Operating Hedge and Gross Profitability Premium *

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Abstract

In this paper we explore the hedging effect induced by variable costs in production, and its impact on fundamental risk of firm cash flows and stock returns. The hedging effect varies across firms and is weaker for more profitable firms. This leads to more profitable firms having a higher exposure to aggregate profitability shocks, giving rise to a gross profitability premium. Our model captures coexistence of the negatively correlated gross profitability and value factors, addressing an empirical pattern that poses a challenge to the models relying on operating leverage as the primary source of the value premium.

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1 Introduction

Historically, firms with high gross profitability generate higher future stock returns than firms with low gross profitability. Using quintile portfolio sorts, Novy-Marx (2013) documents the spread of 0.31% per month (3.78% annually) between average returns on the more profitable firms relative to the less profitable firms. This pattern goes against the operating leverage mechanism, invoked by several popular models of the value premium, e.g., Zhang (2005), Carlson, Fisher, and Giammarino (2004), Novy-Marx (2010). In this paper, we show that firms with different levels of gross profitability exhibit heterogeneous cash flow risk, which can be explained by an operating hedge property of the production function. This mechanism is distinct from operating leverage, but follows a similarly basic logic. Some of the production costs faced by firms are relatively stable (fixed costs), and tend to magnify the impact of revenue shocks on firms' cash flows, giving rise to operating leverage. In contrast, other costs (variable costs) can be highly cyclical. Such cyclical input costs tend to lower the net risk of firms' cash flows, creating an operating hedge. This effect is more pronounced for the firms with a higher level of variable costs relative to revenue, i.e., the firms with a lower level of gross profitability. For more profitable firms, the hedging effect of variable costs is weaker, and hence such firms exhibit higher cash flow risk, and higher average returns.

We embed a production function with constant elasticity of substitution (CES) between capital and variable inputs (including energy, raw materials, semi-finished goods, services, production-related labor, etc.) into a structural asset pricing model. A stylized static model conveys the core intuition. Exposed to both aggregate and firm-specific profitability shocks,[1] firms use their capital and purchase variable inputs to produce output. If 1) the price of variable inputs is highly procyclical with respect to aggregate profitability, and 2) the physical capital and variable inputs are complements in the production function, a negative aggregate profitability shock leads to a larger

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[1] Because our model is set in partial equilibrium, we do not spell out the exact origins of the aggregate profitability shock affecting the entire population of firms. These shocks reflect both the technological shocks affecting profitability across all firms, as well as aggregate "demand" shocks, which may originate as shocks to investor beliefs or tastes, or be driven by government spending shocks. What is important for our purposes is that such common profitability shocks create correlated movements in firm profits relative to their output.

proportional reduction in variable costs than in gross output. Thus, variation in variable costs serves as a hedge against aggregate profitability shocks. Furthermore, when the second condition holds, variable costs vary less than firm's revenue in response to firm-specific profitability shocks, so that strength of the hedging effect declines in firm's gross profitability. More profitable firms thus have higher exposure to the aggregate profitability shock relative to less profitable firms. With the aggregate profitability shock carrying a positive price of risk, heterogeneity in firms' cash flow risk induced by the hedging property of variable costs gives rise to a positive gross profitability premium.

Our empirical analysis supports operating hedge as an important source of cross-sectional variation in firms' cash flow risk, particularly in relation to gross profitability. First, using the data on manufacturing industries from National Bureau of Economic Research (NBER) - U.S. Census Bureau's Center for Economic Studies (CES), we find that the aggregate variable costs across manufacturing firms are more volatile than aggregate revenue. Moreover, at the aggregate level, the elasticity of cost with respect to revenue is above one. These findings suggest that variable costs offer a hedge against systematic shocks to firm revenue. This is also a sufficient condition in the model for a positive gross profitability premium. Second, the firm-level gross profit has a higher volatility relative to the firm-level revenue – the opposite of the aggregate-level relation. Furthermore, the elasticity of firm-level profits with respect to sales is above one, indicating that variable costs create an operating leverage effect in response to firm-specific profitability shocks. Interpreted through the lens of our model, this result also implies that the elasticity of substitution between capital and variable inputs is less than one. Third, we use the long-short portfolio based on GP/A quintiles within the Fama and French 30 industries, and the utilization-adjusted total factor productivity shock from Basu, Fernald, and Kimball (2006) and Fernald (2014) as proxies for aggregate profitability shocks.[2] We find that firms with different gross profitability differ in their risk exposures, both in their cash flows and stock returns.

We extend our stylized static model to a dynamic setting to address the coexistence of two

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[2] Both measures strongly comove with macroeconomic variables such as GDP growth and aggregate consumption growth, and carries a positive price of risk, based on the cross-section of industry portfolios constructed by Fama and French.

distinct factors, the gross profitability factor and the value factor (the value-growth return spread), as well as their negative correlation with each other, as highlighted in Novy-Marx (2013). Besides the aggregate and firm-level profitability shocks, we introduce the aggregate and idiosyncratic investment-specific technology (IST) shocks. Prior studies, e.g., Kogan and Papanikolaou (2013), Kogan and Papanikolaou (2014), emphasize that stock prices of firms with a higher value of growth opportunities relative to the value of their assets in place are more exposed to the aggregate IST shocks, and earn lower risk premia. Such firms tend to have higher market-to-book ratios, i.e., they are the “growth” firms. The same mechanism is at work in our model, which means that the value factor in the model loads heavily on the aggregate IST shocks, and is distinct from the profitability factor, which loads heavily on aggregate profitability shocks. To account for the negative correlation between the two factors, we further allow idiosyncratic shocks to firm profitability and growth opportunities to be positively correlated, which means that more profitable firms tend to have better investment opportunities. This parametric assumption allows us to control the correlation between the profitability and value factors in the model: more profitable firms tend to have higher loadings on the aggregate IST shocks, which is the opposite of the value factor; similarly, growth firms tend to be more profitable, loading more heavily on the aggregate profitability shocks, relative to value firms. We should note that, all else equal, the negative correlation between the value and profitability factors in the model drives down the average return premia on the two factors, as they exhibit opposite exposures to the same aggregate economic shocks.

In calibration, our model generates the annualized gross profitability premium (based on the quintile sorts on gross profitability, with value-weighted portfolios) of 3.26%, relative to 3.83% in the data in our extended sample period. Similarly, the value premium is 3.21% in the model, relative to 4.71% in the data. The correlation between the profitability factor and the value factor is -0.25 in the model versus -0.43 in the data. Related to this correlation pattern, the model generates a larger average return spread among profitability-sorted portfolios when including value factor in the asset pricing tests. More generally, our model replicates the failure of the CAPM to price the cross-sections of stocks sorted on gross profitability, and on book-to-market ratio.

Relation to prior literature

Our study uncovers a novel hedging effect induced by the cyclicality of variable costs that have not been explored in the existing literature. Economically, variable costs are featured at the top of the income statement and represent on average about 70% of revenues. Variable costs are a significant component of firms' profits, and their cyclical properties affect firm cash flow risk in an economically significant way. Our paper is closely related to the literature on the effect of operating leverage on asset prices. Zhang (2005) and Carlson, Fisher, and Giammarino (2004) show how operating leverage can generate a value spread in a neoclassical model of firm investment. Novy-Marx (2010) proposes an empirical measure of operating leverage and documents its positive predictive power for cross-sectional stock returns. While these papers deal with the leverage effect induced by fixed costs, we focus on the hedging property of variable input costs.

A recent strand of related literature focuses on the effects of labor costs on stock return risk, emphasizing wage rigidity as a source of operating leverage. For instance, Danthine and Donaldson (2002) show that wage rigidity can induce a strong labor leverage and improve the performance of asset pricing models with production to better match aggregate market volatility and equity premium. Favilukis and Lin (2015) examine the quantitative effect of wage rigidity and labor leverage on both the equity premium and value premium. Donangelo, Gourio, Kehrig, and Palacios (2018) document that firms with high labor shares have higher expected returns than firms with low labor shares. Favilukis, Lin, and Zhao (2017) examine the effect of labor leverage on the credit market. Our paper offers a complementary perspective relative to these prior studies. The above papers focus on stickiness of wages of existing workers (selling, general, and administrative (SG&A) expenses, which include a labor component, also tend to have low cyclicality). We emphasize high cyclicality of variable input costs, which include the cost of labor used in production of finished goods. A more comprehensive description of how labor costs affect firms' cash flow risk should combine the properties of the intensive margin (wage stickiness emphasized in the above studies) with those of the extensive margin (employment dynamics and worker mobility), which is relevant for the properties of variable input costs. Such a deeper dive into the properties of labor costs is beyond the scope of this paper.

Asset pricing implications of our paper connect it closely to the growing literature that investigates the relation between firm stock returns and accounting measures, such as firm profitability and valuation ratios. While value premium has been extensively studied in the literature,[3], the sources of profitability premium are relatively less understood. Kogan and Papanikolaou (2013) show that firm heterogeneity in growth options gives rise to a sizable profitability premium. All cross-sectional return factors in their model are driven by investment-specific technological shocks, and hence their model cannot generate a profitability factor in returns that is negatively correlated with the value factor. Ma and Yan (2015) extend the idea of Garlappi and Yan (2011) and find that the performance of the value and gross profitability strategies varies with credit conditions. Their model has a single firm-level state variable, and, like the models of the value premium emphasizing operating leverage (e.g., Carlson, Fisher, and Giammarino (2004), Zhang (2005)) faces the challenge of generating the coexistence of positive unconditional gross profitability and value premia. Wang and Yu (2015) and Lam, Wang, and Wei (2014) compare the risk-based and behavioral explanations of gross profitability premium and argue that the empirical evidence is more consistent with investors' under-reaction to news on firm fundamentals. Akbas, Jiang, and Koch (2017) find the recent trajectory of a firm's profits predicts future profitability and stock returns. Bouchaud, Krueger, Landier, and Thesmar (2018) propose a theoretical explanation for the profitability premium based on sticky expectations.

2 Data and Empirical Benchmark

We first summarize some empirical evidence on the profitability premium and the value premium. In the following sections, we relate our theoretical model to these empirical results.

We draw stock return data from the Center for Research in Security Prices (CRSP), the accounting data from the Compustat Annual North America. Following Novy-Marx (2013), we

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[3] Studies on the value premium include Lakonishok, Shleifer, and Vishny (1994), Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), Zhang (2005), Lettau and Wachter (2007), Garleanu, Kogan, and Panageas (2012), Choi (2013), Kogan and Papanikolaou (2014), Donangelo (2017), Kogan, Papanikolaou, and Stoffman (2019) among many others.

define the gross profitability, which we refer to as GP/A, as revenue (Compustat item REVT) minus cost of goods sold (Compustat item COGS) divided by total asset (Compustat item AT), that is, (REVT − COGS)/AT. We define book-to-market equity ratio following Fama and French (1992).[4] We remove firms in the financial industries and only keep in our sample firms with a share code (SHRCD) 10 or 11, and exchange code (EXCHCD) of 1, 2, or 3. Our final sample covers the time period from July 1963 to December 2018.

We examine the gross profitability premium by constructing quintile portfolios sorted by GP/A, and report their portfolio characteristics and summaries of stock return properties. Panel A of Table 1 reports the GP/A portfolio characteristics. Consistent with Novy-Marx (2013), we find that high-GP/A firms have lower book-to-market ratios and higher Tobin's Q. They also exhibit higher gross margin, higher investment rate, and higher recent stock returns than low-GP/A firms. The gross margin increases from 0.18 for low-GP/A firms to 0.43 for high-GP/A firms. On the other hand, while financial leverage slightly declines with GP/A, the relation between operating leverage and GP/A is rather weak and nonmonotonic.

