Household Heterogeneity and the Value of Government Spending Multiplier*

Paweł Kopiec†

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Abstract

This paper provides an analytical decomposition of the fiscal multiplier in economy populated with heterogeneous households, uninsured idiosyncratic income risk and frictional product market. Similarly to Auclert (2019), the derived expression consists of interpretable, model-based channels that describe the transmission of government spending shocks by private consumption. Calibrated model is used to estimate the magnitude of multipliers and their structure under alternative fiscal and monetary rules. Analytical and quantitative comparison to the multiplier's formula in economy with identical agents indicates that household heterogeneity plays a crucial role in the propagation of fiscal expenditures shocks.

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†Narodowy Bank Polski, Świętokrzyska 11/21, 00-919 Warsaw, Poland. E-mail: pawel.kopiec@nbp.pl.

1 Introduction

As argued by Ramey (2019), the Financial Crisis of 2007-2008, during which monetary policy transmission was severely impaired, gave rise to a renewed interest in the long-neglected topic which is the stabilizing role of fiscal policy. In particular, stimulus generated by a rise in government expenditures was among several fiscal measures analyzed by economists after the Great Recession and the most basic measure of its effectiveness - the value of the fiscal multiplier - was subject to a heated debate.

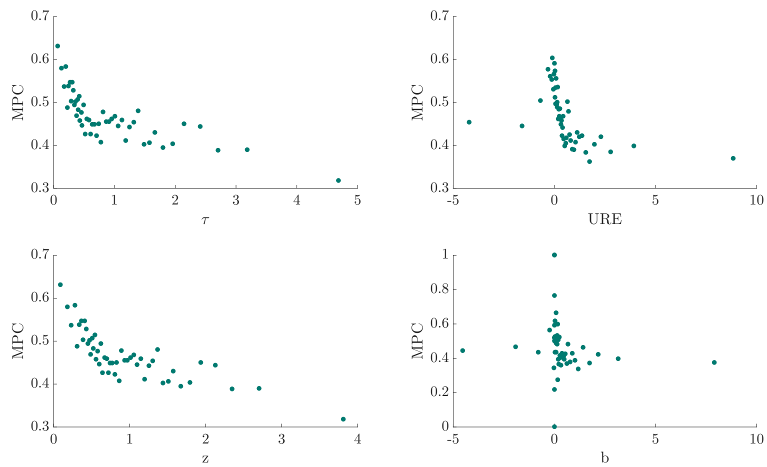
At the same time, a large body of evidence documented an important role of household heterogeneity in the propagation of various types of macroeconomic shocks through the channel of private consumption.[1] To demonstrate how consumption behavior described by the marginal propensity to consume (MPC henceforth) varies across households, I use data from the SHIW survey conducted by the Bank of Italy and plot MPC values for households grouped according to four characteristics in Figure 1.

The observed heterogeneity of consumption behavior, coupled with the standard Keynesian-cross logic according to which the multiplier's value depends crucially on private demand's reaction to additional income generated by the stimulus, is the main motivation of this paper that seeks to understand the role of household inequality for the transmission of government spending shocks through aggregate consumer spending.

To investigate the problem, I use the Bewley-Huggett-Aiyagari model with frictional product market and derive an analytical expression that decomposes the value of fiscal multiplier into several propagation channels that work through household spending behavior. This exercise allows to pin down the exact determinants of fiscal policy transmission by private consumption in an inherently complex, heterogeneous-agent environment. From the technical point of view, the obtained expression resembles the decomposition of aggregate consumption response to exogenous changes in income, real interest rates and prices following a monetary policy shock presented in Auclert (2019). The difference is, however, that the expression derived in this paper has inherently a general equilibrium

[1] See Carroll et al. (2014), Jappelli and Pistaferri (2014), Kaplan and Violante (2014), Krueger et al. (2016) among others.

Figure 1: MPC across Italian households, SHIW survey in 2016



Notation: τ denotes income tax burden, URE is the unhedged interest rate exposure (difference between maturing assets and maturing liabilities), z is pre-tax income and b is the stock of nominal assets.

character that is necessary to analyze the transmission process of fiscal policy, which relies on the feedback loop between income and demand and which is captured by the multiplier.

The key assumption that enables to obtain the multiplier's formula under general equilibrium is the departure from the Walrasian product market present in the standard Bewley-Huggett-Aiyagari framework, which becomes decentralized and features search frictions in my analysis. This formulation of the market for goods is crucial for obtaining the multiplier's formula as it allows to summarize compactly all general equilibrium forces that affect household's consumption choices with only one variable: product market tightness. This, in turn, enables to express the aggregate consumption in a tractable way and to derive the multiplier formula directly from the aggregate resource constraint.[2] Furthermore, this departure from the standard framework allows to relax two other assumptions (in addition to relaxing the assumption about partial equilibrium) that were used in the literature to obtain analytical characterizations in the Bewley-

[2]The role of this property is clarified in Section 2.7.

Huggett-Aiyagari models, such as: i) extreme illiquidity (e.g. Krusell et al. (2011), Werning (2015), Ravn and Sterk (2016), McKay and Reis (2016a)), ii) constant real interest rates (e.g., Auclert et al. (2018), Patterson (2018)). In the context of the studied problem, it is important to relax conditions i) and ii) as they shut off fiscal policy transmission channels working through consumer balance sheets and, additionally, may eliminate some potentially important monetary-fiscal interactions.[3] Except for that, as mentioned by Michaillat and Saez (2015), the assumption about the non-Walrasian product market allows to incorporate price-setting mechanisms featuring various levels of price rigidity in a tractable way.[4] This property, as argued by Hagedorn et al. (2019), is important when investigating fiscal policy transmission because the Keynesian-cross logic relies on the assumption of sticky prices that guarantees that demand shocks are absorbed by adjustment in quantities and not only by changes in prices.

To evaluate the multiplier's value and the size of its components, I use the model calibrated to match the moments characterizing Italian economy, and most crucially, the average MPC among Italian households and its distribution across cash-in-hand deciles. Next, I compute the size of the multiplier and estimate the magnitude of channels of fiscal policy transmission under various scenarios related to monetary policy and government budget management: tax/debt-financed stimulus, constant nominal interest rates and the situation when real rates remain unaffected (as in Auclert et al. (2018)). It turns out, that debt-financing significantly improves the propagation of fiscal spending shock, which is tightly related to the fact that Ricardian equivalence ceases to hold in the model with uninsured income risk. Surprisingly, and in contrast to predictions of the standard New Keynesian model described by Woodford (2011), passive monetary

[3]If the extreme illiquidity is assumed then consumer balance sheets become degenerate and all households have zero assets, so the transmission of shocks through consumer wealth is eliminated. Moreover, under extreme illiquidity, the aggregate liquidity is zero and so is the stock of public debt. This means that monetary policy has no impact on government budget constraint which eliminates some important monetary-fiscal interactions. If real rates are constant then, as shown by Auclert et al. (2018), fiscal multiplier in the Bewley-Huggett-Aiyagari model depends solely on the path of disposable incomes and intertemporal MPCs and, again, both the role of household balance sheets and the interplay between monetary and fiscal policy are eliminated.

[4]Michaillat and Saez (2015) argue that standard price-setting protocols used in macroeconomic models (e.g., Calvo (1983) and Rotemberg (1982)) are complex because they are inherently dynamic (as they rely on the Phillips curve, the Euler equation, and a monetary policy rule). This makes them intractable when deriving closed-form characterizations.

policy (constant nominal rates) does not increase the multiplier's value, and, as explained by means of the derived decomposition method, consumer balance sheets are crucial for that result. Constant real rates (implied by constant nominal rates and constant prices of goods), in turn, improve the propagation of fiscal shock significantly. Finally, to articulate the role of inequality in the propagation of fiscal shocks, I apply the derived formula to compare the size of the multiplier (and its structure) in both heterogeneous and representative agent framework.

This paper is related to several strands of the literature. First, it is associated with works studying the effects of fiscal policy shocks in models with heterogeneous households, in which a significant proportion of agents deviates from the consumption-savings behavior predicted by the permanent income hypothesis and thus exhibits relatively high levels of MPC. There are two main groups of papers within that field: first of them focuses on the role of taxes and transfers (e.g., Oh and Reis (2012), McKay and Reis (2016b), Den Haan et al. (2015)), and the second concentrates on the role of fiscal purchases (e.g., Challe and Ragot (2011), Navarro and Ferriere (2016), Brinca et al. (2017), Hagedorn et al. (2019), Auclert et al. (2018) and Brinca et al. (2019)).

It seems that the closest work to mine is the last one, in which Auclert et al. (2018) characterize analytically the fiscal multiplier using the so-called intertemporal MPCs. Auclert et al. (2018) derive an elegant multiplier's formula in the model populated by unequal consumers under pegged real interest rates and highlight the importance of the disposable income channel for the propagation of higher government spending, which echoes the conclusions from the standard textbook framework. In contrast, in my paper I consider the multiplier under the standard Taylor rule. Using the derived formula, I compare those two scenarios (constant and time-varying real rates) and show that if central bank fails to fix real interest rates, then fiscal shocks are transmitted by additional channels that are absent when real rates remain unchanged. In particular, under the standard Taylor rule, multiplier's size is affected significantly by monetary-fiscal interactions and processes related to consumer balance sheets. Furthermore, I compare the values of multipliers under two scenarios and conclude that the impact of those additional channels is quantitatively important.

As already mentioned, the key ingredient in my analysis is the frictional product market. The presence of a non-Walrasian market for goods in the model can be motivated by the fact that in reality this market functions in a decentralized manner and features frictions (see Michaillat and Saez (2015)). This formulation goes back to a seminal paper by Diamond (1982) who proposed a model with search frictions in the market for goods that is subject to the so-called thick market externality. More recently, frictional product markets were used, among others, in works by Michaillat and Saez (2015), Petrosky-Nadeau and Wasmer (2015), Kaplan and Menzio (2016), Storesletten et al. (2017) and Michaillat and Saez (2019). My paper is tightly related to the last two. First, as Storesletten et al. (2017), I formalize search costs in terms of disutilty from search effort. Second, similarly to Michaillat and Saez (2019), I study the effects of higher fiscal purchases. The most important difference with respect to Michaillat and Saez (2019) is that I consider a model with heterogeneous agents.

The remaining sections of the paper are organized as follows. Section 2 presents the Bewley-Huggett-Aiyagari model with frictional product market. In Section 3 I derive the analytical formula for the multiplier. Section 4 applies the formula the calibrated model to estimate the magnitude of the multiplier and its channels under alternative scenarios related to monetary and fiscal policy. Furthermore, by comparing the transmission of fiscal policy in the model with heterogeneous households with the transmission in "the representative agent limit", I investigate the role of consumer heterogeneity in the propagation of government spending shocks. Section 5 concludes.

2 Model

2.1 Environment

Time is infinite and divided into discrete periods. There are two types of agents in the economy: heterogeneous, self-employed households and government that is composed of two branches: central bank and fiscal authority. There are two markets: a Walrasian market where households trade liquid assets and a decentralized market for consumption goods that features search frictions analogous to those from the standard Diamond-Mortensen-Pissarides model of labor market.

2.2 Households

The model is populated by a continuum of infinitely lived households of measure one. Households are both consumers and producers. Household enters period t with a stock of nominal assets (government bonds) $\tilde{b}_t$. For notational convenience, by b_t I denote the ratio between nominal bonds $\tilde{b}_t$ and price of consumption goods in the previous period p_{t-1}. Consumer's income is a product of two components:

$$z_t \cdot f_t$$

where z_t is the idiosyncratic productivity level (given by the amount of goods or services produced by agent) and f_t denotes the probability at which a unit of good or services supplied by household is matched with a customer and sold. To put it differently, f_t can be thought of as the level of utilization of capacity z_t.[5] Randomness at the individual level is excluded so f_t is equal across households. Rate f_t is endogenous and is an object that is analogous to job-finding rate in the standard Diamond-Mortensen-Pissarides model of frictional product market. Stochastic productivity level z_t is the source of household heterogeneity and follows a Markovian process defined on space Z. By μ_t I denote the distribution of agents over liquid asset holdings and technology levels.

Similarly to Storesletten et al. (2017), agent preferences are given by the instantaneous utility function $\tilde{u}(c_t, v_t)$ where $\tilde{u}_c > 0$, $\tilde{u}_v < 0$, $\tilde{u}_{cc} < 0$, c_t is consumption and v_t is search effort exerted by household in the product market. Alternatively, v_t can be seen as a number of visits made by household to purchase goods from other consumers. Household discounts future utility streams with factor $\beta \in (0, 1)$ and maximizes the following expression:

$$\mathbb{E}_0 \sum_{t \geq 0} \beta^t \cdot \tilde{u}(c_t, v_t)$$

Values of c_t and v_t are related by the following constraint imposed by product market frictions:

$$c_t = q_t \cdot v_t \tag{1}$$

[5] An intuitive description of the frictional product market is provided in Michaillat and Saez (2019), where the economy is populated by people working as butlers for others and using their income to hire their own butlers.

where q_t is the probability with which a visit that is made by household is successful, i.e., it ends with a purchase of a unit of consumption good. Rate q_t is endogenous and bears resemblance to vacancy-filling rate in the Diamond-Mortensen-Pissarides model.

Household pays income tax $\tau\left(z_t\right) \cdot \Theta_t$ where Θ_t is aggregate amount of taxes collected by government and $\tau\left(z_t\right)$ is the individual contribution of household with productivity z_t to Θ_t. Moreover, it is assumed that:

$$\forall_t \int_{B\times Z} \tau\left(z_t\right) d\mu_t\left(b_t, z_t\right) = 1 \text{ and } \forall_{z_t \in Z}\, \tau\left(z_t\right) > 0 \tag{2}$$

where $B = [-\xi, +\infty)$ is set to which individual asset holdings belong and $\xi > 0$ is the value of borrowing constraint which implies:

$$b_{t+1} \geq -\xi \tag{3}$$

for all periods t. Liquid assets earn nominal interest rate i_t that is set by monetary authority.

To complete the description of household's environment, let us denote by Π_t the ratio between current price level p_t and the level in the previous period p_{t-1}:

$$\Pi_t \equiv \frac{p_t}{p_{t-1}}.$$

This allows to formulate household's budget constraint:

$$c_t + \tau\left(z_t\right) \cdot \Theta_t + \frac{b_{t+1}}{1+i_t} = \frac{b_t}{\Pi_t} + z_t \cdot f_t \tag{4}$$

that applies each period.

Similarly to Guerrieri and Lorenzoni (2017) and Auclert et al. (2018), I define time-dependent optimal rules $c_t\left(b, z\right)$, $v_t\left(b, z\right)$ and $b_{t+1}\left(b, z\right)$ of a household with bonds $b_t = b$ and productivity $z_t = z$ (where $b \in B$ and $z \in Z$) that maximizes its utility subject to constraints: 1, 3 and 4.

2.3 Government

Government consists of fiscal authority and central bank. Fiscal authority purchases consumption goods G_t and, since (similarly to households) it operates under product market frictions, its expenditures are subject to the following constraint:

$$G_t = q_t \cdot v_{G,t} \tag{5}$$

where $v_{G,t}$ is the number of visits made by government. Government collects taxes Θ_t according to a time-invariant tax schedule $\{\tau(z)\}_{z \in Z}$, issues bonds $\bar{B}_{t+1}$ to finance purchases G_t and repayment of debt $\bar{B}_t$ issued in the previous period. Consequently, government budget constraint reads:

$$\Theta_t + \frac{\bar{B}_{t+1}}{1+i_t} = \frac{\bar{B}_t}{\Pi_t} + G_t. \tag{6}$$

Monetary authority sets nominal interest rate i_t by following a standard Taylor-type rule that depends on deviations: of aggregate output Y_t and price index Π_t from their levels in stationary equilibrium (denoted by Y and Π, respectively):

$$i_t = \bar{i} + \phi_Y \cdot \left(\frac{Y_t - Y}{Y} \right) + \phi_\Pi \cdot (\Pi_t - \Pi)$$

where $\bar{i} > 0$, $\phi_Y \geq 0$ and $\phi_\Pi \geq 0$. In what follows, I will be considering monetary rules characterized with vector Φ, where:

$$\Phi = \left[\begin{array}{cc} \phi_Y & \phi_\Pi \end{array} \right].$$

2.4 Matching technology and price-setting

It is assumed that the number of successful matches in the product market is governed by a constant returns to scale matching function M that increases in both arguments and depends on the aggregate number of visits made by households and government and on the aggregate output capacity:

$$M\left(\int_{B \times Z} v_t(b,z)\, d\mu_t(b,z) + v_{G,t}, \int_{B \times Z} z d\mu_t(b,z) \right).$$

Product market tightness x_t is given by the ratio between aggregate visits and total production capacity:

$$x_t \equiv \frac{\int_{B\times Z} v_t(b,z)\, d\mu_t(b,z) + v_{G,t}}{\int_{B\times Z} z d\mu_t(b,z)}. \tag{7}$$

Finally, let us turn to the price-setting mechanism. As pointed by Michaillat and Saez (2015), there are two variables on a matching market that equate supply and demand: price and tightness. Putting it differently, as there are two equilibrium variables but only one equilibrium condition (supply equals demand), there are infinitely many combinations of prices and tightnesses that satisfy equilibrium condition, which implies that there are many price mechanisms consistent with market equilibrium.[6] Since there is no universal theory that would pin down prices in a decentralized market that features search frictions I will assume that price index Π_t is a strictly increasing function of x_t:

$$\Pi_t \equiv \Pi(x_t), \ \ \Pi' > 0. \tag{8}$$

It is a relatively mild condition as the exact functional form of Π is not specified. The assumed increasing relationship is tightly related to the following intuition: price level rises when the ratio x_t between aggregate demand (captured by the aggregate number of visits) and aggregate production capacity (captured by aggregate output capacity) rises. In other words, Π_t has an intuitive property and tends to react positively to demand shocks and negatively to supply shocks. This simple formulation of the price-setting mechanism allows to consider various degrees of price stickiness, which is described by the value of derivative Π'.

