Uncertainty and Economic Activity: A Multi-Country Perspective*

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Abstract

This paper develops an asset pricing model with heterogeneous exposure to a persistent world growth factor to identify global growth and financial shocks in a multi-country panel VAR model for the analysis of the relationship between volatility and the business cycle. The econometric estimates yield three sets of empirical results regarding (i) the importance of global growth for the interpretation of the correlation between volatility and growth over the business cycle and the possible presence of omitted variable bias in single-country VARs studies, (ii) the extent to which output shocks drive volatility, and (iii) the transmission of volatility shocks to output growth.

Keywords: Uncertainty, Business Cycle, Global Shocks, Multi-Country Asset Pricing Model, Panel VAR, Identification, Realized Volatility, Impulse Responses.
JEL Codes: E44, F44, G15.

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1 Introduction

It is well-established that empirical measures of uncertainty behave countercyclically in the United States as well as in most other countries.[1] Interpreting correlations is always problematic, as causation can run in both directions. From a theoretical standpoint, uncertainty can cause economic activity to slowdown and even contract through a variety of mechanisms, both on the household side *via* precautionary savings (Kimball (1990)) and on the firm side *via* investment delays or other frictions (see, for instance, Bernanke (1983), Dixit and Pindyck (1994) and, more recently, Bloom (2009), Basu and Bundick (2017), among many others). But it is also possible that uncertainty responds to fluctuations in economic activity or other unobserved effects. Indeed, the theoretical literature highlights mechanisms through which adverse economic conditions can trigger spikes in uncertainty. Examples based on information and financial frictions include Van Nieuwerburgh and Veldkamp (2006), Fostel and Geanakoplos (2012), Kozlowski et al. (2015), and Ilut et al. (2017). Theory, therefore, does not provide a definite guidance on how to interpret the negative correlation between output growth and uncertainty measures that is in the data.

In this paper we base our empirical analysis on realized stock market volatility as a measure of uncertainty and adopt a multi-country perspective. We begin with the empirical observation that realized volatility and output growth are closely correlated not only *within* countries, but also *across* countries. We also observe that this cross-country correlation is much stronger for volatility than for output growth. To interpret these stylized facts, we specify a two-factor augmented multi-country panel VAR model (PVAR). We identify the factors by imposing restrictions on the cross-country correlation of volatility and output growth, consistent with a multi-country version of the Lucas (1978) tree model.

In the theoretical part of the paper we assume that country-specific output growth (namely the dividend or endowment growth process) is determined by a common persistent component with time-varying volatility and heterogeneous loadings, and country-specific business cycle components with a conditionally heteroskedastic variance-covariance matrix. We show that country-specific equity returns and their volatility are driven by more than one common factor, in addition to the common component of the endowment process. Furthermore, assuming that no country is dominant and country-specific business cycle components are weakly cross-correlated, we show that the global growth factor is the only driver of the cross section correlation of country-specific output growth in the theoretical model. As a consequence, by combining all common shocks to volatility other than the world growth factor in a second composite shock, it is possible to capture all the remaining cross-country correlation of the volatility series in a single additional common factor. In the theoretical analysis, we also derive an expression for the covariance between output growth and the volatility of excess returns, showing that it can be negative for plausible values of the coefficient of relative risk aversion, although its magnitude and sign can vary across countries.

[1] For the evidence on the United States see, for example, Schwert (1989) using the volatility of the aggregate stock market return; Campbell et al. (2001) and Bloom et al. (2007) using the volatility of firm-level stock returns; Bloom et al. (2012) and Bachmann and Bayer (2013) using the volatility of plant, firm, industry and aggregate output and productivity; Bachmann et al. (2013) using the behavior of expectations' disagreement. For evidence on other countries, see Baker et al. (2018), Carriere-Swallow and Cespedes (2013), and Nakamura et al. (2017), among others.

Consistent with our theoretical derivations, we then specify a multi-country econometric model in output growth and log-volatility with two common shocks and two country-specific shocks. We assume the first shock, which we refer to as the *global growth shock* is common to both GDP growth and volatility in all countries, while the second shock, which we refer to the *global financial shock,* is only common to the volatility series after controlling for the global growth shock. To identify the two country-specific shocks we use auxiliary assumptions typically used in the literature. Identification of the common shocks, however, does not require any restrictions on the within-country correlation of country-specific volatility and output growth shocks.

More specifically, in line with our theoretical derivations, the global growth and financial shocks are identified by assuming that the country-specific growth innovations are weakly correlated across couturiers, while the innovations of the volatility equations share at least one additional common factor, and no country is large enough to affect all other economies in the world. We show that these identification restrictions not only are consistent with the consumption-based asset pricing theory embedded in our theoretical model, but they also fit well the stylized facts of the data. Specifically, in addition to documenting that they are highly correlated *within* countries, we also show that realized volatility and output growth are closely correlated *across* countries, but this cross-country correlation is much stronger for volatility than for growth. We also establish that global growth shocks are consistently estimated as residuals from the regression of global output growth on its lagged values as well as the lagged values of global volatility, whilst global financial shocks are obtained as residuals from the regression of global volatility on the estimated global growth shocks and the lagged values of global output growth and global volatility.[2]

Our empirical analysis yields three main results. First, we find that a large proportion of the observed negative country-specific correlations between volatility and output growth can be accounted for by shocks to the world growth factor. While unconditionally volatility behaves countercyclically for all but one of the 32 countries in our sample, when we condition on world growth shocks, the correlations between volatility and growth innovations become quantitatively much smaller and statistically insignificant in all countries. This implies that part of the explanatory power attributed to volatility shocks in empirical studies of individual countries (i.e. considered in isolation from the rest of the world economy) might be due to omitted common factors. Indeed, direct evidence on this hypothesis does not contradict it. In line with the insight of our theoretical model, we also show that shocks to the global growth factor correlate closely with proxies for global TFP growth and the world natural interest rate, and more weakly with a measure of global long-run risk.

Second, the paper finds that the time-variation of country-specific volatility is explained largely by shocks to the global financial factor (with a share of forecast error variance being larger than 60 percent) and innovations to country-specific volatility series themselves (with a share of forecast error variance of about 35 percent). Shocks to the global growth factor and to country-specific growth innovations jointly explain less than 5 percent of volatility forecast error variance. This result implies

[2]Note here that one cannot arrive at these estimates by principal component (PC) analysis, where the common factors are estimated as PCs of output growth and/or volatility series considered separately or together, since the PC analysis does not make use of the *a priori* identification of the shocks and, being static in nature, cannot cope with the heterogeneous dynamics of the interactions between volatility and growth across countries.

that the 'endogenous' component of country-specific volatility—namely, the component driven by common and country-specific growth shocks—is quantitatively small.

Third, we find that shocks to the global financial factor explain about 8 percent of the forecast error variance of country-specific output growth, and they have strong and persistent contractionary effects. In contrast, country-specific volatility shocks explain slightly less than 4 percent of the country-specific forecast error growth variance. In our empirical model, the forecast error variance of output growth is explained mainly by innovations to country-specific growth rates themselves (with a share of more than 60 percent) and the world growth shock (with another 25 percent of the total). This third set of results illustrates that even though there might be a very high degree of international comovement in equity prices, such such comovements need not translate into a high degree of explanatory power for real GDP growth. Nonetheless, when global financial shocks occur they can have deep and lasting negative effects, as we record during the recent global financial crisis.

Our paper is closely related to several contributions in the literature on uncertainty and the business cycle.[3] Ludvigson et al. (2015) and Berger et al. (2017) acknowledge that uncertainty has endogenous components and can be driven by the business cycle. We take a common factor approach to modeling the two-way relationship between volatility and growth in a multi-country setting as opposed to a single-country approach. The restrictions that we impose to identify the common factors apply to a cross section of countries, as opposed to a single-country in isolation from the rest of the world, or the global economy as a single closed economy. The identification problem that we pose cannot be addressed in a single-country set up. Furthermore, we show that single-country approaches to the question we study could yield empirically biased estimates due to the omitted common factors. Interestingly, despite the different approaches taken to proxy for uncertainty and to separate endogenous responses to the business cycle from exogenous changes in volatility, we reach very similar conclusions that the endogenous component of volatility is quantitatively small (Ludvigson et al. (2015)) and that first moment shocks can account for the countercyclical nature of volatility (Berger et al. (2017)).

While other papers have highlighted similar patterns of cross-country correlations for equity returns and consumption growth (Tesar (1995), Colacito and Croce (2011), Lewis and Liu (2015)), as far as we are aware, this is the first paper that focuses on cross-country correlations of volatility and output growth. Consistent with Renault et al. (2018), who take an no-arbitrage approach to the pricing of square returns, and Chamberlain and Rothschild (1982) and Sentana (2002), where it is assumed that the idiosyncratic component of returns is weakly correlated, we develop an empirical methodology to determine the number of risk factors necessary to obtain weakly correlated idiosyncratic shocks and estimate them consistently from the data. As in Colacito et al. (2018), we focus on the persistent component of 'dividend' growth or the 'cash flow' risk, as proxied by real GDP growth. Unlike them, however, we take a multi-country perspective and consider the *interaction* among many economies, in addition to their heterogeneous loading on the common factor, thus coming to different conclusions on the importance of global growth for country-specific endowment risks.

[3]The literature is voluminous. See Bloom (2014) for a recent survey. Here we focus only on studies directly related to our paper.

Carriere-Swallow and Cespedes (2013) estimate a battery of 40 VARs for advanced and emerging economies in which the US VIX Index, assumed exogenous, is the only source of interdependence, and identification is achieved by imposing a recursive scheme, country-by-country. Baker et al. (2018) study an unbalanced panel of 60 countries, documenting the counter-cyclicality of different proxies for uncertainty and use measures of natural disasters, terrorist attacks and political events as instruments to quantify the exogenous impact of uncertainty on GDP, without, however, quantifying the importance of activity measures for uncertainty. Hirata et al. (2012) and Mumtaz and Musso (2018) estimate a factor-augmented VAR with factors computed based on data for a large number of countries, and use a recursive identification scheme in which the volatility variable is ordered first. Carriero et al. (2018) estimate a large Bayesian VAR with exogenously driven stochastic volatility to quantify the impact of macroeconomic uncertainty on OECD economies. All these papers, therefore, restrict the direction of economic causation from the outset of the analysis by assuming that the uncertainty proxy used is exogenous. In addition, in our framework, countries interact with each other not only *via* the common factors, but also *via* the covariance matrix of the country-specific volatility and growth innovations, which is an important source of additional spillovers absent in other studies.

Notations: Generic positive finite constants are denoted by C_i for $i = 0, 1, 2, \ldots$. They can take different values at different instances. $\mathbf{w} = (w_1, w_2, ..., w_n)'$ is an $n \times 1$ vector, and $\mathbf{A} = (a_{ij})$ an $n \times n$ matrix, $\varrho_{\max}(\mathbf{A})$ denotes the largest eigenvalue of $\mathbf{A}$, $\|\mathbf{w}\| = \left(\Sigma_{i=1}^{n} w_i^2\right)^{1/2}$ and $\|\mathbf{A}\| = [\varrho_{\max}(\mathbf{A}'\mathbf{A})]^{1/2}$ are the Euclidean norm of $\mathbf{w}$, and the spectral norm of $\mathbf{A}$, respectively. $\boldsymbol{\tau}_T = (1, 1, ..., 1)'$ is a $T \times 1$ vector of ones. If $\{y_n\}_{n=1}^{\infty}$ is any real sequence and $\{x_n\}_{n=1}^{\infty}$ is a sequences of positive real numbers, then $y_n = O(x_n)$, if there exists a positive finite constant C_0 such that $|y_n|/x_n \leq C_0$ for all n. $y_n = o(x_n)$ if $f_n/g_n \to 0$ as $n \to \infty$. If $\{y_n\}_{n=1}^{\infty}$ and $\{x_n\}_{n=1}^{\infty}$ are both positive sequences of real numbers, then $y_n = \mathcal{O}(x_n)$ if there exists $n_0 \geq 1$ and positive finite constants C_0 and C_1, such that $\inf_{n \geq n_0}(y_n/x_n) \geq C_0$, and $\sup_{n \geq n_0}(y_n/x_n) \leq C_1$.

2 Output Growth and Volatility: Measurement and Summary Statistics

We start with the data we use in our empirical analysis and present stylized facts that highlight the cross country correlations of output growth and volatility that form the basis of our theoretical and empirical contributions. To proxy for volatility, we use country-specific quarterly realized volatility (RV_{it}) based on the summation of squared daily equity returns in country i, given by

$$RV_{it} = \sqrt{\sum_{\tau=1}^{D_t} \left[r_{it}(\tau) - \bar{r}_{it}\right]^2}, \tag{1}$$

where $r_{it}(\tau)$ is the asset return during day τ in quarter t in country i, $\bar{r}_{it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{it}(\tau)$, and D_t is the number of trading days in quarter t.[4] In the empirical applications we use log RV_{it} as our country-specific measure of volatility.

There are, of course, a number of other measures that can be used. For example, using a panel data of equity returns on firms or sectors *within* a given country, country-specific volatility can be measured as the cross-sectional dispersion of equity returns. In Section S.4 of the online Supplement to the paper we show that, under fairly general assumptions, realized volatility is closely correlated with this alternative measure and, given that it can be more readily constructed, it is preferable in our application. In the finance literature, the focus of the volatility measurement has now shifted to implied volatility measures obtained from option prices, like the US VIX Index. However, a key input for the implementation of our identification strategy is the availability of country-specific measures for a large number of countries over a long period of time, and implied volatility measures are not yet available for a meaningful number of countries. Moreover, in our robustness analysis, we will show that we find even stronger results by using the US VIX Index rather than our RV measure of US realized volatility. The literature has also used measures based on the dispersion of expectations such as, for instance, the one proposed by Bachmann et al. (2013) in the case of the United States, and by Rossi and Sekhposyan (2015) and Ozturk and Sheng (2018) in the international context. While the data set of Rossi and Sekhposyan (2015) covers a large number of countries, the time series dimension is unbalanced and often not long enough for our purposes. Finally, model based measures, such as those in Jurado et al. (2015) and Ludvigson et al. (2015) could in principle be computed for all countries in our sample, but the data requirements to construct such proxies for many countries over a sufficiently long time period are prohibitive.

The data sources for quarterly real GDP and daily equity prices that we use to measure real output growths and country-specific volatilities are given in Section S.1 of the online Supplement to the paper. To construct a balanced panel for the largest number of countries for which we have sufficiently long time series, we first collect daily stock prices for 32 advanced and emerging economies from 1979 to 2016.[5] We then cut the beginning of the sample in 1993, as daily equity price data are not available earlier for two large emerging economies (Brazil and China) and for Peru. Better quality quarterly GDP data for China also are available from 1993. Our results are robust to excluding these three countries and/or starting the sample in 1988. Moreover, some steps of the empirical analysis can be easily implemented with the unbalanced panel from 1979 without any significant consequences for our main findings. In the online Supplement to the paper, we report summary statistics, showing that realized volatility series are highly persistent (more so than output growth series), but are nevertheless stationary, as required by our analysis.

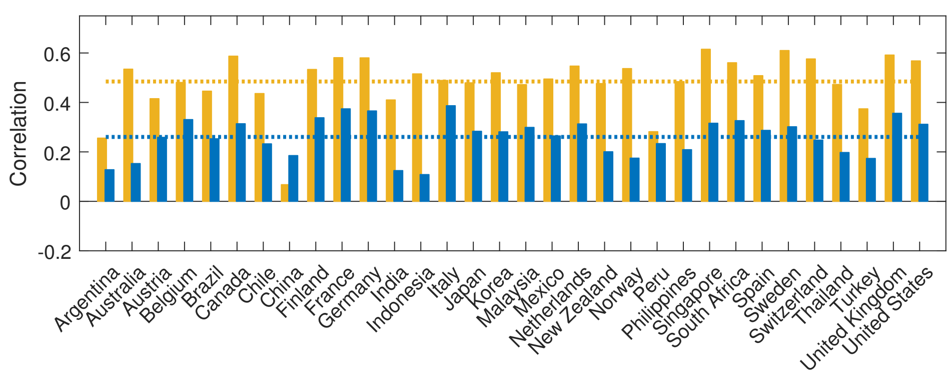
Cross-country correlations of volatility and growth: The differential pattern of cross-country correlations of the growth and volatility *innovations* is crucial for our identification strategy. In order to gauge the extent to which volatility and growth *series* co-move across countries, one can

[4]This is a natural application of within-day measures of volatility based on high frequency within-day price changes. See, for example, Andersen et al. (2001, 2003), and Barndorff-Nielsen and Shephard (2002, 2004).

[5]Daily returns are computed abstracting from dividends, which are negligible by comparison to price changes at this frequency.

use two techniques: standard principal component analysis and pair-wise correlation analysis across countries. The average pair-wise correlation of country i in the panel ($\bar{\rho}_i$) measures the average degree of comovement of country i with all other countries j (i.e. for all $j \neq i$). The average pair-wise correlation *across* all countries, denoted by $\bar{\rho}_N$, is defined as the cross-country average of $\bar{\rho}_i$ over $i = 1, 2, ..., N$. This statistics relates to the degree of pervasiveness of the factors, as measured by the factor loadings. The attraction of the average pair-wise correlation, $\bar{\rho}_N$, lies in the fact that it applies to multi-factor processes, and unlike factor analysis does not require the factors to be strong. In fact, $\bar{\rho}_N$ tends to be a strictly positive number if the panel of series is driven by at least one strong factor, otherwise it must tend to zero as $N \to \infty$. Therefore, non-zero estimates of $\bar{\rho}_N$ are suggestive of strong cross-sectional dependence.[6] Figure 1 plots $\bar{\rho}_i$ for all $i = 1, 2, ...N$ and $\bar{\rho}_N$ for volatility and output growth series (light and dark bars, respectively). It can be seen that, on average across all countries, the average pair-wise correlation for the volatility series is more than twice the average for the growth series, at 0.58 and 0.27, respectively (the two dotted lines). This evidence suggests that both series share at least one strong common factor, but the degree of dependence shared among volatility series is much stronger than among output series.

Figure 1 Average Pair-wise Correlations of Volatility and Growth



Note. For each country, the light (yellow) and the dark (blue) bar show the average pair-wise correlation with the remaining countries in the sample for volatility and GDP growth series, respectively. The dotted lines correspond to the overall average across all countries, equal to 0.55 and 0.27, respectively. Sample period: 1993:Q1-2016:Q4.

For completeness, we also use standard principal component analysis. Principal component analysis yields similar results. The first principal component in our panel of realized volatility series explains 65 percent of the total variation in the log-level of volatility, whilst the first principal component of the growth series accounts for only around 30 percent of total cross-country variations in these series. Thus, both in the case of the pair-wise correlation and principal component analysis, the results point to a much higher degree of cross-country comovements for the volatility series than for the growth series. Our strategy to identify the common factors possibly driving the dynamic relation between volatility and output growth will exploit this differential degree of dependence across countries.

[6]Formal tests of cross-sectional dependence based on estimates of $\bar{\rho}_N$ are discussed in Pesaran (2015) and reported, for our panel data sample, in the sub-section 6.2.

3 A Multi-country Model of Equity Market Volatility and the Business Cycle

In this section we set up a theoretical model to interpret the two common factors that we identify in the data as well as the main empirical results of the paper. The framework is a multi-country version of the Lucas (1978) tree model with complete asset markets, persistent global endowment growth shocks, heterogeneous country loadings on this common factor, time-varying volatilities and cross-country spillovers.

3.1 Endowments and Volatilities

Consider a world consisting of N economies indexed by $i = 1, 2, ...N$, of similar but not necessarily identical size. Each country i is inhabited by an infinitely-lived representative agent endowed at time t with a stochastic stream of a single homogeneous good $\{Y_{i,t+s}, s = 0, 1, 2, ...,\}$ viewed as the economy's measure of real output or GDP, with $\Delta y_{it} = \ln(Y_{it}/Y_{i,t-1})$. We make the following assumptions on the exogenous forces driving $\Delta y_{i,t+1}$:

$$\begin{aligned}
\Delta y_{i,t+1} &= a_i + \gamma_i f_{t+1} + \varepsilon_{i,t+1}, \qquad i = 1, 2, ..., N, &(2)\\
f_{t+1} &= \phi_f f_t + \sigma_t \zeta_{t+1}, &(3)\\
\sigma^2_{t+1} &= \sigma^2(1-\phi_\sigma) + \phi_\sigma \sigma^2_t + \varphi_\chi \chi_{t+1}, &(4)
\end{aligned}$$

where $\sup_i |\gamma_i| < 1$, $|\phi_f| < 1$, $|\phi_\sigma| < 1$, $0 < \sigma^2 < C_0$, a_i and φ_χ are bounded fixed constants, and the innovations ζ_{t+1} and χ_{t+1} have zero means with unit variances. Equation (2) specifies that a country's output growth fluctuates around a deterministic steady state, a_i (interpreted as a country's long-run mean growth rate), and is driven by a common unobserved factor, f_t and the country-specific shock, ε_{it}. The common factor, f_t, which is assumed to follow a stationary first order auto-regressive process, represents the stochastic trend of world growth, possibly driven by productivity (e.g. Aguiar and Gopinath (2007)), and will be loosely referred to as the global growth factor, or 'growth factor' for brevity. As in Bansal and Yaron (2004), f_t is assumed to have mean zero, with conditionally heteroskedastic innovations, but unconditionally homoskedastic variance. Namely, $E(f_t) = 0$, $Var_t(f_{t+1}) = \sigma^2_t$, and $Var(f_{t+1}) = \sigma^2/(1-\phi^2_f)$, where $Var_t(.) = Var(.|\Im_t)$ denotes the conditional variance operator with respect to the non-decreasing information set, $\Im_t$, which at least includes all risky asset returns, r_{it}, as well as Δy_{it} for all $i = 1, 2, ..., N$, and all their lagged values.

The term ε_{it} represents country-specific forces driving the country's business cycles, including technology shocks as well as other demand and supply shocks.[7] Collect the N country-specific shocks in the $N \times 1$ vector $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt})'$ and denote the conditional variance-covariance matrix of $\boldsymbol{\varepsilon}_{t+1}$ by $Cov_t(\boldsymbol{\varepsilon}_{t+1}) = \boldsymbol{\Sigma}_{t\varepsilon}$, where $\boldsymbol{\Sigma}_{t\varepsilon} = (\sigma_{t,ij})$. We assume that the $N \times N$ time-varying

[7] In appendix, we show that this component can be further decomposed as $\varepsilon_{it} = \Delta e_{it} + \Delta\tau_{it}$, where e_{it} is a country-specific technology innovation and τ_{it} is a stationary process representing all country-specific forces driving the country's business cycles, possibly reflecting the effects of aggregate demand shocks, other supply shocks, as well as country-specific uncertainty shocks.

elements of this matrix are such that $Var_t(\varepsilon_{i,t+1}) = \sigma_{t,ii} = \theta_t^2\sigma_{ii} > 0$ and $Cov_t(\varepsilon_{i,t+1}, \varepsilon_{j,t+1}) = \sigma_{t,ij} = \psi_t^2\sigma_{ji}$ for $i \neq j$. With this specification, time-variation in variances of the country-specific shocks differs from the time-variation in covariances, while allowing also for the correlations of the shocks to differ across countries. This permits us to distinguish between the effects of the country-specific shocks, as defined by $\theta_t\sqrt{\sigma_{ii}}$, and the extent to which shocks get transmitted across countries, via $\psi_t^2\sigma_{ji}$, which in our application will play an important role. To keep the analysis manageable, however, we have assumed the patterns of time-variations to be the same across-country pairs. In particular, similarly to the specification of σ_{t+1}^2, we assume that:

$$\theta_{t+1}^2 = \theta^2(1-\phi_\theta) + \phi_\theta\theta_t^2 + \varphi_\varpi\varpi_{t+1}, \tag{5}$$

$$\psi_{t+1}^2 = \psi^2(1-\phi_\psi) + \phi_\psi\psi_t^2 + \varphi_\eta\eta_{t+1}, \tag{6}$$

where $|\phi_\theta|, |\phi_\psi| < 1$, $\theta^2, \psi^2 > 0$, and φ_ϖ, and φ_η are bounded constants, implying the following unconditional moments $E\left(\sigma_{t+1}^2\right) = \sigma^2$, $E\left(\psi_t^2\right) = \psi^2$ and $E\left(\theta_t^2\right) = \theta^2$. Also, without loss of generality, we set $\psi^2 = \theta^2 = 1$, which ensures that $E(\mathbf{\Sigma}_{t\varepsilon}) = \mathbf{\Sigma}_{\boldsymbol{\varepsilon}} = (\sigma_{ij})$.

For tractability, we further assume that the $N+4$ shocks, $\boldsymbol{\varepsilon}_{t+1} = (\varepsilon_{1,t+1}, \varepsilon_{2,t+1}, ..., \varepsilon_{N,t+1})'$, ζ_{t+1}, χ_{t+1}, ϖ_{t+1}, and η_{t+1} are (a) serially independent (over t), (b) distributed independently of each other, and (c) ζ_{t+1}, χ_{t+1}, ϖ_{t+1}, and η_{t+1} are $IIDN(0,1)$ while $\boldsymbol{\varepsilon}_{t+1}\,|\mathcal{F}_t \sim N\left(\mathbf{0}, \mathbf{\Sigma}_{t\varepsilon}\right)$.[8] For future analysis we also introduce here the $N \times 1$ vector of weights, $\mathbf{w_t} = (w_{1t}, w_{2t}, ..., w_{Nt})'$ associated with the $N \times 1$ vector of country-specific shocks, $\boldsymbol{\varepsilon}_{t+1}$, and define the following 'aggregate' output growth shock as $\varepsilon_{w,t+1} = \sum_{i=1}^N w_{ti}\varepsilon_{i,t+1} = \mathbf{w_t}'\boldsymbol{\varepsilon}_{t+1}$.

3.2 Asset Markets, Preferences, and Returns

It will be assumed that the representative agent of country i can trade freely a globally available risk-free bond with known gross return, $R_{f,t+1}$, and N risky country-specific equity claims, defined on the country-specific endowment stream $\{Y_{i,t+s}\}$ with gross returns $R_{i,t+1} = \left(P_{i,t+1} + Y_{i,t+1}\right)/P_{it}$, where P_{it} is the t-dated market price of such claim. We further assume constant relative risk aversion (CRRA) preferences and complete international asset markets in the Arrow-Debreau sense, so that country-specific consumption growth is equalized across countries, and one can use the world endowment growth as the pricing kernel or stochastic discount factor of country i, M_{t+1}, given by:

$$M_{t+1} = \left[\beta\left(\frac{C_{i,t+1}}{C_{it}}\right)^{-\varrho}\right] = \left[\beta\left(\frac{Y_{w,t+1}}{Y_{wt}}\right)^{-\varrho}\right] = \exp\left(\ln\beta - \varrho\Delta\ln Y_{w,t+1}\right), \tag{7}$$

where β $(0 < \beta < 1)$ is the discount rate, ϱ denotes the coefficient of relative risk aversion, and $Y_{w,t+1}$ is world output, defined by $Y_{w,t+1} = \sum_{i=1}^N Y_{i,t+1}$. Denote the world output growth by $g_{w,t+1} =$

[8]Normality is only used to obtain analytically tractable expressions for returns (safe and risky), and is not required for the econometric analysis in our application.

$(Y_{w,t+1}/Y_{w,t})-1$, and note that

$$g_{w,t+1}=\frac{\sum_{i=1}^{N}\left(Y_{i,t+1}-Y_{it}\right)}{\sum_{i=1}^{N}Y_{it}}=\frac{\sum_{i=1}^{N}Y_{it}g_{i,t+1}}{\sum_{i=1}^{N}Y_{it}}=\sum_{i=1}^{N}w_{it}g_{i,t+1}, \tag{8}$$

where $g_{i,t+1}=(Y_{i,t+1}/Y_{it})-1$, $i=1,2,...,N$ are country-specific growth rates, and $w_{it}=Y_{it}/\sum_{j=1}^{N}Y_{jt}$ is the weight of country i in the world economy at time t. Also since $g_{i,t+1}$ and $g_{w,t+1}$ are small they can be well approximated by $g_{i,t+1}\approx\ln\left(1+g_{i,t+1}\right)=\Delta\ln\left(Y_{i,t+1}\right)=\Delta y_{i,t+1}$, and $g_{w,t+1}\approx\ln\left(1+g_{w,t+1}\right)=\Delta\ln\left(Y_{w,t+1}\right)$. Using these results in (8) we obtain

$$g_{w,t+1}\approx\Delta\ln\left(Y_{w,t+1}\right)\approx\sum_{i=1}^{N}w_{it}\Delta y_{i,t+1}.$$

Finally, using the country-specific output growth equations (2), the world growth rate can be written as:

$$\Delta\ln Y_{w,t+1}\approx\sum_{i=1}^{N}w_{it}\Delta y_{i,t+1}=a_{wt}+\gamma_{wt}f_{t+1}+\varepsilon_{w,t+1}, \tag{9}$$

where $a_{wt}=\sum_{i=1}^{N}w_{it}a_i$, $\gamma_{wt}=\sum_{i=1}^{N}w_{it}\gamma_i$, and $\varepsilon_{w,t+1}=\sum_{i=1}^{N}w_{it}\varepsilon_{i,t+1}$.

In what follows, to simplify the notations and without loss of generality we assume the weights are fixed and use w_i, γ_w, α_ω instead of w_{it}, γ_{wt} and a_{wt}, respectively.[9] Then, in view of (7), we have, $M_{t+1}=\exp\left(\ln\beta-\varrho a_w-\varrho\gamma_w f_{t+1}-\varrho\varepsilon_{w,t+1}\right)$, which yields the following expression for the innovation to the (log) stochastic discount factor:

$$m_{t+1}-E_t(m_{t+1})=-\varrho\gamma_w\sigma_t\zeta_{t+1}-\varrho\varepsilon_{w,t+1}. \tag{10}$$

Two remarks are in order here. First, in our model, the price of global growth risk, $\varrho\gamma_w$, depends not only on the degree of risk aversion, ϱ, but also on the average exposure of countries to the global factor, γ_w. Second, equity returns are exposed to a second aggregate risk factor, $\varepsilon_{w,t+1}$ in equation (10), stemming from time-variations in cross-country correlations of idiosyncratic components ε_{it}, captured by θ_{t+1}^2 and its innovations ϖ_{t+1}. As we shall see below, however, this second risk factor and its effects vanish if country-specific shocks are weakly cross correlated, and N is sufficiently large, which is the key assumption made to identify the empirical counterpart of ζ_t from the data in our econometric model.

By arbitrage, the following Euler equation must hold for any (riskless or risky) asset with gross return R_{t+1}: $E_t\left(M_{t+1}R_{t+1}\right)=1$. For the risk free rate, $r_{t+1}^f=1/E_t\left(M_{t+1}\right)$, since by assumption f_t and $\varepsilon_{w,t+1}$ are Gaussian, it follows that:

$$r_{t+1}^f\approx-\ln\beta+\varrho a_w+\varrho\gamma_w\phi_f f_t-\frac{1}{2}\varrho^2\gamma_w^2\sigma_t^2-\frac{1}{2}\varrho^2 Var_t(\varepsilon_{w,t+1}), \tag{11}$$

where $Var_t(\varepsilon_{w,t+1})=\mathbf{w}'\mathbf{\Sigma}_{t\varepsilon}\mathbf{w}$, where $\mathbf{w}=(w_1,w_2,...,w_N)'$. The derivation of country-specific equity

[9] Note that the weights, $\mathbf{w}_t=(w_{1t},w_{2t},...,w_{Nt})'$, as well as a_{wt} and γ_{wt} are included in the information set $\Im_t$.

returns, defined as $r_{i,t+1} = ln(R_{i,t+1})$, is much more involved, and the details are provided in the Appendix. There, it is shown that

$$\begin{aligned} r_{i,t+1} &= b_{0i,N} + b_{1i}f_t + b_{2i}\sigma_t^2 + b_{3i,N}\psi_t^2 + b_{4i,N}\theta_t^2 \\ &\quad + c_{1i}\sigma_t\zeta_{t+1} + c_{2i}\chi_{t+1} + c_{3i,N}\eta_{t+1} + c_{4i,N}\varpi_{t+1} + \varepsilon_{i,t+1}, \end{aligned} \tag{12}$$

where $\{b_{0i}, b_{1i}, b_{2i}, b_{3i,N}, b_{4i,N}\}$ and $\{c_{1i}, c_{2i}, c_{3i,N}, c_{4i,N}\}$ depend on the structural parameters of model and the shocks processes, with some coefficients varying with the size of the cross section, N (see (A13)). Comparing equation (2) and (12), it is clear that, to explain the cross section of equity returns, the model requires at least one more common factor than the cross section of output growth rates, even if ε_{it} were conditionally homoskedastic.

Weak cross-sectional correlation and absence of dominant units: Suppose now that $\mathbf{\Sigma}_{t\varepsilon}$ is such that $\sup_i \sigma_{ii} < C_0$ and $\sup_i \sum_{j=1}^N |\sigma_{ij}| < C_0$, and the weights $w_i > 0$, for $i = 1, 2, ..., N$ are granular, in the sense that they are of order N^{-1} such that $\mathbf{w}'\mathbf{w} = \sum_{i=1}^N w_i^2 = O(N^{-1})$. The first assumption is well understood in finance and allows the country-specific output shocks to be weakly cross-correlated, similar to the arbitrage asset pricing model of Ross (1976) and the approximate factor model of Chamberlain and Rothschild (1982), where it is assumed that the idiosyncratic component of asset returns is weakly correlated. Under this assumption, country-specific risk is fully diversified and will not be priced. The second is a granularity condition that rules out the presence of one or more dominant countries in the cross section sufficiently large that shocks to their outputs affect all other countries in the world.[10]

To see the asset pricing implications of these two assumptions, note first that

$$\begin{aligned} \mathbf{w}'\mathbf{\Sigma}_{t\varepsilon}\mathbf{w} &= \theta_t^2 \sum_{j=1}^N w_j^2 \sigma_{jj} + \psi_t^2 \sum_{j\neq i}^N w_j w_i \sigma_{ij} \\ &\leq \theta_t^2 \left(\sum_{j=1}^N w_j^2\right) sup_i(\sigma_{ii}) + \psi_t^2 \left(\sum_{j=1}^N w_j^2\right) \sup_i \sum_{j=1}^N |\sigma_{ij}| = O(N^{-1}). \end{aligned}$$

As we show in the Appendix, it follows that, as $N \to \infty$, the risk free rate simplifies to

$$r_{t+1}^f \approx -\ln\beta + \varrho a_w + \varrho\gamma_w\phi_f\ f_t - \frac{1}{2}\varrho^2\gamma_w^2\sigma_t^2 + O\left(N^{-1}\right). \tag{13}$$

Similarly, country-specific equity returns will no longer depend on η_{t+1} and ψ_t^2 because, as $N \to \infty$ both $c_{3i,N} = O\left(N^{-1}\right)$ and $b_{3i,N} = O\left(N^{-1}\right)$. This means that the second risk factor in equation (10) associated with time-variations in cross-country correlations vanishes under the assumptions that no single country is dominant and the idiosyncratic shocks are weakly cross-correlated.

