What Explains the Gender Gap Reversal in Educational Attainment?[*]

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Abstract

The reversal of the gender gap in educational attainment is becoming a global phenomenon. Its drivers, however, are not well understood and remain largely untested empirically. This paper develops a unified conceptual framework that allows to formulate and test two main hypotheses for the reversal. It introduces the tail hypothesis, which builds on the lower dispersion of scholastic performance among females observed globally. It also formalizes the mean hypothesis, which states that females' average scholastic performance and returns to education have increased over time relative to males'. The paper theoretically shows that both hypotheses can explain the reversal in our framework. The parameters of the two hypotheses derived from the model are then estimated using educational attainment data from 1950 to 2010 in more than 100 countries. We find that both hypotheses strongly predict the gender gap dynamics in educational attainment when estimated separately. When jointly estimated, accounting for the tail hypothesis significantly increases the model's predictive power. The contribution of the tail hypothesis to the gender gap reversal is particularly strong in developing economies, while the mean hypothesis appears to prevail in high-income countries.

Keywords: Educational attainment, gender gap, gender differences in scholastic performance.

JEL-Codes: I20, J16

1   Introduction

The dramatic increase in educational attainment over the past decades was accompanied by a striking and puzzling phenomenon. As individuals stayed longer in school over time, females not only caught up with males' education levels, but progressively attained higher levels of schooling. This phenomenon, sometimes referred to as the gender gap reversal in education, already took place in virtually all high-income countries. It is also observed in a growing majority of low and middle-income countries. Although the reversal is becoming close to universal, its origins are not well understood.

A lot of attention in the literature has been devoted to explaining the gender gap in labor market market outcomes. In contrast, the gender gap reversal in educational attainment has been under-studied. A few contributions, confined to the US context, have proposed possible explanations for this phenomenon (Goldin, Katz and Kuziemko 2006; Cho 2007; Chiappori, Iyigun and Weiss 2009 and Fortin, Oreopoulos and Phipps 2015). However, to the best of our knowledge, there has been no attempt to model and test alternative hypotheses for this reversal using a unified conceptual framework. This paper aims at filling this gap.

Understanding the origins of the educational gender gap reversal is important in itself, but also to explain gender dynamics in other areas, particularly in the labor market. For policy purposes, it is also important to identify whether differences in educational outcomes between genders originate from distortions such as social barriers and discrimination or, instead, from optimizing behaviors based on potential gender differences in preferences or traits.[1] Learning about the origins of the gender gap reversal can help determine whether policy interventions could help addressing the growing disadvantage of boys in school, and identify potential areas of interventions.

The paper starts by establishing that the gender gap reversal in education is becoming a global phenomenon. Using time-series on enrollment and completion rates in education by gender in more than 100 countries, we show that the reversal occurred in virtually all high-income countries, but also in a rapidly growing proportion of lower-income countries. We also evidence that the reversal

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[1]For a summary of evidence on gender differences in preferences, see Croson and Gneezy (2009).

is not confined to tertiary enrollment as evidenced by previous literature, but also took place at the primary and secondary level.[2] While females have historically represented the minority of tertiary, secondary and primary education completers, they have become the majority over time.

The second contribution of the paper is to develop a simple conceptual framework to account for the gender gap reversal in educational attainment. In our framework, differences in the distributions of scholastic performance between genders play a central role.[3] The model builds on the micro foundations of investment in schooling of Card (1994), before aggregating individual schooling decisions to the macro level. It has three building blocks. First, at the micro level, the optimal number of years of schooling attained by individuals increases with their scholastic performance. Second, the benefits of investing in schooling are allowed to vary over time and between genders. Third, we introduce two alternative assumptions regarding the distribution of scholastic performance by gender. These distributional assumptions, together with an increase in the economy-wide benefits of investing in schooling, generate either a tail dynamics or a mean dynamics, depending on the assumption made. Our model allows to combine these two dynamics within a unified conceptual framework.

A central contribution of the paper is to propose and model the tail dynamics hypothesis within our conceptual framework, and to show that it can generate the gender gap reversal in education. A key result of our model under the tail hypothesis is that a higher variance in scholastic performance among males is sufficient to produce the gender gap reversal, independently of the relative mean scholastic performance of males and females. We show that the tail hypothesis implies first a gender gap reversal in favor of females, but ultimately gender equality in educational attainment as the net benefits of education increase over time and enrollment becomes universal. The hypothesis is novel in the literature and builds on the findings of Machin and Pekkarinen (2008) who find that girls exhibit a lower variance in mathematics and reading test scores relative to boys in virtually all OECD countries.[4] It also formalizes the intuition of Becker, Hubbard and Murphy (2010a) and

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[2]The latter applies mostly to low-income countries.

[3]In the remainder of the paper, scholastic performance refers to students' performance in achievement tests as measured by standardized test scores, while educational attainment refers to the highest level of schooling attained by an individual, e.g secondary school completion.

[4]There also exists an abundant literature in psychology reporting similar findings for different dimensions of abilities or traits, such as Frasier (1919), Hedges and Nowell (1995), or Jacob (2002), among many others.

Becker, Hubbard and Murphy (2010*b*), who suggest that the lower variance of non-cognitive skills among females can generate a higher elasticity of enrollment to returns to schooling.

We also model the mean dynamics hypothesis within our framework. Contrary to the tail hypothesis, the predictions of the mean hypothesis depend directly on the dynamics of gender-specific means, and gender equality in educational attainment is only achieved if gender-specific means are the same. The mean dynamics hypothesis can be modeled in two alternative ways within our framework. It first can be formalized through the mean benefit hypothesis (MBH), which posits that the net benefits of schooling for females have increased more rapidly over time than for males (Goldin, Katz and Kuziemko 2006 and Chiappori, Iyigun and Weiss 2009). Second, it can be modeled through the mean performance hypothesis (MPH), which claims that the mean performance of females in achievement tests has increased over time relative to males' (Cho 2007). We show that both hypotheses are algebraically similar in our framework, and can produce a reversal of the gender gap in educational attainment.

A final contribution of the paper is empirical. Building on our conceptual framework, we are able to estimate the respective contributions of the two hypotheses to the gender gap reversal. This is, to the best of our knowledge, the first paper that formally quantifies the contributions of alternative hypotheses within a unified framework. Using our model, we show that each of the mean and tail dynamics hypothesis can be expressed by a reduced-form equation and summarized by two key parameters. We then fit each hypothesis using harmonized time-series data on educational attainment by gender from 1950 to 2010 in a large sample of countries. We estimate each hypothesis' parameters derived from our theoretical framework first separately, and then jointly.

We find that the model under either parameter strongly predicts the dynamics of the gender gap in tertiary enrollment in the full sample. Accounting for both parameters in the estimation strongly increases predictive power, as opposed to specifications with only one of the two hypotheses parameters. For secondary school completion, however, the predictive power of the mean hypothesis is lower than that of the tail hypothesis in the full sample. Once we split our sample between developing and high-income countries, we find that the respective explanatory power of the two hypotheses varies in the two groups. While the tail hypothesis predominates in the developing

country sample, the mean hypothesis has stronger explanatory power in the high-income country sample.

Our findings indicate that while the tail and mean dynamics have both been at play, the former has a strong and independent explanatory power that has not been uncovered by previous literature, especially in the context of developing economies. Our model estimates for the tail hypothesis parameter are also close to the male-to-female variance ratio in test scores estimated from international student assessments. In addition, our model parameter estimates are fairly robust across specifications. This provides further comfort on the validity of the tail hypothesis in accounting for the gender gap reversal in educational attainment.

The paper is organized as follows. Section 2 presents empirical facts on the gender gap reversal in educational attainment that motivate the paper. Section 3 lays out our conceptual framework. Section 4 formulates two main hypotheses for the gender gap reversal within this framework. Section 5 describes our empirical methodology to estimate the contribution of the two hypotheses to the gender gap reversal. Section 6 presents our empirical results and discusses their implications. Section 7 concludes.

2   Empirical Motivation

2.1   Data

The data used throughout the paper is from the Barro-Lee educational attainment database. The dataset consists of harmonized country-level data on educational attainment constructed from nationally representative surveys and census data. The data is reported at five-year intervals from 1950 and 2010. It is also disaggregated into five-year age groups. The availability of age-specific data is valuable in our context, as it allows to construct the dynamics of educational attainment of specific age cohorts over time. Educational attainment is also reported separately for males and females, which allows to follow the evolution of educational attainment across cohorts by gender. Time-series of educational attainment are available for a total of 146 countries in the database,

which include 36 OECD economies and 110 non-OECD countries.[5] Educational attainment of the adult population in the database is broken down into seven levels of schooling: no formal education, incomplete primary, complete primary, lower secondary, upper secondary, incomplete tertiary, and complete tertiary.

To describe time trends in educational attainment by gender, we focus on three main levels of educational attainment: primary school completion, secondary school completion, and enrollment in tertiary education. As the data is reported every five years from 1950 to 2010, we have a maximum of 13 data points per country, for a given level of educational attainment. We exclude eight countries from the original Barro-Lee dataset, due to highly inconsistent data and missing values on educational attainment.[6]

We compute completion and enrollment rates for 5-year-band birth cohorts born from 1921 to 1990, by gender.[7] We measure the enrollment rate in tertiary education at time t as the share of the population age 25-29 that has ever enrolled in tertiary education. The secondary education completion rate is measured as the share of the population age 20-24 at time t that has completed secondary education and above. Finally, primary education completion rates are measured as the share of the population age 15-19 at time t that has completed primary education and above.[8] The female-to-male ratio is measured as the enrollment or completion rate of females over that of males for a given level of educational attainment.

Table I displays the summary statistics of enrollment and completion rates for primary, secondary and tertiary education in OECD and non-OECD countries from the Barro-Lee database in 1950 and 2010. As documented in Barro and Lee (2013) and Lee and Lee (2016), the data shows a large increase in educational attainment over the period covered. In addition, and more importantly in the context of the paper, enrollment and completion rates have increased more rapidly for females

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[5]In case of missing observations over time, data points were obtained by forward and backward extrapolation of the census/survey observations on attainment. See Barro and Lee (2013) and Lee and Lee (2016) for a complete description of the dataset construction and methodology.

[6]Excluded countries are Bostwana, Burundi, Cameroon, Gabon, Guyana, Mozambique, Libya and Yemen.

[7]For primary education, we compute completion rates for individuals born between 1936 to 1995, from 1931 to 1990 for secondary school completion and from 1921 to 1980 for tertiary education.

[8]The choice of these age brackets was motived by the fact that individuals would be expected to have enrolled in or completed the corresponding level of education by these ages, if they ever did.

than for males for all three levels of educational attainment. This pattern is observed in both OECD and non-OECD countries.

2.2 The Gender Gap Reversal in Educational Attainment

Several papers have reported a convergence followed by a reversal in the relative number of females attending tertiary education in the US (Charles and Luoh 2003; Goldin, Katz and Kuziemko 2006, Chiappori, Iyigun and Weiss 2009; Becker, Hubbard and Murphy 2010*a*; Becker, Hubbard and Murphy 2010*b*; Autor and Wasserman 2013).[9] In table II and Figure I, we look at the evolution of the female-to-male ratio in tertiary enrollment worldwide, by grouping country-level data into seven world regions.[10] As shown in Column 5 and 6 of Table II, young males in 1950 used to outnumber females among participants to tertiary education in virtually all countries in the sample (92%). In contrast, by 2010, 73% of countries in the sample had seen their gender imbalances in tertiary enrollment revert from a male majority to a female majority. Even among non-OECD countries, 66% had experienced the gender gap reversal in tertiary enrollment by 2010. In advanced economies, Europe and Central Asia and Latin America, the share of countries where females outnumber males among tertiary education students is over 90%. In contrast, most countries in South Asia and Sub-saharan Africa have yet to experience the gender gap reversal in tertiary enrollment.

When averaged over all countries in the sample, the Barro-Lee data show a reversal in the gender gap in tertiary enrollment, as reported in Panel A of Figure I. For the oldest cohorts born in the 1920s, the average female-to-male ratio in tertiary enrollment for all countries in the sample was around 0.40, meaning that for ten females enrolled in tertiary education, only four females were enrolled. This ratio was fairly similar across all regions of the world. In contrast, among the most recent cohorts, the average female-to-male ratio had reached 1.22 globally, meaning that there were 22% more females enrolled in tertiary education than males worldwide.

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[9]Pekkarinen (2012) has also reported this phenomenon for Scandinavian countries.

[10]This regional grouping follows the World Bank classification of world regions and includes: advanced economies, Latin American and the Caribean, Europe and Central Asia, East-Asia an Pacific, the Middle-East and North Africa, South Asia and Sub-saharan Africa

Out of the seven world regions, five have already crossed the horizontal line of a female-to-male ratio of 1, meaning that they experienced the gender gap reversal in tertiary enrollment. Sub-saharan African and South Asia are the only two regions that have yet to experience the reversal, with a female-to-male ratio in tertiary enrollment remaining below one despite a noticeable increase across cohorts.

Panel A of Figure I also shows that the timing of the reversal varies across regions. The gender gap reversal in tertiary enrollment occurred first in Europe and Central Asia for cohorts born at the beginning of the 1950s. In advanced economies and Latin America American countries, this reversal occurred first for cohorts born around 1960. Among countries in East Asia and the Middle-East and Northern Africa, the reversal only took place among cohorts born in the late 1970s.

Table II and Figure I show that the global gender gap reversal in educational attainment is not limited to tertiary education enrollment. The gender composition of secondary school completers has also reversed from a male majority to a female majority over time. While young females used to outnumber males among secondary school non-completers in only 6% of the countries in the sample in 1950, females now represent the majority of secondary school completers in 71% of countries in the sample. As for tertiary enrollment, the gender gap reversal in secondary school completion already occurred in five out of the seven world regions. The share of countries that have experienced the reversal is highest in advanced economies (92%) and lowest in Subsaharan Africa (38%). As for tertiary enrollment, Europe and Central Asia is the first region to have experienced the gender gap reversal in secondary school completion.