[Insert Table 1 Here]

Panel B of Table 1 reports the value-weighted portfolio returns and the asset pricing test results, including the CAPM and Fama and French (1992) three-factor model tests, for the quintile portfolios sorted by GP/A.[5] High-profitability firms have higher average returns than low-profitability firms. The annualized return spread is 4.5% per year with a Sharpe ratio of 0.43. CAPM fails to capture the gross profitability premium, as the difference in market beta between high and

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[4] Other variables include: firm size is the market cap at the end of previous June. Momentum is the prior 2-12 month returns. Financial leverage is the sum of total debt in current liability (Compustat item DLC) and total long-term debt (Compustat item item DLTT), divided by the sum of DLC, DLTT, and firm's market cap. Operating leverage is defined as XSGA/(REVT-COGS), where Compustat item XSGA is the selling, general and administrative expense. Q is the sum of market value, long-term debt (Compustat item DLC), preferred stock redemption value (Compustat item PSTKRV), minus the total inventories (Compustat item INVT) and deferred tax in balance sheet (Compustat item TXDB), divided by gross property, plant and equipment (Compustat item PPEGT). Cash holding is cash and short-term investments (Compustat item CHE) divided by PPEGT. R&D intensity is research and development expense (Compustat item XRD) divided by PPEGT. Investment rate is the capital expenditure (Compustat item CAPX) dividend by lagged PPEGT. Gross margin is the (REVT-COGS)/REVTS.

[5] For the rest of the paper, we only report the result for the value-weighted strategy, but the equal weighted strategy generates a very similar conclusion. These results are available upon request.

low GP/A portfolios is essentially zero. Controlling for Fama and French (1992) three factors further increases the return spread, due to the similarity of high (low) GP/A and growth (value) firms. Indeed, the coefficient of the GP/A spread portfolio on the high-minus-low (HML) value premium factor is −0.44 (t-statistic = −6.34), giving rise to an abnormal return of 6.77% per year (t-statistic = 5.02). These results replicate the finding in Novy-Marx (2013) that the gross profitability is the "other side of value."

Novy-Marx (2013) also documents that the gross profitability premium is stronger within industries, which we confirm in our sample in Table 2. We sort stocks into quintile portfolios based on their gross profitability relative to other firms in the same industry, where we use the Fama and French 30 industry classification. As shown in Panel A of Table 2, most of the patterns in the characteristics of these within-industry GP/A portfolios are consistent with the patterns in unconditional GP/A portfolios from Table 1. For instance, high GP/A stocks within industries have lower B/M, higher prior returns, higher Q and IK, and higher gross margin. In the meanwhile, the relation between GP/A and financial and operating leverages are slightly weakened after controlling for industries.

[Insert Table 2 Here]

Panel B reports the average returns and asset pricing test results for these within-industry GP/A portfolios. Although the average return of the GP/A premium within industries is smaller than the unconditional GP/A premium (3.83% vs 4.5%), their Sharpe ratios are almost identical (around 0.43). Furthermore, once we control for the market factor and the value and size premium factors, the within-industry GP/A premium significantly improves because of the strong negative loadings on these factors. Indeed, the CAPM alpha is 4.99% per year with t-statistic = 3.94, and the Fama and French three-factor model alpha becomes even higher at 7.59% per year with t-statistic = 7.59. Panel C is on the horse race between the unconditional GP/A premium and the within-industry GP/A premium. In Panel C1, we run time series regressions of GP/A portfolio returns on the market factor and a within-industry GP/A factor, which is the long-short portfolio from Panel B, and find that the abnormal return of the GP/A premium is only 0.95% per year and statistically insignificant from zero. In Panel C2, when we reverse the order and regress the within-industry GP/A portfolios onto the market factor and an unconditional GP/A factor, the abnormal return of the within-industry GP/A premium remains statistically significant at 2.9% per year. Therefore, our results show that the unconditional GP/A premium is subsumed by the within-industry GP/A premium, indicating potential industry heterogeneity in technology parameters. For the rest of the paper, we use the within-industry GP/A premium as the benchmark for the GP/A premium.

Table 3 reports the main characteristics (Panel A), returns, and the asset pricing test results (Panel B) for quintile portfolios sorted by the book-to-market ratio. We observe that high book-to-market (value) firms have lower gross profitability than low book-to-market (growth) firms. Value firms also have higher financial and operating leverage (this is in contrast to the findings for the GP/A firms), lower Tobin's Q and gross margin, lower research and development expenditure, and a lower investment rate.

For our sample period, the value-weighted return spread between the high- and low book-to-market portfolios (the value premium) has an average of 4.71% per year, with a Sharpe ratio of 0.35. The unconditional CAPM fails to capture the value premium, while including the HML factor into the pricing model reduces the estimated alpha of the value premium, and makes it statistically insignificant. In an untabulated analysis, we find the correlation between the gross profitability factor and the value factor to be -0.43. These properties of the gross profitability premium, value premium, and the negative correlation between the corresponding return factors, serve as target moments for our economic model in the next section.

[Insert Table 3 Here]

3 A model of operating hedge and profitability premium

In this section, we propose a production-based asset pricing model for the gross profitability premium. Section 3.1 describes a statistic model to illustrate the intuition of our explanation for the gross profitability premium. We substantiate this explanation with empirical evidence in

Section 3.2. In Section 3.3, we extend the static model to a dynamic setting and offer a unified explanation for the negatively correlated gross profitability premium and value premium.

3.1 The static model

In this section we develop a static model, which captures the core intuition of the operating hedge mechanism. We consider a firm using two types of productive inputs: physical capital K and variable inputs E. We assume a production function with a constant elasticity of substitution (CES) between capital and variable inputs. The firm's gross profit π is the difference between revenue and input costs:

$$\pi = \max_{E}(O - PE) = \max_{E}\left[XZ\left(E^{\frac{\eta-1}{\eta}} + K^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} - PE\right], \tag{1}$$

where O is the output, X represents the systematic component of profitability, common to all firms, Z is the idiosyncratic profitability, P is the price of variable inputs, and $\eta > 0$ measures the elasticity of substitution between capital and variable inputs. Firms take the price of variable inputs (P) as given and choose the quantity of variable inputs (E) to maximize the gross profit. The quantity of capital (K) is fixed for simplicity (we introduce capital accumulation in the dynamic version of the model).

The share of the variable inputs (ES) can be calculated from the first–order condition:

$$ES \equiv \frac{PE}{O} = \frac{E^{\frac{\eta-1}{\eta}}}{E^{\frac{\eta-1}{\eta}} + K^{\frac{\eta-1}{\eta}}}. \tag{2}$$

This implies that firm gross profits are given by

$$\pi = XZ\left(E^{\frac{\eta-1}{\eta}} + K^{\frac{\eta-1}{\eta}}\right)^{\frac{1}{\eta-1}} K^{\frac{\eta-1}{\eta}}, \tag{3}$$

and that firm's gross profitability, i.e., the gross profit per unit of capital (GP/A), increases with the firm idiosyncratic profitability Z:

$$\text{GP/A} \equiv \frac{\pi}{K} = XZ\left[\left(\frac{E}{K}\right)^{\frac{\eta-1}{\eta}}+1\right]^{\frac{1}{\eta-1}}. \tag{4}$$

The elasticity of gross profit with respect to the aggregate profitability shock, β_X, is

$$\beta_X \equiv \frac{\partial \log \pi}{\partial \log X} = 1+\left(1-\frac{\partial \log P}{\partial \log X}\right)\left(\frac{E}{K}\right)^{\frac{\eta-1}{\eta}}, \tag{5}$$

where $\frac{\partial \log P}{\partial \log X}$ measures the cyclicality of the variable input price. The above equation shows that the higher $\frac{\partial \log P}{\partial \log X}$ is, the lower or more negative the exposure of a firm to the aggregate profitability shock because of the stronger operating hedging effect.

To demonstrate how a firm's exposure to the aggregate profitability shock varies with its gross profitability, which is monotonically increasing in its profitability Z (see Equation (4)), we take the partial derivative of β_X with respect to $\log Z$:

$$\frac{\partial \beta_X}{\partial \log Z} = (1-\eta)\left(\frac{\partial \log P}{\partial \log X}-1\right)\left(\frac{E}{K}\right)^{\frac{\eta-1}{\eta}}\left[1+\left(\frac{E}{K}\right)^{\frac{\eta-1}{\eta}}\right]. \tag{6}$$

The last two terms in Eq. (6) are always positive, so to obtain a positive gross profitability premium, i.e., $\frac{\partial \beta_X}{\partial \log Z} > 0$, we require $(1-\eta)\left(\frac{\partial \log P}{\partial \log X}-1\right) > 0$.

In the appendix, we show that the difference in the exposures of revenue and variable input to the aggregate profitability shock is given by:

$$\frac{\partial \log O}{\partial \log X} - \frac{\partial \log \pi}{\partial \log X} = \left(\frac{E}{K}\right)^{\frac{\eta-1}{\eta}}(1-\eta)\left(\frac{\partial \log P}{\partial \log X}-1\right), \tag{7}$$

so $\frac{\partial \log O}{\partial \log X} - \frac{\partial \log \pi}{\partial \log X}$ has the same sign as $(1-\eta)\left(\frac{\partial \log P}{\partial \log X}-1\right)$. In addition, since the model has only one aggregate shock that drives the variation in the aggregate revenue and gross profit, $\frac{\partial \log O}{\partial \log X} - \frac{\partial \log \pi}{\partial \log X}$ is proportional to the relative cyclicality of aggregate revenue and aggregate gross profit. When aggregate revenue is more cyclical than aggregate gross profit, the variable input provides an operating hedging effect, so $(1-\eta)\left(\frac{\partial \log P}{\partial \log X}-1\right) > 0$, which in turn implies a positive gross profitability premium. In contrast, if aggregate revenue is less cyclical than the aggregate gross profit, the model implies a negative gross profitability premium.

Likewise, the model also has implications on the relative exposure of revenue and gross profit to the shock to idiosyncratic profitability Z. In the appendix, we show that:

$$\frac{\partial \log O}{\partial \log Z} - \frac{\partial \log \pi}{\partial \log Z} = (\eta - 1)\left(\frac{E}{K}\right)^{\frac{\eta-1}{\eta}}, \tag{8}$$

Unlike the exposure to the aggregate profitability shock, the sign of $\frac{\partial \log O}{\partial \log Z} - \frac{\partial \log \pi}{\partial \log Z}$ depends only on the elasticity of substitution between capital and variable input η. When $\eta < 1$, the response of revenue to idiosyncratic profitability shock is lower than that of gross profit. When $\eta > 1$, revenue has a greater response to an idiosyncratic profitability shock than gross profit.

3.2 Empirical evidence

In this subsection, we analyze empirical evidence related to our proposed economic explanation. The two key parameters for a positive gross profitability premium in our model are the cyclicality of variable inputs prices ($\frac{\partial \log P}{\partial \log X}$) and the elasticity of substitution between capital and variable inputs (η). When $(\frac{\partial \log P}{\partial \log X} - 1)(1 - \eta) > 0$, our model predicts a positive gross profitability premium. We provide empirical evidences for the magnitudes of these two parameter in Subsections 3.2.1 and 3.2.2 using properties of aggregate and firm-level cash flows. We explore the exposures of cash flow and stock returns of gross profitability portfolios to the aggregate profitability shock in Subsection 3.2.3.

3.2.1 Cyclicality of variable costs and gross profits

In this section, we examine the cyclicality of the aggregate variable costs (COGS) and gross profits (GP) relative to aggregate revenues (Rev). Our data on the price and value of aggregate revenues, variable inputs, and gross profits are from the NBER-CES Manufacturing Industry Database, which differentiates prices and costs related to materials (Mat), energy (Eng), production worker wages (Prd), and office worker wages (Off) across 459 four-digit 1987 SIC industries.[6] For each variable (revenue, input, gross profit), we compute its aggregate value by summing up the corresponding values across industries, and its aggregate price index as the weighted average of the price indices across industries using the corresponding one-year lagged industry revenues as the weight. These value and price indices are further deflated by the Consumer Price Index. In line with the definition of the cost of goods sold (COGS) in Compustat, we categorize material costs, energy costs, and production worker wages that are directly related to the production of finished goods as variable cost, and define the gross profits as the difference between revenue and variable cost. We treat office worker wages as part of the fixed cost.

