Default Risk and the Pricing of U.S. Sovereign Bonds

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Abstract

We examine the relative pricing of nominal Treasury bonds and Treasury inflation-protected securities (TIPS) in the presence of United States default risk. Higher bond yields are associated with a higher U.S. credit default swap premium, but more so for TIPS. This leads to a narrower breakeven inflation (BEI). An estimated no-arbitrage model shows BEI is related to differing expectations of loss given default on the two Treasury securities and that most of the relative *mispricing* after the crisis can be attributed to default risk. Our finding suggests credit risk is embedded in the pricing of U.S. sovereign debt.

JEL classification: E4, E6, G12.

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1 Introduction

Does United States Treasury default risk have the same impact on the pricing of all U.S. Treasury obligations? We investigate this question through the lens of the relative pricing of nominal and inflation protected Treasury securities. Our interest is motivated by the pairwise mispricing between nominal and real bonds documented in Fleckenstein, Longstaff and Lustig (2014). The authors show that a strategy replicating inflation-protected securities through inflation swaps, STRIPs, and nominal Treasuries generates large and persistent arbitrage profits. Their empirical analysis suggests that much of the profitability of the strategy is likely to be explained by slow-moving capital that prevented the profits from being arbitraged away. Our study asks if part of this differential might be accounted for by differences in exposures to default risk present in nominal and inflation-protected Treasury securities.

Our investigation of the role of default risk in this pricing differential may be surprising given the frequent treatment of Treasury obligations as default risk-free. However, the financial crisis of 2008-2009 and its aftermath have suggested that this perception may need to change. Chernov, Schmid and Schneider (2019) note that the premium paid to insure U.S. sovereign debt as measured by credit default swap (CDS) spreads rose to nearly 100 basis points during the crisis, and remained elevated since. The authors show that a macro-finance model with a non-trivial probability of sovereign default can replicate this pattern in the United States and other developed markets. Repeated political conflict over the debt ceiling in the United States in 2011, 2013, and 2017 has also contributed to questions about the risk-free status of U.S. sovereign debt. The 2011 conflict led to Standard and Poor's downgrading the status of the United States Treasury as an obligor from AAA to AA+.

Even if a default is possible, why might we see a resulting differential in the pricing of nominal and inflation-protected debt? History suggests that there is considerable uncertainty as to how forms of debt might be treated in the case of a sovereign default. As discussed in Duffie, Pedersen and Singleton (2003), sovereign defaults rarely play out in the way modeled in structural or reduced form models of corporate credit risk. Rather than a single event, a sovereign entity weighs the costs and benefits of continuing to pay its obligations against the reputational cost of default.

When default occurs, it is more likely that the debt will be restructured or renegotiated than that an outright liquidation will take place. This renegotiation involves a considerable amount of uncertainty. Duffie, Pedersen and Singleton (2003) examine the case of the Russian default on its ruble-denominated debt in 1998, and its impact on dollar-denominated MinFins. The authors show that uncertainty around the treatment of this debt, which was considered domestic, relative to foreign Eurobonds, had a large impact on the relative pricing of the obligations. Similarly, Zettelmeyer, Trebesch and Gulati (2019) examine the variation in ultimate recovery in present value terms of holders of Greek debt during the 2012 restructuring. All bondholders were provided the same package of securities, which implied large differences in present value loss given default across holders of different bonds.[1]

With this background in mind, our empirical work examines the question of whether the spread between like-maturity inflation swaps (ILS) and breakeven inflation (BEI), the difference between yields of nominal and inflation-protected U.S. Treasuries, is correlated with default risk. Using spreads on CDS written on U.S. Treasury obligations to proxy for overall risk of default, we find a statistically and economically significant relation between the first difference of the ILS-BEI spread and CDS spreads. Specifically, over the period 2008 through 2015, a one standard deviation increase in the CDS spread (16 basis points) is associated with a 4 basis point increase in the first difference of hedged breakeven inflation, about 10% of the average ILS-BEI spread throughout our sample.[2] This relation is not simply a manifestation of dislocation during the financial crisis; the result holds in the subsample from 2010 onward. Furthermore, we demonstrate that the relation is robust to controls for liquidity and slow-moving capital.

In order to better understand the source of this covariation, we derive an affine model of defaultable nominal and inflation-protected sovereign debt following Monfort et al. (2017*a*). In this model, a sovereign entity issues multiple bonds with differences in the possible loss given default on these bonds. As discussed above, renegotiation in the Russian and Greek cases involved considerable uncertainty as to the ultimate losses suffered by holders of different bonds. We view

[1] In Figure 5 of Zettelmeyer, Trebesch and Gulati (2019) titled "Bond-by-bond haircuts, by remaining duration," the authors show significant heterogeneity across maturities in haircuts suffered by holders of nominal Greek debt, ranging from 20% to 90%.

[2] Hedged breakeven inflation is defined as the spread between the inflation-linked swap rate and the Treasury-based breakeven inflation rate of the same maturity (ILS-BEI).

our modeling approach as a convenient way to capture the uncertainty of whether a default event would trigger a default on all U.S. Treasury obligations and the ultimate recovery of present value of the bonds under consideration.[3] Our model produces closed-form pricing formulas, and we show that in the context of the model, the spread between inflation-linked swaps and breakeven inflation rates in the model is related to the relative loss given default on nominal and inflation-protected securities.

We estimate the parameters of the model by minimum sum of squares on a month-by-month basis. Our empirical targets are the time series of the five-year U.S. CDS spreads, the ILS and ILS-BEI spreads at maturities 2-,5-, and 10-y, and the year-on-year inflation rate. We force the estimation to perfectly match CDS spreads, and ask the model to fit all other observables as much as possible. Our results indicate that the model is able to simultaneously capture most of the variations in credit default swap spreads and breakeven rates of inflation. Specifically, while the CDS spread is matched by construction, credit risk factors are able to capture between approximately 50% and 100% of the total ILS-BEI variation after the crisis period, with the remainder explained by illiquidity issues in the TIPS market. Further, our estimates indicate that the market perception of the loss rate on TIPS is about 8 percentage points higher than that of nominal bonds. The results support our conjecture that the pricing of nominal and inflation-protected securities are affected by exposure to default risk.

Our paper contributes to at least three broad strands of the fixed income literature. The first area to which we contribute is the relative pricing of nominal and inflation-protected securities. Fleckenstein, Longstaff and Lustig (2014) document apparent no-arbitrage violations in the pricing of nominal and inflation-protected securities. Specifically, they show that an arbitrage strategy using nominal Treasuries, TIPS and inflation swaps generated large arbitrage profits during the financial crisis, and that these profits were present before and after the crisis period. Their empirical investigation suggests that the arbitrage arises due to slow-moving capital; a lack of arbitrage activity in the Treasury market allows the profits to persist. The hedged ILS-BEI spread utilized

[3] The law is not clear on whether cross-default provisions apply to U.S. Treasury debt. According to Schwarz (2014), "It is unclear whether any U.S. debt securities contain cross-default clauses. The statute setting fort procedures for the U.S. government to issue debt securities makes no mention of these types of clauses ..."

in our empirical analysis is closely related to the mispricing that they document. Our results suggest that part of this mispricing is related to credit risk.[4] Other papers investigate the joint pricing of nominal bonds and TIPS term structure under the assumption that they are virtually risk-free. Reduced-form affine pricing models include, among others Buraschi and Jiltsov (2005), Ang, Bekaert and Wei (2008), Chernov and Mueller (2012), Haubrich, Pennacchi and Ritchken (2012), and Roussellet (2019). While these papers differ by their approaches, their goal is to extract inflation premia from observable market data. Hordahl and Tristani (2012) look at Euro-area data instead of the U.S. Christensen, Lopez and Rudebusch (2012) and Fleckenstein, Longstaff and Lustig (2017) extract deflation probabilities from real Treasuries. Campbell, Sunderam and Viceira (2016) look at the correlation between real bonds and stock returns. We differ from this literature in explicitly considering the role of default risk in the pricing of these bonds.

A related second strand of literature estimates the liquidity premium embedded in TIPS with respect to nominal bonds. Grishchenko and Huang (2012) construct inflation risk premia employing only TIPS yields and control for the liquidity premium between TIPS and nominal bonds. Pflueger and Viceira (2016) suggest that there is a large and economically significant liquidity premium that affects the relative pricing of nominal and real bonds. Their evidence includes both U.S. and U.K. nominal and inflation-indexed bond prices. In the same vein, Abrahams et al. (2016) decompose real and nominal yields into liquidity, inflation, and real interest rate risk components in an affine term structure model. They conclude that forward breakeven inflation is primarily driven by risk and liquidity premia. D'Amico, Kim and Wei (2018) again propose a substantial liquidity premium as the primary factor driving the wedge between TIPS yields and real risk-free rates, thus causing distortions in the term structure of breakeven inflation. Andreasen, Christensen and Ridell (2017) identify liquidity risk in TIPS with the average deviation across bonds from what a no-arbitrage pricing model would predict. Recent studies also use the ILS-BEI spread as a proxy for liquidity risk only (see e.g. Christensen and Gillan (2018) or Moench and Vladu (2018)). This is because very high liquidity is usually attributed to the swap market in the U.S. (see for instance Driessen, Nijman and Simon (2017) or Camba-Mendez and Werner (2017)). An interpretation of Fleckenstein and

[4]The authors note that inflation-protected securities are not necessarily default risk-free, but suggest that since CDS do not distinguish between nominal and inflation-protected debt, default risk is unlikely to explain the arbitrage profits. See also Simon (2015) for a related analysis in the Euro-Area context.

Longstaff (2019) would suggest that the ILS-BEI could be related to intermediary balance sheet constraints. Our evidence suggests that considering the probability of sovereign default further contributes to the understanding of the breakeven spread.

The third strand of literature investigates the role of default risk in the pricing of sovereign securities and CDSs. On the empirical front, Ang and Longstaff (2013) estimate an affine multi-factor model of U.S. and European state and country credit default swaps and conclude that systemic sovereign risk is strongly linked to financial market variables. The authors observe that the estimated U.S. systemic credit risk-neutral default intensities spiked at the beginning of 2009, immediately after the onset of the financial crisis. Augustin and Tédongap (2016) provide an extended analysis for a cross-section of 38 different countries and show that both financial and macroeconomic risks are priced in sovereign CDSs. Similarly, Chernov, Schmid and Schneider (2019) note that CDS spreads of sovereign debt securities in the U.S. rose during the financial crisis and remained elevated in subsequent years. The authors construct a macrofinance model in which CDS premia reflect default probabilities. The authors show that their model is able to generate the high premium paid to insure U.S. sovereign debt. Their results suggest that high CDS spreads can arise in an equilibrium framework with default risk. Other explanations for CDS premia include dealers' counterparty risk and concerns about the protection seller (see Siriwardane (2019)), or financial regulation (see Klingler and Lando (2018)). Arora, Gandhi and Longstaff (2012) show that counterparty risk is priced in the CDS market using data covering the height of the 2008 crisis, but the magnitude is trivial because of the full collateralization of CDS liabilities. A summary of potential drivers of CDS spreads is also provided in survey of Augustin (2018*a*) and the references therein.

These papers are included in the broader-range of so-called intensity-based models (see Duffie and Singleton (1999)). These models allow for a closed-form pricing of both defaultable bonds and associated CDS. Recent examples include Filipovic and Trolle (2013) and Dubecq et al. (2016) who estimate interbank credit risk from LIBOR spreads, and Augustin (2018*b*), who calibrates a model on a large cross-section of sovereign CDS data. Monfort et al. (2017*a*) and Augustin, Chernov and Song (2018) develop new reduced-form frameworks to investigate the joint pricing of Treasuries and delivering payoffs in different currencies (quantos). Our approach builds on their insights by looking at the spread between nominal and inflation-indexed bonds, which corresponds in spirit to a change of currency between cash and consumption units. More generally, our analysis relates to the recent evidence of relative mispricing of Treasuries compared to asset swaps (see e.g. Klingler and Sundaresan (2019) or Augustin et al. (2019)). Another related empirical approach is that of Pan and Singleton (2008) who show that CDS spreads embed information not only about default frequencies but also on recovery rates. We use the same principle in our empirical approach to identify the differential in recovery rates between nominal and real Treasuries. Our paper differs from the previous papers in that it studies the joint pricing of sovereign CDS along with **both** nominal and real debt.

2 The Case of a U.S. Default

Our empirical analysis crucially relies on four notable financial instruments, namely credit default swaps on the U.S. government (CDS), sovereign U.S. nominal and inflation-indexed bonds, and inflation-indexed swaps. This Section details the important institutional features of these market instruments.

2.1 Nominal Bonds and TIPS

The main two instruments of debt issuance for the U.S. government are cash-denominated Treasuries (nominal) and *Treasury inflation-protected securities* (TIPS). Nominal zero-coupon bonds pay their nominal face value to the bond-holder at maturity. In contrast, zero-coupon TIPS holders earn the inflation-adjusted face value of the bond at maturity. Since cumulative inflation tends to be positive, TIPS tend to trade at a premium compared to nominal bonds. For both nominal bonds and TIPS, yields at issuance are determined through an auction process involving numerous market participants. In 2018, total value of outstanding Treasury securities was $15,608 billion, 9% of which are TIPS ($1,413 billion). The TIPS outstanding is comparable in magnitude to each of the respective markets for asset-backed securities, federal agency securities, and U.S. money market instruments.[5]

[5] https://www.sifma.org/resources/research/fixed-income-chart/

The TIPS inflation adjustment is computed using the *seasonally non-adjusted consumer price index* of the entire U.S. (CPI-U). CPI data is published monthly by the Bureau of Labor Statistics with a lag of about one and a half months, making the realized inflation unavailable when TIPS mature. TIPS payments thus include an *indexation lag* — the index used to determine their cashflows is a linear interpolation of CPI-U observed between two and three months before. The inflation-adjusted principal paid back at maturity is calculated by multiplying the face value of the bond by the cumulative *index ratio*. TIPS embed a *deflation floor*, such that they return the full face value even if cumulative inflation realized over the bond lifetime is negative.[6]

Despite the indexation lag, it would be difficult for the U.S. government to inflate away outstanding TIPS. Technically, it would be possible for the sovereign to resort to seignoriage to pay back maturing TIPS and current coupon payments without realizing the consequence of increased inflation. However, the inflation adjustment will materially impact any remaining outstanding TIPS, increasing the future interest payments of the government. Should the U.S. government refuse to honor the TIPS indexation, this would likely trigger a credit event and force the payoff of U.S. CDS contracts (see below). In case of default, nominal bonds and TIPS have the same level of seniority.

Leaving aside the embedded deflation floor, TIPS can be theoretically replicated by combining nominal bonds and inflation-linked swaps (ILS), as shown in Fleckenstein, Longstaff and Lustig (2014). ILS allow for the buyer to earn cumulative inflation in exchange for a fixed rate, relative to the notional agreed upon at inception. Inflation swaps are costless to write, and they are typically zero-coupon. As of April 2012, the average daily brokered inflation swap activity was estimated to be $350 million, concentrated around the 10-year maturity. Importantly, despite a low trading frequency averaging about 2.2 contracts per day, the market for inflation swaps appears fairly liquid, with bid-ask spreads from proprietary data averaging below 3 basis points.[7] Keeping with the standard for swap contracts, ILS are collateralized, thus subject to minimal counterparty risk.

[6] We consider zero coupon bonds in this study. Note however that most of nominal bonds and TIPS issued by the U.S. sovereign are coupon bonds paying on a semi-annual basis, but TIPS are only issued in terms of five, ten, twenty, or thirty years. For TIPS coupon payments, the coupon rate is fixed and paid on the inflation adjusted principal. For coupon payments, there is no deflation floor and the inflation-adjustment is computed using the index ratio realized over the last 6 months.

[7] https://libertystreeteconomics.newyorkfed.org/2013/04/how-liquid-is-the-inflation-swap-market.html and *JPMorgan Investment Insight: Inflation Derivatives.*

In the remainder of the paper, we will assume that ILS are virtually risk-free.

In a frictionless economy, for a given maturity n, no arbitrage implies that the zero-coupon ILS rate is equal to the spread between the nominal and TIPS zero-coupon yields, called *breakeven inflation rate* (BEI):

$$\text{ILS}_t^{(n)} = R_t^{(n)} - R_t^{*(n)} = \text{BEI}_t^{(n)}\,. \tag{1}$$

This measure is the zero-coupon equivalent of Fleckenstein, Longstaff and Lustig (2014), who show that the cash flows of any traded nominal Treasury bond can be replicated by a portfolio of TIPS, U.S. Treasury STRIPS, and inflation swaps. We equivalently call this spread ILS-BEI, *mispricing*, or *hedged breakeven*.

In practice, researchers have observed large deviations from this no-arbitrage relationship over the maturity spectrum. Figures I and II present the 5y series of ILS and BEI and the term structure of the spread between inflation swap rates and zero-coupon BEI, respectively. These deviations from the no-arbitrage relationship are quite persistent, and average between 30 and 36 basis points depending on the maturity. In the midst of the crisis, they reached more than 200 basis points.

Most of this apparent mispricing has been previously attributed to the low liquidity of TIPS relative to nominal bonds and ILS or to slow-moving capital (see e.g. D'Amico, Kim and Wei (2018) or Fleckenstein, Longstaff and Lustig (2014)). Campbell, Shiller and Viceira (2009) suggest the premium is related to the cost of supplying inflation protection and is typical under normal market conditions. Inflation swaps, Treasuries, and TIPS all trade over-the-counter and may be subject to varying liquidity risk or counterparty credit risk in the case of ILS. We argue that the ILS-BEI spread is also significantly related to the risk of default of the U.S. sovereign that we associate with U.S. CDS. In our analysis, we control for all these potential confounding factors in the analysis, and abstract away from the embedded deflation floor in TIPS and the tax-related issues. We note that the deflation floor drives the price of TIPS upward, making the observed TIPS yield lower than the one used in the no-arbitrage argument. This would lead us to *underestimate* the ILS-BEI spread, thus the size of the potential mispricing.

2.2 U.S. Sovereign CDS

Credit default swaps (CDS) are OTC instruments designed to protect bond investors from a contingent *credit event* of the issuing entity. In practice, a bond investor (*protection buyer*) entering a CDS agrees to pay a fixed premium, typically called *CDS spread*, on a regular basis to the *protection seller*, her counterparty. In case of a credit event, the contract terminates and the seller has to deliver the *loss given default* (LGD) realized on the bond to the buyer, making her earn the entire face value of the bond upon default. As standard for swap contracts, the premium is indexed on a notional agreed upon at inception and is set such that the original cost of issuance is zero. While not free from counterparty credit risk, CDS are typically collateralized.

The International Swaps and Derivatives Association (ISDA) provides the reference legal details to define what triggers the termination of a CDS, which type of obligations are considered, and how the LGD and repayment is operates depending on the underlying bond issuer (see ISDA (2003, 2014)). In the case of the U.S. sovereign, a credit event is observed whenever the government either *(i)* fails to repay, *(ii)* repudiates or imposes a moratorium, or *(iii)* restructures any of its borrowed money. This includes in particular any Treasury Bill, Bond or Note, whether nominal or indexed. In our empirical analysis, we identify default with the conditions for which CDS protection are triggered.

In case of a credit event, the LGD is determined through an auction addressed to CDS dealer banks. Participating banks typically submit a bid and ask quote on a $100 face-value bond of the reference entity, and the cross-section of bid-asks is used to determine the final price of the bond, typically below par (see Augustin et al. (2014)).[8]

The settlement of the CDS can be completed either through cash or physical delivery. In the former case, the protection seller delivers a payment equal to the LGD as determined by the auction, multiplied by the notional of the CDS. In the latter case, the protection seller pays the entire notional to the buyer in exchange for an equivalent principal amount of reference bonds. If these bonds have the exact same characteristics as those auctioned, the two deliveries would be equivalent. However, the protection buyer can choose to exchange *any of her reference bonds* with

[8]The final price of the bond resulting from this auction is published by CreditEx (`http://www.creditfixings.com/CreditEventAuctions/results.jsp`).

maturity below 30 years and above the maturity of the CDS contract. This essentially embeds a *cheapest-to-deliver* (CtD) option to the buyer's position, who will likely deliver the lowest dollar price reference obligation available.[9]

U.S. CDS contracts fall under the "Big Bang Protocol" established by ISDA in 2009. In the aftermath of the financial crisis, as the primary industry body overseeing swaps and derivative trading, ISDA pushed swap market participants to adopt the new protocol in an effort to standardize over-the-counter contract parameters.[10] A number of the implemented changes are worth highlighting. First, coupon payments on each contract are fixed at either 100 (investment grade) or 500 basis points (non-investment grade). As a result, there is typically a payment to be made at the initiation of the contract to ensure that the present values of expected cash flows are equal between the buyer's and seller's legs. A second important change stemming from the protocol is the hardwiring of the auction process following credit events such that all protection buyers obtain fair cash payments from protection sellers. Third, the protocol further stipulates the creation of Determinations Committees for determining whether a credit or succession event has occurred in order to reduce disputes between counterparties in case there is ambiguity.

Market participants in the sovereign CDS market range from security dealers, to banks and financial institution, to hedge funds (see e.g. Augustin (2018*a*)). There is evidence that sovereign CDS contracts are used in both a hedging and speculative context. For contracts specifically written on the U.S. sovereign, focusing on the most liquid contracts with 5-years to maturity, price data from Markit shows there is very little pricing movement before the financial crisis of 2008. The premium spiked in 2009, at the height of the crisis, to about 100 basis points and has remained elevated afterward between 20 to 40 basis points.