Finally, we also observe a closure and reversal of the gender gap in primary school completion, although patterns are less clear cut than for secondary and tertiary education. As shown in Panel C of Figure I, the female-to-male ratio in primary completion averaged over all countries in the sample was 0.82 for the oldest cohorts born at the beginning of the 1930s, but had reached a mean of 1.03 among cohorts born at the beginning of the 1990s. When disaggregated by regions, all regions have reached female-to-male ratios in primary school completion around or slightly above one for the latest cohorts, with the exception of South Asia. Similarly, a majority of countries have experienced the gender gap reversal in primary school completion in most regions (Table II).

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The extent of the reversal is however smaller than for secondary school completion and tertiary enrollment. Among advanced economies in particular, the share of countries that experienced the gender gap reversal in primary school completion has hardly changed from 1950 to 2010.

The fact that the gender gap reversal is not as clear cut at the primary level could be explained by several factors. First, in many countries in our sample, primary school completion has been close to universal already for some time. As a result, there has been little variation in the gender ratio which remained around one over our period of study. This is the case for advanced economies, Europe and Central Asia and Latin America (Figure I). A related explanation is that primary schooling has been mandatory for some time for many countries in the sample, which leaves little room for variation in the gender ratio in primary school completion over time. As result, primary school completion is mainly driven by institutional features rather than optimal decisions of investment in schooling by gender.

3 Theoretical Framework

3.1 Micro Foundations

We now develop a conceptual framework that allows to formulate alternative hypotheses for the reversal. The economy is assumed to be populated by of mass individuals that differ in their level of scholastic performance z, which is continuous and perfectly observed by individuals.[11] For the sake of simplicity, a single-period model is assumed where individuals receive the benefits of their investment in schooling in the same period as they invest. Individuals choose years of schooling s to maximize their level of utility U. Building on Card (1994), we simply express the utility function of individuals as:

$$U(s) = B(s) - C(s),$$

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[11] Scholastic performance can be interpreted as the product of a complex combination of individual ability as well as other inputs such as effort and motivation, which can empirically be measured by test scores (Heckman and Kautz, 2012)

where $B(s)$ is the benefit function of schooling, with $B'(s) > 0$ and $B''(s) < 0$. $C(s)$ is the cost function of schooling, which is assumed to be increasing and convex in s. The first-order conditions for the individual maximization problem read as:

$$B'(s) = C'(s),$$

where $B'(s)$ and $C'(s)$ denote the marginal benefits and costs of schooling, respectively. Following Card (1994), we linearize the model by assuming that $B'(s)$ and $C'(s)$ are linear functions of s, with $B'(s)$ having an individual-specific intercept:

$$B'(s) = z_j - k_1 s$$

$$C'(s) = k_2 s,$$

where $k_1 > 0$ and $k_2 > 0$. Intuitively, individuals with higher levels of scholastic performance z_j get higher marginal benefits from schooling.[12] In this framework, the optimal level of schooling s chosen by individual j can be expressed as:

$$s_j^* = z_j \cdot b, \tag{1}$$

where $b \equiv \frac{1}{k_1+k_2}$ is an exogenous technology parameter, which we call net benefits of schooling (hereafter: net benefits). It is identical for all individuals in the economy. b captures the monetary and non-monetary benefits and costs of schooling in the economy. The optimal level of schooling chosen by individual j is therefore strictly increasing in individual scholastic performance z_j.

In this framework, the minimum level of scholastic performance $\bar{z}$ so that individuals attain a given level of schooling $\bar{s}$, such as tertiary education, can be expressed as:

$$\bar{z} = \frac{\bar{s}}{b} \tag{2}$$

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[12]In the original model of Card (1994), the expression for C'(s) also includes an individual-specific intercept, capturing individual-specific circumstances such as access to wealth and network or taste for education. For the sake of simplicity, we abstract from this distinction in our model.

Equation 2 states that individuals whose scholastic performance is below the threshold $\bar{z}$ attain an optimal level of schooling below $\bar{s}$, while individuals whose scholastic performance is equal to or greater than $\bar{z}$ attain $\bar{s}$ and above. It also implies that the scholastic performance threshold is determined by b, common to all individuals in a given cohort. An immediate implication of Equation 2 is:

$$\frac{\partial \bar{z}}{\partial b} \leq 0.$$

In words, the minimum level of scholastic performance required to attain a given level of schooling $\bar{s}$, such as tertiary education, decreases with the net benefits of investing in schooling in the economy.

3.2 Aggregate Educational Attainment and Gender Ratio

We now assume that the economy is populated by successive cohorts. Each cohort consists of a continuum of agents that invest in schooling and differ in their level of scholastic performance z. Let $f_z(\bar{z})$ denote the probability density function of scholastic performance z in the population of a given cohort. The complementary cumulative distribution function of z can be expressed as:

$$G_z(\bar{z}) = \int_{\bar{z}}^{+\infty} f_z(\bar{z}) \, d(\bar{z}).$$

All individuals belonging to the same cohort are exposed to the same value of the exogenous parameter $b_t \equiv \frac{1}{k_{1,t} + k_{2,t}}$, irrespective of their scholastic performance. Given the micro properties of the model summarized in Equation (1) and (2), the share of individuals in the cohort that attain a level of schooling of at least $\bar{s}$ is given by:

$$P(\bar{z}) = 1 - F_z(\frac{\bar{s}}{b}) = G_z(\frac{\bar{s}}{b}) = G_z(\bar{z}). \tag{3}$$

Equation (3) states that the mass of individuals that attain a level of schooling of at least $\bar{s}$ or higher, for a given value of b, is made of all individuals whose scholastic performance is above the scholastic performance threshold $\bar{z}$. Exogenously to individual schooling decisions, b varies across

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cohorts. Equation (3) implies:

$$\frac{\partial P(\bar{z})}{\partial b} \geq 0.$$

The share of individuals that attain a level of schooling equal or higher than $\bar{s}$ therefore increases with the net benefits of schooling b. One key implication at the aggregate level is that, as the net benefits of schooling rise, educational attainment increases and the mean scholastic performance of individuals that attain a given level of schooling decreases. In Appendix A1.3, we show hat this result holds empirically, using US data.

We now introduce the possibility that $G_z(.)$ and $\bar{z}$ differ between genders. Let z_m and z_f be random variables that denote the scholastic performance of males and females, respectively. The scholastic performance threshold $\bar{z}$ is also allowed to vary by gender and denoted $\bar{z}_m$ and $\bar{z}_f$ for males and females, respectively. In our framework, the fraction of the population in the cohort that attains a given level of schooling $\bar{s}$, assuming the total population of males and females is the same, is given by:

$$P(\cdot) \equiv \frac{G_{z_f}(\bar{z}_f) + G_{z_m}(\bar{z}_m)}{2} \tag{4}$$

and the female-to-male ratio among individuals that choose a given level of schooling, denoted $R(\bar{z})$, can be expressed as:

$$R(\cdot) \equiv \frac{G_{z_f}(\bar{z}_f)}{G_{z_m}(\bar{z}_m)} \tag{5}$$

The implications of our model at the aggregate level are fully summarized by Equation 4 and 5. In the remainder of the paper, G_{z_f} and G_{z_m} are assumed to be characterized by their first two moments, where μ and σ^2 denote the mean and variance of the distributions, respectively.

4 Two Main Hypotheses for the Gender Gap Reversal

Within this theoretical framework, we formulate two main hypotheses for the gender gap reversal: the tail dynamics hypothesis (or tail hypothesis), and the mean dynamics hypothesis (or mean hypothesis). The mean dynamics hypothesis itself can be formalized in two alternative ways:

through the mean performance hypothesis (MPH), or through the mean benefit hypothesis (MBH). As shown in Section A3.2, these two sub-hypotheses are algebraically similar within our theoretical framework. We therefore group them together under the mean dynamics hypothesis. Table III summarizes the underlying assumptions of the different hypotheses.

The tail hypothesis states that $G_z(\cdot)$ is gender specific with $\sigma_m > \sigma_f$. In words, the scholastic performance of males and females are characterized by different distributions, and in particular a greater variance among males. The net benefits of schooling b are assumed to be identical between genders. The increase in b over time (or, equivalently, the decrease in the scholastic performance threshold $\bar{z}$) combined with a greater dispersion of scholastic performance among males, produces the reversal in educational attainment.

The mean benefit hypothesis (MBH) claims that the net benefits of schooling differ between genders, i.e. there exist gender-specific $\bar{z}_f$ and $\bar{z}_m$ (or, equivalently in our framework, b_f and b_m) that have different dynamics over time. Prior to the reversal, $\bar{z}_f > \bar{z}_m$ (or, equivalently in our framework, $b_f < b_m$) before $\bar{z}_m$ progressively converges towards $\bar{z}_f$ and surpasses it over time, generating the reversal. The distributions of scholastic performance $G_z(\cdot)$ are assumed to be identical for males and females.

The mean performance hypothesis (MPH) claims that the average scholastic performance of females μ_f has increased more rapidly than for males over time and ultimately surpassed μ_m, producing the gender gap reversal. The variance of z and the net benefits of schooling (or, equivalently, the scholastic performance threshold $\bar{z}$) is assumed to be identical for both genders.

4.1 The Model under the Tail Dynamics Hypothesis

The overwhelming majority of the literature on gender differences in scholastic performance has focused on the gap in mean performance between boys and girls, especially in mathematics.[13] Machin and Pekkarinen (2008), however, show that there also exits systematic and statistically sig-

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[13]See for example Guiso et al. (2008), Niederle and Vesterlund (2010), Dee (2007), Fryer and Lewitt (2010) or Dossi et al. (2019) among many others.

nificant differences in test scores variance between genders. They report that the variance of boys' test scores is larger than that of girls in virtually all 40 countries covered by the 2003 Project for Student International Assessment (PISA).[14] They also find that these results hold for both reading and mathematics test scores, two subjects where the mean gender gap in achievement strongly differs. The average male-to-female test score variance ratio in the PISA sample reported by Machin and Pekkarinen (2008) is very similar for mathematics (1.21) and reading (1.20).[15]

In Figure II, we illustrate that these findings hold in the most recent available wave of the PISA data from 2015, which consists of a larger sample of countries. PISA tests nationally representative samples of 15-year-old students in mathematics and reading, and test scores are comparable across countries. In the 2015 PISA sample of 67 countries, the average ratio of the male test score variance over the female test score variance is 1.17 for both reading and mathematics test scores, close to the estimates reported earlier by Machin and Pekkarinen (2008) using an earlier and smaller sample of countries. The variance ratio ranges from 0.96 in Algeria to 1.62 in Jordan for reading, and from 0.90 in Algeria to about 1.50 in Argentina and Jordan. The test score variance of males is higher than that of females' in 65 countries for mathematics and in 64 countries for reading. The gender difference in test score variance is statistically significant in 58 countries for both reading and mathematics.

Understanding the higher dispersion of boys' test scores is beyond the scope of this paper, but it could be linked to another growing strand of literature suggesting that boys' scholastic performance is more sensitive to inputs in early childhood and at school. Bertrand and Pan (2013) document that boys raised in single-parent families have twice the rates of behavioral and disciplinary issues as boys raised in two-parent families; Autor et al. (2015) show that the boy-girl gap in kindergarten readiness, test scores, truancy, disciplinary problems, disability, juvenile delinquency, and high school graduations is larger in more disadvantaged families. Fan, Fang and Markussen (2015) report that maternal employment during early childhood reduces boys' eventual educational attainment relative to that of girls.

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[14] Test scores can here be thought as the empirical measure of individual scholastic performance z in our theoretical model.

[15] Although this evidence is recently known to economists, long-standing evidence on the larger variance in the distribution of some traits among males has been reported in the psychology literature, such as Frasier (1919), Hedges and Nowell (1995) or Jacob (2002), among many others.

With respect to the impact of school inputs, Autor et al. (2016) find that boys benefit more than girls from cumulative exposure to higher quality schools. Legewie and DiPrete (2012) also show that boys' development or inhibition of anti-school attitudes and behavior is more sensitive to peer socio-economic status, which is correlated with school quality measures. Finally, Cornelissen and Dustmann (2019) show that receiving additional schooling before age five has stronger effects on the cognitive and non-cognitive outcomes of boys from disadvantaged backgrounds relative to girls.

We next formulate the tail hypothesis within our conceptual framework. Let z_m and z_f be random variables that denote the scholastic performance of males and females, respectively, and let $f_z(\bar{z}_m)$ and $f_z(\bar{z}_f)$ denote their density functions. The tail hypothesis assumes that $\sigma_m > \sigma_f$, where the distribution of z_m and z_f are assumed to be invariant over time. Importantly, no assumption is imposed on the relative value of μ_m and μ_f.

In our framework, the share of individuals choosing a level of schooling of at least $\bar{s}$ among the population of a given cohort is given by:

$$P^{TH}(\cdot) \equiv \frac{G_{z_f}(\bar{z}, \mu_f, \sigma_f^2) + G_{z_m}(\bar{z}, \mu_m, \sigma_m^2)}{2},$$

and the female-to-male ratio among individuals that attain a level of schooling of at least $\bar{s}$, denoted $R(\bar{z})$, can be expressed as:

$$R^{TH}(\cdot) \equiv \frac{G_{z_f}(\bar{z}, \mu_f, \sigma_f^2)}{G_{z_m}(\bar{z}, \mu_m, \sigma_m^2)}$$

Panel A of Figure IV displays the distributions of scholastic performance for both genders when $\sigma_m^2 > \sigma_f^2$, together with the scholastic performance threshold $\bar{z}$ that truncates the distributions of those that attain a level of schooling of at least $\bar{s}$.[16] Individuals whose scholastic performance is to the right side of the scholastic performance threshold attain at least $\bar{s}$, while those to the left of the scholastic performance threshold do not. Panel B displays the two corresponding complementary cumulative distribution functions (CCDFs) under this assumption, where the reversal occurs when the two CCDFs cross. Figure V illustrates the relationship between the share of individuals choos-

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[16]For empirical estimations and illustrations, we assume that z_m and z_f are normally distributed, as it is the closest approximation of empirical test score distributions.

ing a level of schooling of at least $\bar{s}$ - referred to as the enrollment or completion rate hereafter - and the gender ratio among those choosing at least $\bar{s}$. The reversal occurs when the ratio $\frac{G_{z_f}(\bar{z})}{G_{z_m}(\bar{z})}$ reaches values above 1.