In Panel A of Table 4, we report the mean and standard deviation of the ratio of various input values to aggregate revenue, and the growth rate in the revenue ($\Delta\,\text{logV(Rev)}$), material costs ($\Delta\,\text{logV(Mat)}$), energy costs ($\Delta\,\text{logV(Eng)}$), production worker wage bills ($\Delta\,\text{logV(Prd)}$), total variable costs ($\Delta\,\text{logV(COGS)}$), gross profits ($\Delta\,\text{logV(GP)}$), and the office worker wage bills ($\Delta\,\text{logV(Off)}$). Material costs are the largest component among all types of inputs, representing on average 54.8% of revenue for the manufacturing industries. Production worker wages account for 10.8% of revenue, whereas energy costs are relatively low, at 1.93%. The ratio of the sum of these three types of variable costs (COGS) to revenue is 67.6%, with a standard deviation of 5%. Therefore, variable cost is an economically important determinant of firm profitability. In comparison, office worker wages account on average for only 6.6% of aggregate revenue.

[Insert Table 4 Here]

Panel A of Table 4 also shows that variable costs are generally more volatile than aggregate revenue. The volatility of aggregate revenue is 5.47% per year, compared to 6.73% for material costs, 7.95% for energy costs, and 4.99% for production worker wages. The combined variable costs (COGS) have annual standard deviation of 6.36%, which is 16% higher than the volatility of revenue growth. In comparison, the volatility of office worker wages is relatively low, 3.27% per year.

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[6] We define the office worker wages as the difference between total payroll and the production worker wages.

Panel B reports the correlation matrix of the growth rates of aggregate revenue, variable costs, and gross profits. The revenue and variable costs are highly correlated, with the correlation between their growth rates of 0.98. This high correlation, together with the high volatility of variable costs, gives rise to a strong hedging effect that explains the lower volatility of gross profits relative to revenue (4.79% vs 5.47%, as we show in Panel A). Panel B also shows that the growth rates of gross profits and revenues have a correlation of 0.83.

Panel C of Table 4 reports the elasticity of variable costs and gross profits with respect to aggregate revenue. In the first specification, we regress the growth rate of COGS on the growth rate of revenue. The point estimate, which is significantly higher than one, indicates that a 1% increase in gross output is associated with a 1.14%, increase in variable costs. Because of the hedging effect from variable costs, the elasticity of gross profits with respect to revenue is only 0.73. In the context of our model, these empirical results indicate that $(1-\eta)\left(\frac{\partial \log P}{\partial \log X} - 1\right) > 0$, and we expect a positive gross profitability premium. In the last specification of Panel C, we examine the elasticity of the price index for COGS with respect to the price index for aggregate revenues. We compute the COGS price index as the weighted average of the price indices for materials, energies, and production worker wages using their lagged values as the weights. The elasticity of $1.47 > 1$ indicates that the variable input price is more procyclical than the output price, which offers a suggestive evidence for $\frac{\partial \log P}{\partial \log X} > 1$. In the next subsection, we use the magnitude of η, inferred from firm-level cash flow elasticities, as further evidence on the cyclicality of the input price, $\frac{\partial \log P}{\partial \log X}$.

Before we move onto the next subsection, we would like to point out that the properties of revenue and variables costs in the NBER-CES dataset are quantitatively similar to those in Compustat, although the latter only covers public firms. When we focus on manufacturing firms in Compustat, the average COGS-to-Rev ratio is 69.8%, compared to 67.6% reported above. The estimated elasticity of COGS (GP) with respect to revenues in manufacturing firms in Compustat is 1.09 (0.8), which is close to the values estimated using the NBER-CES dataset. Furthermore, the correlation between the growth rates of revenue, variable costs, and gross profits between these two datasets is 0.83, 0.83, and 0.73, respectively (untabulated). Since the Compustat database does not separate variables costs into different sources (material, energy, and production worker wage), we only focus on the total variable cost (COGS) in the rest of our analysis.

3.2.2 Cash flow elasticities: aggregate and firm-level evidence

We examine cash flow elasticities at both the aggregate level and firm level in Table 5. The data is annual from 1964 to 2018 from Compustat. Panel A1 of Table 5 reports the summary statistics of the aggregate sales growth (ΔlogASale) and aggregate gross profit growth (ΔlogAGP), which are aggregated from firms in our sample. Consistent with the finding based on the data from the NBER-CES database (Table 4), the aggregate sales growth is more volatile than aggregate gross profit growth (5.99% versus 5.33% per year). When we regress the aggregate gross profit growth onto the aggregate sales growth, the estimated coefficient from the time series regression is 0.77 and close to 0.73 from Table 4, although these two databases have different coverage.

[Insert Table 5 Here]

The result is quite different at the firm level. In Panel B of Table 5, we report the results of the firm-level sales growth and gross profit growth. Panel B1 shows that the firm-level gross profit growth is in fact more volatile than the firm-level sales growth (38.45% versus 34.28%). When we relate firm-level gross profit growth to firm-level sales growth in Fama-MacBeth regressions in Panel B2, the estimated coefficient is 1.07, which is greater than one.

To interpret this difference between the aggregate and firm-level results through the lens of the model, note that the idiosyncratic profitability shock is in general more volatile than the aggregate profitability shock. With that, the impact of idiosyncratic shocks at the firm level may overwhelm the effect of aggregate shocks, so the finding that the sale elasticity of gross profit at the firm level is greater than the sale elasticity at the aggregate level indicates that $\eta < 1$, as illustrated in Eq. 8. Together with the cyclicality condition at the aggregate level, i.e., $(1-\eta)\left(\frac{\partial \log P}{\partial \log X} - 1\right) > 0$, this finding also suggests $\frac{\partial \log P}{\partial \log X} > 1$. Put it differently, although the variable cost tends to reduce a firm's exposure to the aggregate profitability shock through the operating hedging effect, it may actually raise the firm's exposure to the idiosyncratic profitability shock, giving rise to an operating leverage effect at the same time. The different responses of revenues and variable costs to the aggregate and idiosyncratic shocks are key to the mechanism behind a positive gross profitability premium in our model. For firms with high idiosyncratic profitability Z, their revenues are high relative to variable costs (operating leverage), so these firms have higher risk premiums associated with the aggregate profitability shocks (operating hedge) than firms with low idiosyncratic profitability Z.

3.2.3 Cash flow and return risk exposures of GP/A portfolios

In this subsection, we provide empirical evidence on cash flow and return exposures to the aggregate profitability shock across gross profitability portfolios.

In the static model, the gross profitability premium is driven by the risk exposure to the aggregate profitability shock, so the long-short portfolio based on gross profitability quintiles within Fama and French 30 industries can be considered as an empirical factor-mimicking portfolio for this risk factor. As the first test, we examine the cumulative responses in the growth rates of gross profits, revenues, and cost of goods sold of these quintile portfolios from t to $t+K$ (K = 0, 1, 2) following a positive long-short GP/A portfolio return in year t. We normalize the aggregate profitability shock measure to have a unit standard deviation. Table 6 reports the results.

[Insert Table 6 Here]

In Panel A of Table 6, we find the responses of gross profit growth increases from low- to high-GP/A portfolios for the first three years following an aggregate profitability shock. Economically, a one-standard-deviation increase in the return of the factor-mimicking portfolio is associated with a 4.02% contemporaneous decrease in the growth rate of gross profits for the low GP/A portfolio, and a 0.87% contemporaneous increase in the gross profit growth for the high GP/A portfolio.[7] Their difference of 4.89% is 2.38 standard deviations from zero and the cumulative difference further increases to 6.58% in the second year and 6.67% in the third year.

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[7] As shown in Eq 5, firms with sufficiently low Z can have a negative systematic cash flow risk. The intuition is that when $\eta < 1$, variable inputs and capital stock are complements, so as idiosyncratic profitability falls, firms cut input uses but not that much, so their gross margins also decrease and the exposure to the aggregate profitability shock may become negative due to the strong hedging effect from variable costs.

Looking into the source of this cash flow beta difference, we find that the pattern in the sales beta (Panel B) and COGS beta (Panel C) is much weaker. For both betas, the difference between low and high GP/A firms is only statistically significant during the first year (K = 0), and their economic magnitudes are substantially smaller than the gross profit beta difference in Panel A. Therefore, consistent with the economic channel of our theoretical model, it is the compositional difference between sales and COGS, rather than the difference in the cyclicality of sales and COGS across the GP/A portfolios, that creates the cross-sectional difference in their cash flow betas.

Reinforcing our findings on the cash flow beta, the returns of gross profitability portfolios within Fama and French 30 industries also display an increasing pattern in their exposures to the aggregate profitability shock. In Table 7, we use the utilization-adjusted total factor productivity shock (dTFP) from Basu, Fernald, and Kimball (2006) and Fernald (2014) as our proxy for the aggregate profitability shock. This table reports the return betas of the GP/A portfolios from two asset pricing models. In the first model, we augment CAPM with the addition of the TFP shocks in the subsequent year (dTFP(+1)) as the second risk factor.[8] In line with the pattern of the cash flow exposure, we find the return beta increases from -1.84 for low GP/A firms to 1.77 for high GP/A firms, with the difference of 3.6 (t-statistic = 3.6) in this two-factor model. In the second model, we further control for the value premium factor and size premium factor from Fama and French (1992) and find very similar results. The aggregate TFP beta is -1.51 for the low GP/A firms, as compared with 1.54 for the high GP/A firms. The difference in their TFP exposures is 3.2 standard deviations from zero.

[Insert Table 7 Here]

So far, we have established empirical evidence for the different exposures of gross profitability portfolios to the aggregate profitability shock. To generate cross-sectional risk premia among GP/A portfolios, the aggregate profitability shock needs to be a priced risk factor. In the rest of this subsection, we test the prices of risk for the aggregate profitability shock measures, the

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[8] The reason we use the TFP shock in the subsequent year as the aggregate profitability shock measure is partly because there is a lead-lag relation between the stock returns and TFP shocks. In the correlation between the GP/A premium and subsequent one-year TFP growth is much stronger than their contemporaneous correlation, so upon receiving a news about future TFP growth, stock returns respond immediately, followed by real activities.

factor-mimicking portfolio and dTFP, using the Fama and French 17 and 30 industry portfolios as the testing assets. We specify the stochastic discount factor (SDF) as a linear function of the market factor and aggregate profitability shock measure and use GMM to estimate their prices of risk. To save the space, we only report the results from the first-stage estimation using the identity weighting matrix, but the results are quantitatively similar when the optimal weighting matrix is used in the second-stage estimation. Table 8 shows that the two-factor model does a decent job in pricing the industry portfolios. The annualized mean absolute error (MAE) is about 1.2% per year when the factor-mimicking portfolio is used as the proxy for the aggregate profitability shock, and is between 0.89% and 1.05% when dTFP(+1) is used. The over-identification test fails to reject the two-factor model. It is worth noting that this good performance is not trivial for the factor-mimicking portfolio as it is constructed within industries, whereas the testing assets are industry portfolios. More importantly, the estimated price of risk for the aggregate profitability shock is positive and statistically significant for all four specifications. Therefore, the aggregate profitability shock not only plays a key role in driving the gross profitability premium, but also helps to explain the cross-sectional difference in the industry portfolio returns.

[Insert Table 8 Here]

In untabulated analyses, we also investigate the relation between these two aggregate profitability shock measures and standard macroeconomic variables, including GDP growth and durable and nondurable consumption growth at lower frequencies, in the same spirit of Parker and Julliard (2005). We find both the factor-mimicking portfolio return and dTFP have a strong positive comovement with the low-frequency economic growth, which again suggests that the implied price of risk for the aggregate profitability shock is positive.

