Welfare Analysis under Probabilistic Choices in a Rational Expectations Equilibrium Model

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Abstract

We introduce information acquisition uncertainty into an otherwise standard CARA-normal rational expectations equilibrium model with asymmetric information, and analyze the channel through which more informed trading can lead to an overall welfare improvement in the economy. Traders make strategic probabilistic choices about observing a costly private signal of the fundamental value of the risky asset. More informed trading, by resolving payoff uncertainty, makes price more informative but reduces the Sharpe ratio and distorts risk-sharing. However, due to information acquisition uncertainty, traders who become informed receive a net benefit in expected utility, which can dominate the aforementioned negative effects and improve welfare under certain market conditions.

Key words: Social welfare, rational expectations equilibrium, information acquisition uncertainty, probabilistic choices.

JEL Classification: D82, G12, G14

1. Introduction

Two types of traders typically interact in a rational expectations equilibrium (REE) model with asymmetric information, namely *informed* traders who receive a private signal about the fundamental value of the risky asset and *uninformed* traders who try to infer the private signal from the equilibrium price, which serves as a public signal (Grossman and Stiglitz, 1980; Hellwig, 1980; Diamond and Verrecchia, 1981; Admati, 1985). In the endogenous information equilibrium, the usual assumption requires the expected utilities to be identical between informed and uninformed traders, such that a trader would be indifferent to having to pay to observe the private signal or stay uninformed. Under this assumption, a trader receives *zero* net benefit in expected utility from becoming informed, since any utility gain is washed out by the information acquisition cost. As a result, informed trading is often shown to reduce welfare for *two* main reasons.

The first is the so called *Hirshleifer effect.*[1] The idea is that more informed trading brings the asset price closer to its fundamental value, thus distorting risk-sharing among traders with idiosyncratic endowment shocks, leaving traders less incentives to trade for risk-sharing and more incentives to hold on to their initial endowments. The second is the *risk-return effect* (see Allen, 1984; Kurlat and Veldkamp, 2015). Essentially, traders benefit from trading risky assets with high risk and high return; however, more informed trading resolves payoff uncertainty and turns the assets into investments with low risk and low return, which leads to a lower Sharpe ratio. As pointed out by Goldstein and Yang (2017), "*the common theme of both channels is that disclosure harms investors through destroying trading opportunities*". Consequently, in the standard (CARA-Normal type) REE models, the only Pareto-efficient equilibrium is the *no-informed-trading equilibrium*, where the access to private information is shut down.

[1]The name references Hirshleifer (1971) where the role of information in the framework of technological uncertainty is discussed. Allen (1984) is the first to acknowledge the implications of this effect for financial markets, especially in the context of exchange economies.

This paper introduces *information acquisition uncertainty* and *probabilistic choices* into an otherwise standard Grossman-Stiglitz type REE model with asymmetric information as a channel through which informed trading can be welfare-improving. We show that the utility gain by becoming informed is offset but not completely washed out by the information acquisition cost, leading to a *positive* effect on welfare, which under certain market conditions can overcome the negative risk-return and Hirshleifer effects and improve welfare.

In a standard REE model (e.g., Grossman and Stiglitz, 1980), a trader, by paying a fixed cost, can choose to become informed with certainty. However, as pointed out by Mattson and Weibull (2002), "*in most real-life situations, the decision maker cannot guarantee any desired outcome with a probability exactly equal to one*", which is most likely to be the case in financial markets with highly complex and multi-dimensional information structure.[2] For example, a trader may decide to purchase an analyst report, hoping to obtain some valuable information about the fundamental value of the firm. *Ex-post*, the analyst report could turn out to be either informative or completely useless. However, *ex-ante*, the trader expects a higher probability of becoming informed by paying more for a more valuable report. In other words, information acquisition is uncertain. That is, instead of paying to directly observe the payoff-relevant information, traders pay to increase the probability of observing the information, i.e., they make a probabilistic choice.

Our baseline model is the canonical REE model with CARA utility-maximizing traders, where the asset payoff, private signal, and asset supply are normally distributed. To introduce information acquisition uncertainty, we assume each trader chooses the *probability* $p_i \in [0, 1]$ to observe a private signal $\tilde{\theta}$ on the payoff of the

[2]The role of multi-dimensional and complex information markets has been widely discussed in recent literature. Among others, Zhang (2006) discusses information uncertainty and its role in shaping prices, and Veldkamp (2006*b*) considers different information providers with different prices/quality. Gorban, Obizhaeva and Wang (2018) assume the presence of high and low quality signals and uncertainty about the number of high quality informed agents. Breugem and Buss (2018) propose an information choice problem related to institutional investors.

risky asset by paying a variable cost $\mu\, c(p_i)$, where μ is a scalar and $c(\cdot)$ is an increasing and convex function. Thus, the higher the probability of being informed, the larger the cost the trader needs to pay.[3]

Our model can be separated in two stages. In the first stage, each trader chooses strategically a probability p_i^* to become informed. As a result, a certain (random) fraction λ of traders will become informed, meaning that this fraction of traders observes $\tilde{\theta}$, whereas a fraction $1-\lambda$ remains uninformed. In the second stage, the financial market takes place; each trader forms an optimal portfolio conditional on his information set and equilibrium price is determined by the market clearing condition. Finally, the payoff is realized and consumption occurs.

In terms of the *ex-post* welfare, after traders' types are realized, the expected utility improves for those traders who observe $\tilde{\theta}$, and worsens for those who do not. We refer to this *ex-ante* potential benefit of becoming informed as the *asymmetric-information effect*. Traders make their optimal decisions strategically in a Nash equilibrium. In equilibrium, when the asymmetric-information effect dominates the Hirshleifer and risk-return effects, the *ex-ante* welfare can potentially improve for the overall economy from the no-informed-trading equilibrium. We derive necessary and sufficient conditions under which welfare improvement occurs.

To disentangle the Hirshleifer effect from the risk-return and asymmetric-information effects, we propose two formulations of the model. In the baseline model, we turn off the Hirshleifer effect and consider a REE model with exogenous noise demand, in which traders are pure speculators who provide liquidity. In this case, we find that a lower Sharpe ratio and a less precise private signal (about the payoff information) make welfare improvement from the no-informed-trading equilibrium more likely.

The intuition is as follows. Due to the CARA-Normal structure, both the expected utility and Sharpe ratio of the uninformed trader's optimal portfolio decrease faster when the initial Sharpe ratio in the no-informed-trading equilibrium is relatively high. In other words, the risk-return effect is stronger when there are more trading opportunities at stake (measured by high Sharpe ratio in the no-informed-trading

[3]Note that the Grossman-Stiglitz model is a special case if we restrict traders' probabilistic choices to $p_i \in \{0,1\}$.

equilibrium). On the other hand, a lower signal precision means that informed trading resolves less payoff uncertainty and hence leads to a smaller utility gain, i.e., it weakens both the risk-return and asymmetric-information effects. However, we will see that the former always dominates the latter.

Further analysis shows that when an increase in the fraction of the informed traders, λ, improves welfare from the no-informed-trading equilibrium, there exists a unique Pareto-optimal state, $\lambda^* \in (0, 1)$, where traders' welfare is maximized, i.e., the relationship between welfare and λ is *hump-shaped.* Moreover, λ^* decreases with higher signal precision, which is consistent with the above intuition that a higher signal precision worsens the risk-return effect more than the enhancement of the asymmetric-information effect. Furthermore, we show that the welfare attained at λ^* is not monotonically decreasing, but maximized for some intermediate level of signal precision. Intuitively, an overly precise signal resolves too much payoff uncertainty, which is detrimental to welfare. On the other hand, due to a weak asymmetric-information effect, a signal precision close to zero also diminishes the extent to which welfare can improve.

As pointed out by Bond and Garcia (2018), welfare analysis can be compromised by the fact that demand of the noise traders is not explicitly modeled. We overcome this issue by presenting a second formulation of the model, where the noise demand is endogenized. In this setup, traders' optimal demand consists of a speculative component and a hedging component related to the presence of a trader-specific endowment shock. Under this more general setting, we are able to capture the Hirshleifer effect, in addition to the risk-return and asymmetric-information effects.

Intuitively, since informed-trading distorts risk-sharing, which matters more to traders with a relatively large endowment shock (hedgers who demand liquidity), more informed trading tends to be welfare-reducing, i.e., the Hirshleifer effect dominates. On the other hand, for those traders with relatively small endowment shocks (speculators who provide liquidity), since the Hirshleifer effect is relatively weak, informed trading is more likely to be welfare-improving. In other words, more informed trading can have different impacts on trader's expected utility due to different realizations of their endowment shocks. Different from the baseline model, there can be multiple Pareto-optimal equilibria. For example, the no-informed-trading equilibrium is Pareto-optimal since it maximizes the welfare of those hedgers with large endowment shocks. However, an asymmetric-information equilibrium can also be Pareto-optimal since it maximizes the welfare of speculators with relatively small endowment shocks.

As a policy implication, our findings suggest that by levelling the playing field, i.e., making private information inaccessible, one would eliminate both the risk-return and asymmetric-information effects, pushing the economy closer to the no-informed-trading equilibrium, which is not always Pareto-optimal (depending on the Sharpe ratio and signal precision), especially for speculators who provide liquidity in the market (either to noise or hedging demands). On the other hand, the no-informed-trading equilibrium is more likely to be Pareto-optimal in a market with a relatively high Sharpe ratio, e.g., developing and emerging markets.

Related Literature. Our paper is closely related to the literature regarding the role of information on financial markets, which is more concerned about price efficiency (how quickly and accurately prices reflect information) than the social value of information (whether a more informative price benefits investors).[4]

After the pioneering works by Allen (1984) and Laffont (1985), in a more recent contribution, Angeletos and Pavan (2007) use an abstract framework to unveil the efficient use of information and the social value of information. They find that whether increased reliance on public information and whether information is socially valuable depend on the type of economy and information structure. Kurlat

[4]Though information disclosure can improve market quality and efficiency, it can also lead to unintended consequences due to crowding-out of information production. The ambiguous role of information disclosure has led to a large literature on complementarities in information acquisition, including Goldstein and Yang (2015) on different pieces of information about asset value, and Veldkamp (2006*b*) and Veldkamp (2006*a*) on the increasing returns to scale in information supply. Since the seminal work by Diamond (1985), some recent contribution to the literature dealing with the welfare consequences of information disclosure include Medrano and Vives (2004) on inside trading, and Amador and Weill (2010) and Kondor (2012) on information crowding out or distorting the use of other information. A recent review on the topic can be found in Goldstein and Yang (2017).

and Veldkamp (2015) examine the welfare implication of mandatory disclosure by asset issuers to potential buyers about asset quality. They find that, even when asset issuers bear of the cost of information and providing information improves risk allocation, information acquisition can still be welfare-reducing, "*simply because resolving risk reduces returns*". Similar to this paper, Dow and Rahi (2003) endogenize noise trading by using endowment shocks as a risk-sharing motive for trading, in order to have a framework for analyzing the welfare cost and benefit of speculation by privately informed traders. The uninformed trader is assumed to be risk-neutral and acts as a market maker. They also allow asset prices to affect corporate investment decisions.

Other related work includes Bhattacharya and Nicodano (2001) who study whether insider trading can improve risk-sharing among outsiders with stochastic liquidity needs; Goldstein and Leitner (2018) who show that information disclosure may be necessary to prevent market failure; Bond and Garcia (2018) who examine the welfare consequences of indexing; Rahi and Zigrand (2018) who assume traders have heterogeneous and interdependent private valuations for risky assets and find that raising the cost of information that discourages information acquisition can make all traders better off; and Gargano, Rossi and Wermers (2017) who provide empirical evidence that acquiring public information that is not widely available to other agents in the market place can be profitable.

This paper contributes to this literature by showing that, when a trader's decision whether to acquire private information about the asset's payoff becomes a strategic probabilistic choice, informed trading can be welfare-improving in a standard REE model with asymmetric information in a pure-exchange economy, without having to add other trader types, heterogeneity in valuation, or other market frictions.

As said, we introduce uncertainty in information acquisition and assume traders pay to increase their probability of being correctly informed. Put differently, we can think about an information market where investors can have access to different sources of information of different quality: the higher the quality, the higher the probability of being informed.[5] This endogenous information acquisition scheme

[5]For a different approach to modelling an information market, see Veldkamp (2006*b*).

relates our paper to the literature on *information games* inspired by global games (Morris and Shin, 2002).

Distinct from classical global games, the strategy of the players is expressed in terms of the *probability of being informed*. With this respect, our model also resembles literature on *probabilistic choice models* (Mattsson and Weibull, 2002) and classical results in information theory (Hobson, 1969). In Mattsson and Weibull (2002), an individual optimally makes an effort to deviate from the status-quo (a reference probability) and *change the likelihood* of a finite set of possible scenarios in order to get closer to implementing a more desired outcome. Given that the reward is always higher for being informed than uninformed, traders choose their optimal information acquisition strategy to maximize the trade-off between a higher expected reward of being more informed and a higher cost.

We rephrase this game-theoretic setting as a monetary reduction of wealth to take into account the information acquisition cost. We model a two-stage optimization scheme based firstly on a strategic information game and secondly on a classical mean and variance investment decision problem. We characterize a unique Nash equilibrium in the probabilities of traders being informed and a noisy REE in asset pricing. Interestingly, in a recent contribution, Hoff and Stiglitz (2016) discuss the importance of advancing the economic modelling background to allow for endogenization of preferences and behaviors. They argue that an equilibrium of the economy is a joint (endogenous) outcome expressed in terms of *probability of types* and *market prices*. In this respect, our framework can be seen as a first attempt to introduce *endogenization* of types into an otherwise standard exchange economy.

The structure of the paper is as follows. We first introduce the model and traders' optimization problem and then characterize the equilibrium in Section 2. In Section 3, we conduct a welfare analysis and explore the welfare improvement channel through Sharpe ratio, information precision, and market risk premia. Section 4 extends the analysis to explicitly model trading motives as a possible route to endogenous supply. Section 5 concludes and all the proofs and additional discussions are collected in Appendices A and B.