For the reversal to occur under the tail hypothesis, a decrease in the scholastic performance threshold $\bar{z}$ or, equivalently, an increase in the net benefits of education b which translates into an increase in P, is required. Our distributional framework of investment in schooling also implies that the mean scholastic performance of individuals that attain a given level of schooling $\bar{s}$ decreases as the share of population that attains $\bar{s}$ increases. Using US data, Appendix A1.2 shows that the average scholastic performance of individuals attending tertiary education has indeed decreased over time.

Under the assumption that $\sigma_m^2 > \sigma_f^2$, it can be shown that the relationship between the female-to-male ratio among individuals choosing a level of schooling of at least $\bar{s}$, denoted R^{TH}, and the share of individuals that achieve a level of schooling of at least $\bar{s}$, denoted P^{TH}, exhibit three key properties:

Proposition 1. *R^{TH} tends to zero when P^{TH} tends to zero.*

Proposition 2. *R^{TH} tends to one when P^{TH} tends to one.*

Proposition 3. *There exists a value of $P^{TH} \in [0, 1[$ such that $R^{TH} = 1$. This value is unique and always exists.*

Proof. See Appendix A3.

The distributional assumption $\sigma_m^2 > \sigma_f^2$ is a necessary and sufficient condition for Proposition 1 to 3 to hold, without any condition imposed on the relative values of μ_m and μ_f. In addition, Proposition 1 to 3 also hold when z is assumed to follow two-parameter probability distribution functions other than the normal distribution.[17]

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[17]Proofs of Propositions 1 to 3 for distribution functions other than the normal distribution are available upon request.

4.2 The Model under the Mean Dynamics Hypothesis

It can be shown that the mean benefit and mean performance hypotheses are algebraically similar in our framework, under certain conditions (Appendix A3.2). In addition, Goldin, Katz and Kuziemko (2006) argue that the relative increase in females' scholastic performance in upper secondary school in the US was linked to a disproportionate increase in the returns to college education for women. This suggests that these two hypotheses are closely interlinked, and we therefore group them under the broader mean hypothesis in the context if this paper. In the remainder of the paper, we focus on the mean performance hypothesis as it is more straightforward to estimate empirically. The mean benefit hypothesis is however presented in more detail in Appendix A2.1.

A relative increase in females' mean scholastic performance relative to males' over time can generate a reversal in the gender gap in our framework. Such dynamics have been highlighted by Goldin, Katz and Kuziemko (2006), Cho (2007), Fortin, Oreopoulos and Phipps (2015) or Yamaguchi (2018) for the US, and suggested as a potential explanation for the reversal. Goldin, Katz and Kuziemko (2006) use samples of high school graduating seniors from three US longitudinal surveys in 1957, 1972, 1988, among which the last two are nationally representative. Looking at test scores, they find that girls reduced their disadvantage in math, and increased their advantage in reading between 1972 and 1992.

Using three nationally representative longitudinal datasets of high school students, Cho (2007) also looks at the evolution of females' high school performance over time. He reports that females' mean test scores in high school have increased more rapidly than males' over the past three decades. Using a simple Oaxaca decomposition, he finds that women's progress in high school scholastic performance can account for more than half of the change in the college enrollment gender gap over the past thirty years.

Fortin, Oreopoulos and Phipps (2015) use self-reported grades, rather than test scores, to look at the evolution of female high-school scholastic performance over time. Using a sample of 12th graders from the Monitoring the Future (MTF) study in 1976 and 1991, they report that the gender gap in the mean grades of high school seniors remained very stable since the 1970s. They find,

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however, that the mode of girls has shifted upwards over the period 1980–2010 compared to that of boys'. Finally, Yamaguchi (2018) uses using from the Panel Study of Income Dynamics (PSID) and the Dictionary of Occupational Titles (DOT) to study the determinants of the narrowing of the gender wage between 1979 and 1996. He reports a significant increase in women's cognitive and general skills, which he argues contributed to the reduction in the gender wage gap.

We formalize the mean dynamics hypothesis in our framework as an increase in the mean scholastic performance of females relative to males over time. Under this hypothesis, as for the tail hypothesis, there exists a lower bound of scholastic performance $\bar{z}$ such that individuals that attain a given level of schooling $\bar{s}$. Making the normalizing assumption that $\mu_{m_t} = 0$ for all t, the share of individuals in the cohort that attain a level of schooling of at least $\bar{s}$ at time t is given by:

$$P^{MH}(\mu_{f,t}, \bar{z}_t) \equiv \frac{G_{z_m}(\bar{z}_t) + G_{z_{f,t}}(\mu_{f,t}, \bar{z}_t)}{2}.$$

A change from $E[z_{f,t}] < E[z_m]$ to $E[z_{f,t}] > E[z_m]$ is a necessary and sufficient condition for the gender gap reversal in education to occur, when the distributions of scholastic performance by gender are assumed to have the same variance. Under the mean dynamics hypothesis, the gender gap reversal therefore occurs if there is a reversal in the relative average scholastic performance of females relative to males. The gender ratio among individuals that achieve a minimum level of schooling $\bar{s}$ under the mean hypothesis can be expressed as:

$$R^{MH}(\mu_{f,t}, \bar{z}_t) \equiv \frac{G_{z_{f,t}}(\mu_{f,t}, \bar{z}_t)}{G_{z_m}(\bar{z}_t)}, \tag{6}$$

and:

$$\begin{cases} R^{MH} < 1 & \text{when } E[z_{f,t}] < E[z_m] \\ R^{MH} = 1 & \text{when } E[z_{f,t}] = E[z_m] \\ R^{MH} > 1 & \text{when } E[z_{f,t}] > E[z_m] \end{cases}$$

5 Empirical Estimation

From the Barro-Lee database, we observe the share of individuals that enrolled in or completed a given level of schooling $\bar{s}$, in country i and time t. We denote the enrollment or completion rate x_{it}, which is the empirical measure of $P(.)$ in our theoretical framework. From the enrollment or completion rates by gender available in the data, we can also calculate the female-to-male ratio among those that enrolled or completed a given level of schooling in country i and time t, denoted y_{it}. While the original Barro-Lee database consists of 146 countries, we exclude countries that have inconsistent or missing time-series data from our estimation sample.[18] We also exclude countries that have a female-to-male ratio in tertiary enrollment or secondary completion above 4, and countries with age group populations under 10,000 for males or females for any of the years. These sample restrictions result in a balanced panel of 98 countries which we use as our baseline estimation sample. To assess the robustness of our results, we also estimate the model parameters with less restrictive sample restrictions.

We estimate the model parameters by pooling observations of x and y available every five years from 1950 to 2010 for each country in our balanced panel. Our baseline pooled specification therefore exploits variation in educational attainment and gender ratios over time and between countries. Table IV reports the countries with the highest and lowest female-to-male ratio for the three levels of educational attainment. It shows wide cross-country variation in the gender ratio for tertiary enrollment and secondary school completion. As of 2010, the 10th-90th percentile range of the female-to-male ratio in tertiary enrollment spans from 0.59 to 1.73, with a standard deviation of 0.45. For secondary completion, the 10-90th percentile range of the gender ratio goes from 0.80 to 1.24, with a standard deviation of 0.23. We observe less cross-country variation for the gender ratio in primary completion, with a 10th-90th percentile range spanning from 0.93 to 1.13, and a standard deviation of 0.11.

Figure I and Table V show that relative to cross-country variation, time variation in gender ratios for tertiary enrollment and secondary completion is also substantial. The average female-to-male ratio for countries in our sample has increased from about 0.40 in 1950 to about 1.22 in 2010 for

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[18]Many countries in the original sample exhibit a very large drop or increase in the gender ratio in specific years.

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tertiary education, and from 0.55 in 1950 to 1.08 in 2010 for secondary education. The country with the largest increase in the share of females in tertiary education has been Saudi Arabia, from 0.16 to 2.42 over the period 1950-2010. In contrast with tertiary and secondary education, we observe significantly less variation over time for primary completion, with an increase of the mean female-to-male ratio from 0.82 to 1.03 on average in our baseline estimation sample.

We estimate the model parameters for two alternative levels of educational attainment $\bar{s}$: enrollment in tertiary education and secondary school completion. In our model, $P(\cdot)$ and $R(\cdot)$, the theoretical equivalents of x_{it} and y_{it}, are jointly determined by G_{z_f} and G_{z_m}, and by the scholastic performance threshold $\bar{z}$. Unlike $\bar{z}_t$, x_t is observable, and the two distributions are assumed to be fully characterized by the four-parameter vector $\{\mu_m, \sigma_m^2, \mu_f, \sigma_f^2\}$. For tertiary education, estimates are obtained by fitting the total enrollment rate and the female-to-male ratio among individuals that enrolled in tertiary education. For secondary school completion, estimates are derived by fitting the secondary school completion rate and the female-to-male ratio among individuals that completed secondary education. The baseline estimation is conducted over the entire period of 1950-2010, with observations available every five years from the Barro-Lee database.

We do not estimate our model for primary school completion for both conceptual and methodological reasons. As highlighted in Section 2, primary school completion has been universal for many countries over the past decades, where primary schooling has been mandatory. This implies that primary school completion in many countries is the result of institutional features instead of optimal decisions of investment in schooling. This is incompatible with our conceptual framework which assumes that educational attainment is determined by the optimization of individual intertemporal utility functions. In addition, these features generate limited variation in completion rates and gender ratios over time and across countries to estimate our model parameters. As a result, the model estimation using primary completion rates is unlikely to yield reliable and precise estimates.

5.1 Tail Hypothesis Parameter Estimation

In the first specification, only the tail hypothesis is assumed to predict the dynamics of the gender gap in educational attainment, while the mean hypothesis plays no role. From our conceptual framework, the tail hypothesis consists of a total of four parameters $\{\mu_m, \mu_f, \sigma_m^2, \sigma_f^2\}$. Without any loss of generality, the parameters of the model under the tail hypothesis can be reduced to two, by normalizing one of the two probability density functions. We standardize the female probability density function such that $f_{z_f}(\bar{z}_t) \sim N(0, 1)$, and estimate the two moments of the male scholastic performance distribution $\{\mu_m, \sigma_m^2\}$.

The model under the tail hypothesis predicts a unique value $\hat{y}_{it}$, conditional on the triplet $\{x_{it}, \mu_m, \sigma_m^2\}$. To fit the model under the tail hypothesis, we estimate the two parameters $\{\mu_m; \sigma_m^2\}$ from our sample of countries observed in multiple years. From our model, the tail hypothesis after normalization of the scholastic performance distribution for females can be expressed empirically as:

$$y_{it}^{TH} = \frac{G_{z_f}(\bar{z}_{it})}{G_{z_m}(\bar{z}_{it}, \mu_m, \sigma_m^2)} \cdot \exp(\varepsilon_{it}),$$

where $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$. Taking the logarithm of Equation **??** and substituting $\bar{z}_{it} = P^{-1}(x_{it}, \mu, \sigma^2)$ yields the final reduced form:

$$\log y_{it}^{TH} = \log G_{z_f}(P^{-1}(x_{it}, \mu_m, \sigma_m^2)) - \log G_{z_m}(P^{-1}(x_{it}, \mu_m, \sigma_m^2), \mu_m, \sigma_m^2) + \varepsilon_{it}. \quad (7)$$

Equation 7 is a non-linear mapping from x to y defined by the parameters of the males' scholastic performance relative to the females' distribution. As the error term ε_{it} is normally distributed, the model under the tail hypothesis can be fitted numerically by finding the parameter values that minimize the sum of squared errors.

5.2 Mean Hypothesis Parameter Estimation

In the second specification, the tail hypothesis is assumed to play no role and only the mean hypothesis predicts the gender gap dynamics in educational attainment. To estimate the parameters of the mean performance hypothesis, we assume that $\sigma_m^2 = 1$ and introduce the term $t\mu_f$ to allow for an increase in females' average scholastic performance over time relative to males'. This baseline formulation of the mean hypothesis only allows for a linear time progression of the female mean, but it has the advantage of making the model under the mean hypothesis more comparable to the tail hypothesis. In this context, one can think of μ_m as the constant in the model and $t\mu_f$ as the time trend. Thus, $\mu_m - \mu_f$ and $\mu_m - 13\mu_f$ correspond to the relative mean of the males' distribution compared to the females' distribution in periods 1 (1950) and 13 (2010), respectively. From equation 6, the structural model under the mean dynamics hypothesis can be expressed as:

$$y_{it}^{MH} = \frac{G_{z_f}(\bar{z}_{it}, t\mu_f)}{G_{z_m}(\bar{z}_{it}, \mu_m)} \cdot \exp(\varepsilon_{it}),$$

where $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$.

Taking the logarithm and substituting in a similar fashion as for Equation 7 yields the following reduced form:

$$\log y_{it}^{MH} = \log G_{z_f}(P^{-1}(x_{it}, \mu_m, t\mu_f), t\mu_f) - \log G_{z_m}(P^{-1}(x_{it}, \mu_m, t\mu_f), \mu_m) + \varepsilon_{it}. \quad (8)$$

As for the tail hypothesis, we fit the model numerically to estimate the mean hypothesis two-parameter vector $\{\mu_m, \mu_f\}$ so that the sum of squared errors is minimized.

