Understanding the Sources of Macroeconomic Uncertainty

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January 2, 2020

Abstract

We propose a decomposition to distinguish among several notions of uncertainty such as uncertainty about the probability distribution generating the data (broadly speaking, model-misspecification) and uncertainty about the odds of the outcomes when the probability distribution is known (risk). We obtain various measures for these notions of uncertainty: some based on past data (referred to as ex-post measures), while others are forward-looking in nature (referred to as ex-ante measures). We use the US Survey of Professional Forecaster's (SPF) density forecasts to quantify overall uncertainty as well as the evolution of the different components of uncertainty over time and investigate their importance for macroeconomic fluctuations.

Keywords: Uncertainty, Risk, Survey of Professional Forecasters, Predictive Densities.
J.E.L. Codes: C22, C52, C53.[1]

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[1] **Acknowledgements:** We are grateful to T. Clark, D. Giannone and to seminar participants at the Fourth

1 Introduction

The recent financial crisis has renewed interest in measuring uncertainty and studying its macroeconomic effects. Stock and Watson (2012) suggest that liquidity-risk and uncertainty shocks are among the most important factors explaining the decline in U.S. GDP during the Great Recession, accounting for about two thirds of the GDP decline. Given that uncertainty is inherently unobserved, this has sparked a wide research agenda on various measures of uncertainty. The empirical literature has proposed several measures of uncertainty, but does not explain how they relate to each other. However, as shown in Rossi and Sekhposyan (2015), the macroeconomic impact of the various uncertainty measures can be very different from each other – though it is certaintly the case that increased uncertainty, in general, has recessionary effects across the measures used in the literature, these effects can be dramatically different in size and persistence. This observation naturally leads to the question of what exactly the uncertainty indices measure and how they differ from each other.

This paper attempts to study uncertainty in a unified framework. To do so, we introduce a new measure of uncertainty that is based on forecast densities. We focus on forecast densities for output growth to construct a measure of uncertainty that reflects business cycle uncertainty, as business cycles can be proxied by real output growth (Stock and Watson, 1999).[2] Our new measure of uncertainty enables us to make two main contributions to the literature:

(i) The first main contribution is that we use our new measure of uncertainty to distinguish between situations in which agents can accurately predict the odds of the events and situations in which they cannot.[3] The use of forecast densities is key for achieving this goal since it enables quantification of uncertainty pertaining to situations where the odds of the outcomes are known, yet inaccurate.

(ii) The second main contribution is that we provide a decomposition of our uncertainty measure into several components that are related to the uncertainty measures used in the literature. This analysis sheds light on why the various measures of uncertainty differ from each other, and which one is more appropriate to use depending on the goals of the researcher. Again, the use of forecast densities is key to provide a comprehensive decomposition of uncertainty into its sources.

International Symposium in Computational Economics and Finance, the 1st Banque de France – Norges Bank Workshop in Empirical Macroeconomics, the 24th Annual Symposium of the Society for Nonlinear Dynamics and Econometrics, the 9th ECB Workshop on Forecasting Techniques, the IAAE 2016 Annual Conference, the 2016 CEF Conference, the Chicago Fed, the 2017 ASSA Meetings, UCL, York, and Henan Universities for comments. Barbara Rossi gratefully acknowledges financial support from the European Research Council (ERC) grant agreement No 615608.

[2]We also plot an index of inflation uncertainty to show another empirical example of our methodology.

[3]One could interpret these two as risk and Knightian uncertainty, respectively, where Knightian uncertainty is defined as the agents' inability to correctly characterize probability distributions.

In particular, we distinguish between disagreement and aggregate uncertainty. In this respect our contribution is similar to that of Lahiri and Sheng (2010), who consider the relationship between aggregate uncertainty and disagreement over the business cycle, yet measure it in terms of uncertainty and disagreement about the mean of the distribution, as opposed to the whole distribution. Our approach further enables us to distinguish between measures of realized volatility, ex-ante uncertainty (measured by volatility embedded in the survey forecasts) and bias. These various components have all been used in the literature as measures of uncertainty. Our approach, on the other hand, enables us to distinguish among them and understand their relationship with each other.

Several of the components mentioned above have been of interest on their own. For example, Patton and Timmermann (2010) study disagreement among professional forecasters, but do not relate disagreement to measures of uncertainty, while Lahiri and Sheng (2010) consider the relationship of aggregate uncertainty and disagreement over the business cycle, yet they do not distinguish between risk and uncertainty. Jo and Sekkel (2017) and Jurado, Ludvigson and Ng (2015) use the forecast-error-variance-based measures of uncertainty, while D'Amico and Orphanides (2014) consider ex-ante measures of risk for inflation forecasting.

In addition to our main contributions, we also study how uncertainty and its sources resolve over time as the agents get closer in time to the event. For example, Patton and Timmermann (2010) study the resolution of disagreement over time; disagreement is only one of the components of uncertainty: we investigate both how important disagreement is as a source of overall uncertainty over time, as well as how the other components of uncertainty resolve over time. Furthermore, we document the macroeconomic impact and transmission of the various sources of uncertainty that we identify.

Lastly, one of the components of our decomposition measures how accurately the agents can predict the odds of future events. Agents may not be able to accurately predict the odds of the events for various reasons, among which: the possibility that they are not rational; the possibility that they have different information sets or different models; or the possibility that there is Knightian uncertainty.

It is important to note that the existing literature has focused mainly on quantifying and understanding uncertainty associated with point forecasts, for example by mapping uncertainty to forecasters' prediction errors. Though the individual point forecasts are on average consistent with the weighted mean of their predictive probability distributions (see Lambros and Zarnowitz, 1987), predictive distributions undoubtedly contain more information. Our goal is to take advantage of the richer information content of probabilistic forecasts to distinguish among various sources of uncertainty. Thus, an important difference between this paper and the existing literature is that we use the probabilistic forecasts provided by the U.S. Survey of Professional Forecasters (SPF) to measure and decompose uncertainty.[4] We focus mainly on output growth forecasts: since output growth is indicative of business cycle fluctuations, our analysis provides an overall measure of macroeconomic uncertainty; in addition, we also discuss inflation uncertainty measures that might help understand why monetary policy affects short and long term interest rates differently (Wright, 2011).

The paper is structured as follows. The next two sections present our density-forecast-based uncertainty measures and the decompositions we investigate in this paper. Section 4 discusses the SPF data used for the empirical implementation, while Section 5 presents the empirical results. In Section 6 we analyze the macroeconomic impact of the various sources of uncertainty. In Section 7 we extend our results to the analysis of inflation uncertainty, while Section 8 concludes.

2 An Uncertainty Index Based on Density Forecasts

The uncertainty index we propose in this paper measures the distance, on average across forecasters, between the forecast distribution provided by an individual forecaster and the perfect forecast corresponding to the realization, where both are represented by cumulative distribution functions (CDFs). We denote by x_{t+h} a random variable equal to one when the actual realization y_{t+h} is below some threshold r, and it is zero otherwise: $x_{t+h}(r) \equiv 1(y_{t+h} < r)$.[5] Note that $x_{t+h}(r)$ is defined over the support r, $r \in \mathbb{R}$; by varying r, we can focus on different parts of the predictive distribution. Let $P_{s,t+h|t}(r)$ be the $s-th$ forecaster's predictive distribution that the event $x_{t+h}(r)$ equals one, i.e. $P_{s,t+h|t}(r) = P(x_{t+h}(r) = 1|\Omega_{s,t})$, where $s = 1, ..., N$ and $\Omega_{s,t}$ is the information set available to forecaster s at time t. Thus, $P(x_{t+h}(r) = 1|\Omega_{s,t}) = P(y_{t+h} < r|\Omega_{s,t})$ is a CDF. We measure the s-th forecaster's uncertainty as the Mean Squared Forecast Error (MSFE) of his/her probabilistic forecast about a particular outcome, i.e.:[6]

$$u_{s,t+h|t}(r) = E_Q\left(x_{t+h}(r) - P_{s,t+h|t}(r)\right)^2, \quad (1)$$

where Q_{t+h} is the true cumulative density and E_Q denotes the expectations with respect to true probability density (dQ_{t+h})[7] Note that $P_{s,t+h|t}$ is potentially different from Q_{t+h}. The latter is the true probability distribution while the former is the forecaster's probability distribution.

[4] Our analysis can be done with any predictive density. We choose to use predictive densities from the SPF since they are produced by professional forecasters monitoring a wider range of indicators rather than a specific parametric model. Furthermore, the SPF is known for its superior forecasting performance from a point forecasting point of view, as shown in Giannone, Reichlin and Small (2008) and McCracken, Owyang and Sekhposyan (2015), among others. Ganics, Rossi and Sekhposyan (2018) further investigate the informativeness of the SPF density forecasts (used in a similar manner as in this paper), and find that it is competitive relative to a wide range of popular alternatives.

[5] This notation is consistent with Hersbach (2000).

[6] In the forecasting literature, this MSFE is known as the Brier score.

[7] To simplify notation, we assume in this section that N is fixed over time, although in the empirical application we let N vary.

Note that when the forecaster's distribution is the same as the true distribution, i.e. when there is no misspecification, then our proposed measure of uncertainty simplifies to a variance measure typically used to characterize risk. More specifically, when $P_{s,t+h|t}(r) = Q_{t+h}$,then

$$\mathbf{u}_{s,t+h|t}(r) = \sum_{x=0,1} \left(x_{t+h}(r) - P_{s,t+h|t}(r)\right)^2 \mathbf{Q}_{t+h}(r) \tag{2}$$

$$= \left(1 - P_{s,t+h|t}(r)\right)^2 P_{s,t+h|t}(r) + P_{s,t+h|t}(r)^2 \left(1 - P_{s,t+h|t}(r)\right) \tag{3}$$

$$= \left(1 - P_{s,t+h|t}(r)\right) P_{s,t+h|t}(r) \tag{4}$$

Similarly to Jo and Sekkel (2017), Jurado, Ludvigson and Ng's (2015) measure, eq. (1) is a MSFE; however, it is a MSFE applied to a forecast distribution for a given binary event. As such, it measures the unpredictable component associated with each possible value in the domain of the predictive distribution. In fact, $u_{s,t+h|t}(r)$ compares the probability that forecaster s assigns to the different states of nature with the realization, while error-based measures à la Jurado, Ludvigson and Ng (2015) only compare the point forecast with the realization.[8] Note that our measure of uncertainty is based on predictive densities (instead of point forecasts) for two reasons. The first is that our uncertainty measure attempts to capture distributional misspecification, which requires to distinguish situations where agents can accurately predict the odds from situations where they cannot. It would be thus impossible to attempt to capture such measure using point forecasts. The second reason is that uncertainty related to quantiles other than the mean and the variance of the distribution might be important, as pointed out in the recent literature on rare disasters, which has been shown to resolve certain puzzles in the asset market literature (e.g. Barro, 2006 and Farhi and Gabaix, 2016, among others) and have a close association with overall uncertainty (Orlik and Veldkamp, 2015).

The overall measure of uncertainty is then defined as the average of the individual uncertainty across forecasters:

$$u_{t+h|t}(r) = \frac{1}{N} \sum_{s=1}^{N} E_Q \left(x_{t+h}(r) - P_{s,t+h|t}(r)\right)^2 .$$

As mentioned above, by varying r we can explore measures of uncertainty in different parts of the predictive density. We focus on an overall measure of uncertainty (which we label "Uncertainty") that integrates the squared probability forecast errors over the whole domain of the distribution,

[8] In fact, if one associates the value $r \in \mathbb{R}$ with the corresponding quantile of the distribution, our uncertainty index measures an average squared error for that quantile.

that is:[9]

$$U_{t+h|t} = \int_{-\infty}^{+\infty} u_{t+h|t}\,(r)\,dr. \tag{5}$$

A graphical interpretation is provided in Figure 1. In the figure, the actual realization equals -2, denoted by a vertical bar on the left panel; the predictive density is the Normal distribution. The panel on the right shows the CDF of the Normal distribution, as well as that of the perfect forecast, evaluated at the realization $y_{t+h} = -2$. Thus, the ideal CDF equals zero for $r < y_{t+h}(= -2)$ and one otherwise. At the realization, the distance between the CDFs of the perfect forecast and the forecasted distribution, $x_{t+h}(r) - P_{t+h|t}(r)$, is depicted by a hollow vertical bar. Our measure of uncertainty in eq. (5) squares this measure and integrates it over the various values of r (and averages it over time).

Note that, even if an individual is certain about a future event, i.e. he/she puts very high probability on a particular future outcome, our definition (eq. 1) implies that there is uncertainty if that event does not happen. Our definition of uncertainty in eq. 1 will capture that, and it will, on average, be associated with situations where "individuals cannot estimate probabilities reliably."[10] Thus, we distinguish between risk, where probabilities can be accurately assigned, and misspecification, where the information is too imprecise to be adequately summarized by the probabilities. In our context, if the forecaster's predictive distribution is $1(y^* < r)$ (and so the forecaster is certain that y^* will happen) and the true predictive distribution chosen by nature is $1(y^{**} < r)$, then there is misspecification, which will affect our overall uncertainty measure since it is one of its components.

Zarnowitz and Lambros (1987) also emphasize how uncertainty differs from disagreement. In our framework, the diffuseness of a forecaster's predictive density is not the only uncertainty in the economy: that would be true if the forecasters' predictive densities were accurately describing the odds, i.e. when $P_{s,t+h|t}\,(r)$ is the same as $Q_{t+h}\,(r)$. When they are not, that is when the forecasters' predictive densities are different from the nature's predictive density, giving rise to uncertainty associated with distributional misspecification. In other words, this additional source of uncertainty cannot be something that the forecasters are aware of: it is uncertainty about events that the forecasters could not have predicted by their probability distributions, so it will not show up in the diffuseness of forecasters' predictive densities.

[9] Our measure of uncertainty derives from the Continuous Rank Probability Score (CRPS), which is widely used to assess the quality of forecast distributions in statistics. In fact, the CRPS is the integral of Brier scores (Hersbach, 2000, eq. 7). In this particular case it could be viewed as an average CRPS across the forecasters. Note that eq. (5) is the negative of the CRPS, as defined in Gneiting and Raftery (2007). Note the similarity with the stochastic loss distance in Diebold and Shin (2015), which is equivalent to an expected loss. Galvao and Mitchell (2019), instead, use the difference between ex-ante and ex-post CRPS as a measure of Knightian uncertainty.

[10] Note that this definition of uncertainty is reminiscent of what kaynes calls Knightian uncertainty, cfr Kaynes (1921, chp. 6).

INSERT FIGURE 1

3 The Sources of Uncertainty

This section presents our main decompositions of uncertainty into its sources.

3.1 Aggregate Uncertainty and Disagreement

One of the goals of this paper is to link existing measures of uncertainty based on aggregate data with uncertainty measures based on disagreement among forecasters. To do so, we define an aggregate cumulative probability density ($\left\{P_{t+h|t}\left(r\right)\right\}_{r\in\mathbb{R}}$), which is related to the individual ones ($\left\{P_{s,t+h|t}\left(r\right)\right\}_{r\in\mathbb{R}}$) by:

$$P_{t+h|t}\left(r\right)=\frac{1}{N}\sum_{s=1}^{N}P_{s,t+h|t}\left(r\right). \tag{6}$$

The corresponding uncertainty measure for the aggregate predictive density is:

$$u_{t+h}^{A}\left(r\right)\equiv\int\left(x_{t+h}\left(r\right)-P_{t+h|t}\left(r\right)\right)^{2}dQ_{t+h}.$$

Appendix A shows that we can decompose the overall uncertainty measure as follows:

$$u_{t+h|t}\left(r\right)=\int\left(x_{t+h}\left(r\right)-P_{t+h|t}\left(r\right)\right)^{2}dQ_{t+h}+\int\frac{1}{N}\sum_{s=1}^{N}\left(P_{t+h|t}\left(r\right)-P_{s,t+h|t}\left(r\right)\right)^{2}dQ_{t+h} \tag{7}$$

$$=u_{t+h|t}^{A}\left(r\right)+d_{t+h|t}\left(r\right), \tag{8}$$

where $d_{t+h|t}\left(r\right)\equiv\frac{1}{N}\sum_{s=1}^{N}\int\left(P_{t+h|t}\left(r\right)-P_{s,t+h|t}\left(r\right)\right)^{2}dQ_{t+h}$ measures the disagreement between individual forecast densities and the aggregate forecast density, and it is similar to the disagreement defined in Patton and Timmermann (2010) for point forecasts. Lahiri and Sheng (2010, eq. 18) discuss a similar decomposition for point forecasts.

