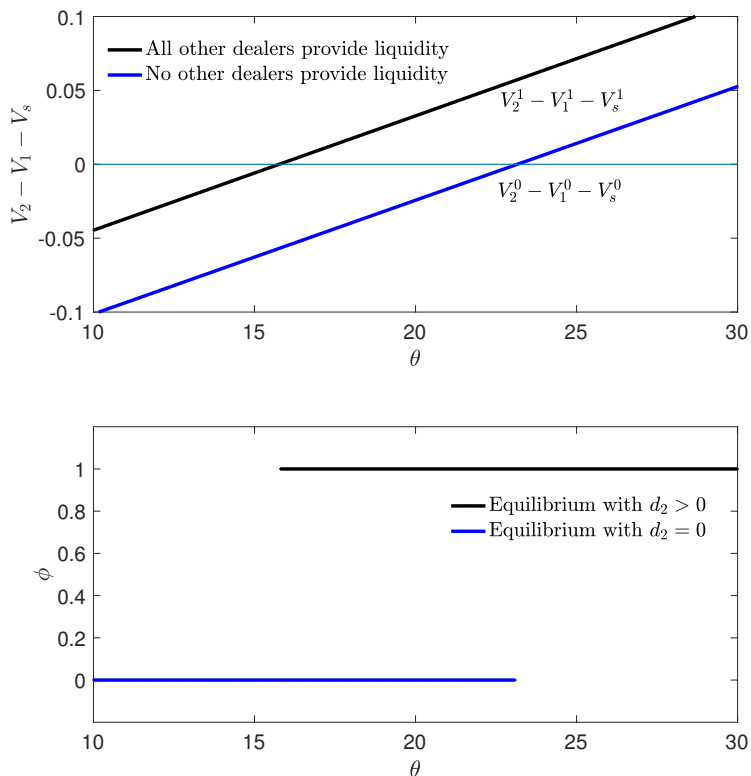
Figure 3: The correspondence of payoff gains and equilibrium multiplicity (view in color)

The top panel plots the gains from trade between a type-1 dealer and a seller against asset fundamental θ, under the distribution when all other type-1 dealers trade with sellers (black) and under the distribution when all other type-1 dealers do not trade with sellers (blue). The bottom panel plots the equilibrium probability of the trade between a type-1 dealer and a seller against asset fundamental θ. Parameters: $n = 1$, $\eta = 1$, $r = 1$, $c = 0.2$, $\rho = 2.5$, $\alpha = 5$, and $\lambda = 0.01$.

provision decisions, and both pure-strategy equilibria exist.

Similarly, if we fix an asset fundamental θ but vary the inventory cost c, we can show that both pure-strategy equilibria exist when c takes medium values.

Proposition 3 provides a concrete answer to the motivating question of this paper. When the economy falls in the parameter region where both equilibria exist, a switch between the two pure-strategy equilibria suggests that dealer balance sheet utilization and liquidity provision may decline for non-fundamental reasons. This also suggests non-fundamental volatility in dealer liquidity provision. These predictions are consistent with empirical evidence, for example, that liquidity provision by U.S. corporate bond dealers declined during 2010-2014 despite the lack of a negative fundamental shock and before the implementation of the Volcker Rule in 2015 (e.g., Bessembinder et al., 2018). As suggested by our model,

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such non-fundamental liquidity decline and volatility may have indeed come from the dealers worrying about each other's liquidity provision decisions.

A natural question is how type-1 dealers' coordination motives arise, that is, what are the economic forces behind them. In the next two subsections, we explore the source of coordination motives by analyzing economically relevant equilibrium outcomes including trading prices and market liquidity measures.

Importantly, to make the analysis transparent, we focus on parameter regions where the interdealer trading friction is sufficiently large, that is, λ is sufficiently close to 0, for the rest of Section 4. Economically, this amounts to saying that interdealer trading is sufficiently unlikely to happen when an equilibrium with $d_2 > 0$ happens.[13] In this case, a type-2 dealer would have to hold the inventory and incur the high inventory cost for longer. Taking this into account, a type-1 dealer will be more reluctant to provide liquidity. Because of this, the type-1 dealer in question may only provide liquidity when other type-1 dealers' liquidity provision decisions change the market conditions to the extent that it finds liquidity provision to be profitable enough. This renders type-1 dealers' liquidity provision decisions sensitive to each other. Thus, a sufficiently large interdealer trading friction allows us to illustrate dealers' coordination motives in liquidity provision most clearly. Looking ahead, we resume considering broader parameter regions where the interdealer trading friction can be either large or small in Section 5. There, we show that a sufficiently small interdealer trading friction (i.e., a sufficiently large λ) can indeed render a type-1 dealer's liquidity provision decision less strategically complementary to other type-1 dealers' and thus reduce the coordination motives.

4.2   Coordination motives and seller reservation value

To help uncover the source of coordination motives behind type-1 dealers' liquidity provision decisions, we first compare the seller's reservation values over the two types of equilibrium with $d_2 > 0$ and with $d_2 = 0$, that is, V_s^1 and V_s^0. (As standard in the OTC asset pricing literature, the reservation value of an agent is defined as the difference between the value of owning and not owning an asset.)

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[13]Interdealer trading always happens in an equilibrium with $d_2 > 0$ as long as $\lambda > 0$, however small λ is.

As well understood in the literature (e.g., Rubinstein and Wolinsky, 1987), the seller's reservation value is a key driver of dealers' liquidity provision decision. Economically, it is the lowest possible price at which a seller is willing to sell to a dealer, or equivalently, the lowest possible bid price that a dealer has to pay when providing liquidity. Thus, it is instructive to compare V_s^1 and V_s^0 to illustrate type-1 dealers' coordination motives. If $V_s^1 < V_s^0$, a liquidity-providing type-1 dealer is more able to pay a lower price to the seller and enjoy a higher profit when other type-1 dealers provide liquidity, rendering the specific type-1 dealer's liquidity provision decision strategically complementary to other type-1 dealers'.[14] Thus, we examine under what conditions $V_s^1 < V_s^0$ happens.

CONDITION 1. *The seller is sufficiently impatient in the sense that* $2(\alpha + \eta)r \geqslant \alpha n$.

PROPOSITION 4. *Suppose the interdealer trading friction is sufficiently large (i.e., λ is sufficiently close to 0). When Condition 1 holds, the reservation value of a seller V_s is lower in an equilibrium with $d_2 > 0$ than that in a comparable equilibrium with $d_2 = 0$, that is,* $V_s^1 < V_s^0$.

Proposition 4 implies that when the seller is sufficiently impatient in the sense that its discount rate r or the default shock rate η is large enough, the seller has a lower reservation value when more type-1 dealers provide liquidity. Intuitively, a more impatient seller needs to sell its asset more urgently. Thus, it is more willing to accept a lower price in exchange for more type-1 dealers providing liquidity, which allows the asset to be more quickly sold. As a result, when other type-1 dealers are willing to provide liquidity (which results in a distribution of agents with $d_2 > 0$), the type-1 dealer in question is also more able to bid a lower price to buy from the seller and to enjoy a higher profit. We call this the seller reservation value effect. This effect contributes to the coordination motives.

We provide a numerical example in Figure 4 to illustrate Proposition 4 and the seller reservation value effect, using the same parameters as those in Figure 3. With those parameters, Condition 1 holds. Thus, consistent with Proposition 4, Figure 4 shows that the reservation value of the seller is unambiguously lower in an equilibrium with $d_2 > 0$ than that in an equilibrium with $d_2 = 0$.

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[14]Note that $V_s^1 < V_s^0$ is not a sufficient condition for conditions (4.1) and (4.2) to hold jointly. The purpose of comparing V_s^1 and V_s^0 is to illustrate the coordination motives, rather than to give a condition with respect to exogenous parameters for equilibrium multiplicity to happen.
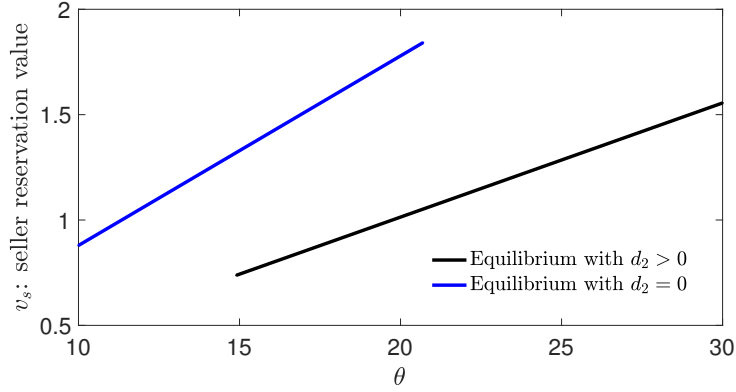
Figure 4: Equilibrium seller reservation value (view in color)

This graph plots equilibrium seller reservation value V_s against asset fundamental θ. Equilibrium multiplicity arises. Parameters: $n = 1$, $\eta = 1$, $r = 1$, $\rho = 2.5$, and $\alpha = 5$.