5.3 Joint Estimation

In the third specification, we allow for both the tail and mean hypotheses to jointly predict the dynamics of the gender gap in educational attainment. This specification allows to quantify the respective explanatory power of each of the two hypotheses, once the influence of the other hy-

pothesis is also accounted for. To test for each of the two hypotheses while controlling for the other hypothesis' parameter, we jointly estimate the three key parameters of the model $\{\mu_m, \mu_f, \sigma_m^2\}$ using the following equation:

$$\log y_{it}^{BH} = \log G_{z_f}(P^{-1}(x_{it}, \mu_m, t\mu_f, \sigma_m^2), t\mu_f)-$$

$$\log G_{z_m}(P^{-1}(x_{it}, \mu_m, t\mu_f, \sigma_m^2), \mu_m, \sigma_m^2) + \varepsilon_{it}. \tag{9}$$

We also estimate a second version of Equation 9, where we also control for GDP per capita. This allows to check whether our main results are not driven by the overall economic development in each country and over time. The data on PPP-adjusted per capita income data are sourced from the Penn World Tables. The drawback of this specification is that it has to be estimated in a smaller sample than our baseline sample, as GDP per capita is not available for all years and all countries included in the baseline sample, particularly for early years. In addition, as per-capita income data is missing for many countries in years prior to the 1960s, we estimate the model parameters in the GDP sample starting from the 1960s rather than the 1950s.

6 Results

6.1 Baseline results

6.1.1 Tertiary enrollment

The model parameter estimates for tertiary enrollment are reported in the upper panel of Table VI. Column 1 and 2 report the parameter estimates of the tail and mean hypotheses, respectively, when they are estimated in separate equations. Both the female mean shift per period μ_f and the male standard deviation σ_m parameters are statistically significantly different from their null hypothesis values of 0 and 1, respectively. In addition, the value of the R-squared for the tail and mean dynamics hypothesis specifications reported in column 1 and 2 are very similar. This suggests that

both hypotheses have roughly the same explanatory power in accounting for the dynamics of the gender gap in tertiary enrollment.

Column 3 of Table VI reports the estimation results when the two hypotheses' parameters are jointly estimated using equation 9. Both the tail and mean hypotheses' parameter estimates are individually highly significant for tertiary education. Accounting for both the tail and mean hypotheses' parameters increases the explanatory of the model: the R-squared increases to 0.52 compared to the specification with the tail hypothesis parameters only (0.47) or with the mean hypothesis parameters only (0.46). The model explanatory power therefore gains from jointly accounting for both hypotheses.

Columns 4 to 6 of Table VI display the estimation results from the same regressions as in column 1 to 3, but only for countries where GDP per capita data is available for all years covered by the Barro-Lee data. The results reported in column 4 to 6 are consistent with the baseline estimates reported in column 1 to 3. For tertiary enrollment, both the mean and tail hypothesis parameters are highly statistically significant when estimated in separate regressions (column 4 and 5) as well as jointly (column 6). As in the baseline sample, the model goodness of fit increases in column 6 when both hypotheses' parameters are included (0.56), compared to R-squared of the specifications in column 4 (0.50) and column 5 (0.48), where only one of the two hypotheses is accounted for.

In Column 7, GDP per capita is added to equation 9 as an additional explanatory variable. The rationale behind this approach is to check whether our main results are not driven by changes in economic development over time that could be associated with both enrollment rates in education and gender ratios. The explanatory power of both the tail and mean hypotheses' parameters remain statistically significant at the one percent level, and the R-squared is only marginally improved by the inclusion of GDP per capita.

Figure VI displays the fit of our model when the tail and mean hypotheses are separately and jointly estimated. This visual evidence brings additional insights on the extent to which each of the hypotheses accurately predicts the gender gap reversal in educational attainment. In the upper panel, the tail hypothesis fitted for tertiary education predicts a relatively late reversal of the gender gap with respect to the total enrollment rate in tertiary education, while the mean hypothesis

predicts an earlier reversal. When both hypotheses are combined, the reversal is predicted to occur for an even lower value of the total enrollment rate (0.20), which is the closest to what is empirically observed. According to the Barro-Lee data, the average female-to-male ratio in tertiary enrollment reached one when the global enrollment rate in tertiary education was between 0.14 and 0.17, for our baseline sample of 98 countries.

6.1.2   Secondary completion

The lower panel of table VI reports the model estimates for secondary education. In contrast with tertiary education estimates, the R-squared of the tail hypothesis specification is higher (0.51) than that of the mean hypothesis (0.45). The tail hypothesis has therefore stronger explanatory power compared to the mean hypothesis for secondary education dynamics. In addition, the R-squared reported in column (1) and (3) are identical. Thus, the mean hypothesis parameter adds little explanatory power compared to the specification with the tail hypothesis only. Additionally, the magnitude of the mean hypothesis parameter μ_f in column 2 decreases more than threefold from 0.022 to 0.06 once the tail hypothesis parameter is also accounted for, although it remains statistically significant at the 5% level. These results overall indicate that the mean dynamics hypothesis adds little additional explanatory power to the tail hypothesis for secondary school completion.

In the restricted sample where GDP per capita is also available, as in the baseline estimations, the R-squared is noticeably larger in the tail hypothesis' specification in column 4 (0.53) compared to the mean hypothesis' specification in column 5 (0.45). The R-squared of the specification with both hypotheses' parameters (column 6) is also identical to that of the specification with the tail hypothesis only (column 4). The mean hypothesis parameter μ_f also decreases more than twofold once the tail hypothesis is also accounted for. The R-squared is virtually unchanged by the inclusion of GDP per capita. The coefficient on GDP per capita is however positive and statistically significant at the one percent level, and the magnitude of both the tail and mean hypotheses parameters is to some extent reduced, suggesting that our baseline estimates were to some extend capturing the influence of overall economic development. Our main results are however quite

robust to controlling for GDP per capita.

The visual evidence displayed in Figure VI shows that for secondary completion, the mean hypothesis predicts a very early reversal of the gender gap, while the tail hypothesis and the joint hypotheses predict the reversal to take place at a much higher value of the secondary completion rate. This late reversal is closer to what is observed in the Barro-Lee dataset, where the global gender gap reversal in secondary completion occurs when the global secondary completion rate is of about 0.50.

6.2   Robustness

The sample restrictions we apply to the original Barro-Lee dataset of over 140 countries aim at increasing the precision and reliability of our estimates. However, one may argue that those could affect the composition of our sample and drive our results. To assess whether our results are robust to using a broader sample, we estimate the model parameters on a larger pool of countries where less restrictive sample restrictions are imposed. In particular, we include countries for which a large drop or jump in the gender ratio is observed in specific years, which increases the estimation sample from 98 to 113 countries. The results of the model estimates in this larger sample are reported in Table VII. As shown in the table, the estimation results in this expanded sample are similar to the ones reported for the baseline estimation in Table VI.

6.3   Model parameter estimates by level of economic development

The respective predictive power of the tail and mean hypothesis may vary by level of economic development. We assess whether our baseline results hold in both OECD and non-OECD countries by splitting the sample into these two country groups and separately estimating the model parameters in these two subsamples.

Table VIII and IX report the model estimates in the sample of OECD and non-OECD countries, respectively. The respective explanatory power of the mean ad tail hypothesis somehow differ in

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the OECD and non-OECD sample. In the non-OECD sample, the tail hypothesis has stronger explanatory than the mean hypothesis for both secondary and tertiary education dynamics. In addition, as in the full sample, the explanatory power of the model is strongly increased for tertiary enrollment when both hypotheses are jointly accounted for.

In contrast, the estimates reported in table VIII suggest that the explanatory power of the mean hypothesis is stronger in the OECD sample. For tertiary enrollment, the R-squared of the model under the mean hypothesis only (column 2 and 4) is larger than that of the tail hypothesis only (column 1 and 3). In addition, accounting for both hypotheses jointly in column 3, 6 and 7 only marginally increases predictive power. Results for secondary completion reported in the lower panel are similar. The mean hypothesis has stronger explanatory power individually, and accounting for the tail hypothesis only marginally increases predictive power. The tail hypothesis parameter remains individually highly statistically significant in all specifications for tertiary enrollment, but turns insignificant in some specifications for secondary completion once the mean hypothesis parameter is also accounted for.

6.4   Magnitude of the parameter estimates

In our preferred specification that also controls for GDP per capita and where both hypotheses' parameters are accounted for, estimates for the male-to-female variance ratio vary between 1.12 and 1.36.[19] In the OECD sample, which is closest to that of Machin and Pekkarinen (2008), estimates for σ^2 range between 1.12 and 1.30. The magnitude of the model estimates is therefore close to the female-to-male variance ratio in test scores estimated by Machin and Pekkarinen (2008) using PISA 2003, which is 1.21 for reading and 1.20 for mathematics.[20] In addition, the tail hypothesis parameter estimates are quite similar when derived from the secondary school completion or tertiary enrollment time-series. The internal and external consistency of our estimates provides further comfort on the predictive power and validity of the tail hypothesis. One should note, however,

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[19]The female-to-male variance ratio is the square of the estimate for the tail hypothesis parameter, which is the ratio of the female and male standard deviation in scholastic performance.

[20]In the latest PISA 2015 data, estimates average female-to male variance ratios is 1.17 for both mathematics and reading, in a larger sample of 67 countries.

that the model estimates in specifications where only the tail hypotheses parameter is included and GDP per capita is not controlled for tend to be larger than the estimates of Machin and Pekkarinen (2008). This suggests that not accounting for the role of the mean hypothesis in the model may lead to overestimating the tail hypothesis parameter value.

6.5 Interpretation and discussion

In the full sample of both high-income and developing economies, our results highlight that the combined forces of the tail and mean hypotheses have been jointly contributing tertiary enrollment dynamics. For secondary completion, however, the mean hypothesis appears to play a weaker role. The joint influence of the mean and tail hypotheses for tertiary enrollment dynamics could help explain why the extent of the gender gap reversal in education has been stronger for tertiary enrollment than for secondary school completion. As evidenced in Section 2, the female-to-male ratio in tertiary enrollment increased faster over time and reached higher levels than the gender ratio among secondary school completers.

Overall, the significance of the tail hypothesis parameter estimate σ is quite robust across estimation samples. Its value estimated using our model is close to prior estimates of male-to-female test score variance ratios from international student assessments, and is fairly stable across specifications. This provides comfort on its validity and suggests that it has been an important determinant of the gender gap reversal in educational attainment worldwide. Accounting for the tail hypothesis parameter in the model strongly increases predictive power for the gender gap in tertiary enrollment, compared to a specification with the mean hypothesis only. Its explanatory power is stronger than that of the better known mean hypothesis in the full sample, and is especially strong in developing economies. We therefore find strong evidence that the tail hypothesis has played a role in the dynamics of the gender gap in educational attainment, which is a key contribution of the paper.

Our model estimates also evidence that the mean hypothesis, put forward by previous literature, has strong explanatory power for the gender gap reversal in educational attainment. The mean

hypothesis parameter estimate is significant in virtually all specifications, although it appears to add less explanatory power to the model in the sample of developing economies, especially for secondary completion. In contrast with the tail hypothesis, its role in the gender gap dynamics appear to to be stronger in high-income countries, while the tail hypothesis predominates among developing economies.

Our findings indicate that the respective contributions of the mean and tail hypotheses to the gender gap reversal somehow differ in high-income and developing economies. Explanations for the gender gap reversal suggested by previous literature have been confined to the mean hypothesis, and have been proposed in the context of high-income countries, primarily in the US. In this context, Goldin, Katz and Kuziemko (2006) and Chiappori, Iyigun and Weiss (2009) invoke the progressive removal of female career barriers over time. Goldin, Katz and Kuziemko (2006) associate the removal of barriers to female employment to a progressive change in fertility and marriage patterns, driven by the access to reliable contraception (Goldin and Katz, 2002). Chiappori, Iyigun and Weiss (2009) argue that technological progress has freed women from many domestic tasks, which disproportionally increased the labor market and marriage market returns of schooling for females. Similarly, Black and Spitz-Oener (2010), Olivetti and Petrongolo (2016) and Ngai and Petrongolo (2017) have argued that technological change and secular labor demand shifts favored occupations and industries disproportionately employing female college graduates. This would in turn increase the returns to higher level of schooling for women.

Such shifts in technology and gender norms have not taken place to the same extent in developing countries (World Bank, 2012). They may have occurred only in recent years or, in some case, are yet to take place. Therefore, the conditions under which the reversal occurs according to the mean hypothesis may not necessarily hold in developing countries. This could explain why the mean hypothesis appears to play a prominent role in the context of high-income countries relative to developing countries.

7   Conclusion

This paper contributed to a better understanding of the driving forces behind the gender gap reversal in education, both theoretically and empirically. We developed a unified conceptual framework to account for the gender gap reversal in education, and formulated two main hypotheses for the reversal within our framework We showed that both the mean and tail dynamics can explain the gender gap reversal in education theoretically. Empirically, we found evidence for both hypotheses having predictive power to account for the gender gap reversal reversal. However, the role played by the tail hypothesis parameter in the gender gap dynamics appears more robust empirically.

Our results indicate that the lower variance in scholastic performance among females has been a driver of the observed gender gap reversal in education, a finding that has not been uncovered by previous literature. The larger variability of men's test scores observed empirically remains mostly unexplained, and could be explored by further research. Our findings also imply that the over-representation of boys at the bottom end of school scholastic performance may be of concern for policy makers. Delving into the origins of the over-representation of boys at the bottom of the scholastic performance distribution could help addressing boys' growing educational disadvantage. Tackling boys' increasing is important as early school dropouts have been linked to poor labor market performance, higher poverty incidence but also higher crime rates, which men have been shown to be more prone to commit.

This paper suggests that when looking at gender differences in observable outcomes, it is important to go beyond the analysis of means by looking at entire distributions. Fundamentally, when analyzing educational outcomes by gender, the researcher is always looking at truncated distributions. In such distributions, the mean is a function of the dispersion of the underlying distribution. Since evidence shows a higher dispersion of test scores among males, this effect needs to be accounted for, whenever educational performance is discussed by gender. Our findings also suggest that the larger variance of test scores among males may be relevant to explain gender differences in other areas. Building on this fact could be a direction for future research in labor economics and other fields of economics.