Note that the decomposition in eq. (8) holds for a particular threshold r, thus it accounts for a forecast error associated with the binary outcome $1\left(y_{t+h}<r\right)$. The overall measure of uncertainty accounts for uncertainty at all possible values of r by considering the integral of the decomposition in eq. (8) over r. Thus, we have "Uncertainty" decomposed into "Aggregate Uncertainty" and

"Disagreement":[11]

$$
\begin{aligned}
U_{t+h|t} &\equiv \int_{-\infty}^{\infty} u_{t+h|t}(r)\,dr = \int_{-\infty}^{\infty} u^A_{t+h|t}(r)\,dr + \int_{-\infty}^{\infty} d_{t+h|t}(r)\,dr \\
&\equiv \underbrace{U^A_{t+h|t}}_{\text{"Aggregate Uncertainty"}} + \underbrace{D_{t+h|t}}_{\text{"Disagreement"}}
\end{aligned}
\tag{9}
$$

3.2 Uncertainty and Risk

As shown in Appendix A, we can further decompose the aggregate uncertainty, $U^A_{t+h|t}(r)$ into components that measure mean bias, dispersion of probability forecasts, realized risk and a covariance term between the forecasted and the ideal distribution as follows:

$$
\begin{aligned}
u^A_{t+h}(r) = u^A_{t+h}(r) &= V\left(x_{t+h}(r)\right) + Var(p_{t+h|t}(r)) \\
&+ \left[E\left(p_{t+h|t}(r)\right) - E\left(x_{t+h}(r)\right)\right]^2 - 2Cov(x_{t+h}(r)\,P_{t+h|t}(r)),
\end{aligned}
\tag{10}
$$

where $E(.)$, $V(.)$ and $Cov(.)$ denote the expectation, variance and covariance under the true probability density, Q_{t+h}. Since the covariance term turns out to be rather small empirically, we summarize aggregate uncertainty with the following additive decomposition:

$$
U^A_{t+h|t} \approx \underbrace{B_{t+h|t}}_{\text{"Mean-Bias"}} + \underbrace{V_{t+h|t}}_{\text{"Dispersion"}} + \underbrace{Vol_{t+h|t}}_{\text{"(Realized) Risk"}}
\tag{11}
$$

where:

- $B_{t+h|t} \equiv \int_{-\infty}^{\infty}\left(\left[E\left(P_{t+h|t}(r)\right) - E\left(x_{t+h}(r)\right)\right]^2\right)dr$, where $E\left(P_{t+h|t}(r)\right) \equiv \int P_{t+h|t}(r)\,dQ_{t+h}$ and $E\left(x_{t+h}(r)\right) \equiv \int x_{t+h}(r)\,dQ_{t+h}$, is the mean squared bias of the forecast distribution;

- $V_{t+h|t} \equiv \int_{-\infty}^{\infty} V(P_{t+h|t}(r))dr$, where $V(p_{t+h|t}(r)) \equiv \int\left(\left[P_{t+h|t}(r) - \left(\int P_{t+h|t}(r)\,dQ_{t+h}\right)\right]^2\right)dQ_{t+h}$, is the uncertainty about the ex-ante subjective probabilities in the aggregate distributional forecast;

- $Vol_{t+h|t} \equiv \int_{-\infty}^{\infty} V\left(x_{t+h}(r)\right)dr$, where $V\left(x_{t+h}(r)\right) \equiv \int\left(x_{t+h}(r) - \int x_{t+h}(r)\,dQ_{t+h}\right)^2 dQ_{t+h}$, represents the realized variance of the binary outcome, $x_{t+h}(r) \equiv 1\left(y_{t+h} < r\right)$, and thus stands for the inherent risk in the data.

The three component decomposition in eq. (11) has an interesting interpretation. We view the realized volatility component $Vol_{t+h|t}$ as a measure of the underlying uncertainty in the data, and

[11] A reason why the aggregate probability distribution, measured with a simple average of the individual probability distributions, is a good measure of aggregate uncertainty is the fact that, as in the context of point forecasts, combinations constructed by simple averages typically result in more accurately calibrated densities. Furthermore, the average of probability distributions is a measure widely used in a variety of central banks and policy institutions.

thus a measure of realized risk. On the other end, we view the bias component $B_{t+h|t}$ as a measure of how distant the predictive density is from the perfect prediction on average, while the dispersion, $V_{t+h|t}$, measures the variability in the predictive density. As we will show, $V_{t+h|t}$ is empirically small, so it can be ignored. The bias component, $B_{t+h|t}$, could have several causes: agents' irrationality, or agents using a misspecified model of the economy, or being subject to Knightian uncertainty. We isolate the components that measure agents' uncertainty about possible future events, given the predictive distribution of those events, from uncertainty capturing the inaccuracy of the agents' models and implied predictive distributions from the truth. In the context of this discussion, we assume that disagreement across agents, which can arise due to many reasons such as informational frictions, preferences, etc., is an artifact of misspecification. In general, the components $B_{t+h|t}$ and $D_{t+h|t}$ capture these misspecifications. The realized variance or realized volatility, instead, is a measure of risk. To summarize, we have the following decomposition:

$$U_{t+h|t} \approx \underbrace{Vol_{t+h|t}}_{\text{"(Realized) Risk"}} + \underbrace{B_{t+h|t} + D_{t+h|t}}_{\text{"Miscalibration Uncertainty"}} . \tag{12}$$

3.3 Ex-ante Vs. Ex-post Uncertainty

Our proposed measure of uncertainty, $U_{t+h|t}$, as well as aggregate uncertainty $U^A_{t+h|t}$, are constructed using realizations of the data. Thus, it is interesting to refine our measure by distinguishing between an ex-ante component (that does not include the realizations) and an ex-post component (which does). More insights on how the expected mean and the variance embedded in the forecast distribution affect our measure of uncertainty can be obtained under additional assumptions. Let the aggregate predictive distribution for the forecast of y_{t+h} made at time t be Normal with mean $\mu_{t+h|t}$ and variance $\sigma^2_{t+h|t}$ and the data be i.i.d.[12] We have the following "Ex-ante/Ex-post" decomposition:

$$U^A_{t+h|t} = \int \underbrace{\left[2\sigma_{t+h|t}\phi\left(\frac{y_{t+h} - \mu_{t+h|t}}{\sigma_{t+h|t}}\right) + \left(y_{t+h} - \mu_{t+h|t}\right)\left(2\Phi\left(\frac{y_{t+h} - \mu_{t+h|t}}{\sigma_{t+h|t}}\right) - 1\right)\right]}_{\text{"Ex-post"}} dQ_{t+h} \tag{13}$$

$$- \underbrace{\frac{\sigma_{t+h|t}}{\sqrt{\pi}}}_{\text{"Ex-ante"}} \tag{14}$$

where $\phi(.)$ and $\Phi(.)$ denote the PDF and the CDF of the Normal distribution, respectively. The proof is provided in Appendix A and follows Gneiting and Raftery (2007).[13]

[12] Since the data are assumed to be i.i.d. in this sub-section, we could omit the time subscripts; however, we decided to keep them to make the notation consistent with the rest of the paper.

[13] Note that even if $U^A_{t+h|t}$ is the difference of two components, it is always positive; thus, the ex-post component is always bigger than the ex-ante one.

The rightmost component in eq. (13), $\sigma_{t+h|t}/\sqrt{\pi}$, is the only component that is not affected by the realization, so we refer to it as the "ex-ante" measure of uncertainty. In fact, as the proof suggests, this is the component that arises from the average distance of random draws from a given predictive distribution. Moreover, it is a function of the standard deviation of the forecaster's density forecasts, and a common measure used in the uncertainty literature as a measure of ex-ante uncertainty. Note that the ex-ante measure of uncertainty is simply $\sigma_{t+h|t}/\sqrt{\pi}$, which, under Normality, is a monotone function of the width of the predictive distribution. Thus, the ex-ante measure is linked to the inter-quantile range measure proposed by Zarnowitz and Lambros (1987), among others.[14]

Note that, more generally, for any predictive distribution, the "ex-ante" measure of uncertainty is the same as $\int_{-\infty}^{+\infty} E\left[\left(x_{t+h}\left(r\right)-P_{s,t+h|t}\left(r\right)\right)^{2}|\Omega_{s,t}\right]dr=\int P_{s,t+h|t}\left(r\right)\left(1-P_{s,t+h|t}\left(r\right)\right)dr$.[15] Note that, from eqs. (11) and (13), we have that $\textit{Ex-post} \approx B_{t+h|t}+V_{t+h|t}+Vol_{t+h|t}+\textit{Ex-ante}$. Thus, the ex-post measure of aggregate uncertainty combines the bias component , $B_{t+h|t}$, realized risk (measured by the volatility in the economy, $Vol_{t+h|t}$), ex-ante risk (measured by the variance of the predictive densities of the forecasters, *Ex-ante*) and dispersion, $V_{t+h|t}$. Note the difference between $V_{t+h|t}$ and *Ex-ante*: the first measures the variability of the probability distribution, while the second measures the width of the distribution at a particular point in time. Thus, if the aggregate density forecast does not change over time, $V_{t+h|t}$ would be zero. However, *Ex-ante* will not be zero as long as the forecasters provide a distributional forecast.

We should note that there is a major difference between the two decompositions in that the "Ex-ante"/"Ex-post" decomposition is written in terms of the moments of the original predictive distribution, while the "Bias/(Realized) Risk" decomposition is in terms of binary outcomes summarized by $x_{t+h}\left(r\right)$. As such, the latter decomposition could be applied to general situations (general forms of distribution and non-i.i.d. data), while the former one relies heavily on the assumption of Gaussianity and independence in the underlying predictive distribution. Ganics, et al. (2019), D'Amico and Orphanides (2014) and Giordani and Soderlind (2003) provide empirical support in favor of Gaussianity for the Survey of Professional Forecasters.

A general note that applies to all proposed decompositions is that the resulting components are not orthogonal to each other. This is in line with the rest of the empirical literature which typically finds that a variety of uncertainty measures, constructed from different sources and measuring different aspects of uncertainty, are correlated with each other.

[14] For a Gaussian distribution, the inter-quantile range is 1.34σ.

[15] In fact, in the Not-for-Publication Appendix, we report results based on this alternative formula and show that they are identical.

4 The Data

We use density forecasts from the Survey of Professional Forecasters (SPF) to calculate our uncertainty measures. The Federal Reserve Bank of Philadelphia provides the aggregate (mean probability distribution) forecasts, as well as the underlying disaggregate density forecasts of a panel of professional forecasters.[16] We use the real GNP/GDP growth density forecasts to extract measures of macroeconomic uncertainty, as real GNP/GDP fluctuations are indicative of the state of the business cycle, and therefore are representative of macroeconomic uncertainty (Stock and Watson, 1999).

In the SPF data set, forecasters are asked to assign a probability value (over pre-defined intervals) to inflation and output growth for the current and the following (one-year-ahead) calendar years. The growth rate is defined as the rate of change in the average GDP from one year to another. The forecasters update the assigned probabilities for the current-year and the one-year-ahead forecasts on a quarterly basis. Thus, by construction, SPF forecasters provide four quarterly forecasts of the same target variable each year; this type of forecasts are typically referred to in the literature as "fixed-event" or "moving-horizon" forecasts. Being fixed-event forecasts, their horizon changes over the quarter. We use the method proposed by Dovern et al. (2012) to transform the SPF fixed-event forecasts into fixed-horizon forecasts by constructing a weighted average of the current-year and next-year forecasts. In detail, for each quarter the survey contains a pair of "fixed-event" density forecasts for the current-year, which we label $\widehat{f}^{FE}_{t+k|t}$, and for the next-year, which we label $\widehat{f}^{FE}_{t+k+4|t}$. The four-quarter-ahead (fixed-horizon) forecast at time t, which we label $\widehat{f}^{FH}_{t+4|t}$, is calculated as the average of the two fixed event forecasts using weights that are proportional to their share of the overlap with the forecast horizon. Let k denote the number of quarters from time t until the end of the year. In quarter one, $k = 4$, while in quarter four, $k = 1$. Thus, for example, in the third quarter of the year, the four-step-ahead fixed-horizon forecast overlaps with the current year forecasts and next year forecasts 50% of the time, respectively. Thus, it would be the weighted average of the two-fixed event forecasts with weights equal to 2/4 and 2/4. Thus, in general, for $k = 1, 2, 3, 4$:

$$\widehat{f}^{FH}_{t+4|t} = \frac{k}{4}\widehat{f}^{FE}_{t+k|t} + \frac{4-k}{4}\widehat{f}^{FE}_{t+k+4|t}. \tag{15}$$

Note that the literature suggest alternatives to this weighting, but only for point forecasts. For instance, Knueppel and Vladu (2016) propose weights for aggregating the fixed-event point forecasts which minimize the mean squared forecast error loss of the fixed-horizon forecast. This methodology requires a researcher to take a stand on the data generating process and is not directly applicable to our case of density weighting. It is important to note that our results are not driven by the particular method that we use to transform fixed-event into fixed-horizon forecasts. Ganics,

[16]The composition of the forecasters can change over time.

et. al. show that these forecasts result in competitve densities when applied to aggregate density forecasts on average, they are also competitive relative to varoius parametric and non-parametric alternatives. While we report our main results based on the procedure outlined above to use all the available quarterly observations, we will also show results based on yearly fixed horizon forecasts based only on the forecasts on the first quarter of the year, which are virtually identical.

INSERT FIGURE 2 HERE

Panels A and B in Figure 2 show the evolution of the current and next year densities over time. The figures plot the mean as well as several quantiles of the distribution, together with the realization. Panel C, on the other hand shows the fixed horizon forecast, eq. (15). The fixed-horizon forecast is by construction less smooth than the fixed-event forecasts. However, both share the same feature that ex-ante uncertainty was higher earlier in the sample, in the sense that both density forecasts have a wider distribution prior to the mid-1980s relative to the later part of the sample; this suggest that forecasters noticed the Great Moderation starting mid-1980s. There appears to be no dramatic shift in the forecasted densities after the Great Recession. Some descriptive statistics on the SPF distributions is provided in Appendix B.

The analysis of SPF probability distributions is complicated since the SPF questionnaire has changed over time in various dimensions: there have been changes in the definition of the variables, the intervals over which probabilities have been assigned, as well as the time horizon for which forecasts have been made. To mitigate the impact of these problematic issues, we truncate the data set and consider only the period 1981:III-2014:II.[17]

As noted, our uncertainty measure depends on realizations. The realized values of output growth are from the real-time data set for macroeconomists, also available from the Federal Reserve Bank of Philadelphia. We use the four-quarter-ahead growth rates of output and prices calculated from the first release of the realization. For instance, in order to get the 4-quarter-ahead realization at the start of our sample, 1981:III, we calculate the growth rate between 1982:III and 1981:III using the 1982:IV vintage of the data.

The probability density of the data as well as expectations are estimated based on actual realizations using a rolling window.

5 The Dynamics of Uncertainty over Time, and Its Sources

Figure 3, Panel A, shows the evolution of our estimated measure of uncertainty and its components, aggregate uncertainty and disagreement, over time. The figure highlights two interesting facts:

[17]We focus on quarterly data. See instead Ferrara and Guérin (2015) for a high-frequency analysis of uncertainty shocks.

disagreement is, in magnitude, only a small portion of the overall measure of uncertainty;[18] in addition, it is trending down until the financial crisis of 2007; this is in sharp contrast with the overall measure of uncertainty, as well as aggregate uncertainty, which have clear spikes in the early 1980s, early 2000s and the financial crisis. Thus, using disagreement as a measure of uncertainty may result in underestimating both the overall level of uncertainty in the economy as well as its fluctuations over time, as currently the level of disagreement is similar to what it was in the mid-1990s and lower than its value in the late 1980s. In addition, most would agree that the early 2007-2008 were probably the most uncertain times in the latest decades; while disagreement increases during that period, it peaks only much later, after the end of the recession, in 2009. Thus, disagreement (i.e., the component of Knightian uncertainty due to disagreement among forecasters) may not be a timely measure of uncertainty. Note that this result is not an artifact of constructing disagreement measures based on density forecasts: Sill (2014, Figure 1) shows a similar delay. In particular, Sill (2014) plots the dispersion of the mean one-year-ahead real GDP growth rate forecasts measured by the inter-quantile range: the first peak in the disagreement does not appear until the middle of the recession.

