Demagogues and the Fragility of Democracy*

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Abstract

We investigate the susceptibility of Democracy to demagogues, studying tensions between far-sighted representatives who guard the long-run interests of voters, and office-seeking demagogues who cater to voters' short-run desires. Parties propose how to allocate resources between consumption and investment. Voters base electoral choices on current period utilities derived from policies and valence shocks. With log utility, investments converge to zero when depreciation goes to one, even though, absent demagogues, the economy would grow forever. For higher relative risk aversions, the economy always faces a risk of death spirals, in which once capital drops too low, it inevitably falls toward zero.

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1 Introduction

“The republican principle,” wrote Hamilton in Federalist No. 71, “does not require an unqualified complaisance to every sudden breeze of passion, or to every transient impulse which the people may receive from the arts of men, who flatter their prejudices to betray their interests.” To the contrary, Hamilton argued, when “the interests of the people are at variance with their inclinations, it is the duty of the persons whom they have appointed to be the guardians of those interests, to withstand the temporary delusion.... conduct of this kind has saved the people from very fatal consequences of their own mistakes, and procured...their gratitude to the men who had courage and magnanimity enough to serve them at the peril of their displeasure.” Still, if such magnanimous representatives cause too much displeasure, they would lose election to those who will implement those “temporary delusions”, paying “obsequious court to the people; commencing demagogues, and ending tyrants.”

In this paper, we study the tension highlighted by Hamilton between far-sighted, magnanimous representatives who guard the long-run interests of voters and office-seeking demagogues who cater to voters’ short-run desires. In particular, we characterize the long-run outcomes of democracy in a country populated by a short-sighted majority. Demagogues and short-sighted voters have long been considered inter-related vices of republican governments. For example, “Madison’s [belief] about democracy was based on [one] about human beings: man, by nature, preferred to follow his passion rather than his reason; he invariably chose short-term over long-term interests” (Middlekauff (2007), p. 678). Indeed, researchers define demagogues by this characteristic. According to Guiso et al. (2018), parties led by demagogues “champion short-term policies while hiding their long-term costs.”[1] What is not well-understood is how demagogues distort the behavior of far-sighted parties, and how a democratic country emerges from the long-run confrontation between selfish demagogues and socially benevolent, but pragmatic parties.

We analyze the dynamic political competition between a far-sighted, benevolent party that seeks to maximize voter welfare, and an office-motivated demagogue, who only cares about winning. The two parties and a representative short-sighted median voter interact over time in an infinite horizon setting. To capture tensions between short- and long-term considerations, we model the political decision process as an investment problem in which parties propose how to allocate existing resources between current consumption and investment in capital that facilitates greater future con-

[1] Guiso et al. (2018) use the term populists. Historically, populists were referred as demagogues but now these terms are often used interchangeably.

sumption. The investment technology exhibits constant returns to scale, and capital depreciates at a constant rate, so absent sufficient investment, the capital stock and hence future consumption fall.

Each period, the benevolent party and demagogue propose investments (and hence consumption) for that period. The voter, who has CRRA preferences, assesses investment policies solely based on his current period utility. In addition to policies, parties are distinguished by their valences, which capture other electoral concerns. The net valence is uncertain. Even if both parties choose the same policy, the demagogue, who may typically have a low valence, still has at least a slim chance of winning the election. After parties propose investments, valences are realized, and the median voter picks his preferred candidate. The winner's investment policy is implemented, period payoffs are realized, and the game proceeds to the next period with a capital stock consisting of the depreciated capital plus any new investment.

Absent a demagogue, the benevolent party acts like a social planner. Investments more than compensate for capital depreciation, and capital stocks grow without bound over time. Left on its own, the benevolent party internalizes voters' utility from future consumption, winning the race between the production and destruction of capital. Its investment policy reflects the primitives of the economy in natural ways, for example, by reducing investments as the depreciation rate falls.

Demagogues, even ones who are likely to have low valences, change all that. Demagogues propose all consumption and no investment to appease short-sighted voters in order to maximize their electoral chances. Now, the benevolent party faces a dilemma. It can ignore the demagogue in its policy choices, but it does so at its own peril by risking electoral support. Alternatively, it can appropriate the demagogue's political agenda, but "trying to beat a...populist insurgency by becoming one...turns out to be a fool's errand...[as it] has a huge...economic cost."[2] In our model, the benevolent party trades off higher investment against higher chances of winning election. The voter views policies with higher current consumption more favorably, and the demagogue proposes no investment. Greater investments benefit voters in the long run, but the benevolent party needs to win in order to invest. This leads the benevolent party to shift its platform toward increased consumption to improve its chances of implementing a policy that is better than the demagogue's.

The policy choices of Huey Long and President Roosevelt illustrate the benevolent party's dilemma. In the midst of the Great Depression, Long proposed a high progressive tax, and dis-

[2]NY times Op Ed, https://www.nytimes.com/2018/07/16/opinion/to-defeat-far-right-nationalists-dont-try-to-imitate-them.html

tributing the revenue to every American family, 5,000 dollars each, supposedly enough for a home, a car, and a radio. Each family would also be guaranteed an annual income of $2,500, topped off by shorter working hours, better veterans' benefits, education subsidies, and pensions. FDR's assessment was that Long "hopes to defeat the Democratic party and put in a reactionary Republican. That would bring the country to such a state by 1940 that Long thinks he would be made dictator.... it is an ominous situation." In response, FDR proposed a Second New Deal that included a Wealth Tax Act designed "to save our system" from the "crackpot ideas" of demagogues. FDR knew that his proposal was bad for the economy, but "I am fighting Communism, Huey Longism, ...," he said, indicating a belief that the consequences of losing were worse (Kennedy (1999), p. 241-7).

We first consider voters who have logarithmic utility. We establish that with log utility the benevolent party's equilibrium probability of winning does not depend on the level of the capital stock. In turn, this lets us derive the benevolent party's value function, and then solve for the equilibrium. We establish that the benevolent party proposes to save a constant share of capital.

One expects that political competition from the demagogue would induce the benevolent party to reduce its proposed investment to raise its chance of winning. What is surprising is the extent to which the benevolent party may mimic even a very unpopular demagogue who has little chance of winning. Indeed, we find that the benevolent party's investment policy converges to zero as the depreciation rate goes to 1, regardless of the demagogue's expected valence disadvantage. In contrast, with no competition, the benevolent party's investment is highest rather than lowest when depreciation is high, and it is always bounded away from zero. The benevolent party may be very likely to win the election, but by almost co-opting the demagogue's policy, an economic meltdown occurs as capital stocks converge to zero in the long run. We term such outcomes *death spirals*.

We identify necessary and sufficient conditions for death spirals to occur with probability one. With normally-distributed valences, death spirals emerge if the benevolent party lacks a valence advantage, unless capital depreciates very slowly. Even when depreciation is low enough to avoid death spirals, a demagogue's impact can be large. For example, with normal distributed valences that give the benevolent party a two standard deviation expected advantage, investment falls as depreciation rises and the investment gap with the no-competition benchmark remains about 50% even when capital does not depreciate. With a three standard deviation valence advantage and no depreciation, the welfare loss measured by the certainty equivalent consumption level is almost 70%. Indeed, if the demagogue's expected valence disadvantage is *seven* standard deviations (so the demagogue is about 1/1000 times less likely to win the election if both choose the same platform than to win the Powerball lottery) the welfare shortfall remains 2% for a depreciation rate of 0.6.

Our initial analysis presumes that the demagogue only cares about winning the current election, but results extend to forward-looking demagogues. In particular, the unique linear Markov Perfect equilibrium corresponds to that with a myopic demagogue, as does the equilibrium of the limiting finite horizon economy. The infinite horizon economy can also support equilibria in which each party proposes to invest more, where the proposed investments raise a demagogue's chances of winning, using threats to revert to the 'myopic equilibrium'. However, such equilibria involve implausible levels of cooperation, and there are surprising limits to the feasible extent of cooperation. Given a minimal equilibrium refinement that the benevolent party wants its policy implemented rather than the demagogue's, we prove that regardless of the discount factor, such trigger strategies cannot support equilibrium outcomes that are close to the first best that obtains without a demagogue.

Our initial analysis also assumes fully-myopic voters. However, results extend if voters are only partially myopic, just under-weighting future consequences in their electoral choices. With less-myopic voters, the electoral risk to the benevolent party of proposing greater investment falls, and a demagogue proposes positive investment if it maximizes the median voter's perceived welfare. Nonetheless, we prove that the demagogue still proposes to invest nothing when the median voter is sufficiently myopic. More generally, as long as the valence distribution is log concave, the benevolent party's proposed investments rise continuously with the median voter's discount factor, and welfare losses remain large for plausible parameterizations.

We then consider CRRA utilities with degrees of relative risk aversion that exceed one, consistent with estimates from macroeconomic models. Such risk aversions amplify the adverse consequences of low capital stocks—when consumption becomes small, utility goes to $-\infty$ more quickly. As a result, the benevolent party's optimal investment ceases to be linear in capital, and its probability of winning depends on capital stocks. While one can no longer determine the functional form of the value function, we identify a lower bound on the marginal benefit of raising investment. This bound lets us characterize the benevolent party's investment policy and the economy's long-run fate. We identify a threshold on capital stocks below which the economy enters a self-enforcing death spiral that leads to zero capital and consumption in the long run. Thus, the paradoxical result arises that when the benevolent party is more concerned about low consumptions (its coefficient of risk aversion exceeds one) it may mimic the demagogue's non-investment policy even more closely.

The intuition for this result is that when the capital stock and hence period consumption are very low, the marginal utility of increased consumption is very high. This creates a strong electoral pressure for the benevolent party to propose lower investments lest it lose to the even worse no-investment policy of the demagogue. If capital stocks ever fall too low, the benevolent party proposes less investment than is needed to compensate for capital depreciation even if it wins; this vicious cycle of low capital stocks and low investment lead the economy into oblivion.

Moreover, capital stocks can always fall—the demagogue always has a chance of winning. This means that an economy is always just a few bad draws away from dropping capital below the critical death spiral threshold and hence from an economic meltdown. Our analysis emphasizes the shocks on the real economy generated by election outcomes. These shocks take the form of a victory by a demagogue that reduces capital.[3] This analysis also implies that, consistent with observation, democracies in developing economies with less capital are more susceptible to meltdowns, because fewer shocks are needed to drop capital below the point where meltdowns become inevitable.

This vicious downward cycle is not a typical poverty trap in which people are too poor to produce, thereby perpetuating economic misery. A benevolent party, free from the electoral pressure of demagogues, would invest more than enough to compensate for depreciation. It is the demagogue's presence that drives this cycle. We call this the *populist trap*. There is a second dimension to this trap. A demagogue maximizes his probability of winning the current election by maximizing spending, thereby reducing future capital. But, reducing future capital then amplifies the utility difference between the demagogue's zero-investment policy and any fixed investment policy. Thus, paradoxically, by damaging capital stocks and destroying social capital associated with institutions and property rights, the demagogue can improve his future chances of winning.

We establish an important converse to these results, characterizing how the demagogue's presence alters investment when capital is high. One might conjecture that because competition from the demagogue reduces the value of capital, it always induces the benevolent party to propose reduced investments. This conjecture is false. Facing less dire electoral concerns when capital stocks are high, the benevolent party proposes even more investment than it would absent a demagogue to insure against the possibility that the demagogue may win. Thus, paradoxically when capital is low, the benevolent party over-spends (compared to a social planner free of electoral pressure);

[3]Of course, adding standard macroeconomic shocks such as real business cycle shocks or wars can also drive capital below the critical death spiral threshold.

but when capital is high, it over-invests, choosing an austerity policy. For example, this result can reconcile why the far-right EKRE party in Estonia[4] "promised to slash taxes," but "the two main parties support continued austerity policies, which have left Estonia with the lowest debt level of any Eurozone country but have caused anger in rural communities."

We conclude by providing a more uplifting counterpoint to our general theme of despair. We establish that *if* a democracy is lucky enough at the outset to grow its capital stock to a sufficient level *and* the expected valence disadvantage of demagogues is sufficiently high, then there is always a positive probability that capital stocks never fall below that high level. This result highlights the value of good leaders in young democracies as they can build enough of a cushion of capital stock that the country can then withstand the negative shock of a rare future demagogue.

2 Related Literature

Bisin et al. (2015) develop a three-period model with voters who use hyperbolic discounting. Voters understand their self-control issues, and can use illiquid assets to prevent overspending in the second period. They show how two office-motivated candidates can undo this commitment device by via excessive debt accumulation, hurting voters. In contrast, in our model voters are unaware of how investment affects future consumption. If we only had office-motivated candidates as in their model, no investments would be made. The questions we address are whether a benevolent party with a stochastic valence advantage will implement policies that are close to socially desirable, and how deviations from optimal policies add up over time. That is, we study the extent to which Hamilton's notion of a good political leader can be effective in the presence of a demagogue.

Guiso et al. (2018) define a party as populist if it champions short-term policies while hiding their long-term costs. Using data from European countries between 2004 and 2016 they show that hard economic times lead to increased support for populists and populist policies, and that establishment parties' policies grow more populist in nature. Our theoretical model assumes that demagogues can hide the long-term consequences of economic policies from voters, as in their paper. Consistent with their empirical findings, we show that as long as risk aversion coefficients exceed one, following hard times, the attraction of populist policies rises and established parties

[4] https://www.bbc.com/news/world-europe-47430993 "Estonia general election: Opposition party beats Centre rivals," March 3, 2019.

become more populist in response, increasingly mimicking populists.

Acemoglu et al. (2013) build a model in which a lobby that favors the wealthy can try to bribe politicians to choose policies that favor the wealthy. In their model, populists are not susceptible to bribes and signal this by choosing extreme left-wing policies. In contrast, in our model the defining feature of demagogues is that, to maximize the probability of winning election, they champion short-term policies that appeal to short-sighted voters, consistent with Guiso et al. (2018).

To be able to distinguish between demagogues and regular politicians, one needs a model in which their preferences and hence policies differ in meaningful ways. Our analysis shows how the political process itself generates endogenous shocks to the economy. Fundamental to these shocks is that there is policy divergence, so election outcomes matter, introducing dynamic stochastic economic distortions. We are not aware of other papers that share these features.