4.3 Coordination motives and market liquidity

To further illustrate the source of coordination motives behind type-1 dealers' liquidity provision decisions, we define and analyze two empirically relevant notions of market liquidity. These two notions capture two different but tightly linked aspects of market liquidity.

The first notion of market liquidity is customer-dealer trading volume, which captures the units of asset being intermediated in a given time. It is one of the most commonly used liquidity measures in empirical work.

DEFINITION **1.** *The customer-dealer trading volume per unit of time is defined by the units of asset intermediated per unit of time:*

$$q \doteq n - \eta b.^{15} \tag{4.3}$$

The following proposition suggests that higher intermediary liquidity provision improves market liquidity by increasing customer-dealer trading volume:

PROPOSITION **5.** *The customer-dealer trading volume per unit of time is higher in an equilibrium with $d_2 > 0$ than that in a comparable equilibrium with $d_2 = 0$, that is, $q^1 > q^0$.*

Proposition 5 is intuitive. When more type-1 dealers provide liquidity by holding a high

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[15]Note that we may equivalently define $q \doteq n - \eta s$ because $b = s$ holds in any equilibrium. The same applies to Definition 3 below.

inventory, the dealer sector can absorb more flows from the customers and hence intermediate more assets during any given period of time. We call this the trading volume effect.[16] Figure 5 illustrates the trading volume effect using the same parameters as those in Figure 3.
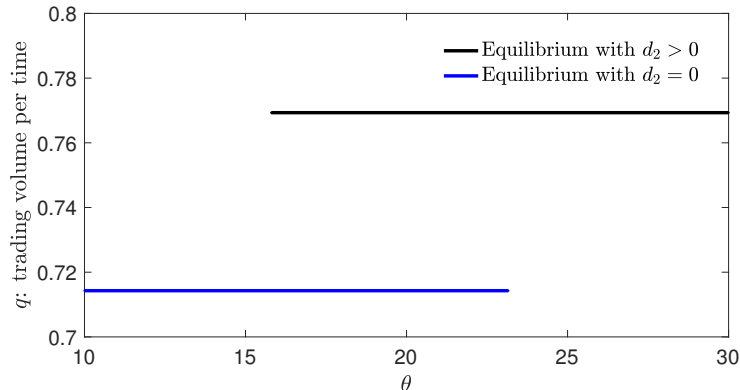

Figure 5: Equilibrium trading volume (view in color)

This graph plots equilibrium customer-dealer trading volume q against asset fundamental θ. Equilibrium multiplicity arises. Parameters: $n = 1$, $\eta = 1$, $r = 1$, $\rho = 2.5$, and $\alpha = 5$.

We then consider the second notion of market liquidity: dealer markup. It corresponds to the bid-ask spread, another commonly used liquidity measure in empirical work:

DEFINITION 2. *The dealer markup per unit of trade Δ_i is defined by the difference between the buyer-dealer price $p_{b,i+1}$ and the seller-dealer price $p_{s,i}$ for the same dealer who conducts a pair of intermediated trade:*

$$\Delta_i \doteq p_{b,i+1} - p_{s,i} \,, \tag{4.4}$$

where $i \in \{0, 1\}$ denotes the dealer type.

As Definition 2 indicates, in principle, markup depends on the dealer type. However, we can show that the markup per unit of time can be re-expressed as:

$$\Delta = \frac{(\theta - V_b) - V_s}{2} \,, \tag{4.5}$$

regardless of dealer type. This expression (4.5) is intuitive; it indicates that what the dealer captures from a pair of intermediated transactions is half of the difference between the buyer's

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[16]As shown in the proof of Proposition 5, to analytically show this trading volume effect does not require the interdealer trading friction to be sufficiently large. It generally holds for any $\lambda > 0$.

reservation value $\theta - V_b$ and the seller's reservation value V_s, which economically captures the joint gains from the pair of intermediated transactions.

PROPOSITION **6.** *Suppose the interdealer trading friction is sufficiently large (i.e., λ is sufficiently close to 0). Dealer markup per unit of trade is lower in an equilibrium with $d_2 > 0$ than that in a comparable equilibrium with $d_2 = 0$, that is, $\Delta^1 < \Delta^0$.*

Proposition 6 implies that dealer markup per trade decreases when more type-1 dealers provide liquidity. This is consistent with empirical evidence (e.g., Choi and Huh, 2017) that the average bid-ask spreads for large, principal trades (which actually occupy dealer balance sheet space) are higher when intermediary liquidity provision is lower. Economically, as more type-1 dealers incur the balance sheet space cost to provide liquidity, the dealer sector jointly incurs a higher inventory cost, and thus the joint gains from the pair of intermediated transactions decrease. As shown by expression (4.5), this immediately implies a lower dealer markup. We call this the markup effect. Figure 6 illustrates the markup effect using the same parameters as those in Figure 3.
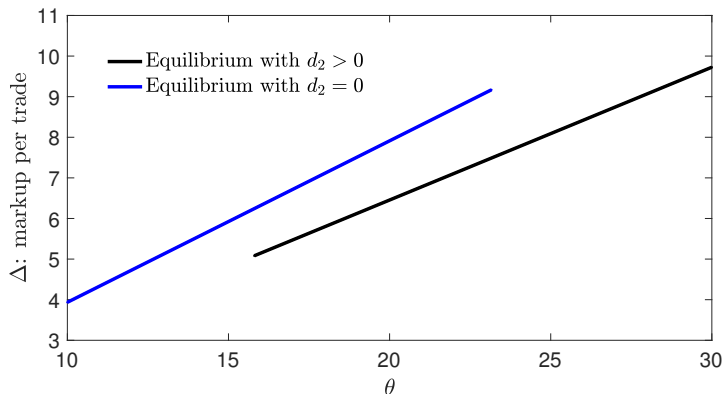

Figure 6: Equilibrium dealer markup (view in color)

This graph plots equilibrium dealer markup Δ against asset fundamental θ. Equilibrium multiplicity arises. Parameters: $n = 1$, $\eta = 1$, $r = 1$, $\rho = 2.5$, and $\alpha = 5$.

Propositions 5 and 6 together provide an aspect to illustrate the coordination motives in dealer liquidity provision. This comes from one important observation from Propositions 5 and 6: although both suggest that market liquidity unambiguously increase as type-1 dealers provide more liquidity, the impact on dealer revenue per unit of time is ambiguous.

Specifically, a dealer's revenue per time, which is the product of trading volume per time and markup per trade, depends on how the trading volume and markup change over the two equilibria. When the trading volume effect dominates the markup effect, a type-1 dealer's revenue per time from buying from sellers will increase if other type-1 dealers also buy from sellers. This contributes to coordination motives in dealer liquidity provision because a dealer cares about its revenue per time to compensate for its inventory cost at any given time.

Unfortunately, beyond Propositions 5 and 6, we are not able to analytically derive clean general conditions with respect to exogenous model parameters (similar to Condition 1) to determine when the trading volume effect dominates the markup effect. This challenge stems from our modeling innovation of allowing for three inventory positions $\{0, 1, 2\}$, which requires solving a complicated quartic inequality for model parameters. Since we believe Propositions 5 and 6 are sufficiently instructive and modeling these three inventory positions is essential to our framework, we decide to make this modeling trade-off.