Chernov, Schmid and Schneider (2019) provide a detailed discussion on the determinants of U.S. sovereign CDS spread beyond credit risk. For instance, the majority of U.S. CDS contracts are denominated in euros, and there is a small foreign exchange premium embedded in the spread.

[9]In the context of the Greek crisis, CDS contracts and the associated auction mechanism played a minor role in the resturcturing process. As highlighted by Zettelmeyer, Trebesch and Gulati (2019), the credit event was triggered only after the preemptive debt restucturing. Therefore, the CDS auction took place after the bond exchange, and the resulting auction price fell in place with the new bond price in the secondary market. To be certain, CDS coverage of Greek sovereign debt was very low, at less than 2%. One would not expect the outcome of the bond auction to dictate terms of the restructuring.

[10]BIS Quarterly Review, December 2010. "The Big Bang in the CDS Market"

U.S. dollar denominated contracts did not start trading until August 2010 and volumes are thin relative to euro contracts. Additionally, there is uncertainty in the cheapest-to-deliver option due to the bond auction protocol conditional on default occurring. Lastly, the U.S. CDS spread should contain a liquidity premium component due to the relative scarcity of the instrument compared to other sovereign CDS contracts. A combination of these factors contribute to the U.S. sovereign CDS premium.

In the context of our project, we use the U.S. sovereign CDS premium as a proxy for default risk to study the relative pricing of nominal Treasury bonds and inflation-protected bonds. We present several robustness tests in the Appendix to rule out the possibility these non-credit risk-related factors can simultaneously generate differential prices in U.S. sovereign bonds.

2.3 CDS-Implied LGD and Effective LGD

In practice, there can be a significant difference in the LGD faced by uncovered and covered bond position holders. This discrepancy is due to the auction process determining the LGD used for CDS purposes. Let us assume that upon trigger of a U.S. sovereign credit event, the auction determines that the reference bond is worth 75 cents per dollar, yielding an auction-based LGD of 25 cents. In the case of physical delivery, an investor holding a covered position can sell her bond at par to the protection seller and receives one dollar.

An interesting case arises for cash delivery, where the protection seller delivers 25 cents to the protection buyer but the latter holds onto her bond. The government then determines an effective LGD which can be different from 25 cents. If the effective LGD is 20 cents, the protection buyer is left with one dollar and five cents. This effect is similar to the CtD option for physical delivery and can lead the CDS-implied LGD to be greater than the effective LGD for the bond holder.

We aware of the existence of these effects, yet we leave them aside in the empirical analysis for practical reasons. Indeed, while it is straightforward to obtain the LGD resulting from the CDS auction for past credit events, obtaining the effective LGD is a daunting task. In addition, the case of a U.S. credit event has not been observed for the last hundred years and attempting to input any figure would be pure conjecture. It should also be noted that we use CDS spreads merely as a proxy for the default risk of the U.S. sovereign, which allows us to abstract away from these specifics and assume that the CDS exactly embeds the effective LGD determined by the government, and is known in advance.

3 Empirical Analysis

In this section we test our main hypothesis that exposure to default risk may influence the relative pricing of nominal and inflation-protected sovereign obligations. Specifically, we test whether the ILS-BEI spread is related to the CDS spread. We examine variation in these quantities over the full sample period and a subperiod that does not include the financial crisis of 2007-2009.

3.1 Data

The spreads between breakevens and inflation-linked swaps are constructed in two steps. We use the data described in Gurkaynak, Sack and Wright (2006) and Gurkaynak, Sack and Wright (2010) for nominal and inflation-protected smoothed zero-coupon bonds respectively. The BEI variable is the difference between the former and the latter. We collect inflation swap data from Bloomberg and subtract the BEI from the swap spread to obtain our mispricing variable, the ILS-BEI spread. EUR-denominated CDS spread data are obtained from Markit. Our focus is on the five-year maturity for CDS contracts as this is the most liquid CDS tenor. Our data are sampled daily from January 2008 to October 2015 (full sample).[11]

We depict the time series of U.S. sovereign credit default swap spreads and the ILS-BEI spread in Figure I, panel (b). As documented in Chernov, Schmid and Schneider (2019), CDS spreads soar to 100 basis points in the wake of the Lehman Brothers bankruptcy, timing that is similar to that of the large increase in ILS-BEI. Our conjecture is that this event, and the crisis that followed caused investors to reprice the probability of a U.S. sovereign default and the recovery on Treasury and TIPS in a default scenario. The spread is volatile in 2010-2013 before becoming quiescent from about 2014 onward. Notably, the spread spikes to more than 40 basis points in the days prior to

[11] Our results are qualitatively the same when using USD-denominated CDS contracts after they began trading in 2010. While data on EUR-denominated CDS are available prior to 2008, U.S. CDS exhibit virtually no variation and volume in the pre-sample period and the quotes are often unchanged for weeks at a time and average between one and two basis points.

the resolution of the the budget showdown of 2013, which threatened to lead to a U.S. sovereign default (we expand on this particular issue in Section 3.7).

Summary statistics for these data are provided in Table I. Over the full sample period, both the ILS-BEI and U.S. CDS spread averaged over 30 basis points (36 and 33 basis points respectively). The ILS-BEI is approximately twice as volatile as the CDS spread, ranging from -1 to 210 basis points. In contrast to the CDS spread, the ILS-BEI declines both on average and in volatility in the post-crisis period, which we define as January 1, 2010 and beyond. Thus, even in the post-crisis period, the U.S. CDS spread averages 34 basis points, considerably greater than its pre-crisis levels. The unconditional correlation between the five-year ILS-BEI and CDS is about 0.3.

3.1.1 Regression controls

In addition to possible fears of default risk, numerous factors may play a role in the observed ILS-BEI spread — namely, heightened counterparty credit risk associated with inflation swap transactions, liquidity concerns, increases in perceived quantities and prices of risk, and a deterioration in arbitrage capital available to deploy in financial markets. Each of these potentially confounding factors would be expected to play an outsized role influencing the components of the ILS-BEI spread at the peak of the financial crisis. We examine the role of several variables in order to investigate alternative possibilities.

HPW Noise and *TIPS Noise* serve as our measures of arbitrage capital as proposed in Hu, Pan and Wang (2013)[12]. *LIBOR-OIS* measures counterparty credit risk. The off/on the run differential in nominal bonds (*OTR Difference*) is a proxy for liquidity in these markets while the *VIX* index is often viewed as a market measure of the prevailing price of risk in financial markets. A detailed description of each variable can be found in Appendix A.1.

3.2 Empirical Results

We employ panel regressions for our empirical analysis. We include first differences of the ILS-BEI spread across five tenors: 2, 3, 5, 7, and 10 years as the dependent variable. As standard in the

[12]The HPW Noise measure is sourced from Jun Pan's website and we thank Richard Crump for providing the TIPS Noise series.

fixed income pricing literature, we assume that all interest rates at all maturities are second-order stationary despite a high persistence. Indeed, it is hard to believe that either the mean or variance of U.S. interest rates will follow an explosive path. Our baseline specification is thus given by:

$$\Delta\left(\mathrm{ILS}-\mathrm{BEI}\right)_{n,t}=\alpha+\rho\cdot\left(\mathrm{ILS}-\mathrm{BEI}\right)_{n,t-1}+\gamma\cdot\mathrm{CDS}_t+\boldsymbol{\beta}^{\top}\cdot\mathbf{X_t}+w_t+\varepsilon_{n,t}\,, \tag{2}$$

where $n=\{2,3,5,7,10\}$, $\mathbf{X_t}$ represents the set of relevant controls, and w_t is the week-time fixed effects. Our formulation is merely equivalent to a level-level specification, but allows us directly interpret the estimated coefficients and R^2 produced by the model in terms of ILS-BEI spread changes instead of the persistent level, and stationarity will be verified if ρ is negative and different from zero.

Tables II, III, and IV present regression results for the full sample, the crisis sample, and the post-crisis sample, respectively. The full sample period spans the beginning of 2008 to October 2015. We start the sample in 2008 due to the fact that U.S. sovereign CDS contracts were thinly traded prior to the 2008 financial crisis. All regressions contain observations at the daily frequency where data is available for consecutive trading days in all markets. We employ week-time fixed effects. Column (8) in the related tables shows results from the full regression specification with both week and tenor fixed effects, as well as all the control variables listed in Section 3.1.1. Finally, in each regression, the U.S. CDS spread is the main explanatory variable, but we include the lagged ILS-BEI spread to ensure the persistence of the dependent variable is not driving our results.

As shown in the top row of Table II, all coefficient loadings on the U.S. CDS spread are positive and highly significant across the columns. The point estimate of 0.196 in Column (1) is statistically significant at the 1% level and suggests that a 16 basis point increase in U.S. CDS spreads (one standard deviation) translates into an approximately 3.2 basis point increase in the first difference of the ILS-BEI spread. This represents approximately 10% of the mean ILS-BEI suggesting the results are economically significant as well.

Table II also demonstrates that it is essential to include lagged ILS-BEI spread as an explanatory variable since coefficient loadings are negative and highly significant regardless which control variables are used. It should not be surprising that the ILS-BEI spread is highly persistent. In

Appendix Table A3 we perform the regressions with all variables constructed in first differences obtaining similar results. Amongst the control variables, LIBOR-OIS spread in Column (4), and VIX in Column (6) are marginally significant (between 5% and 10% statistical significance). The TIPS Noise measure in Column (3) and the OTR Difference in Column (5) are negative and highly significant. A higher value of TIPS Noise indicates greater deviations in the TIPS yield curve. Although we would expect this to reflect poorer liquidity on the TIPS market and thus to increase the ILS-BEI spread, we note that all liquidity measures are highly correlated and it is hard to extract a clean interpretation for each single coefficient. In addition, the significance is driven by the crisis period as shown in Columns (3) and (5) of Table III. The loading of first differences in ILS-BEI on TIPS Noise and OTR Difference becomes insignificant in Columns (3) and (5) of Table IV. The R^2 for all specifications represents around 15% of the variance of the ILS-BEI changes. When all variables are included, in Column (7) of Table II, the coefficient on CDS increases to 0.228 (3.6bps per CDS standard deviation), and HPW Noise becomes statistically significant. Given the high positive correlation with TIPS Noise this result is not surprising. The estimated coefficient loading on the CDS spread actually increases from Column (1) to Column (7) when controls are included. Finally, the addition of a tenor fixed effect does not affect the regression outcomes in Column (8) suggesting our results are not driven by a particular maturity on the yield curve.

3.3 Sub-sample Analysis

We focus first on the crisis period between 2008 and 2009. We examine the degree to which the crisis influences our conclusions by separating the sample into a crisis period, which we specify as January, 2008 through December, 2009, and a post-crisis period from January, 2010 onward. Results for the crisis period are presented in Table III. Our results carry through during the crisis sample. Depending on the specification, CDS coefficients range from 0.21 to 0.27. Although their statistical significance is weaker (5-10% level) during the crisis period relative to the full sample, the point estimates on the U.S. CDS spread are greater, which implies a more pronounced effect between sovereign default risk and first differences in the ILS-BEI spread.

For the post-crisis period, the results depicted in Table IV are essentially unchanged as well. Again, depending on the specification, CDS coefficients range from 0.16 to 0.17 and are significant at the 1% level, indicating more precise estimates than in the crisis sample. The point estimates across all columns are roughly five times greater than their standard errors.

Our interpretation of these results is that determinants of the U.S. CDS spread comove strongly with daily changes in ILS-BEI. While CDS spreads may be driven by a number of different factors, including actual default risk and liquidity effects, we view the evidence here as sufficiently suggestive to indicate ILS-BEI is influenced by credit risk. The relationship appears to persist during periods of high volatility and strained financial conditions (crisis) as well under conditions associated with normally functioning markets.

3.4 Components of ILS-BEI

To confirm that the credit risk influence comes mainly from the TIPS side, we regress the changes in the components of ILS-BEI separately on U.S. CDS spread in the full sample. Table V documents the regression results. The dependent variables in columns (1), (2), and (3) are, respectively, first differences in TIPS yields, nominal Treasury yields, and ILS swap premia. To be consistent with previous tests, we employ tenors of 2-, 3-, 5-, 7-, and 10-years on all the dependent variables. The explanatory variables include the 5-year CDS spread, lagged dependent variables, as well as standard controls used in Tables II to IV.

Changes in both TIPS yields and nominal yields load positively and significantly on the CDS spread, in Columns (1) and (2), whereas the ILS spread shows no significant correlation with the CDS spread in Column (3). This suggests that the ILS-BEI spread comoves with the CDS spread because of the reaction of the real and nominal term structures to sovereign default risk. Second, the coefficient loading of differences in TIPS yields on CDS is larger than the coefficient loading of differences in nominal yields on CDS. This implies that the BEI narrows as CDS spread increase, and that the ILS-BEI spread increases.

Results shown in Table V provide some comfort that the CDS spread is indeed producing differential impact on sovereign bond yields and not on the inflation swap. This is crucial evidence that sovereign default risk can impact the relative pricing of TIPS and nominal bonds.

3.5 Default Risk and Liquidity

Pflueger and Viceira (2016) suggest that much of the spread between nominal and inflation-protected bond yields arises as a premium for liquidity. In their analysis, they find that the portion of breakeven inflation that is related to liquidity rather than inflation expectations accounts on average for 69bps of the spread between nominal and inflation-protected securities. While we endeavor to control for liquidity in our earlier analysis, in this section we explicitly examine the contribution of CDS to the liquidity premium that they document.

The authors measure the liquidity premium by breaking the differential in the yield on nominal and inflation-protected securities on a set of liquidity variables and measures of inflation expectations:

$$\mathrm{BEI}_t^{(n)} = a_1 + \mathbf{a}_2^\top \mathbf{X}_t + \mathbf{a}_3^\top \boldsymbol{\pi}_t^e + \varepsilon_{n,t}, \tag{3}$$

where $\mathbf{X}_t$ is a vector of liquidity-related variables and $\boldsymbol{\pi}_t^e$ is a vector of measures of inflation expectation. The liquidity premium is measured as $\hat{L}_t = -\hat{\mathbf{a}}_2^\top \mathbf{X}_t$. We follow their approach, using the breakeven inflation between 10-year nominal and inflation-protected securities as our dependent variable. We describe the independent variables in Appendix A.2.1. One key point is that the liquidity variables $\mathbf{X}_t$ include the ILS-BEI spread as a proxy.

Results of the analysis are presented in Table VI. In the first column of Panel A, we present an analysis complementary to that of Pflueger and Viceira (2016). Consistent with their analysis, and with intuition, inflation expectation variables are positively related to breakeven inflation. The coefficient on the ILS-BEI is negative; the authors interpret the result as suggesting that the pronounced decrease in breakeven inflation during the financial crisis reflected security market disruption and constraints on levered investors. The authors find that one cannot reject the hypothesis that the coefficient is equal to negative one. However, in our results, the point estimate of the coefficient is more than two standard errors from one. This result suggests that, consistent with our results above, the ILS-BEI may reflect more than just constraints on levered market participants.

In the second column, we add the CDS spread to the regression. Three observations emerge. First, the CDS spread is negatively and significantly related to breakeven inflation. To the extent that default risk may have differential impact on nominal and inflation-protected Treasury securities, the negative coefficient suggests that yield spreads on the two securities tighten when default risk increases. This may reflect a flight to the relative safety of nominal Treasuries or a drop in the prices of inflation-protected securities. Second, the coefficients on the remaining variables, with the exception of ILS-BEI, are materially unaffected. Third, after controlling for CDS, one can no longer reject the hypothesis that the coefficient on the ILS-BEI is equal to negative one, consistent with the results in Pflueger and Viceira (2016). Thus, the results indicate that both the BEI and the ILS-BEI reflect co-movement with CDS spreads due to credit risk.

Our final analysis of the liquidity premium directly regresses the estimated liquidity premium on the CDS spread. Results are presented in Panel B. As shown in the Table, CDS spreads on Treasury securities are positively and statistically significantly related to the liquidity premium, explaining approximately 9% of its variation. This result suggests that part of the liquidity premium documented in Pflueger and Viceira (2016) may in fact reflect compensation for credit risk. However, the majority of the variation in the premium is unrelated to variation in CDS spreads, indicating that both liquidity and credit risk jointly play a role in understanding the differential pricing of TIPS and nominal Treasury securities.

3.6 Cheapest-to-Deliver

In the case of a credit event, the cheapest-to-deliver (CtD) obligation of the reference entity (here, the U.S. sovereign) will be a key determinant of the recovery rate on the underlying asset. In the auction process described in Section 2.2, all obligations deemed *deliverable* by the Determinations Committee are eligible to be sold. The protection buyer is incentivized to deliver the cheapest outstanding reference obligation and hence the final price at auction will largely be determined by the prevailing dealer quotes of this obligation. Therefore, CDS premia and the expected price of the CtD obligation would be expected to have a negative correlation. The CtD obligation may vary considerably over time and depend on the sovereign distance to default.

We examine the effects of CtD empirically in Table VII. We assume the general methodology of Klingler and Lando (2018) to identify the cheapest nominal Treasury on each day of our sample and include the following as a control variable:

$$CtD_t = 100 - \min(Price_t).$$

The regression result is essentially unchanged from our primary specification in Table II. In Column (1), CtD shows a weak, insignificant relationship with changes in ILS-BEI. The coefficient on CtD is negative and significant in Column (3) where we include all controls as well as a week and tenor fixed effect. Based on the construction of the CtD variable, a lower nominal bond price (higher yield) on any day is associated with a higher value of CtD. The results suggest higher Treasury yields are consistent with a narrower ILS-BEI, yet the coefficient on US CDS is essentially unchanged. There exists some ambiguity over what would be considered a deliverable obligation in the case of a sovereign credit event as this would ultimately be decided by the Determinations Committee. It is with minimal disagreement that nominal bonds up to a 30-year maturity would be accepted, yet the role of inflation-linked bonds, zero-coupon STRIPS, or sovereign guarantees (where U.S. sovereign gurantees the repayment of debt issued by another party) is less clear. Our empirical results are consistent whether we include or exclude TIPS in the definition of CtD.

3.7 Debt Ceiling Episodes

Another potential explanation driving our empirical findings is the political impasse seen in the U.S. Congress over the last decade.[13] Specifically, unyielding debt ceiling negotiations on whether or not to raise the borrowing capacity of the federal government impact the credit-worthiness of the U.S. An episode in 2011 led to the U.S. credit-rating downgrade from AAA to AA+, which implies these events can affect U.S. sovereign CDS spreads. To the extent that these negotiations have a differential impact on Treasury and TIPS yields, they may simultaneously cause the widening of CDS spreads and the ILS-BEI spreads.

We identify four debt ceiling episodes during our sample period: August 2011, February 2013, the partial government shutdown in October 2013, and February 2014. We examine the evolution of U.S. CDS spreads and ILS-BEI spreads around the two months event window before and after

[13]We thank Luis Viceira for this suggestion.

the day on which each of these episodes were resolved. The August 2011 negotiations ended in the Budget Control Act of 2011, signed into law on August 2, 2011. The February 2013 episode led to the The No Budget, No Pay Act of 2013, signed into law on February 4, 2013. The 2013 partial shutdown was resolved by the Continuing Appropriations Act, signed into law on October 17, 2013. Lastly, the February 2014 debt debate resulted in the Temporary Debt Limit Extension Act, signed into law on February 12, 2014. By studying the dynamic changes of the 5-year CDS premium and the 5-year ILS-BEI spread around these resolution dates, we can discern if these debt ceiling episodes are material events in generating the positive correlation between them.

Figure III plots the relative change of the CDS spread and the ILS-BEI spread around these four event dates. Relative change is calculated based on the value immediately before the start of the event window. Two observations are apparent. First, the CDS spread (blue line) tends to increase during the negotiation period leading up to the passage of the law. After each of the event day, the CDS spread declines. This is consistent with the notion that the CDS spread is capturing the probability of a technical default happening, thus triggering protection. The exception is the last episode in February of 2014 where the CDS spread does not change much during the entire event window. Second, the movement of the ILS-BEI spread is inconclusive during the event window generally speaking. Outside of the February 2013 episode, the ILS-BEI spread tends to fluctuate before the passage of the funding bill. Furthermore, the narrowing of the ILS-BEI spread at the same time when the CDS spread is widening, as seen in the first two months of 2013, is incompatible with the positive regression coefficients estimated in Table II on U.S. CDS.

Taken together, debt ceiling episodes collected in Figure III do not appear to be a source of the positive correlation between the CDS spread and the ILS-BEI spread in our sample period. Although the possibility of a technical default by the U.S. government seems to push up the cost of buying CDS protection, it does not change bond yield differentials meaningfully around these episodes.

4 Modeling Nominal and Inflation-Protected Debt with Default Risk

In this section, we discuss the pricing of nominal and inflation-protected sovereign bonds, assuming that there is a possibility of a credit event interrupting the promised payments of the securities. Of particular interest is the spread between inflation-linked swaps and the breakeven inflation rate. We first develop a stylized example to explain the possible mechanisms underlying the link between credit risk and ILS-BEI. Then we develop a full affine pricing model to complement panel regression results in Subsection 3.2. With the aid of the model, we further decompose the ILS-BEI spread into its credit and liquidity components to examine the dynamic contribution of the credit factors during the after the crisis.

4.1 Detailing the Possible Mechanisms

We assume that the risk of a credit event can influence the price of nominal bonds and TIPS issued by the U.S. sovereign, thus the BEI. We develop a stylized example and detail some assumptions for which the spread could arise. Details of computations are presented in Appendix A.3.