3.3 The full dynamic model and simulation analysis

The previous two subsections illustrate and provide empirical evidence for our proposed economic mechanism for the gross profitability premium. One drawback of the static model is its couterfactural implication for the value premium. Intuitively, in this simple setup, more profitable firms also have higher valuation ratios and higher risk premiums than less profitable firms due to the exposure to the aggregate profitability shock, so the static model predicts that firms with higher valuation ratios (growth firms) have higher average returns than firms with lower valuations ratios (value firms). This implies a negative value premium. To reconcile the coexistence of a positive profitability premium and a positive value premium, we propose a full dynamic model by incorporating features in Kogan and Papanikolaou (2014) into our illustrative static model and study its quantitative implications for the gross profitability premium and the value premium.

There are a continuum of firms in the cross section. The basic unit of production is projects. Each project uses capital, which is normalized to be 1, and E_{jt} units of variable inputs. Projects are identical within a firm, and firms differ in their profitability in producing outputs. They also differ in their firm-specific investment shocks, capturing different arrival rates of incoming projects. The production function of a project takes the CES form. For each project of firm j, the gross profit π_{jt} from this project is the difference between revenue and input costs:

$$\pi_{jt} = Y_t \max_{E_{jt}} \left[X_t Z_{jt} \left(E_{jt}^{\frac{\eta-1}{\eta}} + 1 \right)^{\frac{\eta}{\eta-1}} - P_t E_{jt} \right] \tag{9}$$

where Z_{jt} is idiosyncratic profitability, X_t is the stationary component of aggregate profitability that is the same as X_t in the static model, Y_t is the permanent component of the aggregate profitability that also captures the cointegrated co-movement of gross profits and input prices (we refer to Y_t shock as the aggregate growth shock), P_t is the price of variable inputs, normalized by Y_t, η measures the elasticity of substitution between capital and variable inputs. As we see below, although the inclusion of he aggregate growth shock better matches the aggregate moments, it does not drive the cross-sectional risk premium. Firms take the process for P_t as given and choose variable inputs to maximize profits within each period. In our setting, π_{jt} is the gross profitability for firm j at time t, due to the normalization of capital per project to one unit.

The first order condition implies that the maximized gross profit from a project is:

$$\pi_{jt} = \left(E_{jt}^{\frac{\eta-1}{\eta}} + 1 \right)^{\frac{1}{\eta-1}} Z_{jt} X_t Y_t. \tag{10}$$

Defining the number of projects (or capital stock) operated by firm j as K_{jt}, the total gross profit is thus $K_{jt}\pi_{jt}$.

Firms accumulate capital with arrivals of new projects. The law of motion for capital stock is:

$$K_{jt+1} = (1-\delta)K_{jt} + \delta S_t A_{jt} K_{jt}, \tag{11}$$

where S_t and A_{jt} measure the aggregate and firm-specific investment shocks, respectively, which jointly capture the intensity of new project arrivals, and δ is the depreciation rate. We allow A shocks and Z shocks to be positively correlated, consistent with the empirical observation that more profitable firms also have greater investment opportunities. As shown below, the cross-sectional variation in book-to-market ratio is mainly driven by A_{jt}. In addition, firms with higher A_{jt} have higher exposures to S_t than firms with lower A_{jt}. This is the channel for a positive value premium in the model.

For given processes governing the pricing kernel (M), the input price, the aggregate profitability shock (X), the aggregate growth shock (Y), the aggregate investment shock (S), the idiosyncratic profitability shock (Z), and firm-specific investment shock (A), which we specify below, a firm's value can be written recursively as:[9]

$$\begin{aligned} V_{jt} &= K_{jt}\pi_{jt} + E_t[M_{t+1}V_{jt+1}] \\ &\text{s.t. } (10) - (11) \end{aligned} \tag{12}$$

Using the lower case variables to represent the logarithmic transformation of the corresponding level variables, we assume the exogenous variables x, s, z, and a follow AR(1) processes, respectively:

$$x_{t+1} = \rho_x x_t + (1-\rho_x)\bar{x} + \sigma_x \epsilon^x_{t+1} \tag{13}$$

$$s_{t+1} = \rho_s s_t + (1-\rho_s)\bar{s} + \sigma_s \epsilon^s_{t+1} \tag{14}$$

$$z_{jt+1} = \rho_z z_{jt} + (1-\rho_z)\bar{z} + \sigma_z \epsilon^z_{jt+1} + \mu_z \tag{15}$$

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[9] We abstract from fixed cost, which is included in the exogenous leverage ratio as discussed below.

$$a_{jt+1} = \rho_a a_{jt} + \sigma_a \epsilon^a_{jt+1} \tag{16}$$

and y follows a random walk

$$\Delta y_{t+1} = \sigma_y \epsilon^y_{t+1}. \tag{17}$$

We assume the variable input price P_t is a function of the stationary component of the aggregate profitability X_t as follows:

$$\log P_t = \log p_0 + p_1 \log X_t, \tag{18}$$

where $p_0 > 0$ and $p_1 > 0$ capture the level and the cyclicality of the variable input price.

The pricing kernel is a function of the three aggregate shocks, ϵ^x_t, ϵ^y_t, and ϵ^s_t:

$$M_{t+1} = \exp\left(-r_f - \gamma_x \sigma_x \epsilon^x_{t+1} - \gamma_y \sigma_y \epsilon^y_{t+1} - \gamma_s \sigma_s \epsilon^s_{t+1} - \frac{1}{2}\gamma_x^2 \sigma_x^2 - \frac{1}{2}\gamma_y^2 \sigma_y^2 - \frac{1}{2}\gamma_s^2 \sigma_s^2\right), \tag{19}$$

where $\gamma_x > 0$, $\gamma_y > 0$, and $\gamma_s < 0$ are the prices of risks for the aggregate profitability shock, the aggregate growth shock, and the aggregate investment shock, respectively, and r_f is the risk-free rate.

Since the economy is homogenous of degree one with respect to capital stock, it can be shown that $V_{jt}(X_t, Y_t, S_t, Z_{jt}, A_{it}, K_{it}) = K_{jt} v_{jt}(X_t, Y_t, S_t, Z_{jt}, A_{it})$. The firm value can be re-written as:

$$v_{jt} = \pi_{jt} + E_t(M_{t+1} v_{jt+1})[(1-\delta) + \delta A_{jt} S_t] \tag{20}$$

The normalized value function v_{jt} is also the market-to-book for firm j at time t. Since y follows a random walk, we can further simplify this problem as:

$$\hat{v}_{jt} = \hat{\pi}_{jt} + E_t[M_{t+1} \hat{v}_{jt+1} \exp(\mu_y + \sigma_y \epsilon^y_{t+1})][(1-\delta) + \delta A_{jt} S_t] \tag{21}$$

where $\hat{\pi} = \pi/Y$ and $\hat{v} = v/Y$. Since the firm value is linear in Y, the shock to economic growth contributes to the equity premium, but not to the risk premium in the cross section. The normalized firm value $\hat{v}$ is a function of four state variables: X_t, S_t, Z_{jt}, and A_{jt}. We also assume an exogenous leverage ratio ϕ, which encompass the effect of both financial and operating leverage.

3.3.1 Calibration and the optimal solution

We solve the problem numerically using value function iterations at a monthly frequency. We simulate the model 100 times with each sample representing 3,000 firms and 600 months. Table 9 reports the parameter values used in our benchmark calibration.

[Insert Table 9 Here]

Consistent with the literature on the real business cycles (e.g., Kydland and Prescott (1982), Cooper and Haltiwanger (2006)), we set the depreciation rate is 1% per month (or 12% per year), and risk-free rate is 0.25% per month (or 3% per year). We normalize $\bar{x} = 0$, and set $\rho_x = 0.98$, $\sigma_x = 0.04$, $\sigma_y = 0.027$ to approximately match the volatility of the aggregate market returns, the volatility of aggregate variable costs growth relative to aggregate sales growth, the volatility of aggregate sales-aggregate variable costs ratio, and the autocorrelations of the aggregate profitability and aggregate book-to-market ratio. We choose $\bar{s} = -0.146$, $\rho_s = 0.9685$, $\sigma_s = 0.026$ to approximately match the mean, standard deviation, and autocorrelation of the aggregate investment rate of 11.5%, 0.95%, and 0.69, respectively. We set η, the elasticity of substitution between capital and variable inputs, to 0.3, $\rho_z = 0.97$, $\sigma_z = 0.09$, $\rho_a = 0.98$, $\sigma_a = 0.111$, and $\rho_{az} = 0.18$ to approximately match the cross-sectional distribution of GP/A, book-to-market ratios, gross profit margin across GP/A and BM quintile portfolios, and the cross-section standard deviation of firm-level stock returns. We choose $p_0 = 0.588$ and $p_1 = 1.39$ to match the level of aggregate GP/A, and the average ratio of aggregate sale to aggregate variable costs. We set $\gamma_x = 15$, $\gamma_y = 7$, and $\gamma_s = -10$ to match the equity premium and aggregate book-to-market ratio. These values are also in line with Kogan and Papanikolaou (2013, 2014). Finally, following Boldrin, Christiano, and Fisher (2001), we choose a leverage ratio ϕ of 1.67, potentially capturing both operating leverage and financial leverage. Table 10 compares the key moments from the simulated data and empirical data. The parameter values chosen match very well the key moments from the simulated data of the model and the actual data.

[Insert Table 10 Here]

Figure 1 plots the value and policy functions of the calibrated model. The top left panel plots the firm value (or the firm's market-to-book) against the two aggregate state variables (x and s). Consistent with our intuition, total firm value increases with both x and s, so the aggregate profitability shock (positively) and aggregate IST shock (negatively) contribute to the equity premium, given the signs of the prices of risk for these two aggregate risk factors. The top middle panel shows the gross profitability (GP/A) against the two firm-specific state variables (z and a). The strong positive relation between GP/A and z indicates that sorts on GP/A create cross-sectional dispersions in z and in the exposure to x. The top right panel plots the relation between the optimal variable input (E), i.e., the policy function, against z and a. Firms with high idiosyncratic profitability z use more variable inputs, whereas the relation between variable input and idiosyncratic investment opportunity a is almost flat.

[Insert Figure 1 Here]

The bottom panels of Figure 1 illustrates the firm value (or the firm's market-to-book), the value of assets-in-place (VAP), and the value of growth options (VGO) against the two idiosyncratic state variables (z and a). We compute the value of assets-in-place by eliminating future project arrivals using value function iterations, and the value of growth options is the difference between the total firm value and the value of assets-in-place. The bottom panels show that the value of assets-in-place increases with idiosyncratic profitability, but its exposure to the idiosyncratic IST shocks is much lower. On the other hand, the value of growth options is more sensitive to the idiosyncratic investment shocks. Taken together, while the firm value (the market-to-book ratio) increases with both idiosyncratic profitability and investment shocks, its sensitivity to the latter is higher, indicating that book-to-market differentials are mainly driven by cross-sectional dispersion in a. The difference in the exposure to the aggregate investment shocks between assets-in-place and growth options gives rise to a composition effect as in Kogan and Papanikolaou (2013, 2014).

3.3.2 Implications for profitability premium and value premium

Tables 11 and 12 provide quantitative results on the portfolio analysis. Panel A of these two tables reports the characteristics of the GP/A and BM quintile portfolios. In our model, high (low) GP/A firms look like growth (value) firms. For instance, Table 11 Panel A shows that high GP/A firms have a logBM of -1.38, as compared to -0.95 for low GP/A firms. Similarly, Table 12 Panel A shows that value firms have a GP/A of 0.07, as compared to 0.35 for growth firms. Therefore, the model reproduces the empirical fact that gross profitability behaves like the other side of value. Panel A of Tables 11 also confirms that the cross-sectional variation in GP/A is mainly driven by the idiosyncratic profitability shock Z (GP/A increases in Z), whereas the variation in BM is driven by both A and Z (Panel A of Table 12). Compared with value firms, growth firms have higher A and Z because positive shocks to both A and Z increase the firm value. Panel A of Tables 11 and 12 also shows that high GP/A firms and growth firms have higher gross margin (GM) and variable input-capital ratio (EK) than low GP/A firms and value firms. Intuitively, following positive firm-specific profitability shocks, firms use more variable inputs, and because the substitution between capital and variable inputs (η) is inelastic, revenue increases more than input cost. As a result, both EK and GM increase with GP/A.