We highlight that although Proposition 4 and Propositions 5 and 6 explore the coordination motives in dealer liquidity provision from two different aspects, they are economically consistent. One may wonder why, in an equilibrium with $d_2 > 0$, a type-1 dealer is able to bid a lower price to the seller (suggested by Proposition 4), but ultimately enjoys a lower markup per trade (suggested by Proposition 6). This is because, as we show in Proposition 10 in Appendix A, the buyer's reservation value is also lower in an equilibrium with $d_2 > 0$, that is, $\theta - V_b^1 < \theta - V_b^0$. We call this decrease the buyer reservation value effect. Since the buyer reservation value is the highest price at which a buyer is willing to buy from a dealer, Proposition 10 implies that a liquidity-providing dealer is likely to receive a lower bid price from the buyer in an equilibrium with $d_2 > 0$. Because dealer markup increases in buyer reservation value while decreases in seller reservation value as shown in (4.5), the buyer reservation value effect offsets the seller reservation value effect, contributing to the decrease in dealer markup in an equilibrium with $d_2 > 0$. Propositions 4 and 10 thus jointly implies that a stronger seller reservation effect is associated with a weaker markup effect. This observation helps us reconcile the economic messages in Sections 4.2 and 4.3: Propositions 5 and 6 suggest that the coordination motives are stronger when the volume effect dominates the markup effect, and Proposition 4 indeed suggests one scenario when the markup effect

is weaker and may be dominated, that is, when the seller reservation value effect is stronger.

Finally, our framework also delivers two more empirically relevant equilibrium outcomes: the average length of intermediation chains and the aggregate inventory held by the dealer sector. In our framework, we define the length of intermediation chains L as the number of dealers through which an asset is intermediated from a seller to a buyer. Note that L is a random variable in the equilibrium with $d_2 > 0$. We define the aggregate inventory I as the units of asset held by all the types of dealers. We have the following straightforward result, though we note that our goal is not to directly map these equilibrium outcomes to data but rather to qualitatively compare them across the two types of equilibrium.

COROLLARY **3.** *Intermediation chains are longer and aggregate dealer inventory is higher in an equilibrium with $d_2 > 0$ than those in a comparable equilibrium with $d_2 = 0$, that is, $\mathbb{E}[L^1] > L^0$ and $I^1 > I^0$.*

5   The role of interdealer trading

A natural question in our framework is how the efficiency of interdealer trading affects the coordination of dealer liquidity provision. When it becomes easier for dealers to trade with each other, would it become more or less likely for multiple equilibria to occur? That our model parsimoniously capturing the function of interdealer trading allows us to answer this question concretely.

As Sections 4.2 and 4.3 already elaborate, the coordination motives in dealer liquidity provision are stronger when the interdealer trading friction is sufficiently large (i.e., λ is sufficiently close to 0). When it is harder for dealers to trade with each other, a type-1 dealer will be more reluctant to buy from a seller. This is because the difficulty of interdealer trading implies that the dealer would have to hold the high inventory and bear the inventory cost by itself for a relatively longer time (before being contacted by a buyer). In this case, only when other type-1 dealers are willing to provide liquidity and sufficiently decrease seller reservation value (as illustrated in Section 4.2) or increase customer-dealer trading volume (as illustrated in Section 4.3), this type-1 dealer in question can enjoy a high enough profit per unit of time and thus is willing to provide liquidity. This effectively makes a type-1 dealer's

liquidity provision decision more strategically complementary to each other's, leading to stronger coordination motives and ultimately multiple equilibria.

Now we consider the opposite case when dealers can trade with each other relatively easily, that is, when the interdealer trading friction is sufficiently small. Proposition 7 suggests that the equilibrium with $d_2 > 0$ is the unique trading equilibrium regardless of the asset fundamental θ or inventory cost c, provided that a trading equilibrium exists.

PROPOSITION **7.** *For given parameters $n, \eta, \alpha,$ and ρ, there exists a threshold $\bar{\lambda} > 0$ such that when $\lambda > \bar{\lambda}$, the equilibrium with $d_2 > 0$ is the unique trading equilibrium regardless of θ and c.*

Intuitively, a more efficient interdealer market renders it easier for the dealers to offload their high inventory holdings. This makes every type-1 dealer more willing to provide liquidity and hold a high inventory regardless of other dealers' liquidity provision decisions. Thus, one dealer's liquidity provision decision becomes less strategically complementary to other dealers'. If the interdealer trading friction is sufficiently small, the endogenously emerged interdealer market and the resulting fast interdealer trading can eliminate the coordination motives among type-1 dealers' liquidity provision decisions, thereby eliminate any possible coordination failure.

Proposition 7 delivers new implications on the function of the interdealer market. As well understood in the literature, more efficient interdealer trading allows dealers to share risks more efficiently (e.g., Ho and Stoll, 1983, Viswanathan and Wang, 2004). Our results further suggest that more efficient interdealer trading, by effectively lowering dealer inventory cost, can reduce potential coordination failures in dealer liquidity provision. Hence, a well-functioning interdealer market may help reduce non-fundamental volatility in dealer liquidity provision and help improve financial stability. Since we interpret a smaller interdealer trading friction as a stronger interdealer trading relationship, Proposition 7 is also consistent with the empirical evidence that interdealer trading relationships help stabilize the market in crisis times (e.g., Di Maggio, Kermani and Song, 2017).

We also provide numerical comparative statics in Figure 7 to illustrate Proposition 7, using the parameters $n = 1$, $\eta = 1$, $r = 1$, $\rho = 2.5$, and $\alpha = 5$ (the same as those in Figure 3 except for varying λ's). In Figure 7, we use yellow color to draw the parameter regions in the

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(θ, c)—space where multiple equilibria happen. We note that, to keep the figure focused, we do not specifically indicate the existence and type of equilibrium in the blue-colored regions. A trading equilibrium may or may not exist in the blue-colored regions, but if it exists, it must be a unique equilibrium with either $d_2 > 0$ or $d_2 = 0$.
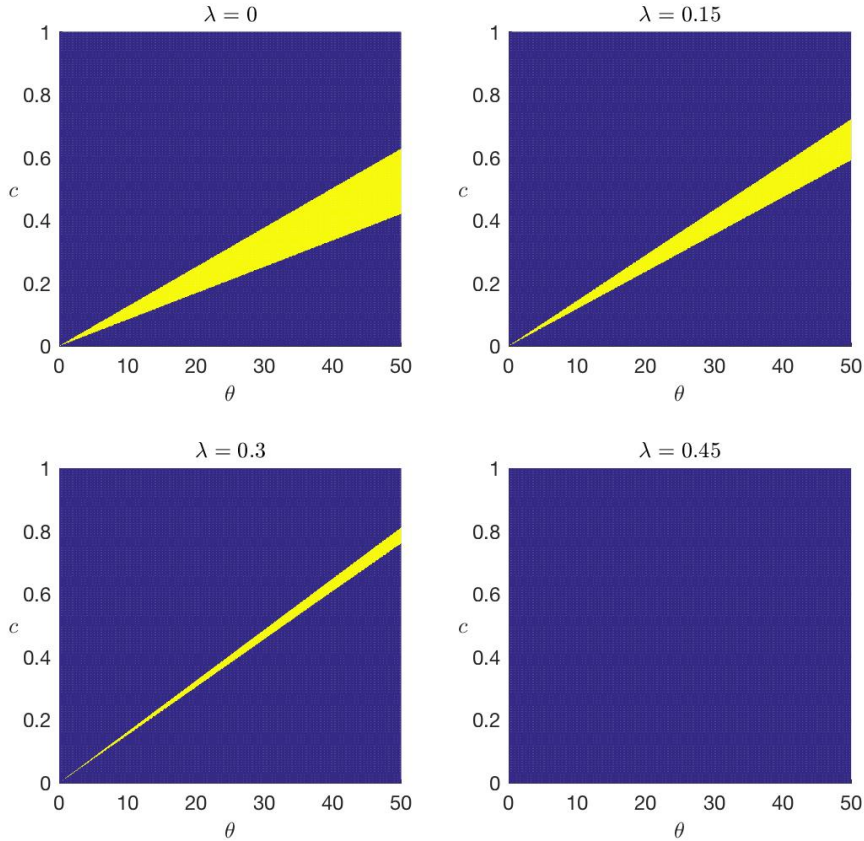

Figure 7: Interdealer market friction and market fragility (view in color)

The yellow-colored, highlighted region in each panel plots the parameter region where equilibrium multiplicity arises over the (θ, c)-parameter space. To keep the figure focused, the existence and type of equilibrium in the blue-colored regions is not specifically indicated. A trading equilibrium may or may not exist in the blue-colored regions, but if it exists, it must be a unique equilibrium with either $d_2 > 0$ or $d_2 = 0$. Parameters: $n = 1$, $\eta = 1$, $r = 1$, $\rho = 2.5$, and $\alpha = 5$.