We consider zero-coupon investments. Denote by $R_0^{(n)}$ and $R_0^{*(n)}$ the zero-coupon yields on (risky) sovereign bonds of maturity n, nominal and inflation-protected, respectively. Their risk-free equivalents are respectively denoted by $r_0^{(n)}$ and $r_0^{*(n)}$, such that:

$$\mathrm{ILS}_0^{(n)} = r_0^{(n)} - r_0^{*(n)} . \tag{4}$$

Every year, the government declares default on nominal bonds and TIPS with respective (risk-neutral) probabilities p and p^*. These events lead to the termination of the respective bond and the loss of LGD and LGD*, respectively. In case of default, nominal bond and TIPS holders thus earn $1-\mathrm{LGD}$ and $\Pi_{0:\tau}-\mathrm{LGD}^*$, respectively, per unit of principal, where $\Pi_{0:\tau}$ is the gross cumulative inflation and τ is the default date. Standard no-arbitrage asset pricing shows that, for the one-year maturity:

$$\text{ILS}_0^{(1)} - \text{BEI}_0^{(1)} \simeq p^* e^{-\text{ILS}_0^{(1)}} \text{LGD}^* - p\text{LGD}\,. \tag{5}$$

This spread is positive whenever the ILS-weigthed loss given default on TIPS is greater than the one on nominal bonds. If we assume that default probabilities and LGDs are the same for both bonds, the ILS-BEI spread is proportional to $(e^{-\text{ILS}_0^{(1)}} - 1)$ which is typically negative since ILS rates tend to be positive, and cannot explain the positive mispricing.

In the following, we assess the plausibility of several explanations by calibrating an example such that we can reproduce the average magnitude of the ILS-BEI spread and the CDS spread. We assume that the nominal rate is at 3%, the ILS is at 1.5%, $\Pi_{0:1} \sim \mathcal{N}(1+\mu_\pi, \sigma_\pi)$ with $\mu_\pi = 1.5\%$ and $\sigma_\pi = 1\%$, the nominal LGD is 25 cents per dollar and the CDS spread is 50bps. We assume that both bonds are potentially deliverable in case of default, and that the risk-neutral distribution of inflation is . We find the one-year CDS spread as:

$$s_0^{(1)} = \text{PD}_0 \times \left[\text{LGD} + (\Delta\text{LGD} - \mu_\pi)\,\Phi\left(\frac{\Delta\text{LGD} - \mu_\pi}{\sigma_\pi}\right) + \sigma_\pi f\left(\frac{\Delta\text{LGD} - \mu_\pi}{\sigma_\pi}\right)\right], \tag{6}$$

where PD_0 is the probability of default on at least one type of bonds, $\Delta\text{LGD} = \text{LGD}^* - \text{LGD}$ and $f(\cdot)$ and $\Phi(\cdot)$ are the PDF and CDF and a standard Gaussian variable, respectively.

Continuing Swap Payments after Default: If the government defaults either on nominal bonds or TIPS during the first year, the corresponding bonds will be terminated. In contrast, the inflation swap will continue running to maturity. This leads the ILS to be relatively more attractive than a BEI long-short position. For 2-year maturity bonds, and the same default probability and LGD for both bonds, we have:

$$\text{ILS}_0^{(2)} - \text{BEI}_0^{(2)} \simeq \frac{p}{2}\left[\text{LGD}\left(1 - p + e^{r_0^{(1)}}\right)\left(e^{-2\text{ILS}_0^{(1)}} - 1\right) + e^{r_0^{(1)}} - e^{r_0^{*(1)}}\right] \tag{7}$$

A 2% default probability is sufficient to obtain the CDS at 50 basis points and the mispricing at 30 basis points.

Simultaneous Credit Events and Different LGDs: Default on both bonds is simultaneous, and $\mathrm{PD}_0 = p = p^*$. As argued above, LGDs have to be different to explain the positive spread, and $\mathrm{LGD}^* > \mathrm{LGD} \cdot e^{\mathrm{ILS}_0^{(1)}}$. We obtain numerically that a 30 basis points ILS-BEI spread requires the TIPS LGD to be 58 cents per inflated dollar.

Partial Default: Instead of defaulting on both nominal and real debt, the government can choose to default on one or the other independently. We have that $\mathrm{PD}_0 = p + p^* - pp^*$ and p does not have to be equal to p^*. Let us assume now that the LGDs on both nominal bonds and TIPS are the same 25 cents per (nominal) dollar of face value. We obtain a positive mispricing as long as $p^* > p \cdot e^{\mathrm{ILS}_0^{(1)}}$. We find numerically that default probabilities of $p^* = 1.61\%$ and $p = 0.39\%$ can explain both the CDS and ILS-BEI magnitudes.

Re-indexation of TIPS: In this case, the government chooses to convert TIPS into their nominal bonds equivalent. The TIPS would then provide only $1 at maturity absent any other event. However, the government can also default on nominal bonds, in which case all bonds including TIPS would provide $1-\mathrm{LGD}$. Putting the two together, we obtain a slightly transformed mispricing, which is given by:

$$\mathrm{ILS}_0^{(1)} - \mathrm{BEI}_0^{(1)} \simeq p^* \left(1 - e^{-\mathrm{ILS}_0^{(1)}}\right) - p\mathrm{LGD} \left(1 - p^* e^{-\mathrm{ILS}_0^{(1)}}\right) . \tag{8}$$

Note that in this case, nominal bonds will always be the CtD and the CDS spread is simply given by $s_0^{(1)} = (p + p^* - pp^*)\mathrm{LGD}$. For our numerical example, there is no solution that satisfies exactly the pricing restrictions for the CDS and the mispricing magnitude, making this case a less satisfying explanation. However there exists a solution if we take a higher LGD, 40cts per dollar for instance.

In sum, we show that the magnitude of the observed ILS-BEI spread in the data can be explained by all of the previous scenarios to a similar extent. Note that our stylized example is trying to explain the entire magnitude of the average ILS-BEI spread through sovereign credit risk. In practice, TIPS yields are contaminated by liquidity risk and the magnitude of the credit-related ILS-BEI spread is likely smaller.

Additionally, not all quantities can be identified from CDS and bond data directly in practice. It is well known for instance that standard credit risk models can have difficulties identifying default probability and loss given default at the same time. It is then very unlikely that we will be able to discriminate between the different scenarios identified above. To simplify the assumptions surrounding the framework, we choose to focus on the case of differential LGDs on nominal bonds and TIPS. This allows us to identify the unique default probability using the CDS data. We detail this further in the Section below.

4.2 The Term structure of Riskless Yields

We consider an economy where the nominal and real term structures are driven by $k_x \times 1$ factors $\mathbf{x_t}$ under the risk-neutral measure $\mathbb{Q}$. More specifically, these factors are partitioned in two blocks such that $\mathbf{x_t} = \left(\mathbf{x_t^{(r)}}^\top, \mathbf{x_t^{(\pi)}}^\top\right)^\top$, where these components are vectors of size $k_x^{(r)}$ and $k_x^{(\pi)}$ respectively. Investors have access to a one-period riskless nominal investment yielding a continuously-compounded interest rate $r_t = -\log\left(D_t^{(1)}\right)$ between t and $t+1$. Investors suffer from high inflation π_t, as given by the growth rate of the CPI-U index between $t-1$ and t. We assume that the nominal rate and inflation risk-neutral dynamics depend on the factors $\mathbf{x_t}$ linearly, such that:

$$r_t = \kappa_0^{(r)} + \boldsymbol{\kappa_r}^\top \mathbf{x_t^{(r)}} \quad \text{and} \quad \pi_\mathbf{t} = \kappa_\mathbf{0}^{(\pi)} + \boldsymbol{\kappa_\pi}^\top \mathbf{x_t^{(\pi)}} . \tag{9}$$

We assume that the risk-neutral dynamics of the factors are given by:

$$\mathbf{x_t} = \boldsymbol{\mu}^\mathbb{Q} + \boldsymbol{\Phi}^\mathbb{Q}\mathbf{x_{t-1}} + \sqrt{\boldsymbol{\Sigma}^\mathbb{Q}}\varepsilon_\mathbf{t}^\mathbb{Q} , \tag{10}$$

where $\varepsilon_t^\mathbb{Q} \overset{i.i.d}{\sim} \mathcal{N}(\mathbf{0}, \mathbf{I_{k_x}})$. In this economy, investors also have access to real bonds (TIPS) at price $D_t^{*(n)}$ that provide one unit of consumption growth at maturity $t+n$. It is well-known that the riskless nominal and real bond yields are obtained in closed-form in that setup. We have:

$$D_t^{(n)} = \exp\left(A_n + \mathbf{B_n}^\top \mathbf{x_t}\right) \quad \text{and} \quad D_t^{*(n)} = \exp\left(A_n^* + \mathbf{B_n^*}^\top \mathbf{x_t}\right) , \tag{11}$$

where the loadings are defined in the recursion of Appendix A.4. The ILS is thus given by:

$$\text{ILS}_t^{(n)} = \frac{1}{n}\left(A_n^* - A_n + (\mathbf{B_n^*} - \mathbf{B_n})^\top \mathbf{x_t}\right) \tag{12}$$

4.3 Default and Liquidity Dynamics

Our modeling framework follows that of Monfort et al. (2017*a*) in modeling risky debt in discrete time. In this framework, sovereign credit events of any kind are represented by the first jump of a non-negative credit-event variable denoted by $\delta_t^{(c)}$. Our modeling of liquidity events mimics the form employed for credit events, as in e.g. Ericsson and Renault (2006), Monfort and Renne (2013) or Dubecq et al. (2016). We assume that liquidity events are represented by the first jump of a liquidity-event variable denoted by $\delta_t^{(\ell)}$. More formally:

$$\tau_c = \min\{\, t \,|\, \delta_t^{(c)} > 0\} \quad \text{and} \quad \tau_\ell = \min\{\, t \,|\, \delta_t^{(\ell)} > 0\}. \tag{13}$$

The risk-neutral dynamics of default and liquidity events are driven by $k_c \times 1$ and $k_\ell \times 1$ vectors of state variables, denoted by $\mathbf{y_t^{(c)}}$ and $\mathbf{y_t^{(\ell)}}$ respectively. Conditionally on the state variables $\mathbf{y}_t$, we assume that the credit and liquidity event variables $\boldsymbol{\delta_t} = \left(\delta_t^{(c)},\, \delta_t^{(\ell)}\right)^\top$ are Gamma-zero distributed:[14]

$$\boldsymbol{\delta_t}\big|\left(\underline{\mathbf{y}_t}, \underline{\delta_{t-1}}\right) \overset{\mathbb{Q}}{\sim} \Gamma_0\left(\begin{pmatrix} \boldsymbol{\gamma}_c^\top \mathbf{y_t}^{(\mathbf{c})} \\ \boldsymbol{\gamma}_\ell^\top \mathbf{y_t}^{(\ell)} \end{pmatrix}; \mathbf{1}\right) \tag{14}$$

where the scaling parameters of the process have been normalized to 1 for identification purposes. Monfort et al. (2017*a*) show that Gamma-zero processes are efficient in representing credit events since they can stay at the value of zero for extended periods of time (no default or liquidity states) and jump to any positive value upon events. Similarly, we assume that $\mathbf{y_t} = \left(\mathbf{y_t^{(c)\top}}, \mathbf{y_t^{(\ell)\top}}\right)^\top$ follows a vector gamma-zero process under the risk-neutral measure.

$$\mathbf{y}_t|\underline{\mathbf{y_{t-1}}} \overset{\mathbb{Q}}{\sim} \Gamma_\mathbf{0}\left(\boldsymbol{\alpha}^\mathbb{Q} + \boldsymbol{\beta}^\mathbb{Q}\mathbf{y}_{t-1}; \mathbf{c}^\mathbb{Q}\right) \tag{15}$$

[14] See Monfort et al. (2017*b*) for details on the gamma-zero process.

Using the same notations as in Monfort et al. (2017*b*), $\boldsymbol{\alpha}^{\mathbb{Q}} \in \mathbb{R}_{+}^{k_c+k_\ell}$ and $\boldsymbol{\beta}^{\mathbb{Q}}$ are respectively the intercept vector and the autoregressive matrix with positive components, and the vector of scale parameters $\mathbf{c}^{\mathbb{Q}} \in \mathbb{R}_{+}^{k_c+k_\ell}$. Combining the dynamics given by Equations (14)-(15), we show in Appendix A.4 that the joint process $(\mathbf{y_t}^\top, \boldsymbol{\delta_t}^\top)^\top$ is affine under the risk-neutral measure. This property will allow us to obtain closed-form pricing formulas.

4.4 Risky Asset Prices

In the following, all nominal securities prices of interest to perform pricing take the form:

$$G_t(n, n_c, n_\ell) = \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{n-1} r_{t+j}\right) \mathbb{1}\left\{\sum_{j=0}^{n_c} \delta_{t+j}^{(c)} = 0\right\} \mathbb{1}\left\{\sum_{j=0}^{n_\ell} \delta_{t+j}^{(\ell)} = 0\right\}\right].$$

The principle is the same for inflation-indexed securities, by just replacing r_{t+j} by $r_{t+j} - \pi_{t+j+1}$. We denote these by $G_t^*(n, n_c, n_\ell)$. Using the lemma provided in Monfort et al. (2017*a*), we show in Appendix A.4 that:

$$\begin{aligned} G_t(n, n_c, n_\ell) &= \exp\left(q_{(n,n_c,n_\ell)} + \mathbf{Q}_{(n,n_c,n_\ell)}^\top \mathbf{z_t}\right), & (16)\\ G_t^*(n, n_c, n_\ell) &= \exp\left(q_{(n,n_c,n_\ell)}^* + \mathbf{Q}_{(n,n_c,n_\ell)}^{*\top} \mathbf{z_t}\right). & (17) \end{aligned}$$

The exact recursions used to obtain the loadings in those equations are closed-form and detailed in the Appendix. Notice that the price of riskless bonds computed above are directly given by $D_t^{(n)} = G_t(n, 0, 0)$ and $D_t^{*(n)} = G_t^*(n, 0, 0)$.

Consider a sovereign state that issues both nominal and inflation-protected debt with maturity n. With some probability, the bond defaults prior to maturity n. Default happens when, at any time $\tau_c \leq t+n$ where the credit-event variable $\delta_t^{(c)}$ jumps from zero to a positive value (see Equation (13)). In the same fashion, liquidity events happen when the liquidity-event variable $\delta_t^{(\ell)}$ jumps from zero to a positive value.

We assume that nominal bonds of the sovereign are unaffected by liquidity events. In the case of a credit event, nominal bondholders get a recovery payment on their investment, $\mathcal{P}_c^{(n)}$. The price of a nominal zero-coupon bond is given by:

$$
\begin{aligned}
B_t^{(n)} &= \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j} \right) \mathcal{P}_c^{(i)} \times \left(\mathbb{1}\left\{ \sum_{j=0}^{i-1} \delta_{t+j}^{(c)} = 0 \right\} - \mathbb{1}\left\{ \sum_{j=0}^{i} \delta_{t+j}^{(c)} = 0 \right\} \right) \right] \\
&+ \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{n-1} r_{t+j} \right) \mathbb{1}\left\{ \sum_{j=0}^{n} \delta_{t+j}^{(c)} = 0 \right\} \right]. \qquad (18)
\end{aligned}
$$

Equation (18) simply states that the price of the nominal bond is the sum of discounted recovery payments if default happens between $t+i-1$ and $t+1$, and the discounted principal if no default occurs during the lifespan of the bond. An inflation-indexed bond is priced similarly, with the difference that its payoff is indexed to a reference inflation index, denoted by π_t. We assume that the bond's recovery payment upon default may differ from that of the nominal bond, and designate this recovery payment $\mathcal{P}_c^{*(n)}$. Similarly, the recovery payment in case of liquidity event is given by $\mathcal{P}_\ell^{*(n)}$. The price of the inflation-indexed bond is then given by:

$$
\begin{aligned}
B_t^{*(n)} &= \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}} \Bigg[\exp\left(-\sum_{j=0}^{i-1} r_{t+j} \right) \left(\mathcal{P}_c^{*(i)} + \mathcal{P}_\ell^{*(i)} \right) \mathbb{1}\left\{ \sum_{j=0}^{i-1} \mathbf{e}_2^\top \boldsymbol{\delta}_{t+j} = 0 \right\} \\
&- \mathcal{P}_c^{*(i)} \exp\left(-\sum_{j=0}^{i-1} r_{t+j} \right) \mathbb{1}\left\{ \sum_{j=0}^{i-1} \mathbf{e}_2^\top \boldsymbol{\delta}_{t+j} + \delta_{t+i}^{(c)} = 0 \right\} \\
&- \mathcal{P}_\ell^{*(i)} \exp\left(-\sum_{j=0}^{i-1} r_{t+j} \right) \mathbb{1}\left\{ \sum_{j=0}^{i-1} \mathbf{e}_2^\top \boldsymbol{\delta}_{t+j} + \delta_{t+i}^{(\ell)} = 0 \right\} \Bigg] \\
&+ \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{n-1} r_{t+j} - \pi_{t+j+1} \right) \mathbb{1}\left\{ \sum_{j=0}^{n} \mathbf{e}_2^\top \boldsymbol{\delta}_{t+j} = 0 \right\} \right], \qquad (19)
\end{aligned}
$$

where $\mathbf{e_2} = (\mathbf{1}, \mathbf{1})^\top$. Following Duffie and Singleton (1999) and to be consistent with the CDSs, we assume the recovery of face value assumption (RFV), which states that in case of default, recovery payments are proportional to the face value of the bond, by a factor equal to the recovery rate. For TIPS, we assume that the principal is continuously adjusted by the realized inflation. The total recovery payments write:

$$\mathcal{P}_c^{(n)} = \rho_c\,, \quad \mathcal{P}_c^{*(n)} = \rho_c^* \exp\left(\sum_{j=0}^{n-1} \pi_{t+j+1}\right)\,, \quad \text{and} \quad \mathcal{P}_\ell^{*(n)} = \rho_\ell^* \exp\left(\sum_{j=0}^{n-1} \pi_{t+j+1}\right)$$

We show in the Appendix that under these assumptions, using equation (18)-(19), bond prices at time t can be approximated by:

$$B_t^{(n)} = \exp\left[\mathcal{A}_n(\rho_c) + \mathcal{B}_n(\rho_c)^\top \mathbf{z_t}\right] \quad (20)$$

$$B_t^{*(n)} = \exp\left[\mathcal{A}_n^*(\rho_c^*, \rho_\ell^*) + \mathcal{B}_n(\rho_c^*,\, \rho_\ell^*)^\top \mathbf{z_t}\right]\,, \quad (21)$$

where $\mathbf{z_t} = \left(\mathbf{x_t}^\top,\, \mathbf{y_t}^\top\right)^\top$, and assuming that the bond has survived to time t, and where the loadings can be obtained as:

$$\begin{aligned}
\mathcal{A}_n(\rho_c) &= q_{(n,n,0)} + \rho_c \sum_{i=1}^{n} \left(q_{(i,i-1,0)} - q_{(i,i,0)}\right) \\
\mathcal{B}_n(\rho_c) &= \mathbf{Q}_{(n,n,0)} + \rho_c \sum_{i=1}^{n} \left(\mathbf{Q}_{(i,i-1,0)} - \mathbf{Q}_{(i,i,0)}\right)\,,
\end{aligned}$$

and,

$$\begin{aligned}
\mathcal{A}_n^*(\rho_c^*,\, \rho_\ell^*) &= q_{(n,n,0)}^* + \sum_{i=1}^{n} \left((\rho_c^* + \rho_\ell^*) q_{(i,i-1,i-1)} - \rho_c^* q_{(i,i,0)} - \rho_\ell^* q_{(i,i,0)}\right) \\
\mathcal{B}_n^*(\rho_c^*,\, \rho_\ell^*) &= \mathbf{Q}_{(n,n,0)}^* + \sum_{i=1}^{n} \left((\rho_c^* + \rho_\ell^*) \mathbf{Q}_{(i,i-1,i-1)} - \rho_c^* \mathbf{Q}_{(i,i,0)} - \rho_\ell^* \mathbf{Q}_{(i,i,0)}\right)\,.
\end{aligned}$$

The recursions to obtain these loadings are again detailed in Appendix A.4. An immediate object of interest is the breakeven inflation rate BEI(t, h), that is the spread between nominal and TIPS yields. Building on our assumed risk-neutral dynamics, we can write:

$$\text{BEI}_t^{(n)} = \frac{1}{n}\left(\mathcal{A}_n^*(\rho_c^*, \rho_\ell^*) - \mathcal{A}_n(\rho_c) + \left[\mathcal{B}_n(\rho_c^*,\, \rho_\ell^*) - \mathcal{B}_n(\rho_c)\right]^\top \mathbf{z_t}\right) \quad (22)$$

The spread between ILS and BEI is therefore given by:

$$\text{ILS}_t^{(n)} - \text{BEI}_t^{(n)} = \frac{1}{n}\left(A_n^* - \mathcal{A}_n^*(\rho_c^*, \rho_\ell^*) + \mathcal{A}_n(\rho_c) - A_n + \left[\begin{pmatrix} \mathbf{B_n^*} - \mathbf{B_n} \\ \mathbf{0}_{k_c+k_\ell} \end{pmatrix} - \mathcal{B}_n(\rho_c^*,\, \rho_\ell^*) + \mathcal{B}_n(\rho_c) \right]^\top \begin{bmatrix} \mathbf{x_t} \\ \mathbf{y_t} \end{bmatrix} \right). \tag{23}$$

For our risk-neutral dynamics, we can show that the spread between ILS and BEI almost depends entirely on the credit and liquidity factors $\mathbf{y_t}$. This provides us with a clean credit-liquidity decomposition of the term structure of ILS-BEI.

