Social Status and Intergenerational Equality of Opportunity

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Abstract

Is long-term income inequality consistent with equality of opportunity (EOp) ethic? In this paper we study the effectiveness of intergenerational EOp policies in an environment with two social groups and infinite generations of individuals, where the outcomes of one generation define the circumstances of the next. Circumstances in this paper have to do either with different preferences among individuals from different social groups or with both resources and preferences due to these resources. We show that in the former case EOp policies reduce inequality and also the EOp policy is the same as the Utilitarian one. In the latter case, inequality is not reduced and its level depends on the relative population of the two social groups.

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1 Introduction

The publication of Piketty's *Capital* (Piketty, 2014) and other empirical works on the issue of rising income and wealth inequality[1] has drawn the attention of both the mainstream media and economics research. The extent of growth in inequality can be summarised by the title of a recent *Oxfam* report: "Just 8 men own same wealth as half the world" (Oxfam, 2016). A different, but related part of recent empirical work has focused on the inequality of opportunity to education and the limits that social mobility people face due to their background.[2] The aim of this paper is to use insights from the latter literature and answer the question on whether the increasing inequality observed in the data is *fair* according to the most widely accepted ethical liberal view, namely *Equality of Opportunity* (EOp).

Since Rawls' *Theory of Justice* (Rawls, 1971), in both the philosophy and normative economics literature, the question of defining a *just* distribution has focused around the distinction between people's *circumstances* beyond one's control and the choices one makes. This *cut* was made clear by Dworkin (1981) who argued that there are two kinds of personal characteristics: the ones which are related to a person's environment and for which they should not be held responsible for, like parental background, and the ones for which the person should be held responsible for. According to Dworkin, this cut was between the preferences of a person, for which they should be held responsible and their resources for which they should not be held responsible for. Thus, if we assume that there are no differences in talents (or handicaps)[3], for Dworkin the fair and responsibility-sensitive distribution is the one which allocates resources equally among individuals, even if this means inequality of welfare, which would then be due to difference in tastes. Cohen (1989), argued that even though the distinction between circumstances and choices is correct, Dworkin's cut had been misplaced, because individual preferences are also affected by their resources. Based on

[1] For other relevant empirical works on inequality see Piketty and Zucman (2014), Saez and Zucman (2016) and references therein.

[2] For example see Chetty et al. (2016).

[3] In the case of different individual circumstances Dworkin (1981) proposes a no envy insurance scheme.

this, Cohen proposed that the correct *responsibility- sensitive* egalitarian policy should be aiming to equalise, not resources but opportunities for advantage[4].

Fleurbaey (1995) and Bossert (1995) proved that it is not possible for a policy to achieve both (i) full accountability for differences in outcomes[5] which stem from differences in preferences and (ii) full compensation for ability differences. Because of this issue, the economics literature has made "concessions" in at least one of (i) or (ii), leading the *responsibility- sensitive* egalitarian welfare economics, to develop in two broad directions. According to the first approach (Fleurbaey, 2008; Fleurbaey and Maniquet, 2011), differences in skills should be compensated for and individuals should be held responsible for their preferences. The second approach put forward by Roemer (1998) emphasises the fact that individuals' circumstances, also affect their preferences, and thus people should be only held partially responsible for their preferences. According to Roemer (1998), this can be overcome by dividing individuals into types according to the characteristics which are due to circumstances. Then, within a given type, individuals would differ according to the characteristics for which they can be held responsible for relatively to the other individuals in the same type. Assuming that the outcomes are affected by both types of characteristics, then the distribution of outcomes *within* a type will be due decisions that individuals could be *relatively* held responsible for, while the same is not true for the distribution of outcomes across all individuals.

Roemer (1998) argues that the EOp policy should aim to equalise (in some average sense) the achievements (or outcomes) across types but not within types. This would be achieved by dividing the individuals within a type into centiles according to their preferences and then maximising the minimum achievement across individuals in each centile, for each centile across types. Due to the complexity of this approach Roemer proposed a "compromise" solution, according to which the EOp policy maximises a weighted average of the minimal utilities across individuals who have the same preferences. In this way, as Fleurbaey (2008) suggests, the first approach is a middle way between outcome egalitar-

[4]Cohen's notion of equality of opportunity for advantage is a more general case of equality of opportunity for welfare proposed by Arneson (1989).

[5]Outcomes could be levels of advantage (Cohen, 1989; Roemer, 1998), welfare, payoffs etc.

ianism and libertarianism while the second approach is a middle way between outcome egalitarianism and utilitarianism. Along similar lines Van de gaer (1993) proposed a simpler policy, which maximises the average utility of the type for which average utility is lowest.

In order to answer the question of whether increasing inequality can be seen as fair according to EOp, we employ a dynamic model of two social classes and infinite generations. The relative income level of a generation defines what we will call socioeconomic status or simply status. Status affects both the circumstances and preferences of the next generations, through different ways which can be related to different issues such as financial resources for education and/ or inheritance of parental social capital. Given that the outcomes of one generation are affected indirectly not only by the outcomes of the previous one, but also by the outcomes of *all* the previous ones, then a *just* distribution would be the one that maximises the outcome of the worst off individuals of any point in time and if this problem has more than one solutions, the appropriate one, would be the one which maximises the outcomes of the second, third (and so on) worst off.

Our model builds on Piketty (1995, 1998), Roemer (1996, 1998), Roemer and Veneziani (2004) and Loury (1976). More specifically: (i) the assumptions on preferences are similar to Piketty (1995), (ii) status captures the public perception of one's skills or how 'smart' they are as in Piketty (1998), (iii) status affects the marginal return of effort as in Loury (1976) and (iv) the equilibrium concept is an extension of Roemer (1996, 1998) and Roemer and Veneziani (2004). We study the effects of an EOp policy in two different economic environments in order to highlight the importance of taking into account *date of birth* as a circumstance. We show that while if date of birth is not considered as a circumstance inequality grows, this is not the case otherwise.

Even though EOp policies have intergenerational implications, there is very limited work on this aspect. Roemer and Veneziani (2004) have considered the effects of EOp in an intergenerational framework and have showed that EOp for some objective condition is incompatible with human development over time. Roemer and Ünveren (2016) have studied the long-term effects of policies intended to equalise opportunities among different social classes and have showed that private investment in education is a major barrier to equalising opportunities in the long run. The present paper contributes and extends this literature (i) by introducing a more general, equilibrium concept which is relevant for intergenerational policies and (ii) by showing the IEOp policies can lead to different results depending on the economic environment which these are implemented.

The present paper is related to the political theory literature, on intergenerational justice e.g. McKerlie (1989, 2001a,b, 2012), Temkin (1992, 1993), Daniels (1988, 1993, 2008), Bidadanure (2015, 2016) and Galanis and Veneziani (2017). With the exception of Galanis and Veneziani (2017), this literature has focused on the distribution between individuals at different segments of their lives (for example young versus old) without taking into account how the distribution in one generation may have implications for the rest. Contrary to Galanis and Veneziani (2017), in the present paper we do not assume different welfare between different segments of individuals' lives but we allow for groups with different social backgrounds.

Our work also contributes to the literature on status and inequality. The origins of the literature on the effects of status are the seminal works of Rae (1834), Veblen (1922) and Duesenberry (1949) who argued that the consumption patterns of individuals are relative to the patterns of their close environment. The works of Frank (1985), Cole et al. (1992), Robson (1992), Clark and Oswald (1996), Corneo and Jeanne (1999), Moav and Neeman (2010), Becker et al. (2005) and Ray and Robson (2012) show how aiming to appear to have high status, leads to conspicuous consumption motives which can lead to persistent inequality. In the present paper, status plays a different role than in the previous literature. Here status influences the marginal return of effort which can be seen as the effect of differences in availability of financial resources and/ or as difference in social capital. In this way, our approach is closely related to the views of Coleman (1988, 1990, 1994) who have argued that social capital is key in the acquiring human capital and also determines the effectiveness of the latter. Recent works on the effects of social capital include Glaeser et al. (2002) has studied the formation of social capital and (Chou, 2006) and (Jennings and Sanchez-Pages, 2017) who have examineed the role on social capital in relation to growth and conflict respectively. In this way our paper also provides a link between the literature on social capital on one hand and on status and inequality on the other.