The four panels in Figure 7 show that, as the interdealer trading friction becomes smaller, captured by λ increasing, the yellow-colored area where multiplicity happens over the (θ, c)—space becomes smaller. Specifically, as λ takes values from 0 to 0.15 and then to 0.3, multiple equilibria always occur, but become less likely to happen. When the interdealer market becomes sufficiently efficient as $\lambda = 0.45$, the coordination motives get completely

eliminated, and thus multiplicity never happens.

Another important take-away from Figure 7 is that, for a given level of interdealer trading friction λ, equilibrium multiplicity happens only for medium values of θ (given c) and medium values of c (given θ). This is consistent with Corollaries 1 and 2 and the numerical example in Figure 3.

6 Welfare implications

Although we primarily focus on the positive implications of coordination in intermediary liquidity provision, we can naturally define and analyze social welfare in our framework. Definition 3 intuitively captures that social welfare is the difference between the realized gains from trade and inventory costs.

DEFINITION 3. *In any equilibrium, the social welfare per unit of time W is defined by:*

$$W = (n - \eta b)\theta - (d_1 c + d_2 \rho c). \tag{6.1}$$

We present two results on equilibrium welfare, whose proofs directly follow from the proof of Proposition 5 and thus are omitted. Given our focus on intermediary balance sheet space costs, we first show that high intermediary liquidity provision is welfare-improving when the inventory cost is low enough.

PROPOSITION 8. *When c is sufficiently small, $W^1 > W^0$, that is, the equilibrium with $d_2 > 0$ delivers a higher social welfare than a comparable equilibrium with $d_2 = 0$.*

Proposition 8 is straightforward; its intuition is that when c is small, social welfare is predominantly driven by trading volume. In contrast, when c is large, the equilibrium with $d_2 > 0$ may not necessarily deliver higher social welfare despite higher liquidity provision.

We note that Proposition 8 is independent to that our model provides an explanation for the recently observed declines in intermediary liquidity provision. Indeed, empirical studies such as Bessembinder et al. (2018) clearly document the declines in intermediary liquidity provision but do not directly claim whether such declines are socially desirable or undesirable.

Next, we examine how customer characteristics affect welfare.

PROPOSITION **9.** *When α is sufficiently large, or when η is sufficiently small, $W^1 < W^0$, that is, the equilibrium with $d_2 > 0$ delivers a lower social welfare than a comparable equilibrium with $d_2 = 0$.*

Proposition 9 suggests that high intermediary liquidity provision is not socially desirable when customers have a good enough technology to contact the dealers, or when customers' default risk is small. In those cases, intuitively, a low level of intermediary liquidity provision (as that in the equilibrium with $d_2 = 0$) is already sufficient to meet customers' trading needs.

Together, Propositions 8 and 9 suggest that despite the potential for multiple equilibria, which equilibrium is socially desirable depends on dealer balance sheet space costs and customer characteristics. The contrast between Propositions 8 and 9 also suggests that dealer balance sheet space and customer technologies are substitutes. An improvement in customer technologies helps mitigate constraints from limited dealer balance sheet space in terms of maximizing welfare. This view is consistent with the increasing popularity of electronic trading platforms that allow customers to contact dealers more efficiently.

A potentially interesting question is which equilibrium is socially desirable in the parameter regions where both equilibria co-exist. Answering this question analytically may help shed light on whether equilibrium multiplicity justifies any policy interventions. Due to the same modeling trade-off we mentioned at the end of Section 4.3, we leave this question for future research.

7   Conclusion

We propose a dynamic model of asset markets to study decentralized intermediary liquidity provision. We focus on the emergence of multiple equilibria and its liquidity implications.

In a trading equilibrium, when inventory cost is low (high), dealers provide more (less) liquidity by holding more (less) inventory, the market is liquid (illiquid), interdealer trading is active (inactive), and the intermediation chain is longer (shorter). When inventory cost is medium, a dealer is more likely to provide liquidity if other dealers do so and the market becomes more liquid, leading to coordination motives among dealers. The coordination motives in turn lead to multiple equilibria. A switch between the two equilibria implies liquidity

decline and volatility for non-fundamental reasons, thus providing a plausible explanation for why dealer liquidity provision declined even before the implementation of the Volcker rule. A small interdealer trading friction effectively reduces dealers' inventory cost, making a dealer's willingness to provide liquidity less strategically complementary to other dealers' decisions and thereby reducing the possibility of equilibrium multiplicity.

On modeling innovations, we model a fully decentralized, two-tier OTC market structure and simultaneously relax the commonly used $\{0, 1\}$ asset holding restrictions in the literature. Although our model predictions are primarily qualitative, we believe that the way that we capture real OTC market structures may also inform future quantitative research.

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Appendix

A   Additional results

We first prove an additional result that implies a restriction on the equilibrium distribution of agents. This result will be repeatedly used in other proofs.

LEMMA **3.** *In any trading equilibrium with $d_2 > 0$, there must be that $0 < d_I \leqslant \frac{1}{3}$, that is, $d_1 \geqslant \frac{1}{3}$.*

PROOF OF LEMMA **3.** In any trading equilibrium, the mass of type-0 dealers that buy from a seller and become type-1 dealers must equal the mass of type-1 dealers that sell to a buyer and become type-0 dealers:

$$d_0(\beta \frac{s}{b+s} + \lambda d_2) = d_1 \beta \frac{b}{b+s} .$$

Because $b = s$ in any equilibrium, it follows that

$$\frac{\beta}{2} d_0 + \lambda d_0 d_2 = \frac{\beta}{2} d_1 . \tag{A.1}$$

Then by equation (3.3), condition (A.1) thus becomes

$$\lambda d_I^2 + \frac{3}{2} \beta d_I - \frac{\beta}{2} = 0 . \tag{A.2}$$

Solving (A.2) and picking the positive root yields:

$$d_I = \frac{-\frac{3}{2}\beta + \sqrt{\frac{9}{4}\beta^2 + 2\lambda\beta}}{2\lambda} \in \left[0, \frac{1}{3}\right] ,$$

completing the proof.

In particular, Lemma 3 implies

$$\begin{cases} \lim_{\lambda \to \infty} d_I = 0 & \text{if } \lambda \to \infty , \\ d_I = \frac{1}{3} & \text{if } \lambda = 0 , \end{cases}$$

which will be frequently used below.

The next lemma shows that a dealer is contacted by a customer more slowly in an equilibrium with $d_2 > 0$.

LEMMA **4.** *The rate at which a dealer is contacted by a customer β is lower in an equilibrium with $d_2 > 0$ than that in a comparable equilibrium with $d_2 = 0$, that is, $\beta^1 < \beta^0$.*

PROOF OF LEMMA 4. Direct calculation from the inflow-outflow balance equations (3.1) and (3.10) yields:

$$\beta^1 = 2\alpha b^1 = \frac{2\alpha n}{\eta + \alpha(1 - d_I)}, \tag{A.3}$$

and

$$\beta^0 = 2\alpha b^0 = \frac{2\alpha n}{\eta + \frac{\alpha}{2}}. \tag{A.4}$$

By Lemma 3, conditions (A.3) and (A.4) immediately yields the desired result, completing the proof.

Finally, we present a result showing that the reservation value of a buyer $\theta - V_b$ is lower in an equilibrium with $d_2 > 0$. We present the proof at the end of Appendix B because it is built on the other proofs.