2. The Model

There is a continuum of homogenous (price-taking) traders, indexed by $i \in (0,1)$, who can invest in two assets: a risk-free asset with the interest rate normalized to zero, and a risky asset with price $\tilde{P}$ and payoff $\tilde{D} = d + \tilde{\theta} + \tilde{\epsilon}$, where d is a constant, $\tilde{\theta} \sim \mathcal{N}(0, v_\theta)$ is a private signal that can be observed by informed traders, and $\tilde{\epsilon} \sim \mathcal{N}(0, v_\epsilon)$ is the residual uncertainty after $\tilde{\theta}$ is observed; $\tilde{\theta}$ and $\tilde{\epsilon}$ are independent. We first introduce the following assumptions on the cost of information acquisition and traders' preferences.

A1. *Each trader i pays a cost, $\mu\, c(p_i)$, to observe the private signal $\tilde{\theta}$ with a probability $p_i \geq 0$, where $c(p)$ is increasing and convex in $p \in [0,1]$ and $\mu > 0$ is a constant sensitivity to information cost.*[6] *If trader i becomes informed, his information set is denoted by $\mathcal{F}_i = \{\theta, P\}$; otherwise $\mathcal{F}_i = \{P\}$.*

A2. *Each trader has zero initial wealth. Thus, his future wealth is given by $\tilde{W}_i = x_i(\tilde{D} - \tilde{P}) - \mu c(p_i)$, where x_i is the number of shares trader i holds in the risky asset.*

A3. *Each trader maximizes $\mathbb{E}\left[-e^{-\alpha \tilde{W}_i}\right]$ by choosing the optimal portfolio x_i^* and the optimal probability p_i^*, where α is the absolute risk aversion coefficient,*

A4. *The risky asset has a random net supply of $\tilde{z} \sim \mathcal{N}(0, v_z)$, which is unobservable to traders, and independent from $\tilde{\theta}$ and $\tilde{\epsilon}$.*[7]

We postulate that traders can make a probabilistic choice $p_i \in [0,1]$ and pay a variable cost $\mu\, c(p_i)$. As we will see later, the uncertainty in information acquisition has important implications on the social value of private information, i.e., the impact of private information gathering activities on traders' welfare.

2.1. **Information Acquisition Uncertainty and Trading.** There are three dates, $t = 0, 1, 2$. At date $t = 0$, each trader i strategically chooses a probability p_i^* to observe $\tilde{\theta}$ and pays the cost $\mu\, c(p_i^*)$. We refer to this stage as the *information game*. Next, at date $t = 1$, a Boolean random variable $\tilde{\omega}_i$ is drawn independently for each

[6] Starting from Section 3, for tractability in the welfare analysis, we will take $c(p) = p^2$ as a benchmark case.

[7] In Section 4, we model the behaviour of liquidity/noise traders explicitly using endowment shocks. For the baseline model in this section, we simply assume a noisy supply.

trader i with $\mathbb{P}(\tilde{\omega}_i = 1) = p_i^*$ and $\mathbb{P}(\tilde{\omega}_i = 0) = 1 - p_i^*$. If $\tilde{\omega}_i = 1$, the trader observes $\tilde{\theta}$ and becomes informed (type I). Otherwise, $\tilde{\omega}_i = 0$; the trader does not observe $\tilde{\theta}$ and remains uninformed (type U). Then, depending on his type, each trader chooses his optimal demand x_i^* in the risky asset. Finally, at date $t = 2$, supply shock $\tilde{z}$ is realized, $\tilde{P}$ is determined by the market clearing condition, and each trader receives his allocation of shares in the risky asset. Then, the payoff $\tilde{D}$ is realized and consumption occurs.

2.2. **Probabilistic and Portfolio Choices.** Concerning portfolio choice, since the payoff is normally distributed (and the information cost $\mu\, c(p)$, as a sunk cost, does not affect the investment strategy), the standard solution for trader i's optimal holding of the risky asset is given by

$$x_i^* = \begin{cases} x_I^*(\theta, P) = \dfrac{\mathbb{E}[\tilde{D} - P|\theta, P]}{\alpha \mathbb{V}ar[\tilde{D} - P|\theta, P]}, & \mathcal{F}_i = \{\theta, P\}; \\ x_U^*(P) = \dfrac{\mathbb{E}[\tilde{D} - P|P]}{\alpha \mathbb{V}ar[\tilde{D} - P|P]}, & \mathcal{F}_i = \{P\}. \end{cases} \tag{2.1}$$

As for information acquisition, by taking into account the associated cost, trader i makes a probabilistic choice p_i to maximize

$$\mathcal{U}(p_i; \lambda) \equiv [p_i V_I(\lambda) + (1 - p_i) V_U(\lambda)]\, e^{\alpha \mu c(p_i)}, \tag{2.2}$$

where $\lambda = \int_0^1 \omega_i \, di$ is a *state variable* representing the *market fraction of informed traders* who observe $\tilde{\theta}$, and

$$V_I(\lambda) = \mathbb{E}\left\{\mathbb{E}\left[-e^{-\alpha x_I^*(\theta,P)(\tilde{D}-P)}\middle|\theta, P\right]\right\}, \qquad V_U(\lambda) = \mathbb{E}\left\{\mathbb{E}\left[-e^{-\alpha x_U^*(P)(\tilde{D}-P)}\middle|P\right]\right\}$$

are the maximum expected utilities of the informed and uninformed attainable by the optimal portfolios $x_I^*(\theta, P)$ and $x_U^*(P)$, respectively.[8] Note that $V_I(\lambda)$ and $V_U(\lambda)$ depend on λ since the equilibrium price $\tilde{P}$ itself depends on λ.[9] Also, we assume traders take λ as given, or more precisely, each trader forms an expectation about the whole vector $(p_j)_{j \in (0,1)}$; thus traders' probabilistic choices result in a non-cooperative strategic game.

[8]The inner expectation is conditional on the realized signal θ and price P, whereas the outer expectation is taken over all possible realizations of $\tilde{\theta}$ and $\tilde{P}$.

[9]More precisely, in equilibrium, $P_\lambda = h_\lambda(\tilde{\theta}, \tilde{z})$ is a random variable, where h_λ is a deterministic function depending on λ.

Two technical lemmas for market equilibrium are now stated; the first provides the solution to traders' optimal portfolio and information acquisition decisions given the market fraction λ. This result is based on the first order condition argument. The second lemma provides a concavity (second order) condition ensuring that the optimization problem (2.2) is well-defined. For convenience, we denote by

$$\gamma(\lambda) = 1 - \frac{V_I(\lambda)}{V_U(\lambda)}, \qquad 0 < \gamma(\lambda) < 1, \tag{2.3}$$

the relative *utility gain* of becoming informed (by observing $\tilde{\theta}$). We also introduce the following notations, $v_x \equiv \mathbb{V}ar[\tilde{x}], v_{x|\mathcal{F}} \equiv \mathbb{V}ar[\tilde{x}|\mathcal{F}], \sigma_{x,y} \equiv \mathbb{C}ov[\tilde{x}, \tilde{y}], \beta_{x,y} \equiv \sigma_{x,y}/v_x$ and $\rho_{x,y} \equiv \sigma_{x,y}/\sqrt{v_x v_y}$ for any two normally distributed random variables $(\tilde{x}, \tilde{y})$ and information set $\mathcal{F}$. Following the noisy REE literature (Admati, 1985; Admati and Peiderer, 1987), we postulate a linear price

$$\tilde{P} = d + b_\theta \tilde{\theta} - b_z \tilde{z}, \tag{2.4}$$

where b_θ and b_z are two positive coefficients to be determined in equilibrium.

Lemma 2.1. *Assume that traders' expected utility is concave in p_i and that price P is given by* (2.4)*. Then*

(i) *trader i's optimal demand in the risky asset is given by*

$$x_i^* = \begin{cases} x_I^*(\theta, P) = \dfrac{d + \theta - P}{\alpha v_\epsilon}, & \omega_i = 1; \\ x_U^*(P) = \dfrac{d - P}{\alpha v_U}, & \omega_i = 0, \end{cases} \tag{2.5}$$

where

$$v_U = \frac{v_\epsilon + v_{\theta|P}}{1 - \beta_{P,\theta}}, \qquad v_{\theta|P} = (1 - \rho^2_{\theta,P}) v_\theta, \qquad \beta_{P,\theta} = \frac{\sigma_{\theta,P}}{v_P};$$

(ii) *the expected utility of trader i, conditional on type $k \in \{I, U\}$, is given by*

$$V_k(\lambda) = -\frac{1}{\sqrt{1 + \xi_k(\lambda)}}, \tag{2.6}$$

where

$$\xi_I(\lambda) = \frac{(1 - b_\theta^2)^2 v_\theta + b_z^2 v_z}{v_\epsilon}, \qquad \xi_U(\lambda) = \frac{(1 - \beta_{P,\theta})(b_\theta^2 v_\theta + b_z^2 v_z)}{v_U};$$

(iii) *trader i's optimal choice of probability to be informed is given by*

$$p_i^* = g^{-1}\left(\frac{1}{\alpha\mu}\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}\right), \tag{2.7}$$

where $g(p_i) = c'(p_i)$ *and* g^{-1} *is the inverse function of* g.

Assuming that the trader's optimization problem is well-defined (i.e., $\mathcal{U}(p_i;\lambda)$ is concave in p_i), (2.5) gives the optimal demand and (2.6) defines the value function of both the informed ($\omega_i = 1$) and uninformed ($\omega_i = 0$), while (2.7) provides the optimal probabilistic choice to observe $\tilde{\theta}$. In particular,

$$\xi_k(\lambda) \equiv \frac{v_{\chi_k}}{v_{D|\mathcal{F}_k}}, \qquad \chi_k = \mathbb{E}[\tilde{D} - \tilde{P}|\mathcal{F}_k], \qquad v_{\chi_k} = \mathbb{V}ar(\chi_k) = \mathbb{V}ar\left\{\mathbb{E}[\tilde{D} - \tilde{P}|\mathcal{F}_k]\right\}.$$

χ_k and v_{χ_k} are the conditional risk premium and the variance of the conditional risk premium, and therefore $\xi_k(\lambda)$ measures the *squared Sharpe ratio* a trader of type k expects from his portfolio, which directly affects the value function $V_k(\lambda)$. More explicitly, for both informed and uninformed traders, their value functions increase in the respective Sharpe ratios of their optimal portfolios. This relationship helps to explore the risk-return effect for the welfare analysis.

For convenience, we introduce two new parameters:

$$n = \frac{v_\theta}{v_\epsilon}, \qquad \xi_0 = \alpha^2 v_z v_D. \tag{2.8}$$

Parameter n measures the *informativeness* or *precision* of the private signal. Concerning the latter, it can be verified that $\xi_0 = \xi_U(0)$, representing the squared Sharpe ratio of the uninformed when $\lambda = 0$.

Lemma 2.2. *Concerning the function* $\mathcal{U}(p_i;\lambda)$ *defined in* (2.2)*;*

(i) *it is concave in* p_i*, i.e.,* $\mathcal{U}''(p_i;\lambda) < 0$*, if and only if*

$$\frac{\gamma(\lambda)}{g(\lambda)}\left[\frac{2g(\lambda)}{1-p_i\gamma(\lambda)} - \frac{g(p_i)}{1-\lambda\gamma(\lambda)}\right] \le \frac{g'(p_i)}{g(p_i)}, \text{ for all } p_i \in [0,1]; \tag{2.9}$$

(ii) *a sufficient condition for* $\mathcal{U}''(p_i;\lambda) < 0$*,* $\lambda \in [0,1]$*, is given by*

$$\gamma(\lambda) < \frac{1}{2}\min_{p_i\in[0,1]}\left\{\left(\frac{g(p_i)}{g'(p_i)} + \frac{p_i}{2}\right)^{-1}\right\}; \tag{2.10}$$

(iii) *for* $c(p_i) = p_i^2$*,* (2.10) *simplifies to* $\gamma(\lambda) < 1/3$ *which holds for all* $\lambda \in [0,1]$ *as soon as*

$$n < \frac{5}{4}. \tag{2.11}$$

Several features of Lemma 2.2 deserve comments since they are essential in shaping the market equilibrium. First, note that although the general condition (2.9) is rather involved, the sufficient condition in the case of quadratic cost is simply expressed in terms of signal precision n not exceeding a threshold of $5/4$.

Secondly, as soon as one of the concavity conditions in Lemma 2.2 is met, the optimal probability in (2.7) is the same for all traders, i.e., $p_i^* = p^*$ for $i \in (0,1)$. In this respect, the (endogenously determined) cost $\mu\, c(p^*)$ paid for information acquisition is the same for all traders. Having said that, the traders' optimization problem differs significantly from that analyzed in Grossman and Stiglitz (1980), which has the following solution,

$$p^* = \begin{cases} 0, & \gamma(\lambda) < 1 - e^{-\alpha\hat{c}}; \\ 1, & \gamma(\lambda) > 1 - e^{-\alpha\hat{c}}, \end{cases} \tag{2.12}$$

where $\hat{c}$ is a fixed cost.

Note that in (2.12), information equilibrium requires $\gamma(\lambda) = 1 - e^{-\alpha\hat{c}}$ or equivalently $V_I(\lambda)e^{\alpha\hat{c}} = V_U(\lambda)$. In other words, in the Grossman-Stiglitz model without information uncertainty, any utility gain from becoming informed is offset completely by the cost of information acquisition and welfare is simply measured by $V_U(\lambda)$ for both informed and uninformed traders. In contrast, when traders make strategic probabilistic choices under information acquisition uncertainty, the expected utility after cost satisfies $\mathcal{U}(p_i^*, \lambda) \geq \mathcal{U}(0, \lambda) = V_U(\lambda)$. Therefore, having information acquisition uncertainty improves the overall welfare level in comparison to that in Grossman and Stiglitz (1980).