2 Model

2.1 The Economic Environment

The basic structure of our economic environment is closely related to Piketty (1995). Consider an infinitely-lived economy in which each individual lives one period and has one offspring, so that population is constant over time. In every period $t = 1, 2, \ldots$, agents produce and consume a single good by exerting effort e_t. Each individual belongs to a social class depending on their income. Call the high-income social class, the *rich* (r) and the low-income class, the *poor* (p). Let α be the fraction of the population who are poor and $1 - \alpha$ the fraction of the rich agents, with $\alpha > 1/2$. Individual welfare depends positively on consumption c and negatively on effort e:

$$u(c, e) = c - \frac{e^2}{2}, \tag{1}$$

The consumption level of an individual i, is given by

$$c^i = (1 - \tau_t)y_t^i + \tau_t \bar{y}_t, \tag{2}$$

where τ_t is a flat tax rate, chosen according to an *Equality of Opportunity* (EOp) ethic defined below; y_t^i is the pre-tax income of an individual belonging to class i and $\bar{y}_t$ is the average pre-tax income at t:

$$\bar{y}_t = \alpha y_t^p + (1 - \alpha)y_t^r. \tag{3}$$

In Roemer and Ünveren (2016) and Piketty (1995, 1998) the income level of individuals in both classes is given and constant over time but individuals can change class (intergenerational mobility). In Piketty (1995, 1998) the probability is of an individual staying to the upper class is higher if in the previous period they were already there, compared to an individual from the lower class who has exerted the same amount of effort than the first. In Roemer and Ünveren (2016) this probability is also influenced by the level of public education. Hence, in all mentioned works, background plays a key role in mobility, however income level in both classes is exogenously set and constant.

In the present paper there is no possibility for intergenerational mobility on an individual basis and background affects the (pre-tax) income of a class. Background here, is captured by what we call *parental status* or simply *status*. Following the relevant literature[6], status takes the form of a positional good, thus capturing the *relative* socioeconomic positioning. Let s_t^p, $s_t^r \in \mathbb{R}_+$ denote the status of the rich and the poor. Taking into account the positional good nature of status assume that

$$s_t^p + s_t^r = 1, \text{for all } t. \tag{4}$$

For simplicity let $s_t^p = s_t$ and $s_t^r = 1 - s_t$. We assume that $s_1 \in \left(0, \frac{1}{2}\right)$.

Parental status affects a child's opportunities in various ways. As Loury has argued,

> There are many reasons why a child's opportunities to acquire skills vary with the economic success of her parents. For example, the quality of schooling any child receives varies considerably across communities and tends to be higher in the suburbs than in the central city. Where there is housing segregation based on income, and the quality of neighbourhood schools shows a positive correlation with the community's wealth, a child's educational opportunities can be expected to vary directly with parental economic achievements. Further the absence of a perfect capital market for educational loans means that the opportunity for higher education and the quality of that education will be sensitive to an individual's socioeconomic background. (Loury, 1976, p. 155).

[6]See for example Hirsch (1977) and Besley and Ghatak (2008).

Given the role of education in acquiring human capital and the positive relationship between human capital and salaries, it is straightforward to assume that status has *potentially* a positive effect on an individual's (pre-tax) income level. From an alternative (but complementary) viewpoint, status could capture the positive effects of social networks (or social capital). As ?Coleman (1990, 1994) has argued social capital plays the role of the effectiveness of social capital. In both cases status is not able to transform it to income itself, but individuals should also exert effort towards acquiring human capital. In both cases effort will be more or less productive depending on status, hence status could be expressed as the productivity of effort. The assumption that social origins affect effort productivity has also been supported by recent empirical studies (see Hershbein (2016) and Chetty et al. (2016)). Based on this, the income of the poor and rich agents is given, respectively, by:

$$y_t^p = s_t e_t^p, \tag{5}$$

$$y_t^r = (1 - s_t) e_t^r. \tag{6}$$

Optimal Effort

During their lifetime, individuals choose how much effort to exert in order to maximise their welfare, given their status. Hence, each individual from class $i = p, r$ solves

$$\max_{e_t^i} \left\{ c_t^i - \frac{(e_t^i)^2}{2} \right\}, \tag{OE}$$

subject to (2) and either (5) or (6) for a given tax rate τ_t. Thus, the optimal effort of the two classes is

$$e_t^{p*} = (1 - \tau_t) s_t, \tag{7}$$

$$e_t^{r*} = (1 - \tau_t)(1 - s_t). \tag{8}$$

Hence, due to differences in productivity the rich have an incentive to exert more effort than the poor. Plugging (7) and (8) into (5) and (6) respectively, the optimal pre-tax income of the representative individual of each class is:

$$y_t^{p*} = (1 - \tau_t)(s_t)^2, \tag{9}$$

$$y_t^{r*} = (1 - \tau_t)(1 - s_t)^2. \tag{10}$$

Given this, the average optimal income $\bar{y}_t$ is:

$$\bar{y}_t = (1 - \tau_t)[\alpha(s_t)^2 + (1 - \alpha)(1 - s_t)^2]. \tag{11}$$

Thus, given (2), (9), (10) and (11) the consumption of the two classes can be expressed in terms of s_t and τ_t as follows

$$c_t^p = (1 - \tau_t)^2(s_t)^2 + \tau_t(1 - \tau_t)[\alpha(s_t)^2 + (1 - \alpha)(1 - s_t)^2], \tag{12}$$

$$c_t^r = (1 - \tau_t)^2(1 - s_t)^2 + \tau_t(1 - \tau_t)[\alpha(s_t)^2 + (1 - \alpha)(1 - s_t)^2]. \tag{13}$$

Given (7), (8), (12) and (13) the indirect utilities of the two classes are

$$v^p(s_t, \tau_t) = \frac{(1 - \tau_t)^2(s_t)^2}{2} + \tau_t(1 - \tau_t)[\alpha(s_t)^2 + (1 - \alpha)(1 - s_t)^2], \tag{14}$$

$$v^r(s_t, \tau_t) = \frac{(1 - \tau_t)^2(1 - s_t)^2}{2} + \tau_t(1 - \tau_t)[\alpha(s_t)^2 + (1 - \alpha)(1 - s_t)^2]. \tag{15}$$

From (14)-(15), if $s_t > 1/2$, then it immediately follows that $v_t^p \leq v_t^r$ for all $\tau_t \in [0, 1]$, with equality holding only if $\tau_t = 1$

Let τ_t^* denote the tax rate which maximises the utility of the worst off individual(s) in period t: in a static perspective, this would be the requirement of an equal opportunity ethics, given that $v_t^p \leq v_t^r$ for all tax rates. Formally:

$$\max_{\tau_t} \left\{ \frac{(1 - \tau_t)^2(s_t)^2}{2} + \tau_t(1 - \tau_t)[\alpha(s_t)^2 + (1 - \alpha)(1 - s_t)^2] \right\}, \tag{$MP\ 1$}$$

which is solved by

$$\tau_t^* = \frac{(1 - \alpha)(1 - 2s_t)}{(s_t)^2 + 2(1 - \alpha)(1 - 2s_t)}. \tag{16}$$

It is immediate to show that τ_t^* is decreasing in both α and s_t. This implies that *ceteris paribus* higher income inequality in the previous generation and higher relative population of the rich both will lead to a higher tax rate.