2.5 Consistency conditions and market clearing

Probabilities f_t and q_t are induced by matching technology M and due to the assumed constant returns to scale they can be expressed as functions of only one

[6] In particular, in the literature, there are both price-setting protocols grounded in microeconomic theory (e.g., Nash bargaining, competitive search equilibrium described by Moen (1997) or the bargaining game proposed by Stole and Zwiebel (1996)) and ad hoc mechanisms (like perfectly rigid prices in Michaillat and Saez (2015) or wage rules considered in Hall (2005)) imposing a particular functional form on the price-setting process.

argument - x_t:

$$f_t \equiv f(x_t) = M(x_t, 1) \tag{9}$$

$$q_t \equiv q(x_t) = M\left(1, \frac{1}{x_t}\right). \tag{10}$$

The market clearing condition for nominal assets reads:

$$\bar{B}_{t+1} = \int_{B\times Z} b_{t+1}(b,z)\, d\mu_t(b,z) \tag{11}$$

and the resource constraint for consumption goods is:

$$\int_{B\times Z} c_t(b,z)\, d\mu_t(b,z) + G_t = f(x_t) \cdot \int_{B\times Z} z d\mu_t(b,z) \tag{12}$$

where the right hand side is defined as aggregate output Y_t:

$$Y_t \equiv Y(x_t) = f(x_t) \cdot \int_{B\times Z} z d\mu_t(b,z). \tag{13}$$

Observe that changes in aggregate product Y_t are driven solely by shifts in $f(x_t)$ as the average economy-wide productivity is fixed (so, in a sense, Y_t becomes demand-driven). This is a significant departure from the neoclassical paradigm under which output depends solely on the supply side (determined by production factors like capital and labor). This assumption was made by Michaillat and Saez (2015), Michaillat and Saez (2019) and Storesletten et al. (2017) among others.

Evolution of the distribution of agents across asset holdings b and productivity levels z is described by the following equation:

$$\mu_{t+1}(\mathcal{B}', z') = \int_{\{b: b_{t+1}(b,z)\in\mathcal{B}'\}\times Z} \mathbb{P}(z'|z) d\mu_t(b,z) \tag{14}$$

where $\mathcal{B}'$ is a Borel subset of B and $\mathbb{P}(z'|z)$ is transition probability between states z and z' associated with the Markovian productivity process. Finally, I assume the following standardization:

$$\forall_t \int_{B\times Z} z d\mu_t(b,z) = 1 \tag{15}$$

i.e., the average productivity across agents is equal to one and constant over time.

2.6 Equilibrium

We are in position to define equilibrium of the model:

Definition 1. *Given an initial government debt level $\bar{B}_0$, initial distribution μ_0 and exogenous sequences $\{G_t, \bar{B}_{t+1}\}_{t\geq 0}$, a competitive equilibrium is given by paths of prices $\{i_t, \Pi_t\}_{t\geq 0}$, sequences $\{Y_t, f_t, q_t, x_t, \Theta_t\}_{t\geq 0}$, individual policy functions $\{c_t(b,z)\}_{t\geq 0}$, $\{b_{t+1}(b,z)\}_{t\geq 0}$, $\{v_t(b,z)\}_{t\geq 0}$ and distributions $\{\mu_t(b,z)\}_{t\geq 1}$ such that: households optimize, monetary authority follows the Taylor rule Φ, government budget constraint is satisfied and consistency, market-clearing and price-setting conditions hold.*

2.7 Aggregate consumption function

In this part I derive aggregate consumption function that is later exploited to obtain the multiplier's formula.

To this end, a particular type of fiscal shock will be considered henceforth. First, it will be assumed that before period t (which will be referred to as "today") economy is in stationary equilibrium. Second, at the beginning of period t there is an unexpected rise in fiscal purchases that jump from the stationary equilibrium level G to G_t. Moreover, it is assumed that households have perfect foresight about aggregate variables in periods $s \geq t$ (i.e. they formulate rational expectations about the future transition path). This means that the equilibrium concept described in the previous section is narrowed down to the so-called perfect foresight equilibrium.

To formulate a tractable aggregate consumption function in period t, I will express all aggregate objects that are taken as given by consumers while solving their maximization problem "today" as functions of the minimum set of arguments.[7] To this end, let us start with consistency condition 9 which automatically implies that rate f_t is simply a function of x_t. Furthermore, notice that since equation 15 holds aggregate output Y_t can be expressed as a function of x_t because by 13 it simply equals f_t:

$$Y(x_t) = f(x_t). \tag{16}$$

[7]This turns out to be a crucial property, as it allows to apply the Implicit Function Theorem to obtain the multiplier from the budget constraint, as shown later.

Combining this with the assumption about price formation (equation 8) allows to express central bank policy rate i_t as:

$$i(x_t) = \bar{i} + \phi_Y \cdot \left(\frac{Y(x_t) - Y}{Y} \right) + \phi_\Pi \cdot (\Pi(x_t) - \Pi).$$

Observe that since $\Pi'(x_t) > 0$ (see condition 8) and $f'(x_t) > 0$ (by 9 and because M increases in its arguments), function $i(x)$ is increasing:[8]

$$i'(x) \geq 0. \tag{17}$$

which is consistent with the reaction of interest rates to demand and supply shocks in the standard New Keynesian model.

I use constraint 1 that relates consumption c_t and visits v_t to eliminate the latter from the maximization problem of household:

$$v_t = \frac{c_t}{q(x_t)}$$

where the relationship between q_t and x_t follows from condition 10. This allows to reformulate the instantaneous utility function $\tilde{u}$ as follows:

$$u(c_t, x_t) \equiv \tilde{u}\left(c_t, \frac{c_t}{q(x_t)}\right)$$

Furthermore, I impose the following condition on utility function u:

$$u_{cx} = 0 \tag{18}$$

which means that marginal utility from consumption is not affected by the value of product market tightness. The motivation for condition 18 and its role in the analysis are discussed in Section 2.8 in a greater detail.

The assumption about perfect foresight equilibrium automatically implies that paths of fiscal variables $\{G_{s+1}, \bar{B}_{s+1}\}_{s \geq t}$ become known to consumers in period t (right after learning the value of G_t). This idea can be formalized in a concise way

[8] Note that due to the assumed possibility that $\phi_\Pi = \phi_Y = 0$ function $i(x_t)$ is not necessarily strictly increasing (this case will be considered by means of numerical simulations when analyzing passive monetary policy during fiscal expansion in Section 4.3.2).

by the concept of fiscal rule - a mapping Λ that establishes a relationship between the value of government expenditures G_t in period t (which is already known to agents "today") and paths $\{G_{s+1}, \bar{B}_{s+1}\}_{s\geq t}$:[9]

$$\Lambda : \mathbb{R}_+ \to \mathbb{R}_+^{2\times\aleph}, \text{ with } \Lambda(G_t) = \begin{bmatrix} \Lambda_{1,1}(G_t) & \Lambda_{1,2}(G_t) & ... \\ \Lambda_{2,1}(G_t) & \Lambda_{2,2}(G_t) & ... \end{bmatrix} \tag{19}$$

where:

$$G_{t+1} = \Lambda_{1,1}(G_t), \; G_{t+2} = \Lambda_{1,2}(G_t), \; ...$$

$$\bar{B}_{t+1} = \Lambda_{2,1}(G_t), \; \bar{B}_{t+2} = \Lambda_{2,2}(G_t), \; ...$$

where functions in each entry of array Λ are assumed to be differentiable with respect to G_t. In other words, under perfect foresight, agents know fiscal rule Λ and once they learn the value of government expenditures in period t they get a perfect knowledge about transition path $\{G_{s+1}\}_{s\geq t}$ and the way fiscal expansion will be financed: $\{\bar{B}_{s+1}\}_{s\geq t}$.

Note that rule Λ is in fact very general: for the transition path of government purchases it nests a one-time shock to G_t, a permanent shock and an autoregressive shock that returns back to the stationary equilibrium level of fiscal spending G as $s \to +\infty$ as special cases. For the path of public debt, it may represent both a budget neutral and a debt-financed rise in government expenditures as two polar cases.

From now on, I assume that the path of public debt satisfies two sets of conditions. First, it is required that:

$$\forall_{s\geq 1} \; \frac{d\Lambda_{2,s}}{dG_t} \geq 0, \tag{20}$$

i.e., the rise in fiscal purchases cannot be accompanied with a reduction in public debt on the transition path. This condition imposes certain consistency on the fiscal rule Λ as it excludes the coexistence of expansionary government spending and public debt austerity.

Second, it is assumed that Λ is such that the implied path of aggregate tax

[9] By $\aleph$ I denote the cardinality of the set of natural numbers.

burdens (which are specified later with formulas 22 and 23) satisfies:[10]

$$\forall_{s\geq t}\, \frac{d\Theta_s}{dG_t} \geq 0, \tag{21}$$

i.e., the increase of debt during fiscal expansion has an upper limit that prevents from reductions in taxes during expansion.[11]

From what has been said above, given Λ, dependence of Π and i on x and government budget constraint (equation 6), the aggregate revenues from taxes in periods $t+1$, $t+2$... can be expressed as functions of two arguments: product market tightness and government purchases G_t:

$$\Theta_{t+s} \equiv \Theta\left(x_{t+s}, G_t\right) = \frac{1}{\Pi\left(x_{t+s}\right)} \cdot \Lambda_{2,s}\left(G_t\right) - \frac{1}{1+i\left(x_{t+s}\right)} \cdot \Lambda_{2,s+1}\left(G_t\right) + \Lambda_{1,s}\left(G_t\right) \tag{22}$$

for $s \geq 1$. An analogous object in period t is given by:

$$\Theta_t \equiv \Theta\left(x_t, G_t\right) = \frac{1}{\Pi\left(x_t\right)} \cdot \bar{B} - \frac{1}{1+i\left(x_t\right)} \cdot \Lambda_{2,1}\left(G_t\right) + G_t. \tag{23}$$

where I used the fact that the analyzed economy is in stationary equilibrium at the beginning of period t and therefore the amount of public debt that has to be repaid by government at t equals $\bar{B}_t = \bar{B}$.

Finally, it can be inferred from definition 1 that once a rational agent knows the paths of current and future government expenditures, current and future levels of public debt, the initial distribution of households in the economy and monetary rule Φ, he or she is able to retrieve the knowledge of paths of all current

[10]Note that 21 and equations 22 and 23 imply that:

$$\frac{d\Theta_s}{dG_t} = \frac{\partial\Theta_s}{\partial G_t} + \frac{\partial\Theta_s}{\partial x_s} \cdot \frac{dx_s}{dG_t} \geq 0$$

i.e., while setting Λ, government takes into account both direct and indirect effects of G_t on tax revenues and adjusts Θ_s accordingly, so that condition 21 is satisfied.

[11]Note that both 21 and 2 guarantee that a positive shock to G_t is not accompanied by a rise in transfers to some (all) agents. Those conditions are imposed because the main object of interest in this paper is fiscal spending multiplier and therefore I want to isolate the impact of government purchases from other types of stimulative fiscal policies (like transfers). For an alternative approach see Navarro and Ferriere (2016) who consider the case in which fiscal stimulus is accompanied by a rise in the progressivity of income tax (which automatically implies positive transfers to lowest income groups).

and future aggregate variables needed to solve for his/her intertemporal optimal plan. Therefore, the overall utility from that plan at the beginning of period $t+1$ can be summarized with the value function:[12]

$$V_{t+1}\left(b_{t+1}, z_{t+1}, \Lambda\left(G_t\right), \Phi, \mu_{t+1}\left(G_t, \Lambda\left(G_t\right), \Phi\right)\right) \tag{24}$$

This implies that given rules Λ and Φ and the level of government purchases G_t, the maximization problem of household with productivity z and assets b in period t (i.e., when the unexpected shock arrives) can be described as follows:[13]

$$\max_{c_t, b_{t+1}} \left\{u\left(c_t, x_t\right) + \beta \mathbb{E}_{z_{t+1}|z} V_{t+1}\left(b_{t+1}, z_{t+1}, \Lambda, \Phi, \mu_{t+1}\right)\right\} \tag{25}$$

subject to:

$$c_t + \tau\left(z\right) \cdot \Theta\left(x_t, G_t\right) + \frac{b_{t+1}}{1+i\left(x_t\right)} = \frac{b}{\Pi\left(x_t\right)} + z \cdot f\left(x_t\right)$$

$$b_{t+1} \geq -\xi.$$

First order condition associated with problem 25 is:

$$u_c\left(c_t, x_t\right) \geq \left(1+i\left(x_t\right)\right) \cdot \beta \tag{26}$$

$$\times \mathbb{E}_{z_{t+1}|z} \frac{\partial V_{t+1}}{\partial b_{t+1}} \left(\left(1+i\left(x_t\right)\right) \cdot \left(\frac{b}{\Pi\left(x_t\right)} + z \cdot f\left(x_t\right) - c_t - \tau\left(z\right) \cdot \Theta\left(x_t, G_t\right)\right), z_{t+1}, \Lambda, \Phi, \mu_{t+1}\right)$$

which is satisfied with equality when $b_{t+1} > -\xi$.

It is important to highlight the fact that, according to 25, solution to consumer problem depends on fiscal rule Λ. This occurs because, in the model with uninsured idiosyncratic risk, the Ricardian equivalence ceases to hold and hence the way in which government finances fiscal deficits (described by the second row of Λ) resulting from higher purchases in period t, becomes relevant for consumer's consumption-saving behavior.

Furthermore, formulas 24 and 25 show that, given Λ, Φ and G_t, all aggregate objects that affect household's decisions in period t can be expressed as functions of two variables: x_t and G_t.[14] This implies that the only aggregate determinants

[12] This formalization is discussed in a more detailed way in the Appendix.

[13] I have omitted the arguments of Λ and μ_{t+1} to economize on notation.

[14] Observe that the only remaining aggregate variable that is taken as given by households

of a change in consumption policy in period t (driven by an unexpected shock to followed by perfect foresight under rules Φ and Λ) are x_t and G_t, which is reflected by the following notation: $c_t^{\Lambda,\Phi}(b,z|x_t,G_t)$. This, in turn, means that aggregate consumption function in period t is:

$$C_t^{\Lambda,\Phi}(x_t,G_t) \equiv \int_{B\times Z} c_t^{\Lambda,\Phi}(b,z|x_t,G_t)\,d\mu(b,z) \tag{27}$$

and can be summarized solely as a function of x_t and G_t (recall that it is assumed that at the beginning of period t we have $\mu_t = \mu$).[15] Therefore, the economy-wide resource constraint in period t can be rewritten as:

$$C_t^{\Lambda,\Phi}(x_t,G_t) + G_t = Y(x_t)\,. \tag{28}$$

Before proceeding, let us discuss the assumptions that have been made in the analysis so far.

2.8 Discussion of the assumptions

The most significant departure from the canonical Bewley-Huggett-Aiyagari framework in my analysis is the specification of product market that features search frictions. There are several important reasons for which this modification is introduced.