As aforementioned, the optimization scheme of information acquisition and portfolio choice for trader i can be separated in two stages and solved using backward induction. At date $t = 1$, trader i's type is revealed, thus his portfolio choice x_i^*, given his type, can be determined by (2.5) and the value functions, $V_I(\lambda)$ and $V_U(\lambda)$, can be computed given the price coefficients b_θ and b_z which are both functions of the state variable λ in equilibrium. At date $t = 0$, traders play the information game: by averaging the likelihood of becoming informed and forming an expectation about other traders' actions, traders strategically choose optimal strategies, $(p_i^*)_{i\in(0,1)}$. Finally, to close the model, we require $\lambda = \int_0^1 p_i^*\,di = p^*$, i.e., the market fraction of informed traders must be consistent with traders' strategic probabilistic choices in the Nash equilibrium.

2.3. **Information and Asset Market Equilibria.** Before characterizing the information and asset market equilibria, we first define equilibrium in our baseline model.

Definition 2.1. *We say that the probabilities $p^* = (p_i^*)_{i\in(0,1)}$, (expected) market fraction of informed traders, λ, and price $\tilde{P}$ of the risky asset are in equilibrium if*

(i) $p^* = (p_i^*)_{i\in(0,1)}$ *is a Nash equilibrium, meaning that for every $i \in (0,1)$,*[10]

$$\mathcal{U}(p_i^*;\lambda) \geq \mathcal{U}(p_i;\lambda) \qquad \textit{for all } p_i \in [0,1];$$

(ii) *the following* consistency condition *is satisfied*[11]

$$\lambda = \mathbb{E}\left[\int_0^1 \omega_i^*\,di\right] = \int_0^1 p_i^*\,di, \tag{2.13}$$

here ω_i^* is the random variable associated with the optimal probability p_i^*;

(iii) *the price $\tilde{P}$ satisfies market clearing condition*

$$\int_0^1 x_i^*\,di = \lambda x_I^*(\theta, P) + (1-\lambda)x_U^*(P) = \tilde{z}, \tag{2.14}$$

where $x_I^*(\theta,P)$ and $x_U^*(P)$ are given in (2.5).

With the above definition, we now characterize the following REE on the fraction of informed traders and market price.

Proposition 2.3. *Assume condition* (2.9) *holds. Then,*

[10]With a slight abuse of notation, we write $\mathcal{U}(p_i;\lambda)$ in place of $\mathcal{U}(p_i;p_{-i}^*)$, where $p_{-i}^* = (p_j^*)_{j\neq i}$. Indeed, the only payoff-relevant variable for the information game is λ; moreover, having a continuum of traders, the contribution of trader i on the realization of λ is negligible.

[11]At the equilibrium, the expectations are realized so that the fraction of informed, λ, exactly matches the value expected by the traders when using the revealed vector of probabilities p^*.

(i) *the equilibrium market fraction of informed traders is determined by*

$$\lambda = g^{-1}\left(\frac{1}{\alpha\mu}\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}\right); \tag{2.15}$$

(ii) *the linear equilibrium price of the risky asset is given by*

$$\tilde{P} = d + b_\theta\tilde{\theta} - b_z\tilde{z}, \tag{2.16}$$

where

$$b_\theta = \frac{\lambda\bar{v}}{v_\epsilon}, \qquad\qquad b_z = \alpha\bar{v}, \tag{2.17}$$

and

$$\frac{1}{\bar{v}} = \frac{\lambda}{v_\epsilon} + \frac{1-\lambda}{v_U}, \qquad v_U = v_D\left(1+\frac{n\lambda}{\xi_0}\right). \tag{2.18}$$

As previously argued, under the mild concavity conditions stated in Lemma 2.2, the optimal probabilistic choices p_i^* for all $i \in (0,1)$ collapse to the same value p^*, which is the solution to (2.7). Moreover, by virtue of (2.13), we have that $\lambda = p^*$. Note that, from (2.15), there exists a unique cost coefficient μ that satisfies the equilibrium condition for a given λ, i.e., in equilibrium we must have

$$\mu = \frac{1}{\alpha g(\lambda)}\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}. \tag{2.19}$$

Concerning the existence and uniqueness of the Nash equilibrium with respect to parameter μ, intuitively, $\lambda \to 0$ as $\mu \to \infty$; $\lambda = 1$ when μ is small enough; otherwise $\lambda \in (0,1)$. Therefore the equilibrium λ decreases in μ. The following result provides a sufficient condition on the uniqueness.[12] This relationship between the cost sensitivity μ and equilibrium level of informed trading λ helps to provide some policy implications from our findings.

Corollary 2.2. *For* $n < \frac{5}{4}$ *and* $\mu > \bar{\mu} := \frac{1}{2\alpha}\frac{\gamma(1)}{1-\gamma(1)}$, *the equilibrium* λ *as expressed in* (2.15) *is unique and decreasing in* μ. *Therefore, under these conditions, there exists a unique equilibrium* (P,λ) *in the economy where* $\lambda \in (0,1)$ *solves* (2.15) *and* P *is given by* (2.16).

[12] In Appendix B, we provide more general sufficient conditions for the uniqueness to keep our discussion focused on welfare analysis and market implications. In principle, there could be multiple equilibria in λ for the fixed point argument (2.15) even if the optimization problem is well-defined in p^*. We leave this intriguing discussion on multiple equilibria for future research.

A final remark on the perceived aggregate risk as expressed by (2.18): the perceived risk is v_ϵ for the informed and v_U for the uninformed, which is even larger than the total payoff risk v_D. The additional component characterizes the *adverse selection risk* born by the uninformed, which always increases in the level of informed trading λ and is amplified by high information precision n and low Sharpe ratio ξ_0. Therefore, for the uninformed traders, due to the adverse selection, the total payoff risk is amplified by a factor of $(1 + n\lambda/\xi_0)$. Moreover, the aggregate risk $\bar{v}$ is a harmonic mean of the perceived risks of informed and uninformed traders. These observations will help us to better understand the underlying mechanism by which asymmetric information and informed trading affect traders' welfare.

3. Welfare Analysis

By virtue of (2.19), we can measure traders' *welfare* as a function of the state variable λ only, i.e.,

$$\mathcal{W}(\lambda) \equiv \mathcal{U}(\lambda; \lambda) = \bar{V}(\lambda)e^{\Phi(\lambda)}, \tag{3.1}$$

where

$$\bar{V}(\lambda) = \lambda V_I(\lambda) + (1-\lambda)V_U(\lambda), \qquad \Phi(\lambda) = \frac{c(\lambda)}{g(\lambda)}\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}.$$

We now use (3.1) to conduct a welfare analysis. Interestingly, as it will become clear later, V_I, V_U and Φ (hence, the welfare function $\mathcal{W}$ itself), in addition to λ, only depend on n and ξ_0 as defined in (2.8).

We have noted that, at the individual level, a trader who makes a strategic choice about the probability of becoming informed, i.e., p_i, is able to achieve better welfare than in the Grossman-Stiglitz model. Thus, in equilibrium,

$$\mathcal{W}(\lambda) \geq \hat{\mathcal{W}}(\lambda) \equiv V_U(\lambda).$$

Next, we examine the impact of more informed trading, i.e., an increase in λ, on traders' welfare improvement. For tractability, we choose $c(p) = p^2$. Thus, from (3.1), the change in welfare is determined by

$$\mathcal{W}'(\lambda) = e^{\Phi(\lambda)}\Big([V_I(\lambda) - V_U(\lambda)] + [\lambda V_I'(\lambda) + (1-\lambda)V_U'(\lambda)] + \Phi'(\lambda)\bar{V}(\lambda)\Big), \tag{3.2}$$

where

$$\Phi(\lambda) = \frac{\lambda}{2} \frac{\gamma(\lambda)}{1 - \lambda\gamma(\lambda)}.$$

Equivalently, we have from (3.2) and (3.1) the following decomposition on the rate of change in welfare,

$$\frac{\mathcal{W}'(\lambda)}{-\mathcal{W}(\lambda)} = \underbrace{\frac{\lambda V_I'(\lambda) + (1-\lambda)V_U'(\lambda)}{-\bar{V}(\lambda)}}_{\text{risk-return effect}} + \underbrace{\frac{V_I(\lambda) - V_U(\lambda)}{-\bar{V}(\lambda)}}_{\text{asymmetric-information effect}} + \underbrace{\big[-\Phi'(\lambda)\big]}_{\text{marginal cost}}. \tag{3.3}$$

In the following, we analyze each of the three components in (3.3) and characterize the conditions under which more informed trading can actually lead to *welfare improvement.*

3.1. **Risk-Return Effect.** Informed trading makes the price more informative, which resolves payoff uncertainty and also reduces the conditional risk premium for the uninformed traders. As Kurlat and Veldkamp (2015) explain, "*decreasing risk lowers the equilibrium return and systematically raises the assets average price. For welfare, this means that information reduces the assets risk, but also implies lower return. With exponential utility and normally distributed payoffs, the return effect always dominates.*" Indeed, we show in the following lemma that an increase in the state variable λ, which measures the level of informed trading, reduces the Sharpe ratios for both informed and uninformed traders, which in turn reduces their expected utilities.

Lemma 3.1. *After information acquisition uncertainty is resolved, i.e., trader types are realized, the square Sharpe ratios as functions of the state variable λ can be written as*

$$\xi_I(\lambda) = \frac{1 + \xi_U(\lambda)}{\eta(\lambda)} - 1, \qquad \xi_U(\lambda) = \frac{\xi_0^2\,(n\lambda^2 + \xi_0)}{[n(1+n)\lambda^2 + (1+n\lambda)\xi_0]^2}, \tag{3.4}$$

where

$$\eta(\lambda) \equiv \left(\frac{V_I(\lambda)}{V_U(\lambda)}\right)^2 = 1 - \frac{n\xi_0}{(1+n)(\xi_0 + n\lambda^2)} < 1.$$

Moreover, $\xi_I'(\lambda) < 0$, $\xi_U'(\lambda) < 0$ and $\xi_I(\lambda) > \xi_U(\lambda)$.

Next, we examine the magnitude of the risk-return effect, and its dependence on the signal precision n and (square) Sharpe ratio ξ_0. We focus on the benchmark case with no informed trading, i.e., $\lambda = 0$, which is the best-case scenario in regards to welfare in the Grossman-Stiglitz model.

The change in the Sharpe ratio is given by

$$\xi_U'(\lambda) = \xi_U(\lambda)\left[\frac{v_{\chi_U}'(\lambda)}{v_{\chi_U}(\lambda)} - \frac{v_{D|P}'(\lambda)}{v_{D|P}(\lambda)}\right].$$

In the benchmark case when $\lambda = 0$, we have

$$\frac{v_{\chi_U}'(0)}{v_{\chi_U}(0)} = -2n, \qquad v_{D|P}'(0) = 0, \qquad \text{and} \qquad \xi_U'(0) = -2n\xi_0.$$

Furthermore, the risk-return effect, measured at $\lambda = 0$, is given by

$$\lim_{\lambda\to 0}\frac{\lambda V_I'(\lambda) + (1-\lambda)V_U'(\lambda)}{-\bar{V}(\lambda)} = \frac{V_U'(0)}{-V_U(0)} = \underbrace{-2n\xi_0}_{\xi_U'(0)} \times \underbrace{\frac{1}{2(1+\xi_0)}}_{V_U'(\xi_0)/V_U(\xi_0)}. \tag{3.5}$$

From (3.5), the risk-return effect is made up of two components. The first component is the reduction in the square Sharpe ratio, which is more severe when signal precision n and the benchmark Sharpe ratio ξ_0 are high. The second component measures the proportional change in the expected utility due to the change in the square Sharpe ratio ξ_0, which is actually decreasing in ξ_0 since the value function $V_U(\xi_0)$ is concave. Overall, in terms of its dependence on ξ_0, the first component always dominates the second component.

Furthermore, the Sharpe ratio decreases *faster* and hence the negative risk-return effect on welfare is more severe with more informed trading when the private signal is more precise (*informed trading resolves more payoff uncertainty*) and when the initial Sharpe ratio is higher (*more trading opportunities at stake*).

Note that, because of the risk-return effect, traders' welfare in Grossman and Stiglitz (1980) and Allen (1984) is strictly decreasing with the level of informed trading, i.e., $\hat{\mathcal{W}}'(\lambda) = V_U'(\lambda) < 0$.

3.2. **Asymmetric-Information and Marginal Cost Effects.** The second component in (3.3) measures the relative utility gain by becoming informed, which we refer to as the *asymmetric-information effect*. When the *positive* asymmetric-information effect dominates the *negative* risk-return effect, more informed trading leads to welfare improvement. In addition, we must also take into account the marginal cost of information acquisition, measured by $\Phi'(\lambda)$, which is positive when the level of informed trading is low. In fact, in the no-informed-trading equilibrium, we can show that

$$\frac{V_I(0) - V_U(0)}{-\bar{V}(0)} = 1 - \sqrt{\frac{1}{1+n}}, \qquad \Phi'(0) = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1+n}}\right).$$

Therefore, in this case, the asymmetric-information effect is halved by the marginal cost of information acquisition, however the *net* asymmetric-information effect remains positive. This implies that, in general, depending on the trade-off between the risk-return and the net asymmetric-information effects, more informed trading can be welfare improving.

It is interesting to note that, different from the risk-return effect, the net asymmetric-information effect at $\lambda = 0$ only depends on the signal precision n, not the Sharpe ratio ξ_0, in the sense that regardless of how much trading opportunities are at stake, the relative utility gain by becoming informed is the same.

As a comparison, the welfare in the Grossman-Stiglitz model can be written as

$$\hat{\mathcal{W}}(\lambda) = \lambda V_I(\lambda) e^{\hat{\Phi}(\lambda)} + (1-\lambda) V_U(\lambda), \qquad \hat{\Phi}(\lambda) \equiv -\ln\left(\frac{V_I(\lambda)}{V_U(\lambda)}\right)$$

and

$$\hat{\mathcal{W}}'(\lambda) = \underbrace{e^{\hat{\Phi}(\lambda)} V_I(\lambda) - V_U(\lambda)}_{\text{asymmetric-information effect}} + \underbrace{\lambda e^{\hat{\Phi}(\lambda)} V_I'(\lambda) + (1-\lambda) V_U'(\lambda)}_{\text{risk-return effect}} + \underbrace{\lambda e^{\hat{\Phi}(\lambda)} \hat{\Phi}'(\lambda) V_I(\lambda)}_{\text{marginal cost}}. \tag{3.6}$$

From (3.6) the asymmetric-information effect in the Grossman-Stiglitz model is zero, since the equilibrium requires the expected utilities of the informed and uninformed traders to be identical. Moreover, the marginal cost effect is *positive*. Since the information acquisition cost is fixed, cost must reduce to incentivize more traders to become informed. Unfortunately, the positive marginal cost effect is not enough to offset the negative risk-return effect, and thus welfare is always decreasing in the level of informed trading, i.e., $\hat{W}'(\lambda) < 0$. In other words, cheaper access to information intensifies informed trading, which resolves payoff uncertainty leading to a lower Sharpe ratio (for uninformed traders), thus actually harming traders' welfare.