A small macroeconomics literature embeds probabilistic voting à la Persson and Tabellini (2000) in a dynamic model (e.g., Persson and Svensson (1989), Cukierman and Meltzer (1989), Alesina and Tabellini (1990), Song et al. (2012), Battaglini (2014)). These models feature policy convergence: the political process generates ex-ante distortions when parties choose platforms, but with policy convergence it does not matter who wins. Battaglini and Coate (2008) analyze the effect of legislative bargaining on government debt and public good provision in a setting where debt constrains public good investment and pork provision, and each district is represented by a legislator. A first mover advantage in bargaining means that the proposer's identity determines which district receives the most pork, but public good investment and debt levels are unaffected. While these tractable formulations of political competition facilitate other insights, who wins the actual election is irrelevant. In contrast, in our model the election itself generates uncertainty and dynamic economic distortions, which captures the observation that in practice election results matter (e.g., Kelly et al. (2016) analyze the impact of electoral outcomes on forward-looking financial markets).

Demagogues, derived from the Greek (Demos + Agogos) are rabble leaders, who appeal to the people solely to win power for themselves. The term populist is often used interchangeably, but contains aspects that we do not model. For example, according to Müller (2017), populists claim to represent the true people against an elite who controls the levers of government at the expense of the true people. As a result, populists believe it is legitimate to move away from pluralistic democracy, because they, and only they, are the legitimate representatives of the people. In contrast, in our model, majority rule is always preserved. Were a demagogue, instead, able to change the rules underlying fair political competition, making it harder for a benevolent party to regain power, it would strengthen our results.[5] In particular, because its costs of losing would rise, the benevolent party would have even stronger incentives to mimic populist policies.

In our model, increasing current consumption comes at the expense of reducing future consumption. One may argue that citizens should be able to come to understand this link. In reality, this link is less clear because governments can borrow for long periods of time without discernible impacts on consumption. For example, Venezuela has borrowed from Russia on claims to its future oil streams. In practice, it can take decades before adverse impacts become observable. As an abstraction, we model this by assuming that citizens are short-sighted. Other papers that model voters as short-sighted include (Baron and Diermeier, 2001) and (Dal Bó et al., 2017). In a historical companion paper (Bernhardt et al., 2019), we document the concerns of founding fathers of American Democracy about this short-sightedness. We believe that modeling how citizens come to learn that short-sighted excessive spending causes severe damage, and the consequences of that learning are important. In turn, it may matter whether the learning occurs in a country like Venezuela where the rules of the game were bent by the ruling party, or in a country like Greece where they were not. We provide insights into this issue by showing how outcomes change when voters are less myopic.

3 Model

Two parties $P = b, d$ propose capital investments at dates $t = 0, 1, 2, \ldots$ There is a consumption good and a capital good. If capital at date $t-1$ is k_{t-1}, then the amount of the consumption good provided at date t is $f(k_{t-1}) = \phi k_{t-1}$. The consumption good can be invested and effectively turned into the capital good: if $i_t \geq 0$ is invested, and $1 - \rho \in (0, 1)$ is the depreciation rate of capital, then the capital stock at time t becomes $k_t = \rho k_{t-1} + i_t$. A party's policy at date t is a proposed investment i_t.

The median voter's date t utility is $u(c_t) + v_P$, where $c_t = f(k_{t-1}) - i_t$ is consumption, and v_P is a valence shock that measures the utility the voter derives if party P is in power. We focus on constant relative risk aversion utility preferences over consumption, i.e., $u(c) = c^{1-s}/(1 - s)$ for $s > 1$, and $u(c) = \log(c)$ for $s = 1$.[6] We interpret $v_{P,t}$, $P = b, d$ as measuring other, non-economic policy aspects

[5] One can also interpret a demagogue's efforts to weaken democratic institutions and norms in order to achieve short-term objectives as a form of reduced investment in social capital and property rights, with adverse future consequences.

[6] The case where $s < 1$ does not capture the real world feature that citizens grow increasingly desperate when capital falls very low. Were $s < 1$, there would be minimal pressure for the benevolent party to mimic the demagogue

that are out of a party's control. Without loss of generality, we set $v_{b,t} = 0$ and write v_t instead of $v_{d,t}$. We first consider myopic voters who base electoral decisions solely on period utility. This captures the idea that voters are unsophisticated and do not understand the long-term impacts of economic policy (Guiso et al., 2018). We later suppose that voters merely underweight future payoffs.

In contrast, parties are sophisticated and forward looking. Parties discount future payoffs by a discount factor β, where $0 < \beta < 1$. The demagogue, d, only cares about winning; it receives a period payoff of 1 if it wins, and 0, otherwise. The benevolent party, b, only cares about policy, and its period utility from consumption corresponds to that of the median voter. This framework nests a setting in which multiple benevolent parties compete: Each would offer the same economic policy, and when party d loses, the benevolent party with the highest valence is elected.

We maintain the following assumptions.

Assumption 1 *v_t is distributed i.i.d. with a cdf G and a pdf g, where $g(x) > 0$ for all x in the support of G, and* $0 < G(0) < 1$.

The *i.i.d.* assumption largely serves to simplify exposition. The assumption that $0 < G(0) < 1$ means that if b perfectly mimics d's platform, each party wins with strictly positive probability.

Assumption 2 $\beta\rho^{1-s} < 1 < \beta\phi$.

The first inequality ensures that b's discounted expected utility is finite. The second inequality means that investment returns exceed the discount factor, so that party b wants to invest.

The game proceeds as follows. At the beginning of each date t, the parties simultaneously propose policies $i_{P,t} \in [0, f(k_{t-1})]$, $P = b, d$. Then the valence shock v_t is realized and the median voter elects his preferred party. The party that wins the election implements its announced policy, period payoffs are realized, and the game moves to date $t + 1$.

We consider subgame perfect equilibria of the game that satisfy an equilibrium refinement:

Assumption 3 *In any equilibrium, at each time t, party b weakly prefers its own policy to the demagogue's, i.e., party b is at least as well off if his policy wins rather than d's policy, keeping strategies at future dates $t' > t$ unchanged.*

This refinement excludes a trivial and implausible equilibrium in which the demagogue proposes party b's most preferred investment, while the benevolent party promises to invest every-

when capital is low; but the reverse would be true when capital is high.

thing. The median voter would get unboundedly negative utility from b's proposed policy, and hence would choose the demagogue regardless of the valence realization. This is an equilibrium because the demagogue only cares about winning, and the benevolent party only cares about policy, and not about who implements a policy. By proposing to invest everything, party b can effectively guarantee that the demagogue is elected. This is an equilibrium of our stylized model, but it does not describe electoral competition in practice. In particular, it is reasonable to assume that candidates who run for office want to be elected, which is what Assumption 3 imposes.

4 No Political Competition from Demagogues

As a benchmark, we first suppose that the benevolent party faces no competition from the demagogue. Let $\bar{k} = k_{-1} > 0$ be the initial capital stock. An unconstrained benevolent party b solves

$$\max_{i_t} \sum_{t=0}^{\infty} \beta^t u(\phi k_{t-1} - i_t) \;\; \text{s.t.} \quad k_t = \rho k_{t-1} + i_t, \; 0 \le i_t \le \phi k_{t-1}. \tag{1}$$

Because the flow payoff, $u(c)$, is unbounded, the usual results that ensure differentiability of the value function and the contraction mapping results that ensure uniqueness do not apply. The problem also does not map into approaches used when one has constant returns to scale (Stokey et al. (1989), Ch. 4.3). One approach is to exploit the monotonicity of the Bellman operator, find an upper bound for the value function, and show that it converges to its fixed point (Stokey et al. (1989), Theorem 4.14). Our simpler approach exploits the scalability of the value function to establish its functional form. To verify scalability, we recast the problem using k_t as the state variable:

$$V(\bar{k}) = \max_{\{k_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u((\rho + \phi)k_{t-1} - k_t) \;\; \text{s.t.} \quad \rho k_{t-1} \le k_t \le (\rho + \phi)k_{t-1}, \text{ with } k_{-1} = \bar{k}. \tag{2}$$

Let $\{\bar{k}_t\}_{t=0}^{\infty}$ be an optimal sequence. Now, suppose we multiply the initial capital by $\alpha > 0$, so that the initial capital stock is $\hat{k}_{-1} = \alpha\bar{k}$, and consider the sequence $\{\hat{k}_t\}_{t=0}^{\infty}$, where $\hat{k}_t = \alpha\bar{k}_t$. This new sequence, $\{\hat{k}_t\}_{t=0}^{\infty}$, satisfies all the constraints because $\rho\bar{k}_{t-1} \le \bar{k}_t \le (\rho + \phi)\bar{k}_{t-1}$ if and only if $\rho\hat{k}_{t-1} \le \hat{k}_t \le (\rho + \phi)\hat{k}_{t-1}$. Moreover, because $V(\hat{k}_{-1})$ reflects optimization given $\hat{k}_{-1}$ rather than $\bar{k}_{-1}$,

$$V(\hat{k}_{-1}) = V(\alpha\bar{k}) \ge \sum_{t=0}^{\infty} \beta^t u((\rho + \phi)\bar{k}_{t-1} - \bar{k}_t) = \begin{cases} \alpha^{1-s} V(\bar{k}) & \text{if } s > 1, \\ \frac{\log(\alpha)}{1-\beta} + V(\bar{k}) & \text{if } s = 1; \end{cases} \tag{3}$$

Because α and $\bar{k}$ are both arbitrary, we can use $\frac{1}{\alpha}$ instead of α, and $\alpha\bar{k}$ instead of $\bar{k}$ to get

$$V(\bar{k}) \geq \begin{cases} \frac{1}{\alpha^{1-s}} V(\alpha\bar{k}) & \text{if } s > 1, \\ \frac{\log(1/\alpha)}{1-\beta} + V(\alpha\bar{k}) & \text{if } s = 1; \end{cases} \tag{4}$$

Suppose the inequality in (3) were strict. Then, substituting the right-hand side of (3) into (4), yields

$$V(\alpha\bar{k}) > \alpha^{1-s} V(\bar{k}) \geq \alpha^{1-s} \frac{1}{\alpha^{1-s}} V(\alpha\bar{k}) = V(\alpha\bar{k}), \quad \text{for } \; s > 1,$$

a contradiction. The case of $s = 1$ is similar. Thus, we must have

$$V(\alpha\bar{k}) = \begin{cases} \alpha^{1-s} V(\bar{k}) & \text{if } s > 1, \\ \frac{\log(\alpha)}{1-\beta} + V(\bar{k}) & \text{if } s = 1. \end{cases}$$

Substituting $\bar{k} = 1$ and $\alpha = k$ yields

$$V(k) = \begin{cases} k^{1-s} V(1) & \text{if } s > 1, \\ \frac{\log(k)}{1-\beta} + V(1) & \text{if } s = 1; \end{cases}$$

where $V(1)$ depends on s. It remains to show that $V(1)$ is finite. First, consider the lower bound obtained if party b never invests, so that $k_{t-1} = \rho^t$ and $c_t = \phi k_{t-1} = \rho^t \phi$. When $s > 1$,

$$\sum_{t=0}^{\infty} \beta^t \frac{(\phi\rho^t)^{1-s}}{1-s} = \frac{\phi^{1-s}}{1-s} \sum_{t=0}^{\infty} (\beta\rho^{1-s})^t = \frac{\phi^{1-s}}{(1-s)(1-\beta\rho^{1-s})} > -\infty,$$

because $\beta\rho^{1-s} < 1$ by Assumption 2. When $s = 1$,

$$\sum_{t=0}^{\infty} \beta^t \log(\phi\rho^t) = \sum_{t=0}^{\infty} \beta^t (\log(\phi) + t\log(\rho)) = \frac{1}{1-\beta} \log(\phi) + \frac{\beta}{(1-\beta)^2} \log(\rho) > -\infty.$$

Next, consider a strict upper bound obtained by letting the benevolent party invest all of the output, i.e., $k_{t-1} = (\rho + \phi)^t$, but supposing that such saving is 'costless' so that consumption equals the sum of principle capital plus output, i.e., $c_t = (\rho + \phi)k_{t-1} = (\rho + \phi)^{t+1}$. When $s > 1$,

$$\sum_{t=0}^{\infty} \beta^t \frac{((\phi + \rho)^{1+t})^{1-s}}{1-s} = \frac{(\rho + \phi)^{1-s}}{1-s} \sum_{t=0}^{\infty} (\beta(\phi + \rho)^{1-s})^t = \frac{(\rho + \phi)^{1-s}}{1-s} \frac{1}{1 - \beta(\phi + \rho)^{1-s}} < \infty,$$

by Assumption 2, because $\beta\phi > 1$ implies $\beta(\phi + \rho)^{1-s} < 1$. The case where $s = 1$ is similar. Thus, $V(1)$ and the value functions are well defined.

The functional form of the value function implies that at each date, a constant fraction capital is invested, and the remaining constant fraction is consumed. That is, the optimal investment at date t is λk_t, for some $\lambda > 0$ that does not depend on k. To see this, note that we showed that if $\overline{k}_t$ is the optimal sequence of capital stocks starting at $\overline{k}_{-1}$, then $\hat{k}_t = \alpha\overline{k}_t$ is the optimal sequence starting at $\alpha\overline{k}_{-1}$. Thus, if $\bar{i}_t = \overline{k}_t - \rho\overline{k}_{t-1}$ is the optimal sequence of investments starting at $\overline{k}_{-1}$, then $\hat{i}_t = \hat{k}_t - \rho\hat{k}_{t-1} = \alpha\bar{i}_t$ is the optimal sequence starting at $\hat{k}_{-1} = \alpha\overline{k}_{-1}$.

In sum, the scalability of the initial stock of capital in our dynamic optimization problem generates a simple closed-form value function, which is differentiable and concave. Thus, we can characterize the solution either by using the Euler equations, or by posing the problem in its recursive form, and then directly optimizing using the functional form of the value function:

Proposition 1 *With no electoral competition, party b solves the recursive optimization problem*

$$V(k) = \max_{i\in[0,\phi k]} u(\phi k - i) + \beta V(\rho k + i). \tag{5}$$

The value function takes the form

$$V(k) = \begin{cases} v\, k^{1-s} & \text{if } s > 1, \\ v + \frac{\log(k)}{1-\beta} & \text{if } s = 1; \end{cases},$$

where v does not depend on k. If k is the current capital, then investment is $i(k) = \lambda k$, where

$$\lambda = (\beta(\rho + \phi))^{1/s} - \rho > 1 - \rho \tag{6}$$

is increasing in ϕ and β, but decreasing in ρ.