INSERT FIGURE 3 HERE

Panel C in Figure 3 depicts the decomposition of aggregate uncertainty into the various components in decomposition (10), while Panel B, plots overall uncertainty and its decomposition into Knightian uncertainty and realized risk (eq. 12). The figures suggest that realized risk (measured by $Vol_{t+h|t}$) was an important component of uncertainty throughout the last three decades, as was Knightian uncertainty, measured by the mean bias component. Some differences between the two are important to note, however. The realized risk component was high during the latest financial crisis, and sharply decreased as soon as the recession was over; Knightian uncertainty (measured by $B_{t+h|t} + D_{t+h|t}$ in Panel B, and its largest component, the mean bias $B_{t+h|t}$, depicted in panel C) remained persistently high even after the end of the crisis. Thus, overall uncertainty remained persistently high after the end of the latest recession mostly because of forecasters' errors as opposed to risk being high. The role of dispersion in probability forecasts ($V_{t+h|t}$) as well as the co-movement between prediction and realization ($Cov_{t+h|t}$) are negligible for the cyclical dynamics of aggregate uncertainty.

Turning to the ex-ante and ex-post components, depicted in Panel D of Figure 3 together with the aggregate uncertainty measure ($U^A_{t+h|t}$), it is interesting to note that ex-ante uncertainty is quite constant in the 1980s and up to 2007. Thus, movements in uncertainty during that period

[18] The magnitudes of $U_{h+h|t}$ and $U^A_{h+h|t}$ are reported on the y-axis on the left while that of disagreement is reported on the y-axis on the right. The magnitude of disagreement is small. This is due to the fact that, unlike the existing measures of disagreement on point forecasts, we measure disagreement in probabilities, not in the mean forecast.

cannot be attributed to changes in ex-ante uncertainty. Ex-ante uncertainty does increase during the latest recession, but only towards its end, and spikes much later than the peak of the recession. This suggests that measures of volatility in the forecasters' predictive distributions are, themselves, not timely measures of uncertainty.

Panels E, F and G depict the same results but using only fixed horizon forecasts in the first quarter in the year, thus sampling at the yearly frequency. In other words, we avoid Dovern et al.'s (2012) procedure although this considerably shortens the number of available observations. The figures show that our empirical results based on quarterly observations (interpolated using Dovern et al.'s (2012) procedure) are not driven by the specific methodology that we used to obtain the fixed horizon forecasts.

Finally, it is also of interest to investigate how the various components of uncertainty evolve as the forecasters get closer in time to the realization date, that is, as the forecast horizon becomes shorter. We separately consider forecasts for $h = 1, 2, ..., 7, 8$ and compare them with the fixed-event realization. Both uncertainty as well as aggregate uncertainty decrease as the forecast horizon increases (Panel A in Figure 4, top left and right graphs). It may seem counter-intuitive that uncertainty decreases at longer horizons; to understand why, we examine its components. Clearly, disagreement decreases as forecasters get closer to the realization: in fact, disagreement decreases on average as the horizon decreases (cfr. bottom graph in Figure 4, Panel A). This finding is reassuring, as it is reminiscent of what Patton and Timmermann (2010) discovered for point forecasts, and our results show that similar results hold for disagreement calculated on density forecasts. The mean bias also decreases as the horizon decreases (Panel B in Figure 4). On the other hand, the dispersion of the density forecasts increases, thus increasing the aggregate uncertainty. The realized variance and covariance are constant over the horizons, and the latter hovers around zero.

INSERT FIGURE 4 HERE

The most striking patterns are displayed by ex-ante and ex-post uncertainty, depicted in Figure 4, Panel C. Clearly, ex-ante uncertainty decreases monotonically as the forecast horizon decreases; that is, forecasters' predictive densities become more spread out when the forecast horizon increases, thus reflecting more uncertainty in the economy when looking at events that are further in the future. However, there is no clear pattern in ex-post uncertainty. This means that, even though the forecasters' predictive densities become tighter as the realization gets closer in time, the uncertainty in the actual realizations does not diminish, as the size of the forecast errors does not diminish with the horizon. We present detailed empirical evidence that this is indeed the case in Section A.6 in the Not-for-Publication Appendix, where we plot the actual predictive densities and realizations across horizons at several points in time.

Comparing the evolution of the ex-ante uncertainty in Panel C and the dispersion of the aggregate predictive density, $V_{t+h|t}$ in Panel B, we note that, although forecasters, on average, become less confident about the future as the forecast horizon increases, their views about uncertainty does not seem to be updated often for forecasts that are further in the future, thus resulting in the low variability of the predictive distribution over time. Moreover, as the distribution becomes more spread out with the forecast horizon, it has a higher chance of including the realization, thus resulting in a decline in the aggregate and overall uncertainty.

6 Understanding the Measures of Uncertainty in the Literature and Their Macroeconomic Effects

In this section, we use our decomposition to shed some light on why existing measures of uncertainty differ from each other. Understanding why they differ provides important insights on which measure is the most appropriate for a particular analysis. To achieve this goal, in this section we will first quantify the correlation of existing measures of uncertainty with the components in our decomposition; then, we will perform VAR analyses that separately include each of the uncertainty measures in a VAR. Such VARs are typically estimated in the existing literature, where different papers use different measures of uncertainty. We do a similar exercise to answer the following questions: what happens if one replaces the uncertainty measure used in the existing studies with one of our components? which one of our components leads to the largest business cycles fluctuations? In other words, our exercise is not meant to capture shocks in a component of our uncertainty measure independently of another one: it simply investigates the effects of uncertainty when it is measured using a particular component from our decomposition. This exercise is interesting since, as we argue, the various uncertainty measures existing in the literature capture different types of uncertainty and our goal is to understand what these different measures stand for. Thus, replicating the exercises conducted in the literature with the various measures in our decomposition taken one-at-a-time is useful to interpret results obtained in the literature.

The top panel in Figure 5 plots Jurado, Ludvigson and Ng's (2015) uncertainty measure together with Baker, Bloom and Davis' (2016) index.[19] Both indices are standardized for comparison. The figure shows that the former is overall smaller than the latter until 1995, then it becomes overall bigger, and in particular spikes up earlier than the latter during the latest financial crisis of 2007-2008. The lower panel plots the decomposition of our aggregate uncertainty index into ex-ante and ex-post components. The ex-post component is lower than the ex-ante component up to mid-1992, then it becomes systematically larger, and spikes up around 2007-2008, behaving similarly to how the Jurado, Ludvigson and Ng's (2015) behaves relative to Baker, Bloom and Davis (2016). Thus, it seems that the Baker, Bloom and Davis (2016) uncertainty measure is driven more by ex-ante

[19] We are using Jurado, Ludvigson and Ng's (2015) one-year-ahead uncertainty index.

uncertainty, while the Jurado, Ludvigson and Ng (2015) uncertainty measure is clearly affected by ex-post uncertainty, namely uncertainty due to misspecification in the predictions.

INSERT FIGURE 5 HERE

To estimate the effects of the uncertainty and its components on the economy, we estimate a Vector Autoregression (VAR) that includes the specific uncertainty indices (included one at a time), (the log of) employment, the Federal Funds rate and (the log of) stock prices.[20] Importantly, note that the uncertainty indices that depend on realizations are lagged h-periods, to address potential endogeneity related to just the timing of the uncertainty variables that use ex-post data. Note that, when lagged, our four-quarter-ahead uncertainty index is an index based on the nowcast (according to Dovern's procedure).

Identification is achieved via a Cholesky procedure, which follows the order in which the variables are listed. The variables are similar to those in Baker, Bloom and Davis (2016), although ours is at the quarterly frequency. We order the variables as in Jurado et al.'s (2015) benchmark specification, i.e. from slow to fast moving, except that we order uncertainty first. For completeness, we investigate the robustness of our results in a larger VAR in the Not-for-Publication Appendix. To better interpret and compare the magnitude of the effects of the uncertainty indices, the uncertainty indices are standardized by their own means and variances.

Panel A in Figure 6 shows the effects of our uncertainty index on the economy. Clearly, an increase in uncertainty has recessionary effects: both GDP and employment decrease, as well as the interest rate and the S&P 500. Panels B and C describe the effects of each of the components in the decomposition. Panel B shows the effects of a shock to aggregate uncertainty, which is in line with that of uncertainty since aggregate uncertainty is the main determinant of the total. Panel C focuses on disagreement; it also decreases employment although by a smaller magnitude; at the same time, it has no significant effects on the remaining variables.

INSERT FIGURE 6 HERE

Figure 7 shows the effects of uncertainty measured by mean bias, realized volatility and the dispersion in the probability forecasts. The mean bias and realized volatility appear to have recessionary effects (Panels B and D); dispersion in the density forecasts (Panel C) drives down employment, while it increases stock prices and output. It is important to note that, in magnitude, the mean bias and realized volatility have similar macroeconomic impact, though these effects are statistically significant for the first but not for the second.

INSERT FIGURE 7 HERE

[20] Given that our uncertainty index is based on GDP forecasts, we include employment as a proxy for real variables instead of GDP.

The effects of ex-ante and ex-post uncertainty on other macroeconomic variables are depicted in Figure 8. They both lead to decreases in employment, interest rates and stock prices of similar magnitude; an increase in ex-ante uncertainty, however, has a small negative impact effect on GDP, while the medium run effect is positive and small, and the longer run effect is again negative; the effects of ex-post uncertainty on GDP are, instead, negative and large.

INSERT FIGURE 8 HERE

Figure 9 compares the results with those in the existing literature; the latter are also obtained by estimating VARs that include (the log of) real GDP, (the log of) employment, the Federal Funds rate, (the log of) stock prices, and the alternative uncertainty index, which is demeaned and standardized as well. The alternative uncertainty indices that we explore (one-at-a-time) include: Bloom (2009), labeled "VXO"; Baker et al.'s (2016) policy uncertainty index, labeled "BBD"; Jurado, Ludvigson and Ng (2015), labeled "JLN"; and Scotti's (2016) macroeconomic surprise-based uncertainty index.

INSERT FIGURE 9 HERE

Panel A in Figure 9 shows that the VXO and BBD indices have similar effects on the economy, while an increase in uncertainty measured by the Jurado, Ludvigson and Ng's (2015) index are qualitatively similar but much larger in magnitude, and, thus, are similar to the effects that we uncover for our ex-post index. The effects of Scotti's index are again recessionary for GDP, employment and stock markets, and lead to an increase in the interest rate. The effects of this index are small and overall insignificant. The effects of our realized volatility measure are more similar to those of the VXO.

7 Inflation Uncertainty

In this last section, we focus on inflation uncertainty. Understanding inflation uncertainty is important for several reasons. High uncertainty about future inflation, possibly spurred by high inflation itself, may have effects on real variables (Ball, 1990). For example, Gurkaynak and Wright (2012) and Wright (2011) have argued that inflation uncertainty matters because it might help explain the behavior of bond risk premia, and therefore help economists understand why monetary policy differently affects short term rates (the instrument of monetary policy) and the long term rate (the rate that is of interest for investors and consumers). In fact, Wright (2011) has found a positive and strong relationship between long-term inflation uncertainty and bond term premia in a large cross-section of countries. The important policy implication of Wright's (2011) findings is the possibility that eliminating long-run inflation uncertainty might facilitate the transmission of monetary policy to the economy. Also, D'Amico and Orphanides (2014) consider ex-ante measures of risk for inflation forecasting and Caporale et al. (2012) have shown that inflation uncertainty has decreased in the Euro area, possibly due to the fact that inflation decreased steadily since the beginning of the Euro.

Figure 10 depicts our measure of uncertainty (Panel A) and the decompositions (Panels B,C). Inflation uncertainty was high in the early 1980s, possibly due to oil price shocks, and decreased substantially afterwards; typically, it tends to be high around recessions. The behavior over time of uncertainty is very different from that of disagreement, which instead does not necessarily peaks around recession times. While the volatility component is pretty constant over time, the majority of the fluctuations in aggregate inflation uncertainty are associated with the bias component and the ex-post components; interestingly, ex-ante inflation uncertainty seems to have decreased monotonically since the early 1980s.

Our empirical results suggest that the most effective policies to decrease inflation uncertainty are those that influence ex-post uncertainty. In other words, policies should aim at ensuring that ex-post realizations of inflation are in line with the average expected inflation (for example, by minimizing shocks to inflation), not those that decrease the agents' ex-ante uncertainty (i.e. not those that affect the agents' expectation formation process), although the latter can also be effective.

INSERT FIGURE 10 HERE

8 Conclusion

This paper proposes an alternative measure of uncertainty based on survey density forecasts. The new measure has the advantage that it can be used to decompose uncertainty into components that can help researchers understand what existing uncertainty indices measure. In particular, our measure of uncertainty can be decomposed into Knightian uncertainty and realized risk. The latter inherently measure different things, have specific business cycle dynamics and different macroeconomic impact. Moreover, these sources of uncertainty resolve differently across prediction horizons.

Given that our proposed uncertainty index is an ex-post measure of uncertainty, we also decompose it into a component that only reflects ex-ante uncertainty, which we can relate to existing measures of uncertainty based on the inter-quantile spread of the forecast distribution, and a component that measures ex-post uncertainty. Our analysis uncovers that some existing measures of uncertainty capture ex-ante uncertainty (such as existing measures of uncertainty based on policy uncertainty), while others capture ex-post uncertainty.

We also investigate the effects of the sources of uncertainty on the macroeconomy. We find that, while an increase in overall uncertainty has recessionary effects, the effects of the various components of uncertainty differ. For example, disagreement is only a small portion of the overall uncertainty, and may both underestimate and lag the actual degree of uncertainty in the economy; thus it may not be a timely measure of uncertainty. In addition, both realized risk and Knightian uncertainty were important components of uncertainty over the last three decades, although the former sharply decreased as soon as the financial recession of 2007-2008 ended while the latter remained high even after the end of the crisis. This suggests that the high overall uncertainty that persisted after the end of the latest recession was mostly due to agents' being unable to assign the correct probability to the economic outcomes and disagreeing on them, rather than because risk was high. Simulation results from a stylized macroeconomic model suggest that the behavior of uncertainty and its components is largely reconcilable with a macroeconomic model with ambiguity. Ambiguity can be a source of its own in increasing the overall level of uncertainty; alternatively, it can also act as an amplifying mechanism for the increase in the level of risk.

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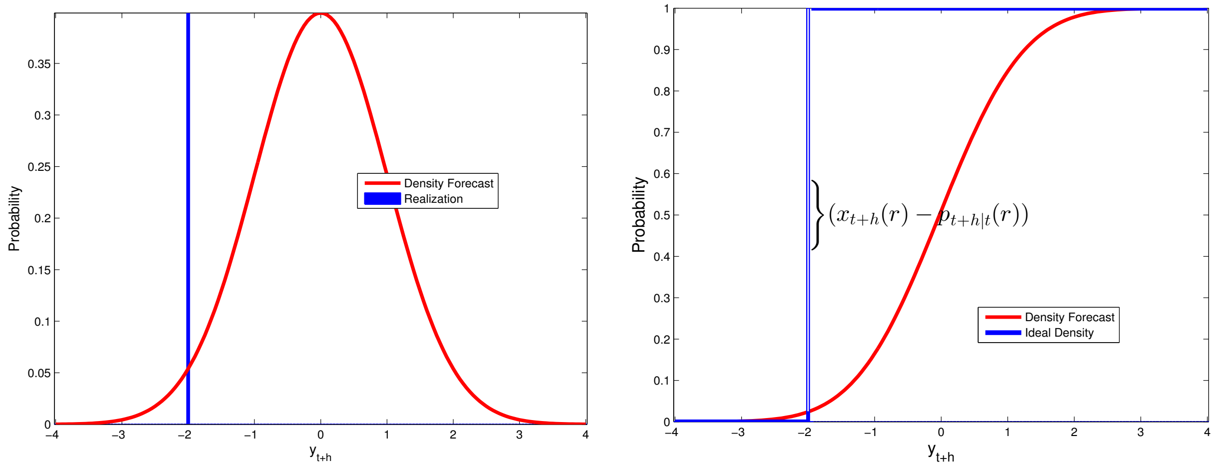
9 Tables and Figures

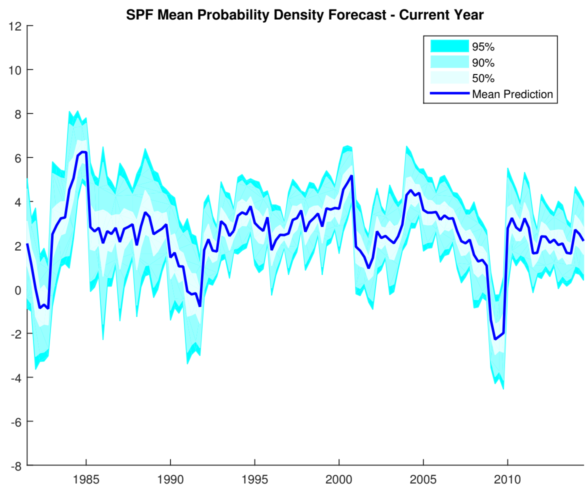
Figure 1: Brier Score Illustration



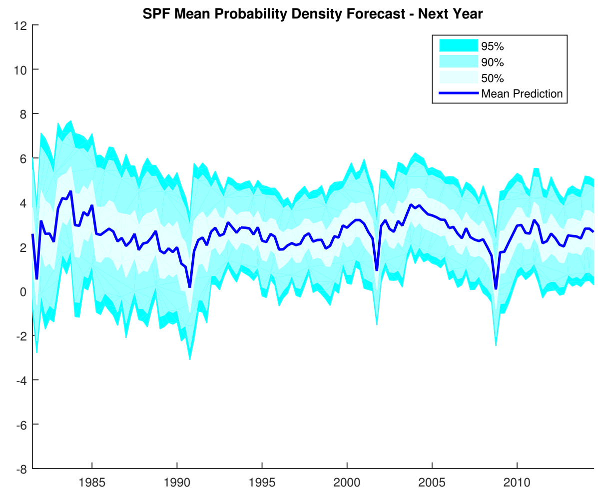
Note: The figure on the left shows the pdf of the predicted distribution together with the realization, $y_{t+h} = -2$. On the right we have the CDFs of the predicted and ideal distribution for the realization. The area between them (denoted by the hollow vertical bar) highlights the distance between the two.