Different from above, due to information acquisition uncertainty, traders pay a variable cost $\mu\, c(p^*)$ depending on their optimal probabilistic choice p^*, which is equal to λ in equilibrium. In this case, cheaper access to information is reflected by a reduction in μ. However, a drop in μ leads to higher $p^* = \lambda$, which can actually increase the overall cost paid, i.e., $\Phi'(\lambda) > 0$, especially when $\lambda \approx 0$. Moreover, the negative marginal cost effect and risk-return effect are offset by the asymmetric-information effect, which is strictly *positive*.

To understand the difference with Grossman and Stiglitz (1980) and Allen (1984), in the current model every trader pays a cost $\mu\, c(p^*)$ for a probability p^* to observe $\tilde{\theta}$. As a result, ex-post after trader types are revealed, the informed traders are strictly better off than the uninformed traders, i.e., $V_I(\lambda)e^{\alpha\mu\, c(p^*)} > V_U(\lambda)e^{\alpha\mu\, c(p^*)}$, while their ex-ante expected utilities are the same, which equals to $\mathcal{U}(p^*;\lambda)$. In other words, under information acquisition uncertainty, utility gain by becoming informed is not entirely washed out by the cost, thus providing a channel through which more informed trading can potentially lead to welfare improvement.

3.3. **Necessary and Sufficient Condition for Welfare Improvement.** We now provide the necessary and sufficient conditions under which the asymmetric-information effect dominates the risk-return and marginal cost effects, and hence more informed trading leads to a welfare improvement.

Proposition 3.2. *In equilibrium, traders' welfare is increasing in the fraction of informed traders, i.e.,* $\mathcal{W}'(\lambda) \geq 0$, *if and only if*

$$\frac{V_U'(\lambda)}{V_U(\lambda)} = \frac{-\xi_U'(\lambda)}{1+\xi_U(\lambda)} \leq \frac{1}{2}\frac{[1-2\lambda\gamma(\lambda)][\gamma(\lambda)+\lambda\gamma'(\lambda)]}{[1-\lambda\gamma(\lambda)]^2}; \tag{3.7}$$

In particular, at $\lambda = 0$, $\mathcal{W}'(0) \geq 0$ *if and only if*

$$\frac{V_U'(0)}{V_U(0)} = \frac{n\xi_0}{1+\xi_0} \leq \frac{1}{2}\gamma(0) = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1+n}}\right). \tag{3.8}$$

Proposition 3.2 provides the general necessary and sufficient condition for welfare improvement. First, when $\lambda \in [0,1]$ condition (3.7) requires the proportional change in the expected utility of the uninformed due to informed trading to be *small* enough. That is, characterized by the relative change in the Sharpe ratio, the risk-return effect must be sufficiently *weak*, which, as previously discussed, occurs when the signal precision and the benchmark Sharpe ratio in the no-informed-trading economy, measured by n and ξ_0 respectively, are low.

The condition becomes more transparent in the benchmark economy with no informed trading, i.e., when $\lambda = 0$. In this case, a lower ξ_0 weakens the risk-return effect and unambiguously makes welfare improvement more probable. On the other hand, the effect of n is more subtle. A lower n weakens the risk-return effect, but at the same time also weakens the asymmetric-information effect. To examine the net effect of n, we can rewrite the condition in (3.8) as

$$\xi_0 \le \frac{1}{1 + 2\left(n + \sqrt{1+n}\right)}. \tag{3.9}$$

Therefore, the net effect of a lower n on the potential of informed trading to improve welfare is *positive* in the benchmark economy with no informed trading. In other words, a less precise signal weakens the risk-return effect more than it weakens the asymmetric-information effect. From the condition in (3.9), we obtain next more explicit sufficient and necessary conditions on welfare improvement.

Corollary 3.1. *On the welfare improvement, suppose condition* (2.11) *is met, then*

(i) *if* $\xi_0 < 2/13$*, then* $\mathcal{W}'(0) > 0$*;*

(ii) *if* $\xi_0 > 1/3$*, then* $\mathcal{W}'(0) < 0$*.*

Corollary 3.1 shows that the benchmark Sharpe ratio ξ_0, which measures the amount of trading opportunities at stake, plays a more important role than the signal precision n, which measures how much payoff uncertainty is resolved by informed trading. Corollary 3.1 (i) shows that a sufficiently small Sharpe ratio ξ_0 ($< 2/13$) ensures welfare improvement from the benchmark case with no informed trading, whereas Corollary 3.1 (ii) shows that informed trading is always detrimental to welfare when the Sharpe ratio ξ_0 is too high ($> 1/3$).

Moreover, since we know that $\mathcal{W}(0) > \mathcal{W}(1)$, if $\mathcal{W}'(0) > 0$, there exists a Pareto-optimal state $\lambda^* > 0$, such that $\mathcal{W}(\lambda) \le \mathcal{W}(\lambda^*)$ for $\lambda \in [0, 1]$. We examine the properties of the Pareto-optimal state λ^* in the following numerical analysis.

3.4. **Pareto Optimal State.** To better understand how welfare improvement and Pareto optimality depend on (n, ξ_0), we conduct a numerical analysis and depict the results in Figure 3.1. Panel (A) plots the parameter regions $\Omega(\lambda)$ expressed in terms of (n, ξ_0) corresponding to a welfare improvement, $\mathcal{W}'(\lambda) > 0$, with the given equilibrium λ. Note that, on the boundary of the *improvement region*, we have $\mathcal{W}'(\lambda^*) = 0$.

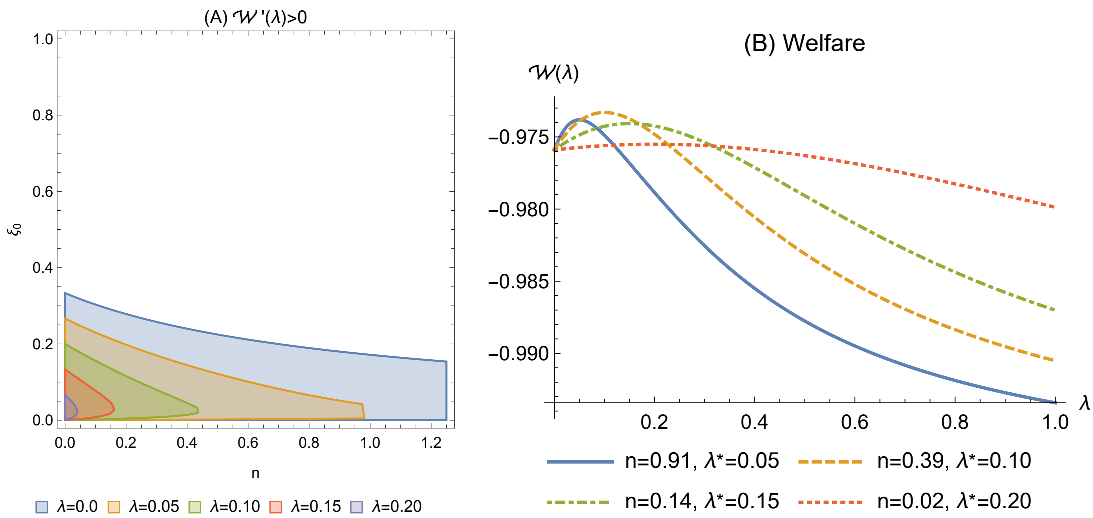

FIGURE 3.1. Panel (A) shows the region marked by $\{n, \xi_0\}$ in which $\mathcal{W}'(\lambda) > 0$ for a given λ. Panel (B) shows $\mathcal{W}(\lambda)$ for $0 \leq \lambda \leq 1$, where $\xi_0 = 0.05$ and n is chosen in such a way that $\mathcal{W}'(\lambda^*) = 0$.

Two remarks are needed. First, consistent with the analytical results in the benchmark economy with no-informed trading, welfare improvement is more likely to occur when signal precision n and benchmark Sharpe ratio are low for any given λ, as Panel (A) shows that the welfare improvement region $\Omega(\lambda)$ shrinks in both n and ξ_0. Second, $\Omega(\lambda)$ is also shrinking in λ, i.e., $\Omega(\lambda_1) \subset \Omega(\lambda_2)$ for $\lambda_1 > \lambda_2$. Generalizing this argument, numerical results suggest that welfare improves if and only if $\mathcal{W}'(0) > 0$.

Overall, at a low level of informed trading λ, the positive asymmetric-information effect is more likely to dominate the risk-return effect. Therefore, more informed trading leads to welfare improvement when both precision of private signal and trading profitability (measured by the Sharpe ratio) are low.

Regarding the welfare function, in Figure 3.1 Panel (B), we fix $\xi_0 = 0.05$ and choose different n on the boundary of the regions plotted in Panel (A) for different values of λ^* such that $\mathcal{W}'(\lambda^*) = 0$. The different plots of $\mathcal{W}(\lambda)$ in Panel (B) show that λ^* indeed represents the unique Pareto-optimal state, under which traders' welfare is maximized. [13] The hump-shaped welfare functions once again corroborate our previous observation related to Panel (A): traders' welfare improves at some informed trading level $\lambda \in (0, 1)$ if and only if $\mathcal{W}'(0) > 0$.

Moreover, with ξ_0 fixed, the Pareto-optimal λ^* is lower for higher signal precision n. Thus, with higher signal precision, traders experience a larger welfare improvement at a low informed trading level (in which a smaller fraction of the population becomes informed due to a higher sensitivity μ to the cost). In other words, if the private signal is very informative and resolves a large part of the uncertainty, the no-informed-trading equilibrium with $\lambda^* = 0$ is more likely to be Pareto-optimal.

On the level of the welfare improvement, Table 1 shows welfare comparison between informed-trading and no-informed-trading equilibria, i.e., $\mathcal{W}(\lambda^*) - V_U(0)$, and the corresponding welfare loss, $V_U(\lambda^*) - V_U(0)$, due to the risk-return effect, for different levels of the Pareto-optimal λ^*. We observe that welfare $\mathcal{W}(\lambda^*)$ at the Pareto-optimal state does not monotonically increase with λ^*. In fact, $\mathcal{W}(\lambda^*)$ has a hump-shaped relationship with n. Therefore, there exists an optimal signal precision, n^*, for which the welfare function is maximized given the Sharpe ratio ξ_0.

In terms of policy implications, our findings suggest that by levelling the playing field, i.e., reducing the level of informed trading by increasing the cost of acquiring private information, one eliminates both the risk-return and asymmetric-information effects, which is not always Pareto-optimal. In many scenarios where acquiring information is very costly (so that the informed trading level in market equilibrium is low), encouraging more informed trading (by lowing the cost) can actually lead to welfare improvement, especially when the Sharpe ratio is relatively low and the information signal precision is at the optimal level.

[13] As already said, by letting μ vary, we obtain a continuum of equilibrium values $\lambda \in [0, 1]$. Among those values, a *unique* λ^* maximizes the welfare function $\mathcal{W}$. Of course, λ^* is related to a unique value of cost sensitivity, given by $\mu^* = \frac{1}{\alpha g(\lambda^*)} \frac{\gamma(\lambda^*)}{1 - \lambda^* \gamma(\lambda^*)}$.

λ^*	0.05	0.10	0.15	0.20
n	0.91	0.39	0.14	0.02
$\mathcal{W}(\lambda^*) - V_U(0)$	20.8	25.9	18.2	3.9
$V_U(\lambda^*) - V_U(0)$	-43.2	-41.9	-26.4	-5.7

TABLE 1. Welfare improvement $\mathcal{W}(\lambda^*) - V_U(0)$ (in basis points) compared with the welfare loss $V_U(\lambda^*) - V_U(0)$ due to the risk-return effect, for different levels of Pareto-optimal λ^*. Here, $\xi_0 = 0.05$.

Furthermore, Table 1 shows that by ignoring the asymmetric-information effect, one could potentially underestimate traders' welfare by a significant amount. For example, for the case of $\lambda^* = 0.10$ ($n = 0.39$), the risk-return effect leads to a welfare loss of 42 basis points (bps); however, there is actually an overall welfare improvement of 26 *bps*. Note that, since $\mathcal{W}(\lambda) > V_U(\lambda)$, a λ fraction of the traders become informed at $t = 1$ with expected utility $V_I(\lambda)e^{\Phi(\lambda)} > V_U(\lambda)$ and a $1 - \lambda$ fraction remains uninformed with expected utility $V_U(\lambda)e^{\Phi(\lambda)} < V_U(\lambda)$. Therefore, *ex-post*, at time $t = 1$, some traders will be worse off than not paying any cost for acquiring information while others will be better off. However, the traders are overall better off. For example, back to the case of $\lambda^* = 0.10$ ($n = 0.39$), we have $V_I(\lambda)e^{\Phi(\lambda)} - V_U(\lambda) = 1302\, bps$ and $V_U(\lambda)e^{\Phi(\lambda)} - V_U(\lambda) = -69\, bps$, thus the asymmetric-information effect is $\bar{V}(\lambda)e^{\Phi(\lambda)} - V_U(\lambda) = 68\, bps$, which overcomes the risk-return effect ($-42\, bps$).