To see this from a different perspective, consider the equity risk premium. From (11) and (12),

[10] This assumption could be relaxed by conditioning the analysis on the dominant unit(s), even though in our empirical analysis, as we shall argue, it will not be necessary to do so.

it is easy, albeit tedious, to derive the risk premium for country i. It is given by

$$E_t\left(r_{i,t+1}-r_{t+1}^f\right)=\varrho\gamma_w c_{1i}\sigma_t^2-\frac{1}{2}c_{1i}^2\sigma_t^2-\frac{1}{2}\sigma_{ii}\theta_t^2+\varrho\left(w_i\theta_t^2\sigma_{ii}+\psi_t^2\sum_{j\neq i}^N w_j\sigma_{ji}\right)-\frac{1}{2}\kappa_{1i}^2G_{iN}^2, \quad (14)$$

where $c_{1i}=\left(\gamma_i-\varrho\gamma_w\kappa_{1i}\phi_f\right)/\left(1-\kappa_{1i}\phi_f\right)$, κ_{i1} is a fixed constant, and G_{iN}^2 is defined by (A23) in the Appendix, which provides further details. The above expression can be written equivalently as[11]

$$E_t\left(r_{i,t+1}-r_{t+1}^f\right)=\varrho\gamma_w Cov_t(r_{i,t+1},f_{t+1})+\varrho Cov_t\left(\varepsilon_{i,t+1},\varepsilon_{w,t+1}\right)-\frac{1}{2}Var_t\left(r_{i,t+1}\right),$$

which can be viewed as a multi-country generalization of result (11) in Bansal and Yaron (2004) for the special case of CRRA preferences. The last two terms in (14) depend on N, and $Cov_t\left(\varepsilon_{i,t+1},\varepsilon_{w,t+1}\right)\to 0$, as $N\to\infty$, if the idiosyncratic shocks are weakly correlated and $w_i=O\left(N^{-1}\right)$. In such a case, the expression for risk premium in (14) simplifies to

$$E_t\left(r_{i,t+1}-r_{t+1}^f\right)=\varrho\gamma_w c_{1i}\sigma_t^2-\frac{1}{2}c_{1i}^2\sigma_t^2-\frac{1}{2}\sigma_{ii}\theta_t^2-\frac{1}{2}\kappa_{1i}^2G_i^2+O(N^{-1}), \quad (15)$$

where G_i^2, defined by (A31), no longer depends on N.

3.3 Econometric Implications

In this section we link the above theoretical results to our empirical analysis by discussing the model's implications for the identification of the common factors and the interpretation of the empirical results.

Identification of the global growth and financial shocks: The assumption of weakly cross correlated country-specific output shocks and absence of dominance will play a crucial role in the econometric identification of the common factors from the data. In the absence of such an assumption, country specific output growths would be driven by the second strong common factor, $\varepsilon_{w,t+1}$, in equation (10), in addition to the global growth factor, f_t, and it would not be possible to identify f_t or its innovation, ζ_t, from the cross section average of country-specific output growths alone as we propose to do in our econometric model.

To see the implications for the identification of the global financial shock, consider the expression for quarterly realized volatility associated to r_{it} given by (1). By substituting in (1) the expression for the equity return r_{it} given by (12) when N is large, it is possible to show that RV_{it}, due to the square in (1), depends on the first-moment innovation (ζ_t), second-moment innovations (χ_t and ϖ_t), as well as their many cross-products (see the Appendix). Thus, unlike country-specific returns, which depend linearly on ζ_t, χ_t and ϖ_t, RV_{it} is a complicated non-linear function of these innovations, and their impacts cannot be traced separately. In our econometric model, all common shocks in (1) except for ζ_t will be combined together in a second common shock, which we will label as the global

[11] This expression can be obtained from (14) recalling that $Cov_t(r_{i,t+1},f_{t+1})=c_{1i}/\sigma_t^2$ from (3) and (12), $Cov_t\left(\varepsilon_{i,t+1},\varepsilon_{w,t+1}\right)=w_i\theta_t^2\sigma_{ii}+\psi_t^2\sum_{j\neq i}^N w_j\sigma_{ji}$ from (A21), and using result (A16) for $Var_t(r_{i,t+1})$.

financial shock.[12]

The theoretical rationale for our identification conditions is best seen in terms of the endowment and return equations (2) and (12), respectively. Under the assumption that country-specific shocks, ε_{it}, are cross-sectionally weakly correlated and no single economy is dominant, the global growth factor, f_t, and hence the global growth shock, ζ_t, defined by (3) can be identified up to an affine transformation using simple or weighted averages of output growths when N is sufficiently large. This result holds irrespective of whether ε_{it} is serially correlated or conditionally heteroskedastic. It is the assumption of weak cross sectional correlation of the country-specific shocks that is critical. But it is clear from the return equation, (12), and the associated realized volatility measure given by (1), that knowing f_t does not identify the remaining common volatility factors, such as σ_t^2, θ_t^2 and ψ_t^2 that enter realized volatility. In our empirical analysis we lump all these volatility components into a single common financial factor that we distinguish from f_t, and identify from country-specific realized volatilities, after filtering out the effects of f_t. In effect, our identification assumptions distinguish between a first factor common to both the growth and volatility series, and all other effects common only to the volatility series, lumped together in a second common financial factor.

Comovement between excess return volatility and output growth: The model does not only provide a clear justification for the identification assumptions used in the empirical analysis, but also helps to interpret the empirical results. One of our key empirical findings is that the negative correlation between return volatility and output growth is much smaller in magnitude once we condition on global shocks. The theoretical counterpart to this correlation is captured by the conditional covariance of excess return volatility and output growth, namely $Cov_t(\tilde{r}^2_{i,t+1}, \Delta y_{it})$, where $\tilde{r}_{i,t+1} = r_{i,t+1} - r^f_{t+1}$. In Appendix, using the solution of the theoretical model, we show that (see (A37))

$$Cov_t\left(\tilde{r}^2_{i,t+1}, \Delta y_{i,t+1}\right) = 2 d_{it} Cov_t\left(r_{i,t+1}, \Delta y_{i,t+1}\right), \tag{16}$$

where (see (A35))

$$Cov_t(\tilde{r}_{i,t+1}, \Delta y_{i,t+1}) = \gamma_i c_{1i}\sigma_t^2 + \sigma_{ii}\theta_t^2, \tag{17}$$

and (see (A38))

$$d_{it} = -\frac{1}{2}\kappa_{1i}^2\left(\varphi_\chi^2 A_{2i}^2 + \varphi_\varpi^2 A_{4i}^2\right) - \frac{1}{2}\left(c_{1i}^2 - 2c_{i1}\varrho\gamma_w\right)\sigma_t^2 - \frac{1}{2}\sigma_{ii}\theta_t^2 + O(N^{-1}). \tag{18}$$

Recalling that $c_{1i} = \left(\gamma_i - \varrho\gamma_w\kappa_{1i}\phi_f\right)/\left(1-\kappa_{1i}\phi_f\right)$, it readily follows that $Cov_t\left(r_{i,t+1}, \Delta y_{i,t+1}\right) > 0$, if

$$\sigma_{ii} + \left(\frac{\sigma_t^2}{\theta_t^2}\right)\frac{\gamma_i\left(\gamma_i - \varrho\gamma_w\kappa_{1i}\phi_f\right)}{\left(1-k_{1i}\phi_f\right)} > 0,$$

which is likely to be satisfied for most countries, so long as ϱ is not too large. Suppose now that ϱ is such that the above condition is met. Then, the sign of $Cov_t\left(\tilde{r}^2_{i,t+1}, \Delta y_{i,t+1}\right)$ depends on that of d_{it}, which is likely to be negative for plausible values of the coefficient of relative risk aversion, ϱ.

[12]Note that in a more general theoretical setting this global financial shock can also include additional factors that may influence realized equity market volatilities, such as market imperfections, bubble effects, or time-varying risk preferences.

But in general the sign and the magnitude of $Cov_t\left(\tilde{r}^2_{i,t+1}, \Delta y_{i,t+1}\right)$ will differ across i, and depend on the structural parameters and the ratio θ^2_t/σ^2_t, namely the realized value of country-specific growth volatility relative to that of global growth volatility. $Cov_t\left(\tilde{r}^2_{i,t+1}, \Delta y_{i,t+1}\right)$ is more likely to be negative when θ^2_t is large relative to σ^2_t. It is clear that the theory alone does not allow us to ascertain the relative importance of country-specific and global shocks for the analysis of correlation between realized volatility and growth, which is the subject of our empirical analysis.

4 A Multi-Country Econometric Framework

We now set up a factor-augmented, multi-country model in which country-specific output growth and realized stock market volatility can be driven by 2 common and $2 \times N$ country-specific shocks. We first discuss the identification of the common shocks in the context of a relatively simple static specification, and then consider model identification and estimation in the context of a more general dynamic setting. Identification of the country-specific shocks, which plays only an auxiliary role in our empirical analysis, will follows conventional approaches and is discussed together with the empirical results.

Consider, without loss of generality, the following first-order panel vector autoregressive (PVAR) model in v_{it} and Δy_{it}, for $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$, typically used in empirical work on volatility and the business cycle at the individual country level (assuming $N = 1$):[13]

$$v_{it} = a_{iv} + \phi_{i,11} v_{i,t-1} + \phi_{i,12} \Delta y_{i,t-1} + e_{iv,t}, \tag{19}$$

$$\Delta y_{it} = a_{iy} + \phi_{i,21} v_{i,t-1} + \phi_{i,22} \Delta y_{i,t-1} + e_{iy,t}. \tag{20}$$

where as before output growth (Δy_{it}) is measured as the log-difference of real GDP, $v_{it} = \ln(RV_{it})$ is the log of realized stock market volatility for country i during quarter t, $e_{iv,t}$ and $e_{iy,t}$ are country-specific reduced-form innovations assumed to be serially uncorrelated. A *single-country* approach to the identification of structural volatility and business cycle shocks in (19)-(20) must impose at least one *a priori* restriction on the covariance matrix of $e_{iv,t}$ and $e_{iy,t}$ or their long-run counterpart or the sign of their impact. Consistent with the theoretical model presented in the previous section, in this paper we posit the following unobservable common-factor representation for the reduced-form PVAR innovations:

$$e_{iv,t} = \lambda_i \zeta_t + \theta_i \xi_t + \eta_{it}, \tag{21}$$

$$e_{iy,t} = \gamma_i \zeta_t + \varepsilon_{it}, \tag{22}$$

where ζ_t and ξ_t are two common shocks, while η_{it} and ε_{it} are two country-specific shocks, by assumption, serially uncorrelated. Note however that, for the purpose of identifying the common shocks, they could be correlated both *within* each country and *across* countries.

Equations (19)-(22), taken together, specify a factor-augmented multi-country PVAR model that

[13] The analysis in this section applies to alternative business cycle indicators or other measures of volatility.

captures the main features of our theoretical model and could be formally shown to approximate its solution.[14] As in the theoretical model, in particular, the econometric model posits that the volatility equations include at least one more common shock, ξ_t, than the output equations, capturing all common components not accounted for by ζ_t. The common shock ζ_t in (22) represents the same innovation as in the theoretical model, and therefore will continue to be labelled "global growth shock". The second common shock, ξ_t, instead, can be seen as a linear combination of all other common shocks in theoretical model, reflecting second and higher-order moment innovations such as χ_{t+1}, η_{t+1}, ϖ_{t+1} and their squares and cross products, as well as changes in non-fundamental aspects of financial markets, such as over-reactions to news due to excessive optimism/pessimism or bubble components, ruled out by the theoretical model. For this reason, we refer to ξ_t as the "global financial shock" or "financial shock" for short. Similarly, while ε_{it} has a more direct mapping into the theoretical model, η_{it} is an all-encompassing country-specific financial shock, broadly defined.

The main idea of the paper is to achieve identification of ζ_t and ξ_t and their loadings, λ_i, γ_i, and θ_i (up to orthonormal transformations) by placing restrictions on the *cross-country* correlations of ε_{it} and η_{it}, while leaving their *within-country* correlation unrestricted. This is a problem that cannot be addressed in a single-country framework, or in any model of the world economy viewed as a single entity. In a single-country model, the common shocks in (19)-(22) cannot be identified even if it is assumed that the idiosyncratic shocks η_{it} and ε_{it} are uncorrelated, or by adding more country-specific variables to the model. Only by adopting a multi-country perspective, we can pose this common factor identification problem and solve it. As an example, consider (19)-(22) with $N = 1$, and take this country to be the US. It is readily seen that the triangular factor representation in (21) and (22) does not impose any restriction on the observed covariance matrix of $(\Delta y_{US,t}, v_{US,t})$.

4.1 Identification of the Common Shocks in a Static Setting

To focus on our approach to the identification of ζ_t and ξ_t, let us drop deterministic components and lagged endogenous variables. Denote world GDP growth and world volatility by $\Delta \bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$, respectively, and suppose that they are measured by the weighted cross section averages of country-specific volatility and growth measures, namely:

$$\Delta \bar{y}_{\omega,t} = \sum_{i=1}^{N} w_i \Delta y_{it}, \qquad \text{and} \qquad \bar{v}_{\omega,t} = \sum_{i=1}^{N} \mathring{w}_i v_{it}, \tag{23}$$

where $\mathbf{w} = (w_1, w_2, ..., w_N)'$ and $\mathring{\mathbf{w}} = (\mathring{w}_1, \mathring{w}_2, ..., \mathring{w}_N)'$ are $N \times 1$ vectors of aggregation weights, which can be the same.[15] We make the following assumptions on the common shocks, ζ_t and ξ_t, their loadings, λ_i, γ_i, and θ_i, the weights, $\mathring{w}_i$ and w_i, and the country-specific innovations, ε_{it} and η_{it}:

[14] In addition to our theoretical reasoning, as we shall see, there is strong empirical support for the econometric model set out in equations (19) to (22).

[15] As we noted earlier, in practice, the weights in $\mathbf{w}$ and $\mathring{\mathbf{w}}$ need not be fixed and could be time-varying, but they must be predetermined.

Assumption 1 *(Common shocks and their loadings) The common unobservable shocks ζ_t and ξ_t have zero means and finite variances, and are serially uncorrelated. The loadings, λ_i, γ_i and θ_i, are distributed independently across i and from the common shocks ζ_t and ξ_t for all i and t, with non-zero means λ, γ and θ ($\lambda \neq 0$, $\gamma \neq 0$, and $\theta \neq 0$), and satisfy the following conditions:*

$$\lambda = \sum_{i=1}^{N} \mathring{w}_i \lambda_i \neq 0, \quad \gamma = \sum_{i=1}^{N} w_i \gamma_i \neq 0 \quad \textit{and} \quad \theta = \sum_{i=1}^{N} w_i \theta_i \neq 0, \tag{24}$$

$$\sum_{i=1}^{N} \lambda_i^2 = \mathcal{O}(N), \quad \sum_{i=1}^{N} \gamma_i^2 = \mathcal{O}(N), \quad \textit{and} \quad \sum_{i=1}^{N} \theta_i^2 = \mathcal{O}(N). \tag{25}$$

Assumption 2 *(Aggregation weights) The weights, w_i and $\mathring{w}_i$, for $i = 1, 2, ..., N$ are fixed non-zero constants such that $\sum_{i=1}^{N} w_i = 1$ and $\sum_{i=1}^{N} \mathring{w}_i = 1$, and satisfy the following "granularity" conditions:*

$$||\mathbf{w}|| = O(N^{-1}), \ \frac{w_i}{||\mathbf{w}||} = O(N^{-1/2}), \tag{26}$$

and

$$||\mathring{\mathbf{w}}|| = O(N^{-1}), \ \frac{\mathring{w}_i}{||\mathring{\mathbf{w}}||} = O(N^{-1/2}). \tag{27}$$

Assumption 3 *(Cross-country correlations) (a) The country-specific shocks, η_{it} and ε_{it}, have zero means and finite variances, and are serially uncorrelated, but can be correlated with each other both within and between countries. (b) Denoting the covariance matrices of the $N \times 1$ vectors $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt})'$ and $\boldsymbol{\eta}_t = (\eta_{1t}, \eta_{2t}, ..., \eta_{Nt})'$ by $\boldsymbol{\Sigma}_{\varepsilon\varepsilon} = Var(\boldsymbol{\varepsilon}_t)$ and $\boldsymbol{\Sigma}_{\eta\eta} = Var(\boldsymbol{\eta}_t)$, respectively, we have:*

$$\varrho_{\max}(\boldsymbol{\Sigma}_{\varepsilon\varepsilon}) = O(1) \quad \textit{and} \quad \varrho_{\max}(\boldsymbol{\Sigma}_{\eta\eta}) = O(1). \tag{28}$$

Assumption 1 is standard in the factor literature (see, for instance, Assumption B in Bai and Ng (2002)). It ensures that ζ_t is strong (or pervasive) for both volatility and growth, and ξ_t is strong for volatility only so that they can be estimated consistently either using principal components or by cross section averages of country-specific observations (see Chudik et al. (2011)).[16] Assumption 2 requires that individual countries' contribution to world growth or world volatility is of order $1/N$. This is consistent with the notion that, since the 1990s, when our sample period starts, world growth and world capital markets have become progressively more diversified and integrated as a result of the globalization process. Part (a) of Assumption 3 is also standard and leaves the causal relation between the idiosyncratic components, ε_{it} and η_{it}, unrestricted. In our model, the correlation between ε_{it} and η_{it} captures any contemporaneous causal relation between volatility and growth at the country level, conditional on ζ_t and ξ_t, on which we do not impose any restrictions for the

[16] In the present context, the use of the cross section average (CSA) estimator of f_t has two advantages. First, it can be directly interpreted as a world growth factor. Second, under Assumptions 1 and 3, the CSA estimator of f_t is consistent so long as N is large, whilst the principal component estimator requires both N and T to be large (See section 19.5.1 of Pesaran (2015)). A comparison of CSA and PC estimates of the common factors in the present static setting can be found in Cesa-Bianchi et al. (2018). But it is important to note that such a simple comparison is not possible once we allow for dynamics and heterogeneity in the country-specific VAR models to be discussed in Section 4.2.

purpose of identifying the common shocks. The second part of Assumption 3 instead is the source of identification in our model and has been discussed at length in the theoretical model. It imposes that the country-specific shocks, ε_{it} and η_{it} are weakly cross correlated, which is in line with the approximate factor model of Chamberlain and Rothschild (1982), and ensures that country-specific shocks can be treated as idiosyncratic for asset pricing purposes.

Under Assumptions 1-3, for N sufficiently large, ζ_t can be identified (up to a scalar constant) by $\bar{y}_{\omega,t} = \sum_{i=1}^{N} w_i \Delta y_{it}$ as:

$$\zeta_t = \gamma^{-1} \Delta \bar{y}_{\omega,t} + O_p\left(N^{-1/2}\right), \tag{29}$$

This result follows noting that, using the definitions in (23), and dropping intercepts and dynamics, the following model for world GDP growth and volatility obtains:

$$\Delta \bar{y}_{\omega,t} = \gamma \zeta_t + \bar{\varepsilon}_{\omega,t}, \tag{30}$$

$$\bar{v}_{\omega,t} = \lambda \zeta_t + \theta \xi_t + \bar{\eta}_{\omega,t}, \tag{31}$$

where $\bar{\varepsilon}_{\omega,t} = \mathbf{w}'\boldsymbol{\varepsilon}_t$, and $\bar{\eta}_{\omega,t} = \mathring{\mathbf{w}}'\boldsymbol{\eta}_t$. Furthermore, $Var\left(\bar{\varepsilon}_{\omega,t}\right) = \mathbf{w}'\boldsymbol{\Sigma}_{\varepsilon\varepsilon}\mathbf{w} \leq \left(\mathbf{w}'\mathbf{w}\right) \varrho_{\max}\left(\boldsymbol{\Sigma}_{\varepsilon\varepsilon}\right)$. Thus, under Assumptions 2 and 3, we have $Var\left(\bar{\varepsilon}_{\omega,t}\right) = O\left(\mathbf{w}'\mathbf{w}\right) = O\left(N^{-1}\right)$, and hence $\bar{\varepsilon}_{\omega,t} = O_p\left(N^{-1/2}\right)$, which allows us to recover ζ_t form $\Delta \bar{y}_{\omega,t}$ up to the scalar $1/\gamma$. Note here that, since volatility has at least one more common factor than growth in the theoretical model, the theoretical model can justify the econometric representation adopted. But on its own it does not give us identification of the world growth factor. To get identification, we need weak cross-country correlation and large N. This is because, with small N, we would not be able to disentangle ζ_t from $\bar{\varepsilon}_{\omega,t}$ (the average of ε_{it} in the equation for output growth above).

The first main empirical result of the paper does not require explicit identification of the second common shock (ξ_t) assumed to be exclusive to the volatility series, v_{it}. But doing so permits exploring other properties of the data that underpin the second and the third main empirical results of the paper summarized in the Introduction. Under our assumptions, ξ_t can be identified from the data as a linear combination of $\Delta \bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$, up to an orthonormal transformation (as $N \to \infty$), given by:

$$\xi_t = \theta^{-1}\left(\bar{v}_{\omega,t} - \frac{\lambda}{\gamma}\Delta \bar{y}_{\omega,t}\right) + O_p\left(N^{-1/2}\right). \tag{32}$$

This result follows immediately from substituting (29) into (31) and applying the same reasoning as before.

4.2 Identification of the Common Shocks in a Dynamic Setting

Identifying the common shocks becomes considerably more complex in the heterogeneous dynamic PVAR specification given by (19) and (20). Let us first consider the much simpler case of a homogeneous model in which $\phi_{i,rs} = \phi_{rs}$ for all i and $r, s = 1, 2$. To simplify the exposition we continue to abstract from the intercepts. In this case the common shocks can be identified as before by averaging the country-specific VARs to obtain:

$$\zeta_t = \gamma^{-1}\left(\Delta\bar{y}_{\omega,t} - \phi_{21}\bar{v}_{\omega,t-1} - \phi_{22}\Delta\bar{y}_{\omega,t-1}\right) + O_p\left(N^{-1/2}\right), \tag{33}$$

$$\xi_t = \frac{\left(\bar{v}_{\omega,t} - \frac{\lambda}{\gamma}\Delta\bar{y}_{\omega,t}\right)}{\theta} - \frac{\left(\phi_{11} - \frac{\lambda}{\gamma}\phi_{21}\right)\bar{v}_{\omega,t-1}}{\theta} - \frac{\left(\phi_{12} - \frac{\lambda}{\gamma}\phi_{22}\right)\Delta\bar{y}_{\omega,t-1}}{\theta} + O_p\left(N^{-1/2}\right), \tag{34}$$

which are obvious generalizations of (29) and (32), respectively. Allowing for heterogeneous dynamics, as needed for empirical implementation, presents a much bigger challenge because it could involve long memory processes as shown by Granger (1980).[17] Therefore, to tackle the general heterogeneous dynamic case requires stronger assumptions on the PVAR coefficients, $\phi_{i,rs}$, and involves higher order lags of $(\bar{v}_{\omega,t}, \Delta\bar{y}_{\omega,t})$.

Specifically, consider the matrix version of equation (19) and (22):

$$\mathbf{z}_{it} = \mathbf{a}_i + \mathbf{\Phi}_i\mathbf{z}_{i,t-1} + \mathbf{\Gamma}_i\boldsymbol{\delta}_t + \boldsymbol{\vartheta}_{it}, \quad \text{for } i = 1,2,...,N;\ t = 1,2,...,T, \tag{35}$$

where $\mathbf{z}_{it} = (v_{it}, \Delta y_{it})'$ with

$$\mathbf{a}_i = \begin{pmatrix} a_{iv} \\ a_{iy} \end{pmatrix},\ \mathbf{\Phi}_i = \begin{pmatrix} \phi_{i,11} & \phi_{i,12} \\ \phi_{i,21} & \phi_{i,22} \end{pmatrix},\ \mathbf{\Gamma}_i = \begin{pmatrix} \lambda_i & \theta_i \\ \gamma_i & 0 \end{pmatrix},\ \boldsymbol{\delta}_t = \begin{pmatrix} \zeta_t \\ \xi_t \end{pmatrix},\ \boldsymbol{\vartheta}_{it} = \begin{pmatrix} \eta_{it} \\ \varepsilon_{it} \end{pmatrix},$$

and consider the following assumptions:

Assumption 4 *(Coefficients) The constants $\mathbf{a}_i$ are bounded, $\mathbf{\Phi}_i$ and $\mathbf{\Gamma}_i$ are independently distributed for all i, the support of $\varrho(\mathbf{\Phi}_i)$ lies strictly inside the unit circle, for $i = 1,2,...,N$, and the inverse of the polynomial $\mathbf{\Lambda}(L) = \sum_{\ell=0}^{\infty}\mathbf{\Lambda}_\ell L^\ell$, where $\mathbf{\Lambda}_\ell = \mathbb{E}\left(\mathbf{\Phi}_i^\ell\right)$ exists and has exponentially decaying coefficients, namely $\|\mathbf{\Lambda}_\ell\| \le C_0\rho^\ell$, with $0 < \rho < 1$.*

We continue to maintain the earlier assumptions, 1, 2 and 3, which ensure the country-specific errors are weakly correlated, and

$$\mathbf{\Gamma} = \mathbb{E}(\mathbf{\Gamma}_i) = \begin{pmatrix} \lambda & \theta \\ \gamma & 0 \end{pmatrix} \tag{36}$$

with $\gamma\theta \neq 0$, which also ensures that $\mathbf{\Gamma}$ is invertible. The additional conditions in Assumption 4 control the effects of aggregation of dynamics across heterogenous units by requiring that $\Lambda_\ell = E\left(\mathbf{\Phi}_i^\ell\right)$ exists and has exponentially decaying coefficients. But it is easily seen that this latter condition holds if it is further assumed that $\sup_i E\|\mathbf{\Phi}_i\| < \rho < 1$. The latter ensures that the time series processes for the aggregates $\Delta\bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$ can be suitably truncated for empirical analyses and are devoid of long memory components. The following proposition provides the identification of the common or global shocks in the general dynamic heterogeneous setting.

Proposition 1 *(Identification of common shocks in the heterogeneous dynamic factor-augmented PVAR model) Consider the models given by (35) for country $i = 1,2,...,N$, and suppose that As-*

[17] For a review and more recent developments see, for example, Chapter 32 in Pesaran (2015).

sumptions 1, 2, 3 and 4 hold. Then:

$$\zeta_t = b_\zeta + \gamma^{-1}\Delta\bar{y}_{\omega,t} + \sum_{\ell=1}^{\infty}\mathbf{c}'_{1,\ell}\bar{\mathbf{z}}_{\omega,t-\ell} + O_p\left(N^{-1/2}\right), \tag{37}$$

$$\xi_t = b_\xi + \theta^{-1}\left(\bar{v}_{\omega,t} - \frac{\lambda}{\gamma}\Delta\bar{y}_{\omega,t}\right) + \sum_{\ell=1}^{\infty}\mathbf{c}'_{2,\ell}\bar{\mathbf{z}}_{\omega,t-\ell} + O_p\left(N^{-1/2}\right), \tag{38}$$

where b_ζ and b_ξ are fixed constants, $\bar{\mathbf{z}}_{\omega,t} = (\bar{v}_{\omega,t}, \Delta\bar{y}_{\omega,t})$, $\{w_i, \text{ for } i = 1, 2, ..., N\}$ are fixed weights that satisfy the granularity Assumption 2, $\mathbf{c}'_{1,\ell}$ and $\mathbf{c}'_{2,\ell}$ are the first and the second rows of $\mathbf{C}_\ell = \mathbf{\Gamma}^{-1}\mathbf{B}_\ell$, where $\mathbf{\Gamma} = \mathbb{E}(\mathbf{\Gamma}_i)$, $\mathbf{B}_\ell$ is defined by $\mathbf{\Lambda}^{-1}(L) = \mathbf{B}_0 + \mathbf{B}_1 L + \mathbf{B}_2 L^2 + ...$, $\mathbf{\Lambda}(L) = \sum_{\ell=0}^{\infty}\mathbf{\Lambda}_\ell L^\ell$, and $\mathbf{\Lambda}_\ell = \mathbb{E}\left(\mathbf{\Phi}_i^\ell\right)$, for all i.

Proof. See Appendix B. ■

Expressions (37) and (38) augment the corresponding results (33) and (34) obtained for the homogeneous case with higher order lags of $(\bar{v}_{\omega,t-\ell}, \Delta\bar{y}_{\omega,t-\ell})$, for $\ell > 1$, to take account of dynamic heterogeneity on the identification of common shocks. The higher order lags are needed to solve the unobserved factors ζ_t and ξ_t in terms of the aggregates $\bar{v}_{\omega,t}$ and $\Delta\bar{y}_{\omega,t}$, when the VAR coefficient matrices, $\mathbf{\Phi}_i$, vary across i. Note that aggregating (35) over i yields

$$\bar{\mathbf{z}}_{\omega,t} = \bar{\mathbf{a}}_w + \sum_{i=1}^{N} w_i\mathbf{\Phi}_i\mathbf{z}_{i,t-1} + \mathbf{\Gamma}\boldsymbol{\delta}_t + O_p\left(N^{-1/2}\right),$$

where $\bar{\mathbf{a}}_w = \Sigma_{i=1}^N w_i\mathbf{a}_i$. It is clear that the second term in the above expression does not reduce to a function of $\bar{\mathbf{z}}_{\omega,t-1}$ unless $\mathbf{\Phi}_i = \mathbf{\Phi}$ for all i. A solution to this aggregation problem is provided in Pesaran and Chudik (2014) by first solving $\mathbf{z}_{it}$ in terms of the common and idiosyncratic shocks and then aggregating the outcomes across i. The procedure introduces the lagged values, $\bar{\mathbf{z}}_{\omega,t-s}$ for $s = 1, 2, ...$, in the determination of $\boldsymbol{\delta}_t$.

4.3 Consistent Estimation of the Factor-augmented, Heterogeneous PVAR Model

As they stand, the expressions given in Proposition 1 for ζ_t and ξ_t are formulated in terms of the observables $\{\bar{\mathbf{z}}_{\omega,t-\ell}, \text{ for } \ell \geq 0\}$, but cannot be used in empirical analysis as they depend on infinite order lags. But, as shown in Pesaran and Chudik (2014) and Chudik and Pesaran (2015), if slope heterogeneity is not extreme (i.e. if the coefficient matrices $\mathbf{\Phi}_i$ do not differ too much across i) and $\mathbf{C}_\ell$ decays exponentially in ℓ, the infinite order distributed lag functions in $\bar{\mathbf{z}}_{\omega,t}$ can be truncated. In practice, Pesaran and Chudik (2014) and Chudik and Pesaran (2015) recommend a lag length ℓ equal to $T^{1/3}$, where T is the time dimension of the panel. So considering a truncated approximation of the unobservable factors in equation (37) and (38) we can derive observable proxies for ζ_t and ξ_t, making also sure that the resultant estimators are orthogonal to each other and with unit variance.[18] As the following proposition illustrates, the latter is achieved simply by choosing coefficients in the linear

[18]Note that ζ_t and ξ_t can be identified only up to a non-singular transformation which we take to be orthonormal, as it simplifies the computation and interpretation of impulse responses and error variance decompositions that we conduct later on in the paper.

regression of $\bar{v}_{\omega,t}$ on $\Delta\bar{y}_{\omega,t}$ such that the observable proxy for the common shocks have zero-means and unit variances.

Proposition 2 *(Consistent estimation of observable orthonormalized common shocks in the heterogeneous dynamic factor-augmented PVAR model) Consider the p^{th} order truncated approximation of the unobservable factors in equation (37) and (38) above, and note that in matrix notations we have:*

$$\boldsymbol{\zeta} = \Delta\bar{\mathbf{y}}_{\omega} + \bar{\mathbf{Z}}_{\omega}\mathbf{C}_1 + O_p\left(N^{-1/2}\right), \tag{39}$$

$$\boldsymbol{\xi} = \bar{\mathbf{v}}_{\omega} - \lambda\Delta\bar{\mathbf{y}}_{\omega} + \bar{\mathbf{Z}}_{\omega}\mathbf{C}_2 + O_p\left(N^{-1/2}\right), \tag{40}$$

where $\boldsymbol{\zeta} = (\zeta_1, \zeta_2, ..., \zeta_T)'$, $\boldsymbol{\xi} = (\xi_1, \xi_2, ..., \xi_T)'$, $\bar{\mathbf{Z}}_{\omega} = (\boldsymbol{\tau}_T, \bar{\mathbf{z}}_{\omega,-1}, \bar{\mathbf{z}}_{\omega,-2}, ..., \bar{\mathbf{z}}_{\omega,-p})$, $\bar{\mathbf{z}}_{\omega,-l} = (\Delta\bar{\mathbf{y}}_{\omega,-l}\bar{v}_{\omega,-l})$, $\Delta\bar{\mathbf{y}}_{\omega,-l} = (\Delta\bar{y}_{\omega,1-l}, \Delta\bar{y}_{\omega,2-l}, ..., \Delta\bar{y}_{\omega,T-l})'$, $\Delta\bar{\mathbf{y}}_{\omega} = \Delta\bar{\mathbf{y}}_{\omega,0}$, $\bar{\mathbf{v}}_{\omega,-l} = (\bar{v}_{\omega,1-l}, \bar{v}_{\omega,2-l}, ..., \bar{v}_{\omega,T-l})'$, $\bar{\mathbf{v}}_{\omega} = \bar{\mathbf{v}}_{\omega,0}$, and p denotes a suitable number of lags (or truncation order). Moreover, consistent estimators of the common shocks, denoted by $\hat{\boldsymbol{\zeta}}$ and $\hat{\boldsymbol{\xi}}$, can be obtained as residuals from the following OLS regressions:

$$\hat{\boldsymbol{\zeta}} = \Delta\bar{\mathbf{y}}_{\omega} - \bar{\mathbf{Z}}_{\omega}\hat{\mathbf{C}}_1, \tag{41}$$

$$\hat{\boldsymbol{\xi}} = \bar{\mathbf{v}}_{\omega} - \hat{\lambda}\hat{\boldsymbol{\zeta}} - \bar{\mathbf{Z}}_{\omega}\hat{\mathbf{C}}_2, \tag{42}$$

where $\hat{\mathbf{C}}_1$ is the OLS estimator of the coefficients in the regression of $\Delta\bar{\mathbf{y}}_{\omega}$ on $\bar{\mathbf{Z}}_{\omega}$, and $\hat{\lambda}$ and $\hat{\mathbf{C}}_2$ are OLS estimators of the coefficients in the regression of $\bar{\mathbf{v}}_{\omega}$ on $\hat{\boldsymbol{\zeta}}$ and $\bar{\mathbf{Z}}_{\omega}$. The estimated shocks have zero means and are uncorrelated, namely $\boldsymbol{\tau}_T'\hat{\boldsymbol{\zeta}} = \boldsymbol{\tau}_T'\hat{\boldsymbol{\xi}} = \hat{\boldsymbol{\zeta}}'\hat{\boldsymbol{\xi}} = 0$.