4.5 CDS Pricing

We assume that a buyer of protection makes periodic payments from time t to maturity n to protect against default on the underlying nominal sovereign bond. The cash flow payment at time $t+i$ conditional on no default is designated as $s_t^{(n)}$. The present value of the stream of cash flows paid by the protection buyer is:

$$\text{PB}_t^{(n)} = s_t^{(n)} \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j} \right) \mathbb{1}\left\{ \sum_{j=0}^{i} \delta_{t+j}^{(c)} = 0 \right\} \right]$$

If the sovereign defaults at time $t+i$, we assume that the protection seller pays the buyer a payment of $1-\rho_c$. The present value of the protection sold is:

$$\text{PS}_t^{(n)} = \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j} \right) (1-\rho_c) \left(\mathbb{1}\left\{ \sum_{j=0}^{i-1} \delta_{t+j}^{(c)} = 0 \right\} - \mathbb{1}\left\{ \sum_{j=0}^{i} \delta_{t+j}^{(c)} = 0 \right\} \right) \right].$$

No arbitrage pricing requires that the present value of the protection bought is equal to the present value of the protection sold. Equating both legs at inception, using the risk-neutral dynamics of Section 4.3 and solving for the swap spread yields:

$$s_t^{(n)} = \frac{(1-\rho_c)\sum_{i=1}^{n}\left[\exp\left(q_{(i,i-1,0)} + \mathbf{Q}_{(i,i-1,0)}^{\top}\mathbf{z_t}\right) - \exp\left(q_{(i,i)} + \mathbf{Q}_{(i,i,0)}^{\top}\mathbf{z_t}\right)\right]}{\sum_{i=1}^{n}\exp\left(q_{(i,i,0)} + \mathbf{Q}_{(i,i,0)}^{\top}\mathbf{z_t}\right)} \tag{24}$$

where all loadings can be computed through closed-form recursions presented in Appendix A.4. As usual, even if the model is affine, the swap rate ends up a closed-form non-affine function of the state variables.

5 Model Estimation

5.1 Data and Estimation Method

Let us first describe our empirical targets. We consider ILS-BEI spreads for 2-, 5-, and 10-years to maturity, and we fit both the term structure of ILS and ILS-BEI spreads. We also consider the 5-year U.S. sovereign CDS spread. We add inflation data computed as the log-change of the CPI-U index provided by the Bureau of Labor Statistics (BLS). The input sample period for the model estimation spans July of 2004 to March of 2015 at the weekly frequency for ease of estimation.

In our baseline specification, we consider one riskless factor ($k_x^{(r)} = 1$), one inflation factor ($k_x^{(\pi)} = 1$), two credit factors ($k_c = 2$) and one liquidity factor ($k_\ell = 1$). Our selection of two factors to represent credit risk is consistent with Augustin (2018*b*). Our estimation only consists in month-by-month curve fitting and is thus only concerned with risk-neutral parameters. Our estimation algorithm relies on non-linear least squares techniques. We first fix a set of parameters θ such that:

$$\theta = \left\{\kappa_0^{(r)},\, \kappa_r,\, \kappa_0^{(\pi)},\, \kappa_\pi,\, \boldsymbol{\mu}^{\mathbb{Q}},\, \boldsymbol{\Phi}^{\mathbb{Q}},\, \boldsymbol{\Sigma}^{\mathbb{Q}},\, \boldsymbol{\alpha}^{\mathbb{Q}},\, \boldsymbol{\beta}^{\mathbb{Q}},\, \boldsymbol{c}^{\mathbb{Q}},\, \boldsymbol{\gamma}_c,\, \gamma_\ell,\, \rho_c,\, \rho_c^*,\, \rho_\ell^*\right\}.$$

Then, for each month, gather all our 8 observable variables (3 ILS, 3 ILS-BEI spreads, inflation and CDS spread) in a vector and estimate the factors by minimizing the weighted sum of squared residuals using Equations (9), (12), (23) and (24).[15] Note that because the CDS pricing formula is non-linear in the factors, we use numerical optimization methods. Since our main interest is to know how much of the ILS-BEI spreads can be explained by credit factors, we subordinate the factor estimation for each date: we first minimize the squared residuals in all factors but liquidity, and then perform a second optimization to find the best liquidity factor. Note that since there is no default or liquidity event throughout the sample, we fix $\delta_t^{(c)} = \delta_t^{(\ell)} = 0$ for all dates.

For parsimony and identification purposes, we impose that $\kappa_r = \kappa_\pi = 1$, $\boldsymbol{\Sigma}^{\mathbb{Q}}$ is a diagonal matrix, $\boldsymbol{\beta}^{\mathbb{Q}}$ is lower triangular, $\boldsymbol{c}^{\mathbb{Q}} = \mathbf{1}_{k_r+k_\ell}$. We end up with a total of 25 parameters that we estimate by minimizing the sum of all dates squared residuals.

5.2 Estimation Results

Table VIII presents the parameter estimates of the model. All parameters are highly significant with the exception of $\kappa_0^{(r)}$. The feedback from the liquidity to credit factors is also significant, such that higher liquidity issues result in higher (risk-neutral) probability of sovereign default. The third panel of Table VIII presents the estimated recovery rates for both nominal and real bonds. We see that our assumption of a differential recovery is supported by the data, and nominal bonds and TIPS show recovery rates of 77% and 69% respectively. Liquidity issues are on average more severe, since the recovery rate associated with liquidity is only 48%. Note that this value is very close to the 50% recovery usually assumed in reduced-form term structure models.

The three latent factors, $\mathbf{y}_t$, filtered by our model are plotted in Figure IV. The first two factors jointly determine the likelihood of default while the thrid factor is the liquidity factor.

[15] For the weights, we normalize all sum of squared residuals by the sample variance of the respective observable variable. We then multiply the sum of squared CDS residuals by 10^4, the sum of squared ILS and ILS-BEI residuals by 10^2. These values are sufficient to impose that the CDS spread is perfectly fitted and that the term structures are fitted as much as possible.

The credit factors allow us to perfectly track the 5y CDS spread. As for the CDS data, most of the movements can be observed after the outbreak of the financial crisis. In contrast, the liquidity factor increases gradually before the crisis and spikes up in 2008. Movements in the liquidity factor are much less pronounced post crisis.

Next, we plot the fit of the model on Figures V-VII and the corresponding R-square values are presented in Table IX. Our model produces a perfect fit of the CDS spread mainly by construction (see Figure VI). For the ILS in Figure V, the model does a tremendous job in capturing the term structure. We obtain R-squares ranging from 91% to 95%. The intuition behind this result is straightforward: because our second riskless factors is used to fit the inflation rate, the first factor is used solely to fit the term structure of inflation swaps. As usual in the term structure literature, one factor is usually sufficient to capture the bulk of fluctuations of the different maturities. For the risky equivalents, the model does a reasonable job in capturing the ILS-BEI spreads (see Figure VII). Because the credit factors are imposed to fit the CDS and the liquidity factor is always positive, the model struggles a bit more for the long maturity. We obtain R-squares of 96% and 88% for the 2y and 5y maturities respectively, while the fitted values for the 10y explain only 34% of the variance of the ILS-BEI spreads. Further inspection on the time-series fit shows that most of this lack of power comes from a poor fit during the pre-crisis period. This is probably linked to our requirement to fit the CDS data perfectly. Indeed, there is little movement in the two default factors during this period, and it is hard for the model to explain the entire term structure with the liquidity factor only.

Overall, the performance of the model is satisfactory. It achieves good fit along four dimensions of data: CDS, inflation, ILS, and BEI. As a result, the estimated model can explain significant variations in the ILS-BEI spread, especially during and following the financial crisis.

5.3 Decomposition of ILS-BEI Spreads

To understand the relative importance of default risk in driving a wedge between ILS and BEI, we fit the ILS-BEI curves at maturities of 2-, 5-, and 10-years by employing only the credit risk factors. Then, we contrast the fitted curves with only the credit risk factors against the fitted ILS-BEI curves in Figure VII (red, dashed lines). The results are plotted in Figure VIII.

Three observations can be discerned from Figure VIII. First, the credit component of the ILS-BEI spread contributes nothing to the overall fit of the curves prior to 2008. This is, again, due to the minimal premium on U.S. CDS contracts before the financial crisis as seen in Figure VI. As a result, the liquidity factor explains the entirety of the ILS-BEI spreads before the crisis across maturities. Second, in the middle of the crisis around September of 2008, the peak of the ILS-BEI spread is mostly driven by the liqudity factor. Going back to Figure VII, the liquidity factor is the dominant component that drives good fit of the model-implied ILS-BEI curves against those in the data during the crisis. Third, the credit component of the ILS-BEI spread is the dominant factor in capturing the variability of the ILS-BEI curves in the data in the post-crisis period. Figure VIII shows this is especially so at long maturities as evidenced by the 10-year ILS-BEI curves with and without the liquidity component.

To put the decomposition in more precise terms, Figure IX plots the percent contribution of the credit component relative to the whole fitted ILS-BEI curves at the same three maturities. The same three implcations from Figure VIII can be visualized here. Namely, the credit component contributes little prior to the 2008 crisis, its contribution is overshadowed by the liquidity component at the height of the crisis, and lastly, it is the main explanatory variable of the ILS-BEI spread in the post-crisis sample up to 2015.

Our asset pricing model validates the results of our panel regressions in Section 3.2. In particular, over the full sample between 2008 and 2015, the U.S. CDS spread driven by the credit factors has positive and significant explanatory power of the ILS-BEI spreads after controlling for liquidity. Moreover, contrasting Column (7) in Tables III and IV, we see that the explanatory power of the credit component (proxied by the CDS spread) is statistically weaker (t-statistic of 1.84) during the crisis period and much stronger (t-statistic of 4.46) in the post-crisis period. Whereas the opposite is true for the liquidity factor (proxied by the OTR Difference) with t-statistic of 3.97 in Table III and t-statistic of 0.52 in Table IV. This is consistent with the decomposition of the fitted ILS-BEI curves shown in Figures VIII and IX.

5.4 Panel Regressions with Credit Factors

As the last exercise to tie our estimated credit factors to the data, we perform panel regressions similar to those used in Subsection 3.2, where ILS-BEI spreads across tenors are projected onto the two credit factors and the liquidity factor constructed here. As before, we estimate the coefficient loading for the sample between 2008 and 2015, as well as the two subsamples covering the crisis period and the post-crisis period. The results are shown in Table X. Column (1) is for the entire sample, Column (2) is for the post-crisis sample, and Column (3) is for during the crisis. Following the previous empirical specification, we control for lagged ILS-BEI spreads.

In all three sample periods, while controlling for liquidity, the second credit factor is always positive and significant in explaining the ILS-BEI spread, consistent with the idea that higher default risk drives a wedge between nominal and real Treasury yields that is beyond inflation risk. Moreover, we notice the first credit factor is positive and significant in Columns (1) and (2) but not in Column (3), which is during the financial crisis. This is consistent with the observation that, in the estimated model, the liquidity factor is the primary driver of the ILS-BEI dynamics during the crisis, whereas in the post-crisis sample, the importance of the credit factors is accentuated. Overall, the panel regressions with credit and liquidity factors reconfirms our results using CDS spreads.

6 Conclusion

In this paper, we explore the relative pricing of nominal and real U.S. sovereign securities in the presence of credit risk. We argue that while most of the previous studies attribute the mispricing of TIPS to liquidity factors or slow moving capital, credit risk can also represent a significant driver of deviations oftentimes interpreted as violations of no-arbitrage. Our study shows that in the presence of credit risk, the spreads between inflation-linked swaps and breakeven inflation rates reflect differences in the propensity of the sovereign to reimburse nominal and real bonds in case of default. We hypothesize this result is driven by a difference in recovery rates. Our empirical approach shows U.S. CDS spreads are positively correlated with first differences in ILS-BEI spreads after the financial crisis, while controlling for liquidity and potential alternative explanations. We then conduct a more formal empirical analysis through an intensity-based affine asset pricing model. We show that credit risk factors extracted from the CDS are able to explain most of the ILS-BEI yield curve after the financial crisis. Our model estimates confirm the existence of a lower recovery rate for TIPS than for nominal bonds by about 8 percentage points.

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Tables

Table I: **Summary Statistics**

Table I provides summary statistics for the variables used in the regression analysis. Panel A includes the full sample period from January 2008 to October 2015. Panel B is the post crisis subsample from January 2008 to October 2015. $ILS - BEI$ is the difference in the 5-year inflation swap rate and the 5-year breakeven inflation rate (Treasury-TIPS). Both $Tsy\ ZC\ Yield$ and $TIPS\ ZC\ Yield$ are for the 5-year maturity. 5-year $US\ CDS$ spreads are denominated in EUR. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $HPWNoise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. VIX denotes the CBOE Volatility Index. $RepoFails$ is the total of weekly failed deliveries and receipts.

Panel A			**Full Sample**		
	Mean	SD	Min	Max	N
ILS-BEI (bps)	36	30	-1	210	1902
Infl Swap Rate	2.04	0.49	-0.57	3.31	1902
Tsy ZC Yield	1.74	0.70	0.59	3.76	1902
TIPS ZC Yield	0.06	1.04	-1.72	3.88	1902
US CDS (bps)	33	16	6	100	1902
LIBOR-OIS	0.34	0.43	0.06	3.64	1902
HPW Noise	3.51	3.50	0.72	20.47	1902
TIPS Noise	5.93	5.06	2.05	41.8	1902
VIX	21.96	10.44	10.32	80.86	1902
Panel B			**Post Crisis**		
	Mean	SD	Min	Max	N
ILS-BEI (bps)	23	10	-1	59	1403
Infl Swap Rate	2.09	0.29	1.24	2.71	1403
Tsy ZC Yield	1.45	0.51	0.59	2.79	1403
TIPS ZC Yield	-0.41	0.62	-1.72	0.83	1403
US CDS (bps)	34	12	14	63	1403
LIBOR-OIS	0.19	0.09	0.06	0.50	1403
HPW Noise	1.99	0.74	0.72	4.58	1403
TIPS Noise	4.72	1.41	2.05	7.58	1403
VIX	18.40	6.13	10.32	48.00	1403

Table II: **ILS-BEI - January 2008 to October 2015**

Table II shows the results from a panel regression of the change in ILS-BEI on US CDS spreads and various controls using daily observations. The sample period is from January 2008 to October 2015. $ILS-BEI$ is the difference in the inflation swap rate and the breakeven inflation rate (Treasury-TIPS) for 2-, 3-. 5-, 7-, and 10-year tenors. $US\ CDS$ spreads are for the 5-year tenor. $HPWNoise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. $LIBOR-OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in 10-year Treasury par yield from Gurkaynak, Sack and Wright (2006) less the on-the-run 10-year Treasury yield from Bloomberg. VIX denotes the CBOE Volatility Index. Standard errors are reported in parentheses.

Dep Var: Δ *ILS-BEI Spread*	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
US CDS	0.196***	0.195***	0.218***	0.205***	0.186***	0.218***	0.228***	0.228***
	(0.049)	(0.049)	(0.049)	(0.050)	(0.049)	(0.051)	(0.051)	(0.051)
ILS-BEI_{t-1}	−0.178***	−0.178***	−0.177***	−0.178***	−0.178***	−0.178***	−0.177***	−0.183***
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
HPW Noise		0.111					0.637**	0.633**
		(0.283)					(0.291)	(0.290)
TIPS Noise			−1.077***				−1.039***	−1.033***
			(0.172)				(0.182)	(0.182)
LIBOR-OIS				−3.303*			−3.965**	−3.952**
				(1.826)			(1.924)	(1.921)
OTR Difference					−24.810***		−17.395***	−17.457***
					(4.816)		(5.086)	(5.076)
VIX						−0.065*	−0.023	−0.022
						(0.039)	(0.041)	(0.041)
Week	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Tenor	No	No	No	No	No	No	No	Yes
Observations	9147	9147	9147	9147	9147	9127	9127	9127
R^2	0.148	0.148	0.152	0.149	0.151	0.149	0.155	0.158

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table III: **ILS-BEI - January 2008 to December 2009**

Table III shows the results from a panel regression of the change in ILS-BEI on US CDS spreads and various controls using daily observations. The sample period is from January 2008 through December 2009. $ILS - BEI$ is the difference in the inflation swap rate and the breakeven inflation rate (Treasury-TIPS) for 2-, 3-. 5-, 7-, and 10-year tenors. $US\ CDS$ spreads are for the 5-year tenor. $HPWNoise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in the on-the-run 10-year U.S. Treasury and the off-the-run 9-year U.S. Treasury from the Bloomberg on/off-the-run U.S. Treasury curve. VIX denotes the CBOE Volatility Index. Standard errors are reported in parentheses.

Dep Var: Δ ILS-BEI Spread	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
US CDS	0.213*	0.209*	0.258**	0.232*	0.199*	0.258**	0.270**	0.264**
	(0.120)	(0.121)	(0.121)	(0.122)	(0.120)	(0.124)	(0.125)	(0.121)
ILS-BEI_{t-1}	−0.205***	−0.205***	−0.204***	−0.205***	−0.205***	−0.204***	−0.203***	−0.302***
	(0.0108)	(0.0108)	(0.0108)	(0.0108)	(0.0108)	(0.0108)	(0.0108)	(0.0128)
HPW Noise		0.364					1.246*	1.091*
		(0.631)					(0.660)	(0.636)
TIPS Noise			−1.139***				−1.011***	−0.886***
			(0.321)				(0.354)	(0.341)
LIBOR-OIS				−3.374			−3.272	−3.001
				(3.256)			(3.594)	(3.462)
OTR Difference					−44.458***		−32.578***	−34.241***
					(11.188)		(12.240)	(11.792)
VIX						−0.151	−0.072	−0.055
						(0.098)	(0.106)	(0.102)
Week	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Tenor	No	No	No	No	No	No	No	Yes
Observations	2387	2387	2387	2387	2387	2387	2387	2387
R^2	0.168	0.168	0.172	0.168	0.173	0.168	0.177	0.238

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table IV: **ILS-BEI - January 2010 to October 2015**

Table IV shows the results from a panel regression of the change in ILS-BEI on US CDS spreads and various controls using daily observations. The sample period is from January 2010 to October 2015. $ILS-BEI$ is the difference in the inflation swap rate and the breakeven inflation rate (Treasury-TIPS) for 2-, 3-. 5-, 7-, and 10-year tenors. $US\ CDS$ spreads are for the 5-year tenor. $HPWNoise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. $TIPSNoise$ measures average daily deviations in the real yield curve. $LIBOR-OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in 10-year Treasury par yield from Gurkaynak, Sack and Wright (2006) less the on-the-run 10-year Treasury yield from Bloomberg. VIX denotes the CBOE Volatility Index. Standard errors are reported in parentheses.

Dep Var: Δ ILS-BEI Spread	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
US CDS	0.171***	0.170***	0.172***	0.171***	0.172***	0.164***	0.165***	0.167***
	(0.035)	(0.035)	(0.035)	(0.035)	(0.035)	(0.037)	(0.037)	(0.037)
ILS-BEI$_{t-1}$	−0.095***	−0.095***	−0.095***	−0.095***	−0.095***	−0.096***	−0.096***	−0.112***
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.006)
HPW Noise		−0.396*					−0.381*	−0.376
		(0.230)					(0.231)	(0.230)
TIPS Noise			−0.415				−0.377	−0.375
			(0.268)				(0.270)	(0.269)
LIBOR-OIS				−4.591			−3.509	−3.280
				(7.704)			(7.769)	(7.739)
OTR Difference					1.563		2.120	2.056
					(3.667)		(3.737)	(3.723)
VIX						0.022	0.018	0.020
						(0.028)	(0.028)	(0.028)
Week	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Tenor	No	No	No	No	No	No	No	Yes
Observations	6755	6755	6755	6755	6755	6735	6735	6735
R^2	0.100	0.101	0.101	0.101	0.101	0.101	0.102	0.109

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table V: **Components of ILS-BEI Spread - January 2008 to October 2015**

Table V shows the results from a panel regression of changes in TIPS yields, Treasury yields, and ILS spreads on US CDS spreads and various controls using daily observations. The sample period is from January 2008 to October 2015. *US CDS* spreads are for the 5-year tenor. $HPWNoise$ follows Hu, Pan and Wang (2013). *TIPS Noise* measures average daily deviations in the real yield curve. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in 10-year Treasury par yield from Gurkaynak, Sack and Wright (2006) less the on-the-run 10-year Treasury yield from Bloomberg. VIX denotes the CBOE Volatility Index. Standard errors are reported in parentheses.