2.2 Intergenerational Transmission of Status

An individual's status depends on her socio-economic background: the economic, cultural, and social conditions of her upbringing. In the context of our simple model, we focus on economic factors and suppose that the status of a generation depends on the after-tax income of their parents. More specifically we assume that

$$\frac{s_t^p}{s_t^r} = \frac{c_{t-1}^p}{c_{t-1}^r}. \tag{17}$$

This implies that if two individuals with different levels of status exert the same effort, their pre-tax income will be the same as their parents' after tax income. Taking also into account the *positional good* nature of status, by (17) it follows that the status of the poor at t is

$$s_t = \frac{c_{t-1}^p}{c_{t-1}^r + c_{t-1}^p}, \tag{18}$$

while the status of the rich at t is $1 - s_t = \frac{c_{t-1}^r}{c_{t-1}^r + c_{t-1}^p}$. At the optimal effort level at t, the status of each class at $t+1$ is a function of the tax rate and of the status of their parents:[7]

$$s_{t+1} = \frac{s_t^2 + \tau_t(1-\alpha)(1-2s_t)}{1 - 2s_t + 2s_t^2 + \tau_t(1-2s_t)(1-2\alpha)}. \tag{19}$$

Thus, for all $\tau_t \in [0,1]$, if $s_t \in (0, 1/2)$, then $s_{t+1} \in (0, 1/2)$.

Within the intergenerational context, before analysing the general EOp problem, we will consider the special case where only status (but not date of birth) is considered a circumstance.

[7] For the derivation see the Addendum.

3 Equality of opportunity

3.1 Status as the only circumstance

When status is the only circumstance, the EOp tax rate maximises the welfare of the worst off in each period. Call this, the *myopic* EOp (MEOp) programme. Let $\boldsymbol{\tau} = \{\tau_t\}_{t=1}^{\infty}$ with $\tau_t \in [0,1]$ all t be an infinite sequence of tax rates. Noting that $v^p(s_t, \tau_t) \leq v^r(s_t, \tau_t)$ for all $s_t \in (0, 1/2)$ and $\tau_t \in [0,1]$, the MEOp can be written as follows:

$$\max_{\tau_t} v^p(s_t, \tau_t), \tag{MEOp}$$

subject to (19) for all $t \geq 1$ and given s_1.

Let $\boldsymbol{\tau}^* = \{\tau_t^*\}_{t=1}^{\infty}$ denote the solution to MEOp. A solution $\boldsymbol{\tau}^*$ to MEOp is called *stationary* if $s_t = s_1$ for all t. Lemma 1 describes the relation between s_{t+1} and s_t at the solution to MEOp.[8]

Lemma 1. *At the solution to MEOp, s_{t+1} is an increasing and strictly convex function of s_t.*

Proposition 1 characterises stationary solutions to MEOp.

Proposition 1. *Let $s_1 \in \left(0, \frac{1}{2}\right)$. Let $s^*(\alpha) = \frac{1}{2}\left[3 - 2\alpha - \sqrt{4(\alpha-1)^2 + 1}\,\right]$.*

(i) There exists a unique stationary solution to MEOp with $s_t = s^*(\alpha) \in (0, \frac{1}{2})$.

(ii) For $s_1 \neq s^*(\alpha)$, at the solution to MEOp, s_t converges to $s^*(\alpha)$.

Proof

Equation (41) is equivalent to

$$s_{t+1} - s_t = \frac{1}{2}\left[1 - 2s_t + \frac{2s_t - 1}{3 + 2s_t(s_t - 3) + 2\alpha(2s_t - 1)}\right]. \tag{20}$$

(i) At the stationary solution to MEOp $s_{t+1} = s_t$. Then from (20) we get $s_{t+1} = s_t$ for $s_t = \frac{1}{2}$, or for $s_t \in (0, \frac{1}{2})$, the solutions of (20) are the same as the solutions of

$$s^2 + s(2\alpha - 3) + 1 - \alpha = 0, \tag{21}$$

[8] The proofs of all Lemmas are in the Appendix.

and these are

$$s^*(\alpha) = \frac{1}{2}\left[3 - 2\alpha \pm \sqrt{4(1-\alpha)^2 + 1}\,\right]. \tag{22}$$

$\alpha < 1$, $\sqrt{4(1-\alpha)^2+1} > 1$, hence

$$\frac{1}{2}(3 - 2\alpha + \sqrt{4(1-\alpha)^2 + 1}) > 2 - \alpha > 1,$$

which is not possible. This means that the only possible solution is

$$s^*(\alpha) = \frac{1}{2}\left[3 - 2\alpha - \sqrt{4(1-\alpha)^2 + 1}\,\right] \geq 0. \tag{23}$$

The fact that $s^*(\alpha) \in (0, \frac{1}{2})$, derives from (23) and $\alpha \in \left(\frac{1}{2}, 1\right)$

(ii) Let $F(s_t) = s_{t+1} - s_t$. Note that $F(s_t) < 0$ for all $s_t \in (s^*(\alpha), \frac{1}{2})$ and $F(s_t) > 0$ for $s_t \in (0, s^*(\alpha))$ which establishes the remaining result.

□

By Proposition 1, the myopic EOp policy will never lead to equality of income, not even in the limit. This is puzzling since agents are identical in all respects, except their date of birth. Worse still, if inequality is sufficiently low ($s_1 > (s^*(\alpha))$, the myopic MEOp policy requires it to *grow* over time.

If we consider a special case, where status captures educational opportunities, then our result is quite similar to Roemer and Ünveren (2016) where inequality is persistent when education depends only on parental status. What is new, is that in the present model, the relative population shares are important for the level of long run inequality which may rise over time. Indeed, at the solution to MEOp, long run inequality $s^*(\alpha)$ increases with α: a higher fraction of poor agents leads to a higher level of long run inequality.

The graphs below shows the evolution of inequality captured by status, for $\alpha = 0.9$. The vertical axis is s_{t+1}, while the horizontal is s_t. As α increases, $s^*(0.99)$ decreases: so $s^*(0.9) \approx 0.09001$ but $s^*(0.99) \approx 0.0099$ and $s^*(0.999) \approx 0.00099$.