Absent political competition from a demagogue, the benevolent party always grows capital: $\lambda > 1 - \rho$. Optimal investment increases in ϕ and β reflecting that the value of investment is higher if the future is discounted by less or if the savings technology is more productive. The impact of ρ is more subtle because greater depreciation raises the value of investment due to the concavity of payoffs in consumption, but reduces the value of investment because what is saved does not last as long. When $s \geq 1$, increasing ρ, i.e., reducing depreciation, reduces investment—the same level of capital can be achieved with reduced investment when capital depreciates more slowly.

5 The Strategic Problem

5.1 Log Utility with Myopic Demagogues

We now analyze how electoral competition from a demagogue affects the policy choices of a benevolent party and long-run outcomes. We first consider a myopic demagogue who only cares about winning in the current period. We then show that this analysis extends if we focus on linear Markov Perfect Equilibria, or if we consider Subgame Perfect Equilibria of the game with finitely-many periods in the limit where the number of periods is arbitrarily large.

Electoral competition introduces uncertainty, which is captured by the sequence of wins and losses of the benevolent party b. Let $\Omega = \{w, \ell\}^{\mathbb{N}}$ be the sequence representing these wins and losses, and let $W_t \colon \Omega \to \{0, 1\}$ be the random variable that is 1 if party b wins at date t, and is 0, if party d wins. The history at time t is given by the sequence of wins and losses up to time $t-1$. Let $\mathcal{F}_t$ be the filtration of Ω that corresponds to knowing that history. Then the capital stock and the investments $k_t, i_t \colon \Omega \to \mathbb{R}$ must be $\mathcal{F}_t$-measurable.

The probability distribution on Ω (i.e., the distribution over wins and losses) depends on k_t and i_t. To derive this distribution, let $\omega \in \Omega$. A myopic demagogue, who only cares about winning in the current period, has a dominant strategy not to invest. Let $i_t(\omega)$ be party b's date t investment policy given ω. Thus, the median voter in state ω at time t is indifferent between the parties at the net-valence ϵ_t that solves $\log(\phi k_{t-1}(\omega) - i_t(\omega)) = \log(\phi k_{t-1}(\omega)) + \epsilon_t$. Let D_t be the difference in utility that the median voter would derive from the two policy proposals:

$$D_t(\omega) = \log(\phi k_{t-1}(\omega) - i_t(\omega)) - \log(\phi k_{t-1}(\omega)) = \log\left(\frac{\phi k_{t-1}(\omega) - i_t(\omega)}{\phi k_{t-1}(\omega)}\right). \tag{7}$$

Then the probability that b wins at time t in state ω is $G(D_t(\omega))$. Letting $E[\cdot; i, k]$ be the resulting expectation over Ω, party b's optimization problem is

$$\begin{gathered} \max_{i_t} E\left[\sum_{t=0}^{\infty} \beta^t \Big(W_t(\omega)\log(\phi k_{t-1}(\omega) - i_t(\omega)) + (1 - W_t(\omega))\log(\phi k_{t-1}(\omega))\Big); \{i_t(\omega), k_t(\omega)\}_{t\in\mathbb{N}}\right] \\ \text{s.t. } \; k_t(\omega) = \rho k_{t-1}(\omega) + i_t(\omega), \;\; 0 \le i_t(\omega) \le \phi k_{t-1}(\omega), \text{ where } k_t, i_t \text{ are } \mathcal{F}_t \text{ measurable.} \end{gathered} \tag{8}$$

A key observation is that, with log utility, winning probabilities are unchanged by scaling: $D_t = \log\left(\frac{\phi k_{t-1} - i_t}{\phi k_{t-1}}\right) = \log\left(\frac{\phi \alpha k_{t-1} - \alpha i_t}{\phi \alpha k_{t-1}}\right)$. Therefore, just like when there is no political competition, party b's value function is scalable in k. In turn, this implies that its optimal investment is linear in capital.

Lemma 1 *The investment platform problem of party b can be written recursively, with a value function that takes the form* $V(k) = v + \log(k)/(1-\beta)$.

Using Lemma 1, we write party b's optimization as

$$\max_i G(D)\left(\log(\phi k - i) + \frac{\beta}{1-\beta}\log(\rho k + i)\right) + (1 - G(D))\left(\log(\phi k) + \frac{\beta}{1-\beta}\log(\rho k)\right), \tag{9}$$

where $D = \log((\phi k - i)/\phi k)$. Differentiating party b's objective with respect to i yields

$$G(D)\left(-\frac{1}{\phi k - i} + \frac{\beta}{(1-\beta)(\rho k + i)}\right) - \frac{g(D)}{\phi k - i}\left(\log\left(\frac{\phi k - i}{\phi k}\right) + \frac{\beta}{1-\beta}\ \log\left(\frac{\rho k + i}{\rho k}\right)\right).$$

Letting $i = \lambda k$, $D = \log\left(\frac{\phi k - i}{\phi k}\right)$ simplifies to $D = \log\left(\frac{\phi - \lambda}{\phi}\right)$, yielding first-order condition[7]

$$G(D)\left(-\frac{1}{\phi - \lambda} + \frac{\beta}{(1-\beta)(\rho + \lambda)}\right) = g(D)\frac{1}{\phi - \lambda}\left(\log\left(\frac{\phi - \lambda}{\phi}\right) + \frac{\beta}{1-\beta}\left(\log\left(\frac{\rho + \lambda}{\rho}\right)\right)\right). \tag{10}$$

The left-hand side of (10) is the expected marginal effect of increasing savings today on the discounted stream of future consumption. The right-hand side of (10) is the marginal cost of increasing savings, which raises the chance that party d wins. It is the marginal reduction in the probability of winning due to increased savings, $g(D)/(\phi - \lambda)$, times the difference in the payoffs from winning and investing λk, versus losing to party d, which will not invest.

To see how d affects investment, multiply both sides of (10) by $(\phi - \lambda)(1-\beta)(\rho + \lambda)$ to obtain:

$$G(D)\,(\beta\phi - (1-\beta)\rho - \lambda) = g(D)(\rho + \lambda)\left((1-\beta)\log\left(\frac{\phi - \lambda}{\phi}\right) + \beta\log\left(\frac{\rho + \lambda}{\rho}\right)\right). \tag{11}$$

When unconstrained, the benevolent party chooses $\lambda = \beta\phi - (1-\beta)\rho$, which sets the left-hand side of (11) to zero. However, when evaluated at that λ, the right-hand side of (11) is strictly positive. Thus, λ must be reduced. Posed differently, party b maximizes social welfare just as when it did not face electoral competition from a demagogue. However, now, b can only implement its policy if elected, so it invests less to reduce the probability that the demagogue controls the government. Proposition 2 uses (11) to characterize how party b adjusts its savings in response to the demagogue.

Proposition 2 *Equilibrium investment is strictly reduced by political competition from the demagogue. Let $\lambda(\rho)k$ be the equilibrium investment given the depreciation rate $1 - \rho$. Then*

1. $\lim_{\rho\to 0} \lambda(\rho) = 0$*, implying that the winning probability $G(D)$ converges to $G(0)$.*

2. *The share of capital reinvested, $\lambda(\rho)/\phi$, converges to zero at the rate $-G(0)/(g(0)\log(\rho))$ as $\rho \to 0$. Thus, $\lambda(\rho)$ is strictly increasing in ρ for small ρ with*

$$\lim_{\rho\to 0}\frac{\lambda(\rho)}{\phi}\left(-\frac{g(0)}{G(0)}\log(\rho)\right) = 1.$$

[7]Observe that party b's objective function at the lower boundary of investment, $i = 0$, and the upper boundary of investment, $i = \phi k$, are the same. Moreover, the derivative of the objective at the lower boundary, $i = 0$, is positive if and only if $\beta(\rho + \phi) > \rho$, which holds by Assumption 2. Thus, there is an interior maximum.

By how much does political competition drive down party b's optimal investment? Proposition 2 gives a partial answer: it provides an analytical characterization of $\lambda(\rho)$ when the depreciation rate is high, $1-\rho \approx 1$. One might expect that if party b has a very large ex-ante valence advantage, so that it is likely to win election, then it would only marginally reduce its investment relative to the no political competition benchmark. However, the winning probability $G(D)$ is endogenous, and there can be a large marginal benefit of mimicking the demagogue's no-investment policy by lowering λ.

In fact, when the depreciation rate approaches 1, the endogenous winning probability goes to $G(0)$. Even when party b has a large ex-ante valence advantage, as long as $G(0)$ is less than 1, the effect on the benevolent party's investment becomes large when ρ is small. In particular, as ρ goes to zero, so does λ. Thus, the difference between the investment levels chosen by party b with and without political competition from the demagogue becomes maximal: even a remote threat of a demagogue winning can have large effects on the behavior of established parties.

To see why λ must go to zero as ρ goes to zero, observe that $\log((\rho+\lambda)/\rho)$ in equation (11) would go to infinity were λ bounded away from zero. That is, the marginal electoral cost would become arbitrarily large, making it optimal to lower λ. This implies that even though absent competition, investment would be $\beta\phi > 1$, the socially-minded party b becomes so concerned about electoral competition that it drives investment down to zero. To see the rate of convergence of $\lambda(\rho)$ as ρ goes to zero, note that the right-hand side of equation (11) goes to

$$g(0)(\rho+\lambda(\rho))\beta\log\left(\frac{\rho+\lambda(\rho)}{\rho}\right) = \beta g(0)\Big((\rho+\lambda(\rho))\log(\rho+\lambda(\rho)) - (\rho+\lambda(\rho))\log(\rho)\Big).$$

Using $\lim_{x\to 0} x\log(x) = 0$, this simplifies to $-\beta g(0)(\rho+\lambda(\rho))\log(\rho)$; and repeating this step, it simplifies further to $-\beta g(0)\lambda(\rho)\log(\rho)$. The left-hand side of equation (11) converges to $\beta G(0)\phi > 0$. For the right-hand side of equation (11) to converge to this value, $\lambda(\rho)$ must converge to zero at the rate $-G(0)\phi/(g(0)\log(\rho))$. The key term $\log\left(\frac{\rho+\lambda(\rho)}{\rho}\right)$ captures the future costs of losing the election to the demagogue. When the depreciation rate is almost 1, almost all capital disappears if the demagogue wins, generating huge future welfare losses. This effect becomes dominant, determining the rate at which b reduces investment to avoid this outcome.

The demagogue alters equilibrium investment in fundamental ways. Absent a demagogue, investment decreases linearly with ρ (Proposition 1). In contrast, Proposition 2 reveals that if ρ is small, then electoral competition *reverses* how party b behaves: λ *increases* in ρ. That is, as depreciation decreases so more capital survives, the benevolent party paradoxically proposes to save more. $\lambda(\rho)$ may globally increase in ρ, or it can start to fall with ρ once ρ is sufficiently large. This

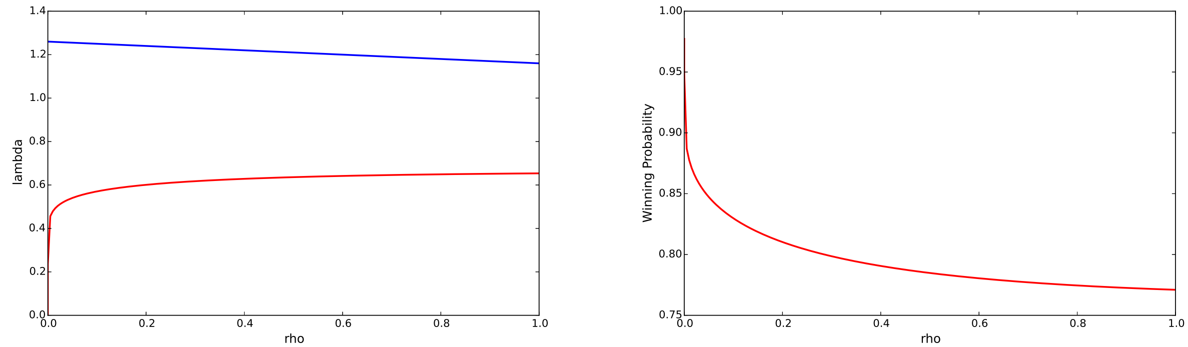

Figure 1: Investment (red), Social Optimal investment (blue), and b's winning probability for $\mu = -1, \sigma = 0.5, \phi = 1.4, \beta = 0.9$.

latter case occurs when the valence distribution G strongly favors party b, so that b is very likely to win. As a result, doing the 'right' thing begins to dominate electoral concerns when ρ is large.

Figure 1 considers a setting where party b has a large normally-distributed valence advantage: b would win with 97.7% probability, if it perfectly mimicked the demagogue. Yet, competition by the demagogue has a major impact. Party b's investment, λ, strictly increases for all values of ρ, while the socially optimal investment is strictly decreasing in ρ. Further, b invests far less than in the absence of a demagogue. One measure of the tradeoff that party b faces is that its equilibrium probability of winning falls below 0.8 for $\rho > 0.4$. That is, to achieve the equilibrium benefits of saving, b reduces its ex-ante electoral advantage by over 20 percent. However, from an outsider's perspective, it looks as if b is pandering to voters, reducing investment by almost 50% from the social optimum. Thus, pandering to short-sighted voters has large welfare costs. This example illustrates a benevolent party's dilemma, as neither ignoring the threat posed by a demagogue nor capitulating to demagoguery himself is a good option.

The threat from demagogue matters even if the demagogue's chance of winning is really remote. Suppose we increase the demagogue's mean valence disadvantage to $\mu = -3.5$, keeping σ at 0.5. If party b perfectly mimics the demagogue, the the demagogue's winning probability is 1.28×10^{-12}, about 1,000 times less likely than winning the Powerball lottery. Nonetheless, party b takes significant risks in equilibrium: the demagogue's winning probability is about 1 in 560 for $\rho = 0.1$, and 1 in 1,350 for $\rho = 0.6$. In these cases, demagogues with a realized valence that is not worse than about -2 can win. In other words, voters may support truly odious demagogues.