Dep Var:	(1) Δ TIPS	(2) Δ Nominal	(3) Δ ILS
US CDS	0.268***	0.123***	0.061
	(0.047)	(0.045)	(0.061)
$TIPS_{t-1}$	−0.015***		
	(0.002)		
Nominal $_{t-1}$		−0.028***	
		(0.002)	
ILS $_{t-1}$			−0.040***
			(0.003)
HPW Noise	−0.735***	−1.910***	−0.489
	(0.266)	(0.254)	(0.345)
TIPS Noise	−1.750***	−0.009	0.541**
	(0.167)	(0.159)	(0.216)
LIBOR-OIS	−28.185***	−20.507***	3.139
	(1.762)	(1.684)	(2.281)
OTR Difference	−19.114***	−22.629***	−20.587***
	(4.651)	(4.449)	(6.026)
VIX	0.077**	−0.454***	−0.568***
	(0.037)	(0.036)	(0.048)
Week	Yes	Yes	Yes
Tenor	Yes	Yes	Yes
Observations	9130	9130	9130
R^2	0.290	0.249	0.176

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table VI: **Liquidity Premia and CDS**

Table VI presents results of an analysis of liquidity premia. In Panel A, we present results from regressions

$$\begin{aligned} BEI_t &= a_1 + a_2 OTR_t + a_3 VOL_t + a_4 ILS - BEI_t + a_5 CPI^e_t + a_6 CFNAI_t + \epsilon_{1t} \\ BEI_t &= b_1 + b_2 OTR_t + b_3 VOL_t + b_4 ILS - BEI_t + b_5 CPI^e_t + b_6 CFNAI_t + b_7 US\ CDS_t + \epsilon_{2t}, \end{aligned}$$

where the dependent variable is breakeven inflation, and the independent variables are OTR, the on-the-run 10-Year Treasury Spread, VOL, the log ratio of volume in the TIPS market to the nominal Treasury market, $ILS - BEI$, the inflation swap-adjusted BEI, CPI^e, the median forecast of 10-year CPI inflation from the Survey of Professional Forecasters, $CFNAI$, the Chicago Fed National Activity Index, and $US\ CDS$, the 5-year credit default swap spread for U.S. Treasury securities. In Panel B, the estimated liquidity premium from Panel A is regressed on the U.S. CDS spread. The liquidity premium is measured as

$$\hat{L}_t = -\left(\hat{a}_2 OTR_t + \hat{a}_3 VOL_t + \hat{a}_4 ILS - BEI_t\right).$$

Newey-West standard errors are reported in parentheses.

Panel A: Breakeven Inflation

Dep Var: BEI	(1)	(2)
OTR	-1.143^{**}	-1.218^{***}
	(0.153)	(0.155)
VOL	-0.438^{***}	-0.472^{***}
	(0.058)	(0.060)
$ILS - BEI$	-1.284^{***}	-1.158^{***}
	(0.103)	(0.109)
CPI^e	0.960^{***}	0.997^{***}
	(0.109)	(0.106)
$CFNAI$	0.027^{*}	0.031^{**}
	(0.014)	(0.014)
$US\ CDS$		-0.214^{***}
		(0.075)
R^2	0.651	0.656

Panel B: Liquidity Premium

Dep Var: $\hat{L}$	
$US\ CDS$	0.608^{***}
	(0.126)
R^2	0.086

Notes: *,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table VII: **Cheapest-to-Deliver**

Table VII shows the results from a panel regression of ILS-BEI on US CDS spreads and various controls using daily observations. Information on the prevailing cheapest-to-deliver outstanding U.S. government nominal bond on each trading day is added as a control as set forth in Klingler and Lando (2018). The sample period is from January 2008 to October 2015. $ILS - BEI$ is the difference in the inflation swap rate and the breakeven inflation rate (Treasury-TIPS) for 2-, 3-. 5-, 7-, and 10-year tenors. $US\ CDS$ spreads are for the 5-year tenor. $HPWNoise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in 10-year Treasury par yield from Gurkaynak, Sack and Wright (2006) less the on-the-run 10-year Treasury yield from Bloomberg. VIX denotes the CBOE Volatility Index. Standard errors are reported in parentheses.

Dep Var: Δ *ILS-BEI Spread*	(1)	(2)	(3)
US CDS	0.184***	0.217***	0.217***
	(0.050)	(0.052)	(0.052)
ILS-BEI$_{t-1}$	−0.178***	−0.177***	−0.183***
	(0.00534)	(0.00533)	(0.00540)
CtD	−0.134	−0.222**	−0.221**
	(0.095)	(0.099)	(0.099)
HPW Noise		0.634**	0.630**
		(0.291)	(0.290)
TIPS Noise		−1.042***	−1.036***
		(0.182)	(0.182)
LIBOR-OIS		−3.798**	−3.786**
		(1.925)	(1.922)
OTR Difference		−17.962***	−18.021***
		(5.091)	(5.082)
VIX		−0.049	−0.048
		(0.043)	(0.042)
Week	Yes	Yes	Yes
Tenor	No	No	Yes
Observations	9147	9127	9127
R^2	0.149	0.155	0.159

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table VIII: Model parameters

These tables presents the parameter estimates from the model of Section 4. All parameters associated with the vector autoregressive gamma process are imposed positive. Standard deviations are obtained through the cross-product approximation. We compute numerical derivatives with the symmetric difference quotient with a step size of 10^{-5}.

	$\boldsymbol{\mu}^{\mathbb{Q}}$	$\boldsymbol{\Phi}^{\mathbb{Q}}$		$\boldsymbol{\Sigma}^{\mathbb{Q}}$
$x_t^{(r)}$	0.208729	0.900778	0.018697	0.140503
	(0.004679)	(0.002899)	(0.002346)	(0.003519)
$x_t^{(\pi)}$	−0.11378	0.274697	0.783866	0.134388
	(0.005167)	(0.005581)	(0.004235)	(0.004247)

	$\boldsymbol{\alpha}^{\mathbb{Q}}$	$\boldsymbol{\beta}^{\mathbb{Q}}$			$\boldsymbol{\gamma}$
$\mathrm{y}_{1,t}^{(c)}$	0.000727	1.094302	0	0	$1.27 \cdot 10^{-10}$
	$(2.67 \cdot 10^{-6})$	(0.00394)	–	–	$(1.87 \cdot 10^{-12})$
$\mathrm{y}_{2,t}^{(c)}$	0.000805	0.342226	0.837805	0	$2.65 \cdot 10^{-6}$
	$(2.79 \cdot 10^{-6})$	(0.00162)	(0.004407)	–	$(4.17 \cdot 10^{-8})$
$\mathrm{y}_t^{(\ell)}$	0.00108	0.351114	0.362457	0.956864	$9.36 \cdot 10^{-10}$
	$(6.08 \cdot 10^{-6})$	(0.001375)	(0.001791)	(0.004784)	$(1.62 \cdot 10^{-11})$

ρ_c	ρ_c^*	ρ_ℓ^*	$\kappa_0^{(r)}$	$\kappa_0^{(\pi)}$
0.76618	0.685344	0.480105	$-8.2 \cdot 10^{-6}$	$-1.3 \cdot 10^{-5}$
(0.000714)	(0.000702)	(0.001158)	$(4.21 \cdot 10^{-6})$	$(4.6 \cdot 10^{-6})$

Table IX: R-squared values from term structure model

	CDS	Inflation	ILS			ILS-BEI spreads		
Maturity	5y		2y	5y	10y	2y	5y	10y
R^2	1	0.893	0.949	0.989	0.912	0.957	0.877	0.342

Table X: **ILS-BEI - Model Credit Factors**

Table X shows the results from a panel regression of monthly changes in ILS-BEI on model-generated credit and liquidity factors. Column (1) includes the sample period from January 2008 through March 2015. Column (2) includes the sample period from January 2010 through March 2015. Column (3) includes the sample period from January 2008 through December 2009. $ILS-BEI$ is the difference in the inflation swap rate and the breakeven inflation rate (Treasury-TIPS) for 2-, 3-. 5-, 7-, and 10-year tenors.

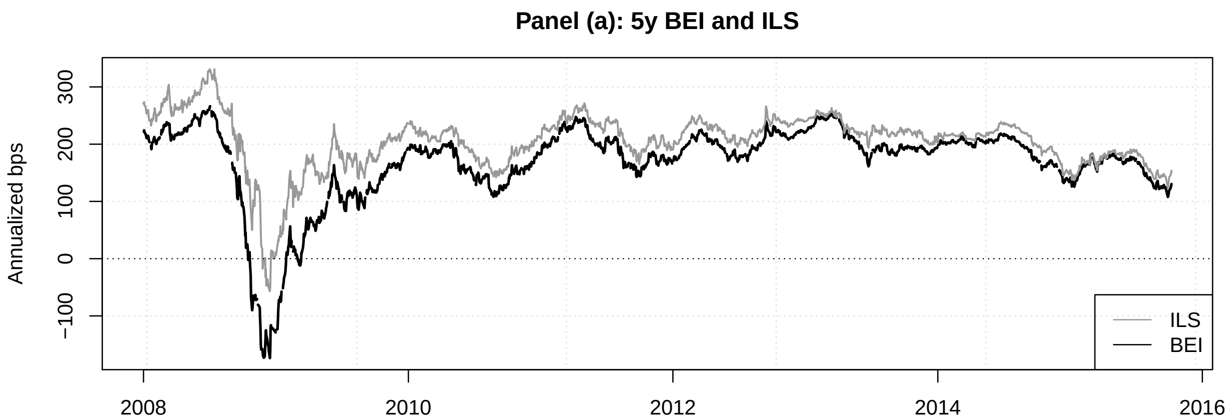
Dep Var: Δ ILS-BEI Spread	(1)	(2)	(3)
ILS-BEI$_{t-1}$	−0.574***	−0.623***	−0.555***
	(0.030)	(0.044)	(0.057)
Credit Factor(1)	1.057***	1.524***	1.039
	(0.338)	(0.519)	(0.660)
Credit Factor(2)	0.002***	0.002***	0.002**
	(0.000)	(0.000)	(0.001)
Liquidity Factor ($\times 10^{-4}$)	0.148***	0.190***	0.148***
	(0.008)	(0.016)	(0.016)
Constant	−0.191	−2.626	−1.390
	(1.343)	(2.329)	(3.825)
Observations	405	285	115
R^2	0.547	0.467	0.573

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Figures

Figure I: Five-year inflation-linked swap and breakeven inflation rate

This figure presents the ILS and BEI daily zero-coupon data for the 5-year maturity from 2005 to 2016 on panel (a). ILS data is taken from bloomberg while nominal and TIPS zero coupon yields are taken from GSW 2006 and 2010 database. Panel (b) presents the spread between ILS-BEI and the CDS spread on the same graph.



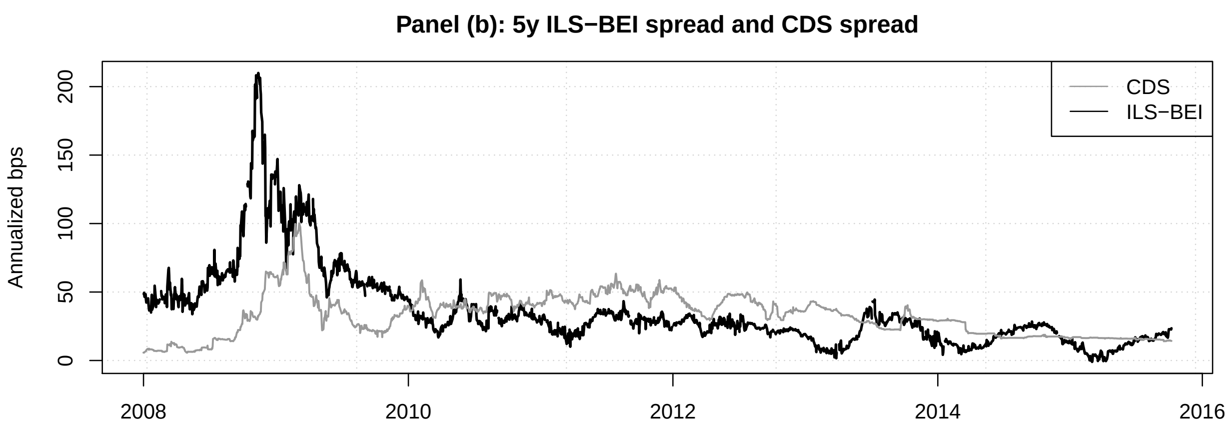


Figure II: Spread between zero-coupon inflation-linked swaps and breakevens

This plot presents daily data of the spread between zero-coupon inflation-linked swaps and breakevens of the corresponding maturity, from January 2008 to October 2015. ILS data is taken from Bloomberg while nominal and TIPS zero-coupon yields are taken from GSW 2006 and 2010 database. Maturities range from 2 years (black line) to 10 years (light grey line).

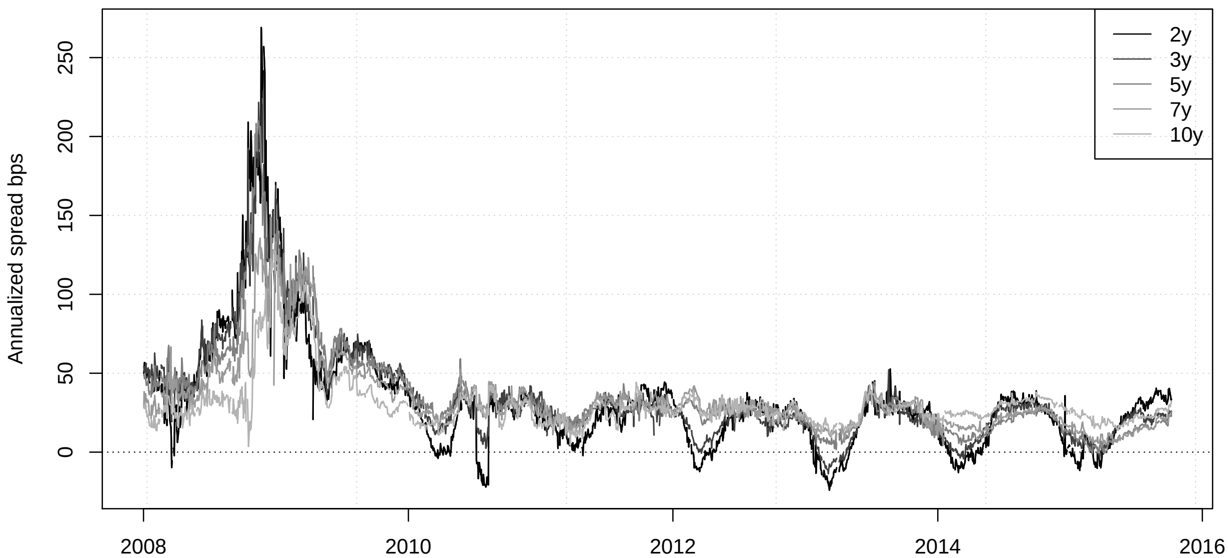


Figure III: Changes in five-year CDS and ILS-BEI data around key policy events

From top left to bottom right we highlight the [-20,+20] trading day windows surrounding the resolution of four key policy events regarding the U.S. debt ceiling and U.S. government shutdown: (1) Budget Control Act of 2011, (2) No Budget, No Pay Act of 2013, (3) Continuing Appropriations Act of 2013, (4) Temporary Debt Limit Extension Act. All changes are relative to the day prior to the [-20,+20] window.

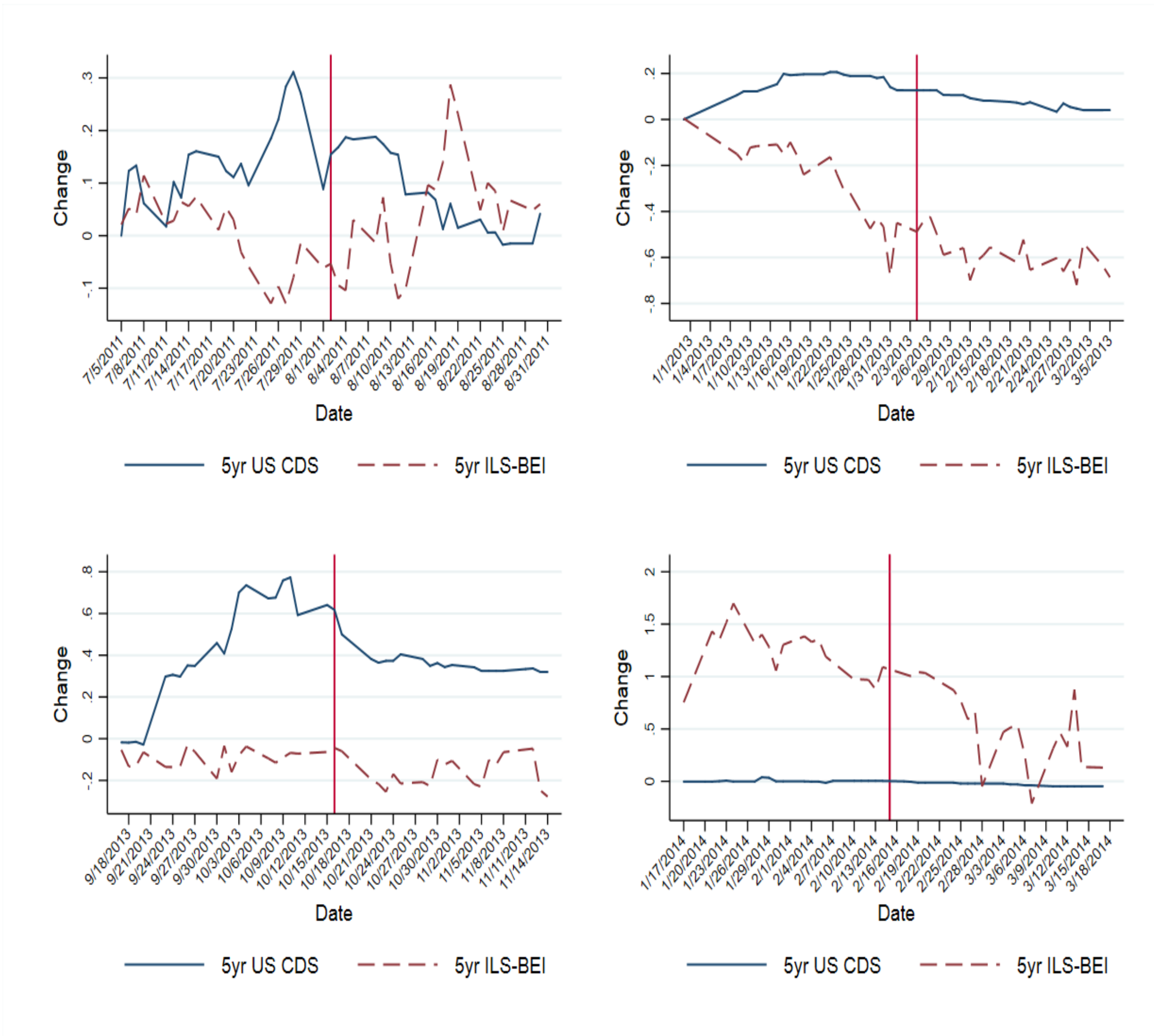


Figure IV: Factors extracted from the NLS estimation in the term structure model

Factors 1 and 2 correspond to credit risk factors while factor 3 is the liquidity factor.

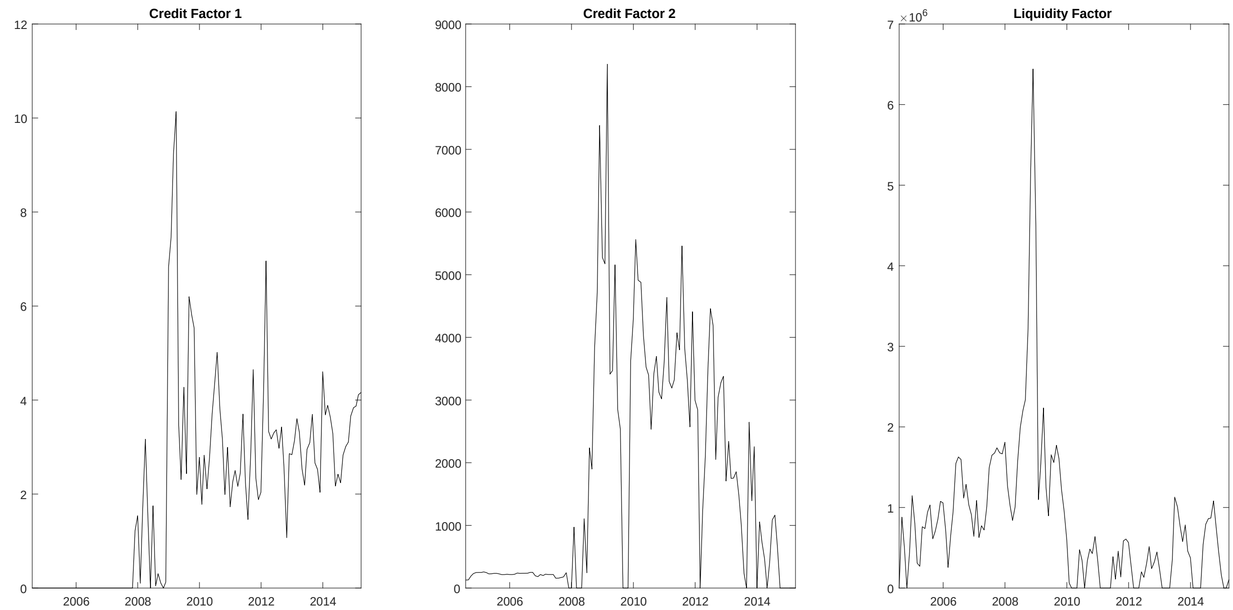


Figure V: Observable inflation linked swaps and model-implied values

Black solid lines represent the observable variables while the red dashed lines are model-implied. All values are in annualized percentage points.