First, and most importantly, the formulation of product market that is used here allows to represent all general equilibrium effects affecting both the supply side of the product market and the demand side with only one variable - product market tightness x. As mentioned, this property enables to compute the analytic formula for the fiscal multiplier by applying the Implicit Function Theorem to equation 28.

Second, there is a long tradition of modeling the productive role of aggregate demand, which is crucial when analyzing the effects of fiscal stimulus, by incorporating search and matching frictions in the product market, which goes back

in period t - the distribution of households μ_t - is a state variable that is fixed and equal to its stationary equilibrium value μ and therefore is not affected by a rise in G_t.

[15] Similar aggregate consumption functions have been derived in Auclert et al. (2018) and Kaplan et al. (2018) among others.

to a seminal contribution by Diamond (1982). This productive role in my model is captured by the fact that changes in aggregate demand (that correspond to changes in the aggregate number of "visits" and thus to changes in market tightness) lead to shifts in capacity utilization $f(x_t)$ (which from equation 13 directly affects $Y(x_t)$) and are not entirely absorbed by changes in prices.

Third, I assume this specification of product market because search and matching protocol allows to incorporate price-setting mechanisms characterized by different levels of price rigidity in a tractable manner. This feature is of particular significance in the context of this paper since, as argued by Hagedorn et al. (2019), price stickiness is an important element that underlies the fiscal multiplier logic which guarantees that suppliers adjust not only prices but also quantities in response to increased government demand.

One technical remark is in order here. In contrast to Michaillat and Saez (2015) and Michaillat and Saez (2019), who model search costs in terms of goods spent by households while making consumer visits, I assume that these costs are captured by disutility from search effort (see, e.g. Storesletten et al. (2017)). I follow this convention because it preserves a standard form of the aggregate resource constraint known from the literature (i.e. without goods spent on search activities on the demand side as in Michaillat and Saez (2015) and Michaillat and Saez (2019)) and thus enables to compare my results with a broader set of theoretical outcomes derived in other works (like Woodford (2011)).

Finally, let us discuss the role of condition 18. As shown in the Appendix, relaxing 18 gives rise to additional mechanism through which private consumption is crowded out (when $u_{cx} < 0$) by government purchases.[16] This is associated with the fact that higher fiscal spending increase market tightness and thus probability q of successful visit faced by consumers is reduced which, in turn, increases the effective cost of search activities. Nevertheless, this "mechanical" crowding out effect is absent in the vast majority of the literature related to fiscal purchases and hence staying in line with it (and thus guaranteeing the compa-

[16]It is easy to check that under relatively mild assumptions about $\tilde{u}$ (i.e. $\tilde{u}_{cv} < 0$ and $\tilde{u}_{vv} < 0$) u always satisfies: $u_{cx} < 0$ because:

$$u_{cx} = -\frac{c \cdot q'}{q^2} \cdot \left(\tilde{u}_{cv} + \frac{\tilde{u}_{vv}}{q} \right) < 0.$$

rability of my analysis with other works) is another argument for excluding the case in which $u_{cx} \neq 0$. In particular, imposing $u_{cx} = 0$ allows to compare the multiplier formula derived in my paper with the one calculated by Woodford (2011). In Section 4 I present functional form of u that satisfies condition 18 which, additionally, excludes wealth effects of search effort, as postulated by Storesletten et al. (2017).

3 Government multiplier: analytical exploration

In this section I present the main result of this paper, i.e. an analytical decomposition of the government spending multiplier in economy populated by heterogeneous households. Recall that I consider an unexpected shock to government expenditures in period t and until its arrival, economy is in stationary equilibrium. Furthermore, it is assumed that agents have perfect foresight about aggregate variables in periods $s > t$.

3.1 Preliminary step

Let us start with a preliminary step in which I derive a general formula for the multiplier in the analyzed economy by applying the Implicit Function Theorem to equation 28:

Lemma 1. *Suppose that economy is in stationary equilibrium at the beginning of period t, government follows fiscal rule Λ and the Taylor rule is characterized with Φ. Then the value of government spending multiplier in period t is:*

$$\frac{dY_t}{dG_t} = \frac{1 + \frac{\partial C_t^{\Lambda,\Phi}}{\partial G_t}}{1 - \frac{\partial C_t^{\Lambda,\Phi}}{\partial x_t} \cdot \frac{1}{f'(x_t)}}. \tag{29}$$

All proofs are delegated to the Appendix. It is now clear, why the assumption about frictional market is crucial for my analysis: if aggregate consumption function depended on more than one endogenous aggregate variables, then it would not be possible to use the Implicit Function Theorem to equation 28 and to characterize the relationship between Y_t and G_t with formula 29.

Although very general, formula 29 already provides us with some insights about the determinants of the multiplier's magnitude. First, notice that its value is affected by both the reaction of private aggregate demand (represented by partial derivatives of C) and the change in capacity utilization $f'(x)$ that drives the response of output. Second, its magnitude depends on both direct effects of government expenditures on private consumption (i.e., $\frac{\partial C^{\Lambda,\Phi}}{\partial G}$) and indirect effects associated with general equilibrium forces summarized with a reaction of private demand to change in product market tightness x (i.e., $\frac{\partial C^{\Lambda,\Phi}}{\partial x}$).

3.2 Analytical decomposition of the government spending multiplier in economy with heterogeneous agents

Before presenting the main result of the paper, I define some additional variables which enable to write down the formula for the multiplier in a heterogeneous agent economy in a concise way.

First, to economize on notation, I suppress the dependence of variables on time and I will use the following aggregation operator $\mathbb{E}_\mu$ to denote the expected value of a variable m in the population distributed according to stationary measure μ:

$$\mathbb{E}_\mu m \equiv \int_{B\times Z} m(b,z)\, d\mu(b,z).$$

where $m(b,z)$ is a variable associated with household that has b of liquid assets and productivity z.

Second, I define the individual marginal propensity to consume (MPC) and the marginal propensity to save (MPS) of household as:

$$MPC \equiv \frac{dc}{dy};\ MPS \equiv \frac{1}{1+i}\cdot\frac{db'}{dy};\ \text{where } y \equiv z\cdot f(x) - \tau(z)\cdot\Theta. \tag{30}$$

i.e., y is household's disposable income.

Similarly to Auclert (2019), let us define the unhedged interest rate exposure as:[17]

$$URE \equiv \frac{b}{\Pi} + z \cdot f - \tau \cdot \Theta - c \tag{31}$$

which decreases with consumer's spending needs. This is because, from the budget constraint $URE = b'/(1+i)$ and therefore agents with high consumption needs who tend to reduce their asset positions b' (e.g. by taking loans) exhibit low URE.

By RRA I denote the relative risk aversion:

$$RRA \equiv -\frac{u_{cc}}{u_c} \cdot c. \tag{32}$$

Moreover, without loss of generality, I standardize:

$$\Pi\left(x\right) = 1 \tag{33}$$

i.e. for the stationary equilibrium level of market tightness x, Π is equal to unity.

Let us define the following variable α:

$$\alpha \equiv \frac{\frac{d\Pi}{dx}}{\frac{dY}{dx}} \tag{34}$$

Since a rise in x can be interpreted as an increase in aggregate demand in the model (recall that output capacity is fixed and normalized to unity), α can be thought of as a value that characterizes the comovement of prices and output resulting from a positive demand shock or the strength of reaction of prices that results from increased government expenditures.

By λ I denote the proportion of additional fiscal spending that is financed with public debt issued in period t:

$$\lambda \equiv \frac{d\Lambda_{2,1}}{dG_t}|_{G_t=G}$$

where the derivative is evaluated at the stationary equilibrium level of govern-

[17]In contrast to Auclert (2019), URE measures the exposure to changes in nominal interest rates and not to shifts in real rates. This is a consequence of the convention followed in this paper which keeps track of nominal interest rates i and prices of goods Π. Equivalently (from the standard Fisher equation), one could re-express the model's equation in terms of real rates and prices of goods which would be identical to the approach presented in Auclert (2019).

ment purchases.

Finally, let us define ι^{Φ} as:

$$\iota^{\Phi} \equiv \phi_{\Pi} \cdot \alpha + \phi_{Y} \tag{35}$$

which measures the responsiveness of monetary policy to changes in output and prices caused by fiscal expansion under policy rule Φ.

The following theorem presents the main result of the paper:

Theorem 1. *Suppose that economy is in stationary equilibrium at the beginning of period t, condition 18 holds, government follows fiscal rule Λ, Taylor rule is characterized by Φ and agents feature perfect foresight about aggregate variables for periods $s > t$. Under those assumptions the formula for the government spending multiplier is:*

$$\frac{dY_t}{dG_t} = \frac{1 + \frac{\partial C_t^{\Lambda,\Phi}}{\partial G_t}}{1 - \frac{\partial C_t^{\Lambda,\Phi}}{\partial x_t} \cdot \frac{1}{f'(x_t)}} \tag{36}$$

where:

$$\frac{\partial C_t^{\Lambda,\Phi}}{\partial x_t} \cdot \frac{1}{f'(x_t)} \equiv \underbrace{-\frac{\iota^{\Phi}}{1+\bar{i}} \cdot \mathbb{E}_{\mu}\left(\frac{MPS \cdot c}{RRA}\right)}_{\textit{Intertemporal substitution channel}(-)} + \underbrace{\frac{\iota^{\Phi}}{1+\bar{i}} \cdot \mathbb{E}_{\mu}\left(MPC \cdot URE\right)}_{\textit{Interest rate exposure channel}(-/+)}$$

$$+ \underbrace{\mathbb{E}_{\mu}\left(MPC \cdot z\right)}_{\textit{Income channel}(+)} \underbrace{- \left(\frac{\iota^{\Phi}}{\left(1+\bar{i}\right)^{2}} - \alpha\right) \cdot \bar{B} \cdot \mathbb{E}_{\mu}\left(MPC \cdot \tau\right)}_{\textit{Debt service costs channel }(-/+)} \underbrace{-\alpha \cdot \mathbb{E}_{\mu}\left(MPC \cdot b\right)}_{\textit{Fisher channel}(-/+)}$$

and:

$$\frac{\partial C_t^{\Lambda,\Phi}}{\partial G_t} \equiv \underbrace{-\left(1 - \frac{\lambda}{1+\bar{i}}\right) \cdot \mathbb{E}_{\mu}\left(MPC \cdot \tau\right)}_{\textit{Taxation channel}(-)} + \underbrace{\beta \cdot \left(1+\bar{i}\right) \cdot \mathbb{E}_{\mu}\left(\frac{MPS \cdot \mathbb{V}^{\Lambda,\Phi}}{u_{cc}}\right)}_{\textit{Expectations channel}(-/+)}$$

where variables without time subscripts denote their values in stationary equilibrium and $(+)$, $(-)$, $(-/+)$ show whether a given channel amplifies, dampens or has an ambiguous impact on the propagation of fiscal shock.

There is one additional variable in Theorem 1 that has not been described yet: $\mathbb{V}^{\Lambda,\Phi}$. From the technical point of view, it measures how change in the expectations about future aggregate variables resulting from higher fiscal purchases affects the slope of V_{t+1} measured along argument b_{t+1} (under rules Λ and Φ).[18]

Let us discuss the forces that affect the value of the multiplier in economy with heterogeneous households. The first channel that appears in the numerator of 36 is related to the increase in taxation needed to finance additional government spending. Obviously, this channel has negative impact on the value of dY/dG as both MPC and τ are positive for all agents. To minimize this effect, government can either increase the proportion of additional government purchases that is financed with debt (i.e. raise λ) or it should levy larger shares in total tax burden τ on households with lower marginal propensities to consume. The latter coupled with empirical observations that richer consumers tend to exhibit smaller *MPCs* (see Figure 1) implies that to dampen the negative impact of higher taxes on the effectiveness of fiscal stimulus government should apply more progressive taxes.[19] Second channel in the numerator is the so-called expectations channel. Its name is motivated by the presence of variable $\mathbb{V}^{\Lambda,\Phi}$. To provide some intuition notice, that if $\mathbb{V}^{\Lambda,\Phi} < 0$ then the expected value function of an agent in $t+1$ flattens along the coordinate b_{t+1} as a result of higher fiscal expenditures G_t. This can be interpreted as a decline in precautionary motives coming from the current rise in G_t. If it is the case for a sufficiently large measure of agents, then rising consumer confidence crowds aggregate consumption in (recall that $u_{cc} < 0$) and amplifies the effects of higher government purchases. Again, it is important to highlight that the dependence of the multiplier on fiscal rule Λ is a consequence of the interplay between market incompleteness and liquidity constraint faced by households. Combined, they imply that Ricardian equivalence does not hold and therefore the way in which dG_t is financed becomes relevant for the consumption response. In Section 4 I demonstrate the relationship between Λ, Φ and $\mathbb{V}^{\Lambda,\Phi}$ by comparing four scenarios associated with different monetary and fiscal rules.

Let us turn to forces that appear in the denominator. First of them is related to intertemporal substitution spurred by monetary policy response during fiscal

[18] The exact formula for $\mathbb{V}^{\Lambda,\Phi}$ is presented in the Appendix in the proof of Theorem 1

[19] This result echoes the conclusion of Navarro and Ferriere (2016) who find that if a more progressive tax schedule accompanies higher fiscal purchases then multipliers in the US are larger.

expansion: if ι^{Φ} is large then central bank counteracts government stimulus more aggressively by raising nominal interest rates and thus creating incentives to save and to reduce private spending. This, in turn, tends to lower the multiplier's value. Monetary authority reaction is prescribed by the Taylor rule and takes place because both price level and output rise during expansion. Second term is associated with the unhedged interest rate exposure of households. Notice that it is an outcome of two mechanisms that go in opposite directions for households from different parts of wealth distribution. On the one hand, $MPC \cdot URE$ is negative for indebted agents that roll over their liabilities. On the other hand it is positive for agents with positive URE (i.e. savers). If the reaction of the former group prevails over the response of the latter then a more responsive monetary policy diminishes the effects of government purchases. In the opposite case, stronger central bank's reaction amplifies the impact of government purchases through wealth effects. Third channel is related to changes in income that accompany fiscal stimulus and, intuitively, it strengthens the effects of fiscal expansion. It is tightly related to a standard, Keynesian feedback loop between household income and consumption that sets in motion the multiplier's mechanism in the standard textbook ISLM model.[20] Fourth force has to do with changes in taxes needed to balance the budget as debt service cost vary.[21] These shifts have two sources: if monetary policy reacts to fiscal stimulus by raising nominal rates significantly then government is forced to issue new debt at lower price and thus has to levy additional taxes to balance the budget. On the other hand, as the stimulus leads to a rise in prices (captured by parameter α) then the nominal public

[20]Indeed, if assumptions related to the derivation of the so-called Keynesian cross (i.e.: i) constant prices, ii) lack of the monetary policy reaction, iii) no dynamic considerations, iv) agents homogeneity and v) irrelevance of the sources of stimulus financing) are imposed on the analyzed framework, then $\alpha = 0$, $\iota^{\Phi} = 0$, $\beta = 0$, $\forall_z\, z = 1$ and $\tau = \lambda = 0$ (by i), ii), iii), iv) and v), respectively). This implies that formula 36 boils down to:

$$\frac{dY_t}{dG_t} = \frac{1}{1 - MPC}$$

which is the standard textbook formula for the Keynesian multiplier.

[21]Observe that this channel is present even if it assumed that stimulus is financed solely with debt. This is because, except for funds needed to finance additional fiscal purchases, government has to balance its budget after changes in i and Π that are induced by its intervention (that can be summarized with the behavior of x). As I assume that the only argument associated with Λ is G_t then, by construction, B_{t+1} does not react to general equilibrium effects captured with x.

debt burden, which has to be repaid by the government in the current period, decreases and gives rise to a downward adjustment in taxes. This can be seen as a version of the Fisher channel that is associated with public liabilities. Household balance sheets are affected by the same force: if consumer has a positive level of nominal wealth then increasing prices impoverish him or her, leading to a cut in expenditures which tends to dampen the effects of fiscal stimulus. Contrarily, for agents with nominal debt, higher prices lower the real value of liabilities and crowd private expenditures in.

Let me point to some distinctive features of my analytical decomposition. First, expression in Theorem 1 bears some resemblance to the result from the seminal work by Auclert (2019) as it contains the averaged cross-products of MPC (or MPS) and individual consumer characteristics. In contrast to his work, however, my result is a general equilibrium outcome. [22] Second, some works (e.g., Kaplan et al. (2018) and Hagedorn et al. (2019), Kopiec (2018)) perform partial equilibrium exercises using numerical methods to decompose the impulse response function of total private consumption into model-based channels. In contrast to those works, not only is Theorem 1 able to decompose the response of output that captures all general equilibrium effects but also, by providing an analytical characterization, it allows to identify the exact determinants underlying fiscal policy transmission channels that work through aggregate consumption.