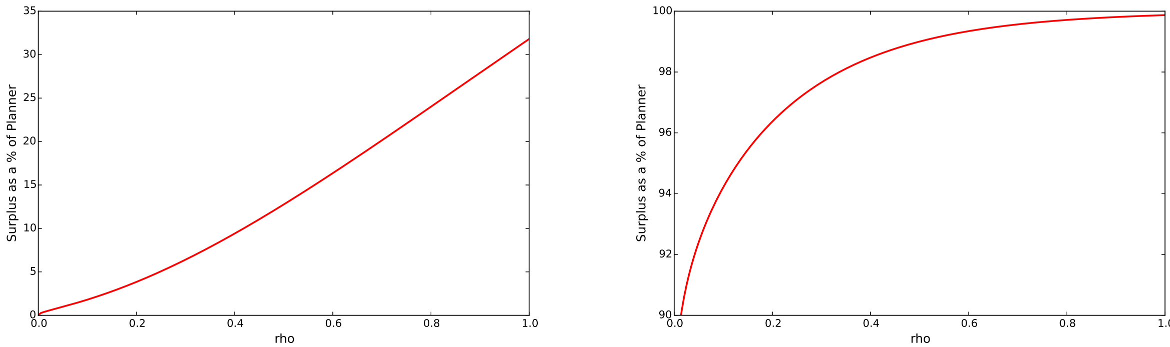

Figure 2: Welfare shortfall due to political competition, $\mu = -1.5$ left panel, and $\mu = -3.5$ right panel; $\sigma = 0.5$, $\phi = 1.4$, $\beta = 0.9$.

Now, investment λ is no longer strictly monotone. Ceteris paribus, strict monotonicity holds for all $\mu \geq -1.735$, but λ decreases for ρ close to 1 when $\mu < -1.735$.

The large impacts of demagogues on optimal investments that arise even when b has a big valence advantage, translate into large welfare losses. To compute this welfare loss, we calculate the constant consumption streams, c_N (no competition) and c_C (competition) that would yield the same ex-ante expected utility.[8] Figure 2 presents the ratio c_C/c_N in percent. In the left panel, party b has a three standard-deviation valence advantage over the demagogue. Nonetheless, the demagogue's presence results in a consumption equivalent that is always less than one third of that in a setting without the demagogue. In the right-panel, party b's valence advantage is seven standard deviations. This extremely remote possibility that the demagogue wins still meaningfully lowers welfare.

Proposition 2 implies that if ρ is small enough, then party b lowers λ below the capital depreciation rate of $1-\rho$. Because party d never re-invests, it follows that the capital stock then converges to zero. We say that the capital stock exhibits a death spiral if it converges to zero.

Corollary 1 *There exists a $\bar{\rho} > 0$ such that if $\rho \leq \bar{\rho}$, then the capital stock k_t exhibits a death spiral, converging to zero as $t \to \infty$.*

To prove Corollary 1 it suffices to show that λ is close to zero when ρ is small. However, death spirals are not limited to such extreme cases. In fact, the probability of a death spiral may go to one even when expected investment, $G(D)(\rho+\lambda(\rho))+(1-G(D))\rho$, exceeds the depreciation rate. A central asset pricing result yields that if $(\rho+\lambda)^{G(D)}\rho^{1-G(D)} < 1$, then the probability of a death spiral goes

[8] These computations do not account for welfare benefits from being able to elect a high-valence demagogue.

to one, but if the inequality is reversed then the probability goes to zero. The logic is direct: starting at any capital stock $\bar{k}$, after t elections of which the benevolent party wins u, the capital stock is

$$\bar{k}(\rho+\lambda)^u\rho^{t-u} = \bar{k}((\rho+\lambda)^{\frac{u}{t}}\rho^{1-\frac{u}{t}})^t.$$

The result then follows from the weak law of large numbers, as $G(D)$ is the probability that b wins.

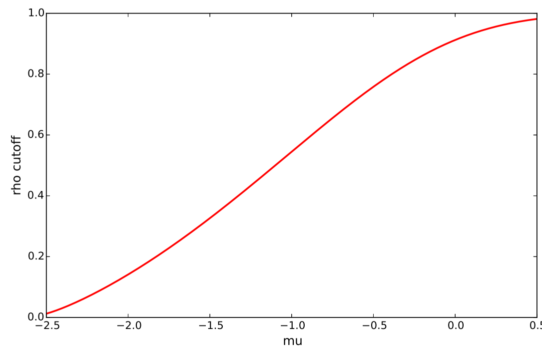

Figure 3: Cutoff value of ρ below which a death spiral occurs, $\sigma = 0.5$, $\phi = 1.4$, $\beta = 0.9$

Figure 3 plots the cutoff level of ρ below which death spirals are inevitable for different values of μ. If no party has a valence advantage, i.e., if $\mu = 0$, then death spirals occur unless ρ is close to one. Such a scenario could reflect a change in how the public perceives demagogues, for example if opprobrium toward demagogues declines due to changes in political discourse.

5.2 Log Utility with Far-sighted Demagogues

To his point, we have focused on demagogues who only care about the present, and thus give citizens exactly what they want at the present, in order to maximize the probability of winning election. If the demagogue is, instead, far sighted and also cares about winning in the future, his interests may no longer be aligned with short-sighted citizens. For example, if high consumption today would lower a demagogue's future chances, then he may propose strictly positive investment, trading off current and future probabilities of election, and no longer giving voters exactly what they want. Were this so, dynamic political competition would reduce the harm generated by a demagogue.[9]

[9]Obviously, we assume that, if elected, the demagogue cannot change the rules of the game while in office. As Levitsky and Ziblatt (2018) discuss, demagogues sometimes damage the "guardrails of democracy," by changing the basic rules and norms of democratic government to their advantage.

We now show that dynamic political competition may not discipline a demagogue. We first consider two natural equilibrium selection criteria and show that the equilibrium with a short-sighted demagogue also obtains with far-sighted demagogues. In particular, the same equilibrium outcomes emerge with a far-sighted demagogue in (1) the unique linear Markov perfect equilibrium, and (2) the unique equilibrium to the stationary limit of a finite-horizon economy. The key as to why a far-sighted demagogue implements short-sighted policies that maximize his probability of winning today is that the demagogue's competitiveness in future elections does not vary with the capital stock.

We then show that the demagogue's behavior can be improved if we allow for non-Markov (e.g., trigger) strategies. The benevolent party is willing to trade off a reduced probability of winning in return for a more far-sighted policy choice by the demagogue. In particular, party b is willing to increase investment toward the optimal level in the absence of electoral competition if the demagogue, in turn, proposes increased investment. In this equilibrium, party b's increased investment makes the demagogue more likely to win, and b gains from the better policies. When the discount factor β is close enough to one, this equilibrium can be supported by the threat of reverting to the 'myopic' equilibrium in which the demagogue does not invest. One might expect that if the discount rate goes to one so that parties care a lot about the long term, then one should be able to get close to the Pareto efficient outcome by using harsh threats to deter deviation. However, implemented equilibrium policies remain bounded away from the first-best—threats can help, but only to a limited extent.

We first formally state the result that equilibrium outcomes with a far-sighted demagogue correspond to those with a myopic demagogue for natural equilibrium selection criteria.

Proposition 3 *In the unique linear MPE, behavior coincides with equilibrium outcomes when the benevolent party is farsighted and the demagogue is myopic.*[10] *Similarly, in a finite-horizon model, as the number of periods goes to infinity, the equilibrium with a far-sighted demagogue coincides with the equilibrium of the infinite-horizon model with a myopic demagogue.*

The state at each date is the capital stock. A Markov strategy for party $P \in \{b, d\}$ is a mapping from that capital stock to an investment, $i_P(k) : [0, \infty) \to [0, \phi k]$. A linear Markov strategy for party P takes the form $i_P(k) = \lambda_P k$, for some $\lambda_P \in [0, \phi]$. A linear Markov perfect equilibrium is a linear strategy profile, (λ_b, λ_d), of mutual best responses in every subgame, where one must verify that the best response to a rival's linear strategy is also linear.

[10]This result holds if we allow for linear strategies in which the coefficients depend on time, so that $i_t^P(k) = \lambda_t^P k$, where λ_t^P depends on t, but not on the history of play.

To prove the result for the finite-horizon model, we proceed by backward induction. In the final period, obviously neither party invests, so election outcomes do not hinge on the capital stock. Continuing inductively, the non-dependence of future electoral outcomes on capital ensure that a demagogue does not want to save, as saving reduces the current probability of winning without affecting future election probabilities. Thus, the demagogue never invests, just as in the myopic setting.

It is immediate that allowing for non-Markov strategies, the demagogue can be induced to use better policies whenever β is sufficiently large. In particular, let b choose the unconstrained optimum, λ^*, and select λ_d so that d's winning probability is marginally larger. Clearly, this is welfare improving from b's perspective. Further, for all β sufficiently close to 1, d does not want to deviate given the threat to revert to the myopic equilibrium after a deviation. What is surprising is that even for β close to one, such trigger strategies cannot support the first best.

Proposition 4 *Let both parties use trigger strategies, where a deviation results in reversion to the linear Markov equilibrium. Then investments are bounded away from first-best levels for all β.*

Party b cannot prefer the demagogue's policy—b wants to win by Assumption 3—so both parties must have chances of winning. But, if both policies are close to first-best levels, the demagogue's winning probability would be too low, so he would want to deviate to zero investment.

At first pass, these trigger strategies in which the benevolent party strategically induces the demagogue to moderate looks like attempts by responsible parties to tame demagogue, for example via inclusion in governing coalitions. However, this is not a situation that our model describes. Instead, we have two separate parties with distinct policies, and any coordination has to occur over time. In fact, rather than taming the demagogue, the responsible party helps the demagogue to win, in the hope that the demagogue will behave more responsibly. In practice, this seems like an unsound strategy, and it is hard to understand how intertemporal coordination of this non-Markovian type would work. Further, the case where β is close to 1 badly approximates reality, because (i) an election winner holds office for a non-trivial term, and (ii) many offices feature term limits.

5.3 Log Utility with Non-Myopic Voters

We now show that our analysis extends when citzens are less myopic, discounting future payoffs using $\beta_m < \beta$. We first consider linear Markov equilibria with a myopic demagogue. Let $\lambda_d k$ and

$\lambda_b k$ be the investment levels chosen by the two parties. The median voter's utility is again scalable in k. As a result, the voter's value function takes the form $V_m(k) = \log(k)/(1-\beta_m) + v_m$.

The median voter's payoff from party b is $\log((\phi-\lambda_b)k) + \beta_m V_m((\rho+\lambda_b)k)$, and the payoff from the demagogue is $\log((\phi-\lambda_d)k) + \beta_m V_m((\rho+\lambda_d)k) + v$. Party b is elected if and only if

$$v \leq D \equiv \log\left(\frac{\phi-\lambda_b}{\phi-\lambda_d}\right) + \frac{\beta_m}{1-\beta_m}\log\left(\frac{\rho+\lambda_b}{\rho+\lambda_d}\right) < 0. \tag{12}$$

Thus, b wins with probability $G(D)$. Party b's optimization problem becomes

$$\max_{\lambda_b} G(D)\left(\log((\phi-\lambda_b)k) + \frac{\beta}{1-\beta}\log((\rho+\lambda_b)k)\right) + (1-G(D))\left(\log((\phi-\lambda_d)k) + \frac{\beta}{1-\beta}\log((\rho+\lambda_d)k)\right). \tag{13}$$

Factoring out k from the above optimization problem, reveals that party b's optimal λ_b solves

$$\max_{\lambda_b} G(D)\left(\log(\phi-\lambda_b) + \frac{\beta}{1-\beta}\log(\rho+\lambda_b)\right) + (1-G(D))\left(\log(\phi-\lambda_d) + \frac{\beta}{1-\beta}\log(\rho+\lambda_d)\right). \tag{14}$$

Similarly, the demagogue solves $\max_{\lambda_d} 1 - G(D)$.

Proposition 5 *The demagogue's optimal investment policy does not depend on the benevolent party's policy choice, and is given by the myopic investment policy for all β_m sufficiently small:*

$$\lambda_d = \begin{cases} 0 & \text{if } \beta_m \leq \frac{\rho}{\phi+\rho}, \\ \beta_m\phi - (1-\beta_m)\rho & \text{if } \beta_m > \frac{\rho}{\phi+\rho}. \end{cases}$$

The benevolent party's optimal choice of λ_b solves

$$\begin{aligned} &G(D)\left(-\frac{1}{\phi-\lambda_b} + \frac{\beta}{(1-\beta)(\rho+\lambda_b)}\right) \\ &= g(D)\left(\frac{1}{\phi-\lambda_b} - \frac{\beta_m}{(1-\beta_m)(\rho+\lambda_b)}\right)\left(\log\left(\frac{\phi-\lambda_b}{\phi-\lambda_d}\right) + \frac{\beta}{1-\beta}\log\left(\frac{\rho+\lambda_b}{\rho+\lambda_d}\right)\right). \end{aligned} \tag{15}$$

The equilibrium converges to the myopic equilibrium as $\beta_m \to 0$. If G is log-concave, then party b's investment policy λ_b is continuous and increasing in $\beta_m < \beta$.

The demagogue's choice of λ_d does not hinge on λ_b reflecting that d always chooses λ_d to appeal maximally to the median voter. This implies that for all β_m sufficiently small, the demagogue chooses the same policy as when the median voter is fully myopic. Thus, the equilibrium is close to the myopic equilibrium when β_m is sufficiently small. That the benevolent party's investment increases when the median voter is less myopic reflects two forces. First, fixing λ_d, the marginal probability that b loses falls as β_m rises, because the median voter attaches more value to future investments; and this reduces the marginal cost of losing, encouraging b to increase investment. Second, once β_m exceeds $\frac{\rho}{\phi+\rho}$, the demagogue increases λ_d as β_m rises. This reduces the difference to b between winning and losing, reducing the cost of losing, and hence encouraging b to invest more.

Even when the median voter is reasonably forward-looking, the demagogue poses a serious electoral threat, inducing the benevolent party to lower its investment, which results in large welfare losses. Concretely, in the setting of Figure 2, where the demagogue's mean valence disadvantage is three standard deviations, when β_m is half of the "true" discount factor, then for any level of ρ, the welfare loss is about 1/3, and the demagogue wins about 5% of the time.

The same equilibrium outcome obtains in the linear Markov equilibrium with a far-sighted demagogue. This reflects that capital does not affect future winning probabilities, so a far-sighted demagogue acts just like a myopic demagogue.

5.4 CRRA preferences and the Inevitability of Death Spirals

The previous subsections establish that when voters have log preferences, the equilibrium probability with which the benevolent party b wins does not vary with the capital stock. As a result, b optimally proposes a constant investment strategy. This means that the occurrence of death spirals depends on model parameters, but not on capital. We now show that this is no longer true if voters are more risk averse, consistent with most macroeconomic calibrations of risk aversion of consumer preferences. In fact, if voters have CRRA utility with relative risk aversion $s > 1$, then regardless of the other parameters describing the economy, for any initial capital stock k, the probability of a death spiral is strictly positive, bounded away from zero. Moreover, death spirals become inevitable once capital levels fall too low.