[Insert Table 11 Here]

[Insert Table 12 Here]

Panel B of Tables 11 and 12 examines the value-weighted returns and asset pricing tests of the GP/A portfolios and BM portfolios. For the asset pricing models, we consider the CAPM and a two-factor model with the market and the value premium factor (HML) as the risk factors.[10] The model generates a positive gross profitability premium of 3.26% per year and value premium of 3.21% per year with a correlation coefficient of -0.25 (untabulated) between the two factors. Thus,

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[10] We use the two-factor model in the simulation to draw parallel to the Fama and French three-factor model in the empirical analysis. We do not include a separate size premium factor as we did in the empirical analysis (Tables 1 and 3) because our theoretical model is a three-factor model and we lack one additional risk factor compared to the data. In the empirical analysis, the gross profitability premium is unable to be captured by the Fama and French three-factor model, so a total of four factors are needed to capture the size, value, and gross profitability premiums in the data.

we are able to generate large profitability and value premia, with negative correlation between the corresponding factors, in a structural asset pricing model with production. None of the two premia are captured by the CAPM. For the gross profitability premium, even though the market beta increases slightly from low to high GP/A firms, the CAPM alpha remains 3.1% per year, which is 5.09 standard errors from zero. For the value premium, the CAPM beta goes in the wrong direction (a negative market beta), so that the abnormal return spread (4.28%) is even greater than the return spread (3.21%). Adding the value premium factor eliminates the abnormal return of the value premium as shown in Table 12 Panel B, but increases the abnormal return of the gross profitability premium even further as shown in Table 11 Panel B. The two-factor model alpha controlling for HML becomes 3.79% per year, which is greater than both the average return and CAPM alpha of the Hi-Lo GP/A portfolio. Therefore, while the model reproduces a coexistence of the profitability premium and the value premium, these two premiums are negatively correlated with each other.

To understand the drivers of the profitability premium and the value premium in our investment-based model, Panel C of Tables 11 and 12 reports the exposures of the GP/A and BM portfolios to the aggregate profitability shock, aggregate growth (Y) shock, and aggregate investment (S) shock. The exposures to the aggregate growth shock is approximately 1.68 for all GP/A and BM portfolios. Therefore, while the aggregate growth shock contributes to the overall equity premium, it does not drive the average stock returns in the cross section. High GP/A firms have higher exposures to the aggregate profitability shock than low GP/A firms, with a difference in $\beta(X)$ of 0.12 (t-statistic = 10.47). So consistent with our earlier discussion, the aggregate profitability shock is the main underlying risk factor for the positive gross profitability premium. In addition, due to the positive correlation between firm-specific profitability shocks and firm-specific investment shocks, profitable firms also have higher exposure to the aggregate investment shocks than less profitable firms, with a difference in $\beta(S)$ of 0.04 (t-statistic = 2.35).

For the value premium, value firms have an exposure to S shocks of 0.42, much lower than that of growth firms (1.06), and their difference is more than 30 standard errors from zero. On the other hand, the exposure to X shocks decreases with book-to-market ratio, with a slightly higher

$\beta(X)$ for growth firms than value firms. Therefore, the asset composition and the exposure to the aggregate investment shocks are the major sources of the positive value premium in our model. Furthermore, the opposite signs in the risk exposures to aggregate profitability and investment shocks between the gross profitability premium and value premium generate a strong negative correlation between these two premiums.

The prediction of our model on the negative correlation between the gross profitability premium and the value premium is also novel and stands out from the existing studies on these cross-sectional phenomena. Most analyses focus on one phenomenon while ignoring the other (e.g., Wang and Yu (2015), Lam, Wang, and Wei (2014), Zhang (2005), Carlson, Fisher, and Giammarino (2004)). One exception is Kogan and Papanikolaou (2013). In their model, both the value premium and gross profitability premium are driven by the asset composition and heterogeneous exposures to the aggregate investment shocks. Specifically, more profitable firms derive much of their values from assets-in-place than growth options, and hence their exposure to the aggregate investment shocks is lower than less profitable firms. Since the aggregate investment shock is also driving the different risk premiums between value and growth firms, the value premium and the gross profitability premium in their model are positively correlated. Different from their model, high GP/A firms in our model have low BM, instead of high BM, than low GP/A firms, so profitable firms look a lot like growth firms, not value firms. In addition, the sources of the GP/A and value premiums are different in our model. While the value premium is due to the heterogeneous exposure to the aggregate investment shock, the gross profitability premium is driven by different exposures to the aggregate profitability shock. Thus, our model breaks the positive correlation between the profitability premium and the value premium.

To further illustrate the mechanism of the model to generate the gross profitability premium and the value premium that is negatively correlated with the profitability premium, we report the returns and risk exposures of the portfolios sorted by the underlying firm-specific state variables Z and A in Table 13. Because of the channel we discussed in Section 3.1, sorting on Z creates a larger return spread of 3.68% per year than the gross profitability premium of 3.26%, as reported in Panel A. The majority of the premium is due to the exposure to the aggregate profitability shock: $\beta(X)$ for the Hi-Lo Z portfolio is 0.13 (t-statistic = 11.28). Furthermore, because of the positive correlation between Z and A shocks, this premium also loads positively on the aggregate investment shock, $\beta(S)$.

[Insert Table 13 Here]

Similarly, Panel B reports the results for the portfolios sorted by firm-specific investment opportunity A. High A stocks have an average return of 4.76% per year, which is 5.09% lower than that of the low A stocks, and the findings on the exposures of these portfolios on the aggregate risk factors indicate that the aggregate investment shock is the dominant source of this risk premium. In addition, this premium is 59% higher than that of the value premium, indicating that book-to-market sorts contain information about Z, which "contaminates" and offsets the negative relation between A and risk premium in the book-to-market sorts.

4 Conclusion

We explore a novel economic channel for heterogeneity in cash flow risk among firms. The hedging effect due to highly procyclical variable costs generates differential exposures to aggregate profitability shocks, which correlate with firm profitability. Less profitable firms benefit more from this hedging effect, and thus exhibit lower cash flow risk, and lower average returns.

We develop a dynamic model, in which the profitability premium coexists with the value premium. The two effects are generated by different economic channels. The value spread reflects in large cross-sectional differences in firm growth opportunities, and thus their heterogeneous exposures to the aggregate investment-specific shocks. The profitability spread is driven primarily by the aggregate profitability shocks. Our model is able to reproduce the negative correlation between the value factor and the gross profitability factor in returns.

Operating leverage has been explored extensively in the prior literature as a mechanism for generating the value premium in the cross-section of stock returns. It is also well known that the logic of operating leverage leads to counterfactual implications for the relation between firm profitability and average returns. In this paper we show that impact of production costs on firm cash flow risk is more nuanced than suggested by the operating leverage channel alone, and variable costs give rise to a first-order hedging effect on firm cash flows. Additional research is needed to better understand the dynamics of firm costs, particularly in relation to the input-output structure of the economy, and cross-sectional differences in production technologies and market power. Our analysis suggests that this would yield useful insights into the properties of stock returns and firm dynamics.

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Appendix

Firm's problem chooses variable inputs to maximize firm operating profits:

$$\begin{aligned}\pi &= \max_E \{O - PE\} \\ &= \max_E \{XZ(E^{\frac{\eta-1}{\eta}} + K^{\frac{\eta-1}{\eta}})^{\frac{\eta}{\eta-1}} - PE\}.\end{aligned} \tag{A.1}$$

where O is firm's revenues. The first-order condition implies that:

$$P = XZE^{\frac{-1}{\eta}} \times (E^{\frac{\eta-1}{\eta}} + K^{\frac{\eta-1}{\eta}})^{\frac{1}{\eta-1}}, \tag{A.2}$$

so variable inputs share is:

$$ES \equiv \frac{PE}{O} = \frac{E^{\frac{\eta-1}{\eta}}}{E^{\frac{\eta-1}{\eta}} + K^{\frac{\eta-1}{\eta}}}. \tag{A.3}$$

Empirically, this measures the ratio of variable cost and revenue. Multiplying both the numerator and denominator of Equation A.3 by $P^{\frac{\eta-1}{\eta}}$ and after some simple algebra, we can show that:

$$\log\left(\frac{K}{PE}\right) = -\eta \log\left(\frac{\pi}{K}\right) + (\eta - 1)P. \tag{A.4}$$

This equation is the basis for estimating η using empirical observables.

In addition, firm's gross profitability equals:

$$\text{GP/A} = XZ\left[\left(\frac{E}{K}\right)^{\frac{\eta-1}{\eta}} + 1\right]^{\frac{1}{\eta-1}}. \tag{A.5}$$

Taking the partial derivative of the logarithm of both sides of Equation A.2 with respect to $\log X$, we have:

$$\frac{\partial \log E}{\partial \log X} = \eta\left(1 - \frac{\partial \log P}{\partial \log X}\right)\left[1 + \left(\frac{E}{K}\right)^{\frac{\eta-1}{\eta}}\right]. \tag{A.6}$$

Taking the partial derivative of the logarithm of both sides of Equation A.2 with respect to

$\log Z$, we have:

$$\frac{\partial \log E}{\partial \log Z} = \eta \left[1 + \left(\frac{E}{K} \right)^{\frac{\eta-1}{\eta}} \right]. \tag{A.7}$$

Therefore, the exposure of firm's gross profit with respect to X, or β_X, is:

$$\begin{aligned} \beta_X \equiv \frac{\partial \log \pi}{\partial \log X} &= 1 + \frac{1}{\eta} \frac{E^{\frac{\eta-1}{\eta}}}{E^{\frac{\eta-1}{\eta}} + K^{\frac{\eta-1}{\eta}}} \times \frac{\partial \log E}{\partial \log X} \\ &= 1 + \left(1 - \frac{\partial \log P}{\partial \log X} \right) \left(\frac{E}{K} \right)^{\frac{\eta-1}{\eta}}. \end{aligned} \tag{A.8}$$

Taking the partial derivative of β_X with respect to $\log Z$:

$$\begin{aligned} \frac{\partial \beta_X}{\partial \log Z} &= \left(\frac{\eta - 1}{\eta} \right) \left(1 - \frac{\partial \log P}{\partial \log X} \right) \left(\frac{E}{K} \right)^{\frac{\eta-1}{\eta}} \frac{\partial \log E}{\partial \log Z} \\ &= (\eta - 1) \left(1 - \frac{\partial \log P}{\partial \log X} \right) \left(\frac{E}{K} \right)^{\frac{\eta-1}{\eta}} \left[1 + \left(\frac{E}{K} \right)^{\frac{\eta-1}{\eta}} \right]. \end{aligned} \tag{A.9}$$

The last two terms on the right-hand-side of the above equation are always positive, so a positive gross profitability premium requires $(\eta - 1) \left(1 - \frac{\partial \log P}{\partial \log X} \right) > 0$. It can be quickly shown that:

$$\frac{\partial \log O}{\partial \log X} = 1 + \eta \left(\frac{E}{K} \right)^{\frac{\eta-1}{\eta}} \left(1 - \frac{\partial \log P}{\partial \log X} \right), \tag{A.10}$$

and

$$\frac{\partial \log (PE)}{\partial \log X} = \frac{\partial \log P}{\partial \log X} \left[1 - \eta + \left(\frac{E}{K} \right)^{\frac{\eta-1}{\eta}} \right] + \eta \left[1 + \left(\frac{E}{K} \right)^{\frac{\eta-1}{\eta}} \right], \tag{A.11}$$

Taking the difference between Equation (A.10) and Equation (A.7), we have

$$\frac{\partial \log O}{\partial \log X} - \frac{\partial \log \pi}{\partial \log X} = \left(\frac{E}{K} \right)^{\frac{\eta-1}{\eta}} \left(1 - \frac{\partial \log P}{\partial \log X} \right) (\eta - 1) \tag{A.12}$$

and taking the difference between Equation (A.12) and Equation (A.10), we have

$$\frac{\partial \log (PE)}{\partial \log X} - \frac{\partial \log O}{\partial \log X} = \left(1 - \frac{\partial \log P}{\partial \log X} \right) (\eta - 1). \tag{A.13}$$

The above two equations suggest that the aggregate variable cost is more procyclical than aggregate revenue and aggregate revenue is more procyclical than gross profit, as long as $(\eta - 1)\left(1 - \frac{\partial \log P}{\partial \log X}\right) > 0$. This turns out to be the same condition for a positive gross profitability premium in the model.