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Tables

Table I: Summary statistics of educational attainment in the sample

		OECD		Non-OECD	
		Panel A: Primary completion rate			
		Males	Females	Males	Females
1950	Mean	0.70	0.68	0.30	0.24
	S.d	0.20	0.21	0.23	0.23
2010	Mean	0.98	0.98	0.80	0.82
	S.d.	0.02	0.02	0.21	0.21
		Panel B: Secondary completion rate			
		Males	Females	Males	Females
1950	Mean	0.22	0.17	0.07	0.04
	S.d.	0.15	0.14	0.08	0.06
2010	Mean	0.79	0.83	0.45	0.49
	S.d.	0.15	0.13	0.26	0.29
		Panel C: Tertiary enrollment rate			
		Males	Females	Males	Females
1950	Mean	0.06	0.02	0.03	0.01
	S.d.	0.04	0.03	0.03	0.02
2010	Mean	0.34	0.45	0.14	0.17
	S.d.	0.15	0.15	0.14	0.17

Notes. Primary and secondary completion rates are calculated for individuals age 15-19 and 20-24 respectively, in a given year. Tertiary enrollment rate are calculated for individuals age 25-29. Statistics exclude eight countries with highly inconsistent time series data or missing values.
Source. Barro-Lee educational attainment database.

Table II: Share of countries that experienced the gender gap reversal in educational attainment

	Primary Completion		Secondary Completion		Tertiary Enrollment		
	1950	2010	1950	2010	1950	2010	N
Advanced economies	0.38	0.46	0.21	0.92	0.00	0.96	24
East Asia and the Pacific	0.05	0.74	0.00	0.63	0.05	0.68	19
Europe and Central Asia	0.50	0.75	0.00	0.85	0.10	0.90	20
Latin America and the Caribbean	0.54	0.83	0.12	0.83	0.08	1.00	22
Middle East and North Africa	0.06	0.50	0.00	0.88	0.06	0.81	18
South Asia	0.00	0.57	0.00	0.42	0.00	0.43	7
Sub-Saharan Africa	0.24	0.59	0.06	0.38	0.17	0.31	28
OECD	0.42	0.53	0.14	0.89	0.00	0.94	36
Non-OECD	0.25	0.63	0.05	0.65	0.11	0.66	102
All countries	0.29	0.64	0.06	0.71	0.08	0.73	138

Notes. The gender gap reversal in educational attainment is defined to occur when the enrollment/completion rate of females for a given level of education is higher than that of males. Primary and secondary completion rates are calculated for individuals age 15-19 and 20-24 respectively, in a given year. Tertiary enrollment rates are calculated for individuals age 25-29. Statistics exclude eight countries with highly inconsistent time series data or missing values.
Source. Barro-Lee educational attainment database.

Table III: Three hypotheses for the gender gap reversal in education

Hypothe-sis	Corresponding model assumption		
	Mean of z	Variance of z	$\bar{z}$ (or, equivalently, b)
Tail hy-pothesis	-	Gender-specific $Var[z_m]$ and $Var[z_f]$	-
MBH	-	-	Gender-specific $\bar{z}_f$ and $\bar{z}_m$ (or, equivalently, gender-specific b_f and b_m)
MPH	Gender-specific $E[z_f]$ and $E[z_m]$	-	-

Notes. z denotes the scholastic performance of individuals, whose distribution can vary between genders in our framework. $\bar{z}$, which can vary between genders, denotes the lower bound of scholastic performance such that individuals enroll into a given level of schooling $\bar{s}$.

Table IV: Female-to-male ratio in educational attainment, top and bottom 5 countries, 2010

Highest female-to-male ratio		Lowest female-to-male ratio	
Tertiary enrollment			
Saudi Arabia	2.42	Central African Republic	0.12
Slovenia	2.29	Benin	0.16
Dominican Republic	2.17	Afghanistan	0.24
Kazakhstan	2.02	Gambia	0.36
Uruguay	2.01	Myanmar	0.36
Secondary completion			
Nepal	1.90	Congo	0.35
Bangladesh	1.79	Afghanistan	0.40
Zimbabwe	1.72	Gambia	0.42
Sudan	1.58	Central African Republic	0.47
Tanzania	1.53	Democratic Republic of the Congo	0.50
Primary completion			
Malawi	1.94	Afghanistan	0.63
Lesotho	1.85	DR of the Congo	0.73
Namibia	1.70	Niger	0.76
Myanmar	1.53	Pakistan	0.82
Mali	1.43	Sierra Leone	0.82

Notes. The female-to-male ratio is calculated as the enrollment/completion rate of females divided by the enrollment/completion rate of males for a given level of education. Primary and secondary completion rates are calculated for individuals age 15-19 and 20-24 respectively, in a given year. Tertiary enrollment rates are calculated for individuals age 25-29. The ranking excludes countries with highly inconsistent time series and missing values, countries with a female-to-male ratio above 4, and countries with a male or female population under 10,000 for the corresponding age group.
Source. Barro-Lee educational attainment database.

Table V: Female-to-male ratio in educational attainment, top and bottom changers between 1950 and 2010

Largest increase in ratio		Largest decrease in ratio	
Tertiary enrollment			
Saudi Arabia	2.26	Central African Republic	-0.87
Slovenia	1.94	Morocco	-0.57
Kazakhstan	1.63	Uganda	-0.48
Venezuela	1.50	Cambodia	-0.35
Croatia	1.47	Niger	-0.28
Secondary completion			
Nepal	1.84	United Kingdom	-0.22
Bangladesh	1.71	Congo	-0.18
Tanzania	1.31	Gambia	-0.17
Sudan	1.30	Canada	-0.13
Tunisia	1.29	USA	-0.13
Primary completion			
Malawi	1.69	Mauritania	-0.86
Uganda	1.06	Dominican Republic	-0.62
Tanzania	1.00	Norway	-0.51
Cambodia	0.97	Bostwana	-0.50
Egypt	0.90	Russia	-0.47

Notes. The female-to-male ratio is calculated as the enrollment/completion rate of females divided by the enrollment/completion rate of males for a given level of education. Primary and secondary completion rates are calculated for individuals age 15-19 and 20-24 respectively, in a given year. Tertiary enrollment rates are calculated for individuals age 25-29. The ranking excludes countries with highly inconsistent time series and missing values, countries with a female-to-male ratio above 4, and countries with a male or female population under 10,000 for the corresponding age group.

Source. Barro-Lee educational attainment database.

Table VI: Model estimates of the tail and mean hypotheses parameters, baseline country sample

	Main sample			Constrained sample			
	Tail hypothesis	Mean performance hypothesis	Both hypotheses	Tail hypothesis	Mean performance hypothesis	Both hypotheses	GDP & Both hypotheses
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Tertiary enrollment							
Male mean (μ_m)	-0.111*** (0.026)	0.540*** (0.010)	0.229*** (0.022)	-0.134*** (0.033)	0.588*** (0.016)	0.192*** (0.024)	0.243*** (0.017)
Female mean shift per period (μ_f)		0.039*** (0.001)	0.029*** (0.002)		0.043*** (0.002)	0.028*** (0.003)	0.026*** (0.004)
Male SD (σ_m)	1.237*** (0.015)		1.142*** (0.010)	1.246*** (0.019)		1.169*** (0.014)	1.166*** (0.009)
$\delta(GDP)$							0.013*** (0.005)
R^2	0.47	0.46	0.52	0.5	0.48	0.56	0.57
N	1,274	1,274	1,274	715	715	715	715
Secondary completion							
Male mean (μ_m)	0.134*** (0.018)	0.465*** (0.034)	0.182*** (0.024)	0.076*** (0.022)	0.554*** (0.014)	0.210*** (0.026)	0.366*** (0.019)
Female mean shift per period (μ_f)		0.022*** (0.005)	0.0064** (0.0030)		0.032*** (0.002)	0.015*** (0.003)	0.012*** (0.003)
Male SD (σ_m)	1.171*** (0.015)		1.156*** (0.014)	1.213*** (0.016)		1.186*** (0.013)	1.116*** (0.010)
$\delta(GDP)$							0.017*** (0.002)
R^2	0.51	0.45	0.51	0.53	0.45	0.53	0.54
N	1,274	1,274	1,274	924	924	924	924

Notes. ***: significant at the 1% level, **: significant at the 5% level, *: significant at the 10% level. Standard errors are bootstrapped with 1000 repetitions and reported in parentheses. The null hypothesis for σ_m is $\sigma_m = 1$. The first three columns are estimated with equations (7), (8) and (9), respectively.

Table VII: Model estimates of the tail and mean hypotheses parameters, expanded country sample

	Main sample			Constrained sample			
	Tail hypothesis	Mean performance hypothesis	Both hypotheses	Tail hypothesis	Mean performance hypothesis	Both hypotheses	GDP & Both hypotheses
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Tertiary enrollment							
Male mean (μ_m)	-0.024 (0.024)	0.492*** (0.009)	0.194*** (0.019)	-0.102*** (0.033)	0.553*** (0.012)	0.165*** (0.022)	0.277*** (0.018)
Female mean shift per period (μ_f)		0.031*** (0.001)	0.020*** (0.002)		0.036*** (0.002)	0.024*** (0.003)	0.019*** (0.004)
Male SD (σ_m)	1.180*** (0.014)		1.125*** (0.009)	1.223*** (0.018)		1.168*** (0.013)	1.133*** (0.012)
$\delta(GDP)$							0.023*** (0.005)
R^2	0.36	0.35	0.39	0.43	0.38	0.46	0.47
N	1,469	1,469	1,469	825	825	825	825
Secondary completion							
Male mean (μ_m)	0.118*** (0.019)	0.462*** (0.026)	0.182*** (0.021)	0.070*** (0.023)	0.546*** (0.014)	0.228*** (0.028)	0.371*** (0.019)
Female mean shift per period (μ_f)		0.023*** (0.004)	0.008*** (0.003)		0.032*** (0.002)	0.016*** (0.003)	0.013*** (0.003)
Male SD (σ_m)	1.167 (0.013)		1.149*** (0.013)	1.210*** (0.017)		1.174*** (0.012)	1.109*** (0.01)
$\delta(GDP)$							0.018*** (0.002)
R^2	0.50	0.44	0.50	0.51	0.43	0.51	0.52
N	1,469	1,469	1,469	957	957	957	957

Notes. ***: significant at the 1% level, **: significant at the 5% level, *: significant at the 10% level. Standard errors are bootstrapped with 1000 repetitions and reported in parentheses. The null hypothesis for σ_m is $\sigma_m = 1$. The sample includes The sample includes the baseline sample of countries and expands that sample with countries with inconsistent data. The first three columns are estimated with equations (7), (8) and (9), respectively.

Table VIII: Model estimates of the tail and mean hypotheses parameters, OECD countries

	Main sample			Constrained sample			
	Tail hypothesis	Mean performance hypothesis	Both hypotheses	Tail hypothesis	Mean performance hypothesis	Both hypotheses	GDP & Both hypotheses
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Tertiary enrollment							
Male mean (μ_m)	-0.291*** (0.059)	0.548*** (0.027)	0.354*** (0.088)	-0.206*** (0.051)	0.608*** (0.023)	0.441*** (0.129)	0.324*** (0.026)
Female mean shift per period (μ_f)		0.060*** (0.005)	0.051*** (0.006)		0.064*** (0.003)	0.054*** (0.007)	0.048*** (0.007)
Male SD (σ_m)	1.391*** (0.032)		1.109*** (0.036)	1.331*** (0.030)		1.086** (0.047)	1.142*** (0.017)
$\delta(GDP)$							-0.001 (0.012)
N	416	416	416	275	275	275	275
R^2	0.53	0.63	0.65	0.46	0.57	0.58	0.58
Secondary completion							
Male mean (μ_m)	0.085** (0.042)	0.331*** (0.063)	0.309 (0.026)	0.060 (0.046)	0.368*** (0.036)	0.308*** (0.029)	0.251*** (0.030)
Female mean shift per period (μ_f)		0.038*** (0.010)	0.035*** (0.004)		0.040*** (0.005)	0.034*** (0.005)	0.021*** (0.007)
Male SD (σ_m)	1.125*** (0.033)		1.016 (0.021)	1.133*** (0.036)		1.038 (0.029)	1.062*** (0.024)
$\delta(GDP)$							0.003 (0.007)
R^2	0.32	0.38	0.38	0.26	0.31	0.32	0.32
N	416	416	416	275	275	275	275

Notes. ***: significant at the 1% level, **: significant at the 5% level, *: significant at the 10% level. Standard errors are bootstrapped with 1000 repetitions and reported in parentheses. The null hypothesis for σ_m is $\sigma_m = 1$. The sample includes only OECD countries within the baseline sample of countries. The first three columns are estimated with equations (7), (8) and (9), respectively.
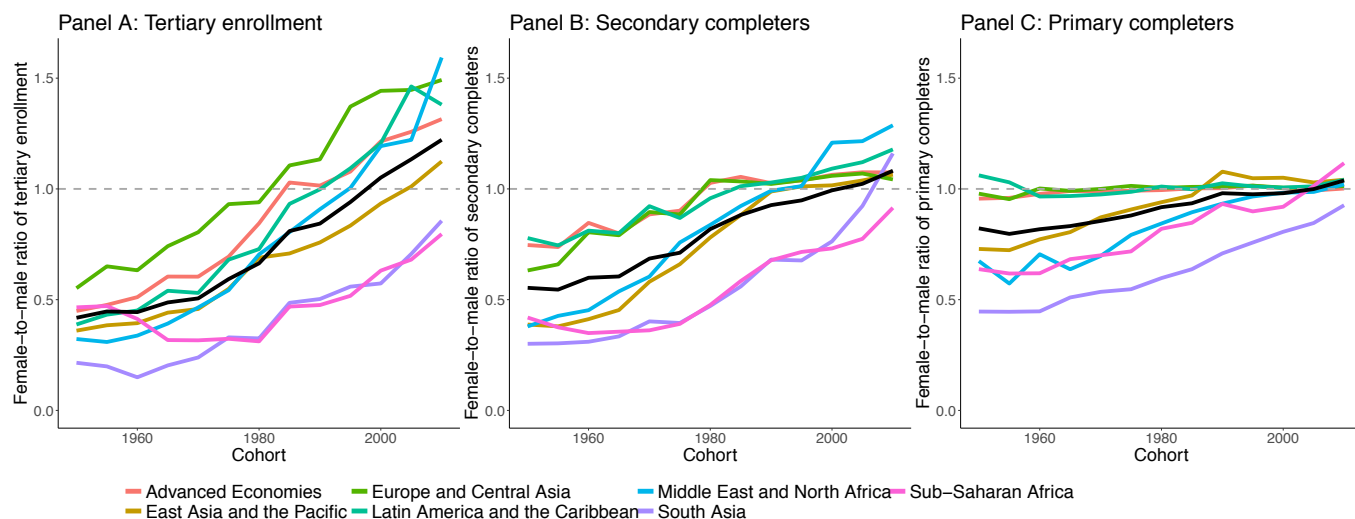
Table IX: Model estimates of the tail and mean hypothesis parameters, Non-OECD countries