Figure 2. The Survey of Professional Forecasters Data

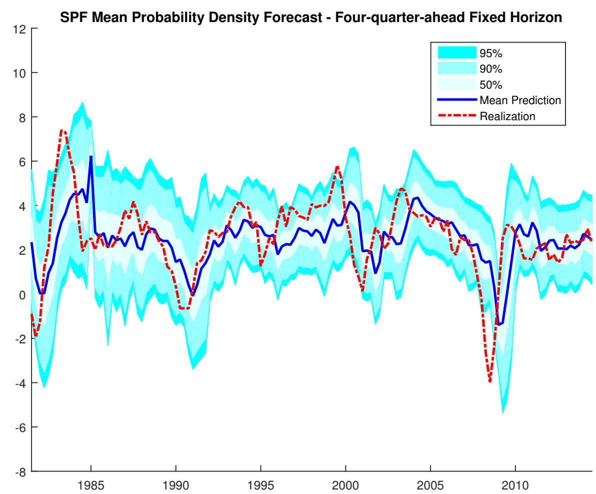
Panel A: Current Year Forecasts



Panel B: Next Year Forecasts



Panel C: Fixed-horizon Forecasts



Note: The figure shows the means and quantiles of the SPF's current year and next year predictive densities for GDP growth, as well as the constructed fixed horizon four-step-ahead predictive density. The four-step-ahead density is constructed from the SPF current year and next year density forecasts based on eq. (15). Panel C also shows the realized value of the GDP growth.

Figure 3: Decomposing Uncertainty

Panel A: Uncertainty, Aggregate Uncertainty and Disagreement

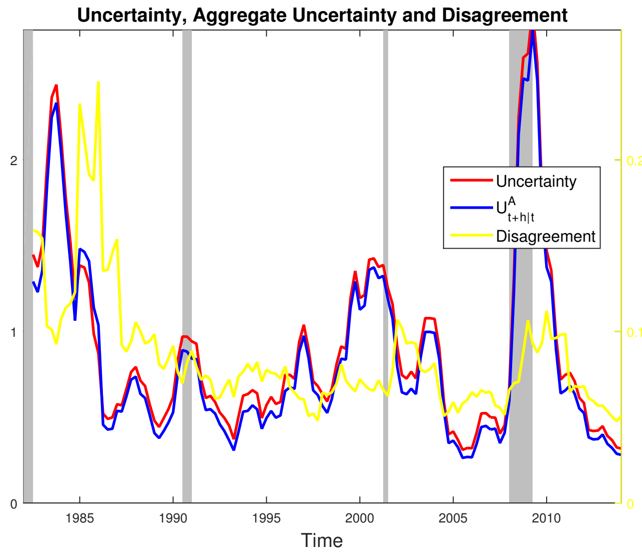


Figure 3: Decomposing Aggregate Uncertainty

Panel B: Decomposition in Eq. (12)

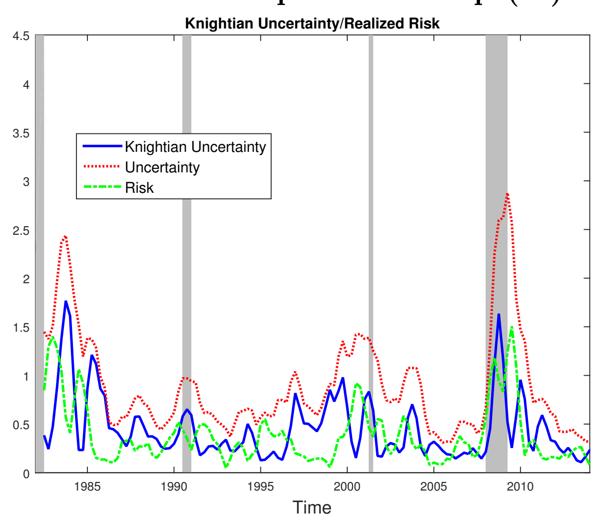
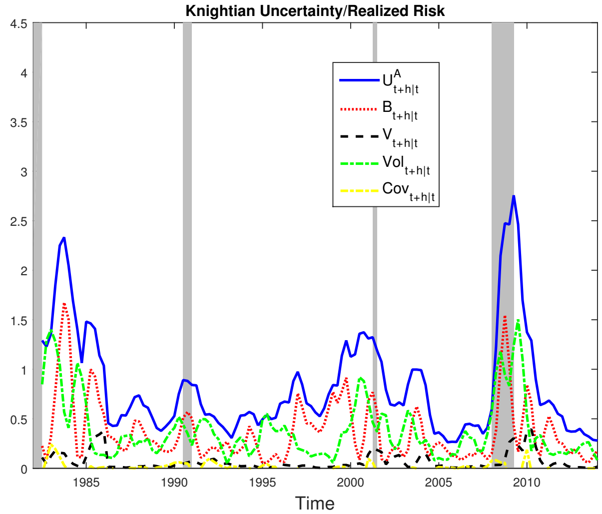
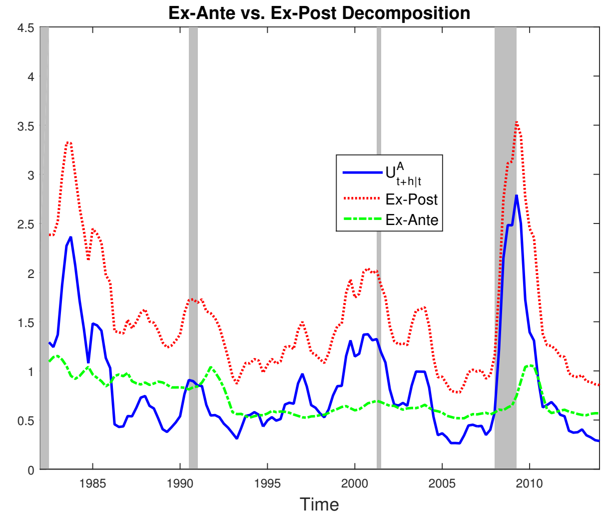


Figure 3: Decomposing Aggregate Uncertainty

Panel C: Decomposition in Eq. (11)



Panel D: Ex-Ante Vs. Ex-Post



Note: Panel A of Figure 3 depicts the evolution of uncertainty, aggregate uncertainty and disagreement (eq. 9) over time. Panel B shows the evolution of the components of aggregate uncertainty based on eq. (12). Panels C and D show the evolution of the components of aggregate uncertainty based on eq. (11) and eq. (13), respectively. Results are based on quarterly data based on fixed horizon forecasts obtained by Dovern et al.'s (2012) procedure.

Figure 3: Decomposing Uncertainty

Panel E: Uncertainty, Aggregate Uncertainty and Disagreement

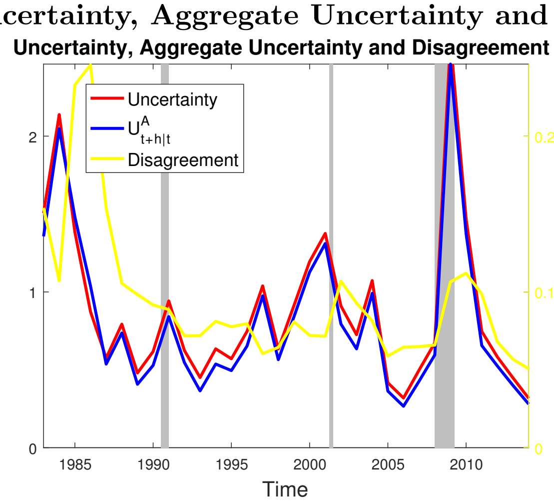
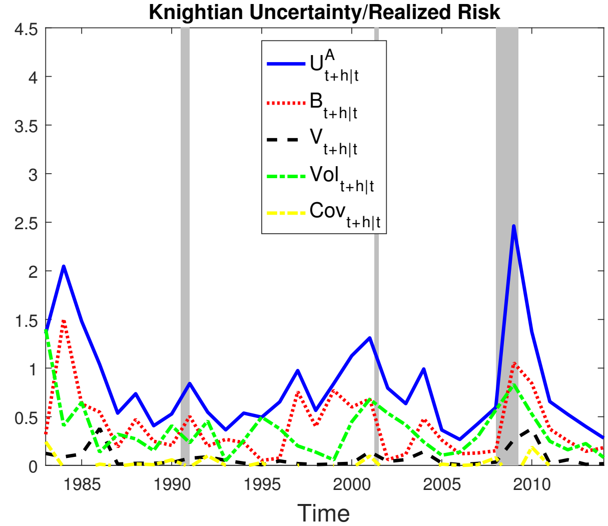
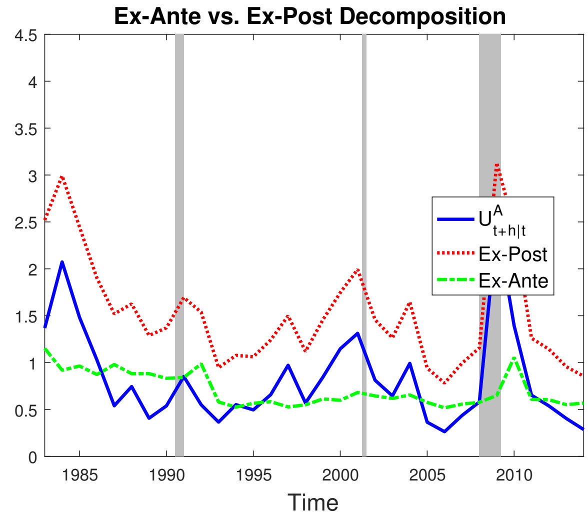


Figure 3: Decomposing Aggregate Uncertainty

Panel F: Knightian Uncertainty Vs. Risk



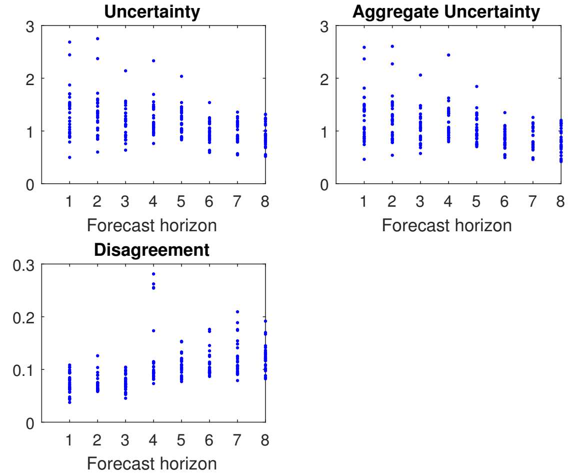
Panel G: Ex-Ante Vs. Ex-Post



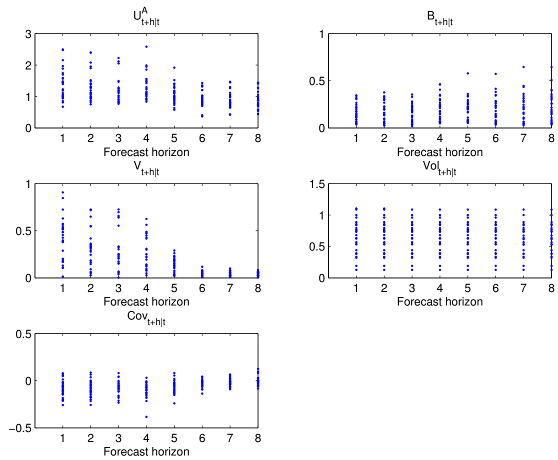
Note: Panel E of Figure 3 depicts the evolution of uncertainty, aggregate uncertainty and disagreement (eq. 9) over time. Panels F and G show the evolution of the components of aggregate uncertainty based on eq. (11) and eq. (13), respectively. Results are based on quarterly data based on fixed horizon forecasts using only the first quarter of the year.

Figure 4: Decomposition of Uncertainty Across Horizons

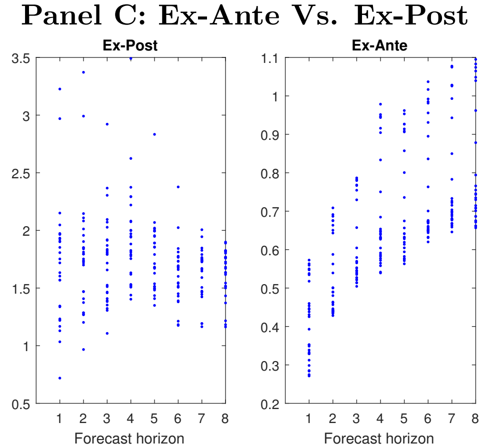
Panel A: Uncertainty, Aggregate Uncertainty, Disagreement



Panel B: Knightian Uncertainty Vs. Risk

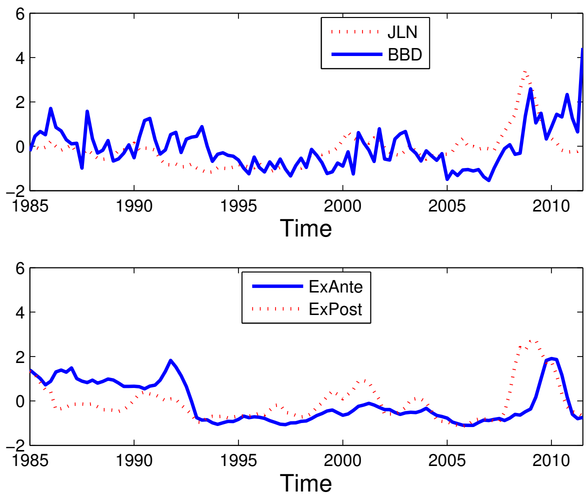


Panel C: Ex-Ante Vs. Ex-Post



Note: Panel A shows uncertainty, aggregate uncertainty and disagreement over time. Panels B and C show the components in decompositions in eqs. (11) and (13), respectively.