Comparing Equation (A.10) with Equation (A.7), we also have the following identity:

$$\frac{\partial \log O}{\partial \log X} - 1 = \eta \left(\frac{\partial \log \pi}{\partial \log X} - 1 \right) \tag{A.14}$$

which has been used in Donangelo, Gourio, Kehrig, and Palacios (2018) to estimate η in the context of labor inputs.

We can also examine how revenue, variable cost, and gross profit respond to the firm-specific profitability shock. It can be shown that:

$$\frac{\partial \log O}{\partial \log Z} = 1 + \eta \left(\frac{E}{K} \right)^{\frac{\eta-1}{\eta}}, \tag{A.15}$$

$$\frac{\partial \log \pi}{\partial \log Z} = 1 + \left(\frac{E}{K} \right)^{\frac{\eta-1}{\eta}}, \tag{A.16}$$

and

$$\frac{\partial \log (PE)}{\partial \log Z} = \eta \left[1 + \left(\frac{E}{K} \right)^{\frac{\eta-1}{\eta}} \right]. \tag{A.17}$$

Therefore,

$$\frac{\partial \log O}{\partial \log Z} - \frac{\partial \log \pi}{\partial \log Z} = (\eta - 1) \left(\frac{E}{K} \right)^{\frac{\eta-1}{\eta}}, \tag{A.18}$$

$$\frac{\partial \log (PE)}{\partial \log Z} - \frac{\partial \log O}{\partial \log Z} = \eta - 1. \tag{A.19}$$

Equations (A.18) and (A.19) state that when $\eta < 1$, the responses of revenue and variable cost to Z shocks are lower than the response of gross profits, giving rise to an operating leverage effect. On the other hand, when $\eta > 1$, the responses of revenue and variable cost to Z shocks are higher than the response of gross profits, giving rise to an operating hedge effect.

Table 1: Gross profitability portfolios

This table reports the characteristics and returns of quintile value-weighted portfolios sorted by gross profitability (GP/A). Panel A reports the average characteristics of GP/A portfolios, including gross profitability (GP/A), log book-to-market (logBM), log market cap (logSize), momentum (Mom), market-based financial leverage (FLev), operating leverage (OLev), Tobin's Q (Q), cash holdings (CH), R&D intensity (RD), investment rate (IK), and gross profit margin (GM). Panel B reports the mean, standard deviation, CAPM test result, and Fama-French three-factor model test result of the quintile GP/A portfolios. The sample is 196307-201812. Newey-West *t*-stats given in parentheses control for heteroskedasticity and autocorrelation.

Panel A: Portfolio characteristics

Portfolio	GP/A	logBM	logSize	Mom	FLev	OLev	Q	CH	RD	IK	GM
Lo	0.08	-0.39	4.50	0.00	0.28	0.90	3.14	0.59	0.74	0.09	0.17
2	0.21	-0.22	4.83	0.05	0.30	0.52	1.14	0.11	0.03	0.11	0.26
3	0.33	-0.37	4.64	0.06	0.22	0.60	1.72	0.16	0.06	0.11	0.30
4	0.46	-0.56	4.59	0.07	0.15	0.67	2.40	0.23	0.10	0.12	0.37
Hi	0.71	-0.77	4.47	0.08	0.08	0.76	2.91	0.28	0.11	0.14	0.44

Panel B: Portfolio returns, CAPM, and Fama-French 3-factor model test

	Lo	2	3	4	Hi	Hi-Lo
Mean	4.01	5.07	7.00	6.25	8.51	4.50
Std	15.58	15.83	15.99	16.86	15.76	10.35
α	-1.80	-0.92	0.79	-0.21	2.68	4.49
	(-1.98)	(-1.08)	(1.33)	(-0.29)	(2.81)	(2.86)
MKT	0.94	0.97	1.01	1.05	0.95	0.00
	(46.78)	(40.75)	(60.83)	(52.74)	(39.98)	(0.08)
α	-2.63	-1.72	0.81	1.12	4.14	6.77
	(-3.08)	(-2.16)	(1.27)	(1.71)	(5.01)	(5.02)
MKT	0.96	1.02	0.99	1.01	0.91	-0.05
	(51.86)	(49.00)	(55.98)	(64.95)	(39.82)	(-1.34)
HML	0.15	0.18	-0.02	-0.27	-0.29	-0.44
	(3.84)	(4.75)	(-0.70)	(-8.73)	(-7.00)	(-6.34)
SMB	0.07	-0.07	0.08	-0.02	-0.04	-0.11
	(1.93)	(-2.05)	(3.42)	(-0.67)	(-1.37)	(-1.96)

Table 2: Gross profitability portfolios with in Fama and French 30 industries
This table reports the characteristics and returns of quintile value-weighted portfolios sorted by gross profitability (GP/A) within Fama and French 30 industries. Panel A reports the average characteristics of within-industry GP/A portfolios, including gross profitability, log book-to-market, log market cap, momentum, market-based financial leverage, operating leverage, Tobin's Q, cash holdings, R&D intensity, investment rate, and gross profit margin. Panel B reports the mean, standard deviation, CAPM test result, and Fama-French three-factor model test result of the quintile GP/A portfolios. Panel C reports the result from the horse race between GP/A premium and within-industry GP/A premium. In Panel C1 (C2), the excess return of GP/A (within-industry GP/A) quintiles are regressed onto the market factor and the within-industry GP/A (GP/A) premium, and the abnormal returns (alphas) are reported. The sample is 196307-201812. Newey-West *t*-stats given in parentheses control for heteroskedasticity and autocorrelation.

Panel A: Portfolio characteristics

Portfolio	GP/A	logBM	logSize	Mom	FLev	OLev	Q	CH	RD	IK	GM
Lo	0.09	-0.19	4.25	0.02	0.29	0.61	2.31	0.35	0.19	0.10	0.18
2	0.22	-0.23	4.72	0.06	0.30	0.58	1.76	0.19	0.10	0.11	0.30
3	0.32	-0.34	4.84	0.07	0.25	0.60	1.71	0.16	0.07	0.11	0.35
4	0.43	-0.49	4.84	0.08	0.18	0.63	2.10	0.20	0.08	0.12	0.38
Hi	0.62	-0.71	4.53	0.08	0.11	0.71	2.74	0.26	0.12	0.13	0.43

Panel B: Portfolio returns, CAPM, and Fama-French 3-factor model test

	Lo	2	3	4	Hi	Hi-Lo
Mean	4.27	6.39	6.16	6.21	8.09	3.83
Std	17.71	16.46	15.24	15.32	14.76	8.86
α	-2.58	-0.09	0.16	0.17	2.41	4.99
	(-3.12)	(-0.19)	(0.35)	(0.36)	(3.94)	(3.94)
MKT	1.11	1.05	0.98	0.98	0.92	-0.19
	(43.28)	(79.33)	(83.90)	(80.48)	(57.90)	(-5.09)
α	-4.15	-0.77	0.13	0.47	3.44	7.59
	(-5.91)	(-1.68)	(0.29)	(0.98)	(6.61)	(7.59)
MKT	1.13	1.05	0.99	1.00	0.91	-0.22
	(55.73)	(90.71)	(81.30)	(86.51)	(66.97)	(-8.05)
HML	0.28	0.11	0.02	-0.03	-0.19	-0.48
	(9.79)	(6.23)	(0.93)	(-1.57)	(-6.50)	(-9.01)
SMB	0.16	0.10	-0.06	-0.11	-0.06	-0.22
	(7.72)	(6.48)	(-2.51)	(-5.57)	(-3.29)	(-6.97)

Panel C: Horse race between GP/A premium and within-industry GP/A premium

Panel C1. Alphas of GP/A portfolios

	Lo	2	3	4	Hi	Hi-Lo
α	-0.49	-0.07	1.02	-0.97	0.45	0.95
	(-0.61)	(-0.09)	(1.73)	(-1.40)	(0.58)	(0.77)

Panel C2. Alphas of within-industry GP/A portfolios

	Lo	2	3	4	Hi	Hi-Lo
α	-1.52	0.39	0.33	0.03	1.38	2.90
	(-2.27)	(0.78)	(0.76)	(0.07)	(2.77)	(3.04)

Table 3: Book-to-market portfolios

This table reports the characteristics and returns of quintile value-weighted portfolios sorted by the book-to-market equity ratio (BM). Panel A reports the average characteristics of BM portfolios, including gross profitability (GP/A), log book-to-market (logBM), log market cap (logSize), momentum (Mom), market-based financial leverage (FLev), operating leverage (OLev), Tobin's Q (Q), cash holdings (CH), R&D intensity (RD), investment rate (IK), and gross profit margin (GM). Panel B reports the mean, standard deviation, CAPM test result, and Fama-French three-factor model test result of the quintile BM portfolios. The sample is 196307-201812. Newey-West t-stats given in parentheses control for heteroskedasticity and autocorrelation.

Panel A: Portfolio characteristics

Portfolio	GP/A	logBM	logSize	Mom	FLev	OLev	Q	CH	RD	IK	GM
Lo	0.41	-1.62	5.19	0.03	0.05	0.66	9.15	0.56	0.27	0.18	0.41
2	0.37	-0.88	5.22	0.05	0.13	0.62	3.44	0.28	0.11	0.14	0.36
3	0.34	-0.45	4.96	0.06	0.21	0.64	1.84	0.18	0.07	0.12	0.32
4	0.30	-0.07	4.49	0.07	0.28	0.66	1.06	0.13	0.05	0.10	0.29
Hi	0.27	0.46	3.54	0.06	0.42	0.73	0.51	0.10	0.03	0.08	0.26

Panel B: Portfolio returns, CAPM, and Fama-French 3-factor model test

	Lo	2	3	4	Hi	Hi-Lo
Mean	5.27	6.03	6.52	8.16	9.98	4.71
Std	17.39	15.44	15.24	15.11	17.32	13.62
α	-1.32	0.01	0.78	2.67	4.07	5.40
	(-1.33)	(0.01)	(0.99)	(2.64)	(3.01)	(2.56)
MKT	1.07	0.98	0.93	0.89	0.96	-0.11
	(45.95)	(56.12)	(41.27)	(29.69)	(23.50)	(-1.92)
α	1.19	0.19	-0.04	0.23	0.33	-0.85
	(2.02)	(0.34)	(-0.06)	(0.34)	(0.38)	(-0.80)
MKT	1.01	0.97	0.96	0.95	1.01	-0.01
	(71.42)	(59.44)	(46.91)	(51.62)	(42.80)	(-0.26)
HML	-0.49	-0.04	0.17	0.47	0.68	1.17
	(-17.24)	(-1.45)	(3.59)	(10.52)	(14.76)	(21.69)
SMB	-0.10	0.03	-0.01	0.11	0.32	0.42
	(-4.54)	(1.27)	(-0.15)	(4.61)	(9.48)	(12.16)

Table 4: The cyclicality of aggregate revenues, costs, and gross profits

This table reports the cyclicality of aggregate revenues, costs, and gross profits. The price indexes and values of aggregate revenues, costs, and gross profits are from the NBER-CES Manufacturing Industry Database. Revenues (Rev) are measured as the total value of shipments. Variable costs (COGS) are defined as the sum of costs for materials (Mat), energy (Eng), and production worker wages (Prd). As a comparison, we also consider office worker wages (Off). These prices and values are further deflated by the Consumer Price Index. Panel A reports the mean and standard deviation of the ratio of each type of costs and gross profits to total revenues, as well as the annual growth rates of revenues ($\Delta\log\mathrm{V(Rev)}$), material costs ($\Delta\log\mathrm{V(Mat)}$), energy costs ($\Delta\log\mathrm{V(Eng)}$), the total variable costs ($\Delta\log\mathrm{V(COGS)}$), gross profits ($\Delta\log\mathrm{V(GP)}$), and office worker wages ($\Delta\log\mathrm{V(Off)}$) in percentages. Panel B reports the correlation matrix of the growth rates in the values of aggregate revenues, COGS, and gross profits. Panel C reports the elasticity of V(COGS) and V(GP) with respect to V(Rev), $\beta^{\mathrm{V}}_{\mathrm{Rev}}$(COGS) and $\beta^{\mathrm{V}}_{\mathrm{Rev}}$(GP), and the elasticity of P(COGS) with respect to P(Rev), $\beta^{\mathrm{P}}_{\mathrm{Rev}}$(COGS), which are estimated from univariate time-series regressions. The Newey-West t-statistics given in parentheses control for heteroskedasticity and autocorrelation. The data are annual from 1958 to 2011.