Characterization of equilibrium behavior becomes far harder when $s > 1$, because the optimal investment as a share of capital is no longer constant. This reflects that for a given proposed investment rate policy λ, the utility that voters derive from the two policies now hinges on k. In particular, $u((\phi - \lambda)k) - u(\phi k) = k^{1-s} \frac{(\phi-\lambda)^{1-s}-\phi^{1-s}}{1-s}$ increases in k for $s > 1$. This means that, ceteris paribus, party b is more likely is to win the election when the capital stock is larger.

The value function describing party b's expected payoffs now hinges in potentially complicated ways on the properties of the valence density. There is no reason for $G(u((\phi-\lambda)k)-u(\phi k))$ to be wellbehaved, so standard arguments used to prove concavity or differentiability of the value function cannot be used. This complicates characterization of the optimal investment rate policy $\lambda(k) = \frac{i(k)}{k}$.

A key step in our analysis is to identify a lower bound on the derivative of party b's value function—if that derivative exists. This lower bound is the derivative of the differentiable value function associated with zero investment at each date, where with zero investment by party b, neither investment rates nor the probabilities of winning vary with the capital stock. This lower bound delivers a lower bound on the marginal value of a positive investment rate. Let $V_N(k)$ denote the benevolent party's value function if investment is zero at each date:

$$V_N(k) = \sum_{t=0}^{\infty} \beta^t (\phi \rho^t k)^{1-s}/(1-s) = \frac{(\phi k)^{1-s}}{1-s} \sum_{t=0}^{\infty} (\beta \rho^{1-s})^t = \frac{(\phi k)^{1-s}}{1-s} \frac{1}{1-\beta\rho^{1-s}},$$

where from Assumption 2, $\beta\rho^{1-s} < 1$. Thus, $V_N(k)$ is differentiable with respect to k and

$$V_N'(k) = \frac{(1-s)V_N(k)}{k}. \tag{16}$$

Lemma 2 *Let $V(k)$ be party b's value function, and let $V_N(k)$ be b's discounted expected payoff, starting with capital stock k when investment is always zero. Then, $V_N(k)$ is differentiable, and*

$$\liminf_{k' \to k} \frac{V(k) - V(k')}{k - k'} \geq V_N'(k).$$

Lemma 2 lets us characterize party b's investment policy. Write b's optimization problem as:

$$V(k) = \max_{\lambda \in [0,\phi]} G(D)\Big(u((\phi-\lambda)k) + \beta V((\rho+\lambda)k)\Big) + (1 - G(D))\Big(u(\phi k) + \beta V(\rho k)\Big), \tag{17}$$

where

$$D = u((\phi-\lambda)k) - u(\phi k) = k^{1-s}\, \frac{(\phi-\lambda)^{1-s} - \phi^{1-s}}{1-s}.$$

The marginal cost of raising investment (i.e., of raising λ) is

$$MC = \frac{g(D)k^{1-s}}{(\phi-\lambda)^s}\Big(u((\phi-\lambda)k) + \beta V((\rho+\lambda)k) - u(\phi k) - \beta V(\rho k)\Big).$$

From Lemma 2, the marginal benefit of raising investment (i.e., of raising λ) satisfies:

$$MB \geq G(D)k^{1-s}\Big(-ku'((\phi-\lambda)k) + k\beta V_N'((\rho+\lambda)k)\Big) = G(D)\Big(-\frac{1}{(\phi-\lambda)^s} + \beta\frac{\phi^{1-s}}{1-\beta\rho^{1-s}}\Big).$$

For any $k > 0$, $MC(\lambda = 0) = 0$, but $MB(\lambda = 0) = G(0)\frac{\beta(\phi+\rho^{1-s})-1}{\phi^s(1-\beta\rho^{1-s})} > 0$. Thus, party b always benefits from marginally raising investment above zero at any $k > 0$. Moreover, $\lim_{k\to 0} \lambda(k) = 0$. Party

b can always do better than $\lambda = 0$. In turn, if the support of valence is bounded then $\lim_{k\to 0} \lambda(k) = 0$. To see this, observe that if $\lim_{k\to 0} \lambda(k) > 0$, then $\lim_{k\to 0} D = -\infty$, and hence with a bounded support $\lim_{k\to 0} G(D) = 0$, i.e., party b would always lose. But, if b always loses, then its payoff equals that from choosing $\lambda = 0$, and we just showed that b can always get a higher payoff. Therefore, $\lim_{k\to 0} \lambda(k) = 0$. By the same logic, $G(D(k)) \in (0, G(0))$. This proves Proposition 6:

Proposition 6 *Party b's optimal investment and probability of winning are strictly positive:* $\lambda(k) > 0$ *and* $0 < G(D(k)) < G(0)$ *for all* $k > 0$. *If G has a finite support,*[11] *then* $\lim_{k\to 0} \lambda(k) = 0$.

Proposition 6 has two implications. Because $\lim_{k\to 0} \lambda(k) = 0$, when k is sufficiently small, $\lambda(k) < 1 - \rho$. Thus, if capital ever falls too low, then it falls at each date thereafter: even if party b wins, investment fails to compensate for depreciation. This reduction in capital, in turn, reduces b's future investment, creating a death spiral for the capital stock. Moreover, even when the initial capital stock is large enough that b's investment exceeds the rate of depreciation, the demagogue may win. If the demagogue wins enough times, depreciation can bring capital stock below the critical threshold that begins a death spiral. Let $\bar{k}$ be that capital threshold, and suppose the initial capital stock is $k > \bar{k}$. If the demagogue wins n consecutive times, then the capital stock falls from k to $\rho^n k$. Thus, a death spiral necessarily occurs if $n > n^*$, where $n^* = \min\{n \in \mathbb{N} \ s.t.\ \rho^n k < \bar{k}\} = \left\lceil \frac{\log(\bar{k}/k)}{\log(\rho)} \right\rceil$. The probability that the demagogue wins n^* consecutive times is at least $(1 - G(0))^{n^*}$. Thus,

Proposition 7 *There exists a capital level* $\bar{k}$ *such that if* $k \le \bar{k}$ *then a death spiral occurs with probability 1. Given any capital stock* $k > \bar{k}$, *the probability of dropping below* $\bar{k}$ *and entering a death spiral exceeds* $(1 - G(0))^{n^*} > 0$, *where* $n^* = \lceil (\log(\bar{k}/k)) / \log(\rho) \rceil$.

This proposition reveals that poor democracies are more vulnerable to demagogues as the requisite n^* for a death spiral is smaller. Interpreting k as including well-defined property rights and the social capital associated with the institutional norms of democracy, this proposition also suggests that younger democracies are more vulnerable.

5.5 CRRA Preferences and Investment in Good Times

We next characterize party b's investment policy choices when capital stocks are large. When capital stocks are small, b invests so little that a death spiral results. In contrast, we now show that when

[11] We still maintain Assumption 1. Thus, $0 < G(0) < 1$.

capital stock are high, b over-invests in the sense that political competition from the demagogue causes it to save even more than it would absent electoral concerns.

This result is perhaps surprising, because competition from the demagogue always reduces the value attached to a given capital stock. One might then think that this *must* reduce the derivative of the value function with respect to capital. To show that this is not so, we show that for large k, solutions of Problem (17) are close to solutions of the recursive optimization problem

$$V_b(k) = \max_{\lambda\in[0,\phi]} G(0)\Big(u((\phi-\lambda)k) + \beta V_b((\rho+\lambda)k)\Big) + (1-G(0))\Big(u(\phi k) + \beta V_b(\rho k)\Big), \tag{18}$$

in which party b's probability of winning is set at its highest level $G(0)$—-and does not depend on its action. Replacing $G(0)$ by 1 in (18) yields the benevolent party's problem when there is no demagogue, whose recursive form is given in (5). Thus, by the same argument as in Proposition 1, the value function takes the form $V_b(k) = \frac{v_b}{1-s}\, k^{1-s}$, and the investment share λ is independent of k. Letting λ^* be that optimal λ, the first-order condition of (18) yields:[12]

$$\lambda^* = \frac{(\beta v_b)^{1/s}\phi - \rho}{1 + (\beta v_b)^{1/s}}. \tag{19}$$

Lemma 3 shows that for large k, solutions of problem (17) are close to this λ^*.

Lemma 3 *Let λ^* be the solution of problem* (18) *given by* (19). *Let $\lambda(k)$ solve problem* (17). *Then for every $\varepsilon > 0$ there exists $\hat{k}$ such that $|\lambda(k) - \lambda^*| < \varepsilon$, for all $k \geq \hat{k}$.*

In view of the lemma, it suffices to compare the benevolent party's problem absent a demagogue to (18). Proposition 1 shows that the value function for the party's problem absent a demagogue is $V_P(k) = k^{1-s}v_P/(1-s)$, where $v_P/(1-s) = V_P(1)$. Because ex-ante utility is maximized when there is no political competition, $V_P(1) > V_b(1)$. Because $1-s < 0$, it follows that $v_P < v_b$. In general, if the value function takes the form $k^{1-s}v/(1-s)$ then

$$\lambda = \frac{(\beta v)^{1/s}\phi - \rho}{1 + (\beta v)^{1/s}}. \tag{20}$$

Note that λ as defined in (20) strictly increases in v. Because $v_P < v_b$ it follows that the solution to problem (18), λ_b, strictly exceeds the no-competition optimum, λ_P. Hence, Lemma 3 implies that $\lambda(k) > \lambda_P$ when k is large. The same argument implies that the solution to (18) decreases in $G(0)$. In other words, if the valence distribution becomes more favorable to the demagogue, then b responds by increasing investment even more when the capital stock is large. Thus:

[12]It is never optimal to invest all output because $u(0) = -\infty$. We show that investment in problem (18) exceeds the unconstrained optimum, so the optimal investment is not 0. Moreover, with $s > 1$, $u(c) < 0$. Thus, $V_b < 0$, i.e., $v_b > 0$.

Proposition 8 *If k is sufficiently large then:*

1. *Party b invests more than a social planner whose policy is always implemented:* $\lambda(k) > \lambda_P$.
2. $\lambda(k)$ *increases with decreases in* $G(0)$*, i.e., with decreases in party b's valence advantage.*

As k grows very large, the probability that b loses goes to $G(0)$, becoming insensitive to b's equilibrium investment. This leads b to 'over-invest' to account for the fact that investments are only made when it wins. This insures against capital being depreciated too low due to a string of wins by the demagogue. In some sense, relative to a scenario with no political competition, the benevolent party pursues "too much" austerity in good times.

We have shown that given any capital stock k, no matter how high, the probability of a death spiral is always strictly positive. In particular, there exists a critical capital stock level $\bar{k}$ such that if k ever drops below $\bar{k}$, then it continues to decrease monotonically toward zero. We now establish a more positive counterpart to this result: *if* a democracy is lucky enough at the outset to grow its capital stock to a sufficient level *and* the expected valence disadvantage of demagogues is sufficiently high, then the probability that electoral competition from the demagogue never drives capital below that high level is bounded strictly away from zero.

Proposition 9 *Let J be the smallest integer solving* $(\beta(\rho+\phi))^{j/s}\rho \geq 1$*, and suppose that the benevolent party's valence is high enough that* $G(0)^J > \frac{1}{2}$*. If a sufficiently high level of capital stock is reached, then the probability,* $P(k)$*, that capital stays above that level forever is bounded away from zero. Letting* $P^* = \frac{2G(0)^J-1}{G(0)^J} > 0$*, for every* $\varepsilon > 0$*, there exists* k^* *such that* $P(k) > P^* - \varepsilon$*, for all* $k \geq k^*$.

The probability capital stocks remain high grow in b's valence advantage, as captured by $G(0)$. For example, if $\phi = 1.4, \beta = \rho = 0.9$, and $s = 1.5$ then with normally distributed valences $N(\mu, \sigma)$, a lower bound on the probability the economy never enters a death spiral, once capital stocks are sufficiently high, is about 81.1% for $\sigma = 0.5$ and $\mu = -0.5$, and about 97.4% when $\sigma = 0.5$ and $\mu = -1$.

We now provide the proof. Proposition 8 showed that there is a threshold $\tilde{k}$ on the capital stock such that when $k > \tilde{k}$, the benevolent party proposes more investment than an unconstrained social planner, i.e., $\lambda(k) > \lambda_P = (\beta(\rho+\phi))^{1/s} - \rho$. Therefore, if $k > \tilde{k}$ and b wins j consecutive times, and then loses to the demagogue, the capital stock will still exceed

$$(\rho + \lambda_P)^j \rho k = (\beta(\rho+\phi))^{j/s}\rho k.$$

Let J be the smallest j such that $(\beta(\rho+\phi))^{j/s}\rho \geq 1$. Beginning at $k > \tilde{k}$, as long as for every time d wins, b has won J times beforehand, the capital stock remains above k. If the probability that b win in a period is G, and the benevolent party's valence is high enough that $G^J > 1/2$, what is a bound on the probability that the capital stock falls below k? This problem is the same as finding the probability of hitting a barrier in a random walk where the probability of going up is $G^J > 1/2$ and the probability of going down is $1 - G^J$. One can show that beginning from one step above 0, the random walk hits the 0 barrier with probability $\frac{1-G^J}{G^J}$. Thus, the random walk remains above the zero barrier with probability $1 - \frac{1-G^J}{G^J} = \frac{2G^J-1}{G^J}$. Lemma 3 establishes that there is a threshold $\hat{k}$ such that when $k > \hat{k}$, the benevolent party's proposed investment is $\lambda(k) < \lambda^* + (\lambda^* - \lambda_P) = 2\lambda^* - \lambda_P$. Because the probability b wins is at least $G\left(k^{1-s}\ \frac{(\phi-\lambda)^{1-s}-\phi^{1-s}}{1-s}\right)$, for every $\epsilon > 0$, there exists a $\hat{k}_\epsilon$ such that for all $k > \hat{k}_\epsilon$, the probability b wins is at least $G(0)-\epsilon$. Combining these observations yields the result.

Summarizing the content of Propositions 6 and 8, if at the outset, an economy has the good fortune of electing benevolent leaders, those leaders may grow the capital by so much that there is a good chance of forestalling a retreat below the current high level. Concretely, enlightened leadership by Washington, Adams, Madison, ... can build enough institutional capital to forestall the adverse effects of later occasionally drawing a demagogue. If, instead, the economy has the misfortune at the outset of drawing a few demagogues, then it may doom the economy forever.