3.3 Special case: fiscal multiplier in the representative agent framework

To highlight the role of household heterogeneity in the propagation of fiscal stimulus, it is useful to study the extreme case in which inequality across agents is eliminated. Moreover, this experiment allows to compare the formula derived in my paper to the one obtained by Woodford (2011) in the standard New Keynesian model with staggered price-setting (as in Calvo (1983)), endogenous labor supply where monetary policy follows a Taylor rule.

Recall that it is assumed that $u_{cx} = 0$ (see condition 18). Functional form of u

[22] The ability of capturing the GE mechanisms comes at some cost, though: contrarily to Auclert (2019), formula 36 contains an element that cannot be expressed as sufficient statistic as it relies on the model's structure (i.e. $\mathbb{V}^{\Lambda,\Phi}$).

which satisfies this condition is presented in Section 4 (see equation 40) and will be used here:

$$u\left(c,x\right)=\log\left(c\cdot\left(1-\frac{\kappa}{q\left(x\right)}\right)\right).$$

Furthermore, to guarantee comparability with Woodford (2011), I make two additional assumptions. First it is assumed that fiscal rule takes the following form:

$$\Lambda^{RA}\left(G_t\right)=\left[\begin{array}{ccc} G & G & ... \\ \bar{B} & \bar{B} & ... \end{array}\right]$$

i.e. fiscal shock lasts for one period and is financed with taxes. Second, the value of parameter α that captures price stickiness in my model is equal to parameter κ in Woodford (2011) which is an increasing function of the proportion of firms that reset their prices according to mechanism described by Calvo (1983).

First assumption implies that in the representative agent economy we have:

$$b=\bar{B}. \tag{37}$$

Additionally, since agents are identical:

$$\tau\left(z\right)=1,\, z=1.$$

Notice that in this situation, the Fisher channel associated with household balance sheet (see formula 36) is exactly offset by the impact of inflation on public debt:

$$\underbrace{\alpha\cdot\bar{B}\cdot\mathbb{E}_\mu\left(MPC\cdot\tau\right)}_{\text{Debt service costs channel: repayment}}-\underbrace{\alpha\cdot\mathbb{E}_\mu\left(MPC\cdot b\right)}_{\text{Fisher channel}}$$
$$=\alpha\cdot\bar{B}\cdot MPC-\alpha\cdot MPC\cdot b=0$$

where the last equality follows from condition 37. This occurs because on the one hand higher inflation decreases household wealth (which imposes a downward pressure on private consumption) but, one the other hand, it decreases the value of public debt that has to be repaid by government which leads to reduction in taxes (which stimulates consumption).[23] The fact that both forces cancel out is

[23] It is assumed that $\bar{B}>0$: government is net debtor and households are net savers.

not surprising because in the representative agent model government liabilities have to be settled by households (so in fact they are household liabilities) and, at the same time government bonds are household assets. This means that any change in the value of liquid assets has no impact on consumer wealth.

Analogously, the impact of a rise in nominal interest rates i, that accompanies the fiscal shock, on consumer balance sheet (captured by URE) during fiscal expansion is offset by a symmetric mechanism that affects government that issues new debt $\bar{B}_{t+1} = \bar{B}$. More specifically, as households are assumed to be net savers in the representative agent case, an increase in i makes the purchase of assets by households cheaper which automatically raises the amount of resources available for consumption. At the same time, however, government issues new debt at lower price $\frac{1}{1+i}$ (i.e., due to monetary expansion obtains less resources from the issuance of $\bar{B}$) which, mechanically, gives rise to budget deficit that under fiscal rule Λ^{RA} is covered with a rise in taxes which, in turn, lowers consumer's disposable income. Again, both effects cancel out in the representative agent framework:[24]

$$\underbrace{\frac{\iota^{\Phi}}{1+\bar{i}} \cdot \mathbb{E}_{\mu}\left(MPC \cdot URE\right)}_{\text{Interest rate exposure channel}} - \underbrace{\frac{\iota^{\Phi}}{\left(1+\bar{i}\right)^{2}} \cdot \bar{B} \cdot \mathbb{E}_{\mu}\left(MPC \cdot \tau\right)}_{\text{Debt service costs channel: issuance}}$$

$$= \frac{\iota^{\Phi}}{\left(1+\bar{i}\right)^{2}} \cdot MPC \cdot \bar{B} - \frac{\iota^{\Phi}}{\left(1+\bar{i}\right)^{2}} \cdot \bar{B} \cdot MPC = 0.$$

Notice that since the stimulus is assumed to be tax-financed and it is a one-time shock, the representative agent economy is back in stationary equilibrium in period $s = t+1$ and therefore V_{t+1} remains unaffected by the intervention which

[24]Note that from definition of URE and from household's budget constraint (equation 6):

$$URE = \frac{b'}{1+i}$$

and as the case of tax-financed stimulus is considered we have $b' = \bar{B}$.

implies:[25]

$$V_{t+1} = V \implies \underbrace{\beta \cdot \left(1+\bar{i}\right) \cdot \mathbb{E}_{\mu}\left(\frac{MPS \cdot \mathbb{V}^{\Lambda,\Phi}}{u_{cc}}\right)}_{\text{Expectations channel}} = 0$$

because the slope of V_{t+1} is not affected by G_t and therefore $\mathbb{V}^{\Lambda,\Phi} = 0$.

Following Woodford (2011) let us by η_u define the inverse of intertemporal elasticity of substitution of private expenditure, which under assumed specification of u equals:

$$\eta_u = \frac{1}{c}.$$

Furthermore, notice that functional form of u implies:

$$RRA = 1.$$

Finally, notice that in the model with identical households, steady state version of the Euler equation implies:

$$\frac{1}{1+\bar{i}} = \beta.$$

The following corollary summarizes those findings and presents a version of formula 36 for the tax-financed multiplier in economy where agents are identical:[26]

Corollary 1. *Let us consider a one-time, budget-neutral increase in government purchases in a representative agent economy in period t. The associated government spend-*

[25]This is because the only aggregate state variable in the representative agent framework that is relevant in case of a one-time fiscal shock financed with taxes is $\bar{B}$ which is constant over time. Hence, by the backward induction:

$$V = \lim_{t \to +\infty} V_t^{\Lambda,\Phi} = ... = V_{t+2}^{\Lambda,\Phi} = V_{t+1}^{\Lambda,\Phi}.$$

[26]Observe that since debt service cost channel cancels out with Fisher and URE channel and because $\tau = z = 1$ then formula 36 becomes:

$$\frac{dY_t}{dG_t} = \frac{1 - MPC}{1 - MPC + \frac{\iota^{\Phi}}{1+i} \cdot MPS \cdot c}$$

which, because $MPS = 1 - MPC$ is equivalent to equation 38 when the definition of Ω is applied.

ing multiplier is given by:

$$\frac{dY_t}{dG_t} = \frac{1}{1 + \beta \cdot \frac{1}{\eta_u} \cdot \iota^{\Phi}}. \tag{38}$$

To put it differently, the only channel through which fiscal stimulus interacts with private demand in the representative agent case (when Λ^{RA} applies) is the intertemporal substitution mechanism which resembles the conclusion presented by Kaplan et al. (2018) for the case of monetary policy in the standard representative agent New Keynesian model.[27] Formula 38 is intuitive: central bank's reaction to fiscal demand shock measured by ι^{Φ} raises interest rates and spurs saving motives across households (described jointly by β and $\frac{1}{\eta_u}$) which decreases the propagation of stimulus.

An analogous object in Woodford (2011) reads (see equation 30 in his paper):

$$\frac{dY_t}{dG_t} = \frac{1}{1 + F\left(\beta \cdot \frac{1}{\eta_u} \cdot \iota^{\Phi}\right)} \tag{39}$$

where F is a strictly increasing function.[28] Comparison of 38 and 39 indicates that the determinants of the output effects of a fiscal shock in the economy with frictional product market populated with equal households are very similar to those in the standard New Keynesian model with endogenous labor supply, which nicely bridges both modeling approaches. This shows, that the consequences of the only non-standard element used in my analysis (i.e., the frictional product market) for the propagation of fiscal shocks are limited.

[27]As shown by Kaplan et al. (2018), the only channel through which monetary policy affects aggregate consumption in the standard New Keynesian model is the intertemporal substitution channel responsible for the so-called direct effects of changes in nominal interest rates on household spending.

[28]More specifically, F is given by:

$$F(x) = \frac{(1-\Gamma) \cdot x}{\beta + \Gamma \cdot x}$$

where $\Gamma \in (0,1)$ is a parameter defined in Woodford (2011) (i.e., the multiplier's value in a frictionless economy). Moreover, α corresponds to κ in his work and because I concentrate on a one-time shock to G the its persistence (denoted by ρ in Woodford (2011)) equals zero. Finally, I derive formula 39 for the log utility from consumption.

4 Household heterogeneity and the multiplier's value: empirical assessment

Recall that, in contrast to Auclert (2019), the so-called sufficient statistic approach cannot be used to estimate the multiplier's value from expression 36. This is because one of the channels - namely the expectations channel - contains an unobserved element that depends on the model's structure. Therefore estimation of multiplier's value in this paper is based on a calibrated model that matches relevant empirical objects. In particular, as formula 36 suggests, to obtain a good estimate of the multiplier it will be important to mimic closely the empirical features of MPC.

4.1 Calibration of the model

The period in the model is equal to one year and the calibration target are moments characterizing Italian economy in 2016. I choose Italy because of the availability of MPC data measured at the household level in the SHIW survey and year 2016 was the last one when the survey was circulated. As the average level of MPC documented therein equals 0.475 can be hardly achieved in the standard incomplete market model then, in what follows, I will consider two alternative modifications suggested by Jappelli and Pistaferri (2014) to solve this problem. First of them assumes that a proportion μ_{HTM} of households are rule-of-thumb (hand-to-mouth) consumers that feature $MPC = 1$. In the second specification there are two groups of consumers of equal measure: patient households characterized with β_H and impatient households with discount factor equal to β_L, where $\beta_H > \beta_L$.[29] Intuitively, raising the value of μ_{HTM} (or, alternatively, lowering β_L) enables to increase the average level of MPC generated by the model. Despite those modifications, formula 36 remains valid.[30]

[29]More precisely, Jappelli and Pistaferri (2014) experimented with a uniform decrease in β across all agents to match empirical value of average MPC. This leads to a drastic and unrealistic increase in real interest rates in stationary equilibrium. To avoid this problem, I follow Auclert (2017) and split the population into two subgroups: patient and impatient households. The value of β_L is calibrated to match the average value of MPC and the calibration target for β_H is the real interest rate.

[30]More precisely, the only change that is needed to guarantee that 36 holds in economy with patient and impatient households is to take $\beta \in \{\beta_L, \beta_H\}$ under the integral associated with

Figure 2: SMM estimation of ϕ_P, σ_P^2, σ_T^2 and μ_{HTM}: average MPC across cash-in-hand deciles in the model and in the data.

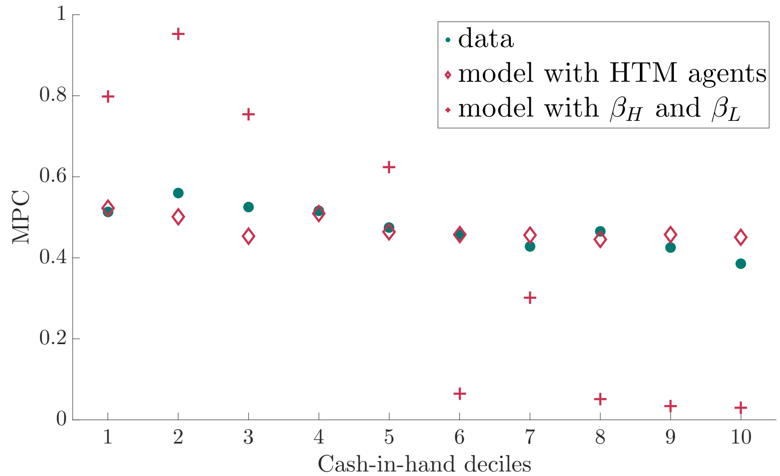


Figure 2 shows how the two calibrated versions of the model match the distribution of MPC across cash-in-hand deciles.[31] The problem with the variant of the model with heterogeneous discount factor is that it generates excessively large differences in MPC across agents (in comparison to empirical evidence) which results from the fact that the difference between β_H and β_L (required to match both the real interest rate and the average MPC) is quite substantial as $\beta_H = 0.98$ and $\beta_L = 0.69$. Therefore, in what follows, I concentrate on the model with a fixed proportion of HTM households.[32]

The assumed functional form for u is:

$$u\left(c,x\right) = \begin{cases} \log\left(c - \frac{\kappa}{\phi} \cdot \left(\frac{c}{q(x)}\right)^{\phi}\right) & \text{if } \sigma = 1 \\ \frac{1}{1-\sigma} \cdot \left[\left(c - \frac{\kappa}{\phi} \cdot \left(\frac{c}{q(x)}\right)^{\phi}\right)^{1-\sigma} - 1\right] & \text{otherwise} \end{cases} \tag{40}$$

which eliminates wealth effects of search effort (see Storesletten et al. (2017)). To guarantee that condition 18 holds, parameters σ and ϕ are set to be equal to one. This, in turn, implies that parameter κ is irrelevant for the equilibrium allocation

expectations channel.

[31] Cash-in-hand in the model is defined as: $b/\Pi + f \cdot z$.

[32] Description of the calibration exercise of the model with heterogeneous discount factors is delegated to the Appendix.

so we do not need to calibrate its value - it occurs because κ and $q(x)$ do not appear in equations that determine equilibrium when $\sigma = \phi = 1$.[33] Moreover, this means that the only way through which product market tightness affects the allocation is $f(x)$. As f is a bijective relationship then the knowledge of f is sufficient to summarize the impact of product market tightness on equilibrium allocation. Therefore, the role of market tightness x can be ignored in further analysis and hence the knowledge of the functional form of matching technology becomes redundant, too.[34] As already explained in subsection 2.4, there are two variables on the matching market that equate supply and demand: price and tightness (or, in the analyzed case f). Therefore, when computing the stationary equilibrium allocation, I treat f as parameter and iterate over interest rate to equate demand and supply of assets.[35] In particular, f is set to be equal to the capacity utilization in Italy in 2016 documented by EUROSTAT.

I use the annual real interest rate equal to 2% to pin down the value of β. I consider a stationary equilibrium in which $\Pi = 1$ and so the value of parameter $\bar{i}$ in the Taylor rule equals the real interest rate.[36] I assume that the two remaining parameters associated with the monetary policy rule take standard, textbook values: $\phi_\Pi = 1.5$ and $\phi_Y = 0.125$ (see Galí (2008)). This allows to define the benchmark monetary rule Φ^0:

$$\Phi^0 = \begin{bmatrix} 0.125 & 1.5 \end{bmatrix}. \tag{41}$$

Aggregate public debt in stationary equilibrium $\bar{B}$ is set to be equal to $1.31 \cdot f$ which is the value corresponding to the level of government debt of 131% GDP in 2016.[37] The comovement between price index and output α is set to be equal 0.51.

[33]To see that notice that when $\sigma = 1$ and $\phi = 1$ then $u_c(c,x)$ is equal to $\frac{1}{c}$. This means that κ does not affect policy functions $c(b,z)$ and $b'(b,z)$ (see the Euler equation) and thus its value becomes irrelevant for the allocation in equilibrium.

[34]This saves up some effort that would be otherwise needed to calibrate parameters associated with M.

[35]When computing the transition path following the shock to government purchases, I switch the roles of f and interest rates in equating demand and supply: I will iterate over the path of capacity utilizations and real interest rates will be pinned down jointly by: Taylor rule and equation 34.

[36]Therefore, as it is the case in the literature, while computing stationary allocation I iterate over $i = \bar{i}$ and keep Π constant and equal one.

[37]Recall that $Y = f$ in the model.

Table 1: Parameters set without model simulations, identical for models with hand-to-mouth households and heterogeneous discount factors.