PROPOSITION **10.** *Suppose the interdealer trading friction is sufficiently large (i.e., λ is sufficiently close to 0). The reservation value of a buyer $\theta - V_b$ is lower in an equilibrium with $d_2 > 0$ than that in a comparable equilibrium with $d_2 = 0$, that is, $\theta - V_b^1 < \theta - V_b^0$.*

B Proofs omitted from the main text

PROOF OF LEMMA 2. First, (3.7) minus (3.6) gives

$$\frac{1}{2}(V_1^1 - V_0^1)(2r + \beta + \lambda d_I) = \frac{\beta}{4}(\theta - V_b) + \frac{1}{2}(V_2^1 - V_1^1)\left(\frac{\beta}{2} + \lambda d_I\right) - c, \tag{B.1}$$

and (3.8) minus (3.7) gives

$$\frac{1}{2}(V_2^1 - V_1^1)(2r + \beta + \lambda d_I) = \frac{\beta}{4}V_s + \frac{1}{2}(V_1^1 - V_0^1)\left(\frac{\beta}{2} + \lambda d_I\right) - (\rho - 1)c. \tag{B.2}$$

Combining (3.4) and (B.1) yields

$$
\frac{1}{2}(V_1^1 - V_0^1)(2r + \beta + \lambda d_I)
$$
$$
= \frac{\beta}{4} \frac{(\eta + r)\theta + \alpha d_I \frac{1}{2}(V_2^1 - V_1^1) + \alpha(1 - 2d_I)\frac{1}{2}(V_1^1 - V_0^1)}{\eta + r + \frac{\alpha}{2}(1 - d_I)} + \frac{1}{2}(V_2^1 - V_1^1)\left(\frac{\beta}{2} + \lambda d_I\right) - c.
$$

Define

$$
B = \frac{\beta}{2} + \lambda d_I + \frac{\beta}{2}\frac{\alpha d_I}{2(\eta + r) + \alpha(1 - d_I)}, \tag{B.3}
$$

and

$$
Q = \frac{\beta(\eta + r)}{2(\eta + r) + \alpha(1 - d_I)}. \tag{B.4}
$$

It follows that

$$
\frac{1}{2}(V_1^1 - V_0^1)(2r + B + Q) = \frac{1}{2}(V_2 - V_1)B + \frac{\theta}{2}Q - c. \tag{B.5}
$$

Similarly, combining (3.5) and (B.2) yields

$$
\frac{1}{2}(V_2^1 - V_1^1)\left(2r + \beta + \lambda d_I - \frac{\beta}{2}\frac{\alpha(1 - 2d_I)}{2(\eta + r) + \alpha(1 - d_I)}\right)
$$
$$
= \frac{1}{2}(V_1^1 - V_0^1)\left(\frac{\beta}{2} + \lambda d_I + \frac{\beta}{2}\frac{\alpha d_I}{2(\eta + r) + \alpha(1 - d_I)}\right) - (\rho - 1)c,
$$

that is,

$$
\frac{1}{2}(V_2^1 - V_1^1)(2r + B + Q) = \frac{1}{2}(V_1^1 - V_0^1)B - (\rho - 1)c. \tag{B.6}
$$

Hence, (B.5) minus (B.6) implies

$$
\frac{1}{2}(2V_1^1 - V_2^1 - V_0^1)(2r + 2B + Q) = \frac{\theta}{2}Q + \underbrace{(\rho - 2)c}_{\geqslant 0}. \tag{B.7}
$$

Because $B > 0$ and $Q > 0$, this implies that $2V_1^1 - V_2^1 - V_0^1 > 0$. Therefore, interdealer trading must happen as $d_2 > 0$, concluding the proof. It is also straightforward to see from (B.4) and (B.7) that

$$
2V_1^1 - V_0^1 - V_2^1 = \frac{(\rho - 2)c + \kappa_1\beta}{\kappa_2},
$$

where

$$\kappa_1 = \frac{(\eta + r)\theta}{4(\eta + r) + 2\alpha(1 - d_I)} > 0$$

and

$$\kappa_2 = r + B + \frac{Q}{2} > 0 \, .$$

PROOF OF PROPOSITION 1. We consider the gains from trade of the five relevant trades in an equilibrium with $d_2 > 0$. We need to show that the following five inequalities all hold:

$$V_2^1 - V_1^1 - V_s^1 \geqslant 0 \, , \tag{B.8}$$

$$V_1^1 - V_0^1 - V_s^1 \geqslant 0 \, , \tag{B.9}$$

$$V_0^1 - V_1^1 + \theta - V_b^1 \geqslant 0 \, , \tag{B.10}$$

$$V_1^1 - V_2^1 + \theta - V_b^1 \geqslant 0 \, , \tag{B.11}$$

$$V_1^1 - V_0^1 + V_1^1 - V_2^1 \geqslant 0 \, , \tag{B.12}$$

when (B.8) is given as a sufficient condition.

We first show that (B.10) holds when (B.8) holds. To see this, first notice that (3.4) implies:

$$V_b = \frac{\frac{1}{2}\alpha(1 - d_I)\theta - \alpha d_I \frac{1}{2}(V_2 - V_1) - \alpha(1 - 2d_I)\frac{1}{2}(V_1 - V_0)}{(\eta + r) + \frac{1}{2}\alpha(1 - d_I)} \, , \tag{B.13}$$

and subsequently,

$$V_0 - V_1 + \theta - V_b = \frac{(\eta + r)\theta + \alpha d_I \frac{1}{2}(V_2 - V_1) - (\eta + r + \alpha d_I)\frac{1}{2}(V_1 - V_0)}{(\eta + r) + \frac{1}{2}\alpha(1 - d_I)} \tag{B.14}$$

Notice that Lemma 3 implies that

$$(\eta + r) + \frac{1}{2}\alpha(1 - d_I) > 0 \, ,$$

in other words, the denominator of (B.14) is always strictly positive. It hence suffices to consider the numerator of (B.14).

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Following the proof for Lemma 2, (B.6) and (B.7) imply

$$\frac{1}{2}(V_2^1 - V_1^1) = (2r + Q)^{-1}\left(\frac{B}{2r + 2B + Q}\left(\frac{Q}{2} + (\rho - 2)c\right) - (\rho - 1)c\right), \qquad \text{(B.15)}$$

where B and Q are defined by (B.3) and (B.4) in the proof of Lemma 2. Equations (B.5) and (B.15) then imply

$$\frac{1}{2}(V_1^1 - V_0^1)(2r + B + Q) = B\left(\frac{1}{2}(V_1^1 - V_0^1) - \frac{\frac{\theta}{2}Q + (\rho - 2)c}{2r + 2B + Q}\right) + \frac{\theta}{2}Q - c. \qquad \text{(B.16)}$$

Therefore,

$$\frac{1}{2}(V_1^1 - V_0^1) = (2r + Q)^{-1}\frac{(2r + B + Q)(\frac{\theta}{2}Q - c) - B(\rho - 1)c}{2r + 2B + Q}. \qquad \text{(B.17)}$$

Similarly, (B.15) can be rewritten as

$$\frac{1}{2}(V_2^1 - V_1^1) = (2r + Q)^{-1}\frac{B(\frac{\theta}{2}Q - c) - (2r + B + Q)(\rho - 1)c}{2r + 2B + Q}. \qquad \text{(B.18)}$$

Combing (B.17) and (B.18), the numerator of (B.14) can be expressed by:

$$\frac{(\eta + r)\left[(2r + 2B + Q)(2r + Q)\theta + B(\rho - 1)c - (2r + B + Q)(\frac{Q\theta}{2} - c)\right] - \alpha d_I\left[(2r + Q)\left(\frac{Q\theta}{2} + (\rho - 2)c\right)\right]}{(2r + Q)(2r + 2B + Q)}$$

$$\text{(B.19)}$$

Since $B > 0$ and $Q > 0$, the denominator of (B.19) is always strictly positive. It hence further suffices to consider the numerator of (B.19). Denote this numerator by

$$Y = (\eta + r)\left[(2r + 2B + Q)(2r + Q)\theta + B(\rho - 1)c - (2r + B + Q)(\frac{Q\theta}{2} - c)\right]$$
$$- \alpha d_I\left[(2r + Q)\left(\frac{Q\theta}{2} + (\rho - 2)c\right)\right] \qquad \text{(B.20)}$$

On the other hand, note that Lemma 3 implies that $d_I \in \left[0, \frac{1}{3}\right]$. Therefore, $\alpha(1 - 2d_I) > 0$ and

$$2(\eta + r) + \alpha(1 - d_I) > 0.$$

Hence, the desired sufficient condition (B.8) implies that

$$2(\eta + r)\frac{1}{2}(V_2^1 - V_1^1) \geqslant \alpha d_I \frac{1}{2}(2V_1^1 - V_2^1 - V_0^1), \tag{B.21}$$

and by (B.7) and (B.18), we have

$$\frac{2(\eta + r)}{2r + Q} \frac{B\left(\frac{\theta}{2}Q - c\right) - (2r + B + Q)(\rho - 1)c}{2r + 2B + Q} \geqslant \frac{\alpha d_I \left(\frac{\theta}{2}Q + (\rho - 2)c\right)}{2r + 2B + Q},$$

which subsequently implies

$$2(\eta + r)\left(B\left(\frac{\theta}{2}Q - c\right) - (2r + B + Q)(\rho - 1)c\right) \geqslant (2r + Q)\alpha d_I \left(\frac{\theta}{2}Q + (\rho - 2)c\right). \tag{B.22}$$

Thus, combining the definition of Y (B.20) and inequality (B.22) yields:

$$
\begin{aligned}
Y &\geqslant (\eta + r)\left((2r + 2B + Q)(2r + Q)\theta + B(\rho - 1)c - (2r + 3B + Q)\left(\frac{Q\theta}{2} - c\right) - 2(2r + B + Q)(\rho - 1)c\right) \\
&\geqslant \left(4r^2 + 4rB + 3rQ + \frac{1}{2}BQ + \frac{1}{2}Q^2\right)\theta + (4r + 3B + 2Q)(\rho - 1)c + (2r + 3B + Q)c \\
&\geqslant 0,
\end{aligned}
\tag{B.23}
$$

where both inequalities rely on $B > 0$ and $Q > 0$. This confirms that (B.10) holds.