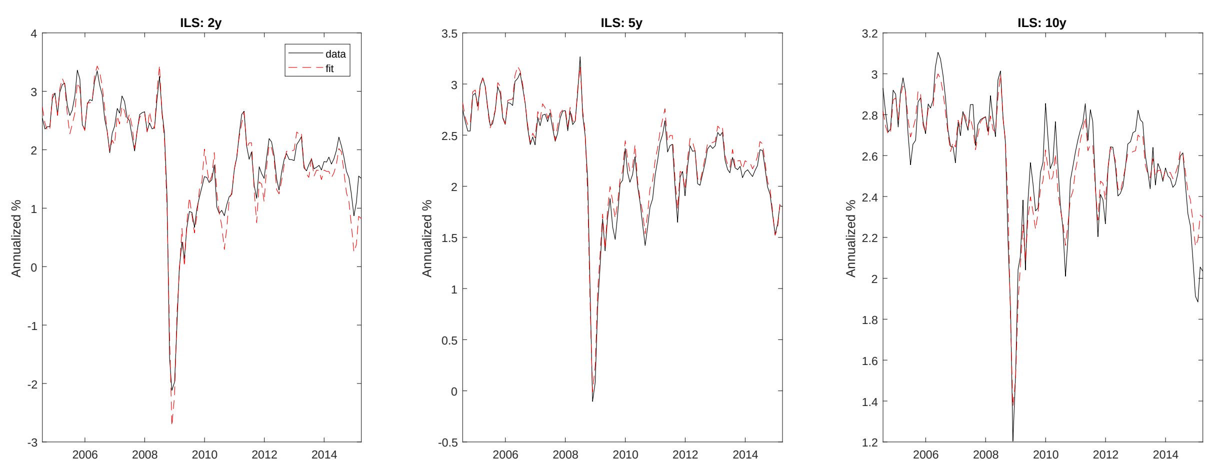


Figure VI: Observable CDS spread and model-implied values

Black solid lines represent the observable variable while the red dashed line is model-implied. All values are in bps.

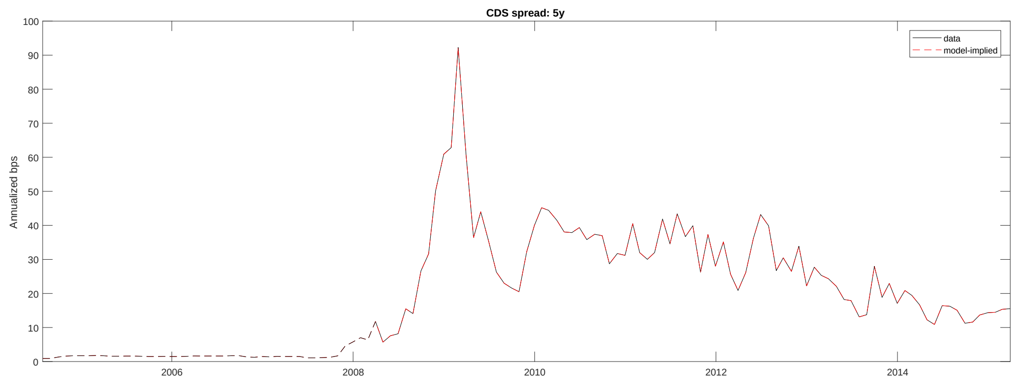


Figure VII: Observable ILS-BEI spreads and model-implied values

Black solid lines represent the observable variables while the red dashed lines are model-implied. All values are in annualized percentage points.

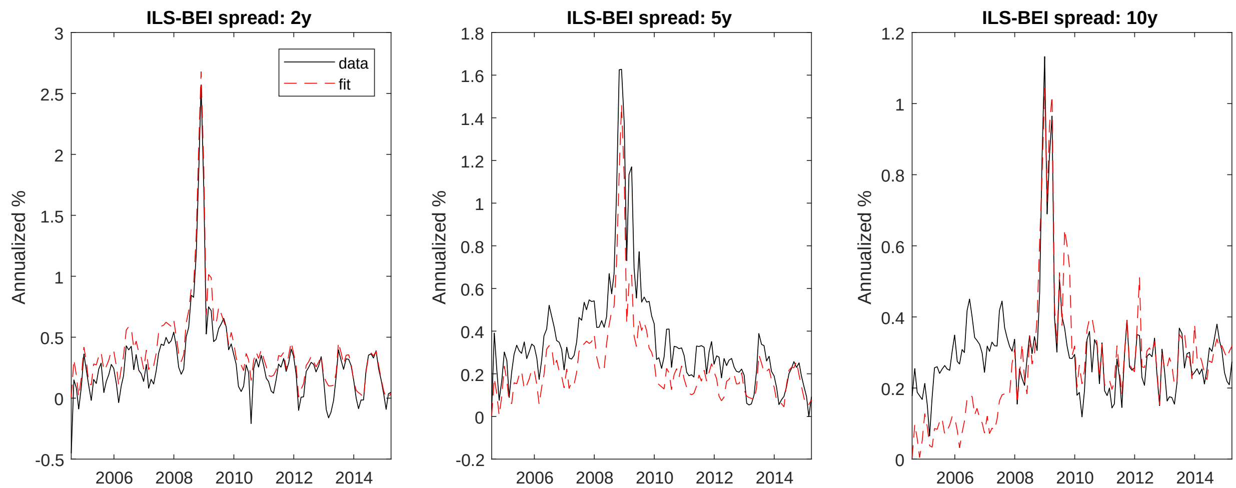


Figure VIII: Decomposition of ILS-BEI spreads

Red dashed lines represent the fitted variables while the blue solid lines represent the component explained by the credit factors. The remaining of the spread is explained by liquidity. All values are in annualized percentage points.

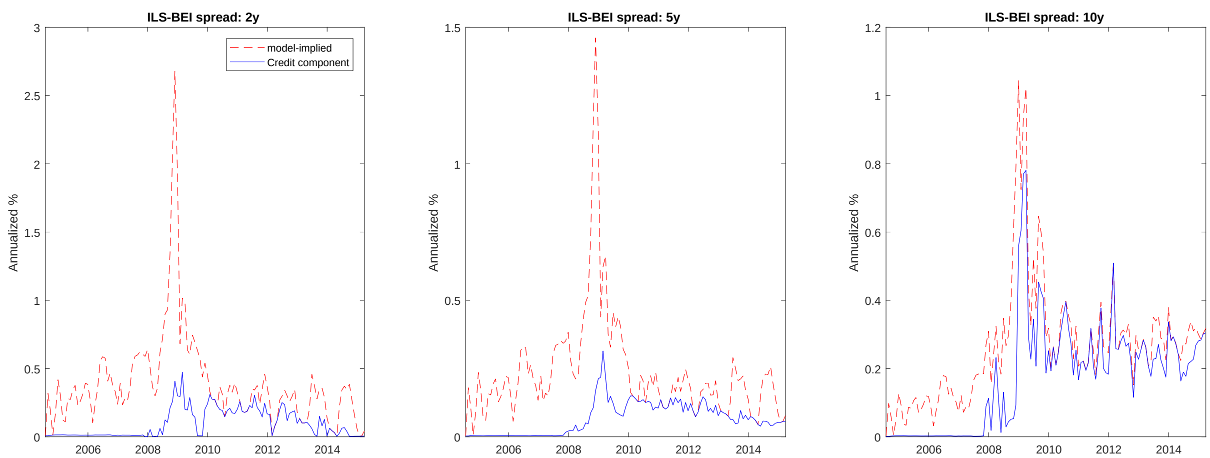
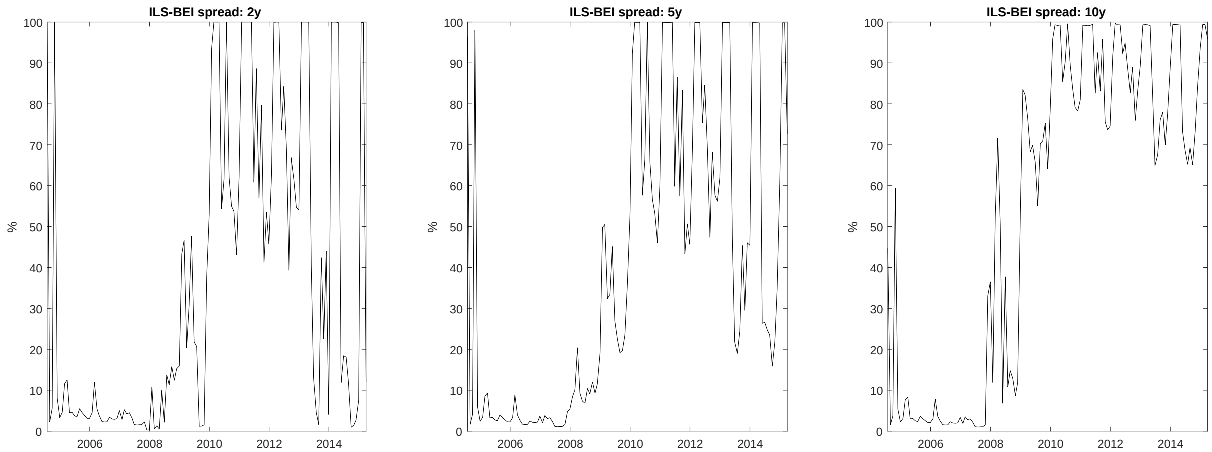


Figure IX: Proportion of ILS-BEI spreads explained by credit

Black solid lines represent the proportion of ILS-BEI spreads explained by the credit factors. The proportion is in ratio of the values fitted by the model. The remaining of the spread is explained by liquidity.



A Appendix

A.1 Variable Descriptions

- HPW Noise, the measure of arbitrage capital availability proposed in Hu, Pan and Wang (2013). This measure is constructed as the root mean squared error in the observed yields of Treasury securities relative to those implied by a Nelson-Siegel-Svensson zero coupon curve across the term structure.[16] The measure takes into account the close relationship between availability of arbitrage capital and liquidity. Fleckenstein, Longstaff and Lustig (2014) posit that the inability of arbitrageurs to immediately eliminate arbitrage may have resulted in the divergence between nominal and inflation-protected securities markets. They suggest that this slow-moving capital hypothesis (Mitchell, Pedersen and Pulvino (2007) and Duffie (2010)) may allow arbitrage profits to persist. This HPW measure, which averages 3.52 basis points, rises to 20.47 basis points during the financial crisis.

- TIPS Noise, is the absolute average fitting error of the Nelson-Siegel-Svensson model estimated on the TIPS yield curve (see Gurkaynak, Sack and Wright (2010)). This variable mimics the HPW Noise measure for the TIPS market as opposed to nominal Treasuries. The series is obtained from the Board of Governors of the Federal Reserve and is computed at the daily frequency.[17] The series move between 0bps and 5 bps before the crisis, spike to 40 bps in late 2008 and average about 5 bps afterward. Since it represents the relative liquidity of TIPS, this measure is well-fitted to control for the slowly moving capital hypothesis of Fleckenstein, Longstaff and Lustig (2014).

- LIBOR-OIS, the spread between LIBOR and the overnight indexed swap rate. As shown in Table I, this spread, which averages 35 basis points over our sample, rose to 364 basis points during the crisis. This rise has been attributed to an increase in per-

[16]These data are obtained from Jun Pan's webpage, http://www.mit.edu/~junpan/

[17]We thank Richard Crump for providing us access to the data.

ceived counterparty credit risk in financial markets. Fleckenstein, Longstaff and Lustig (2014) suggest that their arbitrage profits could arise due to counterparty credit risk, especially if nominal Treasuries are viewed as safe haven assets. However, the authors suggest it is an unlikely explanation for their findings due to the collateralization of of swap contracts (Arora, Gandhi and Longstaff (2012)).

- OTR Difference, the yield difference between the 10-year off-the-run GSW par yield and the generic 10-year on-the-run yield from Bloomberg. During periods of stress, market participants may seek the most liquid securities, on-the-run government benchmark bonds, which accordingly often trade at a premium to an equivalent off-the-run bond.

- VIX, the CBOE volatility index. The VIX is often viewed as a measure of the market's perception of the quantity and/or price of risk in equity markets specifically, and financial markets as a whole. However, Nagel (2012) suggests that an increase in the VIX is associated with a higher premium for liquidity provision, and therefore a reduction in the amount of liquidity in the financial system. The VIX averages 22% over our sample period, with an increase to nearly 81% during the financial crisis.

A.2 Empirical Robustness

A.2.1 Pflueger and Viceira explanatory variables

In the approach of Pflueger and Viceira (2016), liquidity is proxied using three variables: the off-the-run spread (OTR), log relative volume in the TIPS and nominal Treasury markets (VOL), and the synthetic-cash spread, which is our variable ILS-BEI.In their main results, Pflueger and Viceira (2016) use the asset swap spread and use the ILS-BEI for robustness. Results using both variables are similar, and we use the ILS-BEI for simplicity and to complement our earlier results. The off-the-run spread is the difference between the 10 year off-the-run par yield and the 10-year on-the-run nominal yield from Bloomberg (USGG10YR). Relative volume in the two markets is measured using primary dealers' transaction volume from the New York Federal Reserve FR-2004 survey. Inflation expectations are measured using two variables, the median 10-year CPI forecast from the Survey of Professional Forecasters (CPI^e) and the Chicago Fed National Activity Index ($CFNAI$). The CPI forecast is available quarterly, and the $CFNAI$ is available monthly. We create a daily series using the most recently released data.Our results are similar in terms of signs and magnitude regardless of the data frequency; we also examine weekly and monthly data. However, statistical significance of some of the coefficients declines as we sample at coarser data frequencies.

A.2.2 Foreign Exchange Risk

Since the U.S. sovereign CDS contracts are denominated in euros, one could argue that there is foreign exchange risk embedded in CDS contracts which is also driving the variability in the changes in ILS-BEI. As noted by Chernov, Schmid and Schneider (2019), it makes sense for an investor looking for a protection against a U.S. default to obtain a payment in euros instead of dollars since it is likely that the dollar would greatly depreciate. The market for EUR-denominated CDSs is therefore more liquid than for USD-denominated CDSs. This gives rise to a so-called *quanto spread* that has been exploited to measure the depreciation risk upon default. To rule out euro-dollar exchange rate risk as an omitted variable in our baseline results, we perform panel regressions controlling for the exchange rate (risk) between the two currencies: the 5-year EURUSD basis swap spread ($EURUSD$) and the spot exchange rate ($Spot$) between the two currencies.

Table A1 summarizes the results. In columns (1) and (2), we regress changes in ILS-BEI on U.S. CDS spread, and $EURUSD$ or $Spot$, respectively. In Columns (3) and (4), we repeat the regressions after adding the same control variables as those used in Table II. Examining the coefficient loadings on U.S. CDS in the first row, we see that they are all positive and statistically significant across the board. The results presented in Table A1 of the Appendix are essentially unchanged and the coefficient on the CDS ranges from 0.18 to 0.24, as in our baseline specification. We conclude the exchange rate risk in euro CDS contracts is not driving our results.

A.2.3 Fleckenstein, Longstaff and Lustig (2014) Mispricing

Another measure of the relative pricing of nominal bonds vs. real bonds can be found in Fleckenstein, Longstaff and Lustig (2014). The authors replicate a nominal bond by matching the cash flows using a basket of inflation swaps, Treasury Strips, as well as a TIPS with similar maturity and coupon dates. In the absence of any market frictions, the price of the nominal bond and the price of the basket of replicating assets should be exactly the same. Surprisingly, this is not the case in the data, and we refer to the difference as Treasury-TIPS mispricing. Fleckenstein, Longstaff and Lustig (2014) document persistent mispricing between 2004 and 2009 in their sample that can be as high as \$20 per \$100 notional. We reproduce the matched bond pairs from their study and extend the sample period to 2015. We document that mispricing remains in the sample after the financial crisis, and it averages about \$3 per \$100 notional across bond pairs and across time.

We then perform our baseline panel regressions after replacing ILS-BEI with pairwise mispricing as the dependent variable. The results are shown in Table A2, which has a similar format as Table II with two exceptions. First, we add time-to-maturity (TTM) as a control variable in the panel. Second, instead of using a tenor fixed effect, column (7) employs a bond pair fixed effect. We also divide mispricing by 100 to convert mispricing from dollars to cents per \$1 notional. Similar to the first row of Table II, the estimated coefficients on U.S. CDS are positive and statistically significant under all specifications in Table A2. In Columns (6) and (7), with a full slate of control variables, a 1% increase in the CDS spread implies a 0.4 cent increase in the mispricing. This means the nominal bond trades approximately 40 cents rich compared to the basket of inflation swaps, Strips, and TIPS, per \$100 notional. This is also economically significant if you consider that the average mispricing is about \$3. Using the relative pricing of nominal and real bonds from

Fleckenstein, Longstaff and Lustig (2014), we show that sovereign default risk embedded in CDS contracts is strongly correlated with the price differential between Treasury and TIPS. The direction of impact is also consistent with the effect on ILS-BEI: higher CDS spreads are associated with greater downward pressure on TIPS prices relative to Treasury prices.

A.2.4 TIPS Deflation Floors

One feature of TIPS that potentially can produce differential pricing relative to nominal Treasury bonds is the fact that it has a deflation floor. In our study, we show that the BEI spread narrows (ILS-BEI widens) when the CDS spread widens. To the extent that the U.S. CDS spread captures sovereign default risk of the U.S. government, this implies TIPS yields rise more than nominal yields when default risk is elevated. However, if it is the case that the option value of the deflation floor on TIPS is more valuable when default risk is high because deflation is more likely to happen during downturns, then the deflation floor on TIPS actually puts downward pressure on real yields in bad times. Therefore, the fact that we still see a narrowing of the BEI in the data when the CDS spread widens suggests that factors other than the deflation floor feature are driving the wedge between real and nominal bond prices.

A.3 Asset Pricing in the Stylized Example

For one-year maturity bonds, we have the following no-arbitrage relationships on bond prices:

$$\begin{aligned} \exp\left(-R_0^{(1)}\right) &= e^{-r_0^{(1)}}\left[p \cdot (1-\text{LGD}) + (1-p)\cdot 1\right] \\ \exp\left(-R_0^{*(1)}\right) &= e^{-r_0^{(1)}}\mathbb{E}_0^{\mathbb{Q}}\left[p^* \cdot (\Pi_{0:1}-\text{LGD}^*) + (1-p^*)\cdot \Pi_{0:1}\right], \end{aligned}$$

where these expressions are easily obtained combining cashflows and default probabilities. We directly obtain the result for nominal bonds that:

$$\exp\left(-R_0^{(1)}\right) = e^{-r_0^{(1)}}\left[1 - p\text{LGD}\right] \Longleftrightarrow R_0^{(1)} = r_0^{(1)} - \log\left(1 - p\text{LGD}\right) .$$

For TIPS, we note that $e^{-r_0^{(1)}}\mathbb{E}_0^{\mathbb{Q}}\left[\Pi_{0:1}\right] = e^{-r_0^{*(1)}}$ by no-arbitrage. Thus, we directly obtain:

$$\exp\left(-R_0^{*(1)}\right) = e^{-r_0^{*(1)}}\left[p^* \cdot \left(1 - e^{r_0^{*(1)} - r_0^{(1)}}\text{LGD}^*\right) + 1 - p^*\right] = e^{-r_0^{*(1)}}\left[1 - p^* e^{-\text{ILS}_0^{(1)}}\text{LGD}^*\right] .$$

Therefore, the one-year TIPS yield is given by:

$$R_0^{*(1)} = r_0^{*(1)} - \log\left(1 - p^* e^{-\text{ILS}_0^{(1)}}\text{LGD}^*\right) .$$

The one-year ILS-BEI spread is given by:

$$\text{ILS}_0^{(1)} - \text{BEI}_0^{(1)} = \log\left(\frac{1 - p\text{LGD}}{1 - p^* e^{-\text{ILS}_0^{(1)}}\text{LGD}^*}\right) \simeq p^* e^{-\text{ILS}_0^{(1)}}\text{LGD}^* - p\text{LGD} . \tag{25}$$

We now turn to the pricing of the CDS. Assuming that the payment of the protection seller for the one-year CDS is then given by one dollar minus the price of the CTD bond between nominal and TIPS, we obtain the cashflow in case of default as:

$$\text{CF}_1 = 1 - \min\left(1 - \text{LGD}, \Pi_{0:1} - \text{LGD}^*\right) = \max\left(\text{LGD}, \text{LGD}^* + 1 - \Pi_{0:1}\right) .$$

Denoting by $\pi_1 = \Pi_{0:1} - 1$ and $\Delta\text{LGD} = \text{LGD}^* - \text{LGD}$, we have:

$$\text{CF}_1 = \text{LGD} + \max\left(0,\, \Delta\text{LGD} - \pi_1\right) .$$

The CDS spread is thus given by:

$$\begin{aligned} s_0^{(1)} &= \text{PD}_0 \times \mathbb{E}_0^{\mathbb{Q}} \left[\text{LGD} + \max\left(0,\ \Delta\text{LGD} - \pi_1\right) \right] \\ &= \text{PD}_0 \times \left[\text{LGD} + \text{Pr}_0^{\mathbb{Q}} \left(\pi_1 < \Delta\text{LGD}\right) \left(\Delta\text{LGD} - \mathbb{E}_0^{\mathbb{Q}} \left(\pi_1 \middle| \pi_1 < \Delta\text{LGD}\right) \right) \right] , \end{aligned}$$

where PD_0 is the probability of default on at least one type of bonds. Using the fact that $\pi_1 \sim \mathcal{N}(\mu_\pi, \sigma_\pi)$ under the risk-neutral measure, we have:

$$\text{Pr}_0^{\mathbb{Q}} \left(\pi_1 < \Delta\text{LGD}\right) = \Phi\left(\frac{\Delta\text{LGD} - \mu_\pi}{\sigma_\pi} \right) ,$$

and $\pi_1 \middle| \pi_1 < \Delta\text{LGD}$ follows a truncated normal distribution such that:

$$\mathbb{E}_0^{\mathbb{Q}} \left(\pi_1 \middle| \pi_1 < \Delta\text{LGD}\right) = \mu_\pi - \sigma_\pi \frac{f\left(\frac{\Delta\text{LGD} - \mu_\pi}{\sigma_\pi}\right)}{\Phi\left(\frac{\Delta\text{LGD} - \mu_\pi}{\sigma_\pi}\right)} ,$$

where $f(\cdot)$ and $\Phi(\cdot)$ are the pdf and CDF and a standard Gaussian variable, respectively. Putting the results together, we find:

$$s_0^{(1)} = \text{PD}_0 \times \left[\text{LGD} + \left(\Delta\text{LGD} - \mu_\pi\right) \Phi\left(\frac{\Delta\text{LGD} - \mu_\pi}{\sigma_\pi} \right) + \sigma_\pi f\left(\frac{\Delta\text{LGD} - \mu_\pi}{\sigma_\pi} \right) \right] . \tag{26}$$