Parameter	Name	Value	Target/Source
f	Probability of selling output	0.763	Capacity utilization
Π	Price index	1	Standardization
ϕ_Y	Parameter of Taylor rule	0.125	Galí (2008)
ϕ_Π	Parameter of Taylor rule	1.5	Galí (2008)
$\bar{i}$	Parameter of Taylor rule	0.02	Fisher equation
α	Demand-driven comovement of Y and Π	0.51	SVAR evidence
$\bar{B}$	Real public debt	0.99	Debt to GDP ratio
σ	Risk aversion	1	Condition 18
ϕ	Search effort curvature	1	Condition 18
$\{\tau(z)\}_{z\in Z}$	Shares in total tax burden	not reported here	Italian tax system
G	Government purchases	0.28	Equation 6
λ	Stimulus financing rule	$\{0, 1.02\}$	Tax/debt financed dG

This value is based on the SVAR model in which demand shocks are identified with sign restrictions, which is presented in the Appendix in a more detailed way.

The calibration target for ξ (parameter associated with liquidity constraint) is set to match the ratio between aggregate consumer debt and aggregate positive liquid assets of households calculated from the SHIW survey and equal to 0.44.[38]

As already mentioned, parameter μ_{HTM} is calibrated to match the average level of MPC in the SHIW survey. I assume that rule-of-thumb consumers constitute an equal proportion of agents across all states.[39]

To pin down vector $\{\tau(z)\}_{z\in Z}$ I proceed as follows: I first normalize the progressive income tax schedule in Italy with respect to average disposable income

[38]Several works analyzing heterogeneous agent economies (e.g., McKay and Reis (2016b), Krueger et al. (2016) and Kopiec (2018)) standardize the parameter that characterizes the liquidity constraint to zero. Formula 36 shows why this normalization may lead to a distorted picture of the model's reaction to aggregate shocks: if b (and b') is imposed to be non-negative for all agents then it automatically imposes a restriction on the signs of both the interest rate exposure (notice that from the budget constraint $b' = URE$) and the Fisher channel. In particular, in the context of government expenditures shock analyzed here, this assumption implies that Fisher channel always dampens and interest rate exposure channel always amplifies its impact.

[39]In other words, conditional distributions of hand-to-mouth and optimizing agents are the same. Jappelli and Pistaferri (2014) assume that that the fraction of rule-of-thumb consumers is 75 percent in deciles 1–3 of cash-in-hand distribution, 40 percent in deciles 4–7, and 30 percent in deciles 8–10. This gives two additional parameters that are used to achieve calibration targets. In my work I set an equal proportion of hand-to-mouth agents and thus I have two parameters less to match the data. Despite that, I manage to mimic empirical observations reasonably well.

Table 2: Parameters calibrated with the simulated model with a proportion of hand-to-mouth agents

Parameter	Name	Value	Target/Source
β	Discount factor	0.9703	Real interest rate
ξ	Liquidity constraint	-2.2	Ratio of debt to assets
μ_{HTM}	Proportion of HTM agents	0.42	Average MPC
σ_T^2	Variance of transitory shocks	0.05	MPC distribution
σ_P^2	Variance of persistent shocks	0.04	MPC distribution
ρ_P	Autocorrelation of persistent component	0.958	MPC distribution

observed in the data. This automatically gives the income tax thresholds that assign rates from the Italian tax schedule to workers described by z.[40] I can now assign the tax rate $\tilde{\tau}(z)$ from tax schedule to each household which is indexed with productivity z. Simultaneously, given those thresholds and aggregate output (which is equal to capacity utilization in the model) f, I compute the total budget revenues from income tax Θ. To compute the share τ of each household in aggregate tax burden Θ, I divide $\tilde{\tau}(z) \cdot f \cdot z$ (individual amount of tax paid) by Θ. Given Θ, $\bar{B}$, Π and $\bar{i}$, I calculate the value of government purchases G in stationary equilibrium from government budget constraint 6. Parameter λ determines the way in which government finances the stimulus in period t and is set to be equal to 0 in the benchmark simulation (i.e. additional government purchases are financed solely by taxes) and it equals $1+\bar{i}$ when the alternative scenario of financing (i.e. debt-financed stimulus) is considered.

Let us turn to the calibration of the income process that governs changes in z at the individual level. Similarly to Krueger et al. (2016), I assume that productivity follows a process with transitory and persistent components:

$$\begin{cases} \log z' = s + \epsilon_T \\ s' = \rho_P \cdot s + \epsilon_P \end{cases}$$

where by ρ_P I denote the autocorrelation of persistent component, ϵ_T is a tran-

[40]Note that if the corresponding standardization of individual income is performed in the model then:

$$\frac{z \cdot f}{\int z \cdot f d\mu} = \frac{z \cdot f}{f} = z$$

as $\int z d\mu = 1$.

sitory innovation and ϵ_P is the shock that influences the evolution of persistent component s. Parameters ρ_P, σ_P^2 (variance of the shock to persistent component), σ_T^2 (variance of the shock to transitory component) are calibrated using the Simulated Method of Moments to match the average values of MPC (associated with a transitory change in disposable income) across cash-in-hand deciles.[41]

From what was said above, calibrated parameters can be divided into two subgroups. First of them contains those calibrated with reference to the literature and to moments which do not require model simulations and is summarized in Table 1. Second group are values pinned down by model simulations (Table 2).

4.2 Government spending multiplier: benchmark scenario

We are in position to use the formula presented in Theorem 1 and to quantify the impact of channels affecting multiplier's value.

In the benchmark case I assume that: $G_t > G$ and $G_s = G$ for $s > t$ (i.e., fiscal shock lasts for one period) and that stimulus is budget neutral (i.e. λ is equal to zero).[42] This means that fiscal rule Λ^0 in the benchmark scenario is described as follows:

$$\Lambda^0 \left(G_t\right) = \begin{bmatrix} G & G & ... \\ \bar{B} & \bar{B} & ... \end{bmatrix}.$$

In the simulation it is assumed that in period t government purchases increase by 0.1% of the stationary equilibrium level of GDP.[43] Moreover, it is assumed

[41] Given ϕ_P, σ_P^2 and σ_T^2 I use the Rouwenhorst algorithm to discretize the persistent component of the process and I apply the Gauss-Hermite quadrature to approximate the transitory shock. Moreover, observe (see Figure 2) that MPC is not monotonically decreasing with respect to cash-in-hand deciles in the model. This may look a bit surprising but is is driven by the fact that under the assumed specification of idiosyncratic income risk it may occur that agent that exhibits low value of persistent shock and high value of transitory innovation has larger cash-in-hand than agent with high value of persistent shock and low value of transitory innovation. The former tends to have higher MPC than the latter which may give rise to locally increasing relationship between cash-in-hand and MPC.

[42] Observe that I assume that although Italy is a member of the Eurozone, central bank in the simulation reacts to the country-level fiscal shock. This assumption remains plausible if Italy is considered as a large economy among other members of the currency union or if stimulus is coordinated across the Eurozone. The opposite case implies that the ECB does not react to Italian shocks is analogous to the situation in which $\phi_\Pi = \phi_Y = 0$ and it is analyzed later.

[43] I use a relatively small size of the shock to obtain a better approximation of the multiplier (note that in formula 36 dG is an infinitesimal change).

that monetary policy is conducted according to Taylor rule described by Φ^0 (see equation 41).

The value of the multiplier on impact is approximated by the following expression:

$$\frac{dY_t}{dG_t} \approx \frac{Y_t - Y}{G_t - G}.$$

where Y_t is the first element of output transition path.

To estimate the magnitude of multiplier's channels, I will use the following numerical approximation of the MPC of household with assets b and productivity z:

$$MPC\left(b,z\right) \approx \frac{c\left(b,z+\epsilon_T\right) - c\left(b,z\right)}{\left[f \cdot \left(z+\epsilon_T\right) - f \cdot z\right] - \left[\tau\left(z+\epsilon_T\right) - \tau\left(z\right)\right] \cdot \Theta}$$

where the numerator is the difference between stationary equilibrium consumption functions of a household with productivity $z+\epsilon_T$ and of a household with productivity z that have equal levels of asset holdings b. The denominator is the corresponding difference in disposable income. Recall that by ϵ_T I denote the value of a transitory productivity shock. Given $MPC\left(b,z\right)$ and values of c, τ, URE, z, b I compute the magnitudes of: taxation channel, intertemporal substitution channel, interest rate exposure channel, income channel, debt service costs channel and Fisher channel. The size of expectations channel is derived from equation 36 given $\frac{dY_t}{dG_t}$ and values of the remaining channels.[44]

Results are reported in Table 3. The budget-neutral government spending multiplier equals 0.69. First column contains the values of terms appearing in formula 36 that describe the magnitudes of channels through which fiscal stimulus affects aggregate private consumption and, as a consequence, output. To enable the interpretation of those numbers, second column reports the size of the multiplier under a hypothetical scenario, when a given channel is shut off and its value equals zero. If this number exceeds the value of $\frac{dY_t}{dG_t}$ then it means that the corresponding channel crowds private consumption out and dampens the impact of government expenditures on output. If, on the other hand, it is lower than $\frac{dY_t}{dG_t}$ then it implies that consumption is crowded in by a given channel and the effects of stimulus are amplified.

[44] I choose this "indirect" method to estimate the size of the expectations channel because of relatively large approximation errors associated with the computation of mixed derivatives $\mathbb{V}^{\Lambda,\Phi}$ which is required to evaluate this channel directly.

Table 3: Fiscal multiplier: quantitative decomposition, benchmark scenario

	Value	Counterfactual $\frac{dY_t}{dG_t}$
Taxation channel	-0.63	1.95
Expectations channel	-0.03	0.76
Intertemporal substitution channel	-0.13	0.94
Interest rate exposure channel	0.56	0.32
Income channel	0.63	0.30
Debt service costs channel	-0.22	1.22
Fisher channel	-0.34	2.17
MULTIPLIER: $\frac{dY_t}{dG_t}$	0.69	

Decomposition shows that only interest exposure channel and income channel amplify the effects of fiscal stimulus. Their impact on the propagation of government expenditures shock through private consumption is large: in the absence of each of them, the multiplier's value drops by more than 50%. The largest channels that crowd out aggregate consumption and thus dampen the effects of stimulus are: taxation channel and Fisher channel. If closed, each of them generates a rise in the multiplier by approximately 100%.

While the sign of the taxation channel can be easily understood, the sign of the Fisher channel is less intuitive. This is because a conventional wisdom says that rising prices decrease the real value of loans, which coupled with the fact that indebted households exhibit relatively high levels of MPC implies a positive reaction of aggregate consumption. In that context it is therefore surprising that the sign of the Fisher channel in my analysis is negative.[45] This can be explained by the fact that, as Figure 2 demonstrates, the dispersion of MPC across Italian households is not very large. This combined with the observation that aggregate liquid net worth of Italian households is positive (recall that the ratio between absolute values of debt and positive liquid balances is 0.44) implies that positive reaction of consumption of those who benefit from a rise in prices during fiscal expansion (i.e. loan takers) is weaker than the aggregate negative reaction of those those whose net worth depreciates in real terms (i.e. savers). More precisely, the

[45] As demonstrated in the Appendix (Table 7), model with heterogeneous discount factors that fails to replicate empirical pattern from Figure 2 and predicts the sign of the Fisher channel that is consistent with conventional wisdom.

quantitative decomposition of the Fisher channel into responses of debtors and creditors reads:

$$\underbrace{-\alpha \cdot \mathbb{E}_{\mu}\left(MPC \cdot b\right)}_{=-0.34}$$

$$= \underbrace{-\alpha \cdot \mathbb{E}_{\mu}\left(MPC \cdot b | b < 0\right) \cdot \int_{b<0} d\mu\left(b, z\right)}_{=0.28} \underbrace{-\alpha \cdot \mathbb{E}_{\mu}\left(MPC \cdot b | b \geq 0\right) \cdot \int_{b\geq 0} d\mu\left(b, z\right)}_{=-0.62}.$$

Turning to the interest rate exposure channel recall, that from the household budget constraint, URE equals to discounted choice of liquid assets $b'/(1+i)$. As the monetary policy reaction to fiscal stimulus raises the price of new loans then, following the already mentioned common belief, one could expect that because indebted agents exhibit large MPC levels, the URE channel should take negative value. Again, due to a relatively small dispersion of MPC across Italian consumer this intuition fails to be true as illustrated by the following decomposition:

$$\underbrace{\frac{\iota^{\Phi}}{1+\bar{i}} \cdot \mathbb{E}_{\mu}\left(MPC \cdot URE\right)}_{=0.56}$$

$$= \underbrace{\frac{\iota^{\Phi}}{1+\bar{i}} \cdot \mathbb{E}_{\mu}\left(MPC \cdot URE | b < 0\right) \cdot \int_{b<0} d\mu\left(b, z\right)}_{=-0.49}$$

$$\underbrace{+\frac{\iota^{\Phi}}{1+\bar{i}} \cdot \mathbb{E}_{\mu}\left(MPC \cdot URE | b \geq 0\right) \cdot \int_{b\geq 0} d\mu\left(b, z\right)}_{=1.05}$$

i.e., although the monetary contraction during fiscal expansion reduces consumption of indebted agents, there is a concurrent increase in return from savings which crowds in consumption of agents with positive assets.

Finally, let us discuss the remaining propagation channels of fiscal policy shock. First, the monetary policy reaction to higher output and prices raises nominal interest rates which discourages agents from consumption via the intertemporal substitution channel but the quantitative importance of this mechanism is limited. Second, as the amount of public debt in Italy is relatively large then so is the size of the debt service costs channel. Its sign is negative because the rise in prices that reduce the value of public debt does not offset the impact of the monetary policy reaction on price $1/\left(1+i\right)$ of newly issued debt and therefore additional taxes are needed to balance government budget which crowds out private consumption. Third, as a one-time rise in G_t financed with taxes is considered (i.e. Λ^0 is applied), the influence of future equilibrium dynamics on current private spending works solely through their impact on μ_{t+1} (as formulas 24 and 25 suggest) and, as the magnitude of expectations channel suggest, this impact is rather small.

4.3 Government spending multiplier and heterogeneous consumers: alternative scenarios of monetary and fiscal policies

In this part I study the magnitude and the structure of the multiplier under three alternative scenarios: debt-financed stimulus, passive monetary policy (when $i_s = \bar{i}$ for $s \geq t$) and the situation when central bank keeps real interest rates at the constant level (analyzed by Auclert et al. (2018)). Results are displayed in Table 4.

4.3.1 Debt-financed stimulus

Let us start with the case in which additional fiscal spending is financed entirely with public debt. To this end, I assume that the second row of fiscal rule Λ^{DEBT} is specified as follows:

$$\begin{cases} \Lambda_{2,s}^{DEBT}(G_t) = \bar{B} + \left(1+\bar{i}\right) \cdot (G_t - G) & for\ s = 1 \\ \Lambda_{2,s}^{DEBT}(G_t) = \bar{B} + \left(1 - \frac{s}{t'}\right) \cdot \left(1+\bar{i}\right) \cdot (G_t - G) & for\ s \in \{2, 3, ..., t'\} \\ \Lambda_{2,s}^{DEBT}(G_t) = \bar{B} & for\ s > t' \end{cases}$$

i.e., after a rise in period t, government debt is repaid linearly until it attains its stationary equilibrium level $\bar{B}$ in period t' (I set $t' = 5$ in the simulations). Rule Λ^{DEBT} implies that $\lambda = 1 + \bar{i}$.[46] The path of fiscal spending (i.e. the first row of Λ^{DEBT}) is defined as:

$$\Lambda_1^{DEBT} = [G, G, ...]$$

so that the only deviation of Λ^{DEBT} from the benchmark rule Λ^0 is the way in which the stimulus is financed. Monetary rule is the same as in benchmark sce-

[46] Both $\Lambda_{2,1}^{DEBT}$ and the implied value λ are specified so that the value of taxation channel is zero.

Table 4: Fiscal multiplier: quantitative decomposition, alternative scenarios

Channel\Scenario	Benchmark	Debt-financed stimulus	Liquidity trap: $i = \text{const}$	IKC: $r = \text{const}$
Taxation channel	-0.63	0	-0.63	-0.63
Expectations channel	-0.03	-0.39	-0.10	-0.02
Intertemporal substitution channel	-0.13	-0.13	0	0
Interest rate exposure channel	0.56	0.56	0	0
Income channel	0.63	0.63	0.63	0.63
Debt service costs channel	-0.22	-0.22	0.31	0
Fisher channel	-0.34	-0.34	-0.34	0
MULTIPLIER	0.69	1.24	0.68	0.96

nario and is characterized by vector Φ^0.