	Main sample			Constrained sample			
	Tail hypothesis	Mean performance hypothesis	Both hypotheses	Tail hypothesis	Mean performance hypothesis	Both hypotheses	GDP & Both hypotheses
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Tertiary enrollment							
Male mean (μ_m)	-0.086** (0.033)	0.539*** (0.013)	0.189*** (0.023)	-0.142*** (0.042)	0.604*** (0.019)	0.224*** (0.027)	0.285*** (0.034)
Female mean shift per period (μ_f)		0.034*** (0.002)	0.023*** (0.002)		0.040*** (0.002)	0.028*** (0.003)	0.022*** (0.005)
Male SD (σ_m)	1.223*** (0.018)		1.148*** (0.009)	1.245*** (0.023)		1.158*** (0.012)	1.143*** (0.012)
$\delta(GDP)$							0.024** (0.010)
R^2	0.22	0.17	0.35	0.25	0.10	0.36	0.44
N	858	858	858	440	440	440	440
Secondary completion							
Male mean (μ_m)	0.173*** (0.026)	0.501*** (0.015)	0.247*** (0.025)	0.093*** (0.029)	0.606*** (0.016)	0.240*** (0.027)	0.381*** (0.021)
Female mean shift per period (μ_f)		0.023*** (0.002)	0.009*** (0.003)		0.035*** (0.002)	0.015*** (0.003)	0.010*** (0.004)
Male SD (σ_m)	1.154*** (0.017)		1.131*** (0.015)	1.207*** (0.02)		1.170*** (0.014)	1.112*** (0.013)
$\delta(GDP)$							0.022*** (0.004)
R^2	0.49	0.46	0.49	0.48	0.44	0.50	0.51
N	858	858	858	649	649	649	649

Notes. ***: significant at the 1% level, **: significant at the 5% level, *: significant at the 10% level. Standard errors are bootstrapped with 1000 repetitions and reported in parentheses. The null hypothesis for σ_m is $\sigma_m = 1$. The sample includes only non-OECD countries within the baseline sample of countries. The first three columns are estimated with equations (7), (8) and (9), respectively.
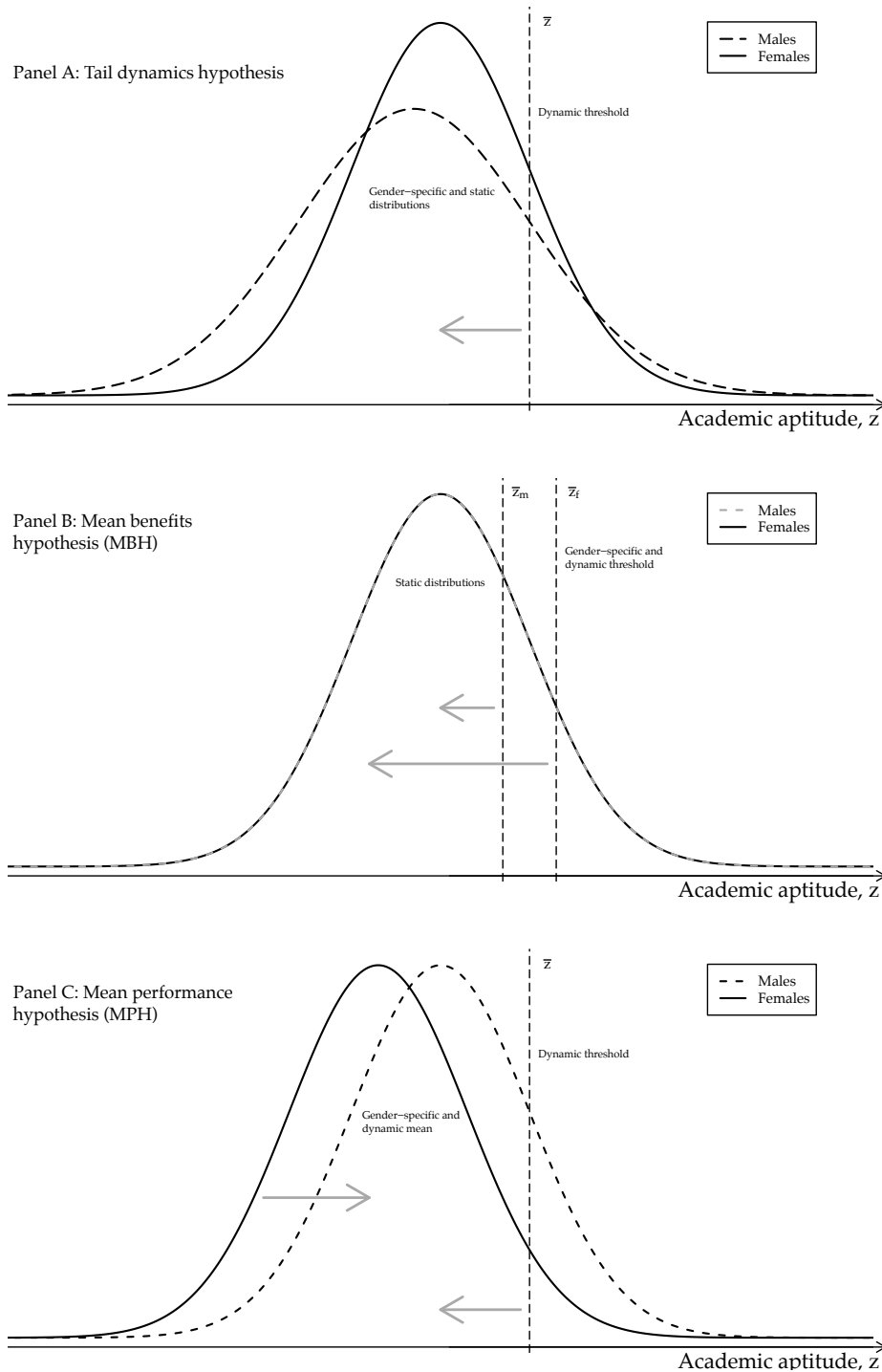
Figures

Figure I: The Gender Gap Reversal in Educational Attainment

Panel A: Tertiary enrollment — Panel B: Secondary completers — Panel C: Primary completers

Legend: Advanced Economies, East Asia and the Pacific, Europe and Central Asia, Latin America and the Caribbean, Middle East and North Africa, South Asia, Sub–Saharan Africa

Notes. The black line represents the unweighted average for all countries in the sample. Averages are unweighted by country size. In each panel, eight countries were dropped for having highly inconsistent time series or missing observations.

Source. Barro-Lee educational attainment database.

Figure II: The ratio of male variance to female variance in test scores in reading (top panel) and mathematics (bottom panel)

Source. Project for International Student Assessment (PISA) 2015.

Figure III: Three Hypotheses for the Gender Gap Reversal in Educational Attainment


Notes. **Panel A:** $\bar{z}$ is the threshold of scholastic performance above which individuals attain a level of schooling of schooling of at least $\bar{s}$, such as tertiary education. The grey arrow indicates a decrease in $\bar{z}$ over time. **Panel B:** The grey arrows indicate a faster decrease of $\bar{z}_f$ relative to $\bar{z}_m$, the female and male-specific scholastic performance thresholds, over time. The distribution of scholastic performance by gender are overlapping as they are identical under the mean benefits hypothesis. **Panel C:** The arrow pointing to the right indicates an increase in mean scholastic performance of females, μ_f, relative to males over time as $\bar{z}$ decreases over time.

Figure IV: Distribution Functions of scholastic performance by Gender under the Tail Hypothesis: Illustration
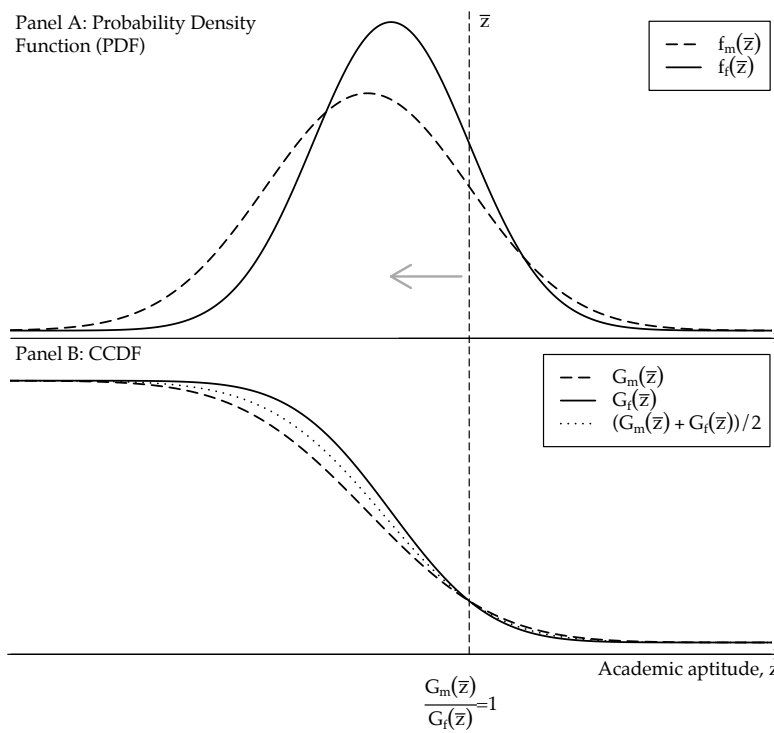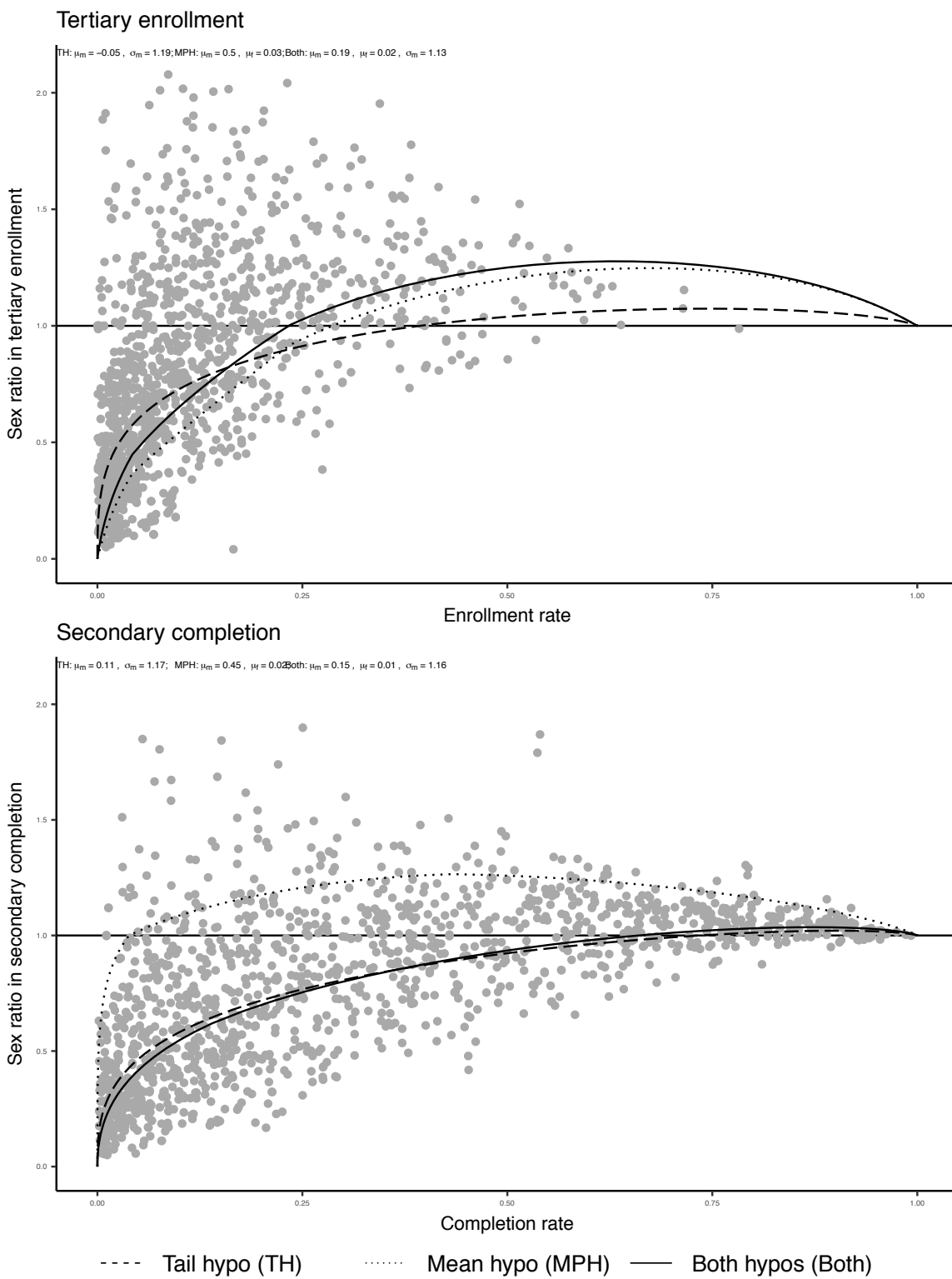
Notes. **Panel A:** The two curves show the probability density functions (pdf) of scholastic performance, z, among males (full line) and females (dashed line), with $\sigma_m^2 > \sigma_f^2$ under the tail dynamics hypothesis and z being normally distributed. **Panel B:** The two curves show the complementary cumulative distribution function (ccdf), resulting from the integration from $+\infty$ to z of $f_{z_f}(z)$ and $f_{z_m}(z)$. The grey arrow indicates a decrease in $\bar{z}$. The gender gap reversal in educational attainment occurs when the ratio of the two CCDFs reaches 1.