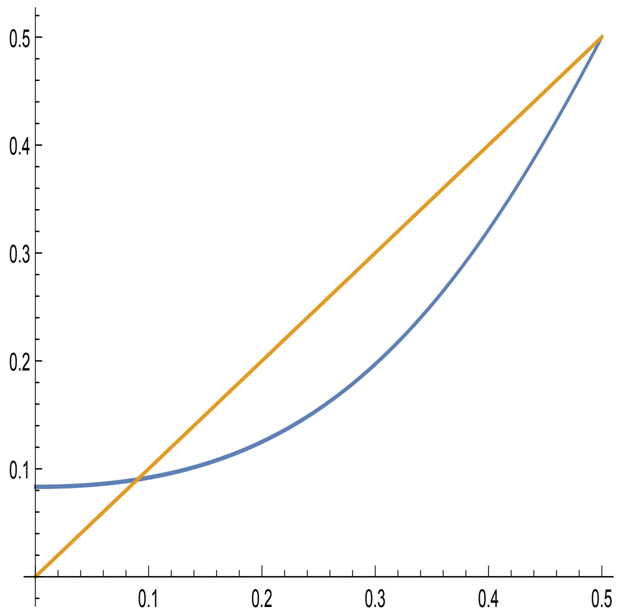


Figure 1: $s^*(0.9) \approx 0.09001$

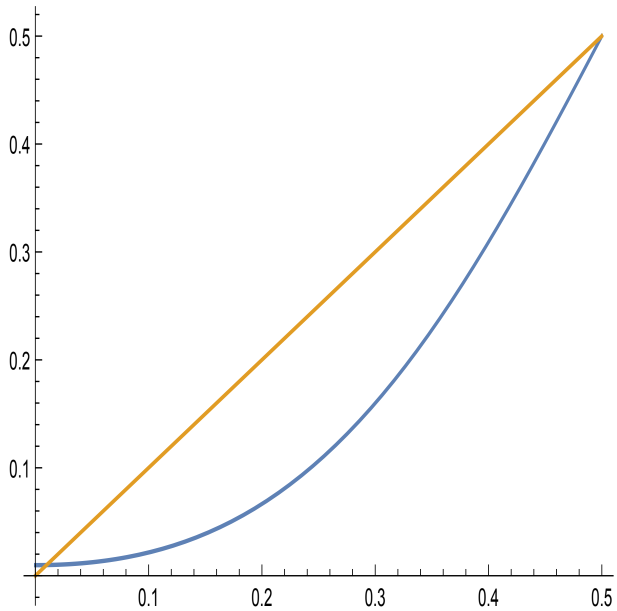


Figure 2: $s^*(0.99) \approx 0.0099$

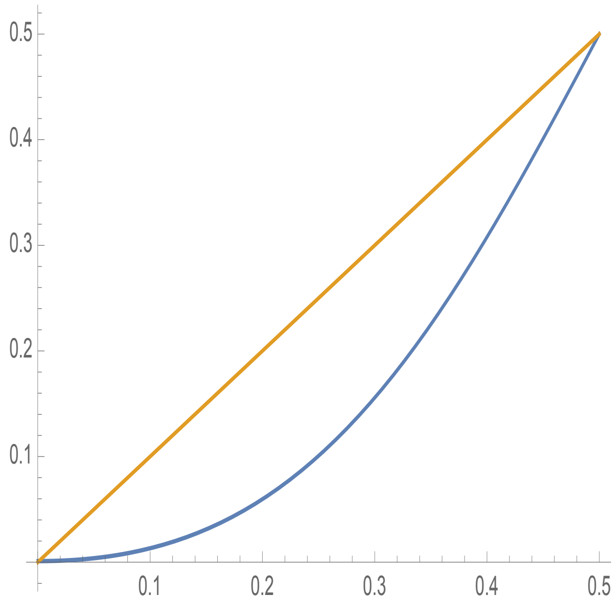


Figure 3: $s^*(0.999) \approx 0.00099$

As shown in Proposition 1 the population shares of rich and poor individuals affect the relative importance of the redistribution channel of taxation compared to the incentive one. A high proportion of poor agents means that *ceteris paribus* they will contribute more to output, which in turn will increase the importance of the incentive channel of taxation and lead to lower taxes and higher inequality (captured by s). Inequality also has welfare implications.

Substituting (19) into (14) we obtain the indirect utility of the poor at the solution to MEOp:

$$v^p(s_t) = \frac{[s_t^2 + (1-\alpha)(1-2s_t)]^2}{2s_t^2 + 4(1-\alpha)(1-2s_t)}. \tag{24}$$

Then we can derive a property of the indirect utility function of the poor.

Lemma 2. *For* $s_t \in \left[s^*(\alpha), \frac{1}{2}\right)$, $\frac{\partial v^p(s_t;\tau_t^*)}{\partial s_t} > 0$.

Proposition 2. *There exist an* $\bar{s}(\alpha)$ *and an* $\bar{\alpha} \in \left(\frac{1}{2}, 1\right)$ *such that*

(i) for any given $\alpha \in \left(\frac{1}{2}, 1\right)$ $\frac{\partial v^p(s_t;\tau_t^*)}{\partial s_t} < 0$, *for* $\alpha < \bar{\alpha}$ *and* $s_t < \bar{s}(\alpha)$, *and* $\frac{\partial v^p(s_t;\tau_t^*)}{\partial s_t} > 0$, *otherwise,*

(ii) $s^*(\alpha) > \bar{s}(\alpha)$ *for all* $\alpha \in \left(\frac{1}{2}, 1\right)$.

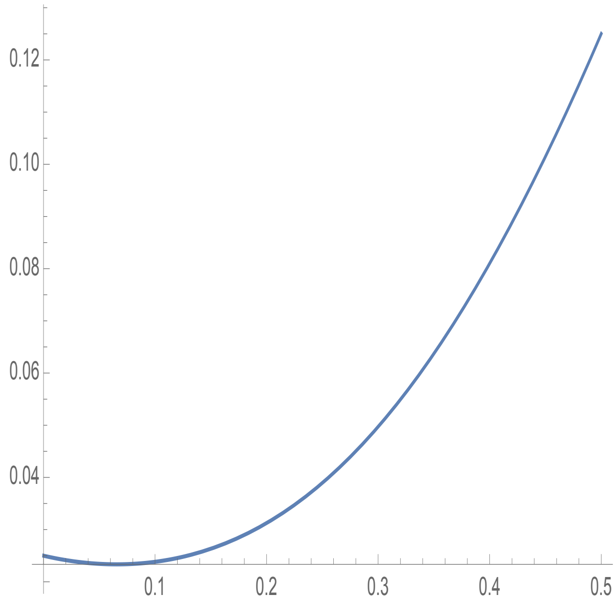
Proof

From Lemma 2 we know that for $s_t \in \left[s^*(\alpha), \frac{1}{2}\right)$, $\frac{\partial v^p(s_t;\tau_t^*)}{\partial s_t} > 0$. From the proof we know that (i) A_2 is increasing in $s_t \in \left(0, \frac{1}{2}\right)$, (ii) also for $s_t = 0$, $A_2 < 0$ and (iii) for $s_t = s^*(\alpha)$, $A_2 > 0$. Therefore (i), (ii), (iii) and continuity imply that there exists $\bar{s}(\alpha)$, such that for $s_t = \bar{s}(\alpha)$, $A_2 = 0$.

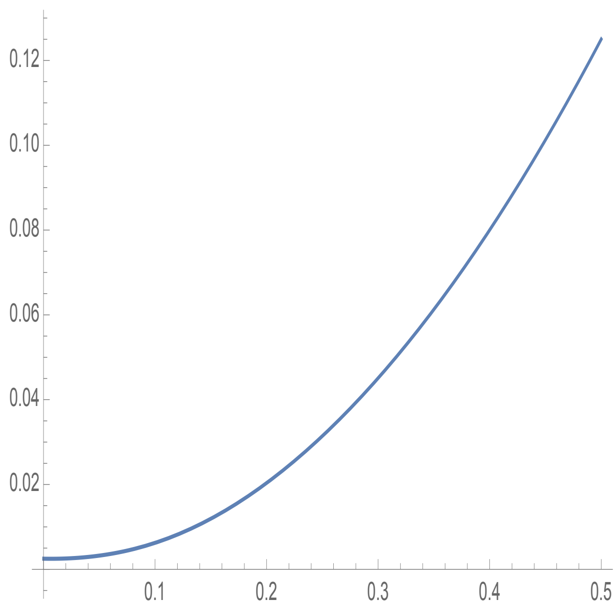
□

The following first three figures show the welfare of the poor at the EOp tax rate (vertical axis) as a function of s_t (horizontal axis) for given values of α; while the fourth one shows the values for which $\frac{\partial v^p(s_t;\tau_t^*)}{\partial s_t} < 0$ as a function of α and s_t. At the fourth graph, $\bar{s}(\alpha)$ is the intersection of the blue surface where $\frac{\partial v^p(s_t;\tau_t^*)}{\partial s_t} = 0$; and orange surface which .