As we can see in Table 4, the fact that government covers a rise in G solely with debt issuance automatically eliminates the taxation channel which was the main factor that decreased the multiplier's value in the benchmark scenario. On the other hand, however, additional debt has to be repaid in periods $s > t$ which implies higher future taxes and translates into a significant drop in the expectations channel. This has a weaker impact on the multiplier's size than the effect of the disappearance of the taxation channel because the Ricardian equivalence does not hold in the studied framework. This gives rise to a significant amplification of the stimulus when additional government purchases are financed with debt: the size of the multiplier almost doubles.

4.3.2 Passive monetary policy

The role of the monetary policy reaction in the propagation of fiscal stimulus has been discussed, among others, by Woodford (2011), who argued that a more accommodative monetary policy rule tends to raise the multiplier's size.[47] The

[47] As pointed by Woodford (2011), the situation in which monetary policy is constrained by zero lower bound can be seen as an extreme case of the accommodative monetary policy which implies that there is not reaction of central bank to shifts in government purchases (i.e., $\phi_Y = \phi_\Pi = 0$). This case, in turn, has been widely discussed in the literature that emerged in the aftermath of the Great Recession (see, e.g., Eggertsson (2011), Christiano et al. (2011)) and the main conclusion from those works is that higher government purchases are more effective in spurring a recovery

intuition behind this result is straightforward: as fiscal policy shock causes a rise of both prices and output then, under a standard parametrization of Taylor rule, nominal interest rates increase. In a representative agent framework with price rigidities, active response of monetary policy translates into a more dynamic rise in real rates and creates stronger incentives to reduce consumption. This channel is closed automatically if central bank does not react to higher G_t. I now reinvestigate the influence of passive monetary policy on the multiplier's value in the model with heterogeneous households.

To this end, I compare the benchmark scenario described by Λ^0 and Φ^0 to the one in which both ϕ_Π and ϕ_Y equal zero:

$$\Phi^{PMP} = \left[\begin{array}{cc} 0 & 0 \end{array} \right].$$

and fiscal rule Λ^0 continues to hold. Results of the simulation are displayed in Table 4.

Intuitively, under monetary rule Φ^{PMP} both the intertemporal substitution channel and interest rate exposure channel are shut off. The latter implies that a powerful mechanism that was at play in the benchmark scenario and that amplified the impact of fiscal stimulus is now missing.

The remaining two differences with respect to the benchmark scenario are: switch of the sign of debt service cost channel and an increase of crowding out of private spending via the expectations channel. First of them is related to the fact that, when monetary policy becomes passive, it is cheaper for government to roll over public debt as the price of newly issued bonds $1/(1+i)$ is higher. The associated automatic downward adjustment of aggregate tax burden Θ, which balances government budget, crowds in private spending. Second difference is related to the deterioration of expectations about future economic conditions under Φ^{PMP}. The impact of this change in expectations channel on the multiplier's value is rather modest.

Overall, the two largest differences in the multiplier's structure between benchmark simulation and the case with passive monetary policy are: the disappearance of the unhedged interest exposure channel (which tends to decrease the multiplier) and the increase in the value of debt service cost channel (which crowd

when economy is in liquidity trap.

private consumption in). Those two forces (together with some minor effects of changes in the expectations and intertemporal substitution channels) almost cancel out and therefore the multiplier's value remains almost unaffected in comparison to benchmark scenario. This result is in stark contrast to conclusions based on a representative agent models with price rigidities, which predicted a significant rise in the propagation of fiscal shocks under passive monetary policy.[48]

4.3.3 Intertemporal Keynesian Cross

Let us turn to the scenario considered in Auclert et al. (2018), under which central bank is able to keep real rates at the constant level along the transition path following a rise in government purchases. To satisfy that condition in my framework, I set $\alpha = 0$ and assume that monetary policy is passive (and follows rule Φ^{PMP}). Those modifications guarantee that neither the price index Π nor the interest rate i change after an increase in fiscal spending, which guarantees constant real interest rates. In addition, fiscal policy is assumed to be conducted according to rule Λ^0 considered in benchmark scenario.

The multiplier's structure under those assumptions is presented in the last column of Table 4. Notice, that the only active channels of fiscal policy transmission are: taxation channel, income channel and expectations channel. This structure is analogous to the Intertemporal Keynesian Cross formula derived by Auclert et al. (2018) who show, that under constant real interest rates, the rise in output resulting from higher government expenditures is determined by the change in individual incomes (i.e., incomes net of taxes) multiplied by iMPCs.[49]

The multiplier's value under constant real interest rates is significantly larger than in benchmark scenario which indicates that ignoring the impact of prices (nominal interest rates and prices of goods) on: i) consumer balance sheets (via the unhedged interest exposure channel and the Fisher channel), ii) monetary-fiscal interactions (described by debt service cost channel), iii) direct effect on consumer decisions (via the intertemporal substitution channel), may lead to large differences in the assessment of the multiplier's value.

[48] A more detailed discussion of that issue is presented in Section 4.4.

[49] The intertemporal aspect of the multiplier's formula which is captured with iMPCs in Auclert et al. (2018) is encapsulated by the expectation channel in my model.

Table 5: Heterogeneity and the propagation of fiscal stimulus: comparison of the heterogeneous agent (HA) and the representative agent (RA) models

Channel\Scenario	HA		RA	
	Benchmark	Liquidity trap: $i = \text{const}$	Benchmark	Liquidity trap: $i = \text{const}$
Taxation channel	-0.63	-0.63	-0.47	-0.47
Expectations channel	-0.03	-0.10	0	0
Intertemporal substitution channel	-0.13	0	-0.42	0
Interest rate exposure channel	0.56	0	0.40	0
Income channel	0.63	0.63	0.47	0.47
Debt service costs channel	-0.22	0.31	-0.16	0.24
Fisher channel	-0.34	-0.34	-0.24	-0.24
MULTIPLIER	0.69	0.68	0.70	1

4.4 Fiscal multiplier - the role of household heterogeneity

To articulate the role of heterogeneity in the propagation of fiscal stimulus, let us now compare the multiplier's structure in the heterogeneous agent model with the representative agent case when policy rules are described by Λ^0 and Φ^0 (the so-called benchmark scenario).

As argued in Section 3.3, applying Theorem 1 to the RA case under rules Λ^0 and Φ^0 yields the following formula for the multiplier:

$$\frac{dY_t}{dG_t} = \frac{1}{1 + \beta \cdot C^{RA} \cdot \iota^{\Phi^0}}$$

where C^{RA} is aggregate consumption in the RA model.[50] As shown in Section 3.3: i) the unhedged interest exposure channel, the debt service cost channel and the Fisher channel cancel out, ii) taxation channel and income channel have opposite impacts of same strength, the only active propagation channel in the model is the intertemporal substitution channel. The corresponding multiplier's value is 0.70, which is very close to the multiplier in the heterogeneous agent model (see Table 5).

[50] Consumption C^{RA} is set to be equal to stationary equilibrium value of C in the heterogeneous agent model for comparison purposes.

It could be therefore erroneously argued that the impact of heterogeneity on the transmission of government spending shock is very limited. There are at least two reasons for which this statement is false. First, as Table 5 demonstrates, the multiplier's structure in the representative agent economy is different from the one in heterogeneous agent case (although the directions of propagation by various channels are in principle the same).[51] Second, and more importantly, when comparing the multiplier's value and structure under the alternative monetary policy rule Φ^{PMP}, it can be concluded that heterogeneity affects not only the composition but also the size of the multiplier (which is is significantly larger when monetary policy becomes passive and agents are identical).

5 Conclusions

This paper presents an analytical decomposition of the government spending multiplier in the Bewley-Huggett-Aiyagari model extended to capture frictional product market. This departure from the standard framework is applied to obtain the multiplier's formula in the heterogeneous agent economy with price rigidities where the central bank follows the Taylor rule.

I use the model, calibrated to match the moments and the distributions observed in Italian data, to quantify the channels that determine the multiplier's value under four scenarios. First of them (also referred to as benchmark simula-

[51]The evaluation of the transmission channels in the representative agent model in the context of Theorem 1 is as follows:

$$\frac{\partial C_t^{\Lambda,\Phi}}{\partial x_t} \cdot \frac{1}{f'(x_t)} \equiv \underbrace{-\frac{\iota^{\Phi}}{1+\bar{i}} \cdot \left(1-\overline{MPC}\right) \cdot C^{RA}}_{\text{Intertemporal substitution channel}(-)} + \underbrace{\frac{\iota^{\Phi}}{1+\bar{i}} \cdot \overline{MPC} \cdot \frac{\bar{B}}{1+\bar{i}}}_{\text{Interest rate exposure channel}(-/+)}$$

$$+ \underbrace{\overline{MPC}}_{\text{Income channel}(+)} \underbrace{-\left(\frac{\iota^{\Phi}}{\left(1+\bar{i}\right)^2}-\alpha\right) \cdot \bar{B} \cdot \overline{MPC}}_{\text{Debt service costs channel }(-/+)} \underbrace{-\alpha \cdot \overline{MPC} \cdot \bar{B}}_{\text{Fisher channel}(-/+)}$$

and:

$$\frac{\partial C_t^{\Lambda,\Phi}}{\partial G_t} \equiv \underbrace{-\overline{MPC}}_{\text{Taxation channel}(-)}$$

by $\overline{MPC}$ I denote the average value of MPC in the SHIW survey. Admittedly, the model with a representative agent can hardly meet empirical evidence about mean MPC so, to guarantee that decomposition containing $\overline{MPC}$ is consistent with a model populated by identical agents, one can again assume that there is a certain proportion of HTM agents.

tion) assumes that additional fiscal purchases are financed with taxes and monetary policy follows a standard Taylor rule. Second scenario is the debt-financed stimulus. Third case describes the propagation of fiscal policy in the situation when monetary policy is passive and the fourth scenario assumes constant real interest rates.

Finally, by comparing the baseline result with its “representative agent limit”, I articulate the role of heterogeneity in the transmission of government spending shocks.

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Appendix

Derivation of function V_{t+1}

Notice, that as Definition 1 suggests, under perfect foresight all information needed by agent to construct V_{t+1} after an increase in G_t in period t is summarized by paths $\{G_{t+1}, G_{t+2}, ...\}$, $\{\bar{B}_{t+1}, \bar{B}_{t+2}, ...\}$ (or, more compactly $\Lambda(G_t)$), monetary rule Φ and distribution μ_{t+1}. By the same token, the last object can be computed by rational agents already in period t given the initial, stationary distribution $\mu_t = \mu$, the knowledge of paths $\{G_t, G_{t+1}, ...\}$, $\{\bar{B}_t, \bar{B}_{t+1}, ...\}$ and Φ. Therefore, under perfect foresight, distribution of agents in period $t+1$ can be represented as a function of G_t, Φ and $\Lambda(G_t)$:[52]

$$\mu_{t+1} = \mu_{t+1}\left(G_t, \Lambda\left(G_t\right), \Phi\right). \tag{42}$$

All this means that the value function in period $t+1$ of an agent with assets b_{t+1} and productivity z_{t+1} can be written as:

$$V_{t+1}\left(b_{t+1}, z_{t+1}, \Lambda\left(G_t\right), \Phi, \mu_{t+1}\left(G_t, \Lambda\left(G_t\right), \Phi\right)\right)$$

Proof of Lemma 1

Suppose that economy is in stationary equilibrium at the beginning of period t, government follows fiscal rule Λ and the Taylor rule is characterized with Φ. Then the value of government spending multiplier in period t is:

$$\frac{dY_t}{dG_t} = \frac{1 + \frac{\partial C_t^{\Lambda,\Phi}}{\partial G_t}}{1 - \frac{\partial C_t^{\Lambda,\Phi}}{\partial x_t} \cdot \frac{1}{f'(x_t)}}.$$

Proof. I omit time subscripts for clarity. First, we use the formula for the deriva-

[52] As argued, to compute μ_{t+1} rational agents need the knowledge of μ_t, Φ and $\{G_t, G_{t+1}, ...\}$, $\{\bar{B}_t, \bar{B}_{t+1}, ...\}$. Note that information about $\bar{B}_t$ is already contained in μ_t (e.g. because $\bar{B}_t = \int_{B\times Z} b\mu_t(b,z)$) and sequences $\{G_{t+1}, G_{t+2}, ...\}$, $\{\bar{B}_{t+1}, \bar{B}_{t+2}, ...\}$ can be summarized with $\Lambda(G_t)$. Therefore, to compute μ_{t+1} agents need Φ, G_t, $\Lambda(G_t)$ and μ_t, where the last object takes a stationary equilibrium value μ and hence can be treated as a constant.

tive of a composite function and the fact that $Y = f$ (see equation 16):

$$\frac{dY}{dG} = \frac{dY}{dx} \cdot \frac{dx}{dG} = \frac{df}{dx} \cdot \frac{dx}{dG}.$$

Next, we apply the Implicit Function Theorem to obtain $\frac{dx}{dG}$ from resource constraint 28 and plug it into equation above and reformulate:

$$\frac{dY}{dG} = \frac{df}{dx} \cdot \frac{dx}{dG} = \frac{df}{dx} \cdot \left(- \frac{1 + \frac{\partial C^{\Lambda,\Phi}}{\partial G}}{\frac{\partial C^{\Lambda,\Phi}}{\partial x} - \frac{df}{dx}} \right)$$

$$= \frac{1 + \frac{\partial C^{\Lambda,\Phi}}{\partial G}}{1 - \frac{\partial C^{\Lambda,\Phi}}{\partial x} \cdot \frac{1}{\frac{df}{dx}}}.$$

$\square$

Proof of Theorem 1

Suppose that economy is in stationary equilibrium at the beginning of period t, condition 18 holds, government follows fiscal rule Λ, Taylor rule is characterized by Φ and agents feature perfect foresight about aggregate variables for periods $s > t$. Under those assumptions the formula for the government spending multiplier is:

$$\frac{dY_t}{dG_t} = \frac{1 + \frac{\partial C_t^{\Lambda,\Phi}}{\partial G_t}}{1 - \frac{\partial C_t^{\Lambda,\Phi}}{\partial x_t} \cdot \frac{1}{f'(x_t)}} \tag{43}$$

where:

$$\frac{\partial C_t^{\Lambda,\Phi}}{\partial x_t} \cdot \frac{1}{f'(x_t)} \equiv \underbrace{-\frac{\Omega}{1+\bar{i}} \cdot \mathbb{E}_\mu \left(\frac{MPS \cdot c}{RRA} \right)}_{\text{Intertemporal substitution channel}(-)} + \underbrace{\frac{\Omega}{1+\bar{i}} \cdot \mathbb{E}_\mu \left(MPC \cdot URE \right)}_{\text{Interest rate exposure channel}(-/+)}$$

$$+ \underbrace{\mathbb{E}_\mu \left(MPC \cdot z \right)}_{\text{Income channel}(+)} \underbrace{- \left(\frac{\Omega}{(1+\bar{i})^2} - \alpha \right) \cdot \bar{B} \cdot \mathbb{E}_\mu \left(MPC \cdot \tau \right)}_{\text{Debt service costs channel } (-/+)} \underbrace{- \alpha \cdot \mathbb{E}_\mu \left(MPC \cdot b \right)}_{\text{Fisher channel}(-/+)}$$

and:

$$\frac{\partial C_t^{\Lambda,\Phi}}{\partial G_t} \equiv \underbrace{-\left(1-\frac{\lambda}{1+\bar{i}}\right)\cdot \mathbb{E}_{\mu}\left(MPC\cdot\tau\right)}_{\text{Taxation channel}(-)} + \underbrace{\beta\cdot\left(1+\bar{i}\right)\cdot \mathbb{E}_{\mu}\left(\frac{MPS\cdot \mathbb{V}^{\Lambda,\Phi}}{u_{cc}}\right)}_{\text{Expectations channel}(-/+)}$$

where variables without time subscripts denote their values in stationary equilibrium and $(+)$, $(-)$, $(-/+)$ *show whether a given channel amplifies, dampens or has an ambiguous impact on the propagation of fiscal shock.*

Proof. We will derive the formulas for $\frac{\partial C_t^{\Lambda,\Phi}}{\partial x_t}$ and $\frac{\partial C_t^{\Lambda,\Phi}}{\partial G_t}$ that appear in the general characterization of the multiplier (equation 29) by aggregating individual partial derivatives $\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t}$ and $\frac{\partial c_t^{\Lambda,\Phi}}{\partial G_t}$. This method, that is based on the application of the Implicit Function Theorem to the first order condition 26 that holds with equality, can be applied to unconstrained agents only (i.e. to those with $b' > -\xi$). The case of the constrained agents is considered at the end of the proof.