Proof. See Appendix B. ■

Since $\hat{\zeta}_t$ and $\hat{\xi}_t$ are the residuals from regressions of $\Delta\bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$ on an intercept and the lagged values $\bar{z}_{\omega,t-1}, ..., \bar{z}_{\omega,t-p}$, it follows that $\hat{\zeta}_t$ and $\hat{\xi}_t$ will have zero (in-sample) means and, for a sufficiently large value of p, will be serially uncorrelated. Therefore, $\hat{\zeta}_t$ and $\hat{\xi}_t$ can be viewed as estimators of the global shocks to the underlying factors, ζ_t and ξ_t. Note also that, in a dynamic setting, the orthogonalized components of $\Delta\bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$, obtained by simply projecting $\bar{v}_{\omega,t}$ on $\Delta\bar{y}_{\omega,t}$, are not the same as our global shocks $\hat{\zeta}_t$ and $\hat{\xi}_t$, because this would ignore the contributions of $\bar{\mathbf{z}}_{\omega,t-\ell}$ for $\ell = 1, 2, ..., p$ to the estimation of ζ_t and ξ_t. As the common shocks depend on lagged variables, it is important to make sure that the past values of $\bar{\mathbf{z}}_{\omega,t}$ are filtered out.

The contemporaneous effects of the common shocks can now be estimated by substituting in (35) the orthogonal factor innovations, $\hat{\zeta}_t$ and $\hat{\xi}_t$, obtained from equations (41) and (42). We can then investigate their dynamic impact and relative importance for country-specific volatility and growth based on the following regressions:

$$v_{it} = a_{iv} + \phi_{i,11}v_{i,t-1} + \phi_{i,12}\Delta y_{i,t-1} + \sum_{\ell=1}^{p}\mathbf{d}_{v,i\ell}'\bar{\mathbf{z}}_{\omega,t-\ell} + \beta_{i,11}\hat{\zeta}_t + \beta_{i,12}\hat{\xi}_t + \eta_{it}, \tag{43}$$

$$\Delta y_{it} = a_{iy} + \phi_{i,21}v_{i,t-1} + \phi_{i,22}\Delta y_{i,t-1} + \sum_{\ell=1}^{p}\mathbf{d}_{\Delta y,i\ell}'\bar{\mathbf{z}}_{\omega,t-\ell} + \beta_{i,21}\hat{\zeta}_t + \varepsilon_{it}. \tag{44}$$

These country-specific equations can be estimated consistently by least squares so long as N and T are sufficiently large. Large N is required so that the probability order $O_p(N^{-1/2})$ in equations (39) and (40) becomes negligible. Large T is required to ensure that the dynamics are estimated accurately.

5 Empirical Results

We are now ready to present our empirical results. We first focus on the contemporaneous within-country correlations of country-specific volatility and growth. We then provide evidence on the question of whether country-specific volatility shocks entail significant endogenous components. Finally, we document the quantitative importance of volatility shocks for output growth.

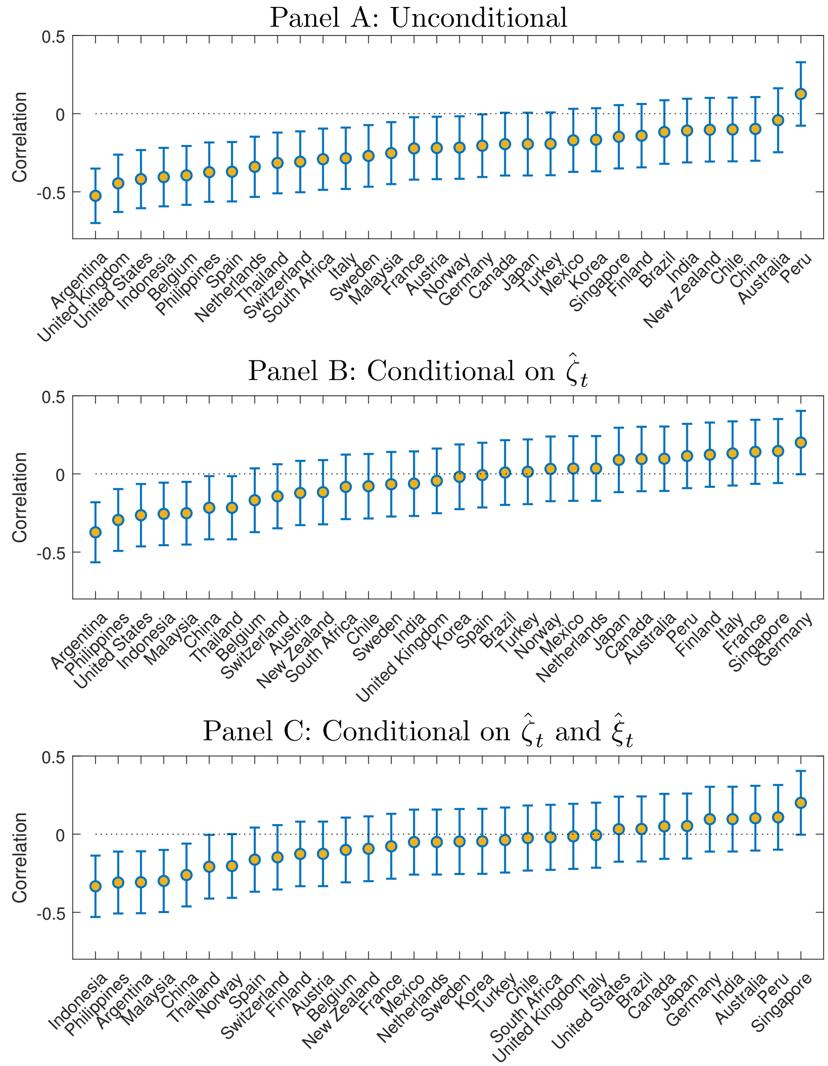
5.1 Country-specific Correlations Between Volatility and Growth Innovations

In this section, we compare the within-country correlations of country-specific volatilities and growth growths with the corresponding correlations of country-specific innovations, η_{it} and ε_{it}, obtained using (43) and (44). Equipped with the estimates of global growth and financial shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, obtained using (41) and (42), we estimate the country-specific VAR models and their growth and volatility innovations by running the OLS regressions in (43) and (44). We consider two alternative specifications of these country-specific VARs. In one specification we condition *only* on the global growth shock, $\hat{\zeta}_t$ in (43)-(44). In another specification, we condition on *both* global growth and financial shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, to obtain the reduced form country-specific shocks $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$ as in (43) and (44). As we will see, conditioning also on the global financial factor, $\hat{\xi}_t$, does not add much to the model's ability to explain within-country correlations.

Figure 2 reports the results. Panel A displays the unconditional correlation coefficients between country-specific realized volatilities and output growths for all the 32 countries in our panel, together with their 95 percent error bands, computed over the period 1993:Q1-2016:Q4. Consistent with the large literature referred to in the Introduction, these correlations are negative, sizable, and statistically significant for most countries. On average across countries, the correlation is about -0.25, ranging from around -0.5 for Argentina to just above zero for Peru. Panel B reports the same correlations when we condition *only* on $\hat{\zeta}_t$ in (43)-(44). In this case, the correlations drop substantially for all countries, to an average of -0.05, and are no longer statistically significant, except in a few cases whose statistical significance is borderline and tend to disappear once we adjust the critical value of the tests to allow for the multiple testing nature of the inference being carried out.[19] In the case of the United States, for instance, the correlation does not vanish, but falls to about half its unconditional value and becomes borderline statistically insignificant when considered in isolation from the other correlations. But it is not statistically different from zero if we allow for multiple testing. Panel C reports the same correlation when we condition on both $\hat{\zeta}_t$ and $\hat{\xi}_t$ and show that the results are virtually identical to those in Panel B, with an average correlation of -0.06.

[19]See Section S.2.4 and Table S.6 of the online Supplement on this.

Figure 2 Country-specific Correlations Between Volatility and Growth Innovations



Note. Panel A displays the unconditional correlations between (log) realized stock market volatility and real GDP growth. Panel B plots the correlation between volatility and growth innovations when we condition *only* on $\hat{\zeta}_t$ in model (43)-(44). Panel C reports the same correlation when we condition on *both* $\hat{\zeta}_t$ and $\hat{\xi}_t$. The dots represent the contemporaneous correlations. The lines represent 95-percent confidence intervals. Sample period: 1993:Q1-2016:Q4.

Overall, the results in Figure 2 suggest that the specification in (43)-(44) can account for a sizable portion of the unconditional association between country-specific volatility and growth. The results in Figure 2 are consistent with the findings of Berger et al. (2017), who show that, after conditioning on a realized equity market volatility shock, the shock to expected future volatility has no effect on output growth in the United States. As we will see below, these results are robust to excluding from the analysis the sample period covering the global financial crisis (i.e. ending the sample period in 2006:Q4 or 2008:Q2), as well as dropping the United States, China, or both countries from the sample.

Omitted variable bias in single-countries VARs: The results in Figure 2 also imply that some of the explanatory power that is typically attributed to uncertainty shocks in empirical studies of individual countries' business cycles considered in isolation from the rest of the world is due to omitted international common factors. It is therefore natural to ask whether impulse responses to country-specific volatility shocks computed from a single-country VAR, as typically estimated in the extant literature, overestimate the impact on output growth if compared to the outcomes from the same shock computed using our multi-country model.

To illustrate the potential for omitted variable bias arising in single-country VAR analyses, we compare impulse responses to country specific-volatility shocks with and without conditioning on our global shocks. Thus, we first estimate single-country VARs in output growth and realized volatility without conditioning on the global shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, and compute impulse responses for a country-specific volatility shock. We then compare them with those obtained from our multi-country model, obtained from the same VARs, but conditioning on $\hat{\zeta}_t$ and $\hat{\xi}_t$.

While the global growth and financial shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, are orthogonal to the country-specific shocks and to each other by construction, the country-specific shocks, $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$, have so far been left unrestricted, and could be correlated, both within and between countries. Thus, in order to compute and interpret an impulse response to a country-specific volatility shock—or the forecast error variance decompositions that we report and discuss below—we need to deal with this second identification problem. As we explain in more details in Section S.2.4 of the online Supplement to the paper, to identify country-specific volatility and growth shocks we exploit the empirical properties of their estimated reduced form correlation matrix, combined with alternative assumptions regarding the causal relation between volatility and growth at the country-specific level. We then show that the inference we draw is reasonably robust to the alternative identification schemes adopted.

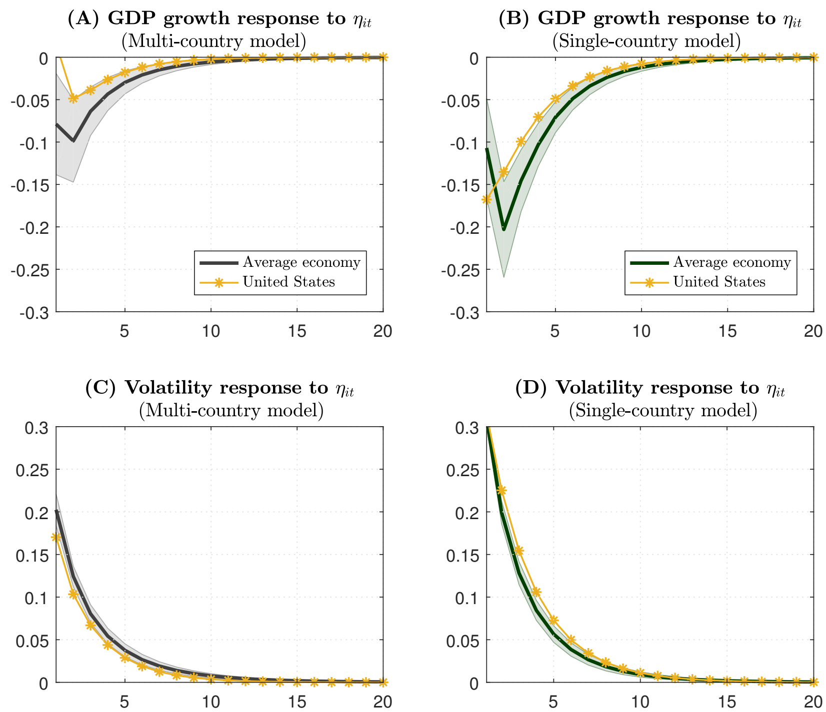
In the baseline specification reported here, we identify country-specific volatility shocks by imposing a block-diagonal covariance matrix, in which the only non-zero off-diagonal elements are the estimated reduced form covariances between volatility and growth innovations of each country-specific block. These within-country blocks are then factorized with a Cholesky decomposition ordering the realized volatility variable first, as often done in the literature, thus assuming that country-specific volatility shocks can have a contemporaneous causal impact on growth variables but not *vice versa*, stacking the odds against our empirical findings.

The results are reported in Figure 3.[20] The panels on the left hand side report the responses to a country-specific volatility shock in our multi-country model. The panels on the right hand side report the responses to a volatility shock in single-country VAR models. All panels report weighted average estimates of impulse responses across country using PPP-GDP weights, as well as impulse responses for the United States, for comparison.

The peak output responses to a country-specific volatility shock obtained from single-country VARs, averaged across countries and for the United States in particular, are much larger than those estimated from our multi-country model. They are about twice as large in the case of the average output responses and approximately a third larger in the case of the volatility responses, providing

[20]The responses to country-specific output growth shocks are not reported to save space, but are available from the authors on request. Robustness to alternative identification assumptions is discussed in the next subsection. The derivation of the impulse responses is given in the online Supplement to the paper.

Figure 3 Multi- and Single-country Responses to Country-specific Volatility Shocks



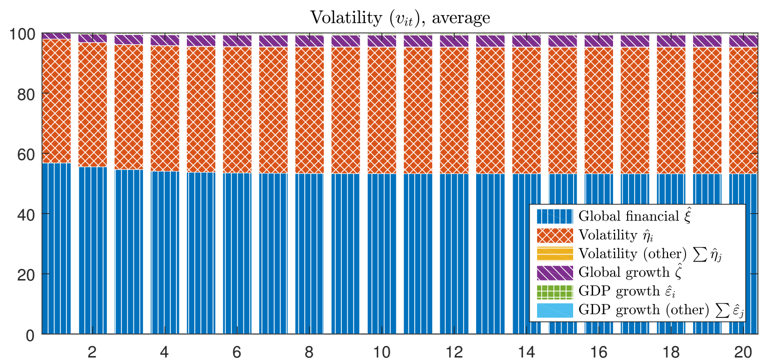
Note. Average (black solid line) and US (yellow line) impulse responses to one-standard deviation to the volatility innovations η_{it}. The average is a PPP-GDP weighted averages of the country-specific responses. The shaded areas are two standard deviations confidence intervals. The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

strong evidence of an omitted variable bias. The reason is intuitive. If there is a common factor in the data, as the pair-wise correlations reported in Figure 1 suggest, it cannot be omitted from the VAR specification. By considering a single VAR country model at a time, the common factor cannot be identified and separately estimated from the country-specific shocks; therefore introducing a bias in the estimated impulse responses. In contrast, in the multi-country framework that we propose we can identify shocks to the common factors, consistent with our multi-country consumption-based asset pricing model and the stylized facts of the data we have reported. The empirical results clearly illustrate the perils of omitting to control for the global components of the international business cycle and the importance of augmenting the single-country VAR models with global variables to account for cross country dependence in the relationship between volatility and the business cycle.

5.2 Is Output Growth Important for Volatility?

Our factor augmented multi-country PVAR model can also be readily used to decompose the forecast error variance of country-specific volatility and growth in terms of the common shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, as well as the 64×1 vector of country-specific shocks, $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$, for $i = 1, 2, ..., 32$. Figure 4 plots the average forecast error variance decomposition (FEVD) of volatility across all countries in our sample. As discussed above in the context of impulse responses, the FEVD for each country volatility is obtained assuming that $\mathbf{\Sigma}_{(\varepsilon,\eta)}$ is block diagonal, with realized volatility ordered first within each country block. The figure reports the 'average' variance decomposition across countries, weighting the country-specific FEVDs with PPP-GDP weights. All results are based on (43)-(44), which include both $\hat{\zeta}_t$ and $\hat{\xi}_t$.[21]

Figure 4 COUNTRY-SPECIFIC VOLATILITY: FORECAST ERROR VARIANCE DECOMPOSITION



NOTE. Block-diagonal covariance matrix, with Cholesky decomposition of the within-country covariance. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common growth shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas with no pattern). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

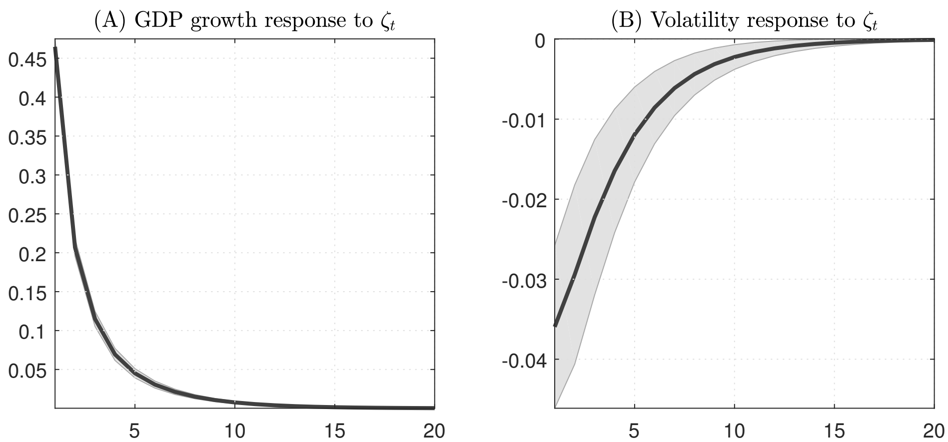
Figure 4 shows that country-specific volatility is largely driven by common financial shocks (blue area with vertical lines) and country-specific volatility shocks (red area with crosses). Together, these two shocks explain about 95 percent of the total variance of realized volatility over time. World growth shocks (purple area with diagonal lines) explain less than 5 percent of the total volatility forecast error variance. Country-specific own growth shocks, as well as all other 31 country-specific foreign growth shocks in the full model, play essentially no role. Results for specific countries, including the United States, are reported in the online Supplement. As can be seen from Figures S.13 to S.16 in the online Supplement, countries behave pretty similarly, with some, though limited, heterogeneity. The US results, in particular, are similar to those for the average economy reported here.

The results also imply that the endogenous component of country-specific volatility, namely the component driven by common and country-specific growth shocks, is quantitatively small. It is worth noting that our average estimated FEVD of country-specific realized volatility is similar to the central estimates of Ludvigson et al. (2015) for their US financial uncertainty measure. In that study, the central estimate of the share of the macroeconomic shock in the forecast error variance of financial uncertainty is estimated at just above 5 percent. However, while Ludvigson et al. (2015) attribute this to the US business cycle (as proxied by a shock to US industrial production), we attribute the outcome largely to the global growth shock, which can be interpreted as an international business cycle factor, as we find that country-specific growth shocks have little or no explanatory power for

[21] The derivation of the FEVDs is provided in the online Supplement to the paper.

country-specific volatility. The results in Figure 4 are also consistent with the Global Financial Cycle hypothesis (e.g. Rey (2013)), which states that a single common factor drives a large portion of the international comovements in asset prices.

Figure 5 Country-specific Volatility and Growth Responses to Global Growth Shocks



Note. Average impulse responses to a one-standard deviation global growth shock, $\hat{\zeta}_t$. The solid lines are the PPP-GDP weighted averages of the country-specific responses. The shaded areas are two standard deviations confidence intervals. The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

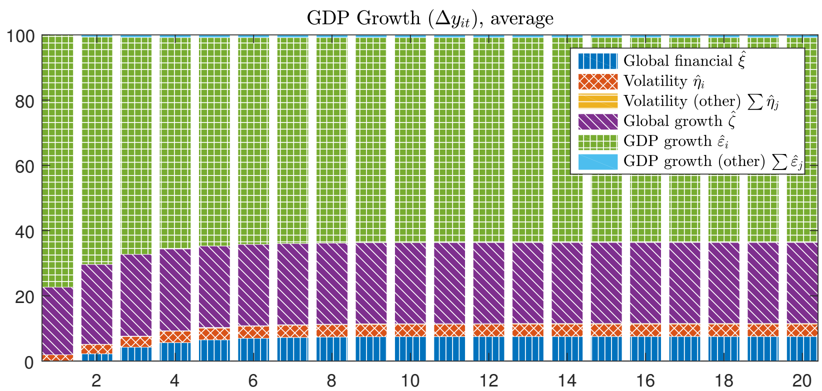
Figure 5 displays the impulse response of country-specific volatility and GDP growth to a global growth shock ($\hat{\zeta}_t$, solid line), together with two-standard-deviation error bands (shaded areas). The figure plots a weighted average (using PPP-GDP weights) of the country responses. We focus on the effects of positive one-standard deviation shocks. The error bands are based on the dispersion of the impulse responses across countries. The Figure shows that a positive global growth shock increases country-specific output growth and lowers volatility.

Panel (A) of Figure 5 shows that, on average, country-specific growth loads positively on $\hat{\zeta}_t$, with persistent effects up to 8-10 quarters. Country-specific output growth increases by about half a percentage point following a one-standard error change in $\hat{\zeta}_t$. Unlike what is reported by Colacito et al. (2018), who finds significant heterogeneity, note here that the error bands are very tight, reflecting relatively homogeneous country responses. In fact, as can be seen from Figure S.11 in the online Supplement, the impulse responses have a similar shape for most countries. Panel (B) of the figure reports the response of realized volatility, which declines following a positive world growth shock, reflecting a highly persistent response to improvements in the world economy.

5.3 Is Volatility Harmful for Growth?

The extent to which volatility shocks could harm output growth is investigated in Figures 6 and 7. Figure 6 reports the average forecast error variance decomposition (FEVD) of output growth across all countries in our sample. Figure 7 plots the impulse responses of volatility and growth to a positive one-standard-deviation global financial shock, $\hat{\xi}_t$. On average across countries, the forecast error variance of country specific GDP growth is driven mostly by global and country-specific growth shocks, with a combined share approaching 90 percent of the total in the long run (green areas with squares and purple area with diagonal lines, respectively). The global growth shock, explains about 25 percent of the total growth forecast error variance, in line with existing results in the international business cycle literature (see, for instance, Kose et al. (2003)). While the country-specific growth shock explains more than 60 percent of the total forecast error variance in the long-run.[22] Thus, output growth shocks seem far more important than global financial shocks and country-specific volatility shocks for output growth.

Figure 6 COUNTRY-SPECIFIC GDP GROWTH: FORECAST ERROR VARIANCE DECOMPOSITION



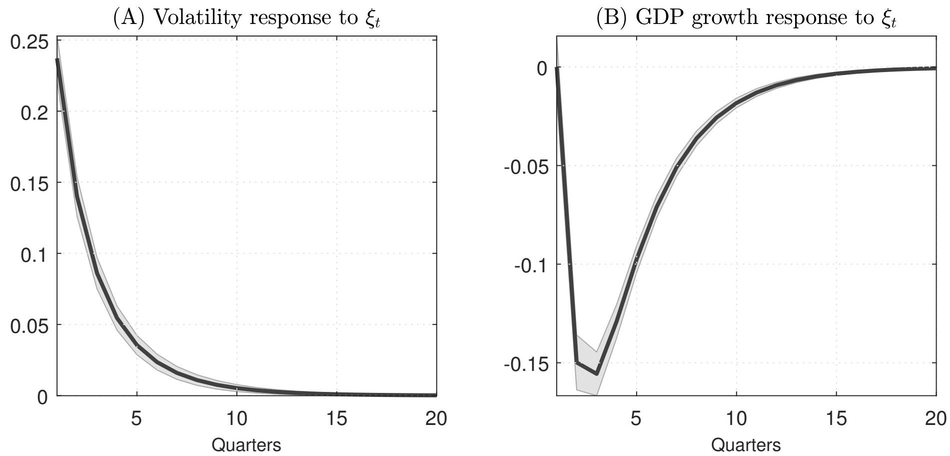
NOTE. Block-diagonal covariance matrix, with Cholesky decomposition of within-country covariance. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common growth shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas with no pattern). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Indeed, the own country-specific volatility shock explains less than 4 percent of the total forecast error variance of GDP growth, while the combination of all other 31 country-specific volatility shocks has essentially no explanatory power for the forecast error variance of country-specific growth. The global financial shock, in contrast, explains a more sizable share of output growth forecast error variance, 7.5 percent, on average across countries using PPP GDP weights. The importance of this shock picks up gradually over the forecast horizon and stabilizes within two years. This is about the same as the share of forecast variance of industrial production explained by the realized volatility shock in the single-country VAR of Berger et al. (2017) at the two-year horizon, but is significantly smaller than the variance share of the financial uncertainty shock in industrial production in the single-country monthly VAR of Ludvigson et al. (2015).

Figure 7 reports the country-specific responses of volatility and growth to a positive global financial shock, $\hat{\xi}_t$. These average responses suggest that a positive shock to $\hat{\xi}_t$ is 'bad news' for the world economy, as volatility increases and growth declines. For a typical one-standard deviation shock to the global financial factor, volatility increases by 25 basis points, while growth declines by about

[22]The share of forecast error variance of country-specific own growth shocks could be decomposed further, but we do not do that here as it requires a more articulated framework.

Figure 7 Country-specific Volatility and Growth Responses to Global Financial Shock



Note. Average impulse responses to a one-standard deviation global financial shock, $\hat{\xi}_t$. The solid lines are the PPP-GDP weighted averages of the country-specific responses. The shaded areas are two standard deviations confidence intervals. The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

15 basis points within two quarters after the shock.[23] Although the peak effect is smaller than the output response to the global growth shock in Panel (A) of Figure 5, the average output growth response to the global financial shock in Panel (B) of Figure 7 is of the same order of magnitude, and sizable. The average responses to the global financial shock are also very persistent, even though there is much more heterogeneity in the country-specific growth responses, as can be seen from Figure S.11 in the online Supplement. These impulse responses thus suggest that global financial shocks can cause large and persistent global recessions.

The variance decomposition results confirm the importance of controlling for global factors in single-country VAR studies of uncertainty and the business cycle. The impulse response results suggest that, even though global financial shocks might not be frequent enough to explain a very large share of output variance, nevertheless when they occur they can have large and persistent negative effects on output growth. For instance, the pattern of shock transmissions in Figures 5 and 7 is consistent with country-specific volatility increasing in response to the large decline in world output in the second part of 2008, with the world recession being amplified by the exceptionally large global financial shocks in the fourth quarter of 2008 and the first quarter of 2009.[24] More specifically, $\hat{\zeta}_t$ dropped by 4 standard deviations during the third quarter of 2008, implying a growth decline in the average economy of 2 percentage points within a quarter. The 2.5 standard deviation shock to $\hat{\xi}_t$ implies an additional 0.4 percentage point output drop during the following quarter, resulting in a total output decline of about 3 percent during the 2008:Q4 and 2009:Q1. By comparison, US real

[23]Note that the delayed growth response to the global financial shock follows from our identification assumptions, but it is not imposed directly on country-specific models.

[24]Our two factor model can also help to explain the seemingly puzzling coexistence of high policy volatility (as in Baker et al. (2016)) and low equity market volatility after the beginning of the Trump administration with a combination of real and financial shocks partially offsetting each other.

GDP dropped by about 3.5 percent over these two quarters.

6 Estimated Global Growth and Financial Shocks

While the theoretical model in Section 3 provides one way to interpret of $\hat{\zeta}_t$ and $\hat{\xi}_t$, as global growth and financial shocks, it is important to check the consistency of such interpretation as well as the plausibility of the identification assumptions made to estimate them. To this end in sub-section (6.1) we report on the correlation of our estimated shocks with a number of indicator variables suggested in the literature, and in sub-section (6.2) we investigate whether the degree of cross-country correlations of country-specific innovations match our identification conditions set out in the second part of Assumption 3.

6.1 Correlations with Observables

Table 1 reports the details of the comparison between our estimated global shocks and several proxies available in the extant literature. Left panel of this table gives the correlations with the global growth shock, and the right column reports the correlations with the global financial shock.

Global growth shock: We first compare our estimates of global growth shock with the TFP measures from Huo et al. (2018). As the TFP data is annual, we temporally aggregate our quarterly global growth shocks to the annual level before computing the correlation. We compare our measure with the changes in global TFP, defined as the simple average of the observations over Canada, the United States, Germany, Japan, and the United Kingdom. The correlation between the global growth shock and global TFP growth is 0.80 for the sample period over which they overlap, and is statistically highly significant. When we use TFP estimates for the United States alone (as opposed to the average of the five countries), this correlation drops to 0.37 and is no longer statistical significant. Huo et al. (2018) also provide a measure of utilization-adjusted TFP for these countries, arguing that it is a better proxy of a genuine technology factor. We therefore repeat the exercise using their adjusted indicator and find qualitatively similar results (i.e. much stronger and much more precisely estimated correlation with the global measure than with the US measure), even though the strength of the correlation is weaker, dropping to 0.46 for the global measure (significant only at the 10% level) and not distinguishable from zero for the United States. This evidence supports an interpretation of $\hat{\zeta}_t$ as a technology factor, even though, as we had anticipated in theoretical section, this global shock can also capture other world 'demand' or supply factors.

Consistent with this interpretation, we then compare our global growth shock with a quarterly measure of the world natural interest rate from Holston et al. (2017)—defined as the real short-term interest rate consistent with the economy operating at its full potential. Similarly to what is done above with TFP, we compute the global measure as the simple average of the natural rate for the United States, Canada, the euro area, and the United Kingdom. The correlation between the global growth shock and changes in the global natural rate is positive and highly statistically significant, amounting to 0.41 for the sample period over which the two measures overlap. Again, when we

Table 1 CORRELATIONS BETWEEN GLOBAL SHOCKS AND OBSERVABLE PROXIES

	Panel A: Global growth shock ($\hat{\zeta}_t$)	
	Global	US
(1) TFP	$0.80^{(a)}$	0.37
(2) TFP Util. Adj.	$0.46^{(c)}$	-0.08
(3) Natural rate of interest (r*)	$0.41^{(a)}$	0.12
(4) Long-run Risk	0.31	0.27
	Panel B: Global financial shock ($\hat{\xi}_t$)	
	Global	US
(5) MM Uncertainty	$0.41^{(a)}$	0.06
(6) RS Uncertainty	$0.20^{(c)}$	0.08
(7) EPU Index	$0.46^{(a)}$	$0.36^{(a)}$
(8) LMN Fin. Uncertainty	n.a.	$0.32^{(a)}$

NOTE. Statistical significance at 1-percent, 5-percent, and 10-percent level is denoted by (a), (b), and (c), respectively. (1) is total factor productivity (TFP) from Huo et al. (2018); (2) is utilization adjusted TFP from Huo et al. (2018); (3) is the natural interest rate from (r*) Holston et al. (2017); (4) is a long-run risk measure from Colacito et al. (2018); (5) is the uncertainty proxy from Mumtaz and Musso (2018); (6) is the uncertainty proxy from Rossi and Sekhposyan (2017); (7) is the economic policy uncertainty (EPU) index for the United States (from Baker et al. (2016)) and global (from Davis (2016)); (8) is the financial uncertainty proxy from Ludvigson et al. (2015). Global measures are computed as averages across countries, with the exception of the MM global uncertainty measure which is derived from a factor model. Correlations computed over longest overlapping sample period. Longest possible sample period is 1993:Q1-2016:Q4.

use only the natural rate for the United States this correlation drops to 0.12 and is not statistically different from zero.

Finally, since the stochastic process (2) for the country-specific endowment in our theoretical model has a similar structure to the long-run risk specification of Bansal and Yaron (2004), we also compare our estimated global growth shock to the proxy for global and US long-run endowment risk of Colacito et al. (2018). We find that our estimated world growth shocks have a correlation of only about 0.3 with both their global and the US measures, and this correlation is not statistically significant.

Global financial shock: We now turn to our estimates of global financial shocks, and begin by comparing them with changes in the global and US uncertainty proxy of Mumtaz and Musso (2018). The lower panel of Table 1 shows that there is a positive and statistically highly significant correlation between our global financial shocks and the global version of Mumtaz and Musso uncertainty proxy, at 0.41, but the association disappears when we use the uncertainty measure specific to the US. A similar picture emerges when comparing our global financial shock with the uncertainty forecast dispersion based proxy constructed by Rossi and Sekhposyan (2017).

The results are different with the news-based world economic policy uncertainty (EPU) index of Davis (2016) and the US counterpart of Baker et al. (2016). In fact, with the EPU index, the association between our global financial shock and the EPU index does not weaken substantially when the comparison is limited to the United States. The strength of the association is similar when we compare our series of global financial shocks with the US financial uncertainty measure of

Ludvigson et al. (2015), with a statistically significant correlation of 0.32. One explanation is that, as Ludvigson et al.'s US financial uncertainty measure is based on information from many US financial variables, picking up a combination of global and country-specific shocks. But as we noted earlier, adding variables to the empirical framework from one country only cannot help identify the global financial factors that seem to be important drivers of volatility and the business cycle around the world.

In sum, the above comparisons provide evidence supportive of an interpretation of the global factors consistent with the theoretical model we set up and the notion that these factors are not primarily driven by the United States as a dominant economy. Our global growth and financial shocks are much more closely correlated with global rather than US proxy variables. Regardless of the ultimate source of the global shocks, whether it is a global demand or supply innovation that triggers a global growth shock, a change in global risk or in risk preferences, the evidence provided does not align well with the possibility that the identified global shocks are primarily driven by US factors.