The final step is to make use of Lemma 2. By Lemma 2, (B.12) holds. Combining (B.8) and (B.12) immediately yields (B.9), and combining (B.10) and (B.12) directly yields (B.11). This completes the proof.

PROOF OF PROPOSITION 2. We take a different idea (than the proof of Proposition 1) to prove Proposition 2. Other than checking the gains from trade for the two relevant trades in an equilibrium with $d_2 = 0$, we directly analyze the HJBs to derive the sufficient conditions.

To begin, HJBs (3.13) and (3.16) imply

$$rV_1^0 = \frac{\beta(\eta + r)V_b^0}{\alpha} - c. \tag{B.24}$$

and HJBs (3.14) and (3.15) imply

$$rV_0^0 = \frac{\beta(\eta + r)V_s^0}{\alpha} \, . \tag{B.25}$$

Moreover, (3.16) minus (3.15) yields

$$\begin{aligned}
r(V_1^0 - V_0^0) &= -\beta\frac{1}{2}(V_1^0 - V_0^0) + \frac{\beta}{2}\left(\frac{1}{2}(\theta - V_b^0) + \frac{1}{2}V_s^0\right) - c \\
&= -\beta\frac{1}{2}(V_1^0 - V_0^0) + \frac{\beta}{4}\left(\theta + (V_2^0 - V_b^0)\right) - c \,,
\end{aligned} \tag{B.26}$$

implying

$$r(V_1^0 - V_0^0) = \frac{\beta}{\alpha}(\eta + r)(V_b^0 - V_s^0) - c \, .$$

Therefore,

$$V_s^0 - V_b^0 = \frac{\alpha}{\beta(\eta + r)}\left(-r(V_1^0 - V_0^0) - c\right) \, . \tag{B.27}$$

Then, (B.26) and (B.27) imply

$$r(V_1^0 - V_0^0) = -\frac{\beta}{2}(V_1^0 - V_0^0) + \frac{\beta}{4}\theta + \frac{\alpha}{4(\eta + r)}\left(-r(V_1^0 - V_0^0) - c\right) - c \, .$$

Therefore,

$$V_1^0 - V_0^0 = \left(r + \frac{\beta}{2} + \frac{r\alpha}{4(\eta + r)}\right)^{-1}\left(\frac{\beta}{4}\theta - (c_1 - c_0)(1 + \frac{\alpha}{4(\eta + r)})\right) \, . \tag{B.28}$$

On the other hand, (B.25) implies

$$V_s^0 = \frac{\alpha}{\beta(\eta + r)}rV_0^0 \, . \tag{B.29}$$

Also note that (B.29) and (3.15) imply

$$rV_0^0 = \frac{\beta}{4}(V_1^0 - V_0^0) - \frac{\beta}{4}V_s^0 = \frac{\beta}{4}(V_1^0 - V_0^0) - \frac{\alpha}{4(\eta + r)}rV_0^0 \, .$$

Therefore,

$$r\left(1 + \frac{\alpha}{4(\eta + r)}\right)V_0^0 = \frac{\beta}{4}(V_1^0 - V_0^0) \, . \tag{B.30}$$

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Define

$$D = 1 + \frac{\alpha}{4(\eta + r)}.$$

Then (B.28) becomes

$$V_1^0 - V_0^0 = \frac{\frac{\beta}{4}\theta - Dc}{\frac{\beta}{2} + rD}. \tag{B.31}$$

Conditions (B.30) and (B.31) imply

$$rDV_0^0 = \frac{\beta}{4}\frac{\frac{\beta}{4}\theta - Dc}{\frac{\beta}{2} + rD}. \tag{B.32}$$

Therefore, $V_0^0 \geqslant 0$ is a sufficient condition for an equilibrium with $d_2 = 0$ to sustain. By (B.32) and the fact that $D > 0$, this further translates to the following inequality:

$$\theta \geqslant 4\beta^{-1}Dc. \tag{B.33}$$

Finally, according to (3.18), we require condition (3.17) to ensure that the trading pattern holds. This completes the proof.

PROOF OF PROPOSITION 4. We consider the limit as $\lambda = 0$ and the desirable result follows from the standard continuity argument. When $\lambda = 0$, direct calculation from the HJBs yields:

$$\begin{aligned} V_s^1 &= \frac{\alpha d_I \frac{1}{2}(V_1^1 - V_0^1) + \alpha(1 - 2d_I)\frac{1}{2}(V_2^1 - V_1^1)}{(\eta + r) + \frac{1}{2}\alpha(1 - d_I)} \\ &= \frac{\theta}{6\alpha^{-1}(\eta + r) + 2} - \frac{r\theta + \rho c}{\alpha^{-1}(\eta + r)(6r + \frac{3}{2}\beta^1) + 2r}, \end{aligned} \tag{B.34}$$

and

$$\begin{aligned} V_s^0 &= \frac{\alpha}{4(\eta + r)(r + \frac{\beta^0}{2}) + r\alpha}\left(\frac{(\eta + r)\beta^0}{4(\eta + r) + \alpha}\theta - c\right) \\ &= \frac{\theta}{8\alpha^{-1}(\eta + r) + 2} - \frac{r\theta + 2c}{\alpha^{-1}(\eta + r)(8r + 4\beta^0) + 2r}. \end{aligned} \tag{B.35}$$

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Therefore,

$$
\begin{aligned}
2(V_s^1 - V_s^0) &= \frac{r\theta}{3\alpha^{-1}(\eta+r)r+r} - \frac{r\theta}{4\alpha^{-1}(\eta+r)r+r} \\
&\quad - \left(\frac{r\theta+\rho c}{\alpha^{-1}(\eta+r)(3r+\frac{3}{4}\beta^1)+r} - \frac{r\theta+2c}{\alpha^{-1}(\eta+r)(4r+2\beta^0)+r} \right) \\
&\leqslant r\theta \left(\frac{\alpha^{-1}(\eta+r)r}{(3\alpha^{-1}(\eta+r)r+r)(4\alpha(\eta+r)r+r)} \right. \\
&\quad \left. - \frac{\alpha^{-1}(\eta+r)(r+2\beta^0-\frac{3}{4}\beta^1)}{(\alpha^{-1}(\eta+r)(3r+\frac{3}{4}\beta^1)+r)(\alpha^{-1}(\eta+r)(4r+2\beta^0)+r)} \right) \\
&\quad -\rho c \frac{\alpha^{-1}(\eta+r)(r+2\beta^0-\frac{3}{4}\beta^1)}{(\alpha^{-1}(\eta+r)(3r+\frac{3}{4}\beta^1)+r)(\alpha^{-1}(\eta+r)(4r+2\beta^0)+r)} \quad \text{(B.36)}
\end{aligned}
$$

Define $A = \alpha^{-1}(\eta+r) > 0$, inequality (B.36) becomes:

$$
2(V_s^1 - V_s^0) \leqslant \left(\frac{1}{r(3A+1)(4A+1)} - \frac{(r+2\beta^0-\frac{3}{4}\beta^1)}{[A(3r+4\beta^1)+r][A(4r+2\beta^0)+r]} \right) Ar\theta. \quad \text{(B.37)}
$$

In the following, we show that the coefficient of $Ar\theta$ in the right hand side of (B.37) is weakly negative when Condition 1 holds. To see this, consider the following quadratic function of A. When Condition 1 holds:

$$
\begin{aligned}
&[A(3r+4\beta^1)+r][A(4r+2\beta^0)+r] - r(3A+1)(4A+1)(r+2\beta^0-\frac{3}{4}\beta^1) \\
&= (-18\beta^0 r + 12\beta^1 r + \frac{3}{2}\beta^0\beta^1)A^2 + (-12\beta^0 r + 6\beta^1 r)A + (-2\beta^0 r + \frac{3}{4}\beta^1 r) \quad \text{(B.38)} \\
&\leqslant 0. \quad \text{(B.39)}
\end{aligned}
$$

To see why inequality (B.39) holds, notice that Lemma 4 immediately implies that the coefficients of A and the constant in the right hand side of (B.38) are strictly negative, that is,

$$
-12\beta^0 r + 6\beta^1 r < 0,
$$

and

$$
-2\beta^0 r + \frac{3}{4}\beta^1 r < 0.
$$

Moreover, Condition 1 implies that the coefficient of A^2 in (B.38) is weakly negative, that

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is,

$$-18\beta^0 r + 12\beta^1 r + \frac{3}{2}\beta^0\beta^1 \leqslant 0 \,.$$

Therefore, (B.39) holds, implying that the coefficient of $Ar\theta$ in the right hand side of (B.37) is weakly negative. This completes the proof.