3.5. **Relationship between Welfare Improvement and Risk Premium.** In this subsection, we try to draw a connection between the condition for the welfare improvement and the expected liquidity cost of the noise demand, $\mathbb{E}\left[-\tilde{z}\tilde{P}\right] = (\alpha\bar{v})v_z$, which is the product of the risk premium $\alpha\bar{v}$ and the variance of the noise demand. Recall that the aggregate (weighted) risk prevailing in the economy, defined in (2.18), has the following form $\bar{v}(\lambda) = \left(\lambda/v_\epsilon + (1-\lambda)/\left[v_D\left(1 + \frac{n\lambda}{\xi_0}\right)\right]\right)^{-1}$. Following the discussion of Proposition 2.3, we know that $\bar{v}(\lambda)$ is not necessarily decreasing in λ and the total risk faced by the uninformed traders is larger than the unconditional variance of the payoff. Intuitively, more informed trading reduces payoff uncertainty; however, it increases adverse selection risk for the uninformed traders, who trade less aggressively knowing that price contains valuable information. The adverse selection risk is then amplified by a low Sharpe ratio ξ_0. Since the total risk is a harmonic mean of the perceived risks of informed and uninformed traders, it is dominated by adverse selection risk when λ is low, and more informed trading can increase the risk premium, i.e., $\bar{v}(\lambda) > v_D$ when λ is low. The next corollary pins down the conditions for this situation to occur and also its connection with welfare improvement.

Corollary 3.3. *On the aggregate risk $\bar{v}$,*

(i) *when $\xi_0 > 1$, it decreases in λ;*

(ii) *when $\xi_0 \leq 1$, it increases in λ if and only if $\lambda \leq \frac{1}{n}\Big[\sqrt{\frac{\xi_0(n+\xi_0)}{1+n}} - \xi_0\Big]$.*

Corollary 3.3 shows that when the Sharpe ratio is relatively large ($\xi_0 > 1$), more informed trading always *reduces* risk premium and thus the expected liquidation cost $\mathbb{E}\left[-\tilde{z}\tilde{P}\right]$, i.e., $\bar{v}'(\lambda) < 0$. In this case, since we know that the no-informed-trading equilibrium with $\lambda = 0$ is Pareto-optimal (see Corollary 3.1 (ii)), more informed trading also reduces traders' welfare, i.e., $\mathcal{W}'(\lambda) < 0$. Therefore, in order for more informed trading to improve traders' welfare, i.e., the Pareto-optimal state $\lambda^* > 0$, it is necessary to have $\bar{v}'(\lambda) > 0$, which requires the *adverse selection* component of the aggregate risk to dominate when the level of the informed trading is low. Therefore, it is likely that at the Pareto-optimal state, $\bar{v}(\lambda^*) > v_D$, and thus liquidity traders are expected to pay more to execute their market orders.

4. Modelling Trading Motives Explicitly

In this section, rather than assuming exogenous noise in supply, we follow Medrano and Vives (2011) and Bond and Garcia (2018) to motivate trading using *endowment shocks*.

Each trader i receives an endowment, $e_i\tilde{D}$, at the end of the trading period. Thus, trader i's future wealth is given by

$$\tilde{W}_i = (x_i + e_i)(\tilde{D} - \tilde{P}) + e_i\tilde{P} - \mu\, c(p_i). \tag{4.1}$$

We assume e_i is known to trader i, whereas other traders only have common knowledge about the distribution function from which e_i is drawn. In particular,

$e_i = \tilde{z} + \tilde{u}_i$, where $\tilde{z} \sim \mathcal{N}(0, v_z)$ is an aggregate endowment shock and $\tilde{u}_i \sim \mathcal{N}(0, v_u)$ is an idiosyncratic shock, thus $v_e \equiv \mathbb{V}ar[\tilde{e}_i] = v_z + v_u$.

4.1. **Optimization problem.** As before, each trader i's objective is to determine the optimal probability p_i^* of observing the private signal θ, in order to maximize his expected utility of terminal wealth,

$$\mathcal{U}(p_i; \lambda, e_i) \equiv [p_i V_I(\lambda, e_i) + (1 - p_i) V_U(\lambda, e_i)]\, e^{\alpha\mu\, c(p_i)}, \tag{4.2}$$

where $V_I(\lambda, e_i) = \mathbb{E}\left\{\mathbb{E}\left[-e^{-\alpha\left(x_I^*(\theta,P,e_i)(\tilde{D}-P)+e_i\right)}\middle|\theta, P, e_i\right]\right\}$ and $V_U(\lambda, e_i) = \mathbb{E}\left\{\mathbb{E}\left[-e^{-\alpha\left(x_U^*(P,e_i)(\tilde{D}-P)+e_i\right)}\middle|P, e_i\right]\right\}$ are trader i's expected utility, depending on whether or not he observes the private signal θ. Note that apart from θ, trader i also has another private signal, which is his own endowment shock e_i. Intuitively, e_i helps trader i to forecast the aggregate endowment shock $\tilde{z}$, which is negatively correlated with the equilibrium price $\tilde{P}$. For example, after observing the same price, a trader who receives a positive endowment shock will infer a larger value for θ than a trader who receives a negative endowment shock.

Conditional on his information set, trader i's optimal portfolio is given by

$$x_i^* = \begin{cases} x_I^*(\theta, P, e_i) = \dfrac{\mathbb{E}[\tilde{D} - P|\theta, P, e_i]}{\alpha\mathbb{V}ar[\tilde{D} - P|\theta, P, e_i]} - e_i, & \mathcal{F}_i = \{\theta, P, e_i\}; \\ x_U^*(P, e_i) = \dfrac{\mathbb{E}[\tilde{D} - P|P, e_i]}{\alpha\mathbb{V}ar[\tilde{D} - P|P, e_i]} - e_i, & \mathcal{F}_i = \{P, e_i\}. \end{cases} \tag{4.3}$$

As before, we conjecture a linear equilibrium price,

$$\tilde{P} = d + b_\theta\tilde{\theta} - b_z\tilde{z}. \tag{4.4}$$

Next, we characterize the solution to the traders' optimization problem. The optimal demand for the uninformed and informed traders are given by

$$x_U^*(P, e_i) = \frac{(1-\kappa)(d - P) - \kappa\beta_{e,P}e_i}{\alpha v_U} - e_i \tag{4.5}$$

and

$$x_I^*(\theta, P, e_i) = \frac{d + \theta - P}{\alpha v_\epsilon} - e_i, \tag{4.6}$$

respectively, where $\kappa = \sigma_{\theta,P}/[v_P - \frac{\sigma_{e,P}^2}{v_e}]$ and $v_U = v_D\left(1 - \frac{\rho_{P,D}^2}{1-\rho_{e,P}^2}\right) = v_D - \kappa\sigma_{\theta,P}$. We now compute expected utilities for the informed and uninformed traders, i.e.,

$V_I(\lambda, e_i)$ and $V_U(\lambda, e_i)$. First, trader i's welfare conditional on his information set is given by

$$\mathbb{E}\left[-e^{-\alpha\tilde{W}_i}|\mathcal{F}_i\right] = -\exp\left\{-\alpha e_i P - \frac{1}{2}\frac{\chi_i^2}{v_i}\right\}, \tag{4.7}$$

where $\chi_i \equiv \mathbb{E}\left[\tilde{D} - \tilde{P}|\mathcal{F}_i\right]$ and $v_i \equiv \mathbb{V}ar\left[\tilde{D} - \tilde{P}|\mathcal{F}_i\right]$. Since, conditional on the endowment shock e_i, the price P and expected excess return χ_i follow a bivariate normal distribution, we can obtain the following expression for trader i's welfare given his own endowment shock.

Proposition 4.1.

$$\mathbb{E}\left\{\mathbb{E}\left[-e^{-\alpha\tilde{W}_i}|\mathcal{F}_i\right]|e_i\right\} = -\exp\left\{-A_0 e_i + \frac{1}{2}A_1 e_i^2\right\}\left(\frac{\nu}{v_i}\right)^{-1/2}, \tag{4.8}$$

where $A_0 = \alpha\, d$, $A_1 = \frac{\alpha^2(v_{P|e}(v_\epsilon+v_\theta)-\sigma_{\theta,P}^2)-\beta_{e,P}^2-2\alpha(v_\epsilon+v_\theta-\sigma_{\theta,P})\beta_{e,P}}{\nu}$, $\nu \equiv v_{\chi_i} + v_i = v_{P|e} + (v_\epsilon + v_\theta) - 2\sigma_{\theta,P}$ *and* $v_{P|e} \equiv \mathbb{V}ar[\tilde{P}|e_i] = v_P - \frac{\sigma_{e,P}^2}{v_z+v_u}$.

From Proposition 4.1, the expected utility gain of becoming informed is *independent* of the realized endowment shock e_i, i.e.,

$$\gamma(\lambda) \equiv \frac{V_I(e_i;\lambda) - V_U(e_i;\lambda)}{-V_U(e_i;\lambda)} = 1 - \sqrt{\frac{v_\epsilon}{v_D - \kappa\sigma_{\theta,P}}}. \tag{4.9}$$

Interestingly, the solution to trader i's optimization problem as in (4.2) does not depend on the trader-specific endowment shock and boils down to (2.7), just as in the baseline model. Also, the concavity condition in p_i, $\mathcal{U}''(p_i;\lambda,e_i) < 0$, is satisfied if (2.11) is true.

4.2. **Equilibrium.** Since the risky asset is in zero net supply, market clearing requires

$$\int_0^1 \left[\lambda x_I^*(\theta, P, e_i) + (1-\lambda)x_U^*(P, e_i)\right] di = 0, \tag{4.10}$$

where λ is the fraction of informed traders. In the next proposition, we determine the coefficient b_θ and b_z in equilibrium.

Proposition 4.2. *Assume that the sufficient condition for concavity expressed in* (2.11) *is satisfied. For given* $\lambda \in [0,1]$, *let* $\Psi \equiv v_z/(v_z + v_u)$, *there exists a linear equilibrium price of the risky asset,*

$$\tilde{P} = d + b_\theta\tilde{\theta} - b_z\tilde{z}, \tag{4.11}$$

where

$$b_\theta = \frac{1}{1 + x^{-2}\frac{v_\theta^{-1}+\lambda v_\epsilon^{-1}}{v_u^{-1}+v_z^{-1}}} + \frac{\lambda}{\frac{v_\epsilon+\lambda v_\theta}{v_\epsilon+v_\theta} + x^2\frac{v_u^{-1}+v_z^{-1}}{v_\epsilon^{-1}+v_\theta^{-1}}}, \tag{4.12}$$

$x \equiv \frac{b_\theta}{b_z}$ *solves*

$$x = \frac{1}{\alpha v_\epsilon}\left(\lambda + \frac{1-\lambda}{\Psi^{-1} + x^{-2}(v_\epsilon^{-1} + v_\theta^{-1})v_u}\right), \tag{4.13}$$

and λ *is the solution of*

$$\lambda = g^{-1}\left(\frac{1}{\alpha\mu}\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}\right), \tag{4.14}$$

where $\gamma(\lambda)$ *is given by* (4.9).

4.3. **Welfare.** The welfare of trader i, assuming $c(p) = p^2$, given his endowment shock e_i, can be measured by

$$\mathcal{W}(\lambda; e_i) \equiv \mathcal{U}(\lambda; \lambda, e_i), \qquad \alpha\mu = \frac{1}{2}\frac{\gamma(\lambda)}{\lambda\left(1-\lambda\gamma(\lambda)\right)}, \tag{4.15}$$

since every trader optimally chooses the same probability $p_i^* = \lambda$ in the Nash equilibrium.

Next, we consider two special cases where $\lambda = 0$ and $\lambda = 1$. Note that for $\lambda = 0$, the equilibrium price becomes $\tilde{P} = d - \alpha(v_\theta + v_\epsilon)\tilde{z}$ and trader i's optimal portfolio is $x_U^*(P, e_i) = \frac{d-P}{\alpha v_D} - e_i$. On the other hand, when $\lambda = 1$, the equilibrium price and trader i's optimal portfolio are given by $\tilde{P} = d + \tilde{\theta} - \alpha v_\epsilon \tilde{z}$ and $x_I^*(\theta, P, e_i) = \frac{d+\theta-P}{\alpha v_\epsilon} - e_i$. The following proposition characterizes traders' overall welfare.

Corollary 4.3. *The welfare of trader i is characterized by Equation* (4.8), *where*

$$A_1 = \frac{\alpha^2(v_\epsilon + v_\theta)\left(2v_z/v_e - (v_z/v_e)^2 + \alpha^2(v_\epsilon+v_\theta)v_{z|e}\right)}{1 + \alpha^2 v_{z|e}(v_\epsilon + v_\theta)}, \qquad \frac{\nu}{v_i} = 1 + \alpha^2(v_\epsilon + v_\theta)v_{z|e} \tag{4.16}$$

when $\lambda = 0$ *and*

$$A_1 = \frac{\alpha^2 v_\epsilon\left(2v_z/v_e - (v_z/v_e)^2 + \alpha^2 v_\epsilon v_{z|e}\right)}{1 + \alpha^2 v_{z|e} v_\epsilon} + \alpha^2 v_\theta, \qquad \frac{\nu}{v_i} = 1 + \alpha^2 v_\epsilon v_{z|e} \tag{4.17}$$

when $\lambda = 1$ *with* $v_{z|e} \equiv \mathbb{V}ar\left[\tilde{z}|e_i\right] = (v_z^{-1} + v_u^{-1})^{-1}$ *and* $\alpha\mu = \frac{1}{2}\frac{\gamma(1)}{1-\gamma(1)}$. *Moreover, traders are always better off in the no-informed-trading equilibrium with* $\lambda = 0$ *than in the full-informed-trading equilibrium with* $\lambda = 1$, *i.e.*,

$$\frac{\mathcal{W}(0; e_i)}{\mathcal{W}(1; e_i)} = \exp\left\{-\frac{1}{2}\frac{\gamma(1)}{1-\gamma(1)} - \frac{1}{2}\alpha^2\left(\frac{v_u}{v_e}e_i\right)^2 v_\theta\right\}\sqrt{\frac{1 + \alpha^2 v_\epsilon v_{z|e}}{1 + \alpha^2(v_\epsilon + v_\theta)v_{z|e}}} \le 1. \tag{4.18}$$

Corollary 4.3 shows that, in terms of welfare, the no-informed-trading equilibrium with $\lambda = 0$ (no traders observe $\tilde{\theta}$) dominates that with $\lambda = 1$ (all traders observe $\tilde{\theta}$), since welfare improves for every trader when λ switches from one to zero, regardless of the realization of endowment shocks. In other words, the full-informed-trading equilibrium is *not* Pareto-optimal. Therefore, the important question is whether the no-informed-trading equilibrium also dominates any other equilibrium with $\lambda \in (0, 1)$. If so, one may conclude that (at least in this particular model setting), the social value of asymmetric information is strictly negative, and traders can be made better off if no one observes $\tilde{\theta}$ (by increasing the cost parameter μ). However, in the following we show (numerically) that this is *not* the case.