6.2 Cross-Country Correlations

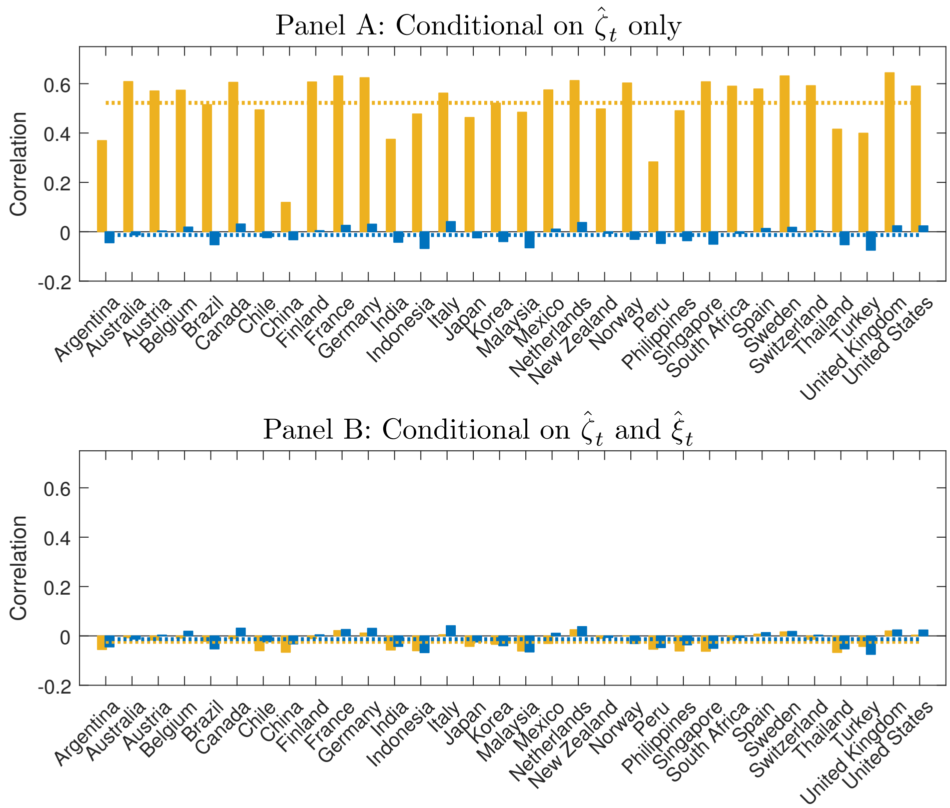
Although the restrictions behind our identification assumptions for the global shocks cannot be formally tested as the model is exactly identified, our multi-country approach permits us to investigate the extent to which the moments of the data restricted by such assumptions are in line with the identification assumptions made.[25] To this end, we explore the degree of cross-country dependence of the estimated residuals from the dynamic regressions (43) and (44), with and without conditioning on the global financial shock, $\hat{\xi}_t$, as it was done to derive the main empirical results of the paper.

Panel A of Figure 8 shows that, if we condition only on $\hat{\zeta}_t$ in (43)-(44), the volatility innovations display average pair-wise correlations pretty much like those of the data reported for all countries in Figure 1. In contrast, the pair-wise correlations of the growth innovations become negligible, with an average across all countries of -0.01. In contrast, Panel B of Figure 8 shows that, if we condition on both $\hat{\zeta}_t$ and $\hat{\xi}_t$, the cross-country correlations of the volatility innovations are now also negligible, as in the case of the growth innovations, with an average pair-wise correlation across all countries equal to -0.02. For instance, in the specific case of the United States, the average pair-wise correlation of the volatility innovations is equal to 0.6 conditioning on $\hat{\zeta}_t$ alone. But it drops to 0.00 if we condition on both factor innovations. By comparison, the US average pair-wise correlation of the growth innovations is 0.02.

Figure 8, therefore, illustrates that, after conditioning on $\hat{\zeta}_t$—which is common to both growth and volatility series—not much commonality is left in the case of growth innovations, but the volatility innovations continue to share strong commonality. Moreover, after conditioning on both $\hat{\zeta}_t$ and $\hat{\xi}_t$, the volatility innovations also appear weakly correlated because of the near-zero average pair-wise correlation across all countries, thus suggesting that only two common shocks are necessary

[25] Note that we can estimate ζ_t and ξ_t consistently by means of the OLS regressions (41) and (42) only under the identification assumptions made. As a result, whilst we can directly estimate pair-wise correlations of volatility and growth series, we cannot examine cross-country pair-wise correlations of their innovations without imposing these identification conditions.

Figure 8 Cross-country Correlation of Country-specific Volatility and Growth Innovations



Note. Country-specific average pair-wise correlation of volatility (yellow, lighter bars) and GDP growth (blue, darker bars) innovations conditional on $\hat{\zeta}_t$ only (Panel A) and on $\hat{\zeta}_t$ and $\hat{\xi}_t$ (Panel B). The volatility measures are based on (1). The dotted lines are the averages across all countries, equal to 0.52 and -0.01 for volatility and growth in Panel A; and equal to -0.02 and -0.01 for volatility and GDP growth in Panel B, respectively. Sample period: 1993:Q1-2016:Q4.

to span their correlations across-countries as we assumed in our theoretical model. It is, therefore, interesting to test whether the two sets of innovations also satisfy a formal definition of *weak* and *strong* dependence, consistent with the stylized facts and as we assumed deriving them.

To formally test for weak and strong cross section dependence, we compute the cross-sectional dependence (CD) test statistic of Pesaran (2015) and the exponent of cross sectional dependence (α) proposed in Bailey et al. (2016). The CD statistic is normally distributed with zero-mean and unit-variance under the null of zero average pair-wise correlations. The critical value is around 2. When the null is rejected, Bailey et al. (2016) suggest estimating the strength of the cross section dependence with an exponent, denoted α in the range $(1/2, 1]$, with unity giving the maximum degree of cross dependence. Any value above $1/2$ and below 1, but significantly different from 1, suggests weak dependence.[26] So, in what follows, we present estimates of α for the volatility and the growth innovations, together with their confidence intervals. For comparison, we also report the same estimates for the (raw) growth and volatility series.

[26]When estimating α one also needs to take into account the sampling uncertainty, which depends on the relative magnitude of N and T, and the null of weak cross dependence, which depends on the relative rates of increase of N

Table 2 TESTING FOR THE STRENGTH OF CROSS-SECTIONAL DEPENDENCE

	CD	Lower 5%	$\hat{\alpha}$	Upper 95%
Data				
v_{it}	104.57	0.94	0.99	1.05
Δy_{it}	55.73	0.87	1.00	1.14
Innovations (conditional on $\hat{\zeta}_t$)				
u_{it}	110.89	0.96	1.00	1.04
ε_{it}	-2.90	0.56	0.62	0.67
Innovations (conditional on $\hat{\zeta}_t$ and $\hat{\xi}_t$)				
η_{it}	-5.12	0.58	0.64	0.70

NOTE. CD is the cross-sectional dependence test statistic of Pesaran (2015). $\hat{\alpha}$ is the estimate of the exponent of cross-sectional dependence as in Bailey et al. (2016), together with its 90-percent confidence interval ('Lower 5%' and 'Upper95%').

The results are summarized in Table 2 and are in strong accordance with the identification assumptions made. The CD test statistic for the growth series is 55.73, with the associated α exponent estimated at 1.00. The CD statistic for the volatility series is even higher at 104.57 with an estimated α of 0.99. The CD statistics and the estimates of α confirm with a high degree of confidence that both series are cross-sectionally strongly correlated, containing *at least* one strong common factor. Conditional only on $\hat{\zeta}_t$, the CD statistic for the country-specific growth innovations ($\hat{\varepsilon}_{it}$) drops to -2.90, close to its critical value under the null of zero average pair-wise correlations, with its exponent of cross-sectional dependence estimated to be 0.62, significantly below 1. In sharp contrast, the CD statistic for the country-specific volatility innovations when we condition only on $\hat{\zeta}_t$ in model (43)-(44) (denoted by $\hat{u}_{it}$) remains close to that of the raw volatility series at 110.89 with an estimated α also not statistically different from unity. However, when we condition on both $\hat{\zeta}_t$ and $\hat{\xi}_t$, the CD statistic for the volatility innovations ($\hat{\eta}_{it}$) also falls to -5.12, with an estimated α of 0.64 and a 95 percent confidence interval of $[0.58, 0.70]$, while the CD statistic and α are the same as before for the growth innovations ($\hat{\varepsilon}_{it}$). The test statistics in Table 2, therefore, accord very well with the assumptions made that the volatility innovations share at least one more, and only one more, strong common factor than the growth innovations.[27]

7 Robustness Analyses

In this section we report on a number of exercises we have conducted to check the robustness of our empirical findings. The results (reported in Section S.2 of the online Supplement to the paper) show that our main findings are not affected by the granularity assumption, the exclusion of the global financial crisis from the sample period, the volatility proxy used, and the assumptions on the error covariance matrix of the multi-country model.

and T.

[27]The estimates of α obtained with the residual-based approach of Bailey et al. (2019a) give very similar results.

Granularity assumption. One important question regarding our results is whether the estimated common shocks might in fact be idiosyncratic shocks to large countries such as the United States and China. To address this issue we re-estimate different versions of the model where we drop (i) the United States, (ii) China, and (iii) both the United States and China from the sample. We obtain essentially the same results. In contrast, when we replaced the estimated global growth and financial factors with US GDP growth and US realized volatility, we were able to control for only a fraction of the cross-country correlation in our data—thus violating our assumptions on weak cross-country correlation of the residuals conditional on the global factors. In this latter case, in particular, the estimated exponents of cross-sectional dependence (α) are not significantly different from one. Taken together, these two sets of results provide clear cut evidence against the United States (or China) driving our empirical results. See Section S.2.1 of the online Supplement.

Weighting scheme. To obtain our baseline results we use equal weights and set $w_i = \mathring{w}_i = 1/N$ in equation (23). Asymptotically, the choice of the weights does not matter (Pesaran, 2006), as long as there is no dominant unit in the set of economies under consideration; an assumption that is in line with the results of the robustness exercise presented above on the granularity assumption. Accordingly, we would expect our results to be reasonably robust to other choices of w_i and $\mathring{w}_i$. In Section S.2.5 of the online Supplement, we show that this is indeed the case when we use PPP-GDP weights instead of equal weights in estimation of global shocks.

Sample period. The results are robust to dropping the period of the global financial crisis from our sample. Specifically, we continue to obtain virtually the same results when we re-estimate our model on a sample that ends in 2006. Importantly, this is true for (i) the cross-sectional dependence of the raw data and the residuals conditional on the global shocks; (ii) the within-country correlation between volatility and GDP growth; and (iii) the role of the global growth and financial shocks in the FEVDs. Only the IRFs to the global growth and financial shocks display some small differences relative to our baseline, in that the size of the impact is slightly smaller, which is not surprising given the dramatic rise in volatility during 2008-2009. See section S.2.2 in the online Supplement.

Realized vs. Implied volatility. In the finance literature, the focus of the volatility measurement has recently shifted to implied volatility measures obtained from option prices, like the US VIX Index. At quarterly frequency, however, the realized volatility of US daily equity returns behaves very similarly to the VIX Index. For example, during the period over which they overlap, our US realized volatility measure and the VIX Index co-move very closely, with a correlation that exceeds 0.9. In addition, to more formally check the robustness of our results to the choice of the volatility measure, we re-estimated our model using the VIX Index as a measure of volatility for the United States (instead of our realized volatility measure). We obtained even stronger results, in that the response of US GDP to a US volatility shock is less negative than in our baseline. This implies that the omitted variable bias from ignoring the global factors is even stronger when using the VIX Index as a proxy for US volatility. See section S.2.3 in the online Supplement.

Alternative Identification Assumptions for Country-Specific Shocks. While in our baseline estimates of the IRFs and FEVDs we assume a block-diagonal covariance matrix for the residuals of the multi-country model (43)-(44), to check our results for robustness we re-estimate the FEVDs under other assumptions on the covariance matrix of country-specific shocks. We consider two alternatives. First, we consider the possibility that the estimated 64 by 64 variance covariance matrix conditional on the estimated global factors shocks, is truly diagonal, as suggested by the empirical evidence discussed in the supplement (see S.2.4). This means assuming that our global growth shocks explains 100 percent of the conditional correlation between country-specific volatility and growth. Second, we refrain altogether from interpreting country-specific volatility and growth shocks as structural, and make use of a general unrestricted error covariance matrix (both within and across countries) and compute the generalized forecast error variance decompositions (GFEVD) of Pesaran and Shin (1998). However, before computing GFEVDs, we use the regularized multiple testing threshold estimator of the error covariance matrix proposed by Bailey et al. (2019b) to obtain a consistent estimator of the 64×64 error covariance matrix of the residuals of the multi-country model.[28] The FEVDs obtained for these alternative specifications of the error covariance matrix of country-specific shocks are very close to the ones reported in the paper. See section S.2.4 in the online Supplement.[29]

8 Conclusions

Volatility behaves counter-cyclically in most countries of the world, but economic theory suggests that causation can run both ways. In this paper, we take a common factor approach in a multi-country setting to study the interrelation between realized equity price volatility and GDP growth without imposing *a priori* restrictions on the direction of economic causation on country-specific volatility and growth shocks.

Based on new stylized facts of the data that we document in the paper, and consistent with a multi-country version of the Lucas tree model with time-varying volatility, a persistent common growth factor, heterogeneous exposure to this common factor, and cross country spillovers, we estimate a multi-country econometric model in output growth and realized volatilities for 32 countries over the period 1993:Q1-2016:Q4. Common growth and financial shocks are identified by assuming different patterns of correlation of volatility and output growth innovations across countries and that no country is large enough to affect all other economies. Evidence based on the estimated innovations of this model accords well with the identification assumptions made.

Empirically, we report three main results. First, shocks to the world growth factor, which are closely associated with changes in proxy for the world TFP growth and the natural rate of interest, account for a sizable portion of the unconditional correlation between volatility and GDP growth. Second, the share of forecast error variance of country-specific volatility explained by this global growth factor and country-specific growth shocks is less than 5 percent. Third, shocks to the financial common factor explain about 8 percent of the country-specific growth forecast error variance, while

[28]This regularized estimator exploits the sparsity of the underlying error covariance matrix. More details are reported in the the online Supplement.

[29]Impulse responses under alternative identification assumptions for country specific shocks are not reported to conserve space, but are also robust.

country-specific volatility shocks explain only half this share of the total forecast error variance of GDP growth; but when a shock to the financial common factor is realized, its negative impact on country-specific GDP growth is large and persistent—as typically estimated in the extant literature.

The theoretical model that we set up and the econometric methodology that we propose can be used to explain cross-country variations in risk premia, to ascertain the relative importance of country-specific and global factors for asset return predictability, and to examine the role of heterogeneity in asset pricing. We regard these as promising areas of future research.

Appendix A: Theoretical Model Details

Output Growth Equations Despite its simplicity, the production side of the economy summarized in equation (2) is consistent with multi-country versions of the international real business cycle models of Backus et al. (1992) and Baxter and Crucini (1995). To see this, assume a standard Cobb-Douglas production function in terms of output per worker denoted by $(Y_{it}/L_{it}) = exp(\tilde{y}_{it})$, $A_{it} = exp(a_{it})$ the country-specific level of technology, L_{it} the labor force, and K_{it} the capital stock in country i, so that we have:

$$\tilde{y}_{it} = \ln(Y_{it}/L_{it}) = a_{it} + \tilde{\alpha}_i \ln(K_{it}/L_{it}) = a_{it} + \tilde{\alpha}_i \log(k_{it}),$$

for $i = 1, 2, \ldots, N$. Further assume that the processes for L_{it} and a_{it} are exogenously given by

$$\ln(L_{it}) - \ln(L_{i,t-1}) = n_i, \text{ and } a_{it} = a_{0i} + \tilde{g}_i t + \gamma_i a_t + e_{it},$$

where the growth rate of the labour force, n_i, is assumed to be fixed, a_{0i} is an initial condition, $\tilde{g}_i$ is a deterministic growth component of a_{it}, a_t is the log-level of a stochastic common technology factor, and e_{it} is the country-specific technology shock, with γ_i measuring the extent to which country i is exposed to the global technology factor a_t. A key result from the stochastic growth literature is that, for all i, $\log(k_{it})$ is ergodic and stationary, in the sense that as t tends to infinity, $\log(k_{it})$ tends to a time-invariant random variable, $\log(k_{it}) = \log(k_i) + \tau_{it}$, where τ_{it} is a stationary process representing all country-specific forces driving the country's business cycles, possibly reflecting the effects of aggregate demand shocks, other supply shocks, as well as country-specific uncertainty shocks (see, for instance, Lee et al. (1997)).[30] So we have $\tilde{y}_{it} = a_{0i} + \tilde{\alpha}_i log(k_i) + \tilde{g}_i t + \gamma_i a_t + e_{it} + \tau_{it}$, and taking first differences we obtain $\Delta\tilde{y}_{it} = \tilde{g}_i + \gamma_i f_t + \varepsilon_{it}$, where $f_t = \Delta a_t = a_t - a_{t-1}$, and $\varepsilon_{it} = \Delta e_{it} + \Delta\tau_{it}$. In terms of log output, $y_{it} = \ln(Y_{it})$, therefore, we have $\Delta y_{it} = y_{it} - y_{i,t-1}$ and $a_i = \tilde{g}_i + n_i$, which is equation (2) in the paper.

Equity Returns To derive the equity return given by (12), where $r_{i,t+1} = \ln(R_{i,t+1})$, $R_{i,t+1} = (P_{i,t+1} + Y_{i,t+1})/P_{it}$, and P_{it} is the price of risky asset in country i at time t, we adopt the Campbell and Shiller (1988) approximation to the (log) one-period gross ex-post return given by:

$$r_{i,t+1} = \kappa_{0i} + \kappa_{1i} z_{i,t+1} - z_{it} + \Delta y_{i,t+1}, \tag{A1}$$

where κ_{0i} and κ_{1i} are fixed constants such that $\left|\phi_f \kappa_{1i}\right| < 1$, and $z_{it} = \ln(P_{it}/Y_{i,t})$. It is possible to find an approximate closed-form solution for z_{it} and hence $r_{i,t+1}$. To accomplish this, similarly to Bansal and Yaron (2004) (BY hereafter) , we guess that the solution for z_{it} has the following linear form in our model's risk factors:

$$z_{i,t} = A_{0i,N} + A_{1i} f_t + A_{2i}\sigma_t^2 + A_{3i,N}\psi_t^2 + A_{4i,N}\theta_t^2, \tag{A2}$$

where $A_{0i,N}$, A_{1i}, A_{2i}, $A_{3i,N}$, and $A_{4i,N}$ are functions of all structural parameters of the model, some of which, under the assumptions made, will be shown to depend on the number of countries in the global economy, N. It is also worth noting that as compared to the single economy country model analyzed by BY, we have three additional risk factors due to the country-specific and cross-country time-varying volatilities.

The risky return $R_{i,t+1}$ satisfies the first order condition $E_t(R_{i,t+1}M_{t+1}) = 1$, where $E_t(\cdot) = E(\cdot|\Im_t)$, $\Im_t$ is the information set, and M_{t+1} is defined by (7) and (9). Using the expression for M_t,

[30] For a discussion of volatility shocks interpreted as demand shocks see Leduc and Liu (2016) and Basu and Bundick (2017).

this first-order condition can be rewritten equivalently as:

$$1 = E_t \left[\exp \left[\ln \left(M_{t+1} R_{i,t+1} \right) \right] \right] = E_t \left[\exp \left(\ln \beta - \varrho a_w - \varrho \gamma_w f_{t+1} - \varrho \varepsilon_{w,t+1} + r_{i,t+1} \right) \right]. \tag{A3}$$

We can now derive the undetermined coefficients, $A_{0i,N}, A_{1i}, A_{2i}, A_{3i,N}, A_{4i,N}$, such that (A3) is satisfied. Substituting (A1) in (A3), using (2) and (A2), and using (3), (4), (5) and (6), we obtain:

$$E_t \left[\exp \left(q_{i,t+1} \right) \right] = \exp \left(-\ln \beta + \varrho a_w - \kappa_{0i} - a_i \right),$$

where:

$$\begin{aligned} q_{i,t+1} &= h_{it} + \left(\varepsilon_{i,t+1} - \varrho \varepsilon_{w,t+1} \right) + \left(\gamma_i - \varrho \gamma_w + \kappa_{1i} A_{1i} \right) \sigma_t \zeta_{t+1} \\ &\quad + \kappa_{1i} \varphi_\chi A_{2i} \chi_{t+1} + \kappa_{1i} \varphi_\eta A_{3i,N} \eta_{t+1} + \kappa_{1i} \varphi_\varpi A_{4i,N} \varpi_{t+1}, \end{aligned}$$

and

$$\begin{aligned} h_{it} &= -\left(1 - \kappa_{1i}\right) A_{0i,N} + \kappa_{1i}(1-\phi_\sigma)\sigma^2 A_{2i} + \kappa_{1i}(1-\phi_\psi)\psi^2 A_{3i,N} + \kappa_{1i}(1-\phi_\theta)\theta^2 A_{4i,N} \\ &\quad + \left[\gamma_i - \varrho \gamma_w - \left(1 - \phi_f \kappa_{1i}\right) A_{1i} \right] f_t \\ &\quad - A_{2i} \left(1 - \kappa_{1i} \phi_\sigma\right) \sigma_t^2 - A_{3i,N} \left(1 - \kappa_{1i} \phi_\psi\right) \psi_t^2 - A_{4i,N} \left(1 - \kappa_{1i} \phi_\theta\right) \theta_t^2. \end{aligned}$$

Under the assumption that the shocks are conditionally Gaussian (see the last paragraph of section 3.1), $q_{i,t+1}$ being a linear function of these shocks would also be Gaussian conditional on $\Im_t$, and we have:

$$E_t \left[\exp \left(q_{i,t+1} \right) \right] = \exp \left(h_{it} \right) \cdot \exp \left[\begin{array}{c} \frac{1}{2} Var_t \left(\varepsilon_{i,t+1} - \varrho \varepsilon_{w,t+1} \right) + \frac{1}{2} \left(\gamma_i - \varrho \gamma_w + \kappa_{1i} A_{1i} \right)^2 \sigma_t^2 \\ + \frac{1}{2} \kappa_{1i}^2 \varphi_\chi^2 A_{2i}^2 + \frac{1}{2} \kappa_{1i}^2 \varphi_\eta^2 A_{3i,N}^2 + \frac{1}{2} \kappa_{1i}^2 \varphi_\varpi^2 A_{4i,N}^2 \end{array} \right],$$

where

$$Var_t \left(\varepsilon_{i,t+1} - \varrho \varepsilon_{w,t+1} \right) = \theta_t^2 B_{iN} + \psi_t^2 C_{iN}, \tag{A4}$$

and

$$B_{iN} = \left(1 - 2 \varrho w_i\right) \sigma_{ii} + \varrho^2 \left(\sum_{j=1}^{N} w_j^2 \sigma_{jj} \right), \tag{A5}$$

$$C_{iN} = \varrho^2 \sum_{j \neq i}^{N} w_j w_i \sigma_{ij} - 2 \varrho \sum_{j \neq i}^{N} w_j \sigma_{ji}. \tag{A6}$$

Thus:

$$\begin{aligned} -\ln \beta + \varrho a_w - \kappa_{0i} - a_i &= \ln \left[E_t \left[\exp \left(q_{i,t+1} \right) \right] \right], \\ &= h_{it} + \frac{1}{2} \left(\theta_t^2 B_{iN} + \psi_t^2 C_{iN} \right) + \frac{1}{2} \left(\gamma_i - \varrho \gamma_w + \kappa_{1i} A_{1i} \right)^2 \sigma_t^2 \\ &\quad + \frac{1}{2} \kappa_{1i}^2 \varphi_\chi^2 A_{2i}^2 + \frac{1}{2} \kappa_{1i}^2 \varphi_\eta^2 A_{3i,N}^2 + \frac{1}{2} \kappa_{1i}^2 \varphi_\varpi^2 A_{4i,N}^2. \end{aligned}$$

Using the expression for h_{it} and matching the terms on both sides of the above equation we have:

$$
\begin{aligned}
-\left(1-\kappa_{1i}\right)A_{0i,N}+\kappa_{1i}(1-\phi_{\sigma})\sigma^2A_{2i}+\kappa_{1i}(1-\phi_{\psi})\psi^2A_{3i,N}+\kappa_{1i}(1-\phi_{\theta})\theta^2A_{4i,N}+...& \\
\frac{1}{2}\kappa_{1i}^2\varphi_{\chi}^2A_{2i}^2+\frac{1}{2}\kappa_{1i}^2\varphi_{\eta}^2A_{3i,N}^2+\frac{1}{2}\kappa_{1i}^2\varphi_{\varpi}^2A_{4i,N}^2+\ln\beta-\varrho a_w+\kappa_{0i}+a_i &= 0, \\
-\left(1-\phi_f\kappa_{1i}\right)A_{1i}+\gamma_i-\varrho\gamma_w &= 0, \\
-\left(1-\kappa_{1i}\phi_{\sigma}\right)A_{2i}+\frac{1}{2}\left(\gamma_i-\varrho\gamma_w+\kappa_{1i}A_{1i}\right)^2 &= 0, \\
-\left(1-\kappa_{1i}\phi_{\psi}\right)A_{3i}+\frac{1}{2}C_{iN} &= 0, \\
-\left(1-\kappa_{1i}\phi_{\theta}\right)A_{4i}+\frac{1}{2}B_{iN} &= 0.
\end{aligned}
$$

Using (A5) and (A6), the above expressions can now be used to solve for A_{1i}, $A_{2i}, A_{3i,N}$ and $A_{4i,N}$:

$$A_{1i} = \frac{\left(\gamma_i-\varrho\gamma_w\right)\phi_f}{1-\kappa_{1i}\phi_f}, \tag{A7}$$

$$A_{2i} = \frac{1}{2}\frac{\left(\gamma_i-\varrho\gamma_w+\kappa_{1i}A_{1i}\right)^2}{1-\kappa_{1i}\phi_{\sigma}}, \tag{A8}$$

$$A_{3i,N} = \frac{1}{2}\frac{\varrho^2\sum_{j\neq i}^N w_jw_i\sigma_{ij}-2\varrho\sum_{j\neq i}^N w_j\sigma_{ji}}{1-\kappa_{1i}\phi_{\psi}}, \tag{A9}$$

$$A_{4i,N} = \frac{1}{2}\frac{\left(1-2\varrho w_i\right)\sigma_{ii}+\varrho^2\sum_{j=1}^N w_j^2\sigma_{jj}}{1-\kappa_{1i}\phi_{\theta}}, \tag{A10}$$

which in turn can be used in (A11) to solve for $A_{0i,N}$:

$$A_{0i,N}=\frac{1}{\left(1-\kappa_{1i}\right)}\left(\begin{array}{c}\ln\beta+a_i-\varrho a_w+\kappa_{0i}+\kappa_{1i}\sigma^2(1-\phi_{\sigma})A_{2i}+\kappa_{1i}(1-\phi_{\psi})\psi^2A_{3iN} \\ +\kappa_{1i}(1-\phi_{\theta})A_{4iN}+\frac{1}{2}\kappa_{1i}^2\varphi_{\chi}^2A_{2i}^2+\frac{1}{2}\kappa_{1i}^2\varphi_{\eta}^2A_{3i,N}^2+\frac{1}{2}\kappa_{1i}^2\varphi_{\varpi}^2A_{4i}^2\end{array}\right). \tag{A11}$$

Equipped with a solution for z_{it}, we can then easily derive $r_{i,t+1}$, its innovation and conditional variance, as well as its realized volatility. Using (A1), (A2) and the assumed processes (2)-(6) we have:

$$
\begin{aligned}
r_{i,t+1} &= \kappa_{0i}+a_i+\gamma_if_{t+1}+\kappa_{1i}\left(A_{0i,N}+A_{1i}f_{t+1}+A_{2i}\sigma_{t+1}^2+A_{3i,N}\psi_{t+1}^2+A_{4i,N}\theta_{t+1}^2\right) \\
&\quad -\left(A_{0i,N}+A_{1i}f_t+A_{2i}\sigma_t^2+A_{3i,N}\psi_t^2+A_{4i,N}\theta_t^2\right)+\varepsilon_{i,t+1},
\end{aligned}
$$

which can be written as equation (12) in the paper (which we reproduce here for convenience):

$$
\begin{aligned}
r_{it} &= b_{0i,N}+b_{1i}f_{t-1}+b_{2i}\sigma_{t-1}^2+b_{3i,N}\psi_{t-1}^2+b_{4i,N}\theta_{t-1}^2 \\
&\quad +c_{1i}\sigma_{t-1}\zeta_t+c_{2i}\chi_t+c_{3i,N}\eta_t+c_{4i,N}\varpi_t+\varepsilon_{it},
\end{aligned} \tag{A12}
$$

where

$$
\left\{\begin{array}{c}
b_{0i,N}=\left[\kappa_{0i}-\left(1-\kappa_{1i}\right)A_{0i,N}+a_i\right]+\kappa_{1i}A_{2i}\sigma^2(1-\phi_{\sigma})+\kappa_{1i}A_{3i,N}(1-\phi_{\psi})+\kappa_{1i}A_{4i,N}(1-\phi_{\theta}) \\
b_{1i}=\gamma_i\phi_f-A_{1i}(1-\kappa_{1i}\phi_f),\ b_{2i}=-A_{2i}(1-\kappa_{1i}\phi_{\sigma}), \\
b_{3i,N}=-A_{3i,N}(1-\kappa_{1i}\phi_{\psi}),\ b_{4i,N}=-A_{4i,N}(1-\kappa_{1i}\phi_{\theta}), \\
c_{1i}=\gamma_i+\kappa_{1i}A_{1i},\quad c_{2i}=\kappa_{1i}A_{2i}\varphi_{\chi},\ c_{3i,N}=\kappa_{1i}A_{3i,N}\varphi_{\eta},\ c_{4i,N}=\kappa_{1i}A_{4i}\varphi_{\varpi}.
\end{array}\right. \tag{A13}
$$

Some of the above coefficients simplifies further:

$$
\begin{aligned}
b_{1i} &= \gamma_w \phi_f \varrho, \quad b_{2i} = -\frac{1}{2}\left(\gamma_i - \varrho\gamma_w + \kappa_{1i}A_{1i}\right)^2, && \text{(A14)} \\
c_{1i} &= \frac{\gamma_i - \varrho\gamma_w \kappa_{1i}\phi_f}{1-\kappa_{1i}\phi_f} = \varrho\gamma_w + \left(\frac{\gamma_i - \varrho\gamma_w}{1-\kappa_{1i}\phi_f}\right). && \text{(A15)}
\end{aligned}
$$

Hence, the innovation to the risky return is given by:

$$
r_{i,t+1} - E_t\left(r_{i,t+1}\right) = c_{1i}\sigma_t\zeta_{t+1} + \kappa_{1i}A_{2i}\varphi_\chi \chi_{t+1} + \kappa_{1i}A_{3i,N}\varphi_\eta \eta_{t+1} + \kappa_{1i}A_{4i,N}\varphi_\varpi \varpi_{t+1} + \varepsilon_{i,t+1},
$$

while the conditional variance of the return is:

$$
Var_t\left(r_{i,t+1}\right) = E_t\Big[r_{i,t+1} - E_t\left(r_{i,t+1}\right)\Big]^2 = \kappa_{1i}^2 G_{iN}^2 + \sigma_{ii}\theta_t^2 + c_{1i}^2\sigma_t^2. \tag{A16}
$$

where

$$
G_{iN}^2 = \varphi_\chi^2 A_{2i}^2 + \varphi_\eta^2 A_{3i,N}^2 + \varphi_\varpi^2 A_{4i,N}^2. \tag{A17}
$$

Risk Premium The risk premium can be derived by noting that the conditional mean of the equity return is given by:

$$
E_t\left(r_{i,t+1}\right) = \pi_{iN} + \left[\gamma_i\phi_f - A_{1i}(1-\kappa_{1i}\phi_f)\right] f_t - A_{2i}(1-\kappa_{1i}\phi_\sigma)\sigma_t^2 - A_{3i,N}(1-\kappa_{1i}\phi_\psi)\psi_t^2 - A_{4i,N}(1-\kappa_{1i}\phi_\theta)\theta_t^2,
$$

where

$$
\pi_{iN} = a_i + \kappa_{0i} - (1-\kappa_{1i})A_{i0,N} + \kappa_{1i}H_{iN}, \tag{A18}
$$

and

$$
H_{iN} = \sigma^2(1-\phi_\sigma)A_{2i} + (1-\phi_\psi)A_{3i,N} + (1-\phi_\theta)A_{4i,N}. \tag{A19}
$$

Subtracting the risk-free rate, Equation (11), we now have:

$$
\begin{aligned}
E_t\left(r_{i,t+1} - r_{t+1}^f\right) &= \pi_{iN} + \ln\beta - \varrho a_w + \left[\gamma_i\phi_f - A_{1i}(1-\kappa_{1i}\phi_f) - \varrho\gamma_w\phi_f\right] f_t && \text{(A20)} \\
&\quad - \left[A_{2i}(1-\kappa_{1i}\phi_\sigma) - \frac{1}{2}\varrho^2\gamma_w^2\right]\sigma_t^2 - A_{3i,N}(1-\kappa_{1i}\phi_\psi)\psi_t^2 \\
&\quad - A_{4i,N}(1-\kappa_{1i}\phi_\theta)\theta_t^2 + \frac{1}{2}\varrho^2 \mathbf{w}'\boldsymbol{\Sigma}_{t\varepsilon}\mathbf{w}.
\end{aligned}
$$

Also since $\varepsilon_{w,t+1} = \sum_{i=1}^N w_i\varepsilon_{i,t+1} = w'\varepsilon_{t+1}$, then

$$
Cov_t\left(\varepsilon_{i,t+1}, \varepsilon_{w,t+1}\right) = w_i\theta_t^2\sigma_{ii} + \psi_t^2\sum_{j\neq i}^N w_j\sigma_{ji}, \tag{A21}
$$

and

$$
\mathbf{w}'\boldsymbol{\Sigma}_{t\varepsilon}\mathbf{w} = \sum_{ij=1}^N w_i w_j \sigma_{t,ij} = \left(\sum_{i=1}^N w_i^2\sigma_{ii}\right)\theta_t^2 + \left(\sum_{i\neq j}^N w_i w_j\sigma_{ij}\right)\psi_t^2.
$$

The expression for $E_t\left(r_{i,t+1} - r_{t+1}^f\right)$ can now be simplified using (A7)-(A10). First note that

$$
\left[\gamma_i\phi_f - A_{1i}(1-\kappa_{1i}\phi_f) - \varrho\gamma_w\phi_f\right] = \left(\gamma_i - \varrho\gamma_w\right)\phi_f - \left(\gamma_i - \varrho\gamma_w\right)\phi_f = 0.
$$

Also letting $c_{1i} = \gamma_i + \kappa_{1i}A_{1i}$, and using (A8) we obtain

$$-\left[A_{2i}(1-\kappa_{1i}\phi_\sigma) - \frac{1}{2}\varrho^2\gamma_w^2\right] = -\frac{1}{2}\left(\gamma_i + \kappa_{1i}A_{1i} - \varrho\gamma_w\right)^2 + \frac{1}{2}\varrho^2\gamma_w^2 = -\frac{1}{2}c_{1i}^2 + \varrho\gamma_w c_{1i}.$$

Finally, using (A9) and (A10) we have

$$A_{3i,N}(1-\kappa_{1i}\phi_\psi) = \frac{1}{2}\left(\varrho^2\sum_{j\neq i}^{N} w_j w_i \sigma_{ij} - 2\varrho\sum_{j\neq i}^{N} w_j\sigma_{ji}\right),\ A_{4i,N}(1-\kappa_{1i}\phi_\theta) = \frac{1}{2}\left(\sigma_{ii} + \varrho^2\sum_{j=1}^{N} w_j^2\sigma_{jj} - 2\varrho w_i\sigma_{ii}\right).$$

Substituting the above result in (A20) we now obtain

$$\begin{aligned} E_t\left(r_{i,t+1} - r_{t+1}^f\right) = \ & \pi_{iN} + \ln\beta - \varrho a_w + \varrho\gamma_w c_{1i}\sigma_t^2 - \tfrac{1}{2}c_{1i}^2\sigma_t^2 \\ & -\tfrac{1}{2}\left(\varrho^2\textstyle\sum_{j\neq i}^{N} w_j w_i\sigma_{ij} - 2\varrho\sum_{j\neq i}^{N} w_j\sigma_{ji}\right)\psi_t^2 \\ & -\tfrac{1}{2}\left(\sigma_{ii} + \varrho^2\textstyle\sum_{j=1}^{N} w_j^2\sigma_{jj} - 2\varrho w_i\sigma_{ii}\right)\theta_t^2 \\ & +\tfrac{1}{2}\varrho^2\left(\textstyle\sum_{i=1}^{N} w_i^2\sigma_{ii}\right)\theta_t^2 + \tfrac{1}{2}\varrho^2\left(\textstyle\sum_{i\neq j}^{N} w_i w_j\sigma_{ij}\right)\psi_t^2, \end{aligned}$$

which after some algebra yields:

$$E_t\left(r_{i,t+1} - r_{t+1}^f\right) = \pi_{iN} + \ln\beta - \varrho a_w + \varrho\gamma_w c_{1i}\sigma_t^2 - \tfrac{1}{2}c_{1i}^2\sigma_t^2 - \tfrac{1}{2}\sigma_{ii}\theta_t^2 + \varrho Cov_t\left(\varepsilon_{w,t+1}, \varepsilon_{i,t+1}\right).$$

The intercept term of the above can also be simplified. Using (A11) we first note that $A_{0i,N}$ can be written as using (A18) and (A19) as

$$(1-\kappa_{1i})A_{0i,N} = \ln(\beta) + a_i - \varrho a_w + \kappa_{0i} + \kappa_{1i}H_i + \frac{1}{2}\kappa_{1i}^2 G_{iN}^2, \tag{A22}$$

where H_i is defined by (A19) and

$$G_{iN}^2 = \varphi_\chi^2 A_{2i}^2 + \varphi_\eta^2 A_{3i,N}^2 + \varphi_\varpi^2 A_{4i,N}^2. \tag{A23}$$

Hence using (A22) in (A18) we have

$$\pi_{iN} = -\ln(\beta) + \varrho a_w - \frac{1}{2}\kappa_{1i}^2 G_{iN}^2. \tag{A24}$$

Using this expression in (A25) now provides the following result for the country-specific risk premia

$$E_t\left(r_{i,t+1} - r_{t+1}^f\right) = c_{1i}\varrho\gamma_w\sigma_t^2 - \tfrac{1}{2}c_{1i}^2\sigma_t^2 - \tfrac{1}{2}\sigma_{ii}\theta_t^2 + \varrho Cov_t\left(\varepsilon_{w,t+1}, \varepsilon_{i,t+1}\right) - \tfrac{1}{2}\kappa_{1i}^2 G_{iN}^2. \tag{A25}$$

The unconditional mean of the excess return is then given by

$$E\left(r_{i,t+1} - r_{t+1}^f\right) = c_{1i}\varrho\gamma_w\sigma^2 - \frac{1}{2}c_{1i}^2\sigma^2 - \frac{1}{2}\sigma_{ii} + \varrho\sum_{j=1}^{N} w_j\sigma_{ji} - \frac{1}{2}\kappa_{1i}^2 G_{iN}^2. \tag{A26}$$

The equilibrium/steady state risk premium given by (A26) is a complicated function of ϱ and all the risk parameters, σ^2, σ_{ii}, φ_χ^2, φ_η^2, and φ_ϖ^2. It also depends on $\varrho\sum_{j=1}^{N} w_j\sigma_{ji}$, which vanishes (as we show below) only if the country-specific shocks are weakly correlated, $w_j = O(N^{-1})$, and N is sufficiently large.