6 Conclusion

Our paper investigates the long-run susceptibility of Democracy to demagogues, studying the tension highlighted by Hamilton between far-sighted, magnanimous representatives who guard the long-run interests of voters, and office-seeking demagogues who cater to voters' short-run desires.

We model the political decision process as a capital investment problem in which politicians propose how to allocate existing resources between current consumption and investment. Voters base political choices on a comparison of the current period utility derived from policy proposals and a stochastic valence shock. The demagogue's sole focus on winning leads him to cater to short-sighted voters by under-investing. The far-sighted politician faces a fundamental tradeoff: He can choose a policy that would be better for voters in the long-run if implemented, but because only the winner gets to choose the policy, he must also be concerned about appealing to short-sighted voters.

A central insight is that even if a demagogue's chance of winning is remote (because his expected valence is very low) his presence can nevertheless have an outsided influence on a far-sighted politician's policy and long run outcomes. This influence can be so large that the economy enters a death spiral with capital declining relentlessly to zero. We show that (1) Bad times in which capital stocks are low bring out populist policies even by benevolent parties. (2) If the capital stock falls below a threshold, there is no coming back as capital continues to depreciate thereafter. Moreover, the economy is always a few bad draws away from entering such a death spiral. Young democracies with limited social capital, and democracies hit by capital-reducing economic shocks are especially vulnerable. (3) If society is lucky enough to elect enough benevolent politicians at the outset when capital stocks are low, the chances that it ever enters a death spiral remain bounded away from 1.

Our results resonate with concerns among the founders of American democracy and their emphasis on public education. Madison, the central architect of American institutions, made this clear at the Constitutional Convention: "I go on this great republican principle, that the people will have virtue and intelligence to select men of virtue and wisdom.... If there be not [virtue among us].... no theoretical checks—no form of government can render us secure. To suppose that any form of government will secure liberty or happiness without any virtue in the people, is a chimerical idea."

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7 Appendix

Proof of Proposition 1. We showed in the text that the value function is differentiable. We have:

$$\begin{aligned} V(k_t) &= \max_{i_t \in [0, \phi k_{t-1}]} u(\phi k_t - i_{t+1}) + \beta V(k_{t+1}) \\ & \quad s.t.\ k_{t+1} = \rho k_t + i_{t+1}. \end{aligned}$$

Let $c_t = \phi k_t - i_{t+1}$, and rewrite the problem as

$$\begin{aligned} V(k_t) &= \max_{c_t \in [0, \phi k_t]} u(c_t) + \beta V(k_{t+1}) \\ & \quad s.t.\ k_{t+1} = (\rho + \phi) k_t - c_t. \end{aligned}$$

The associated first-order condition yields $u'(c_t) = \beta V'(k_{t+1})$. Similarly, $k_{t+2} = (\rho + \phi)k_{t+1} - c_{t+1}$, so $u'(c_{t+1}) = \beta V'(k_{t+2})$. By the Envelope Theorem,$V'(k_t) = \beta(\rho + \phi)V'(k_{t+1})$ and $V'(k_{t+1}) = \beta(\rho + \phi)V'(k_{t+2})$. Putting these together yields

$$\frac{u'(c_{t+1})}{u'(c_t)} = \frac{V'(k_{t+2})}{V'(k_{t+1})} = \frac{1}{\beta(\rho + \phi)}. \tag{21}$$

The date $t + 1$ budget constraint is

$$k_{t+2} = (\rho + \phi)k_{t+1} - c_{t+1} \quad \Leftrightarrow \quad k_{t+1} = \frac{c_{t+1}}{\rho + \phi} + \frac{k_{t+2}}{\rho + \phi}.$$

Next, recursively apply the budget constraint to get

$$\begin{aligned} c_t &= (\rho + \phi)k_t - k_{t+1} \\ &= (\rho + \phi)k_t - \frac{c_{t+1}}{\rho + \phi} - \frac{k_{t+2}}{\rho + \phi} \\ &= (\rho + \phi)k_t - \frac{c_{t+1}}{\rho + \phi} - \frac{c_{t+2}}{(\rho + \phi)^2} - \frac{k_{t+3}}{(\rho + \phi)^2} \\ &\vdots \\ &= (\rho + \phi)k_t - \frac{c_{t+1}}{\rho + \phi} - \frac{c_{t+2}}{(\rho + \phi)^2} - \frac{c_{t+3}}{(\rho + \phi)^3} - \cdots . \end{aligned} \tag{22}$$

Thus, assuming existence, if (21) yields $c_{t+1} \propto c_t$, then it follows that $c_t \propto k_t$.

When $u(c) = \frac{c^{1-s}}{1-s}$, for $s \neq 1$, and $u(c) = \log(c)$, for $s = 1$, then (21) yields

$$\left(\frac{c_t}{c_{t+1}}\right)^s = \frac{1}{\beta(\rho + \phi)} \quad \Leftrightarrow \quad \frac{c_{t+1}}{c_t} = [\beta(\rho + \phi)]^{1/s}. \tag{23}$$

Now, substituting from (23) into (22),

$$c_t = (\rho + \phi)k_t - \frac{[\beta(\rho + \phi)]^{1/s}}{\rho + \phi}c_t - \frac{([\beta(\rho + \phi)]^{1/s})^2}{(\rho + \phi)^2}c_t - \frac{([\beta(\rho + \phi)]^{1/s})^3}{(\rho + \phi)^3}c_t - \cdots .$$

Thus,

$$\begin{aligned} (\rho + \phi)k_t &= c_t + \left(\frac{[\beta(\rho + \phi)]^{1/s}}{\rho + \phi}\right)c_t + \left(\frac{[\beta(\rho + \phi)]^{1/s}}{\rho + \phi}\right)^2 c_t + \left(\frac{[\beta(\rho + \phi)]^{1/s}}{\rho + \phi}\right)^3 c_t + \cdots \\ &= \frac{c_t}{1 - \frac{[\beta(\rho+\phi)]^{1/s}}{\rho+\phi}}, \text{ assuming } \beta(\rho + \phi)^{1-s} < 1, \text{ which is true by Assumption 2.} \end{aligned}$$

Cross-multiplying, yields that consumption is a linear function of k_t.

$$c_t = \left(\rho + \phi - [\beta(\rho + \phi)]^{1/s}\right) k_t.$$

Thus, investment is given by

$$i_{t+1} = \phi k_t - c_t = \left(\phi - \rho - \phi + [\beta(\rho + \phi)]^{1/s}\right) k_t = \left([\beta(\rho + \phi)]^{1/s} - \rho\right) k_t.$$

The FOC hold—investment is positive—since $(\beta(\rho+\phi))^{1/s} > (\beta\phi)^{1/s} > 1 > \rho$ by Assumption 2. ■

Proof of Lemma 1. Party b's discounted payoffs is bounded by the same arguments used for the social planner. In state ω, let $i_t(\omega)$ be an optimal investment given initial capital stock $\bar{k}$, and let $\tilde{i}_t(\omega)$ be an optimal investment given $\alpha\bar{k}$, where $\alpha > 0$. Let $k(\omega)$ and $\tilde{k}(\omega)$ be the associated capital stocks, and $D_t(\omega)$ and $\tilde{D}_t(\omega)$ be the associated valence cutoffs. Now, consider investment $\hat{i}_t(\omega) = \alpha i_t(\omega)$. The constraints of Problem (8) remain satisfied by $\hat{i}_t(\omega)$ when the initial capital stock is $\alpha\bar{k}$. Further, the capital stock is $\hat{k}_t(\omega) = \alpha k_t(\omega)$. Thus, letting $\hat{D}_t(\omega)$ be the associated valence cutoff, $\hat{D}_t(\omega) = D_t(\omega)$, so the expectation $E[\cdot]$ under k_t, i_t, starting at $\bar{k}$, is the same as the expectation $E[\cdot]$ under $\hat{k}_t, \hat{i}_t$, starting at $\alpha\bar{k}$. The expected discounted payoff under $\{\tilde{i}_t(\omega), \tilde{k}_t(\omega)\}_{t=0}^{\infty}$ is higher than under $\{\hat{i}_t(\omega), \hat{k}_t(\omega)\}_{t=0}^{\infty}$ because $\{\tilde{i}_t(\omega), \tilde{k}_t(\omega)\}_{t=0}^{\infty}$ is an optimal solution (the inequality below). Factor α out of the objective (the equality below) to obtain:

$$
\begin{aligned}
&E\left[\sum_{t=0}^{\infty}\beta^t\Big(W_t(\omega)\log(\phi\tilde{k}_{t-1}(\omega)-\tilde{i}_t(\omega))+(1-W_t(\omega))\log(\phi\tilde{k}_{t-1}(\omega))\Big);\{\tilde{i}_t(\omega),\tilde{k}_t(\omega)\}_{t\in\mathbb{N}}\right]\\
&\geq E\left[\sum_{t=0}^{\infty}\beta^t\Big(W_t(\omega)\log(\phi\alpha k_{t-1}(\omega)-\alpha i_t(\omega))+(1-W_t(\omega))\log(\phi\alpha k_{t-1}(\omega))\Big);\{\alpha i_t(\omega),\alpha k_t(\omega)\}_{t\in\mathbb{N}}\right]\\
&=\frac{\log(\alpha)}{1-\beta}+E\left[\sum_{t=0}^{\infty}\beta^t\Big(W_t(\omega)\log(\phi k_{t-1}(\omega)-i_t(\omega))+(1-W_t(\omega))\log(\phi k_{t-1}(\omega))\Big);\{i_t(\omega),k_t(\omega)\}_{t\in\mathbb{N}}\right].
\end{aligned}
$$

Let $V(k)$ denote the expected discounted payoff to party b. Then this argument shows that

$$V(\alpha k) \geq \frac{\log(\alpha)}{1-\beta} + V(k). \tag{24}$$

Because k and α are arbitrary it follows that

$$V(k) = V\left(\frac{1}{\alpha}(\alpha k)\right) \geq \frac{\log\left(\frac{1}{\alpha}\right)}{1-\beta} + V(\alpha k). \tag{25}$$

Suppose that the inequality in (24) is strict. Combining (24) and (25) yields:

$$V(k) \geq \frac{\log\left(\frac{1}{\alpha}\right)}{1-\beta} + V(\alpha k) > \frac{\log\left(\frac{1}{\alpha}\right)}{1-\beta} + \frac{\log(\alpha)}{1-\beta} + V(k) = V(k),$$

a contradiction. Thus, $V(k) = \log(k)/(1-\beta) + V(1)$. ■

Proof of Proposition 2. We first prove that $\lambda(\rho)$ is strictly less than the unconstrained optimum. Equation (6) of Proposition 1 shows that, with log utility ($s = 1$), the social optimum is $\lambda = \beta\phi - (1-\beta)\rho$. Inserting this into equation (11) implies that the left-hand side is zero. We next show that the right-hand side of (11), which represents the marginal cost of increasing λ, is strictly positive. Equation (11) implies that $\lambda > 0$ an optimum. Thus, the dynamic payoff from investing

$\lambda > 0$ exceeds the payoff from investing nothing. The cost to party b of losing the election is $\log((\phi-\lambda)/\phi)+(\beta/1-\beta)\log((\rho+\lambda)/\rho) > 0$. Thus, the marginal cost is strictly positive. Therefore, it is optimal to lower λ from the social optimum, i.e., to set $\lambda(\rho) < \beta\phi - (1-\beta)\rho$ for all $0 < \rho \leq 1$.

Next, we show that $\lim_{\rho\downarrow 0}\lambda(\rho) = 0$. Let $\rho \downarrow 0$. Suppose by way of contradiction that λ remains bounded away from zero. Then there exists a sequence ρ_n, $n \in \mathbb{N}$ such that the associated levels, λ_n converge to $\bar{\lambda} > 0$. As $n \to \infty$, the left-hand side of (11) converges to $G(log((\phi - \bar{\lambda})/\phi))\left(\beta\phi - \bar{\lambda}\right)$. In contrast, the right-hand side of (11) goes to ∞ because $\log((\rho_n + \lambda_n)/\rho_n) \to \infty$, while $\lim_{n\to\infty} g(D)(\rho+\lambda) = g(log((\phi-\bar{\lambda})/\phi))\bar{\lambda}$. Thus, the right-hand side of (11) exceeds the left-hand side for all sufficiently large n, a contradiction to the assumption that λ_n satisfies the first-order condition for ρ_n. Thus, $\lambda(\rho)$ must converge to zero as $\rho \downarrow 0$.