PROOF OF PROPOSITION 5. Consider the equilibrium with $d_2 > 1$ first. The inflow-outflow balance condition (3.1) implies that

$$q^1 \doteq n - \eta b^1 = \alpha b^1 (1 - d_I) \,. \tag{B.40}$$

On the other hand, under the equilibrium with $d_2 = 0$, inflow-outflow balance (3.10) implies that

$$q^0 \doteq n - \eta b^0 = \frac{1}{2}\alpha b^0 \,. \tag{B.41}$$

Notice that Lemma 3 implies that $d_I \leqslant \frac{1}{3}$, that is, $1 - d_I \geqslant \frac{2}{3} > \frac{1}{2}$. Hence, conditions (B.40) and (B.41) jointly imply that $b^1 < b^0$. This immediately implies $q^1 > q^0$ by definition, concluding the proof.

PROOF OF PROPOSITION 6. We consider the limit as $\lambda = 0$ and the desirable result follows from the standard continuity argument. Regardless of the type of the equilibrium, direct calculation based on the definition yields:

$$
\begin{aligned}
\Delta &\doteq p_{b,i+1} - p_{s,i} \\
&= \frac{1}{2}\left((V_{i+1} - V_i + \theta - V_b) - (V_{i+1} - V_i + V_s) \right) \\
&= \frac{1}{2}(\theta - V_b - V_s) \,.
\end{aligned}
\tag{B.42}
$$

Hence, comparing Δ^1 and Δ^0 is equivalent to comparing $V_s^1 + V_b^1$ and $V_s^0 + V_b^0$.

Consider $V_b^1 + V_s^1$ first. Lemma 3 implies that $d_I = \frac{1}{3}$ when $\lambda = 0$. Thus, combining the two value functions (3.4) and (3.5) yields

$$V_b^1 + V_s^1 = \frac{\alpha\theta}{3(\eta + r) + \alpha} \,. \tag{B.43}$$

On the other hand, combining the two value functions (3.13) and (3.14) yields

$$V_b^0 + V_s^0 = \frac{\alpha\theta}{4(\eta + r) + \alpha},$$

which is clearly smaller than $V_b^1 + V_s^1$ as expressed in (B.43). By (B.42), this implies that $\Delta^1 < \Delta^0$ and concludes the proof.

PROOF OF PROPOSITION 7. We consider the limit as $\lambda \to \infty$ and the desirable result follows the standard continuity argument. First, under the equilibrium with $d_2 > 0$, the inflow-outflow balance implies

$$b^1 = \frac{n}{\alpha + \eta}. \tag{B.44}$$

It follows that

$$\beta^1 = 2\alpha b^1 = \frac{2\alpha n}{\alpha + \eta}. \tag{B.45}$$

At the same time,

$$\lim_{\lambda \to \infty} \lambda d_I = \infty,$$

suggesting that the interdealer market is active with the equilibrium mass of type-0 and type-2 dealers being 0. Intuitively, because dealers can contact each other infinitely quickly, any type-2 dealer will immediately trade with a type-0 dealer and then both become type-1 dealers.

Recall the definition of B and Q in the proof for Lemma 2. Direct calculation yields:

$$\lim_{\lambda \to \infty} B = \infty, \tag{B.46}$$

$$\lim_{\lambda \to \infty} Q = \frac{2\alpha n(r + \eta)}{(\alpha + \eta)(2(r + \eta) + \alpha)}. \tag{B.47}$$

Following the argument in the the proof for Proposition 2,

$$\lim_{\lambda \to \infty} V_2^1 - V_1^1 - V_s^1 \geqslant 0$$

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if and only if

$$\theta \geqslant \underline{\theta} = \frac{2\rho c(\alpha + \eta)\left(2(r + \eta) + \alpha\right)}{2n\alpha(r + \eta)}, \tag{B.48}$$

suggesting that an equilibrium with $d_2 > 0$ must exist when the asset fundamental is high enough.

Then, consider the equilibrium with $d_2 = 0$. The inflow-outflow balance implies

$$b^0 = \frac{2n}{\alpha + 2\eta}. \tag{B.49}$$

It follows that

$$\beta^0 = 2\alpha b^0 = \frac{4\alpha n}{\alpha + 2\eta}. \tag{B.50}$$

Similar calculation following the argument in the the proof for Proposition 2 shows that

$$\lim_{\lambda \to \infty} V_2^0 - V_1^0 - V_s^0 > 0, \tag{B.51}$$

regardless of θ. Thus, an equilibrium with $d_2 = 0$ never exists.

Therefore, (B.48) and (B.51) jointly imply that when a trading equilibrium exists, it must be an equilibrium with $d_2 > 0$. By continuity, this concludes the proof.

PROOF OF PROPOSITION 10. We consider the limit as $\lambda = 0$ and the desirable result follows from the standard continuity argument. When $\lambda = 0$, direct calculation from the HJBs yields:

$$V_b^1 = \frac{\frac{1}{2}\alpha(1 - d_I)\theta - \alpha d_I \frac{1}{2}(V_2^1 - V_1^1) - \alpha(1 - 2d_I)\frac{1}{2}(V_1^1 - V_0^1)}{(\eta + r) + \frac{1}{2}\alpha(1 - d_I)}, \tag{B.52}$$

which combined with (B.34) yields:

$$V_b^1 - V_s^1 = \frac{\frac{1}{2}\alpha(1 - d_I)\theta - \alpha(1 - d_I)\frac{1}{2}(V_2^1 - V_1^1) - \alpha(1 - d_I)\frac{1}{2}(V_1^1 - V_0^1)}{(\eta + r) + \frac{1}{2}\alpha(1 - d_I)}. \tag{B.53}$$

Similarly, we have

$$V_b^0 = \frac{\alpha}{4(\eta + r)(r + \frac{\beta}{2}) + r\alpha} \left(\frac{(\eta + r)(4r + \beta) + r\alpha}{4(\eta + r) + \alpha} \theta + c \right), \tag{B.54}$$

which combined with (B.35) yields:

$$V_b^0 - V_s^0 = \frac{\alpha}{4(\eta + r)(r + \frac{\beta}{2}) + r\alpha}(r\theta + 2c) \tag{B.55}$$

Combining equations (B.53) and (B.55) yields:

$$
\begin{aligned}
(V_b^0 - V_s^0) - (V_b^1 - V_s^1) &= \left(\frac{\alpha}{4(\eta + r)(r + \frac{\beta^0}{2}) + r\alpha} - \frac{\alpha}{(1 - d_I)^{-1}(\eta + r)(2r + \frac{\beta^1}{2}) + r\alpha} \right) r\theta \\
&+ \left(\frac{2\alpha}{4(\eta + r)(r + \frac{\beta^0}{2}) + r\alpha} - \frac{\alpha\rho}{(1 - d_I)^{-1}(\eta + r)(2r + \frac{\beta^1}{2}) + r\alpha} \right) c. \\
&\leqslant \left(\frac{\alpha}{4(\eta + r)(r + \frac{\beta^0}{2}) + r\alpha} - \frac{\alpha}{(1 - d_I)^{-1}(\eta + r)(2r + \frac{\beta^1}{2}) + r\alpha} \right) (r\theta + 2c),
\end{aligned}
\tag{B.56}
$$

where the last inequality follows from $\rho \geqslant 2$.