Weakly Correlated Country-specific Shocks Consider first the aggregate risk factor $\varepsilon_{w,t+1}$ in the SDF innovation (10) and its conditional variance $Var_t(\varepsilon_{w,t+1})$:

$$Var_t(\varepsilon_{w,t+1}) = \mathbf{w}'\mathbf{\Sigma}_{t\varepsilon}\mathbf{w} = \sum_{ij=1}^{N} w_i w_j \sigma_{t,ij} = \left(\sum_{i=1}^{N} w_i^2 \sigma_{ii}\right)\theta_t^2 + \left(\sum_{i\neq j}^{N} w_i w_j \sigma_{ij}\right)\psi_t^2.$$

Under our assumptions country-specific shocks, ε_{it}, are weakly cross correlated such that $sup_i \sum_{j\neq i}^{N} |\sigma_{ij}| < C_0 < \infty$. Also by standard result on matrix norms we have $\lambda_{\max}(\mathbf{\Sigma}_{t\varepsilon}) \leq sup_i \sum_{j\neq i}^{N} |\sigma_{t,ij}|$ (See, for example Theorem 5.6.9 of Horn and Johnson (1985)). Hence (noting that θ_t^2 and ψ_t^2 are given at time t, and σ_{ii} is bounded)

$$\lambda_{\max}(\mathbf{\Sigma}_{t\varepsilon}) \leq \sup_i \sum_{j=1}^{N} |\sigma_{t,ij}| = \theta_t^2 \sup_i \sigma_{ii} + \psi_t^2 \sup_i \sum_{j\neq i}^{N} |\sigma_{ij}| < C_1, \tag{A27}$$

and it immediately follows that:

$$Var_t(\varepsilon_{w,t+1}) = \mathbf{w}'\mathbf{\Sigma}_{t\varepsilon}\mathbf{w} \leq (\mathbf{w}'\mathbf{w})\,\lambda_{\max}(\mathbf{\Sigma}_{t\varepsilon}) \leq C_1(\mathbf{w}'\mathbf{w}), \tag{A28}$$

which establishes that $Var_t(\varepsilon_{w,t+1}) = O(N^{-1})$, given the granularity of the weight vector $\mathbf{w}$. Moreover, if N is sufficiently large (assuming that $1-\kappa_{1i}\phi_\theta \neq 0$ and $1-\kappa_{1i}\phi_\psi \neq 0$) we also have:

$$A_{0i,N} = A_{0i} + O(N^{-1}),\; A_{3i,N} = O(N^{-1}),\text{ and } A_{4i,N} = A_{4i} + O(N^{-1}), \tag{A29}$$

where

$$A_{4i} = \frac{1}{2}\left(\frac{\sigma_{ii}}{1-\kappa_{1i}\phi_\theta}\right),\; A_{0i} = \frac{1}{(1-\kappa_{1i})}\left(\begin{array}{c} \ln\beta + a_i + \kappa_{0i} - \varrho a_w + \kappa_{1i}\sigma^2(1-\phi_\sigma)A_{2i} \\ +\kappa_{1i}(1-\phi_\theta)\theta^2 A_{4i} + \frac{1}{2}\kappa_{1i}^2\varphi_\chi^2 A_{2i}^2 + \frac{1}{2}\kappa_{1i}^2\varphi_\varpi^2 A_{4i,\infty}^2 \end{array}\right). \tag{A30}$$

To establish the result for $A_{3i,N}$ in (A29), using (A9) note that

$$\begin{aligned} 2(1-\kappa_{1i}\phi_\psi)A_{3i,N} &= \varrho^2 \sum_{j\neq i}^{N} w_j w_i \sigma_{ij} - 2\varrho \sum_{j\neq i}^{N} w_j \sigma_{ji} \\ &= \varrho^2 \mathbf{w}'\mathbf{\Sigma}_\varepsilon \mathbf{w} - \varrho^2 \sum_{i=1}^{N} w_i^2 \sigma_{ii} - 2\varrho \sum_{j\neq i}^{N} w_j \sigma_{ji}, \end{aligned}$$

where $\Sigma_\varepsilon = (\sigma_{ij})$. Then:

$$|A_{3i,N}| \leq \frac{1}{2}\left|\frac{1}{1-\kappa_{1i}\phi_\psi}\right| \left[\varrho^2(\mathbf{w}'\mathbf{w})\,\lambda_{max}(\mathbf{\Sigma}_\varepsilon) + \varrho^2\left(\sup_i \sigma_{ii}\right)(\mathbf{w}'\mathbf{w}) + 2|\varrho| \sup_j |w_j| \sup_i \sum_{j=1}^{N} |\sigma_{ji}|\right].$$

But under the assumptions made $\lambda_{max}(\mathbf{\Sigma}_\varepsilon) < C_0$, $(\mathbf{w}'\mathbf{w}) = O(N^{-1})$, $\sup_j |w_j| = O(N^{-1})$ and $\sup_i \sum_{j=1}^{N} |\sigma_{ji}| < C_1$, which establishes that $|A_{3i,N}| = O(N^{-1})$.

The results for $A_{0i,N}$ and $A_{4i,N}$ follow similarly. Using the above results it is also easily established that

$$\begin{aligned}
b_{0i,N} &= [\kappa_{0i} - (1-\kappa_{1i})\, A_{0i} + a_i] + \kappa_{1i} A_{2i}(1-\phi_\sigma)\sigma^2 + \kappa_{1i} A_{4i}(1-\phi_\theta) + O(N^{-1}), \\
b_{3i,N} &= O\left(N^{-1}\right), \, b_{4i,N} = -\frac{1}{2}\left(\frac{\sigma_{ii}}{1-\kappa_{1i}\phi_\theta}\right) + O(N^{-1}), \\
c_{3i,N} &= O\left(N^{-1}\right), \, c_{4i,N} = \frac{1}{2}\left(\frac{\kappa_{1i}\sigma_{ii}\varphi_\varpi}{1-\kappa_{1i}\phi_\theta}\right) + O(N^{-1}),
\end{aligned}$$

where A_{0i} and A_{4i} are given by (A30). Also note that b_{1i}, b_{2i}, c_{1i} and c_{2i} do not depend on N, and are given as before (see (A13)).

Finally, under weakly cross correlated country-specific shocks we have (using (A25) and (A26))

$$E_t\left(r_{i,t+1} - r^f_{t+1}\right) = \varrho\gamma_w c_{1i}\sigma_t^2 - \tfrac{1}{2}c_{1i}^2\sigma_t^2 - \tfrac{1}{2}\sigma_{ii}\theta_t^2 - \tfrac{1}{2}\kappa_{1i}^2 G_i^2 + O(N^{-1}),$$

where

$$G_i^2 = \varphi_\chi^2 A_{2i}^2 + \frac{1}{2}\left(\frac{\sigma_{ii}\varphi_\varpi^2}{1-\kappa_{1i}\phi_\theta}\right). \tag{A31}$$

Conditional Covariance of Excess Return Volatility and Output Growth. Using equations (2) and (3) we first note that

$$\Delta y_{i,t+1} = a_i + \gamma_i \phi f_t + \gamma_i \sigma_t \zeta_{t+1} + \varepsilon_{i,t+1}, \tag{A32}$$

where f_t and σ_t are known at time t. Also, using (12) we have

$$r_{i,t+1} = \mathfrak{s}_{it} + q_{i,t+1} + c_{1i}\sigma_t\zeta_{t+1} + \varepsilon_{i,t+1}, \tag{A33}$$

where

$$\mathfrak{s}_{it} = b_{0i,N} + b_{1i} f_t + b_{2i}\sigma_t^2 + b_{3i,N}\psi_t^2 + b_{4i,N}\theta_t^2 \tag{A34}$$

is known at time t, and

$$q_{i,t+1} = c_{2i}\chi_{t+1} + c_{3i,N}\eta_{t+1} + c_{4i,N}\varpi_{t+1}$$

is distributed independently of ζ_{t+1} and $\varepsilon_{i,t+1}$. Hence, $Cov_t(r_{i,t+1}, \Delta y_{i,t+1}) = \gamma_i c_{1i}\sigma_t^2 + Var_t(\varepsilon_{i,t+1})$, which in turn implies that[31]

$$Cov_t\left(\tilde{r}_{i,t+1}, \Delta y_{i,t+1}\right) = Cov_t(r_{i,t+1}, \Delta y_{i,t+1}) = \gamma_i c_{1i}\sigma_t^2 + \sigma_{ii}\theta_t^2 \tag{A35}$$

where $\tilde{r}_{i,t+1} = r_{i,t+1} - r^f_{t+1}$, and as before c_{1i} is given by (A15). Also since by assumption $1-k_{1i}\phi_f > 0$, then $Cov_t\left(\tilde{r}_{i,t+1}, \Delta y_{i,t+1}\right) > 0$ if $\gamma_i\left(\gamma_i - \rho\gamma_w\kappa_{1i}\phi_f\right) \geq 0$. We note that this is a sufficient and not a necessary condition. A necessary and sufficient condition is given by

$$\sigma_{ii} + \left(\frac{\sigma_t^2}{\theta_t^2}\right)\frac{\gamma_i\left(\gamma_i - \rho\gamma_w\kappa_{1i}\phi_f\right)}{1-k_{1i}\phi_f} > 0,$$

Consider now the conditions under which the sufficient condition $\gamma_i\left(\gamma_i - \rho\gamma_w\kappa_{1i}\phi_f\right) \geq 0$ holds. The sign of γ_i is not identified separately from the sign of f_t. But we can assume that most economies are either positively or negatively affected by f_t. In that case γ_i and γ_w will have the same sign. If $\gamma_i \geq 0$, since by assumption $w_i > 0$, then $\gamma_w > 0$.[32] Therefore, $Cov_t(r_{i,t+1}, \Delta y_{i,t+1}) > 0$, if

[31] Note that r^f_{t+1} is known at time t, and $Var_t(\varepsilon_{i,t+1}) = \sigma_{ii}\theta_t^2$.

[32] If $\gamma_i < 0$, we can set $\delta_i = -\gamma_i$ with $\delta_i > 0$, and note that $\gamma_i\left(\gamma_i - \rho\gamma_w\kappa_{1i}\phi_f\right) = \delta_i\left(\delta_i - \rho\delta_w\kappa_{1i}\phi_f\right)$, with $\delta_i > 0$

$\gamma_i \geq \rho\gamma_w\kappa_{1i}\phi_f$. As an example consider the homogeneous case where $\gamma_i = \gamma$, and note that in this case $\gamma_w = \gamma$ and $Cov_t(\tilde{r}_{i,t+1}, \Delta y_{i,t+1}) > 0$ for all i if $1 - \rho\kappa_{1i}\phi_f > 0$. This condition is satisfied if $\rho < 1/\kappa_{1i}\phi_f$. Since $\kappa_{1i}\phi_f < 1$ then condition $\rho < 1/\kappa_{1i}\phi_f$ covers values of $\rho > 1$. It is interesting that allowing for country-specific growth volatility helps deliver a positive value for $Cov_t(r_{i,t+1}, \Delta y_{i,t+1})$ even if ϱ is so large as to yield $\gamma_i - \rho\gamma_w\kappa_{1i}\phi_f < 0$.

Consider now the conditional covariance of excess return volatility and output growth and note that (recall that r^f_{t+1} is known at time t)

$$\begin{aligned} Cov_t\left(\tilde{r}^2_{i,t+1}, \Delta y_{i,t+1}\right) &= Cov_t\left[r^2_{i,t+1} + \left(r^f_{t+1}\right)^2 - 2r_{i,t+1}r^f_{t+1}, \Delta y_{i,t+1}\right] \\ &= Cov_t\left(r^2_{i,t+1}, \Delta y_{i,t+1}\right) - 2r^f_{t+1}Cov_t\left(r_{i,t+1}, \Delta y_{i,t+1}\right). \end{aligned} \tag{A36}$$

Using (A32) and (A33), it also readily follows that $Cov_t(r^2_{i,t+1}, \Delta y_{i,t+1}) = 2\mathfrak{s}_{it}Cov_t(r_{i,t+1}, \Delta y_{i,t+1})$, where $\mathfrak{s}_{it}$ is given (A34). Using this result in (A36) we finally obtain

$$Cov_t\left(\tilde{r}^2_{i,t+1}, \Delta y_{i,t+1}\right) = 2d_{it}Cov_t\left(r_{i,t+1}, \Delta y_{i,t+1}\right), \tag{A37}$$

where, $d_{it} = \mathfrak{s}_{it} - r^f_{t+1}$. Now using (A34) and (11)) and noting that the coefficients $b_{0i,N}, b_{1i}, b_{2i}, b_{3i,N}$ and $b_{4i,N}$ are given by (A13), and after some simplifications we obtain

$$d_{it} = -\frac{1}{2}\kappa^2_{1i}\left(\varphi^2_\chi A^2_{2i} + \varphi^2_\varpi A^2_{4i}\right) - \frac{1}{2}\left(\frac{\gamma_i - \varrho\gamma_w}{1 - \kappa_{1i}\phi_f}\right)^2\sigma^2_t + \frac{1}{2}\rho^2\gamma^2_w\sigma^2_t - \frac{1}{2}\sigma_{ii}\theta^2_t + O(N^{-1}).$$

Also, since $c_{1i} = \varrho\gamma_w + \left(\frac{\gamma_i - \varrho\gamma_w}{1-\kappa_{1i}\phi_f}\right)$, the above expression can be written equivalently as

$$d_{it} = -\frac{1}{2}\kappa^2_{1i}\left(\varphi^2_\chi A^2_{2i} + \varphi^2_\varpi A^2_{4i}\right) - \frac{1}{2}\left(c^2_{1i} - 2c_{i1}\varrho\gamma_w\right)\sigma^2_t - \frac{1}{2}\sigma_{ii}\theta^2_t + O(N^{-1}). \tag{A38}$$

Realized Volatility At daily frequency within a given quarter t, the daily returns can be written as:

$$\begin{aligned} r_{it}(\tau) &= b_{0i,N} + b_{1i}f_{t-1}(\tau) + b_{2i}\sigma^2_{t-1}(\tau) + b_{3i,N}\psi^2_{t-1}(\tau) + b_{4i,N}\theta^2_{t-1}(\tau) \\ &\quad + c_{1i}\sigma_{t-1}(\tau)\zeta_t(\tau) + c_{2i}\chi_t(\tau) + c_{3i,N}\eta_t(\tau) + c_{4i,N}\varpi_t(\tau) + \varepsilon_{it}(\tau), \end{aligned} \tag{A39}$$

where $r_{it}(\tau)$ is the return on country i^{th} return on risky asset for day τ in quarter t. Similarly, $f_{t-1}(\tau)$ stands for the realization of factor f_t for day τ in quarter $t-1$, and etc. Therefore, quarterly realized volatility of r_{it}, denoted by RV_{it}, is given by (assuming there are D_t days in quarter t):

$$RV_{it} = \sqrt{\sum_{\tau=1}^{D_t}\left[r_{it}(\tau) - \bar{r}_{it}\right]^2}, \tag{A40}$$

where $\bar{r}_{it} = D_t^{-1}\sum_{\tau=1}^{D_t} r_{it}(\tau)$. It is now easily seen that realized return volatilities are complicated functions of lagged realized volatilities of the growth factor, f_t, realized volatility of the innovations to the growth factor, the realized volatilities of the country-specific shocks and their correlations

and $\delta_\omega = \sum_{i=1}^N w_i\delta_i > 0$.

(when N is finite), and their many interactions. Specifically, $r_{it}(\tau) - \bar{r}_{it}$ is given by:

$$\begin{aligned} & b_{1i}\left[f_{t-1}(\tau) - \bar{f}_{t-1}\right] + b_{2i}\left[\sigma^2_{t-1}(\tau) - \bar{\sigma}^2_{t-1}\right] + b_{3i,N}\left[\psi^2_{t-1}(\tau) - \bar{\psi}^2_{t-1}\right] + b_{4i,N}\left[\theta^2_{t-1}(\tau) - \bar{\theta}^2_{t-1}\right] \dots \\ & + c_{1i}\left[\sigma_{t-1}(\tau)v_t(\tau) - \overline{\sigma_{t-1}v_t}\right] + c_{2i}\left[\chi_t(\tau) - \bar{\chi}_t\right] + c_{3i,N}\left[\eta_t(\tau) - \bar{\eta}_t\right] + c_{4i,N}\left[\varpi_t(\tau) - \bar{\varpi}_t\right] + \left[\varepsilon_{it}(\tau) - \bar{\varepsilon}_{it}\right], \end{aligned}$$

where the bar on the variables denotes the sample mean of daily values in a given quarter. To simplify notation, we set the sample means of the shocks to zero (namely $\overline{\sigma_{t-1}v_t} = 0$, $\bar{\chi}_t = 0$, etc.), and using (A40) we note that the expression for RV^2_{it} is given by the following long expression:

$$\begin{aligned} RV^2_{it} &= b^2_{1i}\textstyle\sum_{\tau=1}^{D_t}\left[f_{t-1}(\tau) - \bar{f}_{t-1}\right]^2 + b^2_{2i}\sum_{\tau=1}^{D_t}\left[\sigma^2_{t-1}(\tau) - \bar{\sigma}^2_{t-1}\right]^2 + b^2_{3i,N}\sum_{\tau=1}^{D_t}\left[\psi^2_{t-1}(\tau) - \bar{\psi}^2_{t-1}\right]^2 \\ & + b^2_{4i,N}\textstyle\sum_{\tau=1}^{D_t}\left[\theta^2_{t-1}(\tau) - \bar{\theta}^2_{t-1}\right]^2 + c^2_{1i}\sum_{\tau=1}^{D_t}\sigma^2_{t-1}(\tau)\zeta^2_t(\tau) + c^2_{2i}\sum_{\tau=1}^{D_t}\chi^2_t(\tau) + c^2_{3i,N}\sum_{\tau=1}^{D_t}\eta^2_t(\tau) \\ & + c^2_{4i,N}\textstyle\sum_{\tau=1}^{D_t}\varpi^2_t(\tau) + \sum_{\tau=1}^{D_t}\varepsilon^2_{it}(\tau) + 2b_{1i}b_{2i}\sum_{\tau=1}^{D_t}\left[f_{t-1}(\tau) - \bar{f}_{t-1}\right]\left[\sigma^2_{t-1}(\tau) - \bar{\sigma}^2_{t-1}\right] \\ & + \text{other cross terms.} \end{aligned}$$

Appendix B: Proofs

Proof of Proposition 1. Using the country-specific models given by (35), and solving for $\mathbf{z}_{it}$ in terms of current and past values of factors and shocks we have:

$$\mathbf{z}_{it} = \boldsymbol{\mu}_i + \sum_{\ell=0}^{\infty}\boldsymbol{\Phi}^{\ell}_i\boldsymbol{\Gamma}_i\boldsymbol{\delta}_{t-\ell} + \boldsymbol{\varkappa}_{it}, \tag{A41}$$

where

$$\boldsymbol{\mu}_i = (\mathbf{I}_2 - \boldsymbol{\Phi}_i)^{-1}\mathbf{a}_i,\ \boldsymbol{\delta}_t = (\zeta_t, \xi_t)',\ \boldsymbol{\varkappa}_{it} = \sum_{\ell=0}^{\infty}\boldsymbol{\Phi}^{\ell}_i\boldsymbol{\vartheta}_{i,t-\ell},\ \text{and}\ \boldsymbol{\vartheta}_{it} = (\eta_{it}, \varepsilon_{it})'. \tag{A42}$$

Assumption 4, ensures that the infinite sums are convergent. Pre-multiplying both sides of (A41) by (w_i) and summing over i yields:

$$\bar{\mathbf{z}}_{\omega t} = \bar{\boldsymbol{\mu}}_{\omega} + \sum_{\ell=0}^{\infty}\mathbf{A}_{\ell,N}\boldsymbol{\delta}_{t-\ell} + \bar{\boldsymbol{\varkappa}}_{\omega t}, \tag{A43}$$

where

$$\begin{aligned} & \bar{\mathbf{z}}_{\omega t} = \textstyle\sum_{i=1}^{N} w_i\mathbf{z}_{it}, \quad \bar{\boldsymbol{\mu}}_{\omega} = \sum_{i=1}^{N} w_i\boldsymbol{\mu}_i, \\ & \mathbf{A}_{\ell,N} = \textstyle\sum_{i=1}^{N} w_i\boldsymbol{\Phi}^{\ell}_i\boldsymbol{\Gamma}_i, \quad \text{and}\ \bar{\boldsymbol{\varkappa}}_{\omega t} = \sum_{i=1}^{N} w_i\boldsymbol{\varkappa}_{it}. \end{aligned} \tag{A44}$$

Under Assumption 3, $\boldsymbol{\varkappa}_{it}$ are cross-sectionally weakly correlated and the weights $w = (w_1, w_2, ..., w_N)'$ are granular. Using the results in Pesaran and Chudik (2014), it readily follows that:

$$\bar{\boldsymbol{\varkappa}}_{\omega t} = O\left(\|\mathbf{w}\|\right) = O\left(N^{-1/2}\right),\ \text{for each}\ t. \tag{A45}$$

Under Assumptions 3 and 4, we also have

$$\mathbb{E}\left(\boldsymbol{\Phi}^{\ell}_i\boldsymbol{\Gamma}_i\right) = \mathbb{E}\left(\boldsymbol{\Phi}^{\ell}_i\right)\mathbb{E}\left(\boldsymbol{\Gamma}_i\right) = \boldsymbol{\Lambda}_{\ell}\boldsymbol{\Gamma},$$

and since $\mathbf{\Phi}_i$ and $\mathbf{\Gamma}_i$ are distributed independently across i, using again results in Pesaran and Chudik (2014) we have:

$$\mathbf{A}_{\ell,N}-\mathbb{E}\left(\mathbf{A}_{\ell,N}\right)=\sum_{i=1}^{N}w_i\left[\mathbf{\Phi}_i^{\ell}\mathbf{\Gamma}_i-\mathbb{E}\left(\mathbf{\Phi}_i^{\ell}\mathbf{\Gamma}_i\right)\right]=O\left(\|\mathbf{w}\|\right)=O\left(N^{-1/2}\right). \tag{A46}$$

Using (A45) and (A46) in (A43), and setting $\mathbf{\Lambda}(L)=\sum_{\ell=0}^{\infty}\mathbf{\Lambda}_{\ell}L^{\ell}$ we now have:

$$\bar{\mathbf{z}}_{\omega t}=\bar{\boldsymbol{\mu}}_{\omega}+\mathbf{\Lambda}(L)\mathbf{\Gamma}\mathbf{f}_t+O_p\left(N^{-1/2}\right).$$

But under Assumptions 3 and 4, $\mathbf{\Gamma}$ and $\mathbf{\Lambda}(L)$ are both invertible and:

$$\boldsymbol{\delta}_t=\mathbf{\Gamma}^{-1}\mathbf{\Lambda}^{-1}(L)\left(\bar{\mathbf{z}}_{\omega t}-\bar{\boldsymbol{\mu}}_{\omega}\right)+O_p\left(N^{-1/2}\right),$$

where $\mathbf{\Lambda}^{-1}(L)=\sum_{\ell=0}^{\infty}\mathbf{B}_{\ell}L^{\ell}$, with $\mathbf{B}_0=\mathbf{\Lambda}_0=\mathbf{I}_2$, and

$$\mathbf{\Gamma}^{-1}=\begin{pmatrix}0 & \gamma^{-1}\\ \theta^{-1} & -\frac{\lambda}{\theta\gamma}\end{pmatrix}.$$

Hence,

$$\begin{aligned}\boldsymbol{\delta}_t &= \mathbf{\Gamma}^{-1}\left(\bar{\mathbf{z}}_{\omega t}-\bar{\boldsymbol{\mu}}_{\omega}\right)+\left(\mathbf{C}_1+\mathbf{C}_2L+\mathbf{C}_3L^2+....\right)\left(\bar{\mathbf{z}}_{\omega,t-1}-\bar{\boldsymbol{\mu}}_{\omega}\right)+O_p\left(N^{-1/2}\right)\\ &= \mathbf{b}+\left(\sum_{\ell=0}^{\infty}\mathbf{C}_{\ell}L^{\ell}\right)\bar{\mathbf{z}}_{\omega,t}+O_p\left(N^{-1/2}\right),\end{aligned}$$

where $\mathbf{C}_{\ell}=\mathbf{\Gamma}^{-1}\mathbf{B}_{\ell}$, for $\ell=0,1,2,...$, and $\mathbf{b}=-\mathbf{\Gamma}^{-1}\mathbf{\Lambda}^{-1}(1)\bar{\boldsymbol{\mu}}_{\omega}$. But given the lower triangular form of $\mathbf{\Gamma}^{-1}$, we have

$$\zeta_t = \gamma^{-1}\Delta\bar{y}_{\omega t}+\sum_{\ell=1}^{\infty}\mathbf{c}_{1,\ell}'\bar{\mathbf{z}}_{\omega,t-\ell}+O_p\left(N^{-1/2}\right), \tag{A47}$$

$$\xi_t = \theta^{-1}\bar{v}_{\omega t}-\left(\frac{\lambda}{\theta\gamma}\right)\Delta\bar{y}_{\omega t}+\sum_{\ell=1}^{\infty}\mathbf{c}_{2,\ell}'\bar{\mathbf{z}}_{\omega,t-\ell}+O_p\left(N^{-1/2}\right), \tag{A48}$$

where $\mathbf{c}_{1,\ell}'$ and $\mathbf{c}_{2,\ell}'$ are the first and the second rows of $\mathbf{C}_{\ell}$, respectively, and $\bar{v}_{\omega t}$, $\Delta\bar{y}_{\omega t}$, $\bar{\mathbf{z}}_{\omega t}$ are defined as above. Consider now $\mathbf{C}_{\ell}$ and note that $\|\mathbf{C}_{\ell}\|\leq\left\|\mathbf{\Gamma}^{-1}\right\|\|\mathbf{B}_{\ell}\|$, where $\left\|\mathbf{\Gamma}^{-1}\right\|$ is bounded for fixed non-zero values of γ and θ. Further, $\mathbf{B}_{\ell}$ is given by the recursions $\mathbf{B}_{\ell}=-\sum_{i=1}^{\ell}\mathbf{\Lambda}_i\mathbf{B}_{\ell-i}$, for $\ell=0,1,...$, with $\mathbf{B}_0=\mathbf{I}_2$, and $\mathbf{B}_{\ell}=0$, for $\ell<0$. Hence, $\|\mathbf{B}_{\ell}\|\leq\sum_{i=1}^{\ell}\|\mathbf{\Lambda}_i\|\|\mathbf{B}_{\ell-i}\|$, where $\|\mathbf{B}_0\|=1$. However,

$$\|\mathbf{\Lambda}_{\ell}\|=\left\|\mathbb{E}\left(\mathbf{\Phi}_i^{\ell}\right)\right\|\leq\mathbb{E}\left\|\mathbf{\Phi}_i^{\ell}\right\|\leq\left(\mathbb{E}\|\mathbf{\Phi}_i\|\right)^{\ell}\leq\rho^{\ell}.$$

Hence, $\|\mathbf{B}_1\|\leq\rho$, $\|\mathbf{B}_2\|\leq\rho^2$, and so on, and as required $\|\mathbf{C}_{\ell}\|\leq\ell\left\|\mathbf{\Gamma}^{-1}\right\|\rho^{\ell}$.[33] ■

Proof of Proposition 2. Consider equations (39) and (40) in the paper. Let $\mathbf{M}_{\bar{Z}_{\omega}}=\mathbf{I}_T-$

[33]Note that for any matrix $\mathbf{A}$, $\|\mathbf{A}^p\|\leq\|\mathbf{A}\|^p$, and for any random variable X, $\|E(X)\|\leq E\|X\|$.

$\bar{\mathbf{Z}}_\omega \left(\bar{\mathbf{Z}}_\omega' \bar{\mathbf{Z}}_\omega\right)^{-1} \bar{\mathbf{Z}}_\omega'$, and note that:

$$\mathbf{M}_{\bar{Z}_\omega} \boldsymbol{\zeta} = \mathbf{M}_{\bar{Z}_\omega} \Delta \bar{\mathbf{y}}_\omega, \qquad \mathbf{M}_{\bar{Z}_\omega} \boldsymbol{\xi} = \mathbf{M}_{\bar{Z}_\omega} \bar{\mathbf{v}}_\omega - \lambda \mathbf{M}_{\bar{Z}_\omega} \Delta \bar{\mathbf{y}}_\omega,$$

since $\mathbf{M}_{\bar{Z}_\omega} \bar{\mathbf{Z}}_\omega = 0$. We set the first normalized vector of innovations, denoted by $\hat{\boldsymbol{\zeta}}$, to $\mathbf{M}_{\bar{Z}_\omega} \boldsymbol{\zeta}$, namely $\hat{\boldsymbol{\zeta}} = \mathbf{M}_{\bar{Z}_\omega} \Delta \bar{\mathbf{y}}_\omega$, and set the second factor, that we label $\hat{\boldsymbol{\xi}}$, as the linear combination of $\mathbf{M}_{\bar{Z}_\omega} \boldsymbol{\zeta}$ and $\mathbf{M}_{\bar{Z}_\omega} \boldsymbol{\xi}$, such that $\hat{\boldsymbol{\zeta}}' \hat{\boldsymbol{\xi}} = 0$. This can be achieved by selecting λ so that:

$$\hat{\boldsymbol{\zeta}}' \hat{\boldsymbol{\xi}} = \Delta \bar{\mathbf{y}}_\omega' \mathbf{M}_{\bar{Z}_\omega} \left(\mathbf{M}_{\bar{Z}_\omega} \bar{\mathbf{v}}_\omega - \lambda \mathbf{M}_{\bar{Z}_\omega} \Delta \bar{\mathbf{y}}_\omega \right) = 0.$$

The value of λ that solves this equation is given by, $\hat{\lambda} = \left(\Delta \bar{\mathbf{y}}_\omega' \mathbf{M}_{\bar{Z}_\omega} \bar{\mathbf{v}}_\omega\right) / \left(\Delta \bar{\mathbf{y}}_\omega' \mathbf{M}_{\bar{Z}_\omega} \Delta \bar{\mathbf{y}}_\omega\right)$, where $\hat{\lambda}$ is the OLS estimator of the coefficient of the regression of $\mathbf{M}_{\bar{Z}_\omega} \bar{\mathbf{v}}_\omega$ on $\mathbf{M}_{\bar{Z}_\omega} \boldsymbol{\Delta} \bar{\mathbf{y}}_\omega$. Hence, the orthogonalized factors are

$$\hat{\boldsymbol{\zeta}} = \mathbf{M}_{\bar{Z}_\omega} \Delta \bar{\mathbf{y}}_\omega, \ \text{ and } \ \hat{\boldsymbol{\xi}} = \mathbf{M}_{\bar{Z}_\omega} \bar{\mathbf{v}}_\omega - \hat{\lambda} \mathbf{M}_{\bar{Z}_\omega} \Delta \bar{\mathbf{y}}_\omega.$$

In practice, this implies that $\hat{\boldsymbol{\zeta}}$ can be recovered as residuals from the OLS regression of $\boldsymbol{\Delta} \bar{\mathbf{y}}_\omega$ on an intercept and $\bar{\mathbf{z}}_{\omega,t-\ell}$, for $\ell = 1, 2, ..., p$:

$$\Delta \bar{\mathbf{y}}_\omega = \bar{\mathbf{Z}}_\omega \hat{\mathbf{c}}_1 + \hat{\boldsymbol{\zeta}}. \tag{A49}$$

While $\hat{\boldsymbol{\xi}}$ can be recovered as residuals from the OLS regression of $\bar{v}_\omega$ on $\hat{\boldsymbol{\zeta}}$, an intercept, and $\bar{\mathbf{z}}_{\omega,t-\ell}$, for $\ell = 1, 2, ..., p$:

$$\bar{\mathbf{v}}_\omega = \hat{\lambda} \hat{\boldsymbol{\zeta}} + \bar{\mathbf{Z}}_\omega \hat{\mathbf{c}}_2 + \hat{\boldsymbol{\xi}}. \tag{A50}$$

■

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Online Supplement to

Uncertainty and Economic Activity: A Multi-Country Perspective

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Introduction

This supplement gives data sources and some summary statistics, and provides details of robustness analysis, country-specific results, and the derivation of impulse responses and error variance decompositions for global and country-specific shocks used in the paper.