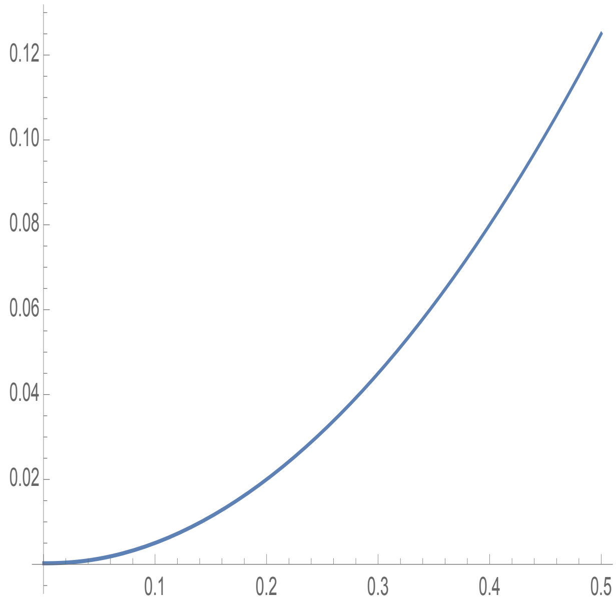
- $\alpha = 0.9$.



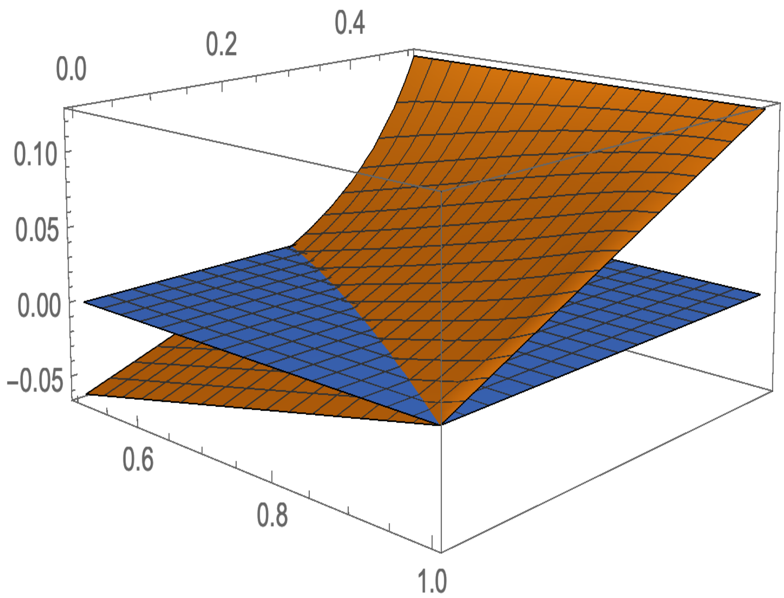
- $\alpha = 0.99$.



- $\alpha = 0.999$.



- $\frac{\partial v^p(s_t;\tau_t^*)}{\partial s_t}$.



Proposition 2 shows that if there are few rich people and the status of the poor is low, then any tax on them is spread over high number of poor agents [...] and a decrease in inequality will lead to a decrease in welfare of the poor at the solution to MEOp. Taxation affects the welfare of individuals via two channels. On the one hand, high taxation means high redistribution, while on the other, high taxation has a negative effect on individual incentives to exert effort. For simplicity call the first channel the *redistribution channel* and the second one the *incentive channel*. The relative strength of these two channels, depends on the population shares of the two classes. For a given level of status inequality, a high (low) proportion of poor individuals means that the benefits of the redistribution channel are relatively low (high) while the effects through the incentive channel are relatively high (low).

Together Propositions 1 and 2 suggest that at the solution to MEOp the welfare of the poor will likely decline over time. If α is relatively high, this decline will be due to the combination of an increase in inequality, captured by decreasing s_t (down to a very low $s^*(\alpha)$), will lead to a reduction in welfare. If α is relatively low (lower than $\bar{\alpha}$), and initial inequality is relatively high ($s_1 < \bar{s}(\alpha)$), at the solution to MEOp, the welfare of the poor will be decreasing until a certain level of inequality ($\bar{s}(\alpha)$) and then will increase as status will be approaching the stationary value $s^*(\alpha)$.

3.2 Date of birth and status as circumstances

Previously, we considered only social class as a circumstance and not date of birth. Given that date of birth *is* a circumstance, the MEOp is extended to an *Intergenerational EOp* (IEOp) programme:

$$\sup_{\boldsymbol{\tau}} \inf_{t} v_t^p, \tag{IEOp}$$

subject to (19) given s_1.

Theorem 1. *For $s_1 \in (\bar{s}(\alpha), s^*(\alpha))$ and $\alpha < \bar{\alpha}$ or for $s_1 \in (0, s^*(\alpha))$ and $\alpha > \bar{\alpha}$, the solution of MEOp is also the solution to IEOp.*

Proof

We know from Propositions 1 and 2, that for $s_1 \in (\bar{s}(\alpha), s^*(\alpha))$ at the solution to MEOp, $s_{t+1} > s_t$. Thus

$$v^p(s_1, \tau_1^*) < v^p(s_2, \tau_2^*) < \ldots < v^p(s^*(\alpha)).$$

Hence, given that the sequence of indirect utilities is increasing, the solution of MEOp is also a solution of IEOp.

□

Proposition 2 states that for sufficient high values of α and/ or s_1, Theorem 1, holds for any $s_1 \in (0, s^*(\alpha))$.

As we have shown, there exists a trade off between redistribution and efficiency, given by the two channels through which the tax rate affects income and welfare. Let $\tilde{\tau}_t(s_t)$ be the tax rate such that $s_{t+1} = s_t$. By equation (19), this is then given by

$$\tilde{\tau}_t(s_t) = \frac{s_t - s_t^2}{1 - s_t - \alpha(1 - 2s_t)}. \tag{25}$$

Note that $\tilde{\tau}_t(s_t) \in [0, 1]$ for all $s_t \in \left(s^*(\alpha), \frac{1}{2}\right)$.[9]

Lemma 3. *Let* $s_t \in \left(s^*(\alpha), \frac{1}{2}\right)$. *Then (i)* $\tilde{\tau}_t > \tau_t^*$ *and (ii)* $\frac{\partial v^p(s_t, \tilde{\tau}_t)}{\partial s_t} > 0$.

This result states that for any level of inequality lower than the level captured by $s^*(\alpha)$, there is a trade off between maximising the welfare of the least well off in a given generation; and keeping at least the same levels of inequality (or status) for the next.

This Lemma is quite intuitive as it states that for any level of income inequality such that s_t is lower than $s^*(\alpha)$, the level of welfare associated with keeping inequality constant, increases with status. We can now state the main result.

Theorem 2. *Let* $s_1 \in \left[s^*(\alpha), \frac{1}{2}\right)$. *At the solution to IEOp,* $\tau_t = \tilde{\tau}_t$ *for all* t.

Fact 1. By the concavity of $v^p(s_t; \tilde{\tau}_t)$ in τ, $\tilde{\tau}_t > \tau^*$ implies $\frac{\partial v^p(s_t;\tilde{\tau}_t)}{\partial \tau_t} < 0$.
Fact 2. Let $s_{t+1} = f(s_t, \tau_t)$. For all $\tau \in (0, 1)$ and all $s_t \in (0, 1)$, $\frac{\partial f(s_t,\tau_t)}{\partial \tau_t} > 0$.
Fact 3. Under appropriate conditions on α and s_1, $\frac{\partial v^p(s_t;\tilde{\tau}_t)}{\partial s_t} > 0$.

[9]For a formal proof of this statement see *constant inequality tax rate* in the Addendum.