In Figure 4.1, Panels (A) and (B) show that the welfare improvement region shrinks in endowment shock e, but expands in the idiosyncratic endowment risk v_u, which has the following implications. First, informed trading is likely to hurt those traders who demand liquidity, whose optimal demand x_i^* is more driven by the endowment shock e_i rather than the speculative component. Intuitively, informed trading distorts risk-sharing, which exerts a negative effect on their welfare; once more, we recognize the *Hirshleifer effect.* Moreover, for those traders with a large hedging demand, the Hirshleifer effect can dominate the asymmetric-information effect.

Second, informed trading is more likely to hurt traders' welfare when individual trader's endowment shock e_i is more informative about the aggregate endowment shock $\tilde{z}$. Intuitively, taking λ as given, a higher correlation between e_i and $\tilde{z}$ helps uninformed traders to make more accurate forecasts about the future payoff $\tilde{D}$ conditional on price P. However, since more payoff uncertainty is resolved, it also brings more distortion to risk-sharing, i.e., both the Hirshleifer and risk-return effects are stronger, leading to a large welfare cost. Furthermore, note that when $v_u \to \infty$ and $e = 0$, the welfare improvement region becomes identical to that under the baseline model in Figure 3.1 Panel (A) and the theoretical results in Proposition 3.2 hold.

Next, Panels (C) and (D) show that the optimal state λ^*, where welfare $\mathcal{W}(\lambda^*, e_i)$ is maximized, differs between traders due to the heterogeneity in endowment shocks

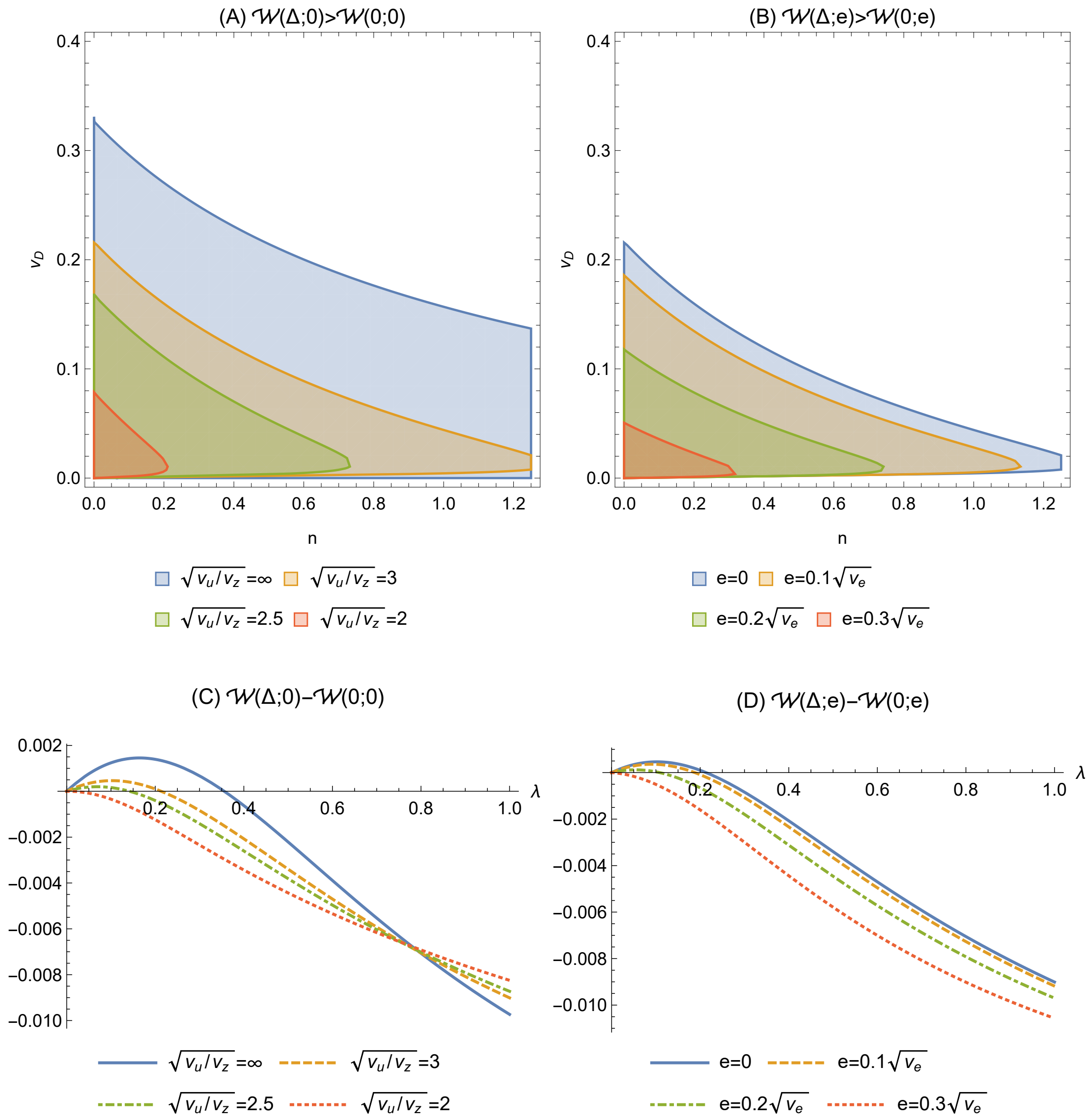

FIGURE 4.1. Panels A and B show that regions marked by $\{n, v_D\}$ in which $\mathcal{W}(\Delta; e) > \mathcal{W}(0; e)$, where $\Delta = 0.01$, $e = 0$ and $\sqrt{v_u/v_z} \in \{\infty, 3, 2.5, 2\}$ in Panels A, and $e/\sqrt{v_e} \in \{0, 0.1, 0.2, 0.3\}$ and $\sqrt{v_u/v_z} = 3$ in Panels B. Panels C and D show the welfare improvement $\mathcal{W}(\Delta; e) - \mathcal{W}(0; e)$, where $n = 0.1$. Here $\alpha = v_z = 1$.

e_i. Since λ^* is decreasing in e_i, λ^* is the largest for purely speculative traders with $e_i = 0$, i.e., liquidity suppliers. For traders with a sufficiently large e_i, $\lambda^* = 0$ becomes more likely the optimal state. Therefore, differently from the baseline model, when the noise demand is endogenized using endowment shocks, there exists not just one, but possibly *multiple* Pareto-optimal states.

In fact, if we define λ_0^* as the optimal state where $\mathcal{W}(\lambda_0^*, 0)$ is the maximum welfare for the purely speculative traders, then any state $\lambda^* \in [0, \lambda_0^*]$ is a Pareto-optimal state. Therefore, unless there is excessive informed trading, i.e., $\lambda > \lambda_0^*$, the asymmetric-information effect can lead to welfare improvement for those traders who are liquidity suppliers with a relatively small hedging demand, despite the negative Hirshleifer and risk-return effects. Thus, in those cases, neither increasing nor reducing the amount of informed trading is Pareto-improving.

5. Conclusion

In this paper, we have examined the effect of information acquisition uncertainty on traders' welfare. When traders make probabilistic choices strategically for observing a costly private signal, more informed trading gives rise to a positive asymmetric-information effect, opening the doors to potential welfare improvement.

We have shown that the asymmetric-information effect can overcome both the risk-return and Hirshleifer (risk-sharing) effects, and thus informed trading can lead to overall welfare improvement. This is more likely to occur when the current level of informed trading, trading profitability (measured by the Sharpe ratio), and signal precision are low, which help to weaken the risk-return and Hirshleifer effects.

Overall, our results suggest that, unless the Sharpe ratio is too high, or there is an excessive amount of informed trading in the market, it is not necessary for regulators to level the playing field by discouraging traders from gathering private information about future payoffs, since a small degree of asymmetric information can actually make traders better off compared to the no-informed-trading equilibrium.

Appendix A. Proofs

A.1. **Proof of Lemma 2.1.** Since traders' terminal wealth $\tilde{W}_i = x_i(\tilde{D} - \tilde{P})$ is normally distributed, given his type, trader i's optimal demand is given by $x_i^* = \frac{\mathbb{E}[\tilde{D}-\tilde{P}|\mathcal{F}_i]}{\alpha \mathbb{V}ar[\tilde{D}-\tilde{P}|\mathcal{F}_i]}$. For the informed trader who observes θ and P, $\mathbb{E}[\tilde{D}-\tilde{P}|\theta, P] = d+\theta-P, \mathbb{V}ar[\tilde{D}-\tilde{P}|\theta, P] = v_\epsilon$. On the other hand, for the uninformed trader who only observes P, $\mathbb{E}[\tilde{D} - \tilde{P}|P] = (1 - \beta_{P,\theta})(d - P), \mathbb{V}ar[\tilde{D} - \tilde{P}|P] = v_\theta(1 - \rho^2_{\theta,P})$. Substitution leads to (2.5). Next, we compute trader i's expected utility given their information set $\mathcal{F}_i$, which yields $\mathbb{E}[-e^{-\alpha \tilde{W}_i}|\mathcal{F}_i] = -\exp\left\{-\frac{1}{2}\frac{\chi_i^2}{v_{D|\mathcal{F}_i}}\right\}$. For the informed, $v_{D|\mathcal{F}_I} = v_\epsilon$ and $v_{\chi I} = \mathbb{V}ar\left[(1-b_\theta)\theta + b_z z\right] = (1 -$

$b_\theta)^2 v_\theta + b_z^2 v_z$, whereas for the uninformed trader, $v_{D|\mathcal{F}_U} = v_{D|P} = v_\epsilon + (1-\rho_{\theta,P}^2)v_\theta = v_\epsilon + (1-\beta_{P,\theta}b_\theta)v_\theta$ and $v_{\chi_U} = \mathbb{V}ar[(1-\beta_{P,\theta})(d-P)] = \mathbb{V}ar[(1-\beta_{P,\theta})(-b_\theta\theta + b_z z)] = (1-\beta_{P,\theta})^2(b_\theta^2 v_\theta + b_z^2 v_z)$. Since the conditional expectation $\chi_i = \mathbb{E}\left[\tilde{D} - \tilde{P}|\mathcal{F}_i\right]$ itself is a normally distributed random variable for both informed and uninformed traders, we can use the following standard result to compute trader i's unconditional expected utility.

Lemma A.1. *Let $X \in \mathbb{R}^n$ be a normally distributed random vector with mean μ and variance-covariance matrix Σ. Let $b \in \mathbb{R}^n$ be a given vector, and $A \in \mathbb{R}^{n\times n}$ a symmetric matrix. If $I - 2\Sigma A$ is positive definite, then $\mathbb{E}\left[\exp\{b^\top X + X^\top AX\}\right]$ is well defined, and given by*

$$
\begin{aligned}
\mathbb{E}\left[\exp\{b^\top X + X^\top AX\}\right] =& |I-2\Sigma A|^{-1/2}\exp\{b^\top\mu + \mu^\top\Sigma\mu \\
&+ \frac{1}{2}(b+2A\mu)^\top(I-2\Sigma A)^{-1}\Sigma(b+2A\mu)\}.
\end{aligned}
$$

Applying Lemma A.1 to the conditional expected utility with $X = \chi_i$, $A = -1/2(v_{D|\mathcal{F}_i})^{-1}$, $\Sigma = v_{\chi_i}$, $b = 0$, $\mu = 0$, some simple but tedious computations lead to the desired result and the expressions for $\xi_I(\lambda)$ and $\xi_U(\lambda)$ in (2.6). Thus, assuming the concavity condition $\mathcal{U}''(p_i,\lambda) < 0$ is satisfied, trader i's optimal choice of p_i is determined by the first order condition, $\alpha\mu g(p_i^*) = -\frac{V_I(\lambda)-V_U(\lambda)}{\lambda V_I(\lambda)+(1-\lambda)V_U(\lambda)} = \frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}$, which leads to (2.7).