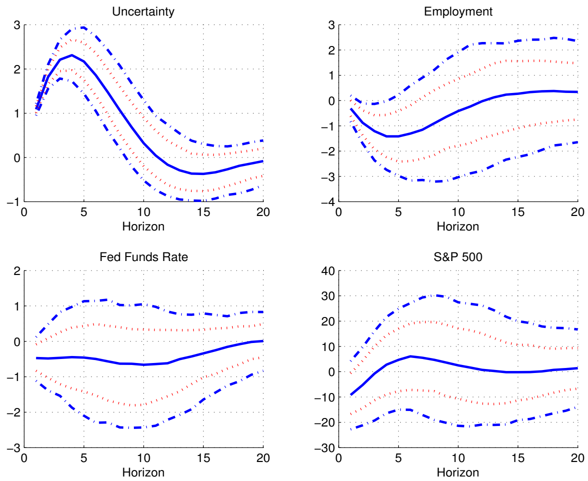
Figure 5. Comparison of Uncertainty Measures



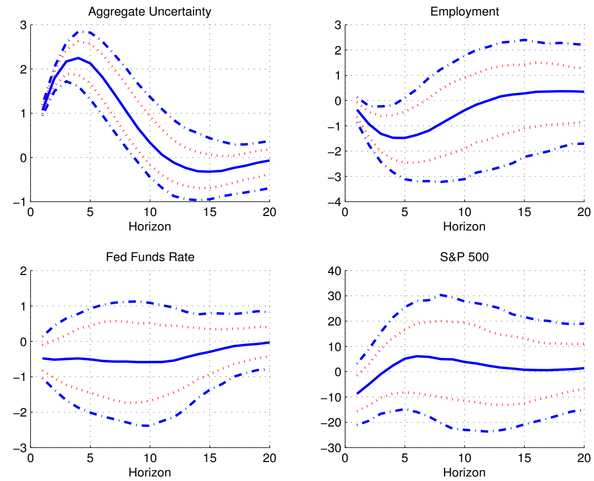
Note: The figure compares the Jurado, Ludvigson and Ng (2015) and Baker, Bloom and Davis (2016) uncertainty indices (top panel) with the ex-ante and ex-post components of our uncertainty measure, eq. (13), depicted in the bottom panel.

Figure 6. The Effects of Uncertainty on the Economy

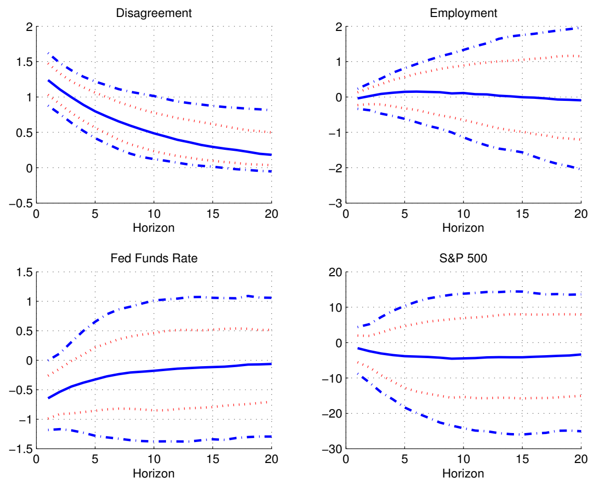
Panel A: Uncertainty



Panel B: Aggregate Uncertainty



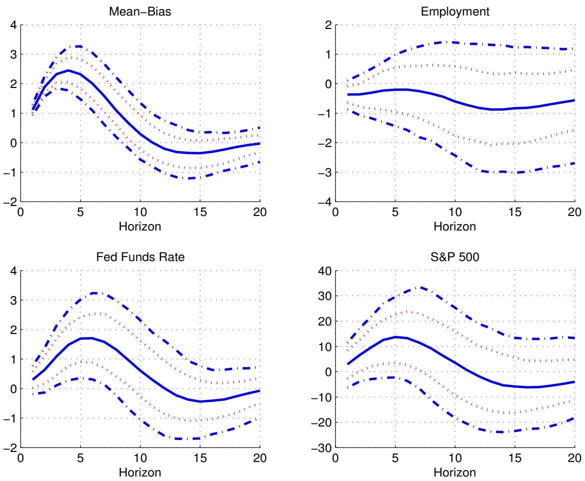
Panel C: Disagreement



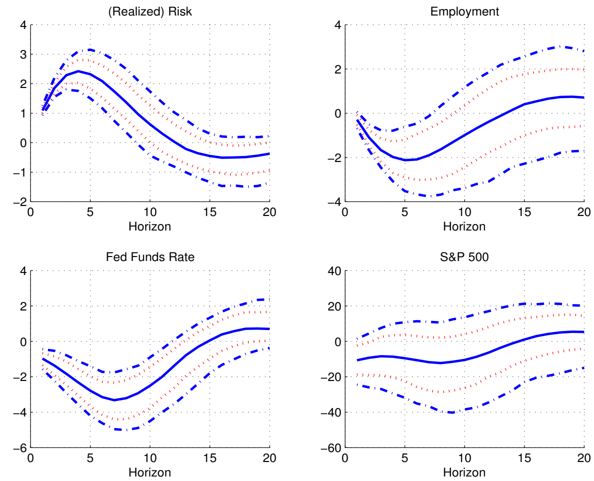
Note: The figure shows the impulse responses of uncertainty, aggregate uncertainty and disagreement shocks. The components are calculated as in eq. (9) All uncertainty measures have been standardized.

Figure 7. The Effects of Uncertainty on the Economy

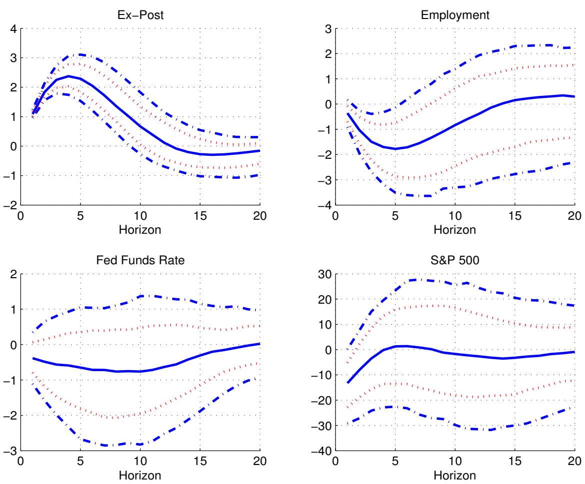
Panel A: Mean Bias



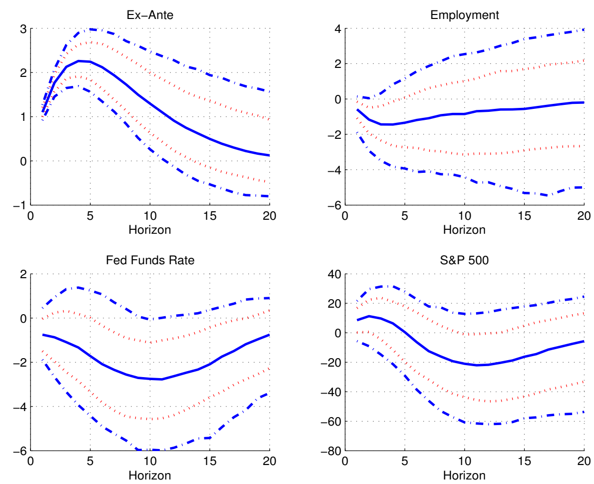
Panel B: Realized Volatility



Panel C: Ex-Post Uncertainty

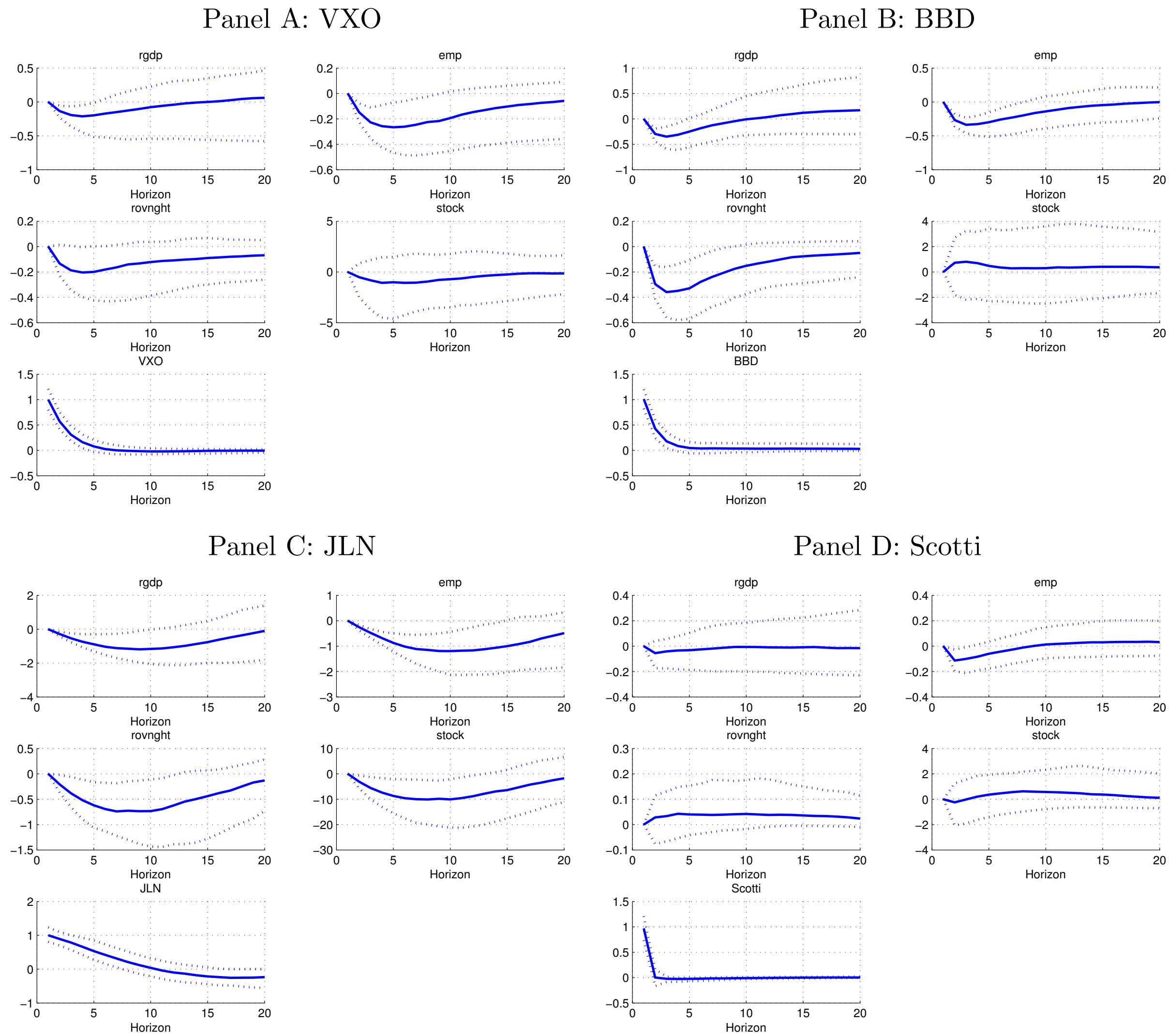


Panel D: Ex-Ante Uncertainty



Note: The figure shows the impulse responses of the aggregate uncertainty components: mean bias (Panel A); realized risk measures (Panel B) – both based on eq. (11); ex-ante (Panel C) and ex-post (Panel D) measures of uncertainty based on eq. (13). The figure also show the 68% (dashed and dotted lines) and 90% (dotted lines) confidence bands based on Kilian's (1999) bootstrap with 500 replications.

Figure 9. The Effects of Uncertainty on the Economy - Alternative Measures



Note: The figure shows the impulse responses for the following uncertainty measures: VXO, JLN, BBD and Scotti. All uncertainty measures have been standardized.

Figure 10: Decomposing Inflation Uncertainty

Panel A: Uncertainty, Aggregate Uncertainty and Disagreement

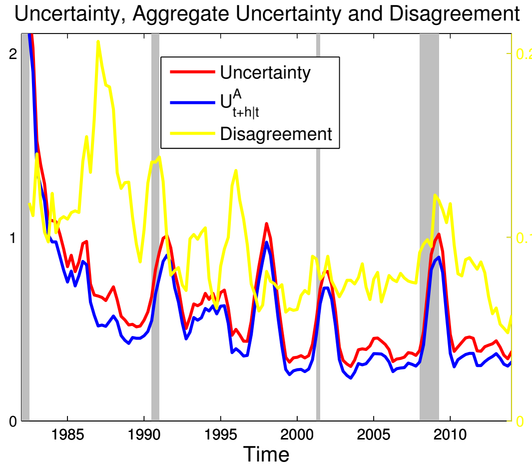
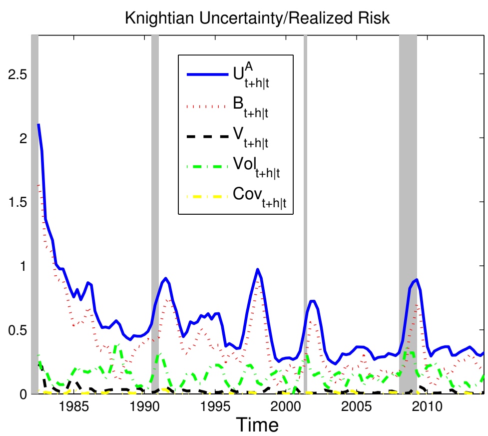
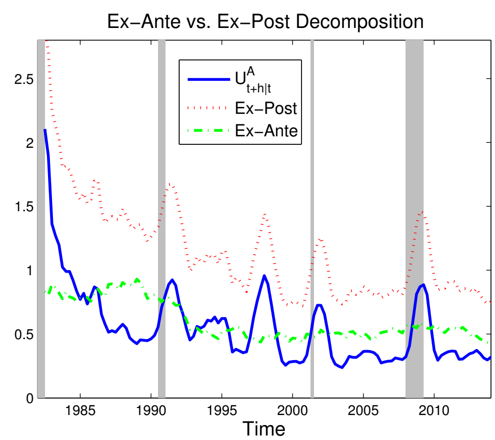


Figure 10: Decomposing Aggregate Inflation Uncertainty

Panel B: Knightian Uncertainty Vs. Risk



Panel C: Ex-Ante Vs. Ex-Post



Note: Panel A depicts the evolution of uncertainty, aggregate uncertainty, as well as disagreement (eq. 9) over time. Panels B and C show the evolution of the components of uncertainty based on eq. (11) and eq. (13), respectively.

Appendix A. Proofs

The appendix provides the proofs for the results in the paper. For simplicity, we write the proof for the unconditional expectation.

Proof of Eq. (8).

$$\int \left[\frac{1}{N}\sum_{s=1}^{N}\left[x_{t+h}\left(r\right)-P_{s,t+h|t}\left(r\right)\right]^{2}\right]dQ_{t+h}$$
$$=\int \left[\frac{1}{N}\sum_{s=1}^{N}\left[x_{t+h}\left(r\right)-P_{t+h|t}\left(r\right)+P_{t+h|t}\left(r\right)-P_{s,t+h|t}\left(r\right)\right]^{2}\right]dQ_{t+h}$$
$$=\int \left(\frac{1}{N}\sum_{s=1}^{N}\left[\left(x_{t+h}\left(r\right)-P_{t+h|t}\left(r\right)\right)^{2}+2\left(x_{t+h}\left(r\right)-P_{t+h|t}\left(r\right)\right)\left(P_{t+h|t}\left(r\right)-P_{s,t+h|t}\left(r\right)\right)\right]\right)dQ_{t+h}$$
$$+\int \left(\frac{1}{N}\sum_{s=1}^{N}\left[\left(P_{t+h|t}\left(r\right)-P_{s,t+h|t}\left(r\right)\right)^{2}\right]\right)dQ_{t+h}$$
$$=\int \left[\frac{1}{N}\sum_{s=1}^{N}\left(x_{t+h}\left(r\right)-P_{t+h|t}\left(r\right)\right)^{2}\right]dQ_{t+h}$$
$$+2\int \left[\frac{1}{N}\sum_{s=1}^{N}\left(x_{t+h}\left(r\right)-P_{t+h|t}\left(r\right)\right)\left(P_{t+h|t}\left(r\right)-P_{s,t+h|t}\left(r\right)\right)\right]dQ_{t+h}$$
$$+\int \left[\frac{1}{N}\sum_{s=1}^{N}\left(P_{t+h|t}\left(r\right)-P_{s,t+h|t}\left(r\right)\right)^{2}\right]dQ_{t+h}$$
$$=\int \left[\left(x_{t+h}\left(r\right)-P_{t+h|t}\left(r\right)\right)^{2}\right]dQ_{t+h}$$
$$+2\int \left[\left(x_{t+h}\left(r\right)-P_{t+h|t}\left(r\right)\right)\frac{1}{N}\sum_{s=1}^{N}\left(P_{t+h|t}\left(r\right)-P_{s,t+h|t}\left(r\right)\right)\right]dQ_{t+h}$$
$$+\int \left[\frac{1}{N}\sum_{s=1}^{N}\left(P_{t+h|t}\left(r\right)-P_{s,t+h|t}\left(r\right)\right)^{2}\right]dQ_{t+h}$$
$$=\int \left[\left(x_{t+h}\left(r\right)-P_{t+h|t}\left(r\right)\right)^{2}\right]dQ_{t+h}$$
$$+2\int \left[\left(x_{t+h}\left(r\right)-P_{t+h|t}\left(r\right)\right)\left(P_{t+h|t}\left(r\right)-\frac{1}{N}\sum_{s=1}^{N}P_{s,t+h|t}\left(r\right)\right)\right]dQ_{t+h}$$
$$+\int \left[\frac{1}{N}\sum_{s=1}^{N}\left(P_{t+h|t}\left(r\right)-P_{s,t+h|t}\left(r\right)\right)^{2}\right]dQ_{t+h}$$
$$=\int \left[\left(x_{t+h}\left(r\right)-P_{t+h|t}\left(r\right)\right)^{2}\right]dQ_{t+h}+0+\int \left[\frac{1}{N}\sum_{s=1}^{N}\left(P_{t+h|t}\left(r\right)-P_{s,t+h|t}\left(r\right)\right)^{2}\right]dQ_{t+h}.$$