Panel A: Summary statistics

	V(Mat)/V(Rev)	V(Eng)/V(Rev)	V(Prd)/V(Rev)	V(COGS)/V(Rev)	V(Off)/V(Rev)
Mean (%)	54.83	1.93	10.80	67.56	6.60
Std (%)	2.11	0.41	2.87	3.62	0.87

	$\Delta\log\mathrm{V(Rev)}$	$\Delta\log\mathrm{V(Mat)}$	$\Delta\log\mathrm{V(Eng)}$	$\Delta\log\mathrm{V(Prd)}$	$\Delta\log\mathrm{V(COGS)}$	$\Delta\log\mathrm{V(GP)}$	$\Delta\log\mathrm{V(Off)}$
Mean (%)	1.87	1.96	20.8	0.0	1.69	2.27	0.86
Std (%)	5.47	6.73	7.95	4.99	6.36	4.79	3.27

Panel B: Correlation matrix

	$\Delta\log\mathrm{V(Rev)}$	$\Delta\log\mathrm{V(COGS)}$	$\Delta\log\mathrm{V(GP)}$
$\Delta\log\mathrm{V(Rev)}$	1.00	0.98	0.83
$\Delta\log\mathrm{V(COGS)}$	0.98	1.00	0.71
$\Delta\log\mathrm{V(GP)}$	0.83	0.71	1.00

Panel C: Elasticity of variable costs and gross profits

	$\beta^{\mathrm{V}}_{\mathrm{Rev}}$(COGS)	$\beta^{\mathrm{V}}_{\mathrm{Rev}}$(GP)	$\beta^{\mathrm{P}}_{\mathrm{Rev}}$(COGS)
Est.	1.14	0.73	1.47
t-stat	(26.83)	(9.06)	(24.52)

Table 5: Cash flow elasticities at aggregate and firm levels

This table reports the cash flow elasticities at the aggregate level and the firm level. In Panel A1, we report the mean and standard deviation of aggregate-level sales growth (ΔlogASale) and aggregate-level gross profits growth (ΔlogAGP). In Panel A2, we estimate the elasticity of AGP with respect to ASale by running the time series regression: $\Delta\text{logAGP} = a + b \times \Delta\text{logASale}$. In Panel B1, we report the mean and standard deviation of the firm-level sales growth (ΔlogSale) and firm-level gross profits growth (ΔlogGP). In Panel B2, we report the elasticity of GP with respect to Sale by running value-weighted Fama-MacBeth regressions $\Delta\text{logGP}_{it} = a_t + b_t \times \Delta\text{logSale}_{it}$ using lagged revenue as the weight and report the time series average of b_t. The Newey-West t-statistics in Panels A2 and B2 control heteroskedasticity and autocorrelation. The sample is annual from 1964 to 2018.

Panel A: Sales growth and gross profits growth at the aggregate level

Panel A1

	ΔlogASale	ΔlogAGP
Mean (%)	2.77	3.08
Std (%)	5.99	5.33

Panel A2

β_{ASale}(AGP)	
Est.	0.77
t-stat	(12.83)

Panel B: Sales growth and gross profits growth at the firm level

Panel B1

	ΔlogSale	ΔlogGP
Mean (%)	7.85	7.01
Std (%)	34.28	38.45

Panel B2

β_{Sale}(GP)	
Est.	1.07
t-stat	(12.97)

Table 6: Cash flow betas of GP/A portfolios within Fama and French 30 industries This table reports the cash flow exposures of the quintile GP/A portfolios within Fama and French 30 industries to the aggregate profitability shock, which is measured by the return of the long-short within-industry GP/A quintiles. We regress the cumulative growth rate of gross profits (Panel A), sales (Panel B), and cost of goods sold (Panel C) of these quintiles from year t to $t+K$ onto the aggregate profitability shock in year t. We consider K = 0, 1, and 2, where K = 0 corresponds to contemporaneous annual regressions. To facilitate interpretations, we standardize the aggregate profitability shock to have a unit standard deviation. Newey-West t-statistics given in parentheses control for heteroskedasticity and autocorrelation. The sample is from 1964 to 2018.

Panel A: Exposures of gross profits

K =	Lo	2	3	4	Hi	Hi-Lo
0	-4.02	-0.49	0.18	0.16	0.87	4.89
	(-1.97)	(-0.67)	(0.25)	(0.27)	(2.49)	(2.38)
1	-6.70	-1.11	-1.46	-0.15	-0.13	6.58
	(-3.20)	(-0.94)	(-1.23)	(-0.14)	(-0.15)	(4.10)
2	-6.74	-1.05	-1.52	0.43	-0.07	6.67
	(-3.11)	(-0.88)	(-1.35)	(0.32)	(-0.09)	(3.39)

Panel B: Exposure of sales

K =	Lo	2	3	4	Hi	Hi-Lo
0	-0.27	1.04	1.23	1.12	1.17	1.45
	(-0.47)	(1.91)	(2.05)	(2.90)	(2.18)	(2.65)
1	-1.25	0.56	-0.08	0.62	0.00	1.25
	(-1.33)	(0.59)	(-0.08)	(0.74)	(0.00)	(1.75)
2	-1.36	-0.10	-0.50	0.93	-0.28	1.08
	(-1.20)	(-0.07)	(-0.37)	(0.69)	(-0.28)	(1.37)

Panel C: Exposures of cost of goods sold

K =	Lo	2	3	4	Hi	Hi-Lo
0	0.46	1.66	1.80	1.76	1.38	0.92
	(0.72)	(2.75)	(2.69)	(2.90)	(1.99)	(2.05)
1	-0.13	1.25	0.64	1.08	0.14	0.28
	(-0.15)	(1.30)	(0.64)	(1.34)	(0.13)	(0.33)
2	-0.27	0.31	0.00	1.24	-0.32	-0.05
	(-0.23)	(0.21)	(0.00)	(0.85)	(-0.25)	(-0.06)

Table 7: Stock return betas of GP/A portfolios within Fama and French 30 industries This table reports the stock return betas of the quintile GP/A portfolios within Fama and French 30 industries to the aggregate profitability shock. We use the aggregate utilization-adjusted total factor productivity shock (dTFP) from Basu, Fernald, and Kimball (2006) and Fernald (2014) as the measure of aggregate profitability shock. The top panel reports the stock return exposure of the within-industry GP/A quintiles to the contemporaneous market factor and dTFP in the subsequent year (dTFP(+1)) from the two-factor model time-series regressions. The bottom panel reports the stock return exposures to the contemporaneous market factor, value premium factor, size premium factor, and dTFP in the subsequent year (dTFP(+1)) from the four-factor model time-series regressions. Newey-West *t*-statistics given in parentheses control for heteroskedasticity and autocorrelation. The sample is from 1964 to 2018.

	Lo	2	3	4	Hi	Hi-Lo
Cons.	-0.44	0.73	0.77	-0.07	0.50	0.94
	(-0.42)	(1.18)	(1.77)	(-0.19)	(0.89)	(0.70)
MKT	1.10	1.00	0.95	0.97	0.96	-0.14
	(19.72)	(32.03)	(31.91)	(33.25)	(33.79)	(-1.82)
dTFP(+1)	-1.84	-0.57	-0.59	0.24	1.77	3.60
	(-2.38)	(-1.38)	(-2.01)	(0.84)	(3.22)	(3.60)

Cons.	-2.34	0.55	0.43	0.01	1.84	4.18
	(-2.40)	(0.67)	(0.78)	(0.02)	(3.61)	(3.69)
MKT	1.13	0.98	0.98	0.99	0.94	-0.19
	(22.83)	(30.22)	(36.21)	(37.47)	(40.55)	(-3.09)
HML	0.23	0.01	0.06	0.00	-0.17	-0.40
	(3.33)	(0.16)	(2.23)	(-0.02)	(-2.04)	(-2.88)
SMB	0.12	0.09	-0.09	-0.06	-0.07	-0.19
	(1.98)	(1.75)	(-1.88)	(-2.09)	(-1.10)	(-1.98)
dTFP(+1)	-1.51	-0.59	-0.46	0.26	1.54	3.05
	(-1.98)	(-1.49)	(-1.62)	(1.00)	(3.22)	(3.21)

Table 8: Pricing of aggregate profitability shocks

This table reports the pricing of the aggregate profitability shock. We test a two-factor model with the market and an aggregate profitability shock measure as the risk factors, and use Fama and French 17 and 30 industry portfolios as the testing assets in GMM stochastic discount factor (SDF) tests. Panel A uses the contemporaneous long-short portfolio return from gross profitability quintiles within Fama and French 30 industries as the measure of aggregate profitability shock. Panel B uses the the aggregate utilization-adjusted total factor productivity shock (dTFP) (e.g., Basu, Fernald, and Kimball (2006) and Fernald (2014)) in the subsequent year as the measure of aggregate profitability shock. We normalize the intercept of the SDF to one and report the annualized mean absolute errors (MAE), the *p*-value associated with the over-identification test, the price of risk (b) from the first-stage GMM estimations. The sample is monthly from January 1964 to December 2018 in Panel A and is annual from 1964 to 2018 in Panel B.

Panel A: GP/A Prm.			Panel B: dTFP(+1)		
Industry	17	30	Industry	17	30
MAE	1.20	1.20	MAE	0.89	1.05
p-value	0.82	0.62	*p*-value	0.85	0.99
b(MKT)	4.19	4.44	b(MKT)	2.17	2.18
	(3.70)	(3.81)		(4.82)	(3.87)
b(GP/A Prm.)	5.97	6.71	b(dTFP(+1))	39.12	41.31
	(2.33)	(2.44)		(2.40)	(3.40)

Table 9: Parameter values

This table reports the parameter values used for the numerical analysis. The model is solved and simulated at a monthly frequency.