S.1 Data Sources and Summary Statistics

Data Sources To construct a balanced panel for the largest number of countries for which we have sufficiently long time series, we first collect daily stock prices (excluding dividends) for 32 advanced and emerging economies from 1979 to 2016. We then cut the beginning of the sample in 1993, as daily equity price data are not available earlier for two large emerging economies (Brazil and China) and for Peru. Better quality quarterly GDP data for China also became available from 1993.[S1]

For equity prices we use the MSCI Index in local currency. We collected daily observations from January 1993 to December 2016. The data source for the daily equity price indices is Datastream. The countries included in the sample are the following: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Finland, France, Germany, India, Indonesia, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, Norway, New Zealand, Peru, Philippines, South Africa, Singapore, Spain, Sweden, Switzerland, Thailand, Turkey, United Kingdom, and United States. The list of Bloomberg tickers is as follows: TOTMKAR, TOTMKAU, TOTMKOE, TOTMKBG, TOTMKBR, TOTMKCN, TOTMKCL, TOTMKCA, TOTMKFN, TOTMKFR, TOTMKBD, TOTMKIN, TOTMKID, TOTMKIT, TOTMKJP, TOTMKKO, TOTMKMY, TOTMKMX, TOTMKNL, TOTMKNZ, TOTMKNW, TOTMKPE, TOTMKPH, TOTMKSG, TOTMKSA, TOTMKES, TOTMKSD, TOTMKSW, TOTMKTH, TOTMKTK, TOTMKUK, TOTMKU.

Real GDP data come from the latest update of the GVAR data set. The data set is balanced and good quality quarterly data are available for all countries in our sample from 1993:Q1 to 2016:Q4. For more details see: https://sites.google.com/site/gvarmodelling/.

Cross-country Correlations The differential pattern of cross-country correlations of the growth and volatility innovations is crucial for our identification strategy. Here we consider the properties of the observed time series as displayed in Figure 1 in the paper. In order to gauge the extent to which volatility and growth series co-move across countries, we use two techniques: standard principal component analysis and pair-wise correlation analysis across countries.

[S1] Note that some steps of the empirical analysis can be easily implemented with the unbalanced panel from 1979. This is the case, for example, for the estimates of factor innovations ($\hat{\zeta}_t$ and $\hat{\xi}_t$), which we report in Section S.2 below.

In a panel of countries indexed by $i = 1, 2, ..., N$, the average pair-wise correlation of country i in the panel ($\bar{\rho}_i$) measures the average degree of comovement of country i with all other countries j (i.e. for all $j \neq i$). The average pair-wise correlation across all countries, denoted by $\bar{\rho}_N$, is defined as the cross-country average of $\bar{\rho}_i$ over $i = 1, 2, ..., N$. This statistic relates to the degree of pervasiveness of the factors, as measured by the factor loadings. To see this, consider equation (2) of our model, $\Delta y_{it} = \gamma_i f_t + \varepsilon_{it}$, where $Var(f_t) = 1$, and $Var(\varepsilon_{it}) = \sigma_{ii}$.[S2] The average pair-wise correlation across all countries is given by:

$$\bar{\rho}_N = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \rho_{ij} = \frac{1}{N(N-1)} \left(\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{ij} - N \right), \tag{S1}$$

where

$$\rho_{ij} = \begin{cases} \frac{\tilde{\gamma}_i}{\sqrt{1+\tilde{\gamma}_i^2}} \frac{\tilde{\gamma}_j}{\sqrt{1+\tilde{\gamma}_j^2}} & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

and $\tilde{\gamma}_i = \gamma_i / \sqrt{\sigma_{ii}}$. Hence

$$\bar{\rho}_N = O\left(\bar{\gamma}_N^2\right), \tag{S2}$$

where $\bar{\gamma}_N = N^{-1} \sum_{i=1}^{N} \tilde{\gamma}_i$ measures the degree of pervasiveness of the factor.

The attraction of the average pair-wise correlation, $\bar{\rho}_N$, lies in the fact that it applies to multi-factor processes, and unlike factor analysis does not require the factors to be strong. In fact, the average pair-wise correlation, $\bar{\rho}_N$, tends to be a strictly positive number if Δy_{it} contains at least one strong factor, otherwise it tends to zero as $N \to \infty$. Therefore, non-zero estimates of $\bar{\rho}_N$ are suggestive of strong cross-sectional dependence.[S3] For completeness, and to show that our analysis is robust to using an alternative methodology, in what follows, we also use standard principal component analysis. (See also Chapter 29 in Pesaran (2015) for more details).

The average pair-wise correlation across all countries for the realized volatility series in Figure 1 is 0.56. In contrast, the average pair-wise correlation across all countries for the growth series at 0.27 is much smaller. Principal component analysis yields similar results. The first principal component in our panel of realized volatility series explains 65 percent of the total variation in the log-level of volatility, whilst the first principal component of the growth series accounts for only around 30 percent of total cross-country variations in these series. Thus, both in the case of the pair-wise correlation and principal component analysis, the results point to a much higher degree of cross-country comovements for the volatility series than for the growth series. As we will see, these differences are even more pronounced in the case of the estimated shocks obtained using equations (43) and (44).

Summary Statistics Table S.1 reports the summary statistics for the realized volatility series for each country in our sample. These results support the use of the log-level of realized volatilities as stationary series in our empirical analysis. Tables S.2 and S.3 give similar summary statistics for log of real GDP and its growth rate, and justifies using the latter as a stationary variable along with the log of realized volatility.

[S2] Under our assumptions $Var_t(\varepsilon_{i,t+1}) = \theta_t^2 \sigma_{ii}$, which gives $Var(\varepsilon_{i,t+1}) = \sigma_{ii}$.

[S3] Formal tests of cross-sectional dependence based on estimates of $\bar{\rho}_N$ are discussed in Pesaran (2015) and reported, for our panel of countries, in the next section.

Table S.1 SUMMARY STATISTICS FOR COUNTRY-SPECIFIC REALIZED VOLATILITY (LOG-LEVEL)

	ARG	AUS	AUT	BEL	BRA	CAN	CHL	CHN	FIN	FRA	DEU
Obs. in quarters	96	96	96	96	96	96	96	96	96	96	96
Mean	-2.12	-2.74	-2.70	-2.68	-2.20	-2.78	-2.85	-2.09	-2.17	-2.48	-2.53
Max	-1.23	-1.42	-1.17	-1.41	-1.12	-1.16	-1.77	-0.60	-1.09	-1.27	-1.23
Min	-2.86	-3.52	-3.38	-3.55	-2.84	-3.53	-3.56	-3.01	-2.87	-3.22	-3.44
St. Dev.	0.36	0.36	0.43	0.45	0.42	0.44	0.38	0.48	0.46	0.38	0.41
Auto Corr.	0.37	0.64	0.66	0.70	0.64	0.70	0.46	0.66	0.77	0.61	0.64
ADF	-3.12^{b}	-2.37	-2.17	-2.69^{c}	-3.55^{a}	-2.64^{c}	-3.19^{b}	-2.69^{c}	-2.31	-2.91^{c}	-2.79^{c}

	IND	IDN	ITA	JPN	KOR	MYS	MEX	NLD	NZL	NOR	PER
Obs. in quarters	96	96	96	96	96	96	96	96	96	96	96
Mean	-2.31	-2.25	-2.36	-2.40	-2.21	-2.76	-2.52	-2.56	-2.98	-2.42	-2.72
Max	-1.27	-1.07	-1.27	-1.13	-1.03	-0.82	-1.46	-1.21	-1.68	-0.93	-1.39
Min	-3.06	-3.19	-3.32	-3.17	-3.06	-3.87	-3.40	-3.33	-3.75	-3.09	-3.79
St. Dev.	0.40	0.43	0.40	0.34	0.48	0.59	0.43	0.46	0.38	0.39	0.49
Auto Corr.	0.58	0.60	0.58	0.32	0.79	0.72	0.66	0.69	0.67	0.61	0.67
ADF	-2.73^{c}	-2.46	-2.52	-3.6^{a}	-1.50	-2.24	-2.36	-2.79^{c}	-3.09^{b}	-2.81^{c}	-2.56

	PHL	SGP	ZAF	ESP	SWE	CHE	THA	TUR	GBR	USA
Obs. in quarters	96	96	96	96	96	96	96	96	96	96
Mean	-2.46	-2.65	-2.53	-2.42	-2.35	-2.67	-2.20	-1.83	-2.67	-2.64
Max	-1.54	-1.44	-1.45	-1.35	-1.21	-1.41	-1.20	-0.90	-1.28	-1.10
Min	-3.22	-3.47	-3.23	-3.18	-3.02	-3.39	-3.12	-2.81	-3.35	-3.40
St. Dev.	0.37	0.45	0.37	0.39	0.40	0.41	0.42	0.45	0.42	0.44
Auto Corr.	0.46	0.68	0.53	0.58	0.66	0.54	0.64	0.73	0.66	0.69
ADF	-2.97^{b}	-2.06	-3.28^{b}	-2.56	-2.59^{c}	-3.23^{b}	-2.25	-1.43	-2.56	-2.27

NOTE. Summary statistics of the log-level of volatility (v_{it}). *ADF* is the Augmented Dickey-Fuller t-statistic computed with 4 lags and a constant, where a, b, and c denote associated p-values at 1-percent, 5-percent, and 10-percent. Sample period 1993:Q1-2016:Q4.

Table S.2 Summary Statistics for Country-specific Real GDP (Log-Level)

	ARG	AUS	AUT	BEL	BRA	CAN	CHL	CHN	FIN	FRA	DEU
Obs. in quarters	96	96	96	96	96	96	96	96	96	96	96
Mean	4.78	4.73	4.66	4.65	4.75	4.67	4.79	5.14	4.65	4.64	4.64
Max	5.20	5.08	4.84	4.82	5.05	4.92	5.24	6.26	4.84	4.77	4.79
Min	4.42	4.30	4.40	4.40	4.39	4.32	4.23	3.99	4.29	4.43	4.47
St. Dev.	0.27	0.23	0.13	0.12	0.20	0.18	0.30	0.72	0.16	0.11	0.09
Auto Corr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
ADF	-0.30	-3.57^a	-1.86	-1.76	-1.83	-2.12	-1.62	-0.24	-2.20	-2.00	-0.72

	IND	IDN	ITA	JPN	KOR	MYS	MEX	NLD	NZL	NOR	PER
Obs. in quarters	96	96	96	96	96	96	96	96	96	96	96
Mean	4.91	4.88	4.60	4.65	4.77	4.79	4.67	4.62	4.98	4.59	4.86
Max	5.73	5.46	4.69	4.75	5.19	5.35	4.96	4.78	5.26	4.80	5.44
Min	4.08	4.37	4.46	4.55	4.19	4.15	4.31	4.36	4.63	4.23	4.24
St. Dev.	0.48	0.31	0.06	0.06	0.28	0.32	0.19	0.12	0.17	0.15	0.35
Auto Corr.	1.00	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
ADF	-0.37	0.64	-2.44	-0.78	-1.63	-0.59	-0.31	-1.97	-0.98	-2.8^c	0.21

	PHL	SGP	ZAF	ESP	SWE	CHE	THA	TUR	GBR	USA
Obs. in quarters	96	96	96	96	96	96	96	96	96	96
Mean	4.84	4.78	4.77	4.66	4.68	4.67	4.82	4.78	4.66	4.66
Max	5.45	5.31	5.07	4.84	4.97	4.85	5.18	5.28	4.86	4.89
Min	4.33	4.06	4.39	4.37	4.36	4.48	4.37	4.27	4.37	4.33
St. Dev.	0.32	0.36	0.22	0.15	0.17	0.12	0.23	0.29	0.14	0.16
Auto Corr.	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	1.00	1.00
ADF	1.96	-1.37	-1.36	-1.65	-0.54	-0.61	-0.68	-0.59	-1.77	-1.65

Note. Summary statistics for the log-level of real GDP (y_{it}). *ADF* is the Augmented Dickey-Fuller t-statistic computed with 4 lags and a constant, where *a*, *b*, and *c* denote associated p-values at 1-percent, 5-percent, and 10-percent. Sample period 1993:Q1-2016:Q4.

Table S.3 Summary Statistics for Country-specific Real GDP (Log-Difference)

	ARG	AUS	AUT	BEL	BRA	CAN	CHL	CHN	FIN	FRA	DEU
Obs. in quarters	95	95	95	95	95	95	95	95	95	95	95
Mean	0.78	0.81	0.46	0.43	0.60	0.63	1.06	2.40	0.55	0.36	0.34
Max	4.04	2.46	2.48	2.23	4.83	1.64	5.91	5.91	4.41	1.56	2.19
Min	-6.35	-0.99	-2.61	-2.12	-5.19	-2.27	-3.38	-1.33	-6.01	-1.70	-4.16
St. Dev.	1.92	0.56	0.92	0.69	1.44	0.61	1.28	1.24	1.32	0.50	0.74
Auto Corr.	0.59	-0.04	0.00	0.24	0.25	0.54	0.12	0.08	0.12	0.47	0.33
ADF	-3.49^{a}	-3.75^{a}	-3.68^{a}	-4.64^{a}	-3.63^{a}	-3.78^{a}	-3.18^{b}	-2.45	-3.88^{a}	-3.19^{b}	-4.65^{a}

	IND	IDN	ITA	JPN	KOR	MYS	MEX	NLD	NZL	NOR	PER
Obs. in quarters	95	95	95	95	95	95	95	95	95	95	95
Mean	1.73	1.14	0.18	0.18	1.05	1.26	0.63	0.44	0.67	0.60	1.27
Max	5.41	5.11	1.79	2.53	3.81	4.65	3.77	1.70	2.54	4.48	4.44
Min	-1.43	-8.17	-3.70	-4.09	-8.94	-7.10	-6.07	-2.10	-1.67	-1.74	-2.52
St. Dev.	1.28	1.76	0.69	1.00	1.48	1.59	1.43	0.65	0.71	1.15	1.36
Auto Corr.	-0.06	0.34	0.42	0.29	0.27	0.33	0.33	0.58	0.24	-0.20	0.28
ADF	-4.66^{a}	-3.47^{a}	-3.3^{b}	-5.08^{a}	-4.74^{a}	-5.34^{a}	-4.1^{a}	-3.04^{b}	-3.94^{a}	-3.46^{a}	-4.2^{a}

	PHL	SGP	ZAF	ESP	SWE	CHE	THA	TUR	GBR	USA
Obs. in quarters	95	95	95	95	95	95	95	95	95	95
Mean	1.17	1.32	0.71	0.50	0.64	0.39	0.86	0.99	0.52	0.59
Max	3.21	6.77	1.86	2.49	2.94	1.98	10.79	6.57	1.41	1.81
Min	-2.44	-3.77	-1.63	-1.57	-3.71	-3.50	-11.97	-11.93	-2.11	-2.18
St. Dev.	0.87	2.03	0.60	0.60	1.12	0.78	2.47	2.91	0.58	0.59
Auto Corr.	0.11	0.23	0.60	0.80	-0.01	0.16	-0.02	-0.01	0.69	0.41
ADF	-4.13^{a}	-5.28^{a}	-2.7^{c}	-2.47	-4.9^{a}	-4.15^{a}	-4.11^{a}	-5.21^{a}	-4^{a}	-3.34^{b}

Note. Summary statistics for the log-difference of real GDP (Δy_{it}). *ADF* is the Augmented Dickey-Fuller t-statistic computed with 4 lags and a constant, where a, b, and c denote associated p-values at 1-percent, 5-percent, and 10-percent. Sample period 1993:Q1-2016:Q4.

S.2 Robustness Analysis

We report here the results from a few exercises showing robustness of our results.

S.2.1 Robustness to Choice of Countries (Granularity Assumptions)

This section compares the results from four robustness exercises with respect to the choice of the countries in our sample with the estimates reported in the paper that are based on all countries. In particular, we consider the following cases: (1) exclude the United States from the sample; (2) exclude China from the sample; (3) exclude the United States and China from the sample; and (4) we treat the United States as the global factor, namely we substitute $\hat{\zeta}_t$ and $\hat{\xi}_t$ with $\Delta y_{US,t}$ and $v_{US,t}$ respectively.

Table S.4 shows that in cases (1), (2), and (3)—i.e. when we exclude the United States, China, or both—the cross-sectional dependence of the country-specific innovations is very similar to the baseline. So, our common factors cannot be driven by shocks to these large economies. This is not true for case (4), i.e. when we treat the US economy as the common factor. In this case, Table S.4 shows that the country-specific GDP growth and volatility innovations display a significant degree of cross-sectional dependence even after conditioning on US GDP growth and US (log) volatility. Consistently with that, the CD test rejects the null of zero average pair-wise correlation of the innovations. In other words, when replacing the common factors $\hat{\zeta}_t$ and $\hat{\xi}_t$ with US GDP growth and US volatility, we can control for some, but not all, the cross-country correlation of the GDP growth and volatility series. Table S.5 reports similar evidence based on 'long-run' (i.e. 12 quarters ahead) forecast error variance decompositions (FEVD). The Table shows that the FEVDs in cases (1), (2), and (3) are very similar to our baseline, while this is not true for case (4).

Table S.4 Cross-Sectional Dependence of the Innovations

	Pairwise Correlation			Exponent of cross-sectional dependence		
	$\hat{\varepsilon}_i$	$\hat{u}_i$	$\hat{\eta}_i$	$\hat{\varepsilon}_i$	$\hat{u}_i$	$\hat{\eta}_i$
Baseline (All countries)	-0.01	0.52	-0.02	0.56 [0.62,0.67]	0.96 [1.00,1.04]	0.58 [0.64,0.70]
Excluding US	-0.02	0.52	-0.02	0.54 [0.60,0.65]	0.96 [1.00,1.04]	0.52 [0.59,0.66]
Excluding China	-0.01	0.55	-0.03	0.57 [0.62,0.68]	0.96 [1.00,1.04]	0.61 [0.68,0.74]
Excluding US & China	-0.01	0.54	-0.03	0.56 [0.61,0.66]	0.96 [1.00,1.04]	0.57 [0.63,0.69]
US as global factor	0.15	0.49	0.27	0.91 [0.99,1.06]	0.96 [0.99,1.03]	0.96 [0.99,1.03]

Note. Pair-wise correlations and exponent of cross-sectional dependence ($\hat{\alpha}$) as in Bailey et al. (2016), together with the associated 90-percent confidence interval in square brackets. Sample period 1993:Q1-2016:Q4.

Table S.5 Forecast Error Variance Decomposition (Long-run)

	FEVD of GDP growth					
	$\hat{\xi}$	$\hat{\eta}_i$	$\sum \hat{\eta}_j$	$\hat{\zeta}$	$\hat{\varepsilon}_i$	$\sum \hat{\varepsilon}_j$
Baseline	7.6	1.9	0.1	24.9	64.6	0.9
Excluding US	7.8	2.0	0.1	24.8	64.5	0.8
Excluding China	7.5	2.0	0.1	25.6	63.9	0.9
Excluding US & China	7.7	2.1	0.1	25.4	63.8	0.9
US as global factor	5.7	2.9	0.2	6.5	83.9	0.9
	FEVD of Volatility					
	$\hat{\xi}$	$\hat{\eta}_i$	$\sum \hat{\eta}_j$	$\hat{\zeta}$	$\hat{\varepsilon}_i$	$\sum \hat{\varepsilon}_j$
Baseline	53.2	41.9	0.1	3.9	0.6	0.2
Excluding US	53.0	42.4	0.1	3.7	0.6	0.2
Excluding China	54.5	40.3	0.1	4.3	0.6	0.2
Excluding US & China	54.3	40.8	0.1	4.0	0.6	0.2
US as global factor	32.8	57.8	0.2	8.5	0.6	0.2

Note. Average across countries with GDP-PPP weights at horizon $h = 12$ quarters. $\hat{\xi}$ is common financial shock; $\hat{\eta}_i$ is country i's volatility shock; $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in country j, for all $j \neq i$; $\hat{\zeta}$ is common growth shock; $\hat{\varepsilon}_i$ is country i's GDP growth shock; $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in country j, for all $j \neq i$. Sample period: 1993:Q1-2016:Q4.

S.2.2 Robustness to the Choice of Sample Periods

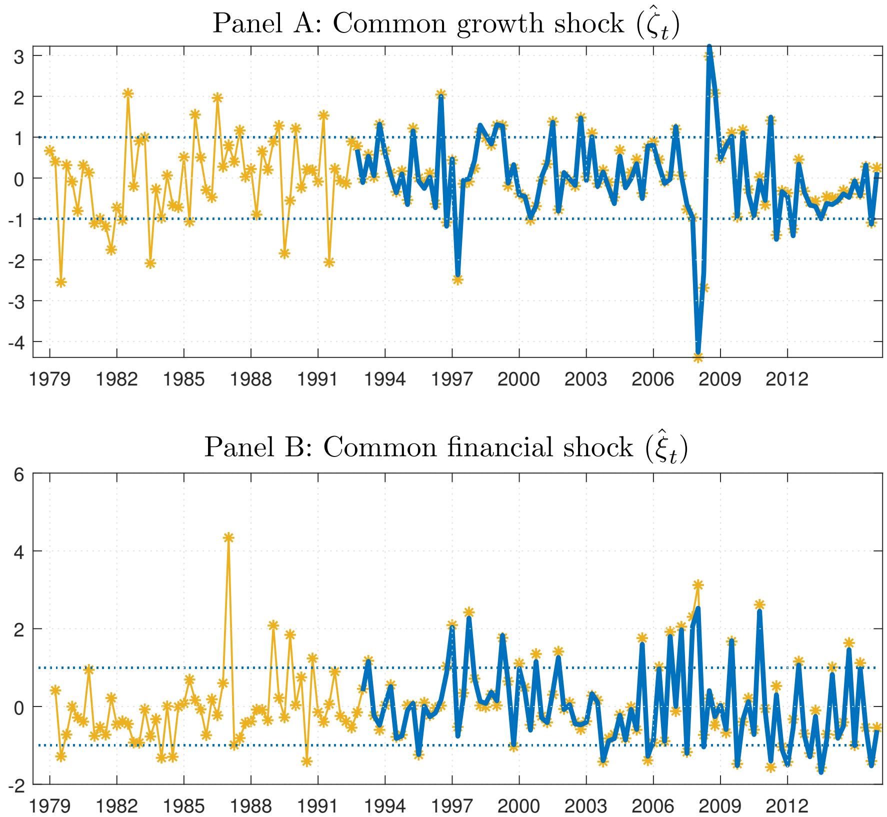
We report here results from a longer unbalanced sample period and when we exclude the global financial crisis period from the sample.

Longer-run Unbalanced Panel Estimates of the Common Shocks. We consider in this section a longer sample period starting from 1979. While for a few emerging economies quarterly GDP data is not available from this starting date, it is possible to interpolate annual series to obtain a balanced sample of GDP growth series at quarterly frequency for all countries considered in our study. For more details see: https://sites.google.com/site/gvarmodelling/. We then collected daily equity prices from January 1979 to December 2016. Note that, over this sample, it is possible to obtain a balanced panel only for 16 economies.

Estimates of the global shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, recovered from the OLS estimation of (41) and (42) are reported in Figure S.1 when estimated using the unbalanced panel from 1979 (thin lines with asterisks), and when we use the balanced panel from 1993 (thick solid lines), so as to better illustrate their time profiles. The figure also reports one-standard deviation bands for the shocks. Note that the shocks are standardized and have zero means and unit in-sample variances. They are also serially uncorrelated and orthogonal to each other by construction. Interestingly, the Jarque-Bera test strongly rejects normality in the case of the growth shocks, with strong evidence of left skewness and kurtosis, and only marginally rejects in the case of the financial shock with mild evidence of right skewness. The figure shows that the largest negative realization of the real common shock was after the second oil shock in 1979, and during the fourth quarter of 2008 after the Lehman Brother's collapse, consistent with prevailing narratives on the characterization of world recessions. Figure S.1 illustrates that the largest realizations of the common financial shock, $\hat{\xi}_t$, coincide with the 1987 stock market crash and the 2008 Lehman Brother's collapse.

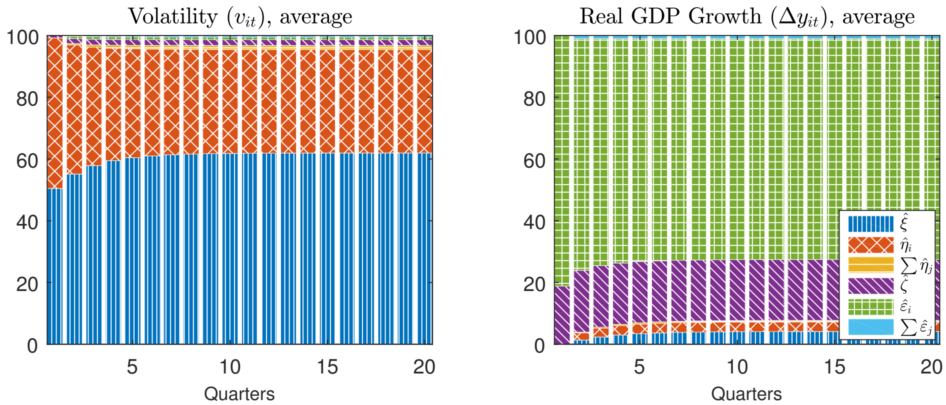
Excluding the global financial crisis. The results are robust to dropping the period of the global financial crisis from our sample. For example, we report in Figures S.2 and S.3 the FEVDs and IRFs that we obtained when re-estimating the sample from 1993 to 2006.

Figure S.1 ESTIMATED COMMON GROWTH AND FINANCIAL SHOCKS



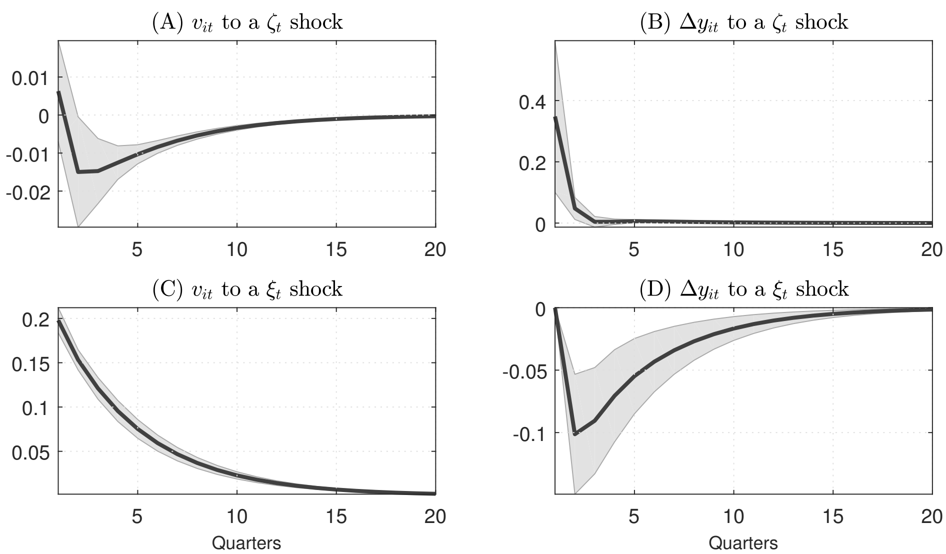
NOTE. The common shocks $\hat{\zeta}_t$ and $\hat{\xi}_t$ are computed using (41) and (42), with one lag of $\mathbf{z}_{it}$, using an unbalanced sample 1979:Q2-2016:Q4 (thin lines with asterisks) and the shorter balanced sample 1993:Q1-2016:Q4 (thick solid lines). The shocks are standardized and the dotted lines are the one-standard deviation bands around the zero mean.

Figure S.2 FEVD - Sample period: 1993-2006



Note. Diagonal covariance matrix. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common growth shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas with no pattern). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2006:Q2.

Figure S.3 Average Country Volatility and Growth Responses to Real and Financial Factor Shocks (In Percent) - Sample period: 1993-2006



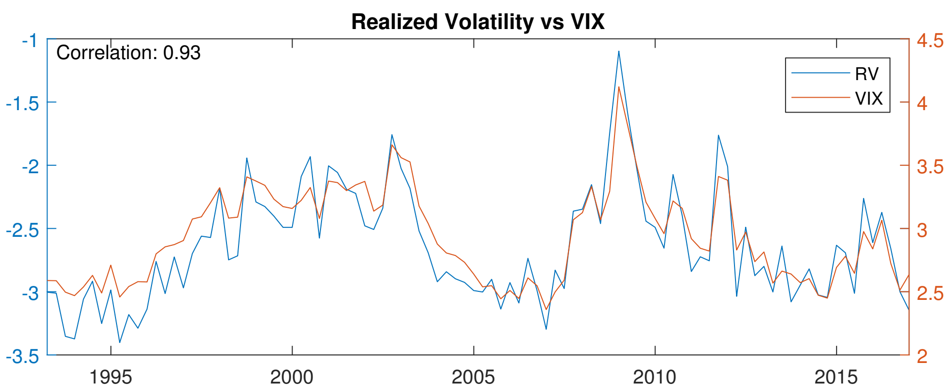
Note. Average impulse responses to one-standard deviation real and financial shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$. The solid lines are the PPP-GDP weighted averages of the country-specific responses. The shaded areas are the two standard deviations confidence intervals. See equations (S12) and (S13) for the derivations and Figure S.11 for the country-specific responses. The horizontal axis is in quarters. Sample period: 1993:Q1-2006:Q2.

S.2.3 Robustness to Choice of Uncertainty Measures: Realized *versus* Implied volatility

At quarterly frequency, the realized volatility of US daily equity returns behaves very similarly to the VIX Index. For example, during the period over which they overlap, our realized volatility measure and the VIX Index co-move very closely, with a correlation that exceeds 0.9. See Figure S.4. In addition, to check more formally the robustness of our results, we re-estimated our model using the VIX Index as a measure of volatility for the United States (instead of our realized volatility measure) and obtain virtually identical results.

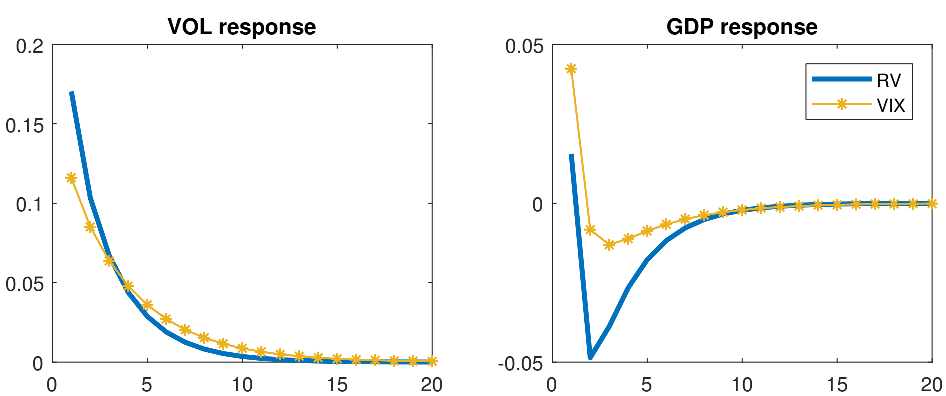
Figure S.5 compares our baseline IRFs of US volatility and US GDP growth to a US country-specific volatility shock (solid blue line) with those obtained from a specification where we used the VIX Index as a measure of US volatility instead of our realized volatility measure (yellow line with asterisks). The comparison shows that, in the robustness exercise, the correlation between US GDP and volatility residuals is even more positive than in our baseline scenario; thus reinforcing our main result.

Figure S.4 United States: VIX Index *versus* RV



Note. Blue line is the (log) realized volatility of equity prices for the United States, as in our baseline model (RV). The red line is the (log) VIX Index (average across days within the quarter). Sample period: 1993:Q1-2016:Q4.

Figure S.5 US Response to US Volatility Shock



Note. US impulse responses to a one-standard deviation shock to US volatility, $\hat{\eta}_{US,t}$. The blue lines are our baseline; the yellow lines with asterisks are obtained from a specification where we used the VIX Index as a measure of US volatility instead of our realized volatility measure. The horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

S.2.4 Robustness to Alternative Identification Assumptions for Country-Specific Shocks

Consider the correlation between volatility and growth innovations within each country. We saw in Figure 2 that, once we condition on the global shocks ($\hat{\zeta}_t$ and $\hat{\xi}_t$), the contemporaneous within-country correlation between $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$ is very small and not statistically significant in most countries. In Figure 8 we also showed that conditional on both $\hat{\zeta}_t$ and $\hat{\xi}_t$, the country-specific shocks $\hat{\varepsilon}_{it}$ and $\hat{\eta}_{it}$ are weakly correlated across countries, with average pair-wise correlations below 0.05. Weak cross-sectional dependence means that, as N grows, the overall average pair-wise correlation must tend to zero; while some pairs of correlations can be different from zero, not all pairs can be so. In practice, this means that most correlation pairs will be very small and the covariance matrix, $\mathbf{\Sigma}_{(\varepsilon,\eta)}$, in the 64 shocks $\hat{\varepsilon}_{it}$ and $\hat{\eta}_{it}$, for $i = 1, 2, ..., N$, must be sparse.