A.2. **Proof of Lemma 2.2.** On the optimization problem in (2.2), let $\bar{V}(\lambda) \equiv \lambda V_I(\lambda) + (1-\lambda)V_U(\lambda)$ and $\bar{V}(p;\lambda) \equiv pV_I(\lambda) + (1-p)V_U(\lambda)$, then we have[14]

$$
\begin{aligned}
\mathcal{U}'(p;\lambda) &= e^{\alpha\mu c(p)}[\alpha\mu g(p)\bar{V}(p,\lambda) + [V_I(\lambda) - V_U(\lambda)]] \\
\mathcal{U}''(p;\lambda) &= \alpha\mu g(p)e^{\alpha\mu c(p)}\left[\left(\alpha\mu g(p) + \frac{g'(p)}{g(p)}\right)\bar{V}(p,\lambda) + 2[V_I(\lambda) - V_U(\lambda)]\right].
\end{aligned}
$$

Therefore, the necessary and sufficient condition for $\mathcal{U}''(p;\lambda) \le 0$ is

$$
\left(\alpha\mu g(p) + \frac{g'(p)}{g(p)}\right)\bar{V}(p,\lambda) + 2[V_I(\lambda) - V_U(\lambda)] \le 0. \tag{A.1}
$$

Note that in equilibrium, $\alpha\mu = -\frac{1}{g(\lambda)}\frac{V_I(\lambda)-V_U(\lambda)}{\bar{V}(\lambda)} = \frac{1}{g(\lambda)}\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}$. Also, note that $\bar{V}(p,\lambda) = [1-p\gamma(\lambda)]V_U(\lambda), V_I(\lambda) - V_U(\lambda) = -\gamma(\lambda)V_U(\lambda)$. Therefore, in equilibrium, (A.1) becomes

$$
\left[\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}\frac{g(p)}{g(\lambda)} + \frac{g'(p)}{g(p)}\right](1-p\gamma(\lambda)) - 2\gamma(\lambda) \ge 0, \tag{A.2}
$$

[14] We drop the subscript i for the remainder of the proof, in order to simplify the notation.

which leads to condition (2.9). Moreover, for a sufficient condition, note that (A.1) can be written as

$$V_I(\lambda) - V_U(\lambda) \le -\frac{1}{2}\left[\frac{g(p)}{g(\lambda)}\frac{V_I(\lambda)-V_U(\lambda)}{-\bar{V}(\lambda)} + \frac{g'(p)}{g(p)}\right]\bar{V}(p,\lambda),$$

which can be written as

$$\underbrace{\left[1 - \frac{1}{2}\frac{\bar{V}(p,\lambda)}{\bar{V}(\lambda)}\frac{g(p)}{g(\lambda)}\right]}_{\le 1}[V_I(\lambda) - V_U(\lambda)] \le -\frac{1}{2}\frac{g'(p)}{g(p)}\bar{V}(p,\lambda). \tag{A.3}$$

Therefore, a sufficient condition for $\mathcal{U}''(p;\lambda) \le 0$ is given by $V_I(\lambda) - V_U(\lambda) \le -\frac{1}{2}\frac{g'(p)}{g(p)}\bar{V}(p,\lambda)$, which is equivalent to

$$\left[1 + \frac{1}{2}\frac{g'(p)}{g(p)}p\right]\underbrace{\frac{V_I(\lambda)-V_U(\lambda)}{-V_U(\lambda)}}_{\gamma(\lambda)} \le \frac{1}{2}\frac{g'(p)}{g(p)} \tag{A.4}$$

that simplifies to condition (2.10). With $c(p) = p^2$, the condition is further simplified to $\gamma(\lambda) < 1/3$ and, finally, to condition (2.11).

A.3. **Proof of Proposition 2.3.** If the sufficient condition for $\mathcal{U}''(p;\lambda) \le 0$ is satisfied, the Nash equilibrium for the choice of probability p_i to observe the private signal $\tilde{\theta}$ must be symmetric, since traders are homogeneous, i.e., $p_i^* = \lambda$ for all $i \in (0,1)$, from which we obtain the equilibrium λ in (2.15). Next, to solve for the equilibrium price P, we substitute the linear equilibrium price $P = d + b_\theta\tilde{\theta} - b_z\tilde{z}$ into traders' optimal demand functions in (2.5), from which we obtain

$$x_I^*(\theta, P) = \frac{d+\theta-P}{\alpha v_\epsilon} \qquad \text{and} \qquad x_U^*(P) = \frac{\left(1 - \frac{b_\theta v_\theta}{b_\theta^2 v_\theta + b_z^2 v_z}\right)(d-P)}{\alpha\left(v_\epsilon + \frac{b_z^2 v_z}{b_\theta^2 v_\theta + b_z^2 v_z}v_\theta\right)}. \tag{A.5}$$

Then, by applying the market clearing condition, $\lambda x_I^*(\theta,P) + (1-\lambda)x_U^*(P) = \tilde{z}$, we obtain the equilibrium price, $\tilde{P} = d + \frac{\lambda\bar{v}}{v_\epsilon}\tilde{\theta} - \alpha\bar{v}\tilde{z}$, where $\frac{1}{\bar{v}} = \frac{\lambda}{v_\epsilon} + (1-\lambda)\left(1 - \frac{b_\theta v_\theta}{b_\theta^2 v_\theta + b_z^2 v_z}\right) / \left[v_\epsilon + \frac{b_z^2 v_z}{b_\theta^2 v_\theta + b_z^2 v_z}v_\theta\right]$. Thus, by matching coefficient to the conjectured equilibrium price, we obtain $b_\theta = \frac{\lambda\bar{v}}{v_\epsilon}$ and $b_z = \alpha\bar{v}$. Since $b_\theta = \lambda b_z/(\alpha v_\epsilon)$, we obtain an explicit solution for $\bar{v}$ by solving

$$\frac{\lambda}{v_\epsilon} + \frac{(1-\lambda)\left(1 - \frac{(\lambda b_z/\alpha)\,v_\theta/v_\epsilon}{(\lambda b_z/\alpha)^2 v_\theta/v_\epsilon^2 + b_z^2 v_z}\right)}{v_\epsilon + \frac{b_z^2 v_z}{(\lambda b_z/\alpha)^2 v_\theta/v_\epsilon^2 + b_z^2 v_z}v_\theta} = \frac{b_z}{\alpha}$$

for b_z and substituting the solution back into the expression for $1/\bar{v}$.

A.4. **Proof of Lemma 3.1.** After traders' types are realized, the Sharpe ratios, $\xi_I(\lambda)$ and $\xi_U(\lambda)$, of the informed and uninformed traders' optimal portfolios are given by (2.6). Substituting the equilibrium value for b_θ and b_z in (2.17) then leads to the desired result in (3.4). Then, it is straightforward to show that $\xi_I(\lambda) > \xi_U(\lambda)$, $\xi_I'(\lambda) < 0$, and $\xi_U'(\lambda) < 0$ for $\lambda \in [0, 1]$.

A.5. **Proof of Proposition 3.2.** In equilibrium, the welfare function is given by $\mathcal{W}(\lambda) = \bar{V}(\lambda)\exp(\Phi(\lambda))$, where $\bar{V}(\lambda) = \lambda V_I(\lambda) + (1-\lambda)V_U(\lambda) = (1-\lambda\gamma)V_U(\lambda)$ and $\Phi(\lambda) = \frac{c(\lambda)}{g(\lambda)}\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}$. Hence

$$\mathcal{W}'(\lambda) = e^{\Phi(\lambda)}(-\bar{V}(\lambda))\left[\frac{\bar{V}'(\lambda)}{-\bar{V}(\lambda)} - \Phi'(\lambda)\right].$$

and $\mathcal{W}'(\lambda) \geq 0$ if and only if $\frac{\bar{V}'(\lambda)}{-\bar{V}(\lambda)} \geq \Phi'(\lambda)$. Note that $\frac{\bar{V}'(\lambda)}{-\bar{V}(\lambda)} = \frac{V_U'(\lambda)}{-V_U(\lambda)} + \frac{\gamma(\lambda)+\lambda\gamma'(\lambda)}{1-\lambda\gamma(\lambda)}$ and $\Phi'(\lambda) = \frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}\left[1 - \frac{c(\lambda)g'(\lambda)}{g^2(\lambda)} + \frac{c(\lambda)}{g(\lambda)}\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}\right] + \frac{c(\lambda)}{g(\lambda)}\frac{\gamma'(\lambda)}{(1-\lambda\gamma(\lambda))^2}$. Therefore $\mathcal{U}'(\lambda) \geq 0$ if and only if

$$\frac{V_U'(\lambda)}{V_U(\lambda)} \leq \frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}\frac{c(\lambda)}{g(\lambda)}\left[\frac{g'(\lambda)}{g(\lambda)} - \frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}\right] + \left(\frac{\lambda}{1-\lambda\gamma(\lambda)} - \frac{c(\lambda)}{g(\lambda)}\frac{1}{(1-\lambda\gamma(\lambda))^2}\right)\gamma'(\lambda).$$

Call $S(\lambda)$ the r.h.s. of the latter inequality. Since $c(p) = p^2$, it follows that

$$S(\lambda) = \frac{1}{2}\left[1 - \frac{\lambda\gamma(\lambda)}{1-\lambda\gamma(\lambda)}\right] + \frac{\lambda}{2}\left[2 - \frac{1}{1-\lambda\gamma(\lambda)}\right]\frac{\gamma'(\lambda)}{\gamma(\lambda)} = \frac{1}{2}\frac{1-2\lambda\gamma(\lambda)}{1-\lambda\gamma(\lambda)}\left[1 + \lambda\frac{\gamma'(\lambda)}{\gamma(\lambda)}\right],$$

which leads to $\frac{V_U'(\lambda)}{V_U(\lambda)} \leq \frac{1}{2}\frac{[1-2\lambda\gamma(\lambda)][\gamma(\lambda)+\lambda\gamma'(\lambda)]}{[1-\lambda\gamma(\lambda)]^2}$. Finally, since $\frac{V_U'(\lambda)}{V_U(\lambda)} = \frac{-\xi_U'(\lambda)}{2[1+\xi_U(\lambda)]}$, we obtain (3.7). At $\lambda = 0$, $\frac{V_U'(0)}{V_U(0)} = \frac{-\xi_U'(0)}{1+\xi_U(0)} = \frac{2n\xi_0}{1+\xi_0}$. Applying this to condition (3.7) at $\lambda = 0$ leads to condition (3.8).

A.6. **Proof of Corollary 3.1.** Note that the necessary and sufficient condition depends only on the signal precision n and the (squared) Sharpe ratio ξ_0. More specifically, the condition (3.8) for $\mathcal{W}'(0) \geq 0$ can be rewritten as

$$\xi_0 \leq \frac{\xi_0^*}{1-\xi_0^*}, \qquad \xi_0^*(n) = \frac{1}{2n}\left(1 - \frac{1}{\sqrt{1+n}}\right). \tag{A.6}$$

It can be verified that $\xi^*(n)$ is a decreasing function of n. Therefore, for $n \leq 5/4$, $\frac{2}{15} = \xi_0^*\left(\frac{5}{4}\right) \leq \xi_0^*(n) < \lim_{n\to 0}\xi_0^*(n) = \frac{1}{4}$. Note that $\xi_0 = 2/13$ when $\xi_0^* = 2/15$ and $\xi_0 = 1/3$ when $\xi_0^* = 1/4$. Therefore, when $\xi_0 \leq 2/13$, the condition (A.6) is satisfied, which implies $\mathcal{W}'(0) \geq 0$. Finally, $\xi_0 \leq 1/3$ becomes a necessary condition for the welfare improvement near $\lambda = 0$.

A.7. **Proof of Corollary 3.3.** From (2.18), we have $\frac{v_\epsilon}{\bar{v}} = \lambda + [1-\lambda]/[1+n+\frac{n(1+n)}{\xi_0}\lambda]$. Then

$$\frac{\partial(v_\epsilon/\bar{v})}{\partial\lambda} = \frac{[1+n+\frac{n(1+n)}{\xi_0}\lambda]^2 - [1+n+\frac{n(1+n)}{\xi_0}]}{[1+n+\frac{n(1+n)}{\xi_0}\lambda]^2}. \tag{A.7}$$

Concerning (i), when $\xi_0 > 1$, condition $\frac{\partial(1/\bar{v})}{\partial\lambda} > 0$ always holds, meaning that the risk premium $\bar{v}$ decreases in λ. Finally, when $\xi_0 \leq 1$, we have $\frac{\partial(1/\bar{v})}{\partial\lambda} \leq 0$ if and only if $1+n+\frac{n(1+n)}{\xi_0}\lambda \leq \sqrt{(1+n)+\frac{n(1+n)}{\xi_0}}$, which leads to the result (ii) of Corollary 3.3.

A.8. **Proof of Proposition 4.1.** In the following proof, we drop the trader specific subscript. First, note that each trader's expected utility conditional on his information set is given by

$$V_K(e;\lambda) \equiv \mathbb{E}\left\{\mathbb{E}\left[-e^{-\alpha\tilde{W}}|\mathcal{F}\right]|e\right\} = \mathbb{E}\left[-\exp\left\{-\alpha eP - \frac{1}{2}\frac{\chi^2}{v}\right\}|e\right], \qquad K \in \{I,U\}, \tag{A.8}$$

where $\chi \equiv \mathbb{E}[\tilde{D}-\tilde{P}|\mathcal{F}]$ and $v \equiv \mathbb{V}ar[\tilde{D}-\tilde{P}|\mathcal{F}]$, respectively. First, since (given the endowment shock e) χ and P follow a bivariate normal distribution with mean vector and covariance matrix given by

$$\mu = \begin{pmatrix} \mu_{\chi|e} \\ \mu_{P|e} \end{pmatrix} \qquad \text{and} \qquad \Sigma = \begin{pmatrix} v_{\chi|e} & \sigma_{(\chi,P)|e} \\ \sigma_{(\chi,P)|e} & v_{P|e} \end{pmatrix}, \tag{A.9}$$

where $\mu_{\chi|e} \equiv \mathbb{E}[\chi|e]$, $\mu_{P|e} \equiv \mathbb{E}[P|e]$, $v_{\chi|e} \equiv \mathbb{V}ar[\chi|e]$, $v_{P|e} \equiv \mathbb{V}ar[P|e]$ and $\sigma_{(\chi,P)|e} \equiv \mathbb{C}ov[\chi,P|e]$. Thus, using Lemma A.1 we can establish the following result,

$$V_K(e;\lambda) = -\exp\left\{-\frac{\mu_{\chi|e}^2 + \alpha e\left[2\nu\mu_{P|e} + 2\mu_{\chi|e}\sigma_{(\chi,P)|e} + \alpha e\left(\sigma_{(\chi,P)|e}^2 - \nu v_{P|e}\right)\right]}{2\nu}\right\}\sqrt{\frac{v}{\nu}}, \tag{A.10}$$

where $\nu = v + v_{\chi|e}$. If the trader is informed ($K=I$), i.e., $\mathcal{F} = \{\theta, P, e\}$, since $\chi = d+\theta-P$ and $v = v_\epsilon$, we obtain that

$$\mu_{\chi|e} = -\beta_{e,P}e, \;\; \mu_{P|e} = d+\beta_{e,P}e, \;\; v_{\chi|e} = v_\theta + v_{P|e} - 2\sigma_{\theta,P}, \;\; \sigma_{(\chi,P)|e} = \sigma_{\theta,P} - v_{P|e}. \tag{A.11}$$

Substituting (A.11) into (A.10) leads to the expected utility of an informed trader in (4.8) with $v = v_\epsilon$. On the other hand, if the trader is uninformed ($K=U$), i.e., $\mathcal{F} = \{P,e\}$, since $\chi = (1-\kappa)(d-P) - \kappa\beta_{e,P}e$ and $v = v_D - \kappa\sigma_{\theta,P}$, $\kappa = \sigma_{\theta,P}/v_{P|e}$, we obtain that

$$\mu_{\chi|e} = -\beta_{e,P}e, \;\; \mu_{P|e} = d+\beta_{e,P}e, \;\; v_{\chi|e} = (1-\kappa)^2 v_{P|e}, \;\; \sigma_{(\chi,P)|e} = -(1-\kappa)v_{P|e}. \tag{A.12}$$

Substituting (A.12) into (A.10) leads to the expected utility of an uninformed trader in (4.8) with $v = v_D - \kappa\sigma_{\theta,P}$.