$\blacksquare$

Proof of Eq. (10).

$$
\begin{aligned}
u^{A}_{t+h}(r) &\equiv \int \left(x_{t+h}(r) - P_{t+h|t}(r)\right)^2 dQ_{t+h} \\
&= \int \left[x_{t+h}(r) - \left(\int x_{t+h}(r)\, dQ_{t+h}\right) + \left(\int x_{t+h}(r)\, dQ_{t+h}\right) - P_{t+h|t}(r)\right]^2 dQ_{t+h} \\
&= \int \left(x_{t+h}(r) - \int x_{t+h}(r)\, dQ_{t+h}\right)^2 dQ_{t+h} + \int \left(\int x_{t+h}(r)\, dQ_{t+h} - P_{t+h|t}(r)\right)^2 dQ_{t+h} \\
&+ 2\int \left[\left(x_{t+h}(r) - \int x_{t+h}(r)\, dQ_{t+h}\right)\left(\int x_{t+h}(r)\, dQ_{t+h} - P_{t+h|t}(r)\right)\right] dQ_{t+h} \\
&= V\left(x_{t+h}(r)\right) + \int \left(\int x_{t+h}(r)\, dQ_{t+h} - P_{t+h|t}(r)\right)^2 dQ_{t+h} - 2Cov(x_{t+h}(r)\, P_{t+h|t}(r)),
\end{aligned}
$$

where the last line follows from the fact that

$$
\begin{aligned}
&2\int \left[\left(x_{t+h}(r) - \int x_{t+h}(r)\, dQ_{t+h}\right)\left(\int x_{t+h}(r)\, dQ_{t+h} - P_{t+h|t}(r)\right)\right] dQ_{t+h} \\
&= 2\int \left[\begin{array}{c} x_{t+h}(r)\left(\int x_{t+h}(r)\, dQ_{t+h}\right) - x_{t+h}(r)\, P_{t+h|t}(r) \\ -\left(\int x_{t+h}(r)\, dQ_{t+h}\right)^2 + P_{t+h|t}(r)\int x_{t+h}(r)\, dQ_{t+h} \end{array}\right] dQ_{t+h} \\
&= 2\int \left[P_{t+h|t}(r)\int x_{t+h}(r)\, dQ_{t+h} - x_{t+h}(r)\, P_{t+h|t}(r)\right] dQ_{t+h} \\
&= 2\left[\left(\int P_{t+h|t}(r)\, dQ_{t+h}\right)\left(\int x_{t+h}(r)\, dQ_{t+h}\right) - \left(\int x_{t+h}(r)\, P_{t+h|t}(r)\, dQ_{t+h}\right)\right] \\
&\equiv -2Cov(x_{t+h}(r)\, P_{t+h|t}(r)).
\end{aligned}
$$

Furthermore, note that $\int \left(\int x_{t+h}(r)\, dQ_{t+h} - P_{t+h|t}(r)\right)^2 dQ_{t+h} = \int \left(\left(\int P_{t+h|t}(r)\, dQ_{t+h}\right) - \left(\int x_{t+h}(r)\, dQ_{t+h}\right)\right)^2 dQ_{t+h}$ $+V(P_{t+h|t}(r))$. The latter result follows from noting that:

$$
\begin{aligned}
&\int \left(\int x_{t+h}(r)\, dQ_{t+h} - P_{t+h|t}(r)\right)^2 dQ_{t+h} \\
&= \int \left(\left[P_{t+h|t}(r) - \left(\int P_{t+h|t}(r)\, dQ_{t+h}\right)\right.\right. \\
&\left.\left.+ \left(\int P_{t+h|t}(r)\, dQ_{t+h}\right) - \left(\int x_{t+h}(r)\, dQ_{t+h}\right)\right]^2\right) dQ_{t+h} \\
&= Var(P_{t+h|t}(r)) + \int \left(\left(\int P_{t+h|t}(r)\, dQ_{t+h}\right) - \left(\int x_{t+h}(r)\, dQ_{t+h}\right)\right)^2 dQ_{t+h} \\
&+ 2\int \left[P_{t+h|t}(r) - \left(\int P_{t+h|t}(r)\, dQ_{t+h}\right)\right] \times \\
&\times \left[\left(\int P_{t+h|t}(r)\, dQ_{t+h}\right) - \left(\int x_{t+h}(r)\, dQ_{t+h}\right)\right] dQ_{t+h} \\
&= Var(P_{t+h|t}(r)) + \int \left(\left(\int P_{t+h|t}(r)\, dQ_{t+h}\right) - \left(\int x_{t+h}(r)\, dQ_{t+h}\right)\right)^2 dQ_{t+h} \\
&\equiv Var(P_{t+h|t}(r)) + \left[E\left(P_{t+h|t}(r)\right) - E\left(x_{t+h}(r)\right)\right]^2
\end{aligned}
$$

since $2\int\left[P_{t+h|t}\left(r\right)-\left(\int P_{t+h|t}\left(r\right)\ dQ_{t+h}\right)\right]\left[\left(\int P_{t+h|t}\left(r\right)dQ_{t+h}\right)-\left(\int x_{t+h}\left(r\right)dQ_{t+h}\right)\right]dQ_{t+h} = 2\left[\left(\int P_{t+h|t}\left(r\right)dQ_{t+h}\right)-\left(\int x_{t+h}\left(r\right)dQ_{t+h}\right)\right]\int\left[P_{t+h|t}\left(r\right)-\left(\int P_{t+h|t}\left(r\right)\ dQ_{t+h}\right)\right]dQ_{t+h}=0.$ ■

Proof of Eq. (13). Our measure of uncertainty is the negative of the expectation of the CRPS (Gneiting and Raftery, 2007). Note that $CRPS(F, y_{t+h}) = -\int_{-\infty}^{\infty}(F(r)-1\{y_{t+h}<r\})^2dr = -\overline{U}_{t+h}^{A}$, where $F(r)$ is the aggregate predictive distribution. Let $G(r)$ denote the ideal distribution, i.e. $G\left(r\right)=1\{y_{t+h}<r\}$. The distributions are conditional distributions, but for notational simplicity we omit the conditionality. Then by Lemma 2.2 of Baringhaus and Franz (2004), we have $\overline{U}_{t+h}^{A}=\int_{-\infty}^{\infty}(F(r)-1\{y_{t+h}<r\})^2dy = E|Y_{1,t+h}-y_{1,t+h}|-\frac{1}{2}E|Y_{1,t+h}-Y_{2,t+h}|-\frac{1}{2}E|y_{1,t+h}-y_{2,t+h}|$, where $Y_{1,t+h}$ and $Y_{2,t+h}$ are i.i.d draws from F, while $y_{1,t+h}$ and $y_{2,t+h}$ are i.i.d. draws from $G\left(r\right)$, and both of these variables have finite expectations. Given Lemma 2.1 of Baringhaus and Franz (2004), $E|y_{1,t+h}-Y_{1,t+h}|=\int_{-\infty}^{\infty}F(r)(1-G(r))dr+\int_{-\infty}^{\infty}G(r)(1-F(r))dr$. Now, for $y_{1,t+h}$ and $y_{2,t+h}$, we have $E|y_{1,t+h}-y_{2,t+h}|=2\int_{-\infty}^{\infty}G(r)(1-G(r))dy=2\int_{-\infty}^{\infty}1\{y_{t+h}<r\}(1-1\{y_{t+h}<r\})dy=0$, where the last equality follows from the fact that, for a particular value of r, either $1\{y_{t+h}<r\}$ or $1-1\{y_{t+h}<r\}$ will be zero, and, thus, the product will be zero. Therefore,

$$\overline{U}_{t+h}^{A}=\int_{-\infty}^{\infty}(F(r)-1\{y_{t+h}<r\})^2dr = E|Y_{1,t+h}-y_{1,t+h}|-\frac{1}{2}E|Y_{1,t+h}-Y_{2,t+h}|. \tag{16}$$

This means we can rewrite aggregate uncertainty as the sum of expected absolute distance measures of random variables coming from the predictive distribution, and that coming from the predictive distribution and the true distribution which generates the realization. If $F(r)$ is the Gaussian distribution, i.e. if $Y_{t+h}\sim iidN(\mu_{t+h},\sigma_{t+h}^2)$, then by the property that the difference of the i.i.d normal random variables is distributed normally (in this case centered around zero with a variance of $2\sigma_{t+h}^2$), and the fact that the absolute value of a mean zero random Normal variable has a half-normal distribution with mean $\frac{2\sigma_{t+h}}{\sqrt{\pi}}$, we have

$$\frac{1}{2}E|Y_{1,t+h}-Y_{2,t+h}|=\frac{\sigma_{t+h}}{\sqrt{\pi}}. \tag{17}$$

To obtain $E|Y_{t+h}-y_{t+h}|$, we use the properties of Dirac delta function. We denote the PDF of y_{t+h} by a Dirac delta function $\delta(r-y_{t+h})$. From the properties of the Dirac function, $E(y_{t+h})=y_{t+h}$ and $Var(y_{t+h})=0$. Then, $Y_{1,t+h}-y_{1,t+h}\sim N(\mu_{t+h}-y_{t+h},\sigma_{t+h}^2)$. By property of the folded Normal distribution, we have:

$$E|Y_{t+h}-y_{t+h}|=\sigma_{t+h}2\varphi\left(-\frac{\mu_{t+h}-y_{t+h}}{\sigma_{t+h}}\right)+(\mu_{t+h}-y_{1,t+h})\left(1-2\Phi\left(-\frac{\mu_{t+h}-y_{t+h}}{\sigma_{t+h}}\right)\right). \tag{18}$$

Substituting (18) and (17) into (16), we get the result:

$$\overline{U}_{t+h}^{A}=\underbrace{\left[2\sigma_{t+h}\phi\left(\frac{y_{t+h}-\mu_{t+h}}{\sigma_{t+h}}\right)+(y_{t+h}-\mu_{t+h})\left(2\Phi\left(\frac{y_{t+h}-\mu_{t+h}}{\sigma_{t+h}}\right)-1\right)\right]}_{\text{“Ex-post”}}-\underbrace{\frac{\sigma_{t+h}}{\sqrt{\pi}}}_{\text{“Ex-ante”}}, \tag{19}$$

and $U^A_{t+h} = \int \overline{U}^A_{t+h} dQ_{t+h}$. ■

Appendix B. Data

As the main text indicates, the fixed-horizon forecasts are constructed as a weighted average of the current and next year forecasts. Figure A1 shows the number of forecasters that provided forecasts for both, current year and next year, as well as the number of forecasters that have provided forecasts for either one of the years, but not both. As the figure shows, the latter group is not large. By limiting our attention to forecasters that provide forecasts for both years we loose 10% of the total number of observations. The maximum per period loss amounts to 30% of forecaster observations. In our sample we have 239 unique forecasters. Out of those, 108 have been providing forecasts more than twelve times. The sample has 31 forecasters that have provided density forecast for 8 or more but less than 12 times, while 37 of them provided forecasts for 4 times and more, but less than 8 times. Thus, the majority of the forecasters in our sample are repeated forecasters, which increases the confidence that our results are not driven by outliers.

D'Amico and Orphanides (2014) highlight the role of approximations in individual predictive distributions. The idea is that many forecasters tend to put a lot of weight on a few bins and zeros on other bins. D'Amico and Orphanides (2014) argue that this could be forecasters' true perceived uncertainty. However, as they suggest, it is also possible that forecasters just use approximations and lump small probabilities into the adjacent bins. To shed some light on this issue, in Figure A2, Panel 1, we show the percentage of forecasters that put probabilities into one bin, two bins and three bins. In general, forecasters with all the probabilities on one bin and two bins are few. However, a non-negligible proportion of forecasters puts all the probabilities on three bins. The proportion of these forecasters is higher prior to 1992:I. This is explained by the structure of the bins at that point. In our sample period, the bin structure for GDP/GNP growth has changed three times. Between 1981:III and 1991:IV there were 6 bins covering [<-2 -2 to -0.1 0 to 1.9 2 to 3.9 4 to 5.9 6+], between 1992:I and 2009:I the bins were [<-2 -2 to -1.1 -1 to -0.1 0 to 0.9 1 to 1.9 2 to 2.9 3 to 3.9 4 to 4.9 5 to 5.9 6+], while since 2009:II the bins have been covering the following intervals [<-3 -3 to -2.1 -2 to -1.1 -1 to -0.1 0 to 0.9 1 to 1.9 2 to 2.9 3 to 3.9 4 to 4.9 5 to 5.9 6+]. Note that in the beginning of the sample the bins have been fairly wide, not giving forecasters opportunities to differentiate among bins. Given that we use a Gaussian approximation, in order to strive for accuracy we adjusted the bins in the period between 1981:III and 1991:IV. The modified grid doubles the number of bins in that period, splitting the original probabilities in each bin uniformly over the newly created ones. Effectively the grid structure in that period becomes the same as in the period between 1992:I and 2009:I. The summary of the number of forecasts with probabilities on one, two and three bins with the modified grid specification is provided in Figure A2, Panel B. The figure shows that, by construction, there are not many forecasts with probabilities on less than or equal to three bins left in the period prior to 1992:I. Moreover, we discarded densities that put all the probability mass on one bin in the calculations. The second source of approximation error that arises when working with SPF probability forecast histograms is the open ended nature of the first and last intervals. In practice, we close these intervals. We assume that the open intervals have the same length as the rest of the intervals in the respective grids. Panels C in Figure A2 shows the proportion of forecasters assigning a probability value on the leftmost and rightmost bins in the survey. On the one hand, these proportions are not negligible, and our choice of dealing with the leftmost and rightmost intervals might have some impact on the overall results. On the other hand, Panel D suggests that the probability value associated with these open intervals is small. Thus, closing the open intervals should not induce a large approximation error. Lastly, since we approximate the histograms with a Gaussian distribution, we use the mid-point approach: when fitting a Gaussian kernel we associate all the probability mass with the midpoint of the interval.