Note that two cases arise on the default probability of the CDS triggering, depending on whether the sovereign automatically defaults on both bonds. In the latter case, the default probability is

$$\text{PD}_0 = p = p^* .$$

In the general case however, the government can choose to default on either nominal bonds or TIPS and the CDS is triggered with probability:

$$\text{PD}_0 = 1 - (1-p)(1-p^*) = p + p^* - pp^* .$$

For the case of continuing swap payments, we look at 2-year maturity instruments. The government has a probability p to default on nominal bonds during the first year, $(1-p)p$ to default during the second year, and $(1-p)^2$ to survive. Hence the pricing of the 2-year nominal bond is:

$$\begin{aligned}\exp\left(-R_0^{(2)}\right) &= e^{-r_0^{(1)}}p\,(1-\text{LGD})+e^{-2r_0^{(2)}}(1-p)\,[1-p\text{LGD}]\\ &= e^{-2r_0^{(2)}}\left[e^{2r_0^{(2)}-r_0^{(1)}}p\,(1-\text{LGD})+(1-p)\,(1-p\text{LGD})\right]\end{aligned}$$

We obtain:

$$R_0^{(2)}=r_0^{(2)}-\frac{1}{2}\log\left(e^{2r_0^{(2)}-r_0^{(1)}}p\,(1-\text{LGD})+(1-p)\,(1-p\text{LGD})\right). \tag{27}$$

For TIPS, we have the same kind of relationship:

$$\begin{aligned}\exp\left(-R_0^{*(2)}\right) &= e^{-r_0^{(1)}}p^*\left(\mathbb{E}_0^{\mathbb{Q}}(\Pi_{0:1})-\text{LGD}^*\right)+e^{-2r_0^{(2)}}(1-p^*)\left[\mathbb{E}_0^{\mathbb{Q}}(\Pi_{0:2})-p^*\text{LGD}^*\right]\\ &= p^*\left(e^{-r_0^{*(1)}}-e^{-r_0^{(1)}}\text{LGD}^*\right)+(1-p^*)\left[e^{-2r_0^{*(2)}}-e^{-2r_0^{(2)}}p^*\text{LGD}^*\right]\\ &= e^{-2r_0^{*(2)}}\left[p^*\left(e^{2r_0^{*(2)}-r_0^{*(1)}}-e^{2r_0^{*(2)}-r_0^{(1)}}\text{LGD}^*\right)+(1-p^*)\left[1-e^{-2\text{ILS}_0^{(2)}}p^*\text{LGD}^*\right]\right]\end{aligned}$$

Thus we have:

$$R_0^{*(2)}=r_0^{*(2)}-\frac{1}{2}\log\left(p^*\left(e^{2r_0^{*(2)}-r_0^{*(1)}}-e^{2r_0^{*(2)}-r_0^{(1)}}\text{LGD}^*\right)+(1-p^*)\left[1-e^{-2\text{ILS}_0^{(2)}}p^*\text{LGD}^*\right]\right). \tag{28}$$

Assuming that both term structures are flat, these expressions simplify:

$$\begin{aligned}R_0^{(2)} &= r_0^{(1)}-\frac{1}{2}\log\left(e^{r_0^{(1)}}p\,(1-\text{LGD})+(1-p)\,(1-p\text{LGD})\right)\\ &= r_0^{(1)}-\frac{1}{2}\log\left(1-p\left[\text{LGD}\left(1-p+e^{r_0^{(1)}}\right)+1-e^{r_0^{(1)}}\right]\right)\\ R_0^{*(2)} &= r_0^{*(1)}-\frac{1}{2}\log\left(1-p^*\left[1-e^{r_0^{*(1)}}+\text{LGD}^*e^{-2\text{ILS}_0^{(1)}}\left(1-p^*+e^{r_0^{(1)}}\right)\right]\right).\end{aligned}$$

Thus, the spread between ILS and BEI is approximately given by:

$$\begin{aligned}
\mathrm{ILS}_0^{(2)} - \mathrm{BEI}_0^{(2)} &= \frac{1}{2}\log\left(\frac{1-p\left[\mathrm{LGD}\left(1-p+e^{r_0^{(1)}}\right)+1-e^{r_0^{(1)}}\right]}{1-p^*\left[1-e^{r_0^{*(1)}}+\mathrm{LGD}^* e^{-2\mathrm{ILS}_0^{(1)}}\left(1-p^*+e^{r_0^{(1)}}\right)\right]}\right)\\
&\simeq \frac{p^*}{2}\left[1-e^{r_0^{*(1)}}+\mathrm{LGD}^* e^{-2\mathrm{ILS}_0^{(1)}}\left(1-p^*+e^{r_0^{(1)}}\right)\right]\\
&\quad - \frac{p}{2}\left[\mathrm{LGD}\left(1-p+e^{r_0^{(1)}}\right)+1-e^{r_0^{(1)}}\right]. \qquad (29)
\end{aligned}$$

If default probabilities and LGDs are the same for nominal bonds and TIPS, we can simplify the previous expression as:

$$\begin{aligned}
\mathrm{ILS}_0^{(2)} - \mathrm{BEI}_0^{(2)} &\simeq \frac{p}{2}\left[\mathrm{LGD}\left(1-p+e^{r_0^{(1)}}\right)\left(e^{-2\mathrm{ILS}_0^{(1)}}-1\right)+e^{r_0^{(1)}}-e^{r_0^{*(1)}}\right]\\
&\simeq \frac{p}{2}\mathrm{ILS}_0^{(1)}\left[1-2\mathrm{LGD}\left(1-p+e^{r_0^{(1)}}\right)\right]
\end{aligned}$$

For any positive ILS rate, we can reproduce the sign of the mispricing as long as the term in the brackets is positive.

Let us turn now to the case of re-indexation. Upon default of TIPS, they transform into nominal bonds instantaneously. For a one-year TIPS, we have:

$$\exp\left(-R_0^{*(1)}\right) = e^{-r_0^{(1)}}\left[(1-p^*)\Pi_{0:1}+p^*\left(1-p+p(1-\mathrm{LGD})\right)\right].$$

If there is a TIPS default, we have to consider two cases, whether there is a simultaneous default on nominal bonds or not. Simplifying the previous expression, we find:

$$\exp\left(-R_0^{*(1)}\right) = e^{-r_0^{*(1)}}\left[(1-p^*)+p^* e^{-\mathrm{ILS}_0^{(1)}}\left(1-p\mathrm{LGD}\right)\right]$$

Therefore, the TIPS yield is given by:

$$R_0^{*(1)} = r_0^{*(1)} - \log\left(1-p^*+p^* e^{-\mathrm{ILS}_0^{(1)}}\left[1-p\mathrm{LGD}\right]\right), \qquad (30)$$

We directly obtain the ILS-BEI spread as:

$$\begin{aligned}
\mathrm{ILS}_0^{(1)} - \mathrm{BEI}_0^{(1)} &= \log\left(\frac{1-p\mathrm{LGD}}{1-p^*\left(1-e^{-\mathrm{ILS}_0^{(1)}}\left[1-p\mathrm{LGD}\right]\right)}\right) \\
&\simeq p^*\left(1-e^{-\mathrm{ILS}_0^{(1)}}\right)-\left(1-p^*e^{-\mathrm{ILS}_0^{(1)}}\right)p\mathrm{LGD} \qquad (31)
\end{aligned}$$

A.4 Recursive formulas

We detail in this Appendix the computations necessary to obtain the recursive pricing formulas.

Riskless yield curves: Remember our distributional assumptions:

$$r_t = \kappa_0^{(r)} + \boldsymbol{\kappa}_{\boldsymbol{r}}^\top \mathbf{x}_\mathbf{t}^{(\mathbf{r})} \qquad (32)$$

$$\pi_t = \kappa_0^{(\pi)} + \boldsymbol{\kappa}_{\boldsymbol{\pi}}^\top \mathbf{x}_\mathbf{t}^{(\pi)} \qquad (33)$$

$$\mathbf{x_t} = \boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{x_{t-1}} + \sqrt{\boldsymbol{\Sigma}^{\mathbb{Q}}}\varepsilon_\mathbf{t}^{\mathbb{Q}}, \qquad (34)$$

where $\varepsilon_t^{\mathbb{Q}} \overset{i.i.d}{\sim} \mathcal{N}\left(\mathbf{0}, \mathbf{I_{k_x}}\right)$. Let us assume that

$$D_t^{(n)} = \exp\left(A_n + \mathbf{B_n}^\top \mathbf{x_t}\right) \quad \text{and} \quad D_t^{*(n)} = \exp\left(A_n^* + \mathbf{B_n^*}^\top \mathbf{x_t}\right). \qquad (35)$$

By no-arbitrage, we have:

$$\begin{aligned}
D_t^{(n)} &= \mathbb{E}_t^{\mathbb{Q}}\left(e^{-r_t}D_{t+1}^{(n-1)}\right) \\
&= \exp\left(-\kappa_0^{(r)} - \boldsymbol{\kappa}_r^\top \mathbf{x}_\mathbf{t}^{(\mathbf{r})}\right)\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(A_{n-1} + \mathbf{B_{n-1}}^\top \mathbf{x_{t+1}}\right)\right] \\
&= \exp\left[-\kappa_0^{(r)} - \boldsymbol{\kappa}_r^\top \mathbf{x}_\mathbf{t}^{(\mathbf{r})} + A_{n-1} + \mathbf{B_{n-1}}^\top\left(\boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{x_t}\right) + \frac{1}{2}\mathbf{B_{n-1}}^\top \boldsymbol{\Sigma}^{\mathbb{Q}}\mathbf{B_{n-1}}\right]
\end{aligned}$$

Thus we directly have:

$$A_n = A_{n-1} - \kappa_0^{(r)} + \mathbf{B}_{\mathbf{n-1}}^{\top}\boldsymbol{\mu}^{\mathbb{Q}} + \frac{1}{2}\mathbf{B}_{\mathbf{n-1}}^{\top}\boldsymbol{\Sigma}^{\mathbb{Q}}\mathbf{B}_{\mathbf{n-1}} \quad \text{and} \quad \mathbf{B_n} = \boldsymbol{\Phi}^{\mathbb{Q}^{\top}}\mathbf{B}_{\mathbf{n-1}} - \begin{pmatrix} \boldsymbol{\kappa_r} \\ \mathbf{0}_{\mathbf{k}_\pi} \end{pmatrix}.$$

For real bonds, using the same reasoning, we have:

$$\begin{aligned} D_t^{*(n)} &= \mathbb{E}_t^{\mathbb{Q}}\left(e^{-r_t+\pi_{t+1}}D_{t+1}^{*(n-1)}\right) \\ &= \exp\left(-\kappa_0^{(r)} - \boldsymbol{\kappa_r}^{\top}\,\mathbf{x_t^{(r)}}\right)\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(A_{n-1}^* + \kappa_0^{(\pi)} + \boldsymbol{\kappa_\pi}\,\mathbf{x_{t+1}^{(\pi)}} + \mathbf{B}_{\mathbf{n-1}}^{*\top}\mathbf{x_{t+1}}\right)\right] \\ &= \exp\left(-\kappa_0^{(r)} + A_{n-1}^* + \kappa_0^{(\pi)} - \boldsymbol{\kappa_r}^{\top}\,\mathbf{x_t^{(r)}}\right)\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(\left(\mathbf{B}_{\mathbf{n-1}}^{*\top} + \left[\mathbf{0}_{\mathbf{k_r}}{}^{\top}, \boldsymbol{\kappa}_{\boldsymbol{\pi}}^{\top}\right]\right)\mathbf{x_{t+1}}\right)\right] \end{aligned}$$

so we directly obtain:

$$\begin{aligned} A_n^* &= A_{n-1}^* - \kappa_0^{(r)} + \kappa_0^{(\pi)} + \left[\mathbf{B}_{\mathbf{n-1}}^* + \begin{pmatrix} \mathbf{0}_{\mathbf{k_r}} \\ \boldsymbol{\kappa}_\pi \end{pmatrix}\right]^{\top}\left(\boldsymbol{\mu}^{\mathbb{Q}} + \frac{1}{2}\boldsymbol{\Sigma}^{\mathbb{Q}}\left[\mathbf{B}_{\mathbf{n-1}}^* + \begin{pmatrix} \mathbf{0}_{\mathbf{k_r}} \\ \boldsymbol{\kappa_\pi} \end{pmatrix}\right]\right) \\ \mathbf{B_n^*} &= \boldsymbol{\Phi}^{\mathbb{Q}^{\top}}\left[\mathbf{B}_{\mathbf{n-1}}^* + \begin{pmatrix} \mathbf{0}_{\mathbf{k_r}} \\ \boldsymbol{\kappa_\pi} \end{pmatrix}\right] - \begin{pmatrix} \boldsymbol{\kappa_r} \\ \mathbf{0}_{\mathbf{k}_\pi} \end{pmatrix}, \end{aligned}$$

All these recursions are starting from initial conditions $A_0 = 0$, $\mathbf{B_0} = \mathbf{0_k}$ and $A_0^* = 0$, $\mathbf{B_0^*} = \mathbf{0_k}$.

One-period MGF: The conditional moment generating function of δ_t and $\mathbf{y_t}$ can be written:

$$\varphi_{t-1}(\mathbf{u},\mathbf{v}) = \mathbb{E}_{t-1}^{\mathbb{Q}}\left[\exp\left(\mathbf{u}^{\top}\mathbf{y_t} + \mathbf{v}^{\top}\boldsymbol{\delta_t}\right)\right] = \exp\left[A(\mathbf{u},\mathbf{v}) + \mathbf{B}(\mathbf{u},\mathbf{v})^{\top}\mathbf{y_{t-1}}\right], \qquad (36)$$

for any vector $(\mathbf{u},\, \mathbf{v})$ such that the expectation exists. The coefficients $A(\mathbf{u},\mathbf{v})$ and $\mathbf{B}(\mathbf{u},\mathbf{v})$ are easily computed as:

$$A(\mathbf{u},\mathbf{v}) \;=\; \boldsymbol{\alpha}^{\mathbb{Q}\top}\frac{\mathbf{c}^{\mathbb{Q}}\left(\mathbf{u}+\tilde{\mathbf{v}}\right)}{\mathbf{1}-\mathbf{c}^{\mathbb{Q}}\left(\mathbf{u}+\tilde{\mathbf{v}}\right)} \tag{37}$$

$$\mathbf{B}(\mathbf{u},\mathbf{v}) \;=\; \boldsymbol{\beta}^{\mathbb{Q}\top}\frac{\mathbf{c}^{\mathbb{Q}}\left(\mathbf{u}+\tilde{\mathbf{v}}\right)}{\mathbf{1}-\mathbf{c}^{\mathbb{Q}}\left(\mathbf{u}+\tilde{\mathbf{v}}\right)}, \tag{38}$$

where the ratio stands for an element-by-element ratio by notation abuse and

$$\tilde{\mathbf{v}} = \left(\frac{v_c}{1-v_c}\boldsymbol{\gamma}_c^\top,\, \frac{v_\ell}{1-v_\ell}\boldsymbol{\gamma}_\ell^\top\right)^\top,$$

Multi-horizon conditional MGF:

$$\begin{aligned}
\psi_{t-1}(\mathbf{u},\mathbf{v},\mathbf{w}) &= \mathbb{E}^{\mathbb{Q}}_{t-1}\left[\exp\left(\mathbf{u}^\top\mathbf{y_t}+\mathbf{v}^\top\boldsymbol{\delta_t}+\mathbf{w}^\top\mathbf{x_t}\right)\right]\\
&= \exp\left[A(\mathbf{u},\mathbf{v})+\mathbf{B}(\mathbf{u},\mathbf{v})^\top\mathbf{y_{t-1}}+\mathbf{w}^\top\left(\mu^{\mathbb{Q}}+\mathbf{\Phi}^{\mathbb{Q}}\mathbf{x_{t-1}}+\frac{\mathbf{1}}{\mathbf{2}}\mathbf{\Sigma}^{\mathbb{Q}}\mathbf{w}\right)\right]\\
&= \exp\left[\widetilde{A}(\mathbf{u},\mathbf{v},\mathbf{w})+\mathbf{B}(\mathbf{u},\mathbf{v})^\top\mathbf{y_{t-1}}+\mathbf{C}\left(\mathbf{w}\right)^\top\mathbf{x_{t-1}}\right]
\end{aligned}$$

which is exponential-affine in $\mathbf{z_t}=\left(\mathbf{x_t}^\top,\, \mathbf{y_t}^\top\right)^\top$.

$$\begin{aligned}
G_t(n,n_c,n_\ell) &= \mathbb{E}^{\mathbb{Q}}_t\left[\exp\left(-\sum_{j=0}^{n-1}r_{t+j}\right)\mathbb{1}\left\{\sum_{j=0}^{n_c}\delta^{(c)}_{t+j}=0\right\}\mathbb{1}\left\{\sum_{j=0}^{n_\ell}\delta^{(\ell)}_{t+j}=0\right\}\right]\\
G^*_t(n,n_c,n_\ell) &= \mathbb{E}^{\mathbb{Q}}_t\left[\exp\left(-\sum_{j=0}^{n-1}r_{t+j}-\pi_{t+j+1}\right)\mathbb{1}\left\{\sum_{j=0}^{n_c}\delta^{(c)}_{t+j}=0\right\}\mathbb{1}\left\{\sum_{j=0}^{n_\ell}\delta^{(\ell)}_{t+j}=0\right\}\right].
\end{aligned}$$

Using the lemma provided in Monfort et al. (2017*a*), we can write:

$$\begin{aligned}
G_t(n,n_c,n_\ell) &= \lim_{u\to+\infty}\mathbb{E}^{\mathbb{Q}}_t\left[\exp\left(-\sum_{j=0}^{n-1}r_{t+j}-u\left(\sum_{j=0}^{n_c}\delta^{(c)}_{t+j}+\sum_{j=0}^{n_\ell}\delta^{(\ell)}_{t+j}\right)\right)\right]\\
G^*_t(n,n_c,n_\ell) &= \lim_{u\to+\infty}\mathbb{E}^{\mathbb{Q}}_t\left[\exp\left(-\sum_{j=0}^{n-1}r_{t+j}-\pi_{t+j+1}-u\left(\sum_{j=0}^{n_c}\delta^{(c)}_{t+j}+\sum_{j=0}^{n_\ell}\delta^{(\ell)}_{t+j}\right)\right)\right]
\end{aligned}$$

Let us denote by $\psi_t^{(n)}(\mathbf{u}_1, \mathbf{v_1}, \mathbf{w_1}, \ldots, \mathbf{u_n}, \mathbf{v_n}, \mathbf{w_n})$ the multi-period conditional MGF of our state variables, i.e.:

$$\psi_t^{(n)}(\mathbf{u}_1, \mathbf{v_1}, \mathbf{w_1}, \ldots, \mathbf{u_n}, \mathbf{v_n}, \mathbf{w_n}) = \mathbb{E}^{\mathbb{Q}}_{\mathbf{t-1}}\left[\exp\left(\sum_{\mathbf{j=1}}^{\mathbf{n}} \mathbf{u_j}^\top \mathbf{y_{t+j}} + \mathbf{v_j}^\top \boldsymbol{\delta}_{\boldsymbol{t+j}} + \mathbf{w_j}^\top \mathbf{x_{t+j}}\right)\right].$$