Figure V: Relationship between the share of individuals that attain a given level of schooling and the female-to-male ratio under the tail Hypothesis: Illustration
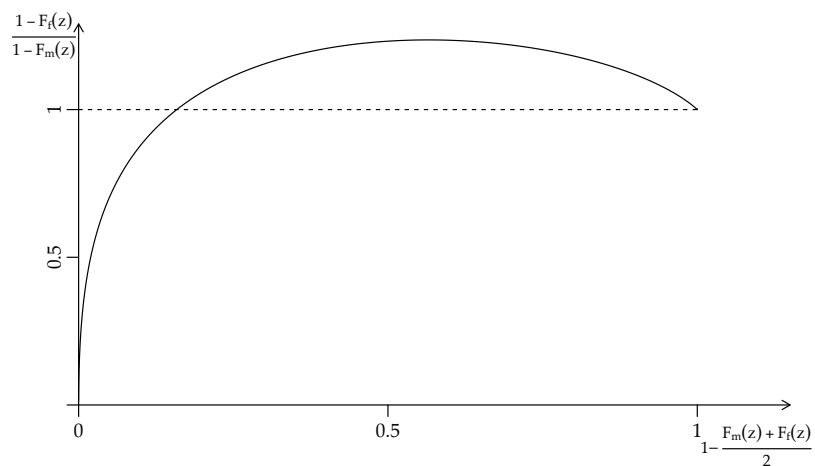
Notes. The x-axis reports the share of individuals that attain a level of schooling of at least $\bar{s}$. The y-axis measures the female-to-male ratio among individuals that attain a level of schooling of at least $\bar{s}$. The gender gap reversal occurs when the female-to-male ratio crosses y=1.

Figure VI: Model fit under the tail and mean hypotheses, baseline country sample

Tertiary enrollment

TH: $\mu_m = -0.05$, $\sigma_m = 1.19$; MPH: $\mu_m = 0.5$, $\mu_f = 0.03$; Both: $\mu_m = 0.19$, $\mu_f = 0.02$, $\sigma_m = 1.13$

Sex ratio in tertiary enrollment

Enrollment rate

Secondary completion

TH: $\mu_m = 0.11$, $\sigma_m = 1.17$; MPH: $\mu_m = 0.45$, $\mu_f = 0.02$; Both: $\mu_m = 0.15$, $\mu_f = 0.01$, $\sigma_m = 1.16$

Sex ratio in secondary completion

Completion rate

- - - - Tail hypo (TH)   .... Mean hypo (MPH) ——— Both hypos (Both)

Appendix

A1 Empirical Support for the Model's Assumptions

A1.1 The Positive Association between scholastic performance and Level of Schooling

The positive relationship between scholastic performance and higher levels of schooling is a key assumption of the micro model. We test for the validity of this assumption empirically by using test score data of 10th graders at age 15 for a representative sample of the US population, that was then followed over time beyond high school. The data is from the Educational Longitudinal study (ELS) which started in 2002 when individuals where in 10th grade, and allows to link test scores in 10th grade to whether individuals ever attended post-secondary education in the following years. Using this data, we estimate a linear probability model of Equation (1), where the dependent variable is tertiary education attendance, denoted H, which is a function of the latent variable s^* in the model:

$$H(s^*) = \begin{cases} 1 & \text{if } s^* \geq \bar{s} \\ 0 & \text{if } s^* < \bar{s} \end{cases}$$

We therefore estimate b in our model from the binary equivalent of Equation (1) for tertiary education, from the regression:

$$H_j = \beta z_j + \varepsilon_j \tag{10}$$

Table A1 reports estimated for the empirical relationship between test scores at age 15 and tertiary education attendance, estimated from the US Educational Longitudinal Survey (ELS). It shows that test scores at age 15, a proxy for individual scholastic performance z, is a major determinant of enrollment in tertiary education. The linear probability model coefficient associated with test scores, i.e the estimate for b in our model, is a positive and highly significant predictor of post-secondary education enrollment a few years later.[21]

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[21]Empirically, the propensity to attend tertiary education increases monotonically with test scores rather than shifting upward discontinuously beyond a given threshold. Our framework is a simplified version of educational enrollment decisions. Empirically, agents certainly enroll into education on the basis of not only individual scholastic performance z_j, but also a set of individual circumstances, such as parental income, personal network or taste for schooling that can be correlated with scholastic performance.

Table A1: Model Estimates of the Relationship between scholastic performance and Participation to Tertiary Education, using US Data

	Dependent Variable: Dummy for Tertiary Education Attendance		
	(1)	(2)	(3)
Composite test score	0.449***		
	(0.003)		
Reading test score		0.449***	
		(0.003)	
Mathematics test score			0.450***
			(0.003)
Number of observations	12,538	12,538	12,538

Notes. ***: significant at the 1% level, **: significant at the 5% level, *: significant at the 10% level. Standard errors are reported in parentheses. The regressions are linear probability models run by ordinary least squares. Regressions do not include an intercept. Test scores were standardized to have a mean of 0 and a standard deviation of 1.
Source. US Educational Longitudinal Survey (ELS), base year (2002) and 2nd follow-up (2006).

A1.2 The increase in b over time

A increase in the net benefit of education b, leading to an increase in years of schooling over time, is a key building block of our of our model. To test the validity of this assumption empirically, we estimate a binary version of Equation 1, where the dependent variable is tertiary education attendance, denoted H. H is observable and is a function of the latent variable s^* in the model:

$$H(s^*) = \begin{cases} 1 & \text{if } s^* \geq \bar{s} \\ 0 & \text{if } s^* < \bar{s} \end{cases}$$

We estimate b, the net benefits to tertiary education in our model, from the linear probability model:

$$H_j = \beta z_j + \epsilon_j,$$

where β denotes the estimate of b in our model. According to the Barro-Lee data, the gender gap reversal in participation to tertiary education occurred in the mid 1980s in the US. The data used for the estimation is from another longitudinal surveys of 10th graders, the High school and

Beyond (HS&B), which started in 1980. As for the ELS 2002, this study collects 10th grade test scores that can be linked to tertiary education attendance a few years later, allowing to estimate changes in the value of b over the 20-year period in which the reversal occurred.

Table A2 reports our estimates for b in the US in 1980 and 2002 using Equation (1) from our framework. It shows that the estimated net benefits of tertiary education increased sharply from 1980 to 2002, providing empirical support to the second building block of our model. The null hypothesis of equality between b_{1980} and b_{2002} is strongly rejected by statistical tests. This indicates that individuals with the same level of scholastic performance are more likely to enroll in higher education in 2002 than they were in 1980, which is equivalent to higher net benefits of education in 2002 compared to 1980 in our framework.

Table A2: Model Estimates of the Relationship between scholastic performance and Participation to Tertiary Education in 1980 and 2002, using US Data

	Dep. Variable: Tertiary Education Attendance	
	1980	2002
$\hat{\beta}$	0.278***	0.450***
	(0.008)	(0.005)
Number of Observations	12,493	12,538

Notes. ***: significant at the 1% level, **: significant at the 5% level, *: significant at the 10% level. Standard errors are reported in parentheses. The regressions are linear probability models run by ordinary least squares. Regressions do not include an intercept. Test score are standardized to have a mean of 0 and a standard deviation of 1 in the full sample.
Sources. High School and Beyond, base year (1980) and 2nd follow-up (1984), US Educational Longitudinal Study, base year (2002) and 2nd follow-up (2006).

This evidence is consistent with a large body of literature showing that returns to education, in particular returns to tertiary education, have increased over the past decades. Grogger and Eide (1995) Goldin and Katz (2007) or Acemoglu and Autor (1998) among others provide consistent evidence of a sharp increase of the college wage premium in the US since the beginning of the 1970s.[22] This rise in the college wage premium was also observed in other contexts. Card and

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[22]The college wage premium is defined as the wage of college-educated workers relative to the wage of high-school educated workers.

Lemieux (2000) also report an important increase in the wage premium of university graduates relative to high school graduates in the UK and Canada over the same period.

A1.3 The Negative Relationship between Enrollment Rate and Mean scholastic performance of those who Enroll

Given the positive association between individual achievement z and optimal level of schooling s^* achieved by individuals in our model, a rise in b leads to a simultaneous increase in enrollment, and to a decrease in $\bar{z}$, the scholastic performance threshold for achieving a given level of schooling $\bar{s}$. We do not observe $\bar{z}$ empirically, but one implication of a decrease in $\bar{z}$ with no accompanying increase in the mean of the distribution is a decrease in the mean ability of individuals enrolled, which can be tested. To do so, we estimate the average verbal test score of individuals that have attended tertiary education in the US over the period 1975–2010, using data from the General Social Survey (GSS). From 1974 onwards, the GSS includes a short 10-item multiple choice test assessing vocabulary knowledge of respondents. A measure of educational attainment in years is also reported, allowing to identify individuals that attended post-secondary education. We classified individuals that have completed more than 12 years of schooling as having attended tertiary education.

Figure A1 depicts the evolution of the average verbal score of post-secondary students relative to the entire population, from 1975 to 2010. In 1975, the average cognitive score of students attending post-secondary education was 0.60 standard deviation higher than the average cognitive score of the whole population. This relative difference decreased progressively until 2005 to reach approximately 0.30 standard deviations in 2010. Consistent with the assumptions of the model, this suggests that greater access to post-secondary education, an increase in P in our framework, was accompanied by a decrease in the average test scores of individuals enrolling in tertiary education.

Figure A1: Empirical Relationship between Enrollment Rate in Tertiary Education and Average Verbal Score of Participants in Tertiary Education: 1975-2010
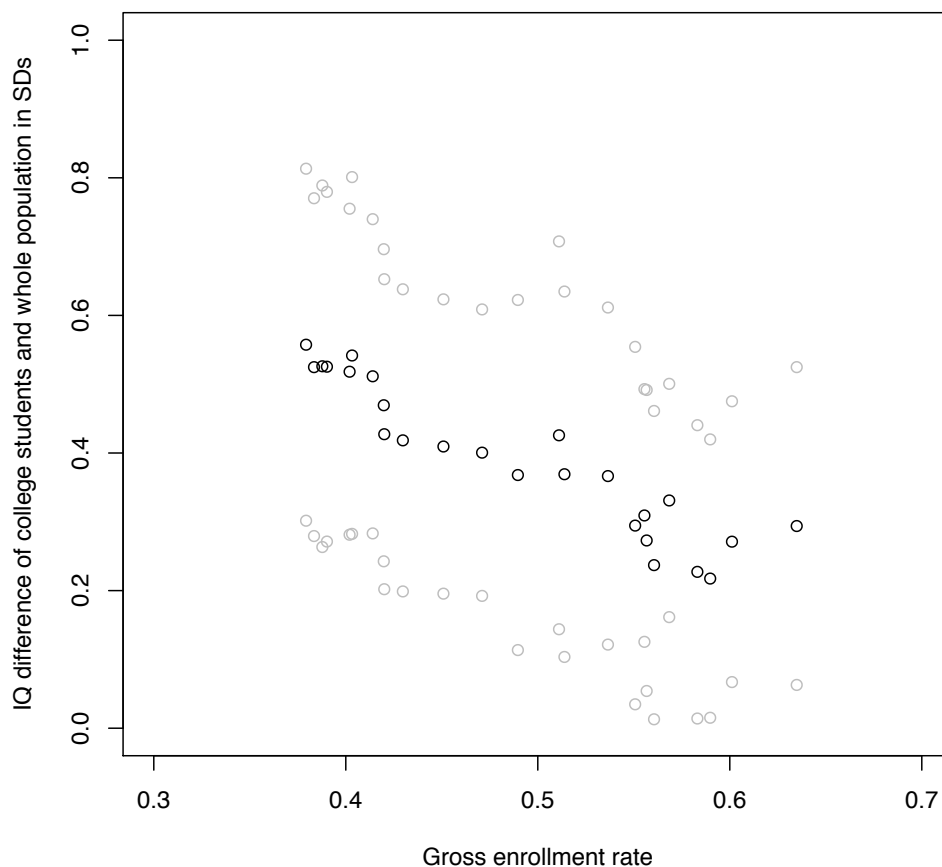
Source. US General Social Survey (1975-2010).
Notes. Dots represent point estimates. Vertical lines represent confidence intervals at the 5%-level.

A2 The Mean Dynamics Hypotheses: Supplementary Material

A2.1 The Mean Benefit Hypothesis (MBH)

Goldin, Katz and Kuziemko (2006) or Chiappori, Iyigun and Weiss (2009) suggested that the returns from education, including labor market and marriage market returns, may have risen more for women over the past decades, which could have driven the gender gap reversal. Both contributions invoke the progressive removal of female career barriers, as a driver of increased returns. Goldin, Katz and Kuziemko (2006) associate the removal of barriers to female employment to a

progressive change in fertility and marriage patterns, driven by the access to reliable contraception (Goldin and Katz (2002)). The authors posit that this raised females' expectations regarding future labor market outcomes, increased labor force participation, and moved female employment out of traditionally female occupations. In a similar spirit, Chiappori, Iyigun and Weiss (2009) argue that technological progress has freed women from many domestic tasks, which disproportionally increased the labor market and marriage market returns of schooling for females. According to both contributions, the disproportionate increase in the benefits of education for females relative to males generated the reversal.