Figure A1: Forecasters with Current Year and Next Year Forecasts

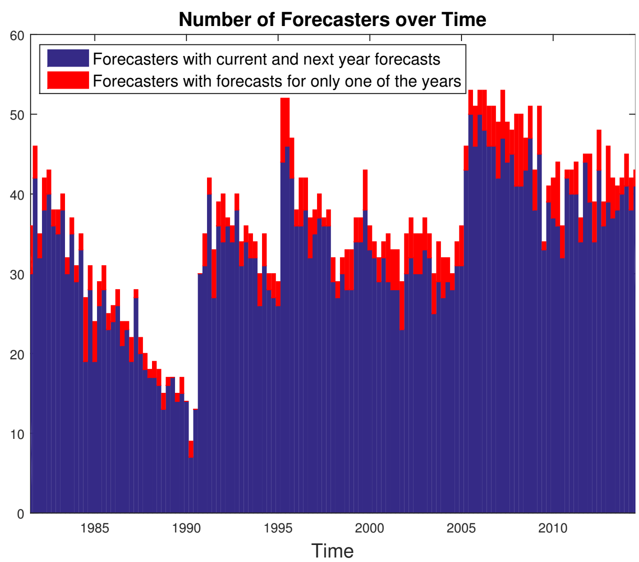
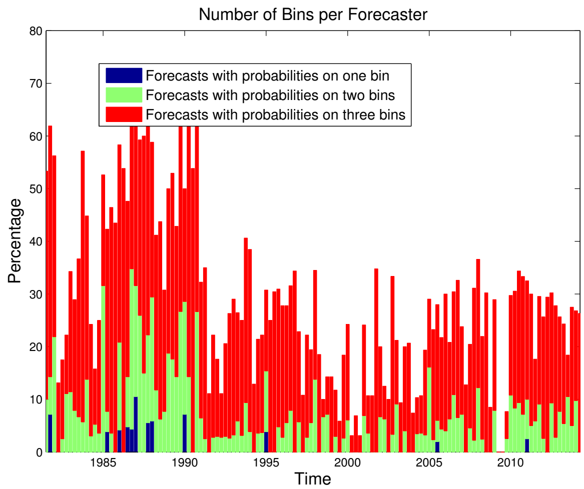
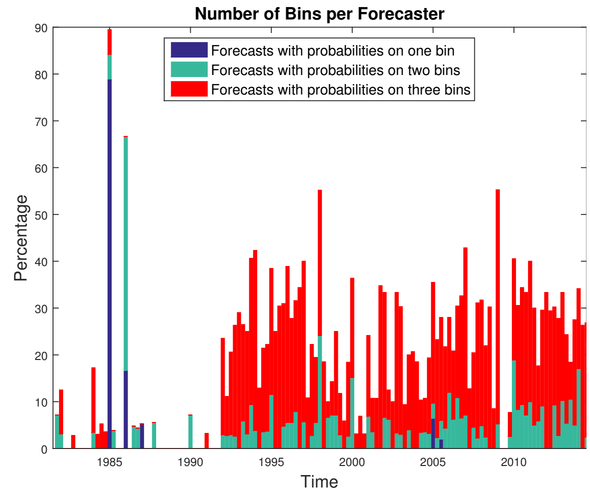


Figure A2: Bin Statistics

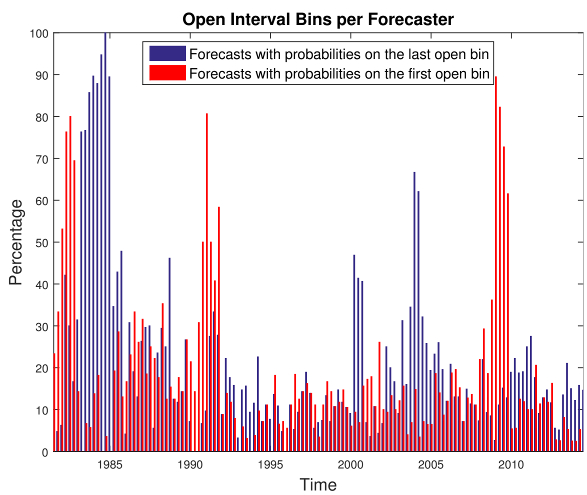
Panel A: Forecasts without Grid Adjustment



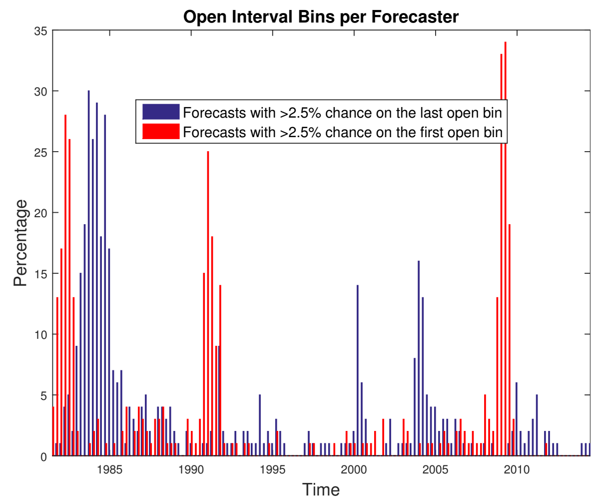
Panel B: Forecasts after Grid Adjustment



Panel C: Forecasts with Open Intervals



Panel D: Forecasts with >2.5% on Open Intervals



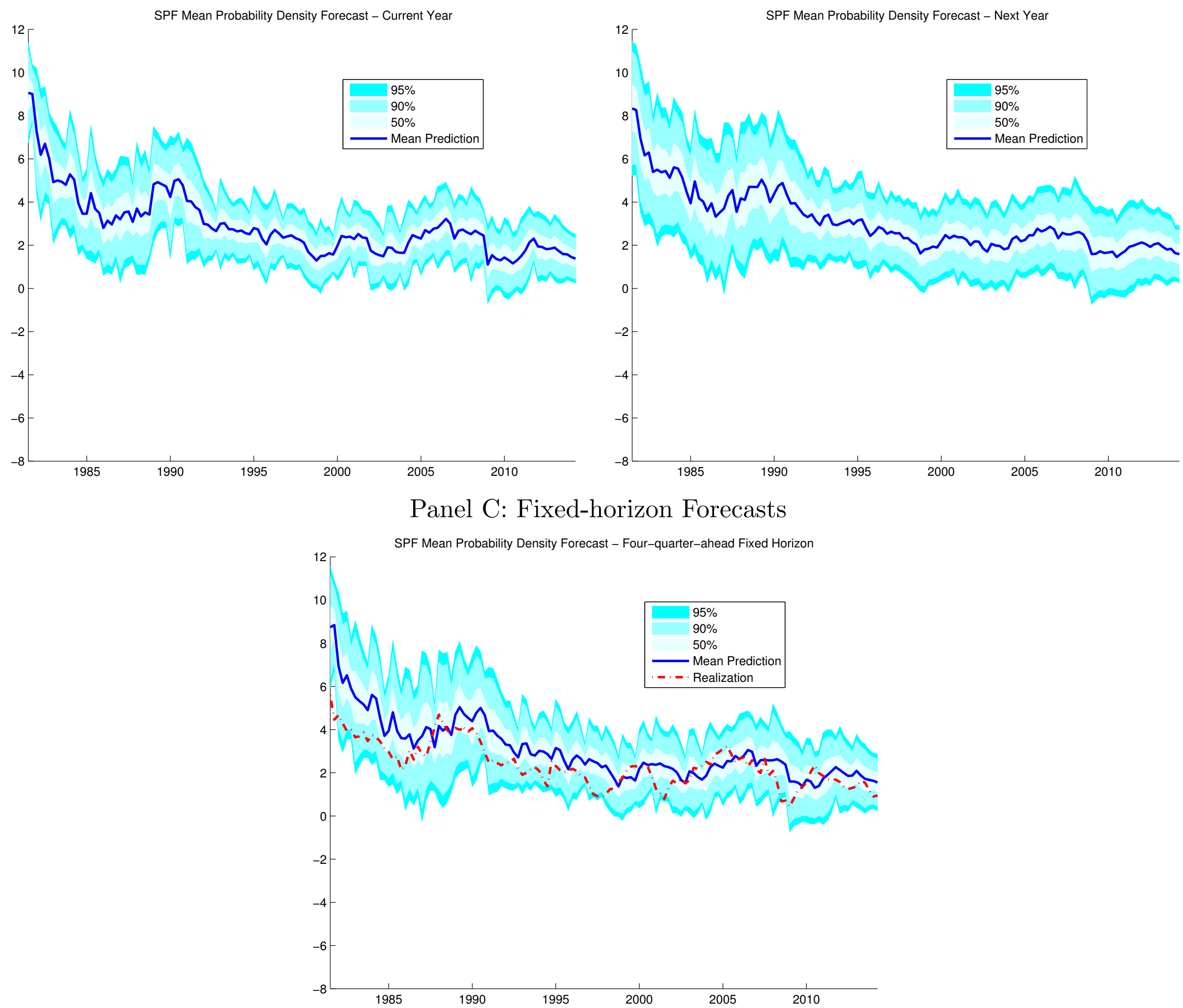
Not-for-Publication Appendix

A.1 Descriptive Analysis of Inflation Forecasts

Figure I. The Survey of Professional Forecasters Data: Inflation

Panel A: Current Year Forecasts

Panel B: Next Year Forecasts



Note: The figure shows the quantiles of the SPF four-step-ahead predictive density, its mean, as well as the realized value of inflation. The four-step-ahead density is constructed from SPF's current year and next year density forecasts based on eq. (15)

A.2 Reliability and Resolution Analysis

Note that an additional, interesting decomposition for $u^{A}_{t+h|t}(r)$ can be obtained following Murphy (1973):

$$u^{A}_{t+h|t}(r) \simeq REL_{t+h|t}(r) - RES_{t+h|t}(r) + V\left(x_{t+h}(r)\right) \tag{20}$$

where:[21]

- $REL_{t+h|t}(r) \equiv E\left(\left[p_{t+h|t}(r) - E\left(x_{t+h}(r)\,|p_{t+h|t}(r)\right)\right]^2\right)$ measures the reliability of the forecast and scores the calibration of the forecast. A forecast is said "reliable" when the observed frequency is consistent with the probabilistic forecast made for a given event. For instance, forecasts that predict a probability of recession of 30 percent will be reliable if the economy effectively enters a recession 30 percent of the time every time such a forecast is made. Hence, reliability measures the unconditional (un)biasedness of the probabilistic forecasts. Because the term is expressed as a squared error, the smaller the calibration error, the better (i.e., the lower) the Brier score.

- $RES_{t+h|t}(r) \equiv \int \left[E\left(x_{t+h}(r)\,|p_{t+h|t}(r)\right) - E\left(x_{t+h}(r)\right)\right]^2$ is the resolution, i.e. the average squared differential of the conditional and unconditional means of the observed outcomes. It captures the "decisiveness" of forecasts by comparing the forecast probability and the long-term average of the underlying process. The larger the term, the lower the Brier score.

As we show below, Eq. (20) holds up to an approximation error that involves within bin variation.

The decomposition can be estimated as follows.

Reliability is estimated as follows. For each t, determine which of the forecast bins $p_{t+h|t}(r)$ falls into. Let $\left\{p^{(k)}_{t+h|t}(r)\right\}$ be the collection of probabilities in the k-th bin and let $p^{E}_{t+h|t}(r)$ denote the unconditional expected value over the bin. We estimate $p^{E}_{t+h|t}(r)$ using a Uniform distribution over the bin, so that $p^{E}_{t+h|t}(r)$ is the midpoint of the bin.[22] In addition, let the number of probabilities in each bin be n_k. Let $\overline{x}_k$ be the average of the realizations conditional on the forecaster having

[21] In this sub-section we use $p_{t+h|t}$ to denote the CDF in order to better link with the notation in the literature. Expectations are with respect to Q_{t+h}.

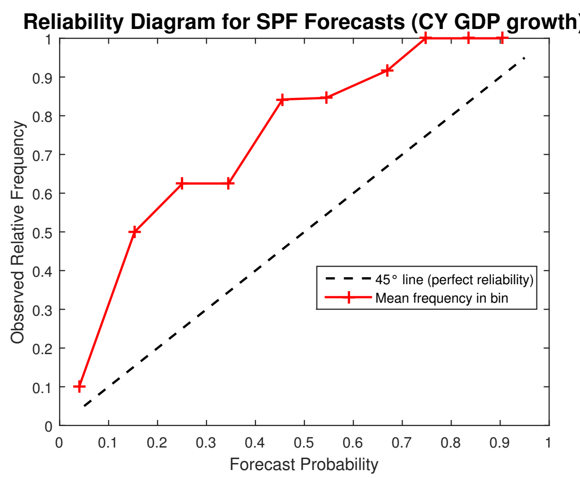
[22] In the 3-terms decomposition that we discuss here, we abstract from within bin variance and within bin covariance; thus, the unconditional expected value over the bin is indeed the midpoint of the bin and all forecasts in the bin are imposed to be equal to the midpoint (so their average is also the midpoint). We derive a 5-terms decomposition which includes within bin variance and within bin covariance (Stephenson, Coelho and Joliffe, 2008). In that case, the reliability will be calculated using the average forecast in the bin without imposing that all forecasts in the bin are equal. That is, $\overline{p}^{(k)}_{t+h|t}(r)$ (which is the average of the collection of probabilities in the k-th bin, $\left\{p^{(k)}_{t+h|t}(r)\right\}$), replaces $p^{E}_{t+h|t}(r)$ in eq. (21).

made the probability forecast associated with the collection of probabilities in bin k, $\left\{p_{t+h|t}^{(k)}(r)\right\}$. Reliability is the average square calibration error, that is,

$$REL(r) = \frac{1}{T}\sum_{k=1}^{K} n_k \left(p_{t+h|t}^{E}(r) - \overline{x}_k(r)\right)^2 . \tag{21}$$

Thus, reliability measures the squared deviation of the predicted probability from the observed outcome conditional probability of the event. This effectively tells the user how often (as a percentage) a forecast probability actually occurred. In theory, a perfect forecasting model will result in forecasts with a probability of $\alpha\%$ being consistent with the eventual outcome $\alpha\%$ of the time. Note that a forecast is reliable if the average square calibration error (REL) is small. Figure II provides intuition to understand reliability. The x-axis reports the forecast probability,[23] while the y-axis reports the observed relative frequency. A reliable forecast would be the 45-degree line, where the observed frequency of realizations equals the forecast probability; the data clearly show departures from reliability in our sample.

Figure II. Reliability Diagram



Notes. The figure plots the reliability diagram for SPF forecasts of current year (CY) GDP growth.

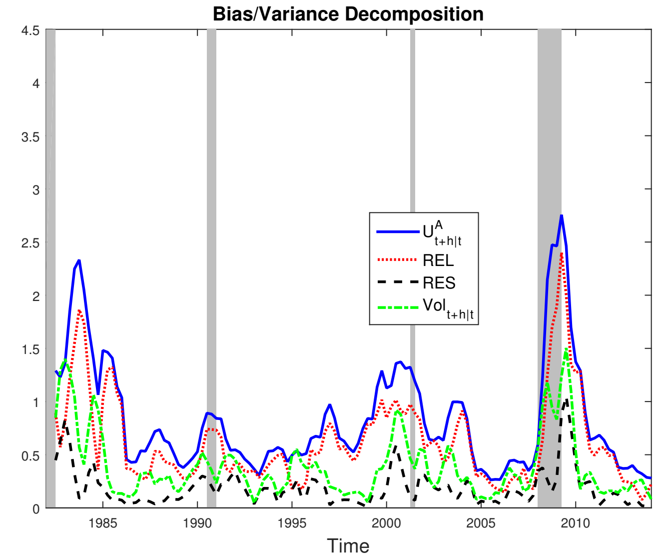
Resolution is the squared average difference between the conditional mean (given the forecast) and the unconditional mean: $RES(r) = \frac{1}{T}\sum_{k=1}^{K} n_k \left(\overline{x}_k(r) - \overline{x}(r)\right)^2$. Note that good forecasts have high resolution.

Figure III shows the evolution of the components of the alternative decomposition over time.[24]

[23]The forecast probability is the mid-point of the bin in the forecast distribution.

[24]Finally, note that the practical implementation of the Brier score involves "binning". Binning smooths the data and makes them less noisy, as larger bins limit the "sparseness" problem (Stephenson et al., 2008). Some information is lost, however, by approximating continuous probability densities with a discrete number of bins.

Figure III. Aggregate Uncertainty, Reliability, Resolution and (Realized) Risk



Notes. The figure displays Aggregate Uncertainty, Reliability, Resolution and Realized Risk.