A first natural property resulting from the affine formulation of the joint process is that the multi-period conditional MGF is an exponential-affine function. It is easy to show that:

$$\begin{aligned}
\psi_t^{(n)}(\mathbf{u}_1, \mathbf{v_1}, \mathbf{w_1}, \ldots, \mathbf{u_n}, \mathbf{v_n}, \mathbf{w_n}) \quad = \quad & \exp\Bigg[\Psi_0^{(n)}(\mathbf{u}_1, \mathbf{v_1}, \mathbf{w_1}, \ldots, \mathbf{u_n}, \mathbf{v_n}, \mathbf{w_n}) \\
+ \quad & \Psi_y^{(n)}(\mathbf{u}_1, \mathbf{v_1}, \mathbf{w_1}, \ldots, \mathbf{u_n}, \mathbf{v_n}, \mathbf{w_n})^\top \mathbf{y_t} \\
+ \quad & \Psi_x^{(n)}(\mathbf{u}_1, \mathbf{v_1}, \mathbf{w_1}, \ldots, \mathbf{u_n}, \mathbf{v_n}, \mathbf{w_n})^\top \mathbf{x_t}\Bigg],
\end{aligned}$$

where:

$$\begin{aligned}
&\Psi_0^{(k)}(\mathbf{u}_1, \mathbf{v}_1, \mathbf{w}_1, \ldots, \mathbf{u}_n, \mathbf{v}_n, \mathbf{w}_n) = \Psi_0^{(k-1)}(\mathbf{u}_1, \mathbf{v}_1, \mathbf{w}_1, \ldots, \mathbf{u}_n, \mathbf{v}_n, \mathbf{w}_n) \\
+ \quad &\widetilde{\mathbf{A}}\left[\mathbf{u}_{n-k+1} + \Psi_y^{(k-1)}(\mathbf{u}_1, \mathbf{v}_1, \mathbf{w}_1, \ldots, \mathbf{u}_n, \mathbf{v}_n, \mathbf{w}_n),\, \mathbf{v}_{n-k+1},\, \mathbf{w}_{n-k+1} + \Psi_x^{(k-1)}(\mathbf{u}_1, \mathbf{v}_1, \mathbf{w}_1, \ldots, \mathbf{u}_n, \mathbf{v}_n, \mathbf{w}_n)\right] \\
&\Psi_y^{(k)}(\mathbf{u}_1, \mathbf{v}_1, \mathbf{w}_1, \ldots, \mathbf{u}_n, \mathbf{v}_n, \mathbf{w}_n) = \mathbf{B}\left[\Psi_y^{(k-1)}(\mathbf{u}_1, \mathbf{v}_1, \mathbf{w}_1, \ldots, \mathbf{u}_n, \mathbf{v}_n, \mathbf{w}_n) + \mathbf{u}_{n-k+1},\, \mathbf{v}_{n-k+1}\right] \\
&\Psi_x^{(k)}(\mathbf{u}_1, \mathbf{v}_1, \mathbf{w}_1, \ldots, \mathbf{u}_n, \mathbf{v}_n, \mathbf{w}_n) = \mathbf{C}\left[\Psi_x^{(k-1)}(\mathbf{u}_1, \mathbf{v}_1, \mathbf{w}_1, \ldots, \mathbf{u}_n, \mathbf{v}_n, \mathbf{w}_n) + \mathbf{w}_{n-k+1}\right]
\end{aligned}$$

Let us denote by $n_{max} = \max(n+1,\, n_c,\, n_\ell)$. Then, $G_t(n,\, n_c,\, n_\ell)$ and $G_t^*(n,\, n_c,\, n_\ell)$ can be rewritten with the multi-horizon conditional MGF:

$$\begin{aligned}
G_t(n,\, n_c,\, n_\ell) \quad &= \quad e^{-r_t-(n-1)\kappa_0^{(r)}}\psi_t^{(n)}(\mathbf{u}_1, \mathbf{v_1}, \mathbf{w_1}, \ldots, \mathbf{u_{n_{max}}}, \mathbf{v_{n_{max}}}, \mathbf{w_{n_{max}}}) \\
G_t^*(n,\, n_c,\, n_\ell) \quad &= \quad e^{-r_t-(n-1)\kappa_0^{(r)}+n\kappa_0^{(\pi)}}\psi_t^{(n)}(\mathbf{u}_1^*, \mathbf{v_1}^*, \mathbf{w_1}^*, \ldots, \mathbf{u}^*_{\mathbf{n_{max}}}, \mathbf{v}^*_{\mathbf{n_{max}}}, \mathbf{w}^*_{\mathbf{n_{max}}})
\end{aligned}$$

where:

$$
\begin{aligned}
\mathbf{u}_j &= \mathbf{0}_{k_c+k_\ell}\\
\mathbf{v}_j &= \begin{cases} -v\,(1,\,1)^\top & \text{if} \quad j \leqslant \min(n_c,\,n_\ell)\\ -v\,(\mathbb{1}\{n_c > n_\ell\},\,\mathbb{1}\{n_\ell > n_c\})^\top & \text{if} \quad j \in (\min(n_c,\,n_\ell), \max(n_c,\,n_\ell)]\\ 0 & \text{if} \quad j \in (\max(n_c,\,n_\ell), n_{max}] \end{cases}\\
\mathbf{w}_j &= \begin{cases} \left(-\kappa_r^\top,\,\mathbf{0}_{k_\pi}^\top\right)^\top & \text{if} \quad j \leqslant n-1\\ 0 & \text{if} \quad j \in [n, n_{max}] \end{cases}
\end{aligned}
$$

and

$$
\begin{aligned}
\mathbf{u}_j^* &= \mathbf{0}_{k_c+k_\ell}\\
\mathbf{v}_j^* &= \mathbf{v}_j\\
\mathbf{w}_j^* &= \begin{cases} \left(-\kappa_r^\top,\,-\kappa_\pi^\top\right)^\top & \text{if} \quad j \leqslant n-1\\ \left(\mathbf{0}_{k_x}^\top,\,-\kappa_\pi^\top\right)^\top & \text{if} \quad j = n\\ 0 & \text{if} \quad j \in [n, n_{max}] \end{cases}
\end{aligned}
$$

where v is a scalar tending to infinity. By a continuity argument, we obtain that both G_t and G_t^* are exponential-affine functions such that:

$$
\begin{aligned}
G_t(n, n_c, n_\ell) &= \exp\left(q_{(n,n_c,n_\ell)} + \mathbf{Q}_{(n,n_c,n_\ell)}^\top \mathbf{z_t}\right) \,,\\
G_t^*(n, n_c, n_\ell) &= \exp\left(q_{(n,n_c,n_\ell)}^* + \mathbf{Q}_{(n,n_c,n_\ell)}^{*\top} \mathbf{z_t}\right) \,,
\end{aligned}
$$

and the loadings are obtained through the recursions defined earlier.

Let us use this result to compute the price of defaultable bonds.

Defaultable bond pricing: Let us first consider nominal bonds.

$$
\begin{aligned}
B_t^{(n)} &= \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j} \right) \mathcal{P}_c^{(i)} \times \left(\mathbb{1}\left\{ \sum_{j=0}^{i-1} \delta_{t+j}^{(c)} = 0 \right\} - \mathbb{1}\left\{ \sum_{j=0}^{i} \delta_{t+j}^{(c)} = 0 \right\} \right) \right] \\
&\quad + \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{n-1} r_{t+j} \right) \mathbb{1}\left\{ \sum_{j=0}^{n} \delta_{t+j}^{(c)} = 0 \right\} \right] \\
&= \lim_{v \to +\infty} \rho_c \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j} \right) \times \left(\exp\left(-v \sum_{j=0}^{i-1} \delta_{t+j}^{(c)} \right) - \exp\left(-v \sum_{j=0}^{i} \delta_{t+j}^{(c)} \right) \right) \right] \\
&\quad + \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{n-1} r_{t+j} \right) \mathbb{1} \exp\left(-v \sum_{j=0}^{n} \delta_{t+j}^{(c)} \right) \right]
\end{aligned}
$$

Conditionally on no default at date t, we have:

$$
\begin{aligned}
B_t^{(n)} &= \rho_c \sum_{i=1}^{n} \left[G_t\left(i,\, i-1,\, 0\right) - G_t\left(i,\, i,\, 0\right) \right] + G_t\left(n,\, n,\, 0\right) \\
&= \rho_c \sum_{i=1}^{n} \left[\exp\left(q_{(i,\, i-1,\, 0)} + \mathbf{Q}_{(i,\, i-1,\, 0)}^{\top} \mathbf{z_t} \right) - \exp\left(q_{(i,\, i,\, 0)} + \mathbf{Q}_{(i,\, i,\, 0)}^{\top} \mathbf{z_t} \right) \right] + \exp\left(q_{(n,\, n,\, 0)} + \mathbf{Q}_{(\mathbf{n},\, \mathbf{n},\, \mathbf{0})}^{\top} \mathbf{z_t} \right)
\end{aligned}
$$

Let us consider a first order Taylor expansion of the previous expression:

$$
\begin{aligned}
B_t^{(n)} &\simeq \rho_c \sum_{i=1}^{n} \left[q_{(i,\, i-1,\, 0)} - q_{(i,\, i,\, 0)} + \left(\mathbf{Q}_{(i,\, i-1,\, 0)} - \mathbf{Q}_{(i,\, i,\, 0)} \right)^{\top} \mathbf{z_t} \right] + 1 + q_{(n,\, n,\, 0)} + \mathbf{Q}_{(n,\, n,\, 0)}^{\top} \mathbf{z_t} \\
&\simeq \exp\left\{ \rho_c \sum_{i=1}^{n} \left[q_{(i,\, i-1,\, 0)} - q_{(i,\, i,\, 0)} \right] + q_{(n,\, n,\, 0)} + \left(\rho_c \sum_{i=1}^{n} \left[\mathbf{Q}_{(i,\, i-1,\, 0)} - \mathbf{Q}_{(i,\, i,\, 0)} \right] + \mathbf{Q}_{(n,\, n,\, 0)} \right)^{\top} \mathbf{z_t} \right\}
\end{aligned}
$$

Thus, we immediately have:

$$
\begin{aligned}
\mathcal{A}_n(\rho_c) &= q_{(n,\, n,\, 0)} + \rho_c \sum_{i=1}^{n} \left[q_{(i,\, i-1,\, 0)} - q_{(i,\, i,\, 0)} \right] \\
\mathcal{B}_n(\rho_c) &= \mathbf{Q}_{(n,\, n,\, 0)} + \rho_c \sum_{i=1}^{n} \left[\mathbf{Q}_{(i,\, i-1,\, 0)} - \mathbf{Q}_{(i,\, i,\, 0)} \right]
\end{aligned}
$$

The proof for the inflation-indexed bonds is similar in spirit:

$$
\begin{aligned}
B_t^{*(n)} &= \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}}\Bigg[\exp\left(-\sum_{j=0}^{i-1} r_{t+j} - \pi_{t+j+1}\right)(\rho_c^* + \rho_\ell^*)\,\mathbb{1}\left\{\sum_{j=0}^{i-1} \mathbf{e}_2^\top \boldsymbol{\delta}_{t+j} = 0\right\} \\
&- \rho_c^* \exp\left(-\sum_{j=0}^{i-1} r_{t+j} - \pi_{t+j+1}\right)\mathbb{1}\left\{\sum_{j=0}^{i-1} \mathbf{e}_2^\top \boldsymbol{\delta}_{t+j} + \delta_{t+i}^{(c)} = 0\right\} \\
&- \rho_\ell^* \exp\left(-\sum_{j=0}^{i-1} r_{t+j} - \pi_{t+j+1}\right)\mathbb{1}\left\{\sum_{j=0}^{i-1} \mathbf{e}_2^\top \boldsymbol{\delta}_{t+j} + \delta_{t+i}^{(\ell)} = 0\right\}\Bigg] \\
&\quad + \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{n-1} r_{t+j} - \pi_{t+j+1}\right)\mathbb{1}\left\{\sum_{j=0}^{n} \mathbf{e}_2^\top \boldsymbol{\delta}_{t+j} = 0\right\}\right] \\
&= \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}}\Bigg[\exp\left(-\sum_{j=0}^{i-1} r_{t+j} - \pi_{t+j+1} + v\mathbf{e}_2^\top \boldsymbol{\delta}_{t+j}\right)(\rho_c^* + \rho_\ell^*) \\
&- \rho_c^* \exp\left(-\sum_{j=0}^{i-1} r_{t+j} - \pi_{t+j+1} + v\mathbf{e}_2^\top \boldsymbol{\delta}_{t+j}\right)\exp\left(-v\delta_{t+i}^{(c)}\right) \\
&- \rho_\ell^* \exp\left(-\sum_{j=0}^{i-1} r_{t+j} - \pi_{t+j+1} + v\mathbf{e}_2^\top \boldsymbol{\delta}_{t+j}\right)\exp\left(-v\delta_{t+i}^{(\ell)}\right)\Bigg] \\
&\quad + \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{n-1} r_{t+j} - \pi_{t+j+1} + v\mathbf{e}_2^\top \boldsymbol{\delta}_{t+j}\right)\exp\left(-v\mathbf{e}_2^\top \boldsymbol{\delta}_{t+n}\right)\right]
\end{aligned}
$$

Assuming no default or liquidity event happened at date t, we obtain:

$$
\begin{aligned}
B_t^{*(n)} &= \sum_{i=1}^{n} \left[(\rho_c^* + \rho_\ell^*) G_t^*(i,\, i-1,\, i-1) - \rho_c^* G_t^*(i,\, i,\, i-1) - \rho_\ell^* G_t^*(i,\, i-1,\, i)\right] \\
&\quad + G_t^*(n,\, n,\, n) \\
&= \sum_{i=1}^{n} \Bigg[(\rho_c^* + \rho_\ell^*)\exp\left(q^*_{(i,i-1,i-1)} + \mathbf{Q}^{*\top}_{(i,i-1,i-1)}\mathbf{z_t}\right) - \rho_c^* \exp\left(q^*_{(i,i,i-1)} + \mathbf{Q}^{*\top}_{(i,i,i-1)}\mathbf{z_t}\right) \\
&- \rho_\ell^* \exp\left(q^*_{(i,i-1,i)} + \mathbf{Q}^{*\top}_{(i,i-1,i)}\mathbf{z_t}\right)\Bigg] + \exp\left(q^*_{(n,n,n)} + \mathbf{Q}^{*\top}_{(n,n,n)}\mathbf{z_t}\right)
\end{aligned}
$$

Again, considering a first order Taylor expansion:

$$
\begin{aligned}
B_t^{*(n)} \;&\simeq\; \sum_{i=1}^{n}\Bigg[(\rho_c^* + \rho_\ell^*)\left(q^*_{(i,\,i-1,\,i-1)} + \mathbf{Q}^{*\top}_{(i,\,i-1,\,i-1)}\mathbf{z_t}\right) - \rho_c^*\left(q^*_{(i,\,i,\,i-1)} + \mathbf{Q}^{*\top}_{(i,\,i,\,i-1)}\mathbf{z_t}\right) \\
&\quad -\; \rho_\ell^*\left(q^*_{(i,\,i-1,\,i)} + \mathbf{Q}^{*\top}_{(i,\,i-1,\,i)}\mathbf{z_t}\right)\Bigg] + 1 + q^*_{(n,\,n,\,n)} + \mathbf{Q}^{*\top}_{(n,\,n,\,n)}\mathbf{z_t} \\
&\simeq\; \exp\Bigg\{\sum_{i=1}^{n}\left[(\rho_c^* + \rho_\ell^*)q^*_{(i,\,i-1,\,i-1)} - \rho_c^* q^*_{(i,\,i,\,i-1)} - \rho_\ell^* q^*_{(i,\,i-1,\,i)}\right] + q^*_{(n,\,n,\,n)} \\
&\quad +\; \left[\sum_{i=1}^{n}\left[(\rho_c^* + \rho_\ell^*)\mathbf{Q}^*_{(i,\,i-1,\,i-1)} - \rho_c^*\mathbf{Q}^*_{(i,\,i,\,i-1)} - \rho_\ell^*\mathbf{Q}^*_{(i,\,i-1,\,i)}\right] + \mathbf{Q}^*_{(n,\,n,\,n)}\right]^\top \mathbf{z_t}\Bigg\}
\end{aligned}
$$

so we obtain:

$$
\begin{aligned}
\mathcal{A}_n^*(\rho_c^*,\,\rho_\ell^*) \;&=\; \sum_{i=1}^{n}\left[(\rho_c^* + \rho_\ell^*)q^*_{(i,\,i-1,\,i-1)} - \rho_c^* q^*_{(i,\,i,\,i-1)} - \rho_\ell^* q^*_{(i,\,i-1,\,i)}\right] + q^*_{(n,\,n,\,n)} \\
\mathcal{B}_n^*(\rho_c^*,\,\rho_\ell^*) \;&=\; \sum_{i=1}^{n}\left[(\rho_c^* + \rho_\ell^*)\mathbf{Q}^*_{(i,\,i-1,\,i-1)} - \rho_c^*\mathbf{Q}^*_{(i,\,i,\,i-1)} - \rho_\ell^*\mathbf{Q}^*_{(i,\,i-1,\,i)}\right] + \mathbf{Q}^*_{(n,\,n,\,n)}
\end{aligned}
$$

Table A1: **Controlling for Foreign Exchange Risk**

Table A1 shows the results from a panel regression of $ILS - BEI$ on US CDS spreads and specifically controlling for Euro-Dollar exchange rate movement using daily observations. The sample period is from January 2008 to October 2015. $US\ CDS$ spreads are for the 5-year tenor. $EURUSD$ denotes the 5-year swap spread of the Euro-Dollar basis swap. *Spot* is the spot exchange rate between the Euro and the Dollar. $HPWNoise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in 10-year Treasury par yield from Gurkaynak, Sack and Wright (2006) less the on-the-run 10-year Treasury yield from Bloomberg. VIX denotes the CBOE Volatility Index. Standard errors are reported in parentheses.

Dep Var: Δ ILS-BEI Spread	(1)	(2)	(3)	(4)
US CDS	0.178***	0.208***	0.213***	0.237***
	(0.051)	(0.050)	(0.052)	(0.052)
EURUSD	−0.076		−0.091*	
	(0.051)		(0.053)	
Spot		10.352		11.461
		(8.067)		(8.313)
ILS-BEI$_{t-1}$	−0.184***	−0.183***	−0.183***	−0.183***
	(0.005)	(0.005)	(0.005)	(0.005)
HPW Noise			0.611**	0.649**
			(0.291)	(0.291)
TIPS Noise			−1.027***	−1.044***
			(0.182)	(0.182)
LIBOR-OIS			−3.881**	−4.145**
			(1.921)	(1.926)
OTR Difference			−17.077***	−17.430***
			(5.081)	(5.076)
VIX			−0.045	−0.008
			(0.043)	(0.042)
Week	Yes	Yes	Yes	Yes
Tenor	Yes	Yes	Yes	Yes
Observations	9142	9147	9127	9127
R^2	0.152	0.152	0.159	0.158

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table A2: **FLL Mispricing - January 2008 to October 2015**

Table A2 shows the results from a panel regression of Fleckenstein, Longstaff and Lustig (2014) Treasury-TIPS pairwise mispricing on US CDS spreads and various controls using daily observations. The sample period is from January 2008 to October 2015. $FLL\ Mispricing$ is the difference between the price of a Treasury bond and the price of a basket of TIPS, inflation swaps, and Treasury strips. $US\ CDS$ spreads are for the 5-year tenor. TTM denotes time-to-maturity. $HPWNoise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in 10-year Treasury par yield from Gurkaynak, Sack and Wright (2006) less the on-the-run 10-year Treasury yield from Bloomberg. VIX denotes the CBOE Volatility Index. Clustered standard errors by bond-pair are reported in parentheses.

Dep Var: FLL Mispricing	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
US CDS	0.665***	0.666***	0.768***	0.631***	0.686***	0.420**	0.501***	0.501***
	(0.203)	(0.203)	(0.226)	(0.187)	(0.219)	(0.172)	(0.181)	(0.180)
FLL-Mis$_{t-1}$	95.408***	95.408***	95.475***	95.411***	95.476***	95.467***	95.473***	94.340***
	(0.752)	(0.752)	(0.762)	(0.751)	(0.762)	(0.758)	(0.761)	(0.782)
TTM	0.003***	0.003***	0.003***	0.003***	0.003***	0.003***	0.003***	−0.111*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.058)
HPW Noise		−0.597					−0.527	−0.483
		(0.876)					(0.904)	(0.911)
TIPS Noise			−5.159***				−5.440***	−5.416***
			(0.728)				(0.835)	(0.834)
LIBOR-OIS				22.849			2.477	2.547
				(13.765)			(10.031)	(9.989)
OTR Difference					3.768		16.987	16.112
					(33.450)		(35.297)	(34.943)
VIX						1.186***	1.196***	1.190***
						(0.283)	(0.264)	(0.263)
Week	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Bond Pair	No	No	No	No	No	No	No	Yes
Observations	43156	43156	41466	43156	41505	41667	41357	41357
R^2	0.976	0.976	0.977	0.977	0.977	0.977	0.977	0.977

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table A3: **First Differences - January 2008 to October 2015**

Table A3 *US CDS* spreads are for the 5-year tenor. *HPWNoise* follows Hu, Pan and Wang (2013). *TIPS Noise* measures average daily deviations in the real yield curve. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. *OTR Difference* is the difference in 10-year Treasury par yield from Gurkaynak, Sack and Wright (2006) less the on-the-run 10-year Treasury yield from Bloomberg. *VIX* denotes the CBOE Volatility Index. Standard errors are reported in parentheses.

Dep Var: Δ ILS-BEI Spread	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Δ US CDS	0.154***	0.153***	0.161***	0.161***	0.141***	0.168***	0.160***	0.160***
	(0.052)	(0.052)	(0.052)	(0.052)	(0.052)	(0.052)	(0.052)	(0.052)
Δ HPW Noise		−0.402					−0.241	−0.241
		(0.254)					(0.256)	(0.256)
Δ TIPS Noise			−1.872***				−1.806***	−1.807***
			(0.205)				(0.207)	(0.207)
Δ LIBOR-OIS				−7.768***			−3.910	−3.910
				(2.713)			(2.835)	(2.836)
Δ OTR Difference					−21.74***		−16.95***	−16.95***
					(3.456)		(3.695)	(3.696)
Δ VIX						−0.0968***	−0.0401	−0.0401
						(0.0367)	(0.0383)	(0.0383)
Week	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Tenor	No	No	No	No	No	No	No	Yes
Observations	9142	9142	9137	9142	9142	9107	9102	9102
R^2	0.040	0.040	0.049	0.041	0.044	0.041	0.053	0.053

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.