Approach 1

Proof

Consider the following optimisation programme

$$\max_{\tau} v^p(s_1, \tau_1), \tag{26}$$

subject to

$$v^p(s_1, \tau_1) - v^p(f(s_t, \tau_t), \tau_{t+1}) \leq 0, \text{all } t > 1. \tag{27}$$

The Langrangean is

$$L(\tau, \lambda) = v^p(s_1, \tau_1) + \sum_{t=1}^{\infty} \lambda_t [v^p(f(s_t, \tau_t), \tau_{t+1}) - v^p(s_1, \tau_1)]. \tag{28}$$

The first order conditions are:[10]

$$\frac{\partial L}{\partial \tau_1} = \frac{\partial v(s_1, \tau_1)}{\partial \tau_1} - \sum_{t=1}^{\infty} \lambda_t \frac{\partial v(s_1, \tau_1)}{\partial \tau_1} + \lambda_1 \frac{\partial v(s_2, \tau_2)}{\partial s_2} \frac{\partial f(s_1, \tau_1)}{\partial \tau_1} \leq 0 \tag{29}$$

$$\frac{\partial L}{\partial \tau_t} = \lambda_t \frac{\partial v(s_t, \tau_t)}{\partial \tau_t} + \lambda_{t+1} \frac{\partial v(s_{t+1}, \tau_{t+1})}{\partial s_{t+1}} \frac{\partial f(s_t, \tau_t)}{\partial \tau_t} \leq 0, \text{all } t > 1 \tag{30}$$

$$\tau_t \frac{\partial L}{\partial \tau_t} = 0, \text{all } t \geq 1 \tag{31}$$

$$v^p(s_1, \tau_1) - v^p(f(s_t, \tau_t), \tau_{t+1}) \leq 0, \lambda_t \geq 0, \lambda_t \left[v^p(s_1, \tau_1) - v^p(f(s_t, \tau_t), \tau_{t+1})\right] = 0. \tag{32}$$

If $\tau_t = \tilde{\tau}_t$ and $s_t = s_1$ all t then

$$\frac{\partial v(s_t, \tau_t)}{\partial \tau_t} = k_1, \frac{\partial v(s_{t+1}, \tau_{t+1})}{\partial s_{t+1}} = k_2, \text{and } \frac{\partial f(s_t, \tau_t)}{\partial \tau_t} = k_3, \text{all } t > 1. \tag{33}$$

Because $\tau_t > 0$ all t, it follows that 37 and 30 must hold as equalities. Then 30 can be written as

$$\lambda_t k_1 + \lambda_{t+1} k_2 k_3 = 0, \text{all } t > 1 \tag{34}$$

[10]Observe that under the conditions of Theorem 1 the first order conditions are satisfied with $\lambda_t = 0$ all t.

or, equivalently,letting $K = -\frac{k_1}{k_2 k_3} > 0$,

$$\lambda_{t+1} = K^{t-1}\lambda_1, \text{all } t > 1. \tag{35}$$

Plugging this into 37, and noting that $\frac{\frac{\partial v(s_2,\tau_2)}{\partial s_2}\frac{\partial f(s_1,\tau_1)}{\partial \tau_1}}{\frac{\partial v(s_1,\tau_1)}{\partial \tau_1}} = -\frac{1}{K}$ we obtain

$$1 - \sum_{t=1}^{\infty} K^{t-1}\lambda_1 - \lambda_1 \frac{1}{K} = 0. \tag{36}$$

If $K < 1$ then the latter expression becomes

$$1 - \frac{1}{1-K}\lambda_1 - \lambda_1 \frac{1}{K} = 0, \tag{37}$$

which gives $\lambda = K(1-K)$. In other words, if $K < 1$ then there exists an infinite sequence of strictly positive multipliers λ_t such that the first order conditions are satisfied when $\tau_t = \tilde{\tau}_t$ and $s_t = s_1$ all t.

□

Approach 2

Proof

Suppose, by way of contradiction, that the path with $\tau_t = \tilde{\tau}_1$, $s_t = s_1$ and $v^p(s_t; \tilde{\tau}_t) = v^p(s_1; \tilde{\tau}_1)$ all t does not solve IEOp. This implies that there exists another path with $v^p(s_t; \tilde{\tau}_t) > v^p(s_1; \tilde{\tau}_1) + \epsilon$ all $t \geq > 1$ for some $\epsilon > 0$.

Therefore consider a perturbation of the path with $\tau_t = \tilde{\tau}_1$, $s_t = s_1$ and $v^p(s_t; \tilde{\tau}_t) = v^p(s_1; \tilde{\tau}_1)$ all t. In the first period, given $\frac{\partial v^p(s_t;\tilde{\tau}_t)}{\partial \tau_t} < 0$ we have

$$dv^p(s_1; \tilde{\tau}_1) = \frac{\partial v^p(s_1; \tilde{\tau}_1)}{\partial \tau_1} d\tau_1 > 0 \Leftrightarrow d\tau_1 < 0. \tag{38}$$

Then, for all $t > 1$ we have

$$dv^p(s_t; \tilde{\tau}_t) = \frac{\partial v^p(s_1; \tilde{\tau}_1)}{\partial s_t} \frac{\partial f(s_1; \tilde{\tau}_1)}{\partial \tau_{t-1}} d\tau_{t-1} + \frac{\partial v^p(s_1; \tilde{\tau}_1)}{\partial \tau_t} d\tau_t. \tag{39}$$

Therefore, at $t = 2$, we have

$$dv^p(s_t;\tilde{\tau}_t) > 0 \Leftrightarrow \frac{d\tau_2}{d\tau_1} > -\frac{\frac{\partial v^p(s_1;\tilde{\tau}_1)}{\partial s_2}\frac{\partial f(s_1;\tilde{\tau}_1)}{\partial \tau_1}}{\frac{\partial v^p(s_1;\tilde{\tau}_1)}{\partial \tau_2}} > 0. \qquad (40)$$

Noting that $d\tau_1 < 0$ it follows that $d\tau_2 < 0$. Furthermore, if $-\frac{\frac{\partial v^p(s_1;\tilde{\tau}_1)}{\partial s_2}\frac{\partial f(s_1;\tilde{\tau}_1)}{\partial \tau_1}}{\frac{\partial v^p(s_1;\tilde{\tau}_1)}{\partial \tau_2}} > 1$ then $|d\tau_2| > |d\tau_1|$. Iterating over t, $d\tau_t < 0$ implies $d\tau_t + 1 < 0$ and $|d\tau_{t+1}| > |d\tau_t|$ all t. This implies that the sequence of perturbations violates the nonnegativity constraint on τ after a finite number of periods and thus is not feasible.

□

Theorem 2 shows that at the solution to IEOp, both the welfare of the poor and inequality stay constant over time, if initial inequality is *lower* than the threshold $s^*(\alpha)$.

4 Conclusion

The aim of this paper has been to provide an answer to whether increasing income inequality can be considered as fair, according to the most mainstream theory of distributive justice, namely *Equality of Opportunity*. Building on the existing literature on EOp, status and inequality, we analysed the implications of intergenerational EOp policies, where the extent of inequality between social classes in a given generation defined the circumstances of the next. In this context the EOp policy should take into account both *inter*- and *intra*-generational inequalities. We have shown that in an economy where inequality of one generation affects the marginal returns of effort of the next one, the IEOp policy cannot increase inequality.