We formulate the mean benefits hypothesis in our framework by allowing the net benefits of education b to differ between genders. Let b_m and b_f denote the net benefits of education for males and females, where b_f and b_m are allowed to have different dynamics over time. $G_z(.)$ is assumed to be identical for males and females. Under the MBH, the optimal level of schooling chosen by individuals conditional on scholastic performance is gender-specific:

$$s^* = z \cdot b_g,$$

or, equivalently, $\bar{z}$, is gender specific:

$$\bar{z}_g = \frac{\bar{s}}{b_g}, \tag{11}$$

where $g = \{m; f\}$, with m standing for males and f for females. Equation 11 states that there exists gender-specific scholastic performance thresholds such that males and females enroll in a given level of schooling $\bar{s}$, denoted $\bar{z}_m$ and $\bar{z}_f$ respectively. The enrollment rate in a given level of schooling $\bar{s}$ reads:

$$P^{MBH}(\bar{z}_f, \bar{z}_m) \equiv \frac{G_z(\bar{z}_m) + G_z(\bar{z}_f)}{2}, \tag{12}$$

Under the MBH, a shift from $b_m > b_f$ to $b_m < b_f$ over time, or equivalently, from $\bar{z}_m < \bar{z}_f$ to $\bar{z}_m > \bar{z}_f$, is a necessary and sufficient condition for the gender gap reversal in education to occur. The gender ratio among individuals enrolled in a given level of education $\bar{s}$ is given by:

$$R^{MBH}(\bar{z}_f, \bar{z}_m) \equiv \frac{G_z(\bar{z}_f)}{G_z(\bar{z}_m)}, \tag{13}$$

and:

$$\begin{cases} R^{MBH} < 1 & \text{if } \bar{z}_m < \bar{z}_f \\ R^{MBH} = 1 & \text{if } \bar{z}_m = \bar{z}_f \\ R^{MBH} > 1 & \text{if } \bar{z}_m > \bar{z}_f. \end{cases}$$

Using the Barro-Lee data, we parameter values of the model by solving the system of two equations (12 and 13) and two unknowns $\{\bar{z}_{f_t}, \bar{z}_{m_t}\}$ for each cohort t, where $P^{MBH}(\bar{z}_{f_t}, \bar{z}_{m_t}) = y_t$ and $R^{MBH}(\bar{z}_{f_t}, \bar{z}_{m_t}) = x_t.$[23]

A3   Technical Appendix

A3.1   The Tail Hypothesis: Mathematical Proofs of Propositions 1 to 3, Normal Distributions

Let $f_{zf}(z)$ and $f_{zm}(z)$ denote the probability distribution functions of scholastic performance z for females and males, respectively. We assume for the sake of the argument that:

$$z_f \sim N(\mu_f, \sigma_f^2)$$

and

$$z_m \sim N(\mu_m, \sigma_m^2),$$

where $\sigma_m^2 > \sigma_f^2$, according to the tail dynamics hypothesis.

Proof of Proposition 1. *The female-to-male ratio $R^{TH}(\bar{z})$ tends to zero when the total enrollment rate $P^{TH}(\bar{z})$ tends to zero.*

First, it is straightforward to see that $\lim_{\bar{z} \to \infty} P^{TH}(\bar{z}) = \frac{G_{z_f}(\bar{z}) + G_{z_m}(\bar{z})}{2} = \frac{0+0}{2} = 0$, where $G_z(\bar{z})$ denotes the complementary cumulative distribution function (or tail distribution function) of scholastic performance z, defined as $\int_{\bar{z}}^{+\infty} f_z(z)\,dz$.

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[23]Estimates are available upon request.

Let us now study $\lim_{\bar{z}\to\infty} R^{TH}(\bar{z})$. Using the analytical expression of the probability distribution function of the normal distribution, the ratio $R^{TH}(\bar{z})$ can be expressed as:

$$R^{TH}(\bar{z}) = \frac{\int_{\bar{z}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_f^2}} e^{\frac{(\bar{z}-\mu_f)^2}{2\sigma_f^2}} \, dz}{\int_{\bar{z}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_m^2}} e^{\frac{(\bar{z}-\mu_m)^2}{2\sigma_m^2}} \, dz}$$

Taking the integrals, one can express the ratio as:

$$R^{TH}(\bar{z}) = \frac{\frac{1}{2}\left(1 - erf\left[\frac{\bar{z}-\mu_f}{\sqrt{2\sigma_f^2}}\right]\right)}{\frac{1}{2}\left(1 - erf\left[\frac{\bar{z}-\mu_m}{\sqrt{2\sigma_m^2}}\right]\right)},$$

where $erf(\cdot)$ denotes the Gauss error function, defined as $erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} \, dt$.

Using the analytical expression of $R^{TH}(\bar{z})$, we get:

$$\lim_{\bar{z}\to\infty} \frac{\frac{1}{2}\left(1 - erf\left[\frac{\bar{z}-\mu_f}{\sqrt{2\sigma_f^2}}\right]\right)}{\frac{1}{2}\left(1 - erf\left[\frac{\bar{z}-\mu_m}{\sqrt{2\sigma_m^2}}\right]\right)} = \lim_{z\to\infty} \frac{1 - (erf\,[\bar{z}])}{1 - (erf\,[\bar{z}])} = \frac{1-1}{1-1} = \frac{0}{0},$$

where the second to last step follows from the fact that $\lim_{\bar{z}\to\infty} erf(\bar{z}) = 1$.

Using the l'Hôpital rule, we take the derivative of the denominator and the numerator to get the following expression:

$$\lim_{\bar{z}\to\infty} \frac{\sigma_m^2}{\sigma_f^2} exp\left\{ \frac{(\bar{z}-\mu_m)^2}{\sigma_m^2} - \frac{(\bar{z}-\mu_f)^2}{\sigma_f^2} \right\} = \lim_{\bar{z}\to\infty} \frac{\sigma_m^2}{\sigma_f^2} exp\left\{ \frac{(\bar{z}-\mu_m)^2\sigma_f^2}{\sigma_m^2\sigma_f^2} - \frac{(\bar{z}-\mu_f)^2\sigma_m^2}{\sigma_f^2\sigma_m^2} \right\}$$

$$= \lim_{\bar{z}\to\infty} \frac{\sigma_m^2}{\sigma_f^2} exp\left\{ \frac{\bar{z}^2\sigma_f^2 - 2\bar{z}\mu_m\sigma_f^2 + \mu_m^2\sigma_f^2 - \bar{z}^2\sigma_m^2 + 2\bar{z}\mu_f\sigma_m^2 - \mu_f^2\sigma_m^2}{\sigma_m^2\sigma_f^2} \right\}$$

$$= \lim_{\bar{z}\to\infty} \frac{\sigma_m^2}{\sigma_f^2} exp\left\{ \frac{\bar{z}}{\sigma_m^2\sigma_f^2} \left[\bar{z}\{\sigma_f^2 - \sigma_m^2\} - 2\mu_m\sigma_f^2 + 2\mu_f\sigma_m^2 + \frac{\mu_m^2\sigma_f^2}{\bar{z}} - \frac{\mu_f^2\sigma_m^2}{\bar{z}} \right] \right\} = 0,$$

since, by assumption under the tail dynamics hypothesis, $\sigma_m^2 > \sigma_f^2$, which are both positive by definition.

Proof of Proposition 2. *The female-to-male ratio $R^{TH}(\bar{z})$ tends to one when the total enrollment rate $P^{TH}(\bar{z})$ tends to one.*

First, it is straightforward to see that $\lim_{\bar{z} \to infty} P^{TH}(\bar{z}) = \frac{G_{z_f}(\bar{z}) + G_{zm}(\bar{z})}{2} = \frac{1+1}{2} = 1$.

Let us now study the behavior of $R^{TH}(\bar{z})$ when $\bar{z}$ tends to $-\infty$:

$$\lim_{\bar{z} \to -\infty} \frac{\frac{1}{2}\left(1 - erf\left[\frac{\bar{z}-\mu_f}{\sqrt{2}\sigma_f^2}\right]\right)}{\frac{1}{2}\left(1 - erf\left[\frac{\bar{z}-\mu_m}{\sqrt{2}\sigma_m^2}\right]\right)} = \lim_{\bar{z} \to -\infty} \frac{1 - (erf[\bar{z}])}{1 - (erf[\bar{z}])} = \frac{1+1}{1+1} = 1,$$

where we use the fact that $\lim_{\bar{z} \to -\infty} erf(\bar{z}) = -1$.

Proof of Proposition 3. *There exists a value of $P^{TH}(\bar{z})$ such that $R^{TH}(\bar{z}) = 1$. This value is unique and always exists.*

Let us now show that given our distributional assumptions, there exists a value of z denoted z^*, such that the numerator and denominator are of equal value, thus the ratio is one. Again, we invoke the ratio:

$$R^{TH}(\bar{z}) = \frac{\frac{1}{2}\left(1 - erf\left[\frac{\bar{z}-\mu_f}{\sqrt{2}\sigma_f^2}\right]\right)}{\frac{1}{2}\left(1 - erf\left[\frac{\bar{z}-\mu_m}{\sqrt{2}\sigma_m^2}\right]\right)}.$$

Since we know that the error function is monotonously increasing on the whole domain, $R^{TH}(\bar{z}) = 1$ when:

$$\frac{\frac{\bar{z}-\mu_f}{\sqrt{2}\sigma_f^2}}{\frac{\bar{z}-\mu_m}{\sqrt{2}\sigma_m^2}} = 1 \Leftrightarrow \frac{\bar{z}-\mu_f}{\sigma_f^2} = \frac{\bar{z}-\mu_m}{\sigma_m^2} \Leftrightarrow \bar{z} = \frac{\mu_m\sigma_f^2 - \mu_f\sigma_m^2}{\sigma_f^2 - \sigma_m^2}.$$

This equation has a unique solution given $\sigma_m^2 > \sigma_f^2$. Since the support of $E(\bar{z})$ is the whole real line, there always exists a value of $\bar{z}$ denoted $\bar{z}^*$ such that:

$$\bar{z}^* = \frac{\mu_m\sigma_f^2 - \mu_f\sigma_m^2}{\sigma_f^2 - \sigma_m^2}.$$

In addition, $\bar{z}^*$ is unique given the vector of exogenous parameters $\{\mu_f, \mu_m, \sigma_f^2, \sigma_m^2\}$.

A3.2 Equivalence of the MPH and MBH hypotheses for uniform distribution within the intersection of their domains

The MPH in our framework models the gender gap reversal such that females initially have lower participation rates in education, due to a lower $E[z]$. Conditional on z, the net benefits of schooling and the optional level of schooling chosen are identical for both genders: $E[s_f^*|z] = E[s_m^*|z]$. The MPH and MBH thus differ in the interpretation of the source of the dynamics in the gender mean differences. They are, in the way they are modeled in our framework, algebraically very similar. It can be shown that if scholastic performance is normally distributed, they are actually identical within the intersection of the domains of the two distributions. As a result, the closer the CCDFs of z are to linear, the more similar the two hypotheses will be in our framework. Here follows the proof to the claim that the MPH is equivalent to the MBH for uniform distribution within the intersection of their domains.

MBH: For each time period t, let $b \equiv a + c$ and $z \sim unif(a, b) = unif(a, a + c)$. Then, $G_z(\bar{z}) = 1 - \frac{\bar{z}-a}{b-a} = 1 - \frac{\bar{z}-a}{a+c-a} = 1 - \frac{\bar{z}-a}{c}$. Also, let (*): $\bar{z}_m \equiv \bar{z}$ and (**): $\bar{z}_f \equiv \bar{z} + a - a_f$. Now, for $\bar{z}_f, \bar{z}_m \in \{a, a+c\} \Leftrightarrow \bar{z} + a - a_f, \bar{z} \in \{a, a+c\}$,

$$E^{MBH}(\bar{z}_f, \bar{z}_m) = \frac{G_z(\bar{z}_m) + G_z(\bar{z}_f)}{2} = \frac{1}{2}\left(1 - \frac{\bar{z}_m - a}{(a+c)-a} + 1 - \frac{\bar{z}_f - a}{(a+c)-a}\right) = \frac{1}{2}\left(2 - \frac{\bar{z}_m + \bar{z}_f - 2a}{2c}\right)$$

$$= 1 - \frac{\bar{z}_m + \bar{z}_f - 2a}{4c} \stackrel{(*,**)}{=} 1 - \frac{2\bar{z} - a - a_f}{4c} \ and$$

$$R^{MBH}(\bar{z}_f, \bar{z}_m) \equiv \frac{G_z(\bar{z}_f)}{G_z(\bar{z}_m)} = 1 - \frac{\bar{z}_f - a}{(a+c)-a} \Big/ 1 - \frac{\bar{z}_m - a}{(a+c)-a} = \frac{c}{c} - \frac{\bar{z}_f - a}{c} \Big/ \frac{c}{c} - \frac{\bar{z}_m - a}{c}$$

$$= \frac{c - \bar{z}_f + a}{c - \bar{z}_m + a} \stackrel{(*,**)}{=} \frac{c - \bar{z} + a_f}{c - \bar{z} + a}$$

MPH: For $\bar{z} + a - a_f, \bar{z} \in \{a, a + c\}$,

$$E^{MPH}(\mu_f, \bar{z}) = \frac{G_{z_m}(\bar{z}) + G_{z_{f,t}}(\mu_f, \bar{z})}{2} = \frac{1}{2}\left(1 - \frac{\bar{z} - a}{(a + c) - a} + 1 - \frac{\bar{z} - a_f}{(a_f + c) - a_f}\right)$$

$$= \frac{1}{2}\left(2 - \frac{2\bar{z} - a - a_f}{2c}\right) = 1 - \frac{2\bar{z} - a - a_f}{4c} = E^{MBH}(\bar{z}_f, \bar{z}_m) \, and$$

$$R^{MPH}(\mu_f, \bar{z}) \equiv \frac{G_{z_{f,t}}(\mu_f, \bar{z})}{G_{z_m}(\bar{z})} = 1 - \frac{\bar{z} - a_f}{(a_f + c) - a_f} \Big/ 1 - \frac{\bar{z} - a}{(a + c) - a} = \frac{c}{c} - \frac{\bar{z} - a_f}{c} \Big/ \frac{c}{c} - \frac{\bar{z} - a}{c}$$

$$= \frac{c - \bar{z} + a_f}{c - \bar{z} + a} = R^{MBH}(\bar{z}_f, \bar{z}_m)$$

Thus, the MBH and MPH are equivalent when scholastic performance is uniformly distributed and when $\bar{z} + a - a_f, \bar{z} \in \{a, a + c\}$. The difference between the two hypotheses originates from the non-linearities in $G_z(\cdot)$.