Before moving to $\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t}$ and $\frac{\partial c_t^{\Lambda,\Phi}}{\partial G_t}$, let us make a preliminary step that turns to be very useful later: notice that the Implicit Function Theorem can be used to derive $\frac{dc_t^{\Lambda,\Phi}}{db_{t+1}^{\Lambda,\Phi}}$ from the first order condition 26 which after rearranging yields:

$$u_c\left(c_t,x_t\right) = \left(1+i_t\right)\cdot\beta\cdot\mathbb{E}_{z_{t+1}|z_t}\frac{\partial V_{t+1}}{\partial b}\left(b_{t+1},z_{t+1},\Lambda,\Phi,\mu_{t+1}\right) \Longrightarrow$$

$$u_{cc}\left(c_t,x_t\right)\cdot dc = \left(1+i_t\right)\cdot\beta\cdot\mathbb{E}_{z_{t+1}|z_t}\frac{\partial^2 V_{t+1}}{\partial b^2}\left(b_{t+1},z_{t+1},\Lambda,\Phi,\mu_{t+1}\right)\cdot db_{t+1}$$

On the other hand, notice that when:

$$G_t - G \to dG_t,$$

where dG_t is an infinitesimal positive number, we get:

$$\lim_{G_t-G\to dG_t}\frac{dc_t^{\Lambda,\Phi}}{db_{t+1}} = \frac{dc}{db'} = \frac{1}{1+\bar{i}}\cdot\frac{MPC}{MPS}$$

which exploits the definitions of MPC and MPS (see equation 30) and which was used by Auclert (2019).

Combining both observations allows to express V_{bb} as a function of u'', MPC,

MPS and $\bar{i}$:

$$\mathbb{E}_{z_{t+1}|z_t}\frac{\partial^2 V_{t+1}}{\partial b^2}\left(b_{t+1}, z_{t+1}, \Lambda, \Phi, \mu_{t+1}\right) \tag{44}$$

$$= \frac{1}{\beta\cdot\left(1+\bar{i}\right)}\cdot u_{cc}\left(c,x\right)\cdot\frac{dc}{db'} = \frac{1}{\beta\cdot\left(1+\bar{i}\right)^2}\cdot\frac{MPC}{MPS}\cdot u_{cc}\left(c,x\right).$$

as $G_t - G \to dG_t$.

I apply the Implicit Function Theorem to 26 to get $\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t}$:

$$\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t} = \left[-\frac{1}{u_{cc}\left(c_t,x_t\right)+\left(1+i\left(x_t\right)\right)^2\cdot\beta\cdot\mathbb{E}_{z_{t+1}|z_t}\frac{\partial^2 V_{t+1}}{\partial b^2}}\right]\cdot\left\{u_{cx}\left(c_t,x_t\right)-i'\left(x_t\right)\cdot\beta\cdot\mathbb{E}_{z_{t+1}|z_t}\frac{\partial V_{t+1}}{\partial b}\right.$$

$$\left.-\left(1+i\left(x_t\right)\right)^2\cdot\beta\cdot\mathbb{E}_{z_{t+1}|z_t}\frac{\partial^2 V_{t+1}}{\partial b^2}\cdot\left\{\frac{1}{\left(1+i\left(x_t\right)\right)}\cdot i'\left(x_t\right)\cdot URE_t-\tau\left(z_t\right)\frac{\partial\Theta_t}{\partial x_t}\left(x_t,G_t\right)-\frac{\Pi'\left(x_t\right)}{\Pi^2\left(x_t\right)}b_t+z_t\cdot f'\left(x_t\right)\right\}\right\}.$$

where I have used the definition of URE (equation 31).

Before reformulating $\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t}$ further, it is useful to apply the Euler equation to reexpress the following term:

$$\frac{-i'\left(x_t\right)\cdot\beta}{u_{cc}\left(c_t,x_t\right)}\cdot\mathbb{E}_{z_{t+1}|z_t}\frac{\partial V_{t+1}}{\partial b} = \frac{-i'\left(x_t\right)\cdot u_c\left(c_t,x_t\right)}{u_{cc}\left(c_t,x_t\right)\cdot\left(1+i\left(x_t\right)\right)}$$

$$= \frac{i'\left(x_t\right)\cdot c_t}{RRA_t\cdot\left(1+i\left(x_t\right)\right)} \tag{45}$$

where I used the definition of RRA_t (equation 32).

Now, I exploit the fact that dG_t is an infinitesimal number, use condition 18 and plug 44 into formula for $\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t}$ and rearrange to get (variables without time subscripts denote stationary equilibrium values):

$$\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t} = \left[-\frac{1}{u_{cc}\left(c,x\right)+\beta\cdot\left(1+\bar{i}\right)^2\cdot\frac{1}{\beta\cdot\left(1+\bar{i}\right)^2}\cdot\frac{MPC}{MPS}\cdot u_{cc}\left(c,x\right)}\right]\cdot\left\{\frac{i'\left(x\right)\cdot c\cdot u_{cc}\left(c,x\right)}{RRA\cdot\left(1+\bar{i}\right)}\right.$$

$$-\beta\cdot\left(1+\bar{i}\right)^2\cdot\frac{1}{\beta\cdot\left(1+\bar{i}\right)^2}\cdot\frac{MPC}{MPS}\cdot u_{cc}\left(c,x\right)$$

$$\left.\cdot\left\{\frac{1}{\left(1+\bar{i}\right)}\cdot i'\left(x\right)\cdot URE-\tau\left(z\right)\cdot\frac{\partial\Theta}{\partial x}\left(x,G\right)-\frac{\Pi'\left(x\right)}{\Pi^2\left(x\right)}\cdot b+z\cdot f'\left(x\right)\right\}\right\}$$

$$= \left[- \frac{1}{u_{cc}(c,x) + \frac{MPC}{MPS} \cdot u_{cc}(c,x)} \right] \cdot \left\{ \frac{i'(x) \cdot c \cdot u_{cc}(c,x)}{RRA \cdot (1+\bar{i})} \right.$$

$$\left. - \frac{MPC}{MPS} \cdot u_{cc}(c,x) \cdot \left\{ \frac{1}{(1+\bar{i})} \cdot i'(x) \cdot URE - \tau(z) \cdot \frac{\partial \Theta}{\partial x}(x,G) - \frac{\Pi'(x)}{\Pi^2(x)} \cdot b + z \cdot f'(x) \right\} \right\}$$

$$= -MPS \cdot \frac{i'(x) \cdot c}{RRA \cdot (1+\bar{i})}$$

$$+ MPC \cdot \left\{ \frac{1}{(1+\bar{i})} \cdot i'(x) \cdot URE - \tau(z) \cdot \frac{\partial \Theta}{\partial x}(x,G) - \frac{\Pi'(x)}{\Pi^2(x)} \cdot b + z \cdot f'(x) \right\}$$

where I have used the fact that $MPC = 1 - MPS$.

Since partial derivative of aggregate consumption with respect to market tightness is divided by $f'(x)$ (common to all agents) in the formula for general multiplier (equation 29), it is useful to calculate (for an infinitesimal dG_t: $f(x_t) = f(x)$):

$$\frac{\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t}}{f'(x_t)} = -MPS \cdot \frac{\frac{i'(x)}{f(x)} \cdot c}{RRA \cdot (1+\bar{i})}$$

$$+ MPC \cdot \left\{ \frac{1}{(1+\bar{i})} \cdot \frac{i'(x)}{f'(x)} \cdot URE - \tau(z) \cdot \frac{\frac{\partial \Theta}{\partial x}(x,G)}{f'(x)} - \frac{\frac{\Pi'(x)}{f'(x)}}{\Pi^2(x)} \cdot b + z \right\}.$$

To proceed with $\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t} / f'(x_t)$ I make several observations. From the differentiation of Taylor rule with respect to x we obtain:

$$i'(x) = \phi_\Pi \Pi'(x) + \phi_Y f'(x)$$

and hence:

$$\frac{i'(x)}{f'(x)} = \phi_\Pi \frac{\frac{d\Pi}{dx}}{\frac{df}{dx}} + \phi_Y = \phi_\Pi \frac{d\Pi}{df} + \phi_Y$$

$$= \phi_\Pi \frac{d\Pi}{dY} + \phi_Y = \phi_\Pi \cdot \alpha + \phi_Y = \iota^\Phi$$

where I have used the definition of α 34, the fact that $f = Y$ and the definition of 35.

Again from the definition of α 34:

$$\frac{\Pi'}{f'} = \alpha.$$

Moreover, from the government budget constraint 23 evaluated in stationary equilibrium:

$$\frac{\partial\Theta}{\partial x}(x,G)=\frac{d}{dx}\left(\frac{1}{\Pi(x)}-\frac{1}{1+i(x)}\right)\cdot\bar{B},$$

from the results for $\frac{i'(x)}{f'(x)}$ and $\frac{\Pi'(x)}{f'(x)}$ derived above:

$$\frac{\frac{\partial\Theta}{\partial x}(x,G)}{f'(x)}=\frac{1}{f'(x)}\left(\frac{\partial}{\partial x}\left(\frac{1}{\Pi(x)}-\frac{1}{1+i(x)}\right)\cdot\bar{B}\right)$$

$$=\frac{1}{f'(x)}\left(-\frac{\Pi'(x)}{\Pi^2(x)}+\frac{i'(x)}{(1+i(x))^2}\right)\cdot\bar{B}$$

$$=\left(-\alpha+\frac{\iota^{\Phi}}{(1+\bar{i})^2}\right)\cdot\bar{B}.$$

where the last equality follows because in the stationary equilibrium I standardize $\Pi(x)=1$ (see equation 33). All these means that $\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t}/f'(x_t)$ can be rewritten as:

$$\frac{\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t}}{f'(x_t)}=-MPS\cdot\frac{\iota^{\Phi}\cdot c}{RRA\cdot(1+\bar{i})}$$

$$+MPC\cdot\left\{\frac{1}{(1+\bar{i})}\cdot\iota^{\Phi}\cdot URE+\tau(z)\cdot\left(\alpha-\frac{\iota^{\Phi}}{(1+\bar{i})^2}\right)\cdot\bar{B}-\alpha\cdot b+z\right\}. \tag{46}$$

Aggregation over all agents yields:

$$\frac{\frac{\partial C_t^{\Lambda,\Phi}}{\partial x_t}}{f'(x_t)}=\left[-\frac{\iota^{\Phi}}{1+\bar{i}}\right]\cdot\mathbb{E}_{\mu}\left(\frac{MPS\cdot c}{RRA}\right)$$

$$+\frac{1}{(1+\bar{i})}\cdot\iota^{\Phi}\cdot\mathbb{E}_{\mu}(MPC\cdot URE)-\left(\frac{\iota^{\Phi}}{(1+\bar{i})^2}-\alpha\right)\cdot\bar{B}\cdot\mathbb{E}_{\mu}(MPC\cdot\tau)$$

$$-\alpha\cdot\mathbb{E}_{\mu}(MPC\cdot b)+\mathbb{E}_{\mu}(MPC\cdot z)$$

which is what we wanted to show.

Before computing $\frac{\partial C_t^{\Lambda,\Phi}}{\partial G_t}$ let us define an auxiliary and helpful mapping:

$$H(G_t,\Lambda(G_t))\equiv[\Lambda(G_t),\mu_{t+1}(G_t,\Lambda(G_t))].$$

Mapping H summarizes compactly the impact of G_t on V_{t+1} that works through: i) the influence on the path of future government spending and levels of public debt (captured by fiscal rule Λ) and ii) the influence of G_t on state variable μ_{t+1} (see equations 24 and 25).

To get the formula for $\frac{\partial C_t^{\Lambda,\Phi}}{\partial G_t}$, we need to compute $\frac{\partial c_t^{\Lambda,\Phi}}{\partial G_t}$ and then aggregate it across all agents. The latter is obtained (analogously to $\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t}$) by applying the Implicit Function Theorem to 26:

$$\frac{\partial c_t^{\Lambda,\Phi}}{\partial G_t} = -\left[\frac{1}{u_{cc}\left(c_t, x_t\right) + \left(1 + i\left(x_t\right)\right)^2 \cdot \beta \cdot \mathbb{E}_{z_{t+1}|z_t} \frac{\partial^2 V_{t+1}}{\partial b^2}}\right]$$

$$\left(\left(1 + i\left(x_t\right)\right)^2 \cdot \beta \cdot \mathbb{E}_{z_{t+1}|z_t} \frac{\partial^2 V_{t+1}}{\partial b^2} \cdot \tau\left(z_t\right) \cdot \frac{\partial \Theta_t}{\partial G_t}\left(x_t, G_t\right)\right.$$

$$\left. - \left(1 + i\left(x_t\right)\right) \cdot \beta \cdot \mathbb{E}_{z_{t+1}|z_t} \frac{\partial V_{t+1}}{\partial b \partial H} \cdot \left\{\frac{\partial H}{\partial G_t} + \sum_{j_1=1}^{2} \sum_{j_2=1}^{+\infty} \frac{\partial H}{\partial \Lambda_{j_1,j_2}} \cdot \Lambda'_{j_1,j_2}\right\}\right).$$

We now apply the relationship between V_{bb} and u_{cc} given by equation 44 and use the fact that dG_t is an infinitesimal number:

$$\frac{\partial c_t^{\Lambda,\Phi}}{\partial G_t} = -\left[\frac{1}{u_{cc}\left(c, x\right) + \beta \cdot \left(1 + \bar{i}\right)^2 \cdot \frac{1}{\beta \cdot \left(1 + \bar{i}\right)^2} \cdot \frac{MPC}{MPS} \cdot u_{cc}\left(c, x\right)}\right]$$

$$\cdot \left(\beta \cdot \left(1 + \bar{i}\right)^2 \cdot \frac{1}{\beta \cdot \left(1 + \bar{i}\right)^2} \cdot \frac{MPC}{MPS} \cdot u_{cc}\left(c, x\right) \cdot \tau\left(z\right) \cdot \frac{\partial \Theta}{\partial G}\left(x, G\right)\right.$$

$$\left. - \left(1 + \bar{i}\right) \cdot \beta \cdot \mathbb{E}_{z'|z} \frac{\partial V_{t+1}}{\partial b \partial H} \cdot \left\{\frac{\partial H}{\partial G} + \sum_{j_1=1}^{2} \sum_{j_2=1}^{+\infty} \frac{\partial H}{\partial \Lambda_{j_1,j_2}} \cdot \Lambda'_{j_1,j_2}\right\}\right)$$

$$= -\left(1 - \frac{\lambda}{1 + \bar{i}}\right) \cdot MPC \cdot \tau\left(z\right) + MPS \cdot \frac{1}{u_{cc}\left(c, x\right)} \cdot \left(1 + \bar{i}\right) \cdot \beta \cdot \mathbb{V}^{\Lambda,\Phi} \tag{47}$$

where:

$$\mathbb{V}^{\Lambda,\Phi} \equiv \lim_{G_t - G \to dG_t} \mathbb{E}_{z'|z} \frac{\partial V_{t+1}}{\partial b \partial H} \cdot \left\{\frac{\partial H}{\partial G_t} + \sum_{j_1=1}^{2} \sum_{j_2=1}^{+\infty} \frac{\partial H}{\partial \Lambda_{j_1,j_2}} \cdot \Lambda'_{j_1,j_2}\right\}$$

and where I have used the fact that $MPS = 1 - MPC$ and the fact that from 23 we obtain:

$$\frac{\partial \Theta}{\partial G}(x, G) = 1 - \frac{\lambda}{1+\bar{i}}.$$

Aggregation of 47 over all agents yields the desired formula for $\frac{\partial C_t^{\Lambda,\Phi}}{\partial G_t}$:

$$\frac{\partial C_t^{\Lambda,\Phi}}{\partial G_t} \equiv -\left(1 - \frac{\lambda}{1+\bar{i}}\right) \cdot \mathbb{E}_\mu (MPC \cdot \tau) + \beta \cdot (1+\bar{i}) \cdot \mathbb{E}_\mu \left(\frac{MPS \cdot \mathbb{V}^{\Lambda,\Phi}}{u_{cc}(c,x)}\right)$$

Let us consider constrained agents now. Observe, that for those households we have $MPC = 1$ and $MPS = 0$ by the definition. I will argue, that formulas from Theorem 1 continue to apply for consumers with $b' = -\xi$ if we plug $MPC = 1$ and $MPS = 0$. Individual consumption is determined directly from the budget constraint:

$$c_t = \frac{b_t}{\Pi(x_t)} + z_t \cdot f(x_t) - \tau(z_t) \cdot \Theta_t(x_t, G_t) + \frac{\xi}{1 + i(x_t)}.$$

Thus, the partial derivative $\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t}$ divided by $f'(x_t)$ reads:

$$\frac{\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t}}{f'(x)} = \frac{-\frac{b \cdot \Pi'(x)}{\Pi^2(x)} + z f'(x) - \tau \cdot \left[-\frac{\Pi'(x)}{\Pi^2(x)} + \frac{i'(x)}{(1+\bar{i})^2}\right] \cdot \bar{B} + URE \cdot \frac{i'(x)}{(1+\bar{i})^2}}{f'(x)}$$

where I have used the fact that $b' = URE$ (see equation 31) and the fact that variables take their stationary equilibrium values when dG_t is infinitesimal. Simplifying and using the fact that in stationary equilibrium $\Pi(x) = 1$:

$$\frac{\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t}}{f'(x)} = -b \cdot \alpha(x) + z + \tau \cdot \left[\alpha(x) - \frac{\iota^\Phi}{(1+\bar{i})^2}\right] \cdot \bar{B} + URE \cdot \frac{\iota^\Phi}{(1+\bar{i})^2}$$

which is identical to formula 46 for the unconstrained agents if we plugged $MPC = 1$ and $MPS = 0$. Similarly, by applying the budget constraint we agents:

$$\frac{\partial c_t^{\Lambda,\Phi}}{\partial G_t} = -\tau \cdot \frac{\partial \Theta}{\partial G} = -\tau \cdot \left(1 - \frac{\lambda}{1+i}\right)$$

for constrained agents, which is identical to formula 47 when $MPC = 1$. All this means that formulas for $\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t}$ and $\frac{\partial c_t^{\Lambda,\Phi}}{\partial G_t}$ for constrained agents are special cases of formulas for the unconstrained agents and thus formulas from Theorem 1 capture the case of constrained agents, too. $\square$

Multiplier formula when $u_{cx} \neq 0$

In this part, I present the formula for the multiplier when condition 18 is relaxed. This gives rise to an additional, "mechanical" channel through which fiscal purchases affect private consumption.