Appendix

Proof of Lemma 1

Substituting (16) into (19), at the solution of MEOp,

$$s_{t+1} = \frac{(1-s_t)^2 - \alpha(1-2s_t)}{2s_t^2 + (3-2\alpha)(1-2s_t)}. \tag{41}$$

Then

$$\frac{\partial s_{t+1}}{\partial s_t} = \frac{2s_t(1-s_t)}{(3-2\alpha-6s_t+4\alpha s_t+2s_t^2)^2} > 0, \tag{42}$$

and

$$\frac{\partial^2 s_{t+1}}{\partial s_t^2} = -\frac{2(-3+2\alpha+6s_t^2-4s_t^3)}{[(1-2s_t)(3-2\alpha)+2s_t^2]^3}. \tag{43}$$

Given that the denominator in the RHS of (43) is positive, for $\frac{\partial^2 s_{t+1}}{\partial s_t^2} > 0$, it is sufficient to show that $-3+2\alpha+6s_t^2-4s_t^3 < 0$, or equivalently that

$$6s_t^2 - 4s_t^3 < 1 + 2(1-\alpha). \tag{44}$$

Note that the LHS of (44) is an increasing function of s_t, hence

$$6s_t^2 - 4s_t^3 < \frac{6}{4} - \frac{4}{8} = 1 < 1 + 2(1-\alpha). \tag{45}$$

□

Proof of Lemma 2

Differentiating (24) we obtain

$$\frac{dv^p(s_t)}{ds_t} = \frac{A_1 A_2}{[2s_t^2 + 4(1-\alpha)(1-2s_t)]^2}, \tag{46}$$

where

$$A_1 = 1 - \alpha - 2s_t + 2\alpha s_t + s_t^2$$

and

$$A_2 = -2 + 4\alpha - 2\alpha^2 + 7s_t - 11\alpha s_t + 4\alpha^2 s_t - 6s_t^2 + 6\alpha s_t^2 + s_t^3.$$

$$\frac{dA_1}{ds_t} = -2 + 2\alpha + 2s_t,$$

Which means that $\frac{dA_1}{ds_t} = 0$ for $s_t = 1 - \alpha$. Note that $\frac{d^2 A_1}{ds_t^2} > 0$, therefore A_1 is minimised for $s_t = 1 - \alpha$. Then we get that the minimum value of $A_1(s_t)$ over $(0, \frac{1}{2})$ is

$$A_{1_{min}} = \alpha(1 - \alpha) > 0.$$

Hence, it is sufficient to prove that also $A_2 > 0$ for all $s_t \in \left[s^*(\alpha), \frac{1}{2}\right)$. In order to prove this we will show that (i) $\frac{dA_2}{ds_t} = 7 - 11\alpha + 4\alpha^2 - 12s_t + 12\alpha s_t + 3s_t^2 > 0$ for all $\alpha \in \left(\frac{1}{2}, 1\right)$ and that (ii) for $s_t = s^*(\alpha)$, $A_2 > 0$

(i) Let $F(\alpha, s_t) := 7 - 11\alpha + 4\alpha^2 - 12s_t + 12\alpha s_t + 3s_t^2$. Note that $\frac{\partial F}{\partial \alpha} = 0$ gives

$$-11 + 8\alpha + 12s_t = 0, \tag{47}$$

and that $\frac{\partial F}{\partial s_t} = 0$ gives

$$-12 + 12\alpha + 6s_t = 0. \tag{48}$$

From (47) and (48) we get $\alpha = \frac{88}{100}$ and $s = \frac{3}{8}$. The second partial derivatives are

$$\frac{\partial^2 F}{\partial \alpha^2} = 8,$$
$$\frac{\partial^2 F}{\partial s_t^2} = 6,$$
$$\frac{\partial^2 F}{\partial s_t \partial \alpha} = 12.$$

Hence, the determinant of the Hessian of $F(\alpha, s_t)$ is strictly negative and the point (88/100, 3/8) is a non degenerate saddle point. Note that $F\left(\frac{88}{100}, \frac{3}{8}\right) > 0$, therefore in order to prove that $F(\alpha, s_t) > 0$ for all $\alpha \in \left(\frac{1}{2}, 1\right)$ and $s_t \in \left(s^*(\alpha), \frac{1}{2}\right)$ it is sufficient to show that the three dimensional graph of $F(\alpha, s_t)$ takes non negative values at the edges of the parallelogram of α, s_t. The following equations represent these edges:

$$F(1, s_t) = 7 - 11 + 4 - 12s_t + 12s_t + 3s_t^2 \geq 0$$

$$F(1/2, s_t) = 7 - 5.5 + 1 - 6s_t + 3s_t^2 = 2.5 - 6s_t + 3s_t^2$$

Note that $F(\frac{1}{2}, s_t)$ is decreasing in s_t over $(0, \frac{1}{2})$ thus it is minimised for $s_t = 1/2$, but $F(1/2, 1/2) = -0.5 + 0.75 > 0$.

$$F(\alpha, 0) = 7 - 11\alpha + 4\alpha^2 = (1-\alpha)[4 + 4(1-\alpha)] > 0$$

$$F(\alpha, 1/2) = 7/4 - 5\alpha + 4\alpha^2$$

$F(\alpha, \frac{1}{2})$ is a parabola and reaches a minimum for $\alpha = 5/8$, for which $F(5/8, 1/2) > 0$.

(ii) If $s_t = s^*(\alpha)$,

$$A_2 = \frac{1}{2}(2\alpha - 1)\left[7 + 4\alpha^2 - 10\alpha + (2\alpha - 3)\sqrt{5 - 8\alpha + 4\alpha^2}\right]$$

$A_2 > 0$ if and only if

$$7 + 4\alpha^2 - 10\alpha > (3 - 2\alpha)\sqrt{5 - 8\alpha + 4\alpha^2}$$

or

$$49 - 140\alpha + 156\alpha^2 - 80\alpha^3 + 16\alpha^4 > 45 - 132\alpha + 152\alpha^2 - 80\alpha^3 + 16\alpha^4$$

which simplifies to

$$4(\alpha - 1)^2 > 0$$

which is always true.

Since claims (i) and (ii) are true, then the result has been proven.

□

Proof of Lemma 3

Part (i). From (16) and (25)

$$\tilde{\tau}_t(s_t) - \tau_t^*(s_t) = \frac{N(s_t)}{D(s_t)},$$

where

$$N(s_t) = -1+2\alpha-\alpha^2+5s_t-9\alpha s_t+4\alpha^2 s_t-8s_t^2+12\alpha s_t^2-4\alpha^2 s_t^2+5s_t^3-4\alpha s_t^3-s_t^4,$$

and

$$D(s_t) = (1-\alpha-s_t+2\alpha s_t)(2-2\alpha-4s_t+4\alpha s_t+s_t^2),$$

We need to show that $\frac{N(s_t)}{D(s_t)} > 0$. To this aim, first, note that

$$1-\alpha-s_t+2\alpha s_t = 1-\alpha+s_t(2\alpha-1) > 0,$$

and

$$2-2\alpha-4s_t+4\alpha s_t+s_t^2 = 2(1-\alpha)-4s_t(1-\alpha)+s_t^2 = 2(1-\alpha)(1-2s_t)+s_t^2 > 0.$$