Next, we show that $\lim_{\rho\downarrow 0}\lambda(\rho)/\rho = \infty$. The left-hand side of (11), i.e., the marginal benefit of saving converges to $G(0)\beta\phi$ as $\rho \downarrow 0$, which is strictly positive. Thus, the right-hand side of (11), i.e., the marginal cost, must also be non-zero in the limit. Again, note that D converges to 0 as $\rho \downarrow 0$. Next, because $\lim_{\rho\downarrow 0}\lambda(\rho) = 0$ it follows that $\log((\phi-\lambda(\rho))/\phi)$ goes to zero. Thus, $(\rho+\lambda(\rho))\log((\rho+\lambda(\rho))/\rho)$ must be non-zero in the limit. Given that both ρ and $\lambda(\rho)$ go to zero, it follows that $\log((\rho+\lambda(\rho))/\rho) = \log(1+\lambda(\rho)/\rho)$ goes to infinity. Thus, $\lambda(\rho)/\rho$ goes to infinity. Taking the limit on both sides of (11) as $\rho \downarrow 0$ therefore yields

$$G(0)\beta\phi = g(0)\beta \lim_{\rho\downarrow 0}\lambda(\rho)\log\left(\frac{\rho+\lambda(\rho)}{\rho}\right).$$

Thus,

$$\begin{aligned} G(0)\phi/g(0) &= \lim_{\rho\to 0}\lambda(\rho)\log\left(\frac{\rho+\lambda(\rho)}{\rho}\right) = \lim_{\rho\to 0}\lambda(\rho)\log\left(\frac{\lambda(\rho)}{\rho}\right) \\ &= \lim_{\rho\to 0}\lambda(\rho)\log(\lambda(\rho)) - \lim_{\rho\to 0}\lambda(\rho)\log(\rho) = -\lim_{\rho\to 0}\lambda(\rho)\log(\rho), \end{aligned}$$

where we have used the fact that $\lim_{x\to 0} x\log x = 0$. Thus,

$$\lim_{\rho\to 0}\frac{\lambda(\rho)}{\phi}\left(-\frac{g(0)}{G(0)}\log(\rho)\right) = 1.$$

That is, $\lambda(\rho)$ goes to zero at the indicated rate. ■

Proof of Proposition 3. We first prove the result for Linear Markov Perfect Equilibria. Let $V_P(k;\lambda_{-P})$ be the value function of party $P = b, d$, when the initial capital stock is k and the other party uses a linear strategy $k' \mapsto \lambda_{-P}\,k'$. Then, (λ_b, λ_d) characterize a linear MPE if and only if they solve the following optimization problems for all values of $k > 0$:

$$\begin{aligned} V_b(k;\lambda_d) = \max_{\lambda_b\in[0,\phi]} & G\left(\log\left(\frac{\phi-\lambda_b}{\phi-\lambda_d}\right)\right)(\log((\phi-\lambda_b)k) + \beta V_b((\rho-\lambda_b)k;\lambda_d)) \\ & + \left(1 - G\left(\log\left(\frac{\phi-\lambda_b}{\phi-\lambda_d}\right)\right)\right)(\log((\phi-\lambda_d)k) + \beta V_b((\rho-\lambda_d)k;\lambda_d)). \end{aligned}$$

$$V_d(k;\lambda_b) = \max_{\lambda_d\in[0,\phi]} G\left(\log\left(\frac{\phi-\lambda_b}{\phi-\lambda_d}\right)\right)(\beta V_d((\rho-\lambda_b)k;\lambda_b)) + \left(1-G\left(\log\left(\frac{\phi-\lambda_b}{\phi-\lambda_d}\right)\right)\right)(1+\beta V_d((\rho-\lambda_d)k;\lambda_b)).$$

Expanding d's value function reveals that it does not depend on the initial capital k:

$$V_d(k;\lambda_b) = \max_{\lambda_d\in[0,\phi]} \sum_{t=0}^{\infty}\beta^t\Big(1-G\Big(\log\Big(\frac{\phi-\lambda_b}{\phi-\lambda_d}\Big)\Big)\Big).$$

Thus, d's strategy is not to invest at all, i.e., to choose $\lambda_d = 0$. But from our analysis of the myopic case, if d does not invest, investment strategies are linear. It remains to show that, if party b chooses a linear strategy, then d's best response is also linear. Using the scalability argument from before, we show that, in fact, if party b chooses a linear strategy, then d's best response is not to invest. Let $V_d(k)$ be d's value function when the initial capital stock is k, and party b uses a linear strategy $\lambda_b k$.

$$V_d(\overline{k}) = \max_{\{i_t^d\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\Big(1-G\Big(\log\Big(\frac{(\phi-\lambda_b)k_{t-1}}{\phi k_{t-1}-i_t^d}\Big)\Big)\Big).$$

Let $\tilde{i}^d$ be d's best response. Now let initial capital be $\alpha\overline{k}$, and consider a strategy of $\hat{i}_t^d = \alpha\tilde{i}^d$. Then,

$$V_d(\alpha\overline{k}) \geq \sum_{t=0}^{\infty}\Big(1-G\Big(\log\Big(\frac{(\phi-\lambda_b)k_{t-1}}{\phi k_{t-1}-\tilde{i}_t^d}\Big)\Big)\Big) = V_d(\overline{k}).$$

Now, use $\frac{1}{\alpha}$ instead of α, and $\alpha\overline{k}$ instead of $\overline{k}$ to get $V_d(\overline{k}) \geq V_d(\alpha\overline{k})$. Thus, $V_d(\overline{k}) = V_d(\alpha\overline{k})$, and hence $V_d(k) = V_d(1)$: if b uses a linear strategy, the demagogue's payoff does not depend on the capital stock. Thus, the future capital stock is irrelevant for the demagogue, so the problem reduces to maximizing the probability of election in the current period. Therefore, the demagogue does not invest. This proves the result for linear Markov perfect equilibria.

Next, we show that our results with a myopic demagogue also hold for finite horizon economies. The game begins at date $t = 0$ with initial capital $k_{-1} = \overline{k}$, and ends at date $T \geq 1$. We proceed by backward induction. At date T, both parties have a dominant strategy not to invest. Thus,

$$i_T^d = i_T^b = 0, \text{ and } V_T(k_{T-1}) = \log(\phi k_{T-1}),$$

where V_T is party b's value function at date T. At date $T-1$, d does not invest because it hurts his current payoff, and it does not change his future payoff. To analyze party b's optimization problem at $T-1$, let $V_{T-1}(k_{T-2})$ be b's value function at date $T-1$, and define $\lambda_{T-1} = i_{T-1}^b/k_{T-2}$. Then

$$\begin{aligned} V_{T-1}(k_{T-2}) = \max_{\lambda_{T-1}\in[0,\phi]} & G\Big(\log\Big(\frac{\phi-\lambda_{T-1}}{\phi}\Big)\Big)\Big(\log((\phi-\lambda_{T-1})k_{T-2}) + \beta V_T((\rho-\lambda_{T-1})k_{T-2})\Big) \\ & + \Big(1-G\Big(\log\Big(\frac{\phi-\lambda_{T-1}}{\phi}\Big)\Big)\Big)\Big(\log(\phi k_{T-2}) + \beta V_T(\rho k_{T-2})\Big). \end{aligned}$$

Using the fact that $V_T(k_{T-1}) = \log(\phi k_{T-1})$ yields

$$\begin{aligned} V_{T-1}(k_{T-2}) = (1+\beta)\log(k_{T-2}) + \max_{\lambda_{T-1}\in[0,\phi]} & G\Big(\log\Big(\frac{\phi-\lambda_{T-1}}{\phi}\Big)\Big)(\log(\phi-\lambda_{T-1}) + \beta\log(\rho-\lambda_{T-1})) \\ & + \Big(1-G\Big(\log\Big(\frac{\phi-\lambda_{T-1}}{\phi}\Big)\Big)\Big)(\log(\phi) + \beta\log(\rho)). \end{aligned} \tag{26}$$

Thus, the optimal λ_{T-1} depends on the model parameters (ρ, ϕ, β), but not on capital. Let $\tilde{V}_{T-1}$ be the value of the maximization problem on the right-hand side of (26). Then

$$V_{T-1}(k_{T-2}) = (1+\beta)\log(k_{T-2}) + \tilde{V}_{T-1}(\beta, \phi, \rho). \tag{27}$$

Next, consider date $T-2$. Again, d chooses $i^d_{T-2} = 0$ because it maximizes his current period payoff (i.e., the winning probability), and his future payoffs do not depend on the capital stock, because party b's behavior, and hence d's winning probabilities do not depend on the capital stock. Now, consider b's optimization at date $T-2$. We repeat the above procedure.

$$\begin{aligned} V_{T-2}(k_{T-3}) = \max_{\lambda_{T-2}\in[0,\phi]} & G\Big(\log\Big(\frac{\phi-\lambda_{T-2}}{\phi}\Big)\Big)(\log((\phi-\lambda_{T-2})k_{T-3}) + \beta V_{T-1}((\rho+\lambda_{T-2})k_{T-3})) \\ & + \Big(1 - G\Big(\log\Big(\frac{\phi-\lambda_{T-2}}{\phi}\Big)\Big)\Big)(\log(\phi k_{T-3}) + \beta V_{T-1}(\rho k_{T-3}). \end{aligned} \tag{28}$$

We substitute (27) into (28), factoring out the term $\log(k_{T-3})$, and rearrange to obtain

$$\begin{aligned} V_{T-2}(k_{T-3}) = {} & (1+\beta+\beta^2)\log(k_{T-3}) + \beta\tilde{V}_{T-1} \\ & + \max_{\lambda_{T-2}\in[0,\phi]} G\Big(\log\Big(\frac{\phi-\lambda_{T-2}}{\phi}\Big)\Big)(\log(\phi-\lambda_{T-2}) + \beta(1+\beta)\log(\rho+\lambda_{T-2})) \\ & + \Big(1 - G\Big(\log\Big(\frac{\phi-\lambda_{T-2}}{\phi}\Big)\Big)\Big)(\log(\phi) + \beta(1+\beta)\log(\rho)). \end{aligned} \tag{29}$$

Thus, the optimal λ_{T-2} does not depend on capital. Define $\tilde{V}_{T-2}$ to be the value of the maximization problem on the right-hand side of (29). Then

$$V_{T-2}(k_{T-3}) = (1+\beta+\beta^2)\log(k_{T-3}) + \tilde{V}_{T-2}(\beta, \phi, \rho) + \beta\tilde{V}_{T-1}(\beta, \phi, \rho).$$

Continuing inductively implies

$$V_{T-n}(k_{T-(n+1)}) = (1+\beta+\cdots+\beta^n)\log(k_{T-(n+1)}) + \tilde{V}_{T-n} + \beta\tilde{V}_{T-(n-1)} + \cdots + \beta^{n-1}\tilde{V}_{T-1},$$

where

$$\begin{aligned} \tilde{V}_{T-k} = \max_{\lambda_{T-k}\in[0,\phi]} & G\Big(\log\Big(\frac{\phi-\lambda_{T-k}}{\phi}\Big)\Big)\Big(\log(\phi-\lambda_{T-k}) + \beta\sum_{i=0}^{k-1}\beta^i\log(\rho+\lambda_{T-k})\Big) \\ & + \Big(1 - G\Big(\log\Big(\frac{\phi-\lambda_{T-k}}{\phi}\Big)\Big)\Big)\Big(\log(\phi) + \beta\sum_{i=0}^{k-1}\beta^i\log(\rho)\Big). \end{aligned} \tag{30}$$

Choose $n = T$ to get

$$V_0(k_{-1}; T) = \sum_{t=0}^{T}\beta^t\log(k_{-1}) + \sum_{t=0}^{T-1}\beta^t\tilde{V}_t.$$

Note that (30) implies that for given β, ϕ and ρ there exists $M > 0$ such that $|\tilde{V}_{T-k}| < M$ for all T and k. Thus, $v = \lim_{T\to\infty}\sum_{t=0}^{T-1}\beta^t\tilde{V}_t$ exists. Recall that $k_{-1} = \overline{k}$. Then

$$V(\overline{k}) = \lim_{T\to\infty} V_0(\overline{k}; T) = \frac{\log(\overline{k})}{1-\beta} + v.$$

Thus, we get the same value function as when d is myopic, establishing the result. ■

Proof of Proposition 4. We construct trigger strategies, in which the two parties offer strategies $\lambda_b(k)$ and $\lambda_d(k)$, unless a party deviates, in which case the parties revert to the unique linear Markov equilibrium. If $D = \log(\phi - \lambda_b(k)) - \log(\phi - \lambda_d(k))$, then d's wins with probability $1 - G(D)$. Let $\hat{\lambda}_b$, $\hat{\lambda}_d = 0$ be the linear Markov equilibrium strategies. Suppose by way of contradiction that there exists a sequence of discount factors β_n and a sequence of strategies $\lambda_{b,n}(k)$ and $\lambda_{d,n}(k)$ that can be supported as a trigger-strategy equilibrium, which converges to the first-best strategies, $\lim_{n\to\infty} \lambda_{b,n}(k) = \lim_{n\to\infty} \lambda_{d,n}(k) = \lambda^*$. Then d's winning probability would converge to $1 - G(0)$.

Let $\hat{\lambda}_{b,n}$, and $\hat{\lambda}_{d,n} = 0$ be the linear Markov equilibrium strategies given discount factor β_n. Let $\hat{D}_n = \log(\phi - \hat{\lambda}_{b,n}) - \log(\phi)$. Given our characterization of the linear Markov equilibrium there exists $\kappa > 0$ such that $\hat{\lambda}_{b,n} \geq \kappa$ for all n. From Assumption 1, $0 < G(0) < 1$ and the density $g(0) > 0$. But then, $\limsup_n 1 - G(\hat{D}_n) < 1 - G(0)$, i.e., d's winning probability would be higher on the 'punishment' path where linear Markov equilibrium strategies are played. However, this means that d would strictly better off deviating to investing nothing for sufficiently large n, a contradiction.

Next observe that (i) $\lim_{n\to\infty} \lambda_{d,n}(k) \neq \lim_{n\to\infty} \lambda_{d,n}(k) = \lambda^*$ violates Assumption 3; (ii) if $\lim_{n\to\infty} \lambda_{d,n}(k) > \lim_{n\to\infty} \lambda_{d,n}(k) = \lambda^*$, then d would want to deviate to investing nothing; and (iii) if $\lim_{n\to\infty} \lambda_{d,n}(k) < \lim_{n\to\infty} \lambda_{d,n}(k) = \lambda^*$, then with strictly positive probability, the demagogue wins and the first-best is not achieved. Thus, the first-best cannot be obtained in the limit. ■

Proof of Proposition 5. The first-order condition describing d's optimal choice of λ_d is:

$$-g(D)\left(\frac{1}{\phi - \lambda_d} - \frac{\beta_m}{(1-\beta_m)(\rho + \lambda_d)}\right) \leq 0, \tag{31}$$

where $\lambda_d = 0$ if the inequality is strict. d can ensure a winning probability of at least $1 - G(0)$ by selecting b's strategy. By assumption $g(D) > 0$, so d's optimal investment is

$$\lambda_d = \max\Big\{\beta_m\phi - (1-\beta_m)\rho, 0\Big\}. \tag{32}$$

The first-order condition describing party b's optimal choice of λ_b is:

$$\begin{aligned} &G(D)\left(-\frac{1}{\phi - \lambda_b} + \frac{\beta}{(1-\beta)(\rho + \lambda_b)}\right) \\ &= g(D)\left(\frac{1}{\phi - \lambda_b} - \frac{\beta_m}{(1-\beta_m)(\rho + \lambda_b)}\right)\left(\log\left(\frac{\phi - \lambda_b}{\phi - \lambda_d}\right) + \frac{\beta}{1-\beta}\, \log\left(\frac{\rho + \lambda_b}{\rho + \lambda_d}\right)\right). \end{aligned} \tag{33}$$

For all β_m sufficiently small, $\lambda_d = 0$, implying that as $\beta_m \to 0$, the first-order condition (33) describing b's optimal investment level converges to that when the median voter is fully myopic. Hence, for all sufficiently small β_m, the equilibrium is close to the myopic equilibrium.