A.9. **Proof of Proposition 4.2.** Substituting the optimal demands $x_U^*(P,e)$ and $x_I^*(\theta,P,e)$ in (4.5) and (4.6) into the market clearing condition (4.10) leads to the following,

$$\frac{(d-P)}{\alpha\bar{v}} + \frac{\lambda}{\alpha v_\epsilon}\tilde{\theta} = \left[1 + (1-\lambda)\frac{\kappa\beta_{e,P}}{\alpha v_U}\right]\tilde{z} \tag{A.13}$$

where $\frac{1}{\bar{v}} \equiv \frac{\lambda}{v_\epsilon} + \frac{1-\lambda}{\tilde{v}_U}$ and $\tilde{v}_U = \frac{v_U}{1-\kappa}$. Thus, the equilibrium price can be written as

$$P = d + \underbrace{\frac{\lambda\bar{v}}{v_\epsilon}}_{b_\theta}\,\tilde{\theta} - \underbrace{\alpha\bar{v}\left[1 + (1-\lambda)\frac{\kappa\beta_{e,P}}{\alpha v_U}\right]}_{b_z}\tilde{z}. \tag{A.14}$$

Therefore, we obtain

$$x \equiv \frac{b_\theta}{b_z} = \frac{1}{\alpha v_\epsilon}\frac{\lambda}{1 + (1-\lambda)\frac{\kappa\beta_{e,P}}{\alpha v_U}},$$

which can be written as

$$x = \frac{1}{\alpha v_\epsilon}\left[\lambda - (1-\lambda)\left(\kappa\beta_{e,P}\frac{v_\epsilon}{v_U}\right)x\right]. \tag{A.15}$$

Since $v_U = v_D - \kappa\sigma_{\theta,P}$, $\kappa = \sigma_{\theta,P}/v_{P|e}$ and $\beta_{e,P} = \sigma_{e,P}/v_e$, also,

$$\sigma_{e,P} = -b_z v_z,\;\; \sigma_{\theta,P} = b_\theta v_\theta,\;\; v_{P|e} = b_\theta^2 v_\theta + b_z^2 v_{z|e},\;\; v_{z|e} = (v_z^{-1} + v_u^{-1})^{-1}, \tag{A.16}$$

we can obtain that

$$-\left(\kappa\beta_{e,P}\frac{v_\epsilon}{v_U}\right)x = \frac{v_z v_\epsilon v_\theta x^2}{v_u v_z v_D + v_e v_\epsilon v_\theta x^2} = \frac{1}{v_e/v_z + x^{-2}v_u(v_\theta^{-1} + v_\epsilon^{-1})}. \tag{A.17}$$

Substituting (A.17) back into (A.15) leads to (4.13). Next, given x, we substitute (A.16) into the expression for b_θ and obtain that

$$b_\theta = \frac{\lambda\bar{v}}{v_\epsilon} = \frac{\lambda b_z\left(v_u v_z v_D + v_e v_\epsilon v_\theta x^2\right)}{b_z x^2 v_z v_\epsilon v_\theta - v_e v_\epsilon v_\theta x(1-\lambda) + b_z v_u(v_\epsilon v_\theta x^2 + v_z v_D\lambda)}. \tag{A.18}$$

Since $b_z = b_\theta/x$, (A.18) can be simplified to $b_\theta = \frac{v_e v_\epsilon v_\theta x^2 + v_u v_z v_D\lambda}{v_e v_\epsilon v_\theta x^2 + v_u v_z(v_\epsilon + v_\theta\lambda)}$, which leads to the expression in (4.12).

A.10. **Proof of Corollary 4.3.** For $\lambda = 0$, since the equilibrium price $\tilde{P} = d - \alpha v_D\tilde{z}$, we have

$$v_{P|e} = \alpha^2 v_D^2 v_{z|e},\;\; \sigma_{\theta,P} = 0,\;\; \beta_{e,P} = -\alpha v_D\frac{v_z}{v_e},\;\; \nu = v_D(1 + \alpha^2 v_D v_{z|e}). \tag{A.19}$$

Substituting (A.19) into (4.8) leads to (4.16). On the other hand, for $\lambda = 1$, since the equilibrium price $\tilde{P} = d + \tilde{\theta} - \alpha v_\epsilon\tilde{z}$, we have

$$v_{P|e} = v_\theta + \alpha^2 v_\epsilon^2 v_{z|e},\;\; \sigma_{\theta,P} = v_\theta,\;\; \beta_{e,P} = -\alpha v_\epsilon\frac{v_z}{v_e},\;\; \nu = v_\epsilon(1 + \alpha^2 v_\epsilon v_{z|e}). \tag{A.20}$$

Substituting (A.20) into (4.8) leads to (4.17).

Appendix B. Existence and uniqueness of Nash equilibrium

This appendix examines the existence and uniqueness of the Nash equilibrium with respect to parameter μ, which measures the sensitivity to the cost of information. For convenience, we define $\xi_1 = \alpha^2 v_z v_\epsilon$. Note that $\xi_1 = \xi_I(1)$, representing the squared Sharpe ratio of informed traders when $\lambda = 1$. Intuitively, in equilibrium, $\lambda \to 0$ as $\mu \to \infty$; $\lambda = 1$ when μ is small enough; otherwise $\lambda \in (0,1)$. This is demonstrated as follows.

Proposition B.1. *Assume $c(p) = p^2$ and condition* (2.11) *holds. Then*

(i) $\lambda = 0$ *as* $\mu \to \infty$*;*
(ii) $\lambda = 1$ *when* $\mu \le \bar{\mu} := \frac{1}{2\alpha}\frac{\gamma_1}{1-\gamma_1}$*, where* $\gamma_1 \equiv \gamma(1) = 1 - \sqrt{\frac{n+\xi_1}{n+\xi_0}}$*;*
(iii) *there exists a unique* $\lambda \in (0,1)$ *when* $\mu > \bar{\mu}$*;*

Moreover, the equilibrium price P, satisfying (2.16)*, is characterized by parameters b_θ and b_z defined in* (2.17) *and* (2.18)*, evaluated at the equilibrium λ.*

Proof: Note that $\gamma(\lambda) \in (0,1)$. With $c(p) = p^2$, from the equilibrium condition $2\alpha\mu\lambda = \gamma(\lambda)/[1-\gamma(\lambda)]$, it is easy to see that $\lambda \to 0$ as $\mu \to \infty$. For $\lambda = 1$, we have $\mu = \frac{1}{2\alpha}\frac{\gamma(1)}{1-\gamma(1)}$. It remains to discuss the case $\mu > \bar{\mu}$. To this aim, note that, in case of $c(p) = p^2$, the fixed point (2.15) is equivalent to

$$\lambda^2 - \frac{1}{\gamma(\lambda)}\lambda + \frac{1}{2\alpha\mu} = 0. \tag{B.1}$$

By defining

$$F_1(\lambda) = \frac{1}{2\gamma(\lambda)} - \frac{1}{2\gamma(\lambda)}\sqrt{1 - \frac{2\gamma^2(\lambda)}{\alpha\mu}}; \quad F_2(\lambda) = \frac{1}{2\gamma(\lambda)} + \frac{1}{2\gamma(\lambda)}\sqrt{1 - \frac{2\gamma^2(\lambda)}{\alpha\mu}},$$

(B.1) can be rewritten as $[\lambda - F_1(\lambda)][\lambda - F_2(\lambda)] = 0$. Assuming $\mu \ge 2\gamma^2(\lambda)/\alpha$ (otherwise the fixed point has no solution and $\lambda = 1$), F_1 and F_2 are well-defined. It is not difficult to show that $0 < F_1(\lambda) \le F_2(\lambda)$. Therefore, since $F_1(0) > 0$, one solution to (B.1) exists if and only if $F_1(1) < 1$. This condition is exactly $\mu > \bar{\mu}$. Finally, concerning uniqueness, note that $dF_1(\lambda)/d\lambda < 0$. Indeed,

$$\frac{dF_1(\lambda)}{d\lambda} = -\frac{\gamma'(\lambda)}{2\gamma^2(\lambda)}\left(1 - \sqrt{1 - \frac{2\gamma^2(\lambda)}{\alpha\mu}}\right) + \frac{\gamma'(\lambda)}{\alpha\mu\sqrt{1 - \frac{2\gamma^2(\lambda)}{\alpha\mu}}} = \frac{\gamma'(\lambda)}{\gamma(\lambda)}\frac{F_1(\lambda)}{\sqrt{1 - \frac{2\gamma^2(\lambda)}{\alpha\mu}}} < 0.$$

Negativity is due to the fact that $\gamma'(\lambda) < 0$, $\gamma(\lambda) > 0$, and $F_1(\lambda) > 0$. By monotonicity, $\lambda = F_1(\lambda)$ provides at most one solution. Therefore, if a second solution $\tilde{\lambda}$ to the fixed point exists, it must solve $\tilde{\lambda} = F_2(\tilde{\lambda})$. By definition $F_2(\lambda) > \frac{1}{2\gamma(\lambda)}$; therefore, as soon as $\gamma(\lambda) < 1/3$, we would have $\tilde{\lambda} = F_2(\tilde{\lambda}) > 3/2$, which is not feasible. This proves that the solution to the fixed point is unique as soon as the sufficient condition for concavity, $\gamma(\lambda) < 1/3$, is satisfied. $\square$

Proposition B.1 provides a sufficient condition for the existence of a unique non-trivial Nash equilibrium $0 < \lambda < 1$. In general, the equilibrium fraction of informed traders is expected to increase as traders become less sensitive to the cost function. Put differently, we expect λ to be decreasing in μ. However, it turns out that such monotonicity is not guaranteed in general.

Proposition B.2. *The equilibrium $\lambda = \lambda(\mu)$ is decreasing in μ if and only if*

$$\frac{\Gamma'(\lambda)}{\Gamma(\lambda)} \leq \frac{g'(\lambda)}{g(\lambda)}, \qquad \Gamma(\lambda) = \frac{\gamma(\lambda)}{1 - \lambda\gamma(\lambda)}; \tag{B.2}$$

or equivalently

$$\frac{\gamma^2(\lambda) + \gamma'(\lambda)}{1 - \lambda\gamma(\lambda)} \leq \frac{g'(\lambda)}{g(\lambda)}. \tag{B.3}$$

For $c(p) = p^2$, condition (B.3) *becomes*

$$\lambda[\gamma^2(\lambda) + \gamma(\lambda) + \gamma'(\lambda)] \leq 1. \tag{B.4}$$

In particular, at $\lambda = 0$, condition (B.4) *is always satisfied; while at $\lambda = 1$, it becomes*

$$\sqrt{\frac{\xi_1 + n}{\xi_0 + n}}\left[1 + \frac{\xi_1 + n}{\xi_0 + n}\right] \leq 3\frac{\xi_1 + n}{\xi_0 + n} + \frac{n^2\xi_1}{[\xi_0 + n]^2}. \tag{B.5}$$

Proof: In equilibrium, $\alpha\mu g(\lambda) = -\frac{V_I(\lambda) - V_U(\lambda)}{V(\lambda)} = \frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)} = \Gamma(\lambda)$. For $\lambda = \lambda(\mu)$, taking the derivative w.r.t. μ, we have $\alpha g(\lambda) = -\lambda'(\mu)[\Gamma'(\lambda) - \frac{g'(\lambda)}{g(\lambda)}\Gamma(\lambda)]$. Therefore $\lambda'(\mu) \leq 0$ if and only if (B.2) holds. Applying $c(p) = p^2$ to condition (B.2) leads to condition (B.3). Clearly, (B.3) holds for $\lambda = 0$. For $\lambda = 1$, condition (B.3) becomes $\gamma_1^2 + \gamma_1 + \gamma_1' \leq 1$. Since $\gamma(\lambda) = 1 - f(\lambda)$, this is equivalent to $1 + f_1^2 \leq 3f_1 + f_1'$. Using the fact that $f(\lambda) = \sqrt{\frac{\xi_1 + n\lambda^2}{\xi_0 + n\lambda^2}}$, we obtain condition (B.5). $\square$

Proposition B.2 provides conditions for the equilibrium $\lambda = \lambda(\mu)$ to be decreasing in μ, or, put differently, it provides a less restrictive condition for the uniqueness of the Nash equilibrium λ. Note that, since $\lambda < 1$ and $\gamma'(\lambda) < 0$, condition (B.4) is always satisfied under condition (2.11). This leads to Proposition B.3.

Proposition B.3. *Consider the optimization problem* (2.2) *with $c(p) = p^2$. Suppose that $\mu > \bar{\mu} := \frac{1}{2\alpha}\frac{\gamma_1}{1-\gamma_1}$ and $\gamma(\lambda) < 1/3$, where $\gamma_1 \equiv \gamma(1) = 1 - \sqrt{\frac{n+\xi_1}{n+\xi_0}}$. Then, there exists a unique equilibrium (P, λ) such that (i) $\lambda \in (0, 1)$ solves* (2.15) *and is decreasing in μ; and (ii) P is given by* (2.16).

The condition $\gamma(\lambda) < 1/3$ for the existence and uniqueness in Proposition B.3 indicates that the relative utility gain of being informed should be small. To better understand this condition, we note from $\gamma'(\lambda) < 0$ that $\gamma(\lambda) \leq \gamma(0) = 1 - 1/\sqrt{1+n}$.

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