On the other hand, we have

$$
\begin{aligned}
(1 - d_I)^{-1}(2r + \frac{\beta^1}{2}) & \\
\leqslant \quad & \frac{3}{2}(2r + \frac{\beta^1}{2}) \\
< \quad & \frac{3}{2}(2r + \frac{\beta^0}{2}) \\
< \quad & 4(r + \frac{\beta^0}{2}),
\end{aligned}
$$

where the first inequality follows from Lemma 3 and the second follows from Lemma 4. This implies that

$$\frac{\alpha}{4(\eta + r)(r + \frac{\beta^0}{2}) + r\alpha} < \frac{\alpha}{(1 - d_I)^{-1}(\eta + r)(2r + \frac{\beta^1}{2}) + r\alpha},$$

in other words, the coefficient of $(r\theta + 2c)$ in the right hand side of (B.56) is negative. Since

$r\theta + 2c > 0$, this immediately implies that

$$V_b^1 - V_s^1 > V_b^0 - V_s^0 \,. \tag{B.57}$$

Therefore, combining (B.57) above and the result that

$$V_b^1 + V_s^1 > V_b^0 + V_s^0$$

in Proposition 6 immediately yields that $V_b^1 > V_b^0$, completing the proof.

Online Appendix

Online Appendix for

The Coordination of Intermediation

Not for publication

We allow the probability at which a trade between a type-1 dealer and a seller happens to be $\phi \in (0, 1)$. In a mixed-strategy equilibrium, a given type-1 dealer has probability ϕ to trade with a seller when other type-1 dealers also trade with sellers with probability ϕ, conditional on a meeting.[1] In this case, an equilibrium is determined by

$$V_2^\phi - V_1^\phi - V_s^\phi = 0$$

and other value functions as prescribed by (3.4), (3.5), (3.6), (3.7), and (3.8). Online Appendix C provides a micro-foundation from which ϕ can be constructed from the staged bargaining game imbedded in our setting, and Online Appendix D provides the formal procedure to solve for the mixed-strategy equilibria, if any.

C Strategy representation of the bargaining game

Since the game played by agents in our framework is essentially a complete information dynamic bargaining game, it is clear that at any steady-state equilibrium, the sub-game played by two meeting agents j and k can be summarized by the following two-strategy (sub-)game:

---

[1]Note that this description of mixed-strategy equilibria also accommodates the pure-strategy equilibria we already considered in the main text.

<center>Agent k</center>

		Accept	Reject
Agent j	Accept	$\frac{G(\{V_i\}_i)}{2}, \frac{G(\{V_i\}_i)}{2}$	$0,0$
	Reject	$0,0$	$0,0$

where $G(\{V_i\}_i)$ denotes the potential gains from trade between the two meeting agents j and k, which are in turn determined endogenously by all the agents' value functions given the steady state of the dynamic game as well as agents' rational expectations of achieving the corresponding steady state. Intuitively, only when the two meeting agents both choose "Accept", trade will happen. In turn, only when the potential gains from trade are positive, the two meetings agents will choose "Accept" simultaneously. On the flip side, at least one agent will choose "Reject" when the potential gains from trade are negative, and thus a trade will not happen. When the potential gains from trade are zero, agents may play mixed strategies, where their equilibrium mixed strategies will be determined by the steady-state distribution of the mass of agents as well as their value functions.

Below we focus on the staged bargaining game played by a type-1 dealer and a seller conditional on a meeting. First, note that Propositions 1 and 2 in the main text suggest that whether a type-1 dealer is willing to buy from a seller and to effectively increase its inventory is the sole criterion to determine which type of equilibrium happens, given the equilibrium distribution of agents and values of other agents in the corresponding equilibrium. To formulate type-1 dealers' strategy as well as their willingness to trade requires us to analyze the bargaining (sub-)game between an type-1 dealer and a searching seller, when they meet each other:

<center>Seller</center>

		Accept	Reject
Type-1 Dealer	Accept	$\frac{V_2-V_1-V_s}{2}, \frac{V_2-V_1-V_s}{2}$	$0,0$
	Reject	$0,0$	$0,0$

As suggested by the bargaining (sub-)game above, the type-1 dealer's liquidity provision decision of whether or not to buy from a seller and increase its inventory is solely determined by whether $V_2 - V_1 - V_s$ is positive or negative, given the endogenously determined value

<center>2</center>

functions in the corresponding equilibrium.

With the preparation above, we are now able to explicitly show how the trading probability ϕ between a type-1 dealer and a seller, conditional on a meeting, can be constructed from the staged bargaining (sub-)game. Specifically, a type-1 dealer's strategy in the above bargaining (sub-)game is $(p, 1 - p)$ while a seller's strategy is $(q, 1 - q)$, whenever they meet each other. In this case, a trade between a type-1 dealer and a seller happens with probability $\phi = pq$ when they meet. Notice that the bargaining (sub-)game itself is not sufficient to determine the equilibrium profile (p, q). Rather, the equilibrium probability pq at which a trade between a type-1 dealer and a seller happens will be determined by the condition $V_2 - V_1 - V_s = 0$ as well as other value functions as prescribed by (3.4), (3.5), (3.6), (3.7), and (3.8).

D   Derivation of the mixed-strategy equilibria

We explicitly derive the mixed-strategy equilibria that we consider in the main text. In any candidate mixed-strategy equilibria with $\phi \in (0, 1)$, the inflow-outflow balance of the dealer sector implies

$$b = s \,, \tag{D.1}$$

$$b\alpha(d_1 + d_2) = n - \eta b \,, \tag{D.2}$$

and

$$s\alpha(\phi d_1 + d_0) = n - \eta b \,. \tag{D.3}$$

Conditions (D.1), (D.2), and (D.3) together imply

$$d_1 + d_2 = \phi d_1 + d_0 \,. \tag{D.4}$$

At the same time, the inflow-outflow balance of type-0 dealers suggests

$$d_0 \left(\beta \frac{s}{b + s} + \lambda d_2 \right) = d_1 \beta \frac{b}{b + s} \,, \tag{D.5}$$

while the inflow-outflow balance of type-2 dealers suggests

$$d_2 \left(\beta \frac{b}{b+s} + \lambda d_0 \right) = d_1 \beta \frac{\phi s}{b+s} \, . \tag{D.6}$$

First, condition (D.4) implies $\phi d_1 + d_0 = 1 - d_0$, that is,

$$d_1 = \phi^{-1}(1 - 2d_0) \tag{D.7}$$

and consequently,

$$d_2 = 1 - d_0 - d_1 = 1 - d_0 - \phi^{-1}(1 - 2d_0) \, . \tag{D.8}$$

Thus, conditions (D.1), (D.5), (D.7), and (D.8) jointly imply

$$(2 - \phi)\lambda d_0^2 + \left(\left(1 + \frac{\phi}{2} \right) \beta - (1 - \phi)\lambda \right) d_0 - \frac{\beta}{2} = 0 \, , \tag{D.9}$$

which determine d_0 under a mixed-strategy equilibrium with ϕ.

To check (D.9) whether has meaningful solutions, define

$$\begin{aligned} D &= \left(\left(1 + \frac{\phi}{2} \right) \beta - (1 - \phi)\lambda \right)^2 + 2\beta(2 - \phi)\lambda \\ &= \left(1 + \frac{\phi}{2} \right)^2 \beta^2 + (1 - \phi)^2\lambda^2 - 2 \left(1 + \frac{\phi}{2} \right) \beta(1 - \phi)\lambda + 2\beta(2 - \phi)\lambda, \end{aligned}$$

where the sum of the last two terms

$$-2 \left(1 + \frac{\phi}{2} \right) \beta(1 - \phi)\lambda + 2\beta(2 - \phi)\lambda = 2\beta\lambda \left(\left(1 - \frac{\phi}{4} \right)^2 + \frac{7}{16}\phi^2 \right) > 0.$$

Therefore, $D > 0$ and

$$d_0 = \frac{(1 - \phi)\lambda - \left(1 + \frac{\phi}{2} \right) \beta + \sqrt{\left(1 + \frac{\phi}{2} \right)^2 \beta^2 + (1 - \phi)^2\lambda^2 + 2\beta\lambda \left(\frac{\phi^2}{2} - \frac{\phi}{2} + 1 \right)}}{2(2 - \phi)\lambda} \, . \tag{D.10}$$

Note that the negative solution is dropped since $d_0 \in [0, 1]$. By (D.7) and (D.8), this fully pins down the distribution of the dealer sector, and further of the customers by (D.5) and (D.6).