Thus $D(s_t) > 0$ for all $s_t \in (0, \frac{1}{2})$ and $\alpha \in (\frac{1}{2}, 1)$. Hence we need to show that $N(s_t) > 0$. $N(s_t)$ can be expressed as

$$N(s_t) = (1-2s_t)^2(s_t-1+\alpha) + \frac{s_t^3(1-s_t)}{1-\alpha}.$$

Let $N_1 = (1-2s_t)^2(s_t-1+\alpha)$ and $N_2 = \frac{s_t^3(1-s_t)}{1-\alpha}$. Note that $N_2 > 0$ and

$$\frac{\partial N_2}{\partial s_t} = \frac{s_t^2(3-4s_t)}{1-\alpha} > 0,$$

while $N_1 < 0$ for $s^*(\alpha) < s_t < 1-\alpha$ and $N_1 \geq 0$ for $1-\alpha \leq s_t < \frac{1}{2}$. Hence $N(s_t) > 0$ for $1-\alpha \leq s_t < \frac{1}{2}$; so we need to prove that this is also the case for $s^*(\alpha) \leq s_t < 1-\alpha$. Note that

$$\frac{\partial N_1}{\partial s_t} = (1-2s_t)(5-6s_t-4\alpha),$$

which means that N_1 is increasing for all $s_t < \frac{5-4\alpha}{6}$ and decreasing for $\frac{5-4\alpha}{6} < s_t < \frac{1}{2}$. This then means that $\frac{\partial N(s_t)}{\partial s_t} > 0$ for $s^*(\alpha) < s_t < \frac{5-4\alpha}{6}$. Given that $\frac{5-4\alpha}{6} > 1-\alpha$. In order to conclude the proof it is sufficient to show that the minimum is $N(s^*(\alpha)) \geq 0$ which is true as $N(s^*(\alpha)) = 0$

Part (ii). For any $\tau_t \in (0,1)$,

$$\frac{\partial v^p(s_t,\tau_t)}{\partial s_t} = s_t(1-\tau_t)^2 + \tau_t(1-\tau_t)[2\alpha s_t - 2(1-\alpha)(1-s_t)] > 0,$$

if and only if $s_t(1-\tau_t) + \tau_t[2\alpha s_t - 2(1-\alpha)(1-s_t)] > 0$.

This is equivalent to

$$\tau_t(2 - s_t - 2\alpha) < s_t. \tag{49}$$

Here we can distinguish two cases: (i) $2 - s_t - 2\alpha < 0$; and (ii) $2 - s_t - 2\alpha > 0$.

(i) If $2 - s_t - 2\alpha < 0$, then (49) holds for any $\tau_t > 0$.

(ii) If $2 - s_t - 2\alpha > 0$, then (49) is equivalent to

$$\tau_t < \frac{s_t}{2 - s_t - 2\alpha}.$$

Thus $\frac{\partial v^p(s_t,\tilde{\tau}_t)}{\partial s_t} > 0$ if and only if

$$\tilde{\tau}_t = \frac{s_t - s_t^2}{1 - s_t - \alpha(1-2s_t)} < \frac{s}{2 - s - 2\alpha},$$

or after some algebra,

$$(1-s_t)^2 < \alpha,$$

which is equivalent to $s_t > 1 - \sqrt{\alpha}$. In order to conclude the proof it is sufficient to prove that

$$s^*(\alpha) = \frac{1}{2}(3 - 2\alpha - \sqrt{4(1-\alpha)^2 + 1}) > 1 - \sqrt{\alpha},$$

or, after some algebra

$$1 - 2\alpha > \sqrt{4(1-\alpha)^2 + 1} - 2\sqrt{\alpha}. \tag{50}$$

The left hand side of (50) is clearly negative. The right hand side is negative if

$$4(1-\alpha)^2 + 1 < 4\alpha,$$

or equivalently if

$$5 + 4\alpha^2 - 12\alpha < 0,$$

which is true for any $\alpha \in (\frac{1}{2}, 1)$. Thus (50) can be expressed as

$$(1 - 2\alpha)^2 < \left(\sqrt{4(1-\alpha)^2 + 1} - 2\sqrt{\alpha}\right)^2,$$

or after some calculations

$$-4 < -4\sqrt{\alpha + 4\alpha(1-\alpha)^2},$$

which is equivalent to

$$4\alpha(1-\alpha)^2 < 1 - \alpha,$$

or

$$4\alpha(1-\alpha) < 1,$$

which means

$$4\alpha^2 - 4\alpha + 1 > 0,$$

which is true for any $\alpha \in \left(\frac{1}{2}, 1\right)$.

□

Addendum

Proof that τ^* is decreasing in α and s.

The derivative with respect to α is:

$$\frac{\partial \tau}{\partial \alpha} = \frac{-(1-2s)[s^2 + 2(1-\alpha)(1-2s)] + 2(1-2s)(1-\alpha)(1-2s)}{[s^2 + 2(1-\alpha)(1-2s)]^2}, \tag{51}$$

which simplifies to

$$\frac{\partial \tau}{\partial \alpha} = \frac{-(1-2s)s^2}{[s^2 + 2(1-\alpha)(1-2s)]^2} < 0.$$

Also, the derivative with respect to s is:

$$\frac{\partial \tau}{\partial s} = \frac{-2(1-\alpha)[s^2 + 2(1-\alpha)(1-2s)] - [2s - 4(1-\alpha)](1-\alpha)(1-2s)}{[s^2 + 2(1-\alpha)(1-2s)]^2}, \tag{52}$$

or

$$\frac{\partial \tau}{\partial s} = \frac{-2s(1-s)(1-\alpha)}{[s^2 + 2(1-\alpha)(1-2s)]^2} < 0.$$

□

Status at optimal effort level

From equations (13), (12) and (18), the status at $t+1$ is

$$s_{t+1} = \frac{(1-\tau_t)^2 s_t^2 + \tau_t(1-\tau_t)[\alpha s_t^2 + (1-\alpha)(1-s_t)^2]}{(1-\tau_t)^2[(1-s_t)^2 + s_t^2] + 2\tau_t(1-\tau_t)[\alpha s_t^2 + (1-\alpha)(1-s_t)^2]},$$

or

$$s_{t+1} = \frac{(1-\tau_t)s_t^2 + \tau_t[\alpha s_t^2 + (1-\alpha)(1-s_t)^2]}{(1-\tau_t)(1-2s_t+2s_t^2) + 2\tau_t[\alpha s_t^2 + (1-\alpha)(1-2s_t+s_t^2)]},$$

or

$$s_{t+1} = \frac{s_t^2 - \tau_t s_t^2 + \tau_t \alpha s_t^2 + \tau_t(1-s_t)^2 - \tau_t\alpha(1-s_t)^2}{(1-\tau_t)(1-2s_t+2s_t^2) + 2\tau_t[\alpha s_t^2 + 1 - 2s_t + s_t^2 - \alpha + 2\alpha s_t - \alpha s_t^2]},$$

or

$$s_{t+1} = \frac{s_t^2 - \tau_t s_t^2 + \tau_t \alpha s_t^2 + \tau_t(1-s_t)^2 - \tau_t \alpha(1-s_t)^2}{(1-\tau_t)(1-2s_t+2s_t^2) + 2\tau_t[1-2s_t+s_t^2-\alpha+2\alpha s_t]},$$

or

$$s_{t+1} = \frac{s_t^2 + \tau_t(1-\alpha)(1-2s_t)}{1-2s_t+2s_t^2+\tau_t(1-2s_t)(1-2\alpha)}.$$

□

Constant inequality tax rate

We need to show that $\tilde{\tau}_t(s_t) = \frac{s_t - s_t^2}{1-s_t-\alpha(1-2s_t)} \in [0,1]$. Note that $s_t - s_t^2 < 1$ given that $s_t \in \left(0, \frac{1}{2}\right)$ and also

$$1 - s_t - \alpha(1-2s_t) = 1 - \alpha + s_t(2\alpha - 1) > 0,$$

due to $\alpha > \frac{1}{2}$. Hence $\tilde{\tau}_t(s_t) > 0$. For $\tilde{\tau}_t(s_t) \leq 1$, it is sufficient to show that

$$s_t - s_t^2 < 1 - s_t - \alpha(1-2s_t),$$

or,

$$-s_t^2 < (1-2s_t)(1-\alpha),$$

which is true for all $s_t \in \left(0, \frac{1}{2}\right)$.

□

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