Adding the assumption that G is log-concave, write (33) as

$$\frac{G(D)}{g(D)}\left(-\frac{1}{\phi-\lambda_b}+\frac{\beta}{(1-\beta)(\rho+\lambda_b)}\right)=\left(\frac{1}{\phi-\lambda_b}-\frac{\beta_m}{(1-\beta_m)(\rho+\lambda_b)}\right)\left(\log\left(\frac{\phi-\lambda_b}{\phi-\lambda_d}\right)+\frac{\beta}{1-\beta}\ \log\left(\frac{\rho+\lambda_b}{\rho+\lambda_d}\right)\right). \tag{34}$$

By log-concavity, the left-hand side is increasing in D, and D is increasing in $\beta_m < \beta$ for fixed λ_b. For $\beta_m < \frac{\rho}{\phi+\rho}$, $\lambda_d = 0$. It follows immediately that the right-hand side of (34) is decreasing in β_m, while the left-hand side is increasing, i.e., the marginal benefit of increased investment now exceeds the marginal cost. As the left-hand side is decreasing in λ_b, while the right-hand side is increasing, it follows that λ_b must increase to equate both sides.

Lastly, we show that for $\beta_m \geq \frac{\rho}{\phi+\rho}$, the right-hand side decreases in β_m. Write the RHS as:

$$\frac{1}{1-\beta}\left(\frac{1}{\phi-\lambda_b}-\frac{\beta_m}{(1-\beta_m)(\rho+\lambda_b)}\right)\left((1-\beta)\log\left(\frac{\phi-\lambda_b}{(1-\beta_m)(\rho+\phi)}\right)+\beta\log\left(\frac{\rho+(1-\beta)\lambda_b}{(1-\beta_m)(\rho+\phi)}\right)\right)$$

$$=\frac{1}{1-\beta}\left(\frac{1}{\phi-\lambda_b}-\frac{\beta_m}{(1-\beta_m)(\rho+\lambda_b)}\right)\Big((1-\beta)\log(\phi-\lambda_b)+\beta\log(\rho+(1-\beta)\lambda_b)-\log((1-\beta_m)(\rho+\phi))\Big).$$

The derivative with respect to β_m has sign

$$\frac{-1}{(1-\beta_m)(\rho+\lambda_b)}\quad\Big(\quad(1-\beta)\log(\phi-\lambda_b)+\beta\log(\rho+(1-\beta)\lambda_b)-\log((1-\beta_m)(\rho+\phi))\Big)$$
$$+\quad\left(\frac{1}{\phi-\lambda_b}-\frac{\beta_m}{(1-\beta_m)(\rho+\lambda_b)}\right)$$

The first line is negative; and so is the second since $\lambda_b > \lambda_d$, and λ_d sets the second line to zero. ∎

Proof of Lemma 2. Pick $k', k > 0$, and without loss of generality, suppose $k' < k$. Let $\alpha = k'/k \in (0,1)$. If $\lambda(k) = \lambda(k')$, then $\alpha^{1-s}V(k) \geq V(\alpha k)$. To see this, observe that if the winning probabilities remained unchanged, then $V(\alpha k) = V(k') = \alpha^{1-s}V(k)$. However, the winning probability is increasing in k, and hence $V(\alpha k) = V(k') \leq \alpha^{1-s}V(k)$. Then,

$$\liminf_{k'\to k}\frac{V(k)-V(k')}{k-k'}\geq\liminf_{\alpha\to 1}\frac{V(k)-\alpha^{1-s}V(k)}{(1-\alpha)k}=\lim_{\alpha\to 1}\frac{V(k)}{k}\frac{1-\alpha^{1-s}}{1-\alpha}=\frac{(1-s)V(k)}{k}$$
$$\geq\frac{(1-s)V_N(k)}{k}=V_N'(k),$$

where the last equality follows from equation (16). ∎

Proof of Lemma 3. Assume by way of contradiction that $\lambda(k)$ does not converge to λ^* as $k \to \infty$. Then, there exists a sequence $k_n \to \infty$ with $\lim_{n\to\infty}\lambda(k_n) \neq \lambda^*$. Without loss of generality, suppose that $\lambda(k_n) < \bar{\lambda} < \lambda^*$ for some $\bar{\lambda}$ (the analysis when $\bar{\lambda} > \lambda^*$ is analogous). From (18),

$$k^{s-1}V_b(k)=\max_{\lambda\in[0,\phi]}G(0)\Big(k^{s-1}u((\phi-\lambda)k)+\beta k^{s-1}V_b((\rho+\lambda)k)\Big)+(1-G(0))\Big(k^{s-1}u(\phi k)+\beta k^{s-1}V_b(\rho k)\Big). \tag{35}$$

Recalling that $V_b(k) = \frac{v_b}{1-s}\, k^{1-s}$ and $i(k) = \lambda k$ yields that the objective function in (35) is strictly concave in λ and does not depend on k. The optimality of $i = \lambda^* k$ plus the strict concavity imply

$$\begin{aligned} & G(0)\Big(k_n^{s-1}u((\phi-\lambda(k_n))k_n)+\beta k_n^{s-1}V_b((\rho+\lambda(k_n))k_n)\Big)+(1-G(0))\Big(k_n^{s-1}u(\phi k_n)+\beta k_n^{s-1}V_b(\rho k_n)\Big) \\ < \;& G(0)\Big(k_n^{s-1}u((\phi-\lambda^*)k_n)+\beta k_n^{s-1}V_b((\rho+\lambda^*)k_n)\Big)+(1-G(0))\Big(k_n^{s-1}u(\phi k_n)+\beta k_n^{s-1}V_b(\rho k_n)\Big) \quad (36) \end{aligned}$$

Note that k_n, outside $\lambda(k_n)$, cancels from both sides of (36). Thus, there is a $\delta > 0$ such that for all k_n:

$$\begin{aligned} & G(0)\Big(k_n^{s-1}u((\phi-\lambda(k_n))k_n)+\beta k_n^{s-1}V_b((\rho+\lambda(k_n))k_n)\Big)+(1-G(0))\Big(k_n^{s-1}u(\phi k_n)+\beta k_n^{s-1}V_b(\rho k_n)\Big)+\delta \\ < \;& G(0)\Big(k_n^{s-1}u((\phi-\lambda^*)k_n)+\beta k_n^{s-1}V_b((\rho+\lambda^*)k_n)\Big)+(1-G(0))\Big(k_n^{s-1}u(\phi k_n)+\beta k_n^{s-1}V_b(\rho k_n)\Big) \quad (37) \end{aligned}$$

From (17),

$$k^{s-1}V(k) = \max_{\lambda\in[0,\phi]} G(D_k)\Big(k^{s-1}u((\phi-\lambda)k)+\beta k^{s-1}V((\rho+\lambda)k)\Big)+(1-G(D_k))\Big(k^{s-1}u(\phi k)+\beta k^{s-1}V(\rho k)\Big), \tag{38}$$

where

$$D_k = u((\phi-\lambda)k)-u(\phi k) = k^{1-s}\,\frac{(\phi-\lambda)^{1-s}-\phi^{1-s}}{1-s}.$$

If $G(D_k)$ is arbitrarily close to $G(0)$ and $k^{s-1}V(k)$ is arbitrarily close to $k^{s-1}V_b(k)$, then (37) implies

$$\begin{aligned} & G(D_{k_n})\Big(k_n^{s-1}u((\phi-\lambda(k_n))k_n)+\beta k_n^{s-1}V((\rho+\lambda(k_n))k_n)\Big)+(1-G(D_{k_n}))\Big(k_n^{s-1}u(\phi k_n)+\beta k_n^{s-1}V(\rho k_n)\Big) \\ < \;& G(D_{k_n})\Big(k_n^{s-1}u((\phi-\lambda^*)k_n)+\beta k_n^{s-1}V((\rho+\lambda^*)k_n)\Big)+(1-G(D_{k_n}))\Big(k_n^{s-1}u(\phi k_n)+\beta k_n^{s-1}V(\rho k_n)\Big). \end{aligned}$$

This contradicts that $\lambda(k_n)$ is the optimal investment share given k_n in (38). This contradiction proves that $\lim_{k\to\infty}\lambda(k_n) = \lambda^*$. It remains to show that when k is arbitrarily large, $G(D_k)$ is arbitrarily close to $G(0)$ and $k^{s-1}V(k)$ is arbitrarily close to $k^{s-1}V_b(k)$. We proceed in four steps.

Step 1. There exists $\hat{\lambda} < \phi$ and $\bar{k} > 0$ such that $\lambda(k) < \hat{\lambda}$ for all $k \geq \bar{k}$. Suppose to the contrary that there exists a sequence $k_n \to \infty$ such that $\lim_{n\to\infty}\lambda(k_n) = \phi$. Recall that $V_N(k)$ is the expected discounted payoff from no investment. The net benefit of investment contingent on winning is

$$u((\phi-\lambda)k_n)-u(\phi k_n)+\beta\left(V((\rho+\lambda)k_n)-V(\rho k_n)\right) < u((\phi-\lambda)k_n)-u(\phi k_n)-\beta V_N(\rho k_n). \tag{39}$$

The inequality follows since utility is negative and $V_N(k) \leq V(k)$. Note that

$$\lim_{n\to\infty} u((\phi-\lambda)k_n)-u(\phi k_n)-\beta V_N(\rho k_n) = \lim_{n\to\infty}\frac{1}{(1-s)k_n^{s-1}}\left((\phi-\lambda(k_n))^{1-s}-\phi^{1-s}-\frac{\beta\phi^{1-s}}{1-\beta\rho^{1-s}}\right) < 0,$$

because $\lim_{n\to\infty}\lambda(k_n) = \phi$ implies $\lim_{n\to\infty}(\phi-\lambda(k_n))^{1-s} = -\infty$, and the other terms are bounded. This implies that the net benefit from investment given by (39) is strictly negative. By choosing $\lambda = 0$ candidate b can guarantee a higher payoff, contradicting the optimality of $\lambda(k_n)$.

Step 2. Recall that $D_k = k^{1-s}\,\frac{(\phi-\lambda)^{1-s}-\rho^{1-s}}{1-s}$. Moreover, Step 1 showed that $\lambda(k) < \hat{\lambda} < \phi$ for all sufficiently large k. Thus, $\lim_{k\to\infty}|G(D_k)-G(0)| = 0$.

Step 3. The social planner's value function takes the form $V_S(k) = \frac{v_S}{(1-s)}k^{1-s}$. Similarly, the expected discounted payoff if no investment takes place takes the form $V_N(k) = \frac{v_N}{(1-s)}\, k^{1-s}$. The value function of (17) is bounded from above and below by these two functions. Therefore,

$$\frac{v_N}{1-s} = k^{s-1}V_N(k) \le k^{s-1}V(k) \le k^{s-1}V_S(k) = \frac{v_S}{1-s}. \tag{40}$$

Thus, for all k, k', k'',

$$\begin{aligned} |k^{s-1}V(k') - k^{s-1}V_b(k'')| &\le |k^{s-1}V(k')| + |k^{s-1}V_b(k'')| = \left(\frac{k}{k'}\right)^{s-1} |k'^{s-1}V(k')| + \left(\frac{k}{k''}\right)^{s-1} |k''^{s-1}V_b(k'')| \\ &\le \left(\left(\frac{k}{k'}\right)^{s-1} + \left(\frac{k}{k''}\right)^{s-1}\right) \left|\frac{v_N}{1-s}\right|. \end{aligned} \tag{41}$$

Now, suppose the capital stock at $t = 0$ is k, and the capital stock at date T is k_T. Then, $\frac{k}{k_T} \le \frac{k}{\rho^T k} = \frac{1}{\rho^T}$. Let k'_T be the capital stock at date T of the program (17), and let k''_T be the capital stock at date T of the program (18). Then, from (41),

$$\begin{aligned} |k^{s-1}\beta^T V(k'_T) - k^{s-1}\beta^T V_b(k''_T)| &\le \beta^T \left(\left(\frac{k}{k'_T}\right)^{s-1} + \left(\frac{k}{k''_T}\right)^{s-1}\right) \left|\frac{v_N}{1-s}\right| \le \beta^T\, 2\left(\frac{1}{\rho^T}\right)^{s-1} \left|\frac{v_N}{1-s}\right| \\ &= (\beta\rho^{1-s})^T\, 2\left|\frac{v_N}{1-s}\right|. \end{aligned} \tag{42}$$

By Assumption 2, $\beta\rho^{1-s} < 1$, so a sufficiently large T, makes the difference $|k^{s-1}\beta^T V(k'_T) - k^{s-1}\beta^T V_b(k''_T)|$ arbitrarily small. Thus, the difference in the tail of expected discounted payoffs, $|k^{s-1}V(k) - k^{s-1}V_b(k)|$, becomes arbitrarily small when T is sufficiently large, independent of k.

Step 4. Finally, we show that the difference in the first $T+1$ periods of $t = 0, \cdots, T$ is small when k is sufficiently large. From step 1, $\max\{\lambda^*, \bar{\lambda}\} < \phi$. Define f and h as functions of $(\lambda_0, \cdots, \lambda_T) \in [0, \max\{\lambda^*, \bar{\lambda}\}]^{T+1}$,

$$\begin{aligned} f(\lambda_0, \cdots, \lambda_T) &= k^{s-1}\sum_{t=0}^{T} \beta^t (G(0)u((\phi - \lambda_t)k_{t-1}) + (1 - G(0))\, u(\phi k_{t-1})) \\ h(\lambda_0, \cdots, \lambda_T) &= k^{s-1}\sum_{t=0}^{T} \beta^t (G(D_{k_{t-1}})u((\phi - \lambda_t)k_{t-1}) + (1 - G(D_{k_{t-1}}))\, u(\phi k_{t-1})), \end{aligned}$$

and suppose $k_{-1} = k$ evolve as specified in the model. f and h are bounded. From Step 2 it follows that for every $\varepsilon > 0$, there exists $\tilde{k}$ such that $|f(\lambda_0, \cdots, \lambda_T) - h(\lambda_0, \cdots, \lambda_T)| < \varepsilon$ for all $k \ge \tilde{k}$.

Let $\lambda^* = (\lambda^*, \ldots, \lambda^*)$ maximize f, and let $\hat{\lambda} = (\hat{\lambda}_0, \ldots, \hat{\lambda}_T)$ maximize h. Then

$$f(\lambda^*) \ge f(\hat{\lambda}) \ge h(\hat{\lambda}) - \varepsilon \ge h(\lambda^*) - \varepsilon \ge f(\lambda^*) - 2\varepsilon.$$

Thus, $|f(\lambda^*) - h(\hat{\lambda})| < 2\varepsilon$, i.e., the difference in utility over the first T periods can be made arbitrarily small by an appropriately large choice of k. This completes the proof. ∎