I proceed analogously to the proof of Theorem 36. It is easy to see that the only modification that has to be introduced is associated with term $\frac{\partial C_t^{\Lambda,\Phi}}{\partial x_t}$. Analogously to case when $u_{cx} = 0$, I apply the Implicit Function Theorem to 26 to get $\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t}$:

$$\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t} = \left[-\frac{1}{u_{cc}\left(c_t, x_t\right) + \left(1 + i\left(x_t\right)\right)^2 \cdot \beta \cdot \mathbb{E}_{z_{t+1}|z_t} \frac{\partial^2 V_{t+1}}{\partial b^2}} \right] \cdot \left\{ u_{cx}\left(c_t, x_t\right) - i'\left(x_t\right) \cdot \beta \cdot \mathbb{E}_{z_{t+1}|z_t} \frac{\partial V_{t+1}}{\partial b} \right.$$

$$\left. - \left(1 + i\left(x_t\right)\right)^2 \cdot \beta \cdot \mathbb{E}_{z_{t+1}|z_t} \frac{\partial^2 V_{t+1}}{\partial b^2} \cdot \left\{ \frac{1}{\left(1 + i\left(x_t\right)\right)} \cdot i'\left(x_t\right) \cdot URE_t - \tau\left(z_t\right) \frac{\partial \Theta_t}{\partial x_t}\left(x_t, G_t\right) - \frac{\Pi'\left(x_t\right)}{\Pi^2\left(x_t\right)} b_t + z_t \cdot f'\left(x_t\right) \right\} \right\}.$$

I plug 44 into formula for $\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t}$, use equation 45, use the fact that an infinitesimal change to G_t (i.e. dG_t) is analyzed and rearrange to get:

$$\frac{\partial c_t^{\Lambda,\Phi}}{\partial x_t} = \left[-\frac{1}{u_{cc}\left(c, x\right) + \beta \cdot \left(1 + \bar{i}\right)^2 \cdot \frac{1}{\beta \cdot \left(1 + \bar{i}\right)^2} \cdot \frac{MPC}{MPS} \cdot u_{cc}\left(c, x\right)} \right] \cdot \left\{ u_{cx}\left(c, x\right) + \frac{i'\left(x\right) \cdot c \cdot u_{cc}\left(c, x\right)}{RRA \cdot \left(1 + \bar{i}\right)} \right.$$

$$- \beta \cdot \left(1 + \bar{i}\right)^2 \cdot \frac{1}{\beta \cdot \left(1 + \bar{i}\right)^2} \cdot \frac{MPC}{MPS} \cdot u_{cc}\left(c, x\right)$$

$$\left. \cdot \left\{ \frac{1}{\left(1 + \bar{i}\right)} \cdot i'\left(x\right) \cdot URE - \tau\left(z\right) \cdot \frac{\partial \Theta}{\partial x}\left(x, G\right) - \frac{\Pi'\left(x\right)}{\Pi^2\left(x\right)} \cdot b + z \cdot f'\left(x\right) \right\} \right\}$$

$$= \left[-\frac{1}{u_{cc}\left(c, x\right) + \frac{MPC}{MPS} \cdot u_{cc}\left(c, x\right)} \right] \cdot \left\{ u_{cx}\left(c, x\right) + \frac{i'\left(x\right) \cdot c \cdot u_{cc}\left(c, x\right)}{RRA \cdot \left(1 + \bar{i}\right)} \right.$$

$$\left. - \frac{MPC}{MPS} \cdot u_{cc}\left(c, x\right) \cdot \left\{ \frac{1}{\left(1 + \bar{i}\right)} \cdot i'\left(x\right) \cdot URE - \tau\left(z\right) \cdot \frac{\partial \Theta}{\partial x}\left(x, G\right) - \frac{\Pi'\left(x\right)}{\Pi^2\left(x\right)} \cdot b + z \cdot f'\left(x\right) \right\} \right\}$$

$$= -MPS \cdot \frac{u_{cx}\left(c, x\right)}{u_{cc}\left(c, x\right)} - MPS \cdot \frac{i'\left(x\right) \cdot c}{RRA \cdot \left(1 + \bar{i}\right)}$$

$$+MPC\cdot\left\{\frac{1}{(1+\bar{i})}\cdot i'(x)\cdot URE-\tau(z)\cdot\frac{\partial\Theta}{\partial x}(x,G)-\frac{\Pi'(x)}{\Pi^{2}(x)}\cdot b+z\cdot f'(x)\right\}$$

To proceed further, it is useful to concentrate on the following functional form of preferences:

$$u(c,x)=\frac{1}{1-\sigma}\cdot\left(c\cdot\left(1-\frac{\kappa}{q(x)}\right)\right)^{1-\sigma}$$

where $\sigma>0$ and $\sigma\neq1$. Note that this is a special case of GHH preferences postulated in equation 40 and that $u_{cx}=0$ is not satisfied.[53] This implies that:

$$u_{cc}(c,x)=-\sigma\cdot\left(c\cdot\left(1-\frac{\kappa}{q(x)}\right)\right)^{-\sigma-1}\cdot\left(1-\frac{\kappa}{q(x)}\right)^{2}$$

and:

$$u_{cx}(c,x)=\left(c\cdot\left(1-\frac{\kappa}{q(x)}\right)\right)^{-\sigma}\cdot\frac{\kappa\cdot q'(x)}{q^{2}(x)}\cdot(1-\sigma)$$

This means that:

$$\frac{u_{cx}(c,x)}{u_{cc}(c,x)}=\left(-\frac{c}{\sigma}\right)\cdot(1-\sigma)\cdot\underbrace{\frac{\kappa}{q(x)-\kappa}}_{\equiv\Upsilon(x)}\cdot\frac{q'(x)}{q(x)}.$$

Plugging this result into derivations from the proof of Theorem 1, leads to the following formula for $\frac{\partial C_t^{\Lambda,\Phi}}{\partial x_t}/f'(x_t)$:

$$\frac{\frac{\partial C_t^{\Lambda,\Phi}}{\partial x_t}}{f'(x_t)}=(1-\sigma)\cdot\frac{\Upsilon(x)}{q(x)}\cdot\frac{q'(x)}{f'(x)}\cdot\frac{1}{\sigma}\cdot\mathbb{E}_{\mu}(MPS\cdot c)+\left[-\frac{\iota^{\Phi}}{1+\bar{i}}\right]\cdot\mathbb{E}_{\mu}\left(\frac{MPS\cdot c}{RRA}\right)$$

$$+\frac{1}{(1+\bar{i})}\cdot\iota^{\Phi}\cdot\mathbb{E}_{\mu}(MPC\cdot URE)-\left(\frac{\iota^{\Phi}}{(1+\bar{i})^{2}}-\alpha\right)\cdot\bar{B}\cdot\mathbb{E}_{\mu}(MPC\cdot\tau)$$

$$-\alpha\cdot\mathbb{E}_{\mu}(MPC\cdot b)+\mathbb{E}_{\mu}(MPC\cdot z)$$

This implies that the only difference between the multiplier described in Theorem 1 and the multiplier derived in the situation when condition $u_{cx}=0$ is relaxed is

[53] I impose $\phi=1$ on 40 so that the "mechanical" channel is a product of two terms: aggregate component that is equal across all households and aggregated cross-products of individual variables. This formulation allows for a better interpretability and enables to express that channel in a similar way to other channels.

the emergence of the channel described with:

$$(1-\sigma)\cdot\frac{Y(x)}{q\left(x\right)}\cdot\frac{q'\left(x\right)}{f'\left(x\right)}\cdot\frac{1}{\sigma}\cdot\mathbb{E}_{\mu}\left(MPS\cdot c\right)$$

Given the general formula for the multiplier 29 and given that $f' > 0$, $q' < 0$, $q\left(x\right) > \kappa$ (and hence $Y > 0$) and because $\sigma > 1$, we conclude that this additional channel crowds private consumption out. The interpretation of this mechanism is the following: if government increases G_t then it automatically raises $v_{G,t}$, needed to achieve the new level of purchases (see equation 5). This, according to condition 7, raises market tightness and, at the same time, decreases the probability of a successful visit $q\left(x\right)$ (because $q' < 0$). This, in turn, raises the effective utility cost of making visits by households, which discourages them from spending and therefore aggregate consumption is crowded out. As mentioned, this "mechanical" channel is absent in works that are standard references in the literature concerning fiscal multipliers so to stay in line with it, I assume $u_{cx} = 0$ (condition 18) in the core text.

Calibration of parameter α with the SVAR model

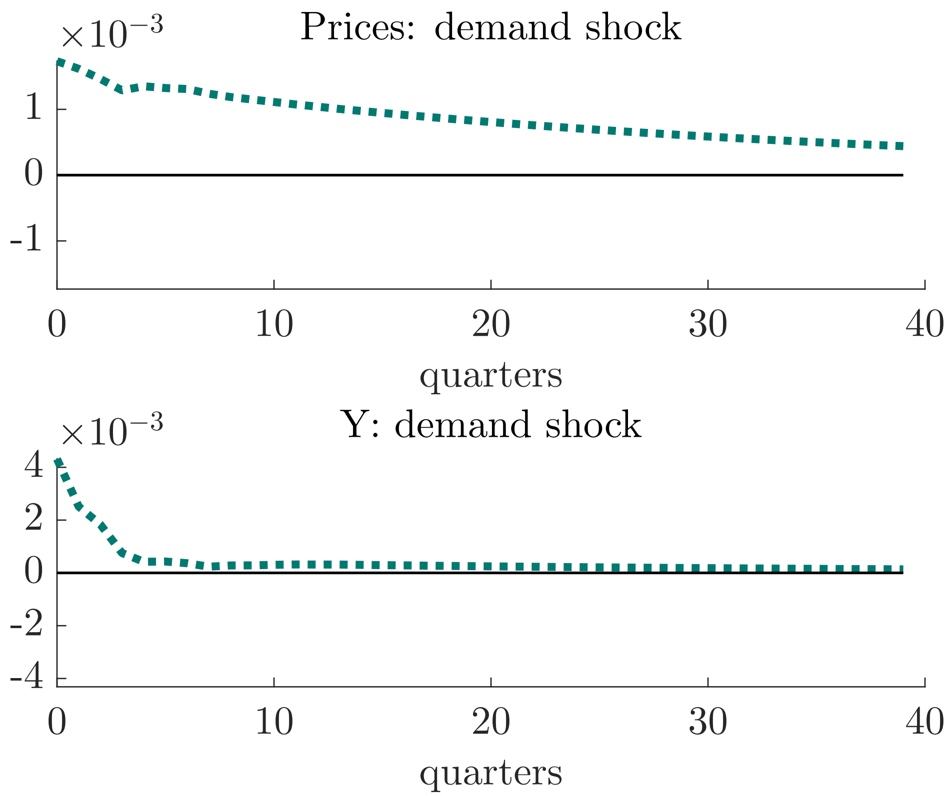
Recall that the value of α is defined as:

$$\alpha \equiv \frac{\frac{d\Pi}{dx}}{\frac{dY}{dx}}$$

and, because x can be thought of as a measure of aggregate demand, α can be interpreted as a measure of comovement of prices and output which results from a positive demand shock. To find an empirical measure of α I use the standard SVAR model (that consists of two variables: output and prices and four lags chosen with standard tests). I estimate the model using quarterly data for Italy from 1985 to 2018 (I take first differences to obtain data used in estimation).

To identify demand shocks I use sign restrictions (it is assumed that a positive demand shock increases both price level and output while a positive supply shock raises output and lowers prices). Parameter α is approximated with the ratio between the value of the impulse response function of price index and the value of the impulse response function of output in period 0. Impulse response functions are reported in Figure 3 (I report the median value of all IRFs satisfying the sign restriction at a given date).

Figure 3: SVAR simulation: the impact of demand shock on output and prices



Calibration of the model with patient and impatient households

In this subsection I follow Auclert (2017) and I describe the calibration of the model where agents are grouped into two populations of measure 0.5: patient and impatient households. The majority of parameter values are set at levels that are identical with those in the model with hand-to-mouth consumers (see table 1). There are several exceptions: parameters calibrated with the simulated model.

First, there are two discount factors: β_L (associated with impatient households) and β_H (associated with patient households) that satisfy $\beta_L < \beta_H$. The value of discount factor β_H is set to match the level of real interest rate. As mentioned, the calibration target for β_L is the average level of MPC documented in

Table 6: Parameters calibrated with the simulated model with heterogeneous discount factors

Parameter	Name	Value	Target/Source
β_H	Discount factor of patient agents	0.9736	Real interest rate
β_L	Discount factor of impatient agents	0.69	Average MPC
ξ	Liquidity constraint	-1.35	Ratio of debt to assets
σ_T^2	Variance of transitory shocks	0.05	MPC distribution
σ_P^2	Variance of persistent shocks	0.04	MPC distribution
ρ_P	Autocorrelation of persistent component	0.958	MPC distribution

the SHIW survey. Introducing discount factor heterogeneity changes the stationary distribution of households significantly (in comparison to the model with hand-to-mouth consumers) and thus to match the ratio between aggregate debt and aggregate positive assets we need to reparametrize the tightness of liquidity constraint captured with ξ.

The remaining parameters: ρ_P, σ_P^2, σ_T^2 were used to match the distribution of MPC across cash-in-hand deciles in the model with rule-of-thumb consumers. The problem with the variant of the model with heterogeneous discount factor is that it generates excessively large differences in MPC across agents (in comparison to empirical evidence) irrespectively of values assigned to parameters ρ_P, σ_P^2, σ_T^2. Figure 2 provides an example of the distribution of MPC generated by the model with β_L and β_H in which ρ_P, σ_P^2, σ_T^2 take values identical to those calibrated for the model with hand-to-mouth agents. As discussed later, the fact that the model with heterogeneous discount factors fails to mimic this pattern has some important consequences for the evaluation of fiscal policy effectiveness.

Table 7: Decomposition of fiscal multiplier, model with heterogeneous discount factors

	Value	Counterfactual $\frac{dY_t}{dG_t}$
Taxation channel	-0.42	0.79
Expectations channel	-0.08	0.50
Intertemporal substitution channel	-0.24	0.54
Interest rate exposure channel	-0.50	0.76
Income channel	0.43	0.31
Debt service costs channel	-0.14	0.49
Fisher channel	0.29	0.35
MULTIPLIER: $\frac{dY_t}{dG_t}$	0.43	