Indeed, when we apply the threshold estimation procedure of Bailey et al. (2019b) to the whole set of distinct off-diagonal elements of $\mathbf{\Sigma}_{(\varepsilon,\eta)}$ we find that only 57 out of 2016 off-diagonal elements are statistically different from zero. Table S.6 shows that, of these 57, about half are positively correlated and the other half are negatively correlated, with an average value that is close to zero. Most notably, there is no surviving within-country contemporaneous correlation between volatility and growth. There are also very few significant GDP-GDP correlation pairs (i.e., $\hat{\varepsilon}_{it}$ with $\hat{\varepsilon}_{jt}$), with no obvious regional pattern of comovements. There are a few significant pairs of volatility-volatility correlations (i.e. $\hat{\eta}_{it}$ with $\hat{\eta}_{jt}$), but involving only a handful of countries, with no evidence of a dominant role for the United States. Finally, there are only two significant GDP-volatility correlation pairs (i.e. $\hat{\varepsilon}_{jt}$ with $\hat{\eta}_{it}$), again revealing no specific patterns.

Note that even a block diagonal estimated reduced form covariance matrix (where all cross-country innovations correlations are zero), would not imply that innovations $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$ can be interpreted as 'structural' country-specific volatility and growth shocks. As is well known, there always exists an orthonormal transformation of η_{it} and ε_{it} that leads to the same forecast error variance decomposition. It is therefore important to complement this evidence with some explicit assumption about the 64×64 matrix of correlations.

In our baseline estimates of the FEVDs we assume a block-diagonal covariance matrix for the residuals of the multi-country model (43)-(44), where we assume that within each country volatility shocks affect output growth contemporaneously (but not vice versa).

To check the robustness of our results we re-estimate the FEVDs with two alternative sets of assumptions on the covariance matrix of country-specific innovations. First, we assume that the only source of interdependence among all growth and volatility series are the global real and financial shocks $\hat{\zeta}_t$ and $\hat{\xi}_t$ and that country-specific volatility and growth shocks have no contemporaneous impact on growth or volatility series within and across countries. In other words, we assume that the reduced form innovations are also structural. Then, we also consider the case in which we refrain from interpreting these innovations structurally.

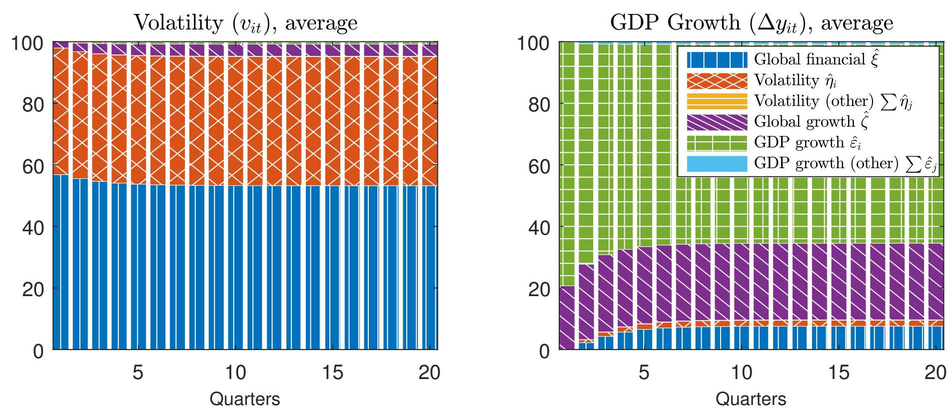
Table S.6 Non-zero Elements of the Regularized Error Covariance Matrix Estimate

All Significant			Between-county correlations		
Country - Variable Pairs		Corr	$\hat{\varepsilon}_{it}, \hat{\varepsilon}_{jt}$	$\hat{\eta}_{it}, \hat{\eta}_{jt}$	$\hat{\varepsilon}_{it}, \hat{\eta}_{jt}$
ARG VOL	NLD VOL	-0.34		ARG,NLD	
AUS VOL	NZL VOL	0.37		AUS,NZL	
AUT GDP	PHL GDP	-0.36	AUT,PHL		
BEL VOL	ITA VOL	0.48		BEL,ITA	
BEL VOL	NLD VOL	0.56		BEL,NLD	
BEL VOL	CHE VOL	0.38		BEL,CHE	
BEL VOL	GBR VOL	0.50		BEL,GBR	
BRA VOL	MEX VOL	0.45		BRA,MEX	
CAN VOL	NOR VOL	0.44		CAN,NOR	
CHL VOL	FRA VOL	-0.35		CHL,FRA	
CHL VOL	NLD VOL	-0.35		CHL,NLD	
CHL VOL	ESP VOL	-0.36		CHL,ESP	
FIN VOL	SWE VOL	0.51		FIN,SWE	
FIN GDP	ITA GDP	0.37	FIN,ITA		
FRA VOL	DEU VOL	0.62		FRA,DEU	
FRA VOL	IND VOL	-0.38		FRA,IND	
FRA VOL	IDN VOL	-0.43		FRA,IDN	
FRA VOL	ITA VOL	0.44		FRA,ITA	
FRA VOL	MEX VOL	-0.43		FRA,MEX	
FRA VOL	NLD VOL	0.67		FRA,NLD	
FRA VOL	PHL VOL	-0.35		FRA,PHL	
FRA VOL	SGP VOL	-0.44		FRA,SGP	
FRA VOL	ESP VOL	0.58		FRA,ESP	
FRA VOL	SWE VOL	0.48		FRA,SWE	
FRA VOL	THA VOL	-0.37		FRA,THA	
FRA VOL	GBR VOL	0.60		FRA,GBR	
DEU VOL	ITA VOL	0.37		DEU,ITA	
DEU VOL	MEX VOL	-0.38		DEU,MEX	
DEU VOL	NLD VOL	0.62		DEU,NLD	
DEU VOL	ESP VOL	0.48		DEU,ESP	
DEU VOL	GBR VOL	0.37		DEU,GBR	
IDN VOL	PER GDP	-0.36			IDN,PER
IDN VOL	PHL VOL	0.38		IDN,PHL	
IDN VOL	SGP VOL	0.35		IDN,SGP	
IDN VOL	THA VOL	0.39		IDN,THA	
IDN VOL	GBR VOL	-0.36		IDN,GBR	
IDN GDP	KOR GDP	0.35	IDN,KOR		
ITA VOL	MYS VOL	-0.38		ITA,MYS	
ITA VOL	NLD VOL	0.53		ITA,NLD	
ITA VOL	PHL VOL	-0.34		ITA,PHL	
ITA VOL	SGP VOL	-0.36		ITA,SGP	
ITA VOL	ESP VOL	0.56		ITA,ESP	
ITA VOL	GBR VOL	0.39		ITA,GBR	
KOR GDP	MYS GDP	0.47	KOR,MYS		
MYS VOL	SGP VOL	0.43		MYS,SGP	
MYS VOL	SWE VOL	-0.48		MYS,SWE	
MEX VOL	NLD VOL	-0.36		MEX,NLD	
NLD VOL	PER VOL	-0.35		NLD,PER	
NLD VOL	ESP VOL	0.46		NLD,ESP	
NLD VOL	CHE VOL	0.50		NLD,CHE	
NLD VOL	GBR VOL	0.70		NLD,GBR	
NOR VOL	PER GDP	0.36			NOR,PER
PHL VOL	SGP VOL	0.48		PHL,SGP	
SGP VOL	ESP VOL	-0.39		SGP,ESP	
SWE VOL	GBR VOL	0.37		SWE,GBR	
CHE VOL	GBR VOL	0.46		CHE,GBR	
THA VOL	GBR VOL	-0.38		THA,GBR	

Note. Non-zero elements of the regularized error covariance matrix estimate proposed by Bailey et al. (2019b). Sample period: 1993:Q1-2016:Q4.

Diagonal Covariance Matrix and Orthogonal Decomposition. We assume that all elements of the variance covariance matrix of the country-specific shocks are truly zero after conditioning on the common shocks across countries. The results for this specification are given in Figure S.6 and can be seen to be virtually identical to the estimates obtained for the diagonal error covariance matrix reported in Figures 4 and 6 in the paper. This is perhaps not surprising given that the correlations between the country-specific innovations, once the effects of the common shocks are removed, are very small as in Figure 2 in the paper.

Figure S.6 Forecast Error Variance Decomposition of Country-specific Shocks - Diagonal Error Covariance Matrix (In Percent)



Note. Diagonal covariance matrix. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common growth shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas with no pattern). The horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

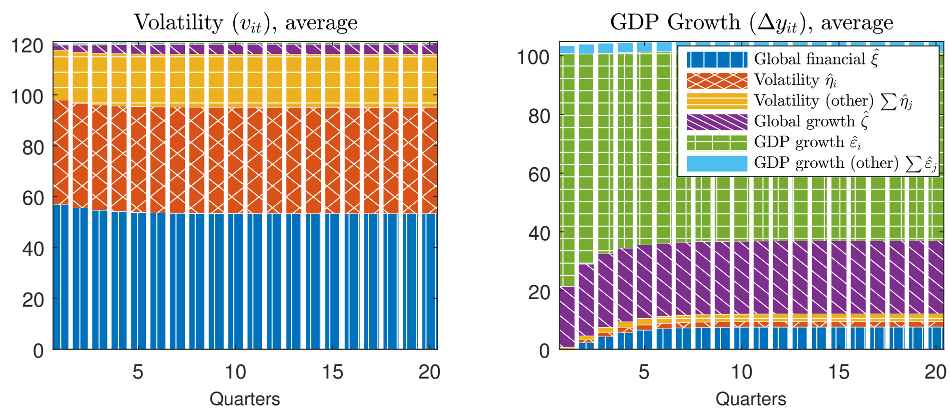
Thresholding the Country-specific Error Covariance Matrix and Generalized Error Variance Decomposition. Here we allow for a fully estimated (64×64) correlation matrix or country-specific errors, both within and across countries, and compute the Generalized Forecast Error Variance Decompositions (GFEVDs). However, given the large size of this matrix, we regularize it by computing a threshold estimator following Bailey et al. (2019b), who developed a procedure based on results from the multiple testing literature. Specifically, we first test for the statistical significance of each of the 2016 distinct off-diagonal elements of the (64×64) matrix. We then set to zero all those elements that are not statistically significant, using suitably adjusted critical values to allow for the large number of tests that are being carried out. We then finally compute the GVEDs by using the regularized estimates as set out in Section S.5 below.

The estimated generalized forecast error variance decompositions (GFEVDs), reported in Figure S.7, are consistent with those obtained assuming a diagonal or block-diagonal error covariance matrix.[S4] Relative to the results with diagonal or block-diagonal covariance matrix in Figures S.6 and 4 and 6, the contribution of foreign country-specific volatility (growth) shocks, $\sum \hat{\eta}_j$ ($\sum \hat{\varepsilon}_j$), to domestic volatility (growth) is now larger, but the spillover effects of foreign volatility shocks to growth (and foreign growth shocks to volatility) remain negligible. Moreover, global financial shocks and domestic country-specific volatility shocks continue to explain the bulk of the forecast error variance of volatility. Similarly, global growth shocks and the country-specific growth shocks remain the main drivers of the forecast error variance of growth.

[S4]Notice here that the GFEVDs need not sum to 100 as the underlying shocks are not orthogonal.

We interpret the above results as strong evidence of robustness of our conclusions reached by assuming a diagonal or block-diagonal error covariance matrix. In particular, it remains the case that common or country-specific output growth shocks have a small quantitative importance for volatility, and home and foreign country-specific volatility shocks have little or no quantitative consequence for output growth.

Figure S.7 GENERALIZED FORECAST ERROR VARIANCE DECOMPOSITION OF COUNTRY-SPECIFIC SHOCKS - ESTIMATION OF REGULARIZED FULL ERROR COVARIANCE MATRIX (IN PERCENT)



NOTE. Threshold estimator of the population covariance matrix. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common growth shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas with no pattern). The horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

S.2.5 Robustness to Weighting Scheme

In this subsection we assess the role of the weights used in our analysis. First note that, asymptotically, the weights do not matter (Pesaran, 2006), as long as there is no dominant unit in the cross section (on the absence of dominant units in our sample, see the evidence provided in Section S.2.1). Consistently with that, we show here that our results are robust to an alternative weighting scheme.

Recall here that in our baseline we used equal weights to estimate the factors, that is we assumed w_i and $\mathring{w}_i$ in Equation 23 to be $1/N$.[S5] Alternatively, one could have used PPP-GDP weights to construct the global variables and estimate the factors. Also, while in principle time-varying weights could be used, we focus here on simple weights based on the average PPP-GDP weight over the full sample period. The average PPP-GDP weights are reported in Table S.7. Clearly the US and China stand out as the largest economies in our sample. The average weight is about 3 percent and the the standard deviation across countries is about 5 percent.

As in the main text, Figures S.8, S.9, and S.10 report the contemporaneous correlations between the country-specific volatility and growth innovations, the impulse responses, and the forecast error variance decompositions obtained when using PPP-GDP weights for the construction of the factors. A comparison with the same figures in the main text shows that our results are virtually unchanged when using this alternative weighting scheme.

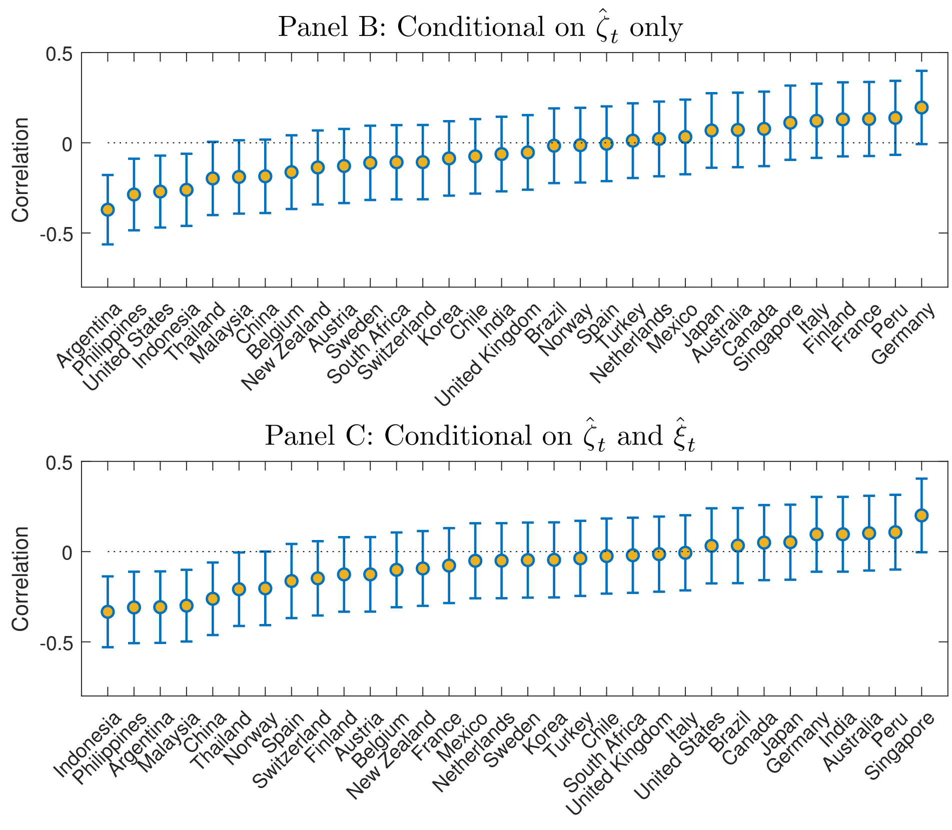
[S5] Remember these are the weights used to construct the global variables (as shown in Equation 23), not the weights used to aggregate results in a single average estimate, as reported in the impulse response and forecast error variance decomposition analysis (as shown in Section S.5.3).

Table S.7 PPP-GDP weights

Argentina	1.1%	Malaysia	0.9%
Australia	1.3%	Mexico	2.6%
Austria	0.6%	Netherlands	1.1%
Belgium	0.7%	New Zealand	0.2%
Brazil	4.1%	Norway	0.4%
Canada	2.1%	Peru	0.4%
Chile	0.4%	Philippines	0.8%
China	16.0%	Singapore	0.5%
Finland	0.3%	South Africa	0.9%
France	3.7%	Spain	2.2%
Germany	5.2%	Sweden	0.6%
India	7.2%	Switzerland	0.6%
Indonesia	2.9%	Thailand	1.3%
Italy	3.3%	Turkey	1.8%
Japan	7.4%	United Kingdom	3.6%
Korea	2.2%	United States	23.7%

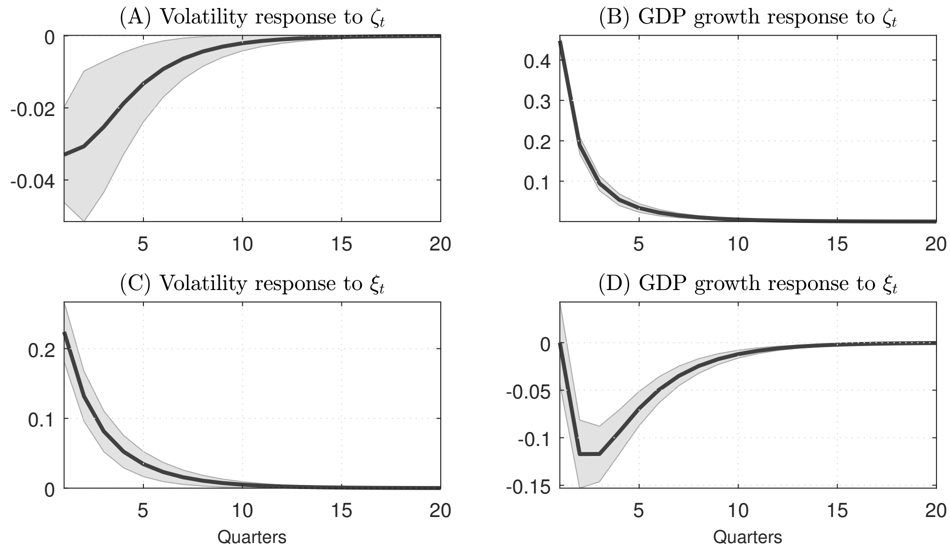
NOTE. PPP-GDP weights based on average PPP-GDP figures over the 1993:Q1-2016:Q4 period.

Figure S.8 Country-specific Correlations Between Volatility and Growth Innovations (PPP-GDP Weights)



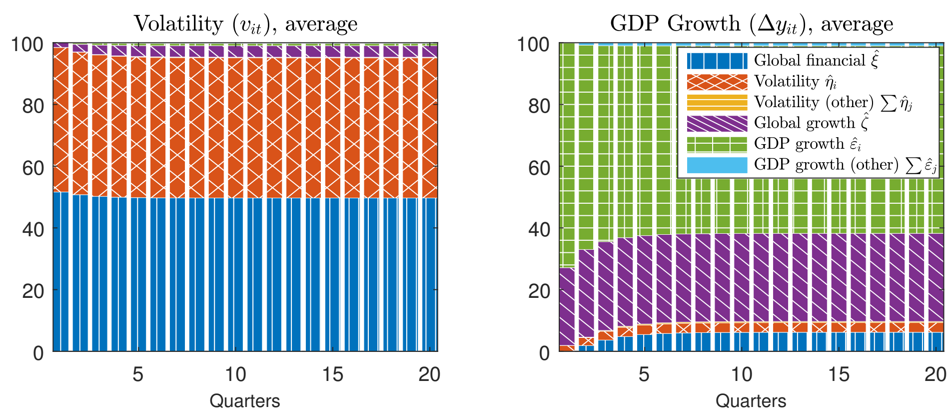
NOTE. Panel A displays the unconditional correlations between (log) realized stock market volatility and real GDP growth. Panel B plots the correlation between volatility and growth innovations when we condition only on $\hat{\zeta}_t$ in model (43)-(44). Panel C reports the same correlation when we condition on both $\hat{\zeta}_t$ and $\hat{\xi}_t$. The dots represent the contemporaneous correlations. The lines represent 95-percent confidence intervals. Sample period: 1993:Q1-2016:Q4.

Figure S.9 Average Country Volatility and Growth Responses to Real and Financial Factor Shocks (PPP-GDP Weights)



Note. Average impulse responses to one-standard deviation real and financial shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$. The solid lines are the PPP-GDP weighted averages of the country-specific responses. The shaded areas are two standard deviations confidence intervals. The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.10 Forecast Error Variance Decomposition of Country-specific Shocks (PPP-GDP Weights)

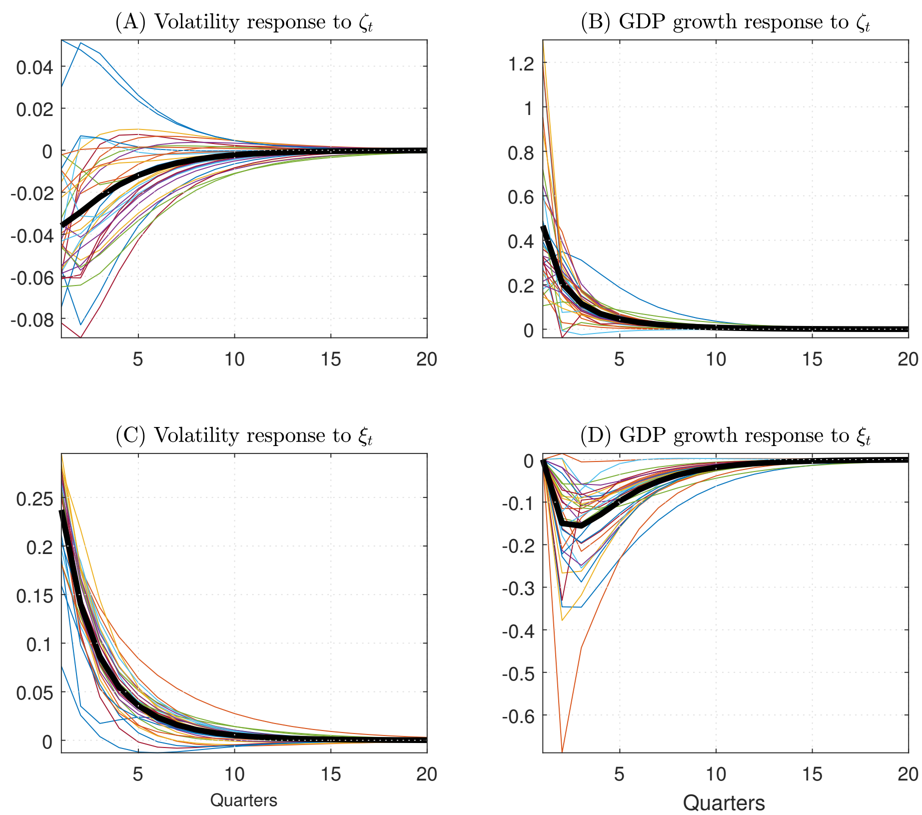


Note. Block-diagonal covariance matrix, with Cholesky decomposition of within-country covariance. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common growth shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas with no pattern). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

S.3 Country-specific Results

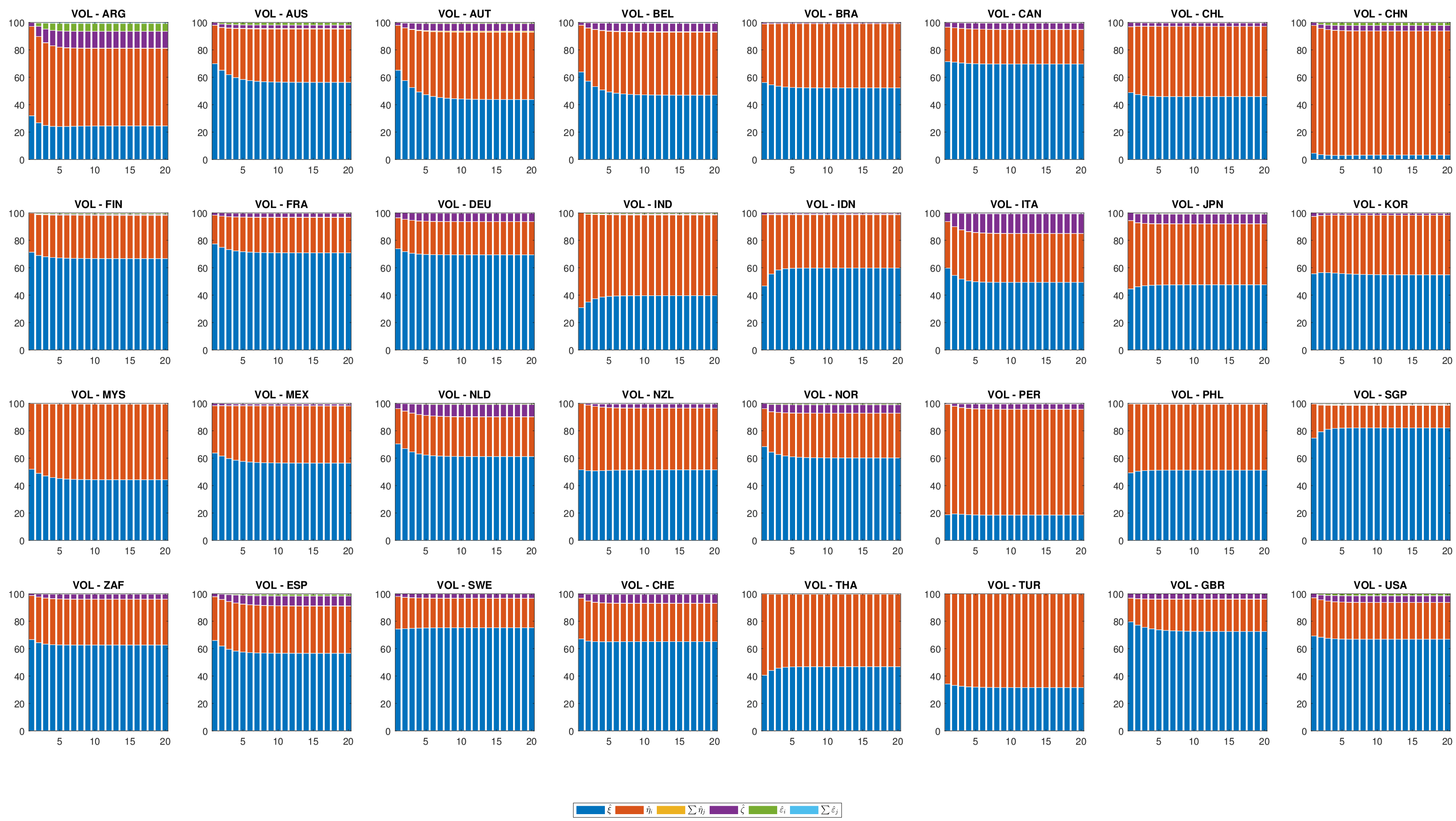
In this section we report selected country-specific results, namely the individual country impulse response functions and forecast error variance decompositions. Figure S.11 plots the country-specific impulse response of volatility and growth to a positive, one-standard-deviation shock to the global shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$. We can see from Figure S.11 that for most countries the impulse responses have a very similar profile. Figures S.12 to S.17 report forecast error variance decompositions for each country, for both volatility and growth, computed with different assumptions on the covariance matrix of the volatility and growth innovations. As can be seen the estimates are very similar across countries and for all the three schemes assumed for the error covariances.

Figure S.11 Country-specific Volatility and Growth Impulse Responses to Common Real and Financial Shocks



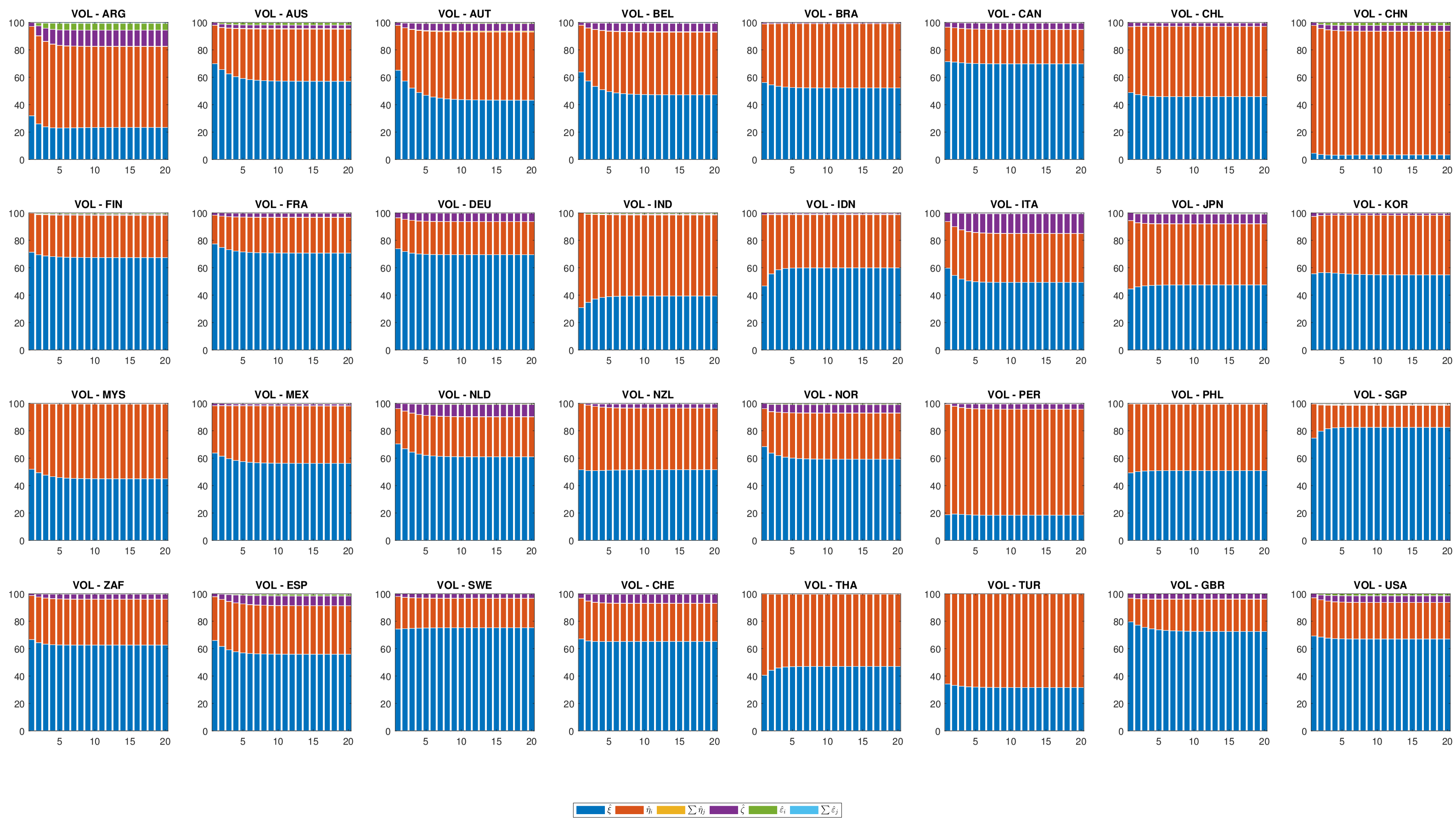
Note. One standard deviation shocks to $\hat{\zeta}_t$ and $\hat{\xi}_t$. Thin lines are individual country responses. The solid lines are the PPP-GDP weighted averages, as the ones reported in the main text. The horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.12 Forecast Error Variance Decomposition of Country-Specific Volatility Shocks - Diagonal Error Covariance Matrix



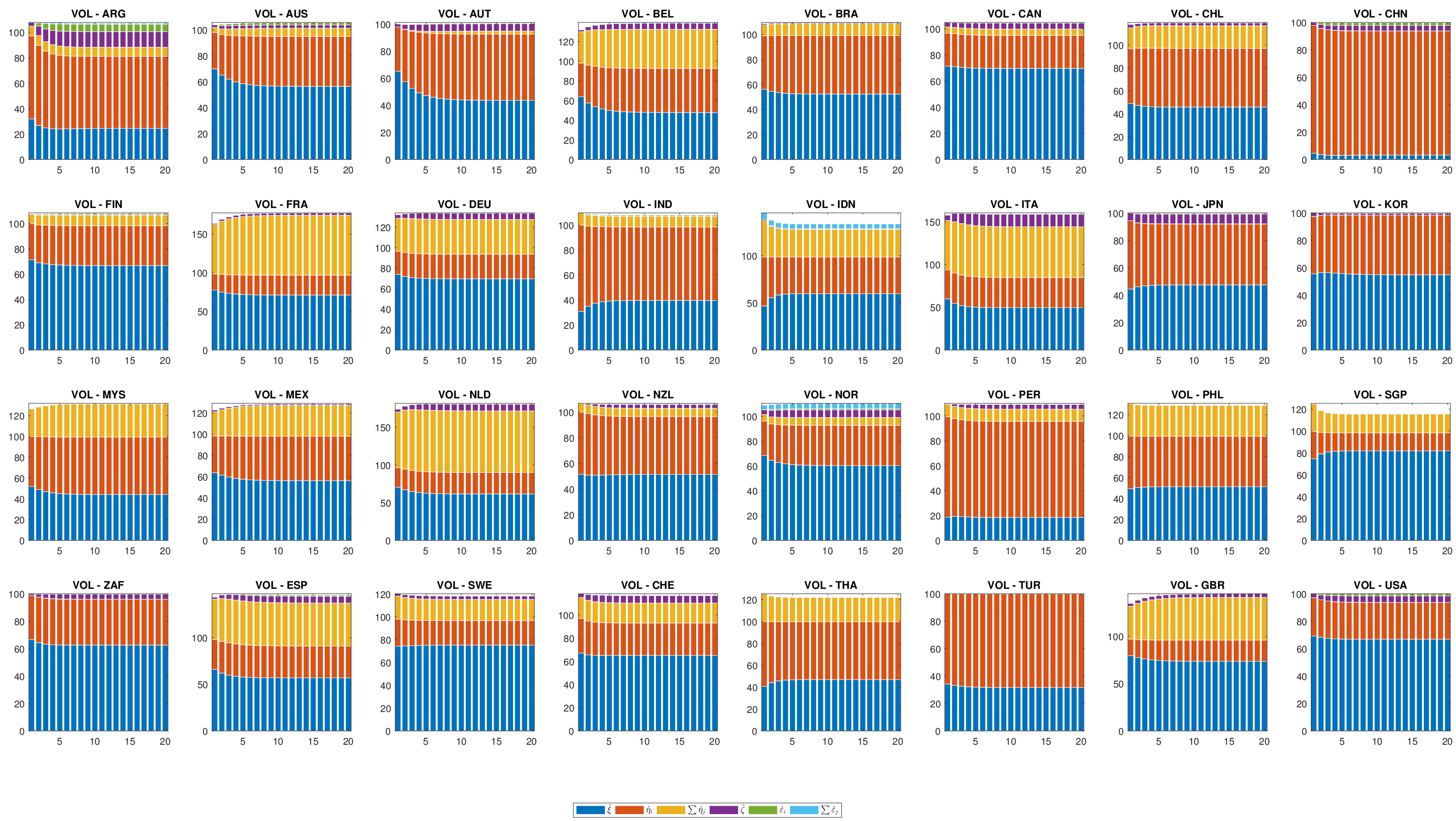
Note. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.13 Forecast Error Variance Decomposition of Country-Specific Volatility Shocks - Block Diagonal Error Covariance Matrix