Parameter	Description	Value
γ_x	Price of risk for aggregate profitability shocks	15
γ_y	Price of risk for aggregate growth shocks	7
γ_s	Price of risk for aggregate investment shocks	-10
η	Elasticity of substitution between capital and variable inputs	0.3
δ	Depreciation rate	0.01
r_f	Risk-free rate	0.0025
$\bar{x}$	Unconditional aggregate profitability	0
ρ_x	Persistence of aggregate profitability shocks	0.98
σ_x	Conditional volatility of aggregate profitability shocks	0.04
σ_y	Conditional volatility of aggregate growth shocks	0.027
$\bar{s}$	Unconditional aggregate investment opportunity	-0.146
ρ_s	Persistence of aggregate investment shocks	0.9685
σ_s	Conditional volatility of aggregate investment shocks	0.026
$\bar{z}$	Unconditional idiosyncratic profitability	0.85
ρ_z	Persistence of idiosyncratic profitability shocks	0.97
σ_z	Conditional volatility of idiosyncratic profitability shocks	0.09
ρ_a	Persistence of idiosyncratic investment shocks	0.98
σ_a	Conditional volatility of idiosyncratic investment shocks	0.111
ρ_{az}	Correlation between idiosyncratic profitability and investment shocks	0.18
p_0	Logarithm of the level of variable inputs price	0.588
p_1	Cyclicality of variable inputs price w.r.t. aggregate profitability shock	1.39
ϕ	Leverage ratio	1.67

Table 10: Moments

This table reports the moments of interest from model simulation. The model is solved at a monthly frequency. 100 samples are simulated with each sample representing 600 months and 3,000 firms. Cross sample means of these moments are reported.

Moment	Data	Model
Average annual aggregate GP/A	0.24	0.21
AR(1) of aggregate annual GP/A	0.87	0.92
Average book-to-market ratio	0.52	0.42
AR(1) of aggregate book-to-market	0.91	0.89
Average annual aggregate investment rate	11.5%	11.1%
Standard deviation of annual aggregate investment rate	0.95%	0.996%
AR(1) of annual aggregate investment rate	0.69	0.72
Average aggregate sales - aggregate variable costs ratio	1.45	1.46
Volatility of aggregate sales - aggregate variable costs ratio	0.05	0.05
Volatility of aggregate variable costs growth/volatility of aggregate sales growth	1.12	1.14
Correlation between aggregate sales growth and aggregate variable costs growth	0.985	0.99
Value-weighted annual market premium	6.38%	7.50%
Value-weighted annual market volatility	16.27%	18.36%
Equal-weighted annual market premium	9.82%	8.10%
Equal-weighted annual market volatility	22.21%	18.04%
Cross-sectional std. of monthly firm-level stock returns	13.0%	13.2%

Table 11: GP/A portfolios: Model

Panel A of this table reports the characteristics, including the gross profitability (GP/A), log book-to-market ratio (logBM), idiosyncratic profitability (Z), idiosyncratic investment opportunity (A), gross profit margin (GM), and variable input-capital ratio (EK) of the GP/A quintile portfolios. Panel B reports the mean and standard deviation the value-weighted returns, and the asset pricing test results based on CAPM and a two-factor model with market (MKT) and the value premium factor (HML) as the risk factors. Panel C reports the risk factor exposures of the GP/A portfolios to the aggregate profitability shock $\beta(X)$, aggregate growth shock $\beta(Y)$, and aggregate investment shock $\beta(S)$. The portfolio returns, standard deviations, and the abnormal returns are annualized. The model is simulated for 100 samples, with each sample representing 3,000 firms and 600 months. The Newey-West t-statistics given in parentheses control for heteroskedasticity and autocorrelation.

Panel A: Portfolio characteristics

	GP/A	logBM	Z	A	GM	EK
Lo	0.00	-0.95	0.38	-0.36	0.02	0.07
2	0.03	-1.06	0.67	-0.28	0.11	0.28
3	0.11	-1.14	0.85	-0.23	0.19	0.47
4	0.25	-1.22	1.02	-0.18	0.28	0.63
Hi	0.64	-1.38	1.32	-0.10	0.41	0.84

Panel B: Portfolio returns and asset pricing tests

	Lo	2	3	4	Hi	Hi-Lo
Mean	5.89	6.21	6.76	7.58	9.15	3.26
Std	17.07	17.18	17.24	17.29	17.45	4.36
α	-1.28	-1.02	-0.50	0.30	1.82	3.10
	(-3.52)	(-2.77)	(-1.38)	(0.82)	(4.79)	(5.09)
MKT	0.99	1.00	1.00	1.00	1.01	0.02
	(156.47)	(156.56)	(159.51)	(159.28)	(155.94)	(2.03)
$R^2(\%)$	97.66	97.67	97.73	97.76	97.60	0.89
α	-1.66	-1.19	-0.53	0.39	2.13	3.79
	(-4.52)	(-3.14)	(-1.41)	(1.01)	(5.48)	(6.20)
MKT	1.00	1.00	1.00	1.00	1.00	0.00
	(152.58)	(148.73)	(150.60)	(149.71)	(146.77)	(-0.22)
HML	0.09	0.04	0.01	-0.02	-0.07	-0.16
	(4.90)	(2.31)	(0.38)	(-1.10)	(-3.78)	(-5.17)
$R^2(\%)$	97.79	97.70	97.74	97.77	97.68	7.15

Panel C: Risk exposures

	Lo	2	3	4	Hi	Hi-Lo
$\beta(X)$	0.03	0.04	0.06	0.09	0.15	0.12
	(3.15)	(4.65)	(6.18)	(9.33)	(15.86)	(10.47)
$\beta(Y)$	1.68	1.68	1.68	1.68	1.68	0.00
	(136.07)	(128.21)	(123.17)	(120.72)	(119.17)	(0.21)
$\beta(S)$	0.71	0.73	0.74	0.75	0.75	0.04
	(55.28)	(53.89)	(51.75)	(51.14)	(50.65)	(2.35)

Table 12: Book-to-market portfolios: Model

Panel A of this table reports the characteristics, including the gross profitability (GP/A), log book-to-market ratio (logBM), idiosyncratic profitability (Z), idiosyncratic investment opportunity (A), gross profit margin (GM), and variable input-capital ratio (EK) of the book-to-market (BM) quintile portfolios. Panel B reports the mean and standard deviation the value-weighted returns, and the asset pricing test results based on CAPM and a two-factor model with market (MKT) and the value premium factor (HML) as the risk factors. Panel C reports the risk factor exposures of the BM portfolios to the aggregate profitability shock $\beta(X)$, aggregate growth shock $\beta(Y)$, and aggregate investment shock $\beta(S)$. The portfolio returns, standard deviations, and the abnormal returns are annualized. The model is simulated for 100 samples, with each sample representing 3,000 firms and 600 months. The Newey-West t-statistics given in parentheses control for heteroskedasticity and autocorrelation.

Panel A: Portfolio characteristics

	GP/A	logBM	Z	A	GM	EK
Lo	0.35	-1.72	1.06	0.53	0.28	0.62
2	0.27	-1.31	0.97	0.04	0.24	0.56
3	0.20	-1.11	0.87	-0.25	0.21	0.48
4	0.14	-0.93	0.77	-0.52	0.17	0.39
Hi	0.07	-0.68	0.58	-0.95	0.11	0.23

Panel B: Portfolio returns and asset pricing tests

	Lo	2	3	4	Hi	Hi-Lo
Mean	5.39	7.77	8.13	8.46	8.60	3.21
Std	18.68	17.06	16.67	16.42	16.15	6.97
α	-2.41	0.57	1.11	1.57	1.87	4.28
	(-4.85)	(1.94)	(3.44)	(4.17)	(4.09)	(4.71)
MKT	1.07	0.99	0.97	0.95	0.93	-0.15
	(126.64)	(198.21)	(174.78)	(147.64)	(118.88)	(-9.48)
$R^2(\%)$	96.37	98.51	98.09	97.32	95.90	13.14
α	-0.17	0.10	0.14	0.20	-0.17	0.00
	(-1.14)	(0.35)	(0.54)	(0.82)	(-1.14)	
MKT	1.00	1.01	1.00	1.00	1.00	0.00
	(377.05)	(200.49)	(221.16)	(227.38)	(377.05)	
HML	-0.52	0.11	0.23	0.32	0.48	1.00
	(-74.05)	(8.37)	(19.54)	(28.11)	(67.43)	
$R^2(\%)$	99.69	98.69	98.88	98.91	99.59	100.00

Panel C: Risk exposures

	Lo	2	3	4	Hi	Hi-Lo
$\beta(X)$	0.09	0.09	0.08	0.06	0.05	-0.04
	(7.82)	(12.74)	(12.95)	(11.69)	(9.67)	(-3.58)
$\beta(Y)$	1.68	1.68	1.68	1.68	1.68	0.00
	(100.49)	(160.24)	(183.31)	(202.48)	(227.95)	(0.19)
$\beta(S)$	1.06	0.71	0.60	0.52	0.42	-0.63
	(59.74)	(64.85)	(63.80)	(61.33)	(57.33)	(-35.08)

Table 13: Portfolios sorted by Z and A: Model

Panel A of this table reports the mean and standard deviation the value-weighted returns, and the asset pricing test results based on CAPM, and the risk factor exposures to the aggregate profitability shock $\beta(X)$, aggregate growth shock $\beta(Y)$, and aggregate investment shock $\beta(S)$ of portfolios sorted by firm-specific profitability Z and portfolio sorted by firm-specific investment opportunity A. The portfolio returns, standard deviations, and the abnormal returns are annualized. The model is simulated for 100 samples, with each sample representing 3,000 firms and 600 months. The Newey-West t-statistics given in parentheses control for heteroskedasticity and autocorrelation.

Panel A: Portfolios sorted by Z

	Lo	2	3	4	Hi	Hi-Lo
Mean	5.76	6.19	6.59	7.46	9.44	3.68
Std	17.06	17.18	17.27	17.29	17.44	4.43
α	-1.41	-1.03	-0.68	0.18	2.11	3.52
	(-3.79)	(-2.77)	(-1.84)	(0.50)	(5.47)	(5.67)
MKT	0.99	0.99	1.00	1.00	1.01	0.02
	(154.03)	(156.11)	(155.98)	(161.66)	(153.51)	(2.03)
$R^2(\%)$	97.58	97.58	97.65	97.80	97.52	0.92
$\beta(X)$	0.03	0.03	0.05	0.08	0.16	0.13
	(3.17)	(3.98)	(5.61)	(9.00)	(16.87)	(11.28)
$\beta(Y)$	1.68	1.68	1.68	1.68	1.68	0.00
	(136.37)	(128.88)	(122.06)	(120.93)	(118.88)	(0.24)
$\beta(S)$	0.71	0.73	0.74	0.75	0.75	0.04
	(55.64)	(53.09)	(51.08)	(51.54)	(50.51)	(2.08)

Panel B: Portfolios sorted by A

	Lo	2	3	4	Hi	Hi-Lo
Mean	9.85	9.02	8.19	7.48	4.76	-5.09
Std	16.12	16.39	16.67	17.09	18.76	7.34
α	3.15	2.15	1.16	0.26	-3.07	-6.21
	(6.49)	(5.55)	(3.61)	(0.89)	(-5.92)	(-6.47)
MKT	0.92	0.95	0.97	0.99	1.08	0.15
	(111.96)	(141.96)	(174.53)	(199.87)	(121.93)	(9.47)
$R^2(\%)$	95.40	97.12	98.09	98.54	96.10	13.11
$\beta(X)$	0.08	0.08	0.08	0.08	0.07	-0.01
	(16.27)	(15.07)	(13.49)	(11.26)	(6.41)	(-0.65)
$\beta(Y)$	1.68	1.68	1.68	1.68	1.68	0.00
	(224.66)	(204.38)	(185.61)	(158.22)	(100.03)	(-0.27)
$\beta(S)$	0.39	0.50	0.60	0.72	1.08	0.69
	(52.59)	(60.61)	(65.06)	(65.74)	(60.58)	(38.10)

Figure 1: Value functions

This figure plot the value function of the total firm value (Value), the value of assets-in-place (VAP), the value of growth opportunities (VGO), gross profitability (GP/A), and optimal variable input (E) against aggregate profitability (x), aggregate investment opportunities (s), idiosyncratic profitability (z), and idiosyncratic investment opportunities (a).