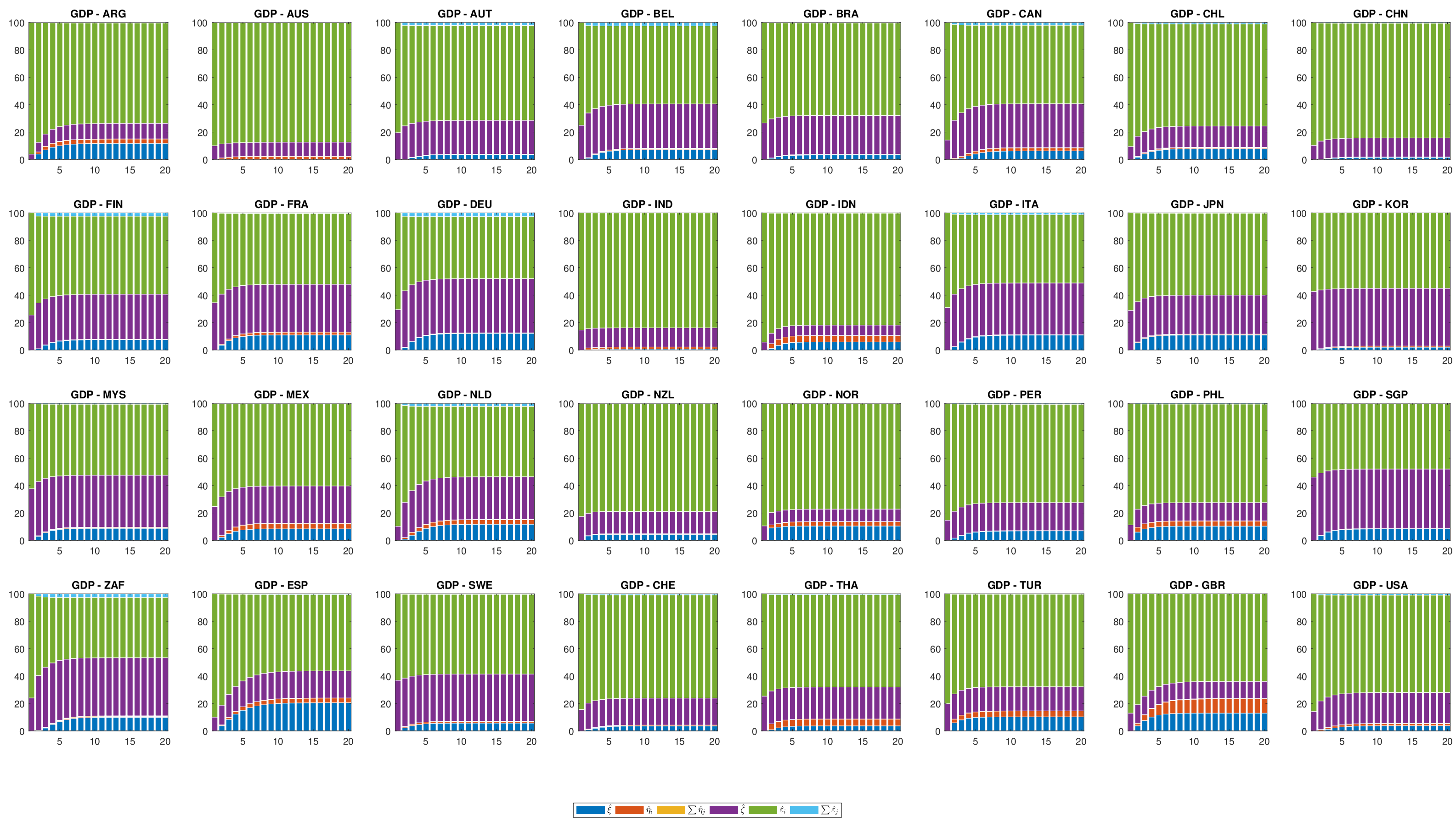
Note. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.14 Generalized Forecast Error Variance Decomposition of Country-Specific Volatility Shocks - Regularized Estimation of Full Error Covariance Matrix



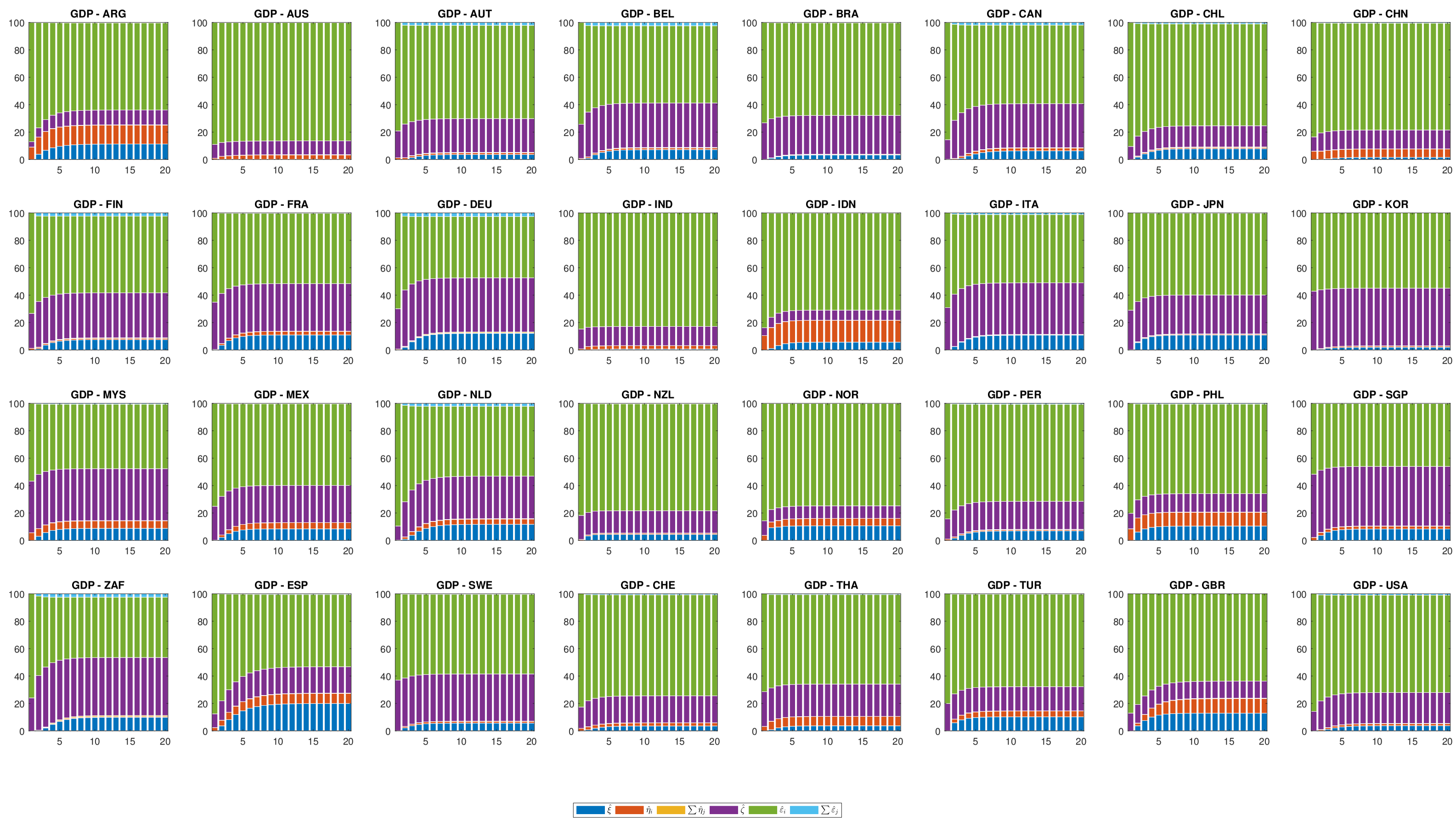
Note. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.15 Forecast Error Variance Decomposition of Country-Specific Growth Shocks - Diagonal Error Covariance Matrix



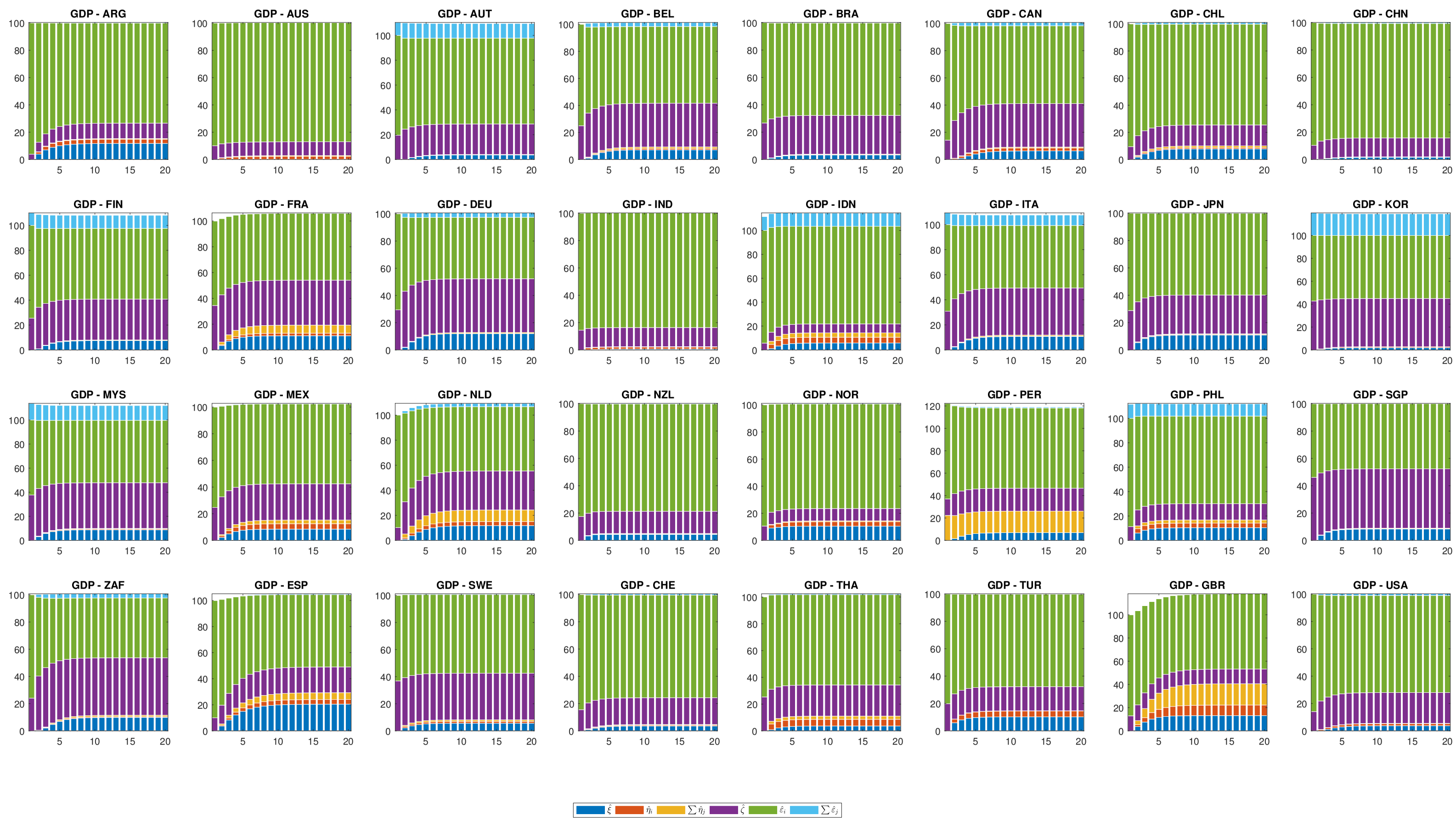
Note. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.16 FORECAST ERROR VARIANCE DECOMPOSITION OF COUNTRY-SPECIFIC GROWTH SHOCKS - BLOCK DIAGONAL ERROR COVARIANCE MATRIX



NOTE. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.17 Generalized Forecast Error Variance Decomposition of Country-Specific Growth Shocks - Regularized Estimation of Full Error Covariance Matrix



Note. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

S.4 Realized Volatility versus Cross-sectional Dispersion

As noted in the paper, if we consider a panel of country-specific equities (e.g. of firms or sectors within a country), a different measure of uncertainty can be computed as the cross-sectional dispersion of equity prices. In this section we show that this concept is closely related to the realized volatility measure we consider. To illustrate the point with the data that we use in our application, we derive results at the 'country-specific versus world level' rather than 'firm-specific versus country level'.[S6] Specifically, we compare the cross-sectional dispersion of equity returns across countries with the realized volatility of 'world' equity returns.

Define the daily cross-country dispersion of equity returns as:

$$\sigma_{cdt} = \sqrt{D_t^{-1} \sum_{\tau=1}^{D_t} \sum_{i=1}^{N} w_i \left[r_{it}(\tau) - \bar{r}_t(\tau)\right]^2}, \tag{S1}$$

and the daily realized volatility of world equity returns as:

$$\sigma_{rvt} = \sqrt{D_t^{-1} \sum_{i=1}^{N} \sum_{\tau=1}^{D_t} w_i \left[r_{it}(\tau) - \bar{r}_{it}\right]^2}, \tag{S2}$$

where $r_{it}(\tau) = \Delta \ln P_{it}(\tau)$ and $\bar{r}_{it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{it}(\tau)$ is the average daily price change over the quarter t, and D_t is the number of trading days in quarter t; and w_i is the weight attached to country i. To establish the relation between these two measures it is easier to work with their squares:

$$\sigma_{rvt}^2 = D_t^{-1} \sum_{i=1}^{N} \sum_{\tau=1}^{D_t} w_i \left[r_{it}(\tau) - \bar{r}_{it}\right]^2, \quad \sigma_{cdt}^2 = D_t^{-1} \sum_{\tau=1}^{D_t} \sum_{i=1}^{N} w_i \left[r_{it}(\tau) - \bar{r}_t(\tau)\right]^2.$$

Note also that

$$\sigma_{rvt}^2 = D_t^{-1} \sum_{i=1}^{N} \sum_{\tau=1}^{D_t} w_i r_{it}^2(\tau) - \sum_{i=1}^{N} w_i \bar{r}_{it}^2,$$

and

$$\sigma_{cdt}^2 = D_t^{-1} \sum_{\tau=1}^{D_t} \sum_{i=1}^{N} w_i r_{it}^2(\tau) - \sum_{i=1}^{N} w_i \left(D_t^{-1} \sum_{\tau=1}^{D_t} \bar{r}_t^2(\tau) \right).$$

Hence, since $\sum_{i=1}^{N} w_i = 1$, it follows that

$$\sigma_{cdt}^2 - \sigma_{rvt}^2 = \sum_{i=1}^{N} w_i \bar{r}_{it}^2 - D_t^{-1} \sum_{\tau=1}^{D_t} \bar{r}_t^2(\tau),$$

where as before $\bar{r}_{it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{it}(\tau)$, and $\bar{r}_t(\tau) = \sum_{i=1}^{N} w_i r_{it}(\tau)$.

Suppose now that daily returns have the following single-factor structure:[S7]

$$r_{it}(\tau) = \beta_i f_t(\tau) + \varepsilon_{it}(\tau),$$

[S6] Our analysis holds at the firm-specific versus country level as well.

[S7] The analysis readily extends to more general multiple factor settings.

where the factor is strong in the sense that (Bailey et al. (2016))

$$\lim_{N\to\infty}\sum_{i=1}^{N} w_i\beta_i = \bar{\beta} \neq 0, \text{ and } \lim_{N\to\infty}\sum_{i=1}^{N} w_i\beta_i^2 = \sigma_\beta^2 + \bar{\beta}^2 > 0.$$

The idiosyncratic components, $\varepsilon_{it}(\tau)$, are assumed to be independently distributed from $\beta_i f_t(\tau)$, cross-sectionally weakly correlated, and serially uncorrelated with zero means and finite variances. Also let:

$$\lim_{D_t\to\infty} D_t^{-1}\sum_{\tau=1}^{D_t} f_t^2(\tau) = h_{f_t}^2.$$

We now note that

$$\begin{aligned}
\sum_{i=1}^{N} w_i\bar{r}_{it}^2 &= \left(\sum_{i=1}^{N} w_i\beta_i^2\right)\bar{f}_t^2 + \left(\sum_{i=1}^{N} w_i\bar{\varepsilon}_{it}^2\right) + 2\left(\sum_{i=1}^{N} w_i\beta_i\bar{\varepsilon}_{it}\right)\bar{f}_t \\
&= \left(\sigma_\beta^2 + \bar{\beta}^2\right)\bar{f}_t^2 + O_p\left(D_t^{-1/2}\right) + O_p\left(N^{-1/2}\right),
\end{aligned}$$

where $\bar{f}_t = D_t^{-1}\sum_{\tau=1}^{D_t} f_t(\tau)$, and $\bar{\varepsilon}_{it} = D_t^{-1}\sum_{\tau=1}^{D_t}\varepsilon_{it}(\tau)$. Also

$$\begin{aligned}
D_t^{-1}\sum_{\tau=1}^{D_t}\bar{r}_t^2(\tau) &= D_t^{-1}\sum_{\tau=1}^{D_t}\left[\bar{\beta}f_t(\tau) + \bar{\varepsilon}_t(\tau)\right]^2 \\
&= \bar{\beta}^2\left[D_t^{-1}\sum_{\tau=1}^{D_t} f_t^2(\tau)\right] + D_t^{-1}\sum_{\tau=1}^{D_t}\bar{\varepsilon}_t^2(\tau) + 2D_t^{-1}\sum_{\tau=1}^{D_t}\bar{\beta}\bar{\varepsilon}_t(\tau)f_t(\tau) \\
&= \bar{\beta}^2 h_{f_t}^2 + O_p\left(N^{-1/2}\right) + O_p\left(D_t^{-1/2}\right).
\end{aligned}$$

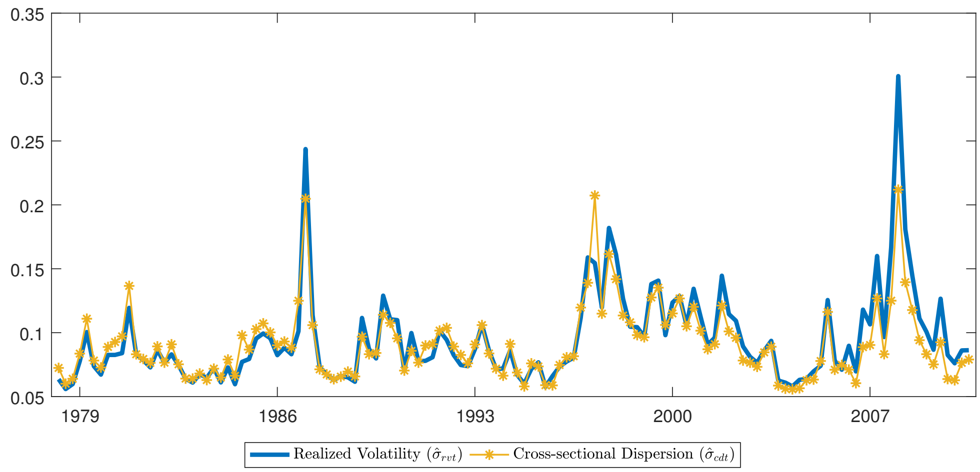
Hence

$$\begin{aligned}
\sigma_{cdt}^2 - \sigma_{rvt}^2 &= \left(\sigma_\beta^2 + \bar{\beta}^2\right)\bar{f}_t^2 - \bar{\beta}^2 h_{f_t}^2 + O_p\left(N^{-1/2}\right) + O_p\left(D_t^{-1/2}\right) \\
&= \sigma_\beta^2\bar{f}_t^2 - \bar{\beta}^2\sigma_{f_t}^2 + O_p\left(N^{-1/2}\right) + O_p\left(D_t^{-1/2}\right).
\end{aligned}$$

where $\sigma_{f_t}^2 = \left(h_{f_t}^2 - \bar{f}_t^2\right) \geq 0$, is the variance of the common factor. This expression shows that, under fairly general assumptions (and for N and D_t sufficiently large) we would expect the cross-sectional dispersion measure to be closely related to asset-specific measures of realized volatility when the factor loadings, β_i, are not too dispersed across countries. The results also show that the relative magnitudes of the cross section dispersion and realized volatility measures depend on the relative values of $\sigma_\beta^2\bar{f}_t^2$ and $\bar{\beta}^2\sigma_{f_t}^2$.

Figure S.18 compares world realized volatility (σ_{rvt}, light thick line) and cross-sectional dispersion (σ_{cdt}, dark thin line), computed as in equations (S2) and (S1), respectively, with equal weights. Their sample correlation over the 1979:Q1 to 2016:Q4 period is 0.92. Figure S.18 suggests that the two measures are very closely related, which is in line with the evidence provided by Bloom et al. (2012).

Figure S.18 Realized Volatility and Cross-sectional Dispersion



Note. World realized volatility of equity returns ($\hat{\sigma}_{rvt}$) and cross-sectional dispersion of equity returns across countries ($\hat{\sigma}_{cdt}$), computed as in equations (S2) and (S1), respectively. Both measures are expressed at quarterly rates and computed over the 1979:Q2-2016:Q4 period.

S.5 Computing Impulse Responses and Error Variance Decompositions

Consider the factor-augmented country-specific VAR models augmented with lagged cross section averages, $\bar{\mathbf{z}}_{\omega,t-\ell}$, for $\ell = 1, 2, ..., p$ as in equations (43)-(44) in the main text:

$$\mathbf{z}_{it} = \mathbf{\Phi}_i \mathbf{z}_{i,t-1} + \sum_{\ell=1}^{p} \mathbf{d}_{i\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + \boldsymbol{\beta}_i \boldsymbol{\delta}_t + \boldsymbol{\vartheta}_{it}, \text{ for } i = 1, 2, ..., N, \tag{S1}$$

where:

$$\mathbf{d}_{i\ell} = \begin{pmatrix} d_{1v,i\ell} & d_{2v,i\ell} \\ d_{1\Delta y,i\ell} & d_{2\Delta y,i\ell} \end{pmatrix}, \boldsymbol{\beta}_i = \begin{pmatrix} \beta_{i,11} & \beta_{i,12} \\ \beta_{i,21} & 0 \end{pmatrix}, \boldsymbol{\upsilon}_t = \begin{pmatrix} \zeta_t \\ \xi_t \end{pmatrix}.$$

Intercepts are omitted to simplify the exposition. Note also that $\bar{\mathbf{z}}_{\omega,t} = \sum_{i=1}^{N} w_i \Delta \mathbf{z}_{it} = \mathbf{W}\mathbf{z}_t$, where $\mathbf{z}_t = (\mathbf{z}'_{1t}, \mathbf{z}'_{2t}, ..., \mathbf{z}'_{Nt})'$, and $\mathbf{W}$ is a $2 \times 2N$ matrix of weights. Stacking the VARs in (S1) over i we obtain:

$$\mathbf{z}_t = \mathbf{\Phi}\mathbf{z}_{t-1} + \sum_{\ell=1}^{p} \mathbf{d}_\ell \mathbf{W}\mathbf{z}_{t-\ell} + \boldsymbol{\beta}\boldsymbol{\delta}_t + \boldsymbol{\vartheta}_t, \tag{S2}$$

where $\boldsymbol{\vartheta}_t = (\boldsymbol{\vartheta}'_{1t}, \boldsymbol{\vartheta}'_{2t}, ..., \boldsymbol{\vartheta}'_{Nt})'$ and:

$$\mathbf{\Phi} = \begin{pmatrix} \mathbf{\Phi}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Phi}_N \end{pmatrix}, \quad \mathbf{d}_\ell = \begin{pmatrix} \mathbf{d}_{1,\ell} \\ \mathbf{d}_{2,\ell} \\ \vdots \\ \mathbf{d}_{N,\ell} \end{pmatrix}, \quad \boldsymbol{\beta} = \begin{pmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \vdots \\ \boldsymbol{\beta}_N \end{pmatrix}.$$

The high-dimensional VAR in (S2) can now be written as a standard FAVAR(p) model in $2N$ variables:

$$\mathbf{z}_t = (\mathbf{\Phi} + \mathbf{d}_1\mathbf{W})\,\mathbf{z}_{t-1} + \sum_{\ell=2}^{p} \mathbf{d}_\ell \mathbf{W}\mathbf{z}_{t-\ell} + \boldsymbol{\beta}\boldsymbol{\delta}_t + \boldsymbol{\vartheta}_t. \tag{S3}$$

For example, when $p = 1$ we have the FAVAR(1):

$$\mathbf{z}_t = (\mathbf{I}_{2N} - \boldsymbol{\Psi}_1 \mathbf{L})^{-1}(\boldsymbol{\beta}\boldsymbol{\delta}_t + \boldsymbol{\vartheta}_t),$$

where $\boldsymbol{\Psi}_1 = \boldsymbol{\Phi} + \mathbf{d}_1 \mathbf{W}$ and

$$\mathbf{z}_t = (\mathbf{I}_{2N} - \boldsymbol{\Psi}_1 \mathbf{L})^{-1}\boldsymbol{\beta}\boldsymbol{\delta}_t + (\mathbf{I} - \boldsymbol{\Psi}_1 \mathbf{L})^{-1}\boldsymbol{\vartheta}_t.$$

Note that by construction $\boldsymbol{\delta}_t$ and $\boldsymbol{\vartheta}_t$ are orthogonal, and for sufficiently large p, they are serially uncorrelated. Hence, the impulse response of shocks to elements of $\boldsymbol{\delta}_t$ and $\boldsymbol{\vartheta}_t$ can be computed using the following moving average representation:

$$\mathbf{z}_t = \sum_{n=0}^{\infty} \mathbf{A}_n \boldsymbol{\delta}_{t-n} + \sum_{n=0}^{\infty} \mathbf{C}_n \boldsymbol{\vartheta}_{t-n}, \tag{S4}$$

where $\mathbf{A}_n = \boldsymbol{\Psi}_1^n \boldsymbol{\beta}$, and $\mathbf{C}_n = \boldsymbol{\Psi}_1^n$, for $n = 0, 1, 2, ...$

S.5.1 Responses to Common and Country-specific Shocks

Let $\mathfrak{e}_i$ be a selection vector such that $\mathfrak{e}_i' \mathbf{z}_t$ picks the i^{th} element of $\mathbf{z}_t$. Also let $\mathbf{s}_f = (1, 0)'$ and $\mathbf{s}_g = (0, 1)'$, the vectors that select ζ_t and ξ_t from $\boldsymbol{\delta}_t$, namely:

$$\mathbf{s}_f' \boldsymbol{\delta}_t \equiv \zeta_t, \quad \mathbf{s}_g' \boldsymbol{\delta}_t \equiv \xi_t. \tag{S5}$$

Recall now that ζ_t and ξ_t have zero means, unit variances and are orthogonal to each other. Then the impulse responses to a positive unit shock to ζ_t or ξ_t are given by:

$$IR_{i,\zeta,n} = \mathfrak{e}_i' \mathbf{A}_n \mathbf{s}_f \quad \text{and} \quad IR_{i,\xi,n} = \mathfrak{e}_i' \mathbf{A}_n \mathbf{s}_g \quad \text{for } n = 0, 1, 2, ..., \tag{S6}$$

where $\mathbf{A}_n$ is given by the moving average representation, (S4)

To derive impulse response functions for country-specific shocks, namely the j^{th} element of $\boldsymbol{\vartheta}_t$, we need to make assumptions about the correlation between volatility and growth innovations within each country and across countries. Since the elements of $\boldsymbol{\vartheta}_t$ are weakly correlated across countries, they have some, but limited correlations across countries (see Figure 8). We also documented that, conditional on the common factors ζ_t and ξ_t, the country-specific correlation of volatility and growth innovations are statistically insignificant for all except for four countries.

As a first order approximation, therefore, we will assume that the covariance matrix of $\boldsymbol{\vartheta}_t$ in (S3) is diagonal. Under this assumption, the impulse response function of a positive, unit shock to the j^{th} element of $\boldsymbol{\vartheta}_t$ on the the i^{th} element of $\mathbf{z}_t$ is given by:

$$IR_{i,\vartheta_j,n} = \sqrt{\hat{\omega}_{jj}} \mathfrak{e}_i' \mathbf{C}_n \mathfrak{e}_j \ , \tag{S7}$$

where $\mathbf{C}_n$ is given by the moving average representation, (S4), $\hat{\omega}_{jj}$ is the (estimate) of the variance of the j^{th} country-specific shock and $\mathfrak{e}_j$ is a selection vector such that $\mathfrak{e}_j' \mathbf{z}_t$ picks the j^{th} element of $\mathbf{z}_t$.

The above impulse responses can be compared to the generalized impulse responses of Pesaran and Shin (1998). The latter are given by:

$$GIR_{i,\vartheta_j,n} = \frac{\mathfrak{e}_i' \mathbf{C}_n \hat{\Omega} \mathfrak{e}_j}{\sqrt{\hat{\omega}_{jj}}}, \tag{S8}$$

where $\hat{\mathbf{\Omega}} = (\hat{\omega}_{ij})$ is the estimate of the covariance of $\boldsymbol{\vartheta}_t$. The generalized impulse responses allow for non-zero correlations across the idiosyncratic errors. The two sets of impulse responses coincide if the covariance matrix of $\boldsymbol{\vartheta}_t$ is diagonal.

S.5.2 Forecast Error Variance Decompositions

Traditionally, the forecast error variance decomposition of a VAR model is performed on a set of orthogonalized shocks, whereby the contribution of the j^{th} orthogonalized innovation to the mean square error of the n-step ahead forecast of the model is calculated. In our empirical application this is not the case as —even if the country-specific volatility and growth innovations η_{it} and ε_{it} are weakly correlated across countries— some pairs of innovations can still display some non-zero correlation. An alternative approach is to compute Generalized Forecast Error Variance Decompositions (GVD) of Pesaran and Shin (1998). The Generalized Forecast Error Variance Decompositions consider the proportion of the variance of the n-step forecast errors of the endogenous variables that is explained by conditioning on the non-orthogonalized shocks, while explicitly allowing for the contemporaneous correlations between these shocks and the shocks to the other equations in the system.

Let $GVD_{i,\zeta,n}$ and $GVD_{i,\xi,n}$ be the share of the n-step ahead forecast error variance of the i^{th} variable in $\mathbf{z}_t$ that is accounted for by ζ_t and ξ_t, respectively, and $GVD_{i,j}$ the variance share of a generic country-specific shock, then:

$$GVD_{i,\zeta,n} = \frac{\sum_{\ell=0}^{n} \left(\mathfrak{e}_i' \mathbf{A}_\ell \mathbf{s}_f\right)^2}{\sum_{\ell=0}^{n} \mathfrak{e}_i' \mathbf{A}_\ell \mathbf{A}_\ell' \mathfrak{e}_i + \sum_{\ell=0}^{n} \mathfrak{e}_i' \mathbf{C}_\ell \hat{\mathbf{\Omega}} \mathbf{C}_\ell' \mathfrak{e}_i}, \quad n = 1, 2, ..., H, \tag{S9}$$

$$GVD_{i,\xi,n} = \frac{\sum_{\ell=0}^{n} \left(\mathfrak{e}_i' \mathbf{A}_\ell \mathbf{s}_g\right)^2}{\sum_{\ell=0}^{n} \mathfrak{e}_i' \mathbf{A}_\ell \mathbf{A}_\ell' \mathfrak{e}_i + \sum_{\ell=0}^{n} \mathfrak{e}_i' \mathbf{C}_\ell \hat{\mathbf{\Omega}} \mathbf{C}_\ell' \mathfrak{e}_i}, \quad n = 1, 2, ..., H, \tag{S10}$$

$$GVD_{i,j,n} = \frac{\hat{\omega}_{jj}^{-1} \sum_{\ell=0}^{n} \left(\mathfrak{e}_i' \mathbf{C}_\ell \hat{\mathbf{\Omega}} \mathfrak{e}_j\right)^2}{\sum_{\ell=0}^{n} \mathfrak{e}_i' \mathbf{A}_\ell \mathbf{A}_\ell' \mathfrak{e}_i + \sum_{\ell=0}^{n} \mathfrak{e}_i' \mathbf{C}_\ell \hat{\mathbf{\Omega}} \mathbf{C}_\ell' \mathfrak{e}_i}, \quad j = 1, 2, ..., 2N, \quad n = 1, 2, ..., H; \tag{S11}$$

Note that the different assumptions we make on the covariance matrix of all country-specific shocks, $\hat{\mathbf{\Omega}}$, have implications for the error variance decompositions. Specifically, when we assume that (i) $\hat{\mathbf{\Omega}}$ is diagonal or (ii) $\hat{\mathbf{\Omega}}$ is block-diagonal with Cholesky-orthogonalized blocks, the relative importance of shocks to country-specific volatility and growth for all countries (η_{it} and ε_{it}, for $j = 1, 2, ..., 2N$) and shocks to the two common factors ζ_t and ξ_t, is easily characterized as $VD_{i,\zeta,n} + VD_{i,\xi,n} + \sum_{j=1}^{2N} VD_{i,j,n} = 1$. That is the GVD formula coincides with the standard VD formula. In contrast, when we consider an unrestricted covariance matrix $\hat{\mathbf{\Omega}}$, the sum of the variance shares does not necessarily add up to 1.

S.5.3 Average Impulse Responses and Forecast Error Variance Decompositions

As a summary measure of the effects of shocks to the common factors we report the following average measures. Denote the impulse response (or forecast error variance decomposition) of a particular shock on the j^{th} variable in country i at horizon n by $X_{i,j,n}$. Let $\mathbf{w} = (w_1, w_2, ..., w_N)'$ be a vector of fixed weights such that $\Sigma_{i=1}^{N} w_i = 1$. Then the average impulse response (or forecast error variance decomposition) of the shock to variable j, at horizon n, is computed as:

$$X_{\omega,j,n} = \sum_{i=1}^{N} w_i X_{i,j,n}. \tag{S12}$$

and its dispersion is computed by:

$$\sigma_{X_{\omega,j,n}} = \left[\sum_{i=1}^{N} w_i^2 \left(X_{i,j,n} - X_{\omega,j,n} \right)^2 \right]^{1/2}, \tag{S13}$$

assuming country-specific impulse responses or forecast error variance decompositions are approximately uncorrelated.

Supplement References

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