Proof of eq. (20). In practice, the Murphy decomposition requires partitioning the range of forecasts – essentially, the [0,1] line – into K sub-segments. Let r be a number along the real line; let $\overline{p}^{(k)}$ denote the average probability in segment k;[25] and let n_k denote the number of forecast probabilities that fall in the k-th sub-segment, for $k = 1, \ldots, K$. Given all forecasts in the sample,

[25] Alternatively, one could consider $\overline{p}^{(k)}$ as the midpoint of the k-th segment

the Brier score can be broken down as follows:

$$
\begin{aligned}
\frac{1}{T}\sum_{t=1}^{T}[x_{t+h}(r) - P_{t+h|t}(r)]^2 &= \frac{1}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[x_{t+h}^{(j)}(r) - p_{t+h|t}^{(j)}(r)\right]^2 \\
&= \frac{1}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[x_{t+h}^{(j)}(r) - \overline{x}_{t+h}^{(k)}(r) + \overline{x}_{t+h}^{(k)}(r) - \overline{p}_{t+h|t}^{(k)}(r) + \overline{p}_{t+h|t}^{(k)}(r) - p_{t+h|t}^{(j)}(r)\right]^2 \\
&= \frac{1}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[x_{t+h}^{(j)}(r) - \overline{x}_{t+h}^{(k)}(r)\right]^2 + \frac{1}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[\overline{x}_{t+h}^{(k)}(r) - \overline{p}_{t+h|t}^{(k)}(r)\right]^2 \\
&+ \frac{1}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[\overline{p}_{t+h|t}^{(k)}(r) - p_{t+h|t}^{(j)}(r)\right]^2 \\
&+ \frac{2}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[x_{t+h}^{(j)}(r) - \overline{x}_{t+h}^{(k)}(r)\right]\left[\overline{x}_{t+h}^{(k)}(r) - \overline{p}_{t+h|t}^{(k)}(r)\right] \\
&+ \frac{2}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[x_{t+h}^{(j)}(r) - \overline{x}_{t+h}^{(k)}(r)\right]\left[p_{t+h}^{(k)}(r) - p_{t+h|t}^{(j)}(r)\right] \\
&+ \frac{2}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[p_{t+h|t}^{(j)}(r) - \overline{p}_{t+h|t}^{(k)}(r)\right]\left[\overline{x}_{t+h}^{(k)}(r) - \overline{p}_{t+h|t}^{(k)}(r)\right] \\
&= \frac{1}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[x_{t+h}^{(j)}(r) - \overline{x}_{t+h}^{(k)}(r)\right]^2 + \frac{1}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[\overline{x}_{t+h}^{(k)}(r) - \overline{p}_{t+h|t}^{(k)}(r)\right]^2 \\
&+ \frac{1}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[\overline{p}_{t+h|t}^{(k)}(r) - p_{t+h|t}^{(j)}(r)\right]^2 \\
&+ \frac{2}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[x_{t+h}^{(j)}(r) - \overline{x}_{t+h}^{(k)}(r)\right]\left[p_{t+h}^{(k)}(r) - p_{t+h|t}^{(j)}(r)\right].
\end{aligned}
$$

We can already recognize the reliability (REL) in the second term of this decomposition:

$$
\begin{aligned}
REL(r) &= \frac{1}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[\overline{x}_{t+h}^{(k)}(r) - \overline{p}_{t+h|t}^{(k)}(r)\right]^2 \\
&= \frac{1}{T}\sum_{k=1}^{K} n_k\left[\overline{x}_{t+h}^{(k)}(r) - \overline{p}_{t+h|t}^{(k)}(r)\right]^2.
\end{aligned} \tag{22}
$$

The first term can be expressed as follows:

$$\begin{aligned}
\frac{1}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[x_{t+h}^{(j)}(r)-\overline{x}_{t+h}^{(k)}(r)\right]^2 &= \frac{1}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[x_{t+h}^{(j)}(r)-\overline{x}(r)+\overline{x}(r)-\overline{x}_{t+h}^{(k)}(r)\right]^2 \\
&= \frac{1}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[x_{t+h}^{(j)}(r)-\overline{x}(r)\right]^2+\frac{1}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[\overline{x}(r)-\overline{x}_{t+h}^{(k)}(r)\right]^2 \\
&\quad +\frac{2}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[x_{t+h}^{(j)}(r)-\overline{x}(r)\right]\left[\overline{x}(r)-\overline{x}_{t+h}^{(k)}(r)\right] \\
&= \frac{1}{T}\sum_{t=1}^{T}\left[x_{t+h}(r)-\overline{x}(r)\right]^2-\frac{1}{T}\sum_{k=1}^{K}n_k\left[\overline{x}(r)-\overline{x}_{t+h}^{(k)}(r)\right]^2 \\
&\equiv V\left(x_{t+h}\left(r\right)\right)-RES(r).
\end{aligned}$$

Note that because the outcome variable x is binary, the uncertainty term can be expressed as$V\left(x_{t+h}\left(r\right)\right)=\overline{x}(r)(1-\overline{x}(r))$. To summarize, we have decomposed the Brier score in the following way:

$$\begin{aligned}
\frac{1}{T}\sum_{t=1}^{T}[x_{t+h}(r)-p_{t+h|t}(r)]^2 &= V\left(x_{t+h}\left(r\right)\right)+REL(r)-RES(r) \\
&\quad +\frac{1}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[\overline{p}_{t+h|t}^{(k)}(r)-p_{t+h|t}^{(j)}(r)\right]^2 \\
&\quad +\frac{2}{T}\sum_{k=1}^{K}\sum_{j=1}^{n_k}\left[x_{t+h}^{(j)}(r)-\overline{x}_{t+h}^{(k)}(r)\right]\left[p_{t+h}^{(k)}(r)-p_{t+h|t}^{(j)}(r)\right].
\end{aligned}$$

The last two terms measure the variance of forecasts within the sub-segments and the co-movement between forecasts within a segment and their corresponding outcomes. The decomposition therefore writes:

$$\frac{1}{T}\sum_{t=1}^{T}[x_{t+h}(r)-p_{t+h|t}(r)]^2=V\left(x_{t+h}\left(r\right)\right)+REL(r)-RES(r)+WSV(r)+WSC(r).$$

Remark that the last two terms equal zero when all forecasts within the same segment are assumed identical. Because $WSV(r)$ and $WSC(r)$ are quantitatively very small in the data, we will work under the simpler decomposition:

$$\frac{1}{T}\sum_{t=1}^{T}[x_{t+h}(r)-p_{t+h|t}(r)]^2\simeq V\left(x_{t+h}\left(r\right)\right)+REL(r)-RES(r),$$

as per the definitions that we have written. ■

A.3 Results for the Large-Dimensional VAR

This section shows the robustness of our results to a mid-size 11 variable VAR as considered in Jurado, Ludvigson and Ng (2015) specified in the spirit of Christiano, Eichenbaum and Evans (2005). The VAR (11) is in the following variables: log(real GDP), log(employment), log(real consumption), log(PCE deflator), log(real new order), log(real wage), hours, federal funds rate, log(S&P 500 Index), growth rate of M2, and various uncertainty indices discussed in the paper. The variables are downloaded from the 2015-11 version of the FRED-QD (Quarterly Database for Macroeconomic Research) discussed in McCracken and Ng (2015). The labels on the impulse responses carry the mnemonics of the variables in the database described in Table I.

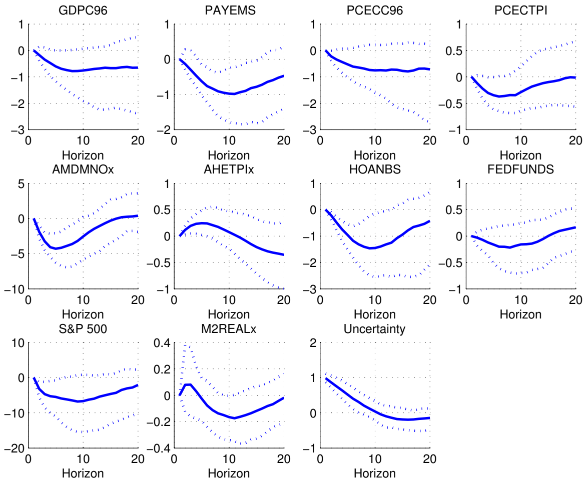
Table I. Description of Variables Included in the VAR

Variable	*Mnemonics*	*Description*
real GDP	GDPC96	Real Gross Domestic Product, 3 Decimal (Billions of Chained 2009 Dollars)
Employment	PAYEMS	All Employees: Total nonfarm (Thousands of Persons)
Real Consumption	PCECC96	Real Personal Consumption Expenditures (Billions of Chained 2009 Dollars)
PCE deflator	PCECTPI	Personal Consumption Expenditures: Chain-type Price Index (Index 2009=100)
real new order	AMDMNOx	Real Manufacturers' New Orders: Durable Goods (Millions of 2009 Dollars),
		deflated by Core PCE
real wage	AHETPIx	Real Average Hourly Earnings of Production and Nonsupervisory Employees:
		Total Private (2009 Dollars per Hour), deflated by Core PCE
hours	HOANBS	Nonfarm Business Sector: Hours of All Persons (Index 2009=100)
federal funds rate	FEDFUNDS	Effective Federal Funds Rate (Percent)
S&P 500 Index	S&P 500	S&P's Common Stock Price Index: Composite
M2	M2REALX	Real M2 Money Stock (Billions of 1982-84 Dollars)

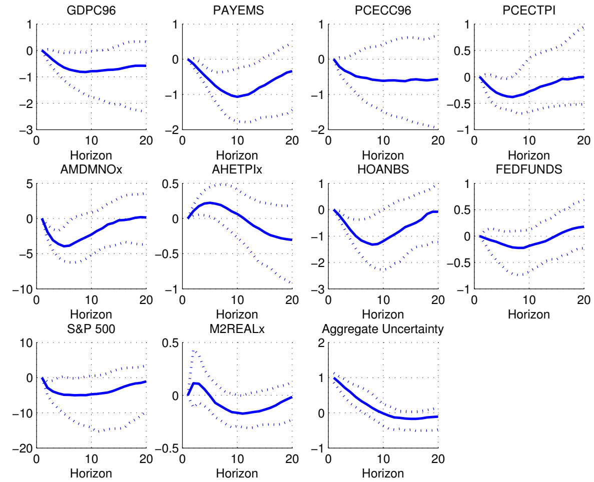
The impulse responses to shocks in the uncertainty indices are displayed in Figures IV-VII. The figures show that the findings are in general the same as those we report in the main text: all uncertainty measures are recessionary in nature. The ex-post measures, as well as realized volatility, have higher impact in magnitude than disagreement or ex-ante uncertainty. Also, even in the large VAR, GDP increases after a shock to dispersion.

Figure IV: Macroeconomic Impact of Uncertainty

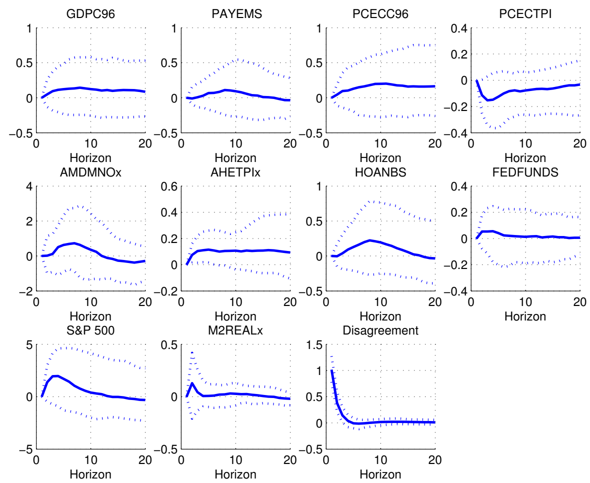
Panel A: Uncertainty



Panel B: Aggregate Uncertainty

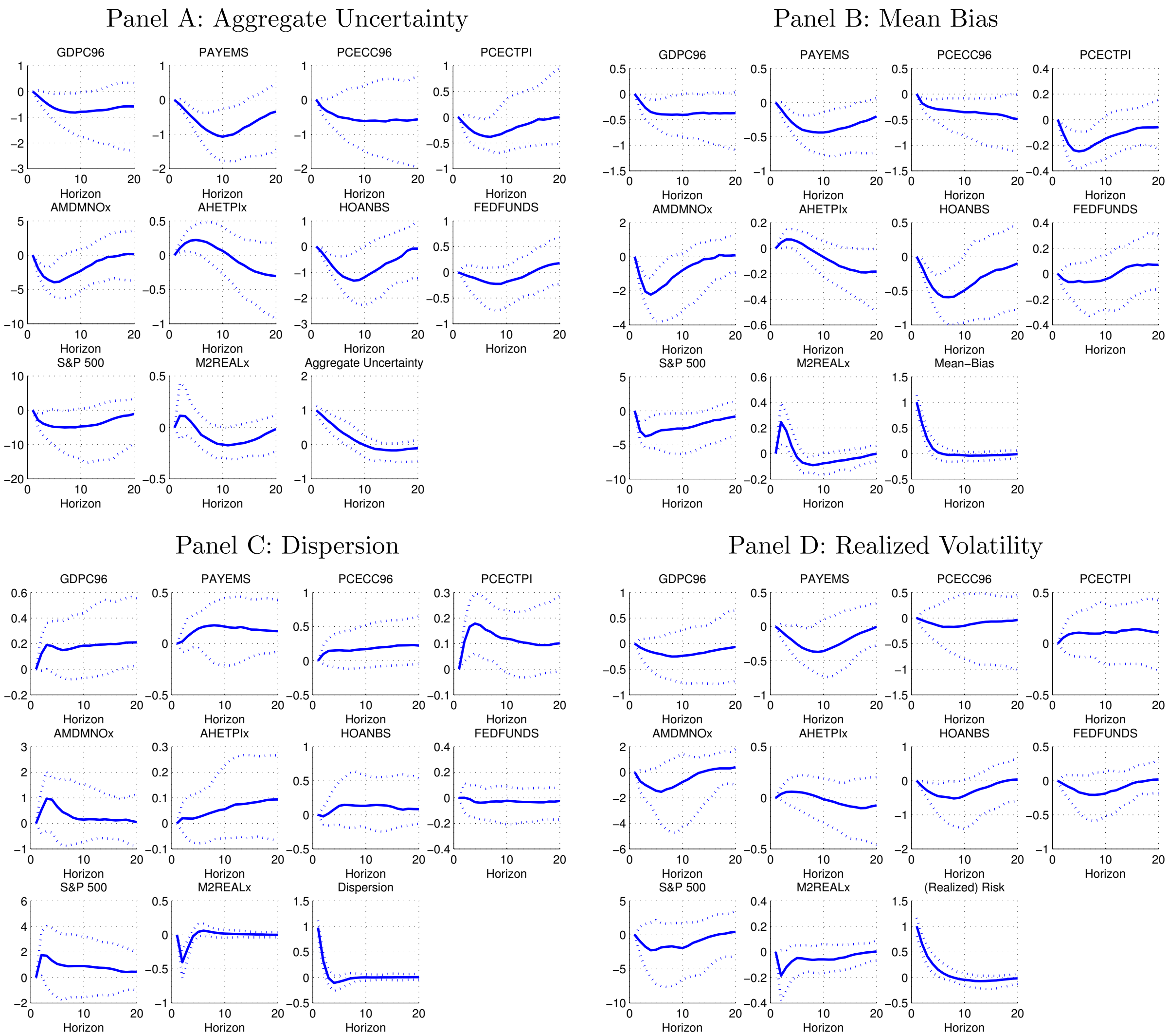


Panel C: Disagreement



Note: The figure shows the impulse responses of uncertainty, aggregate uncertainty and disagreement based on eq. (9).

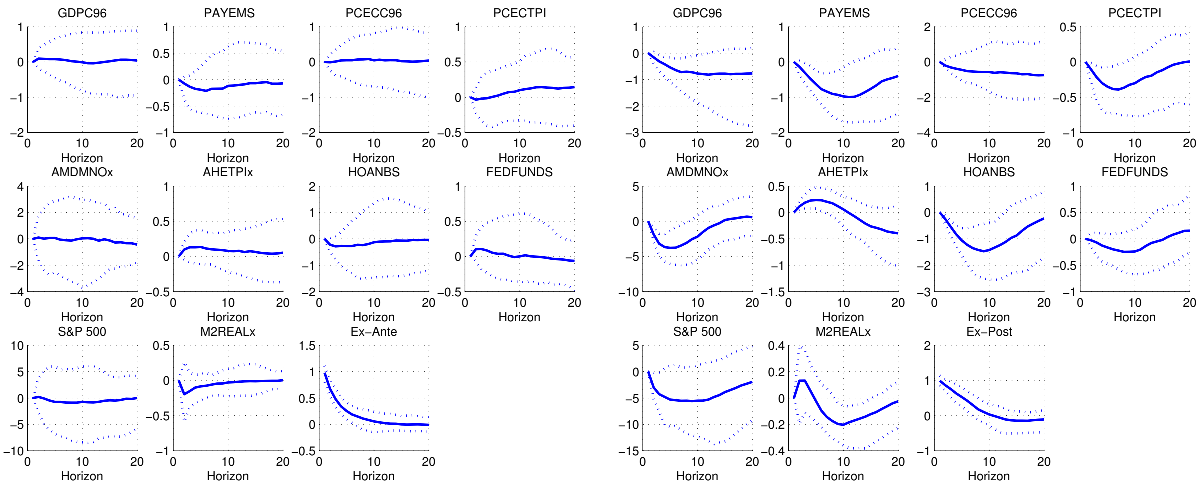
Figure V: Macroeconomic Effect of Uncertainty



Note: The figure shows the impulse responses of the ex-ante and ex-post measures of uncertainty based on eq. (11). The uncertainty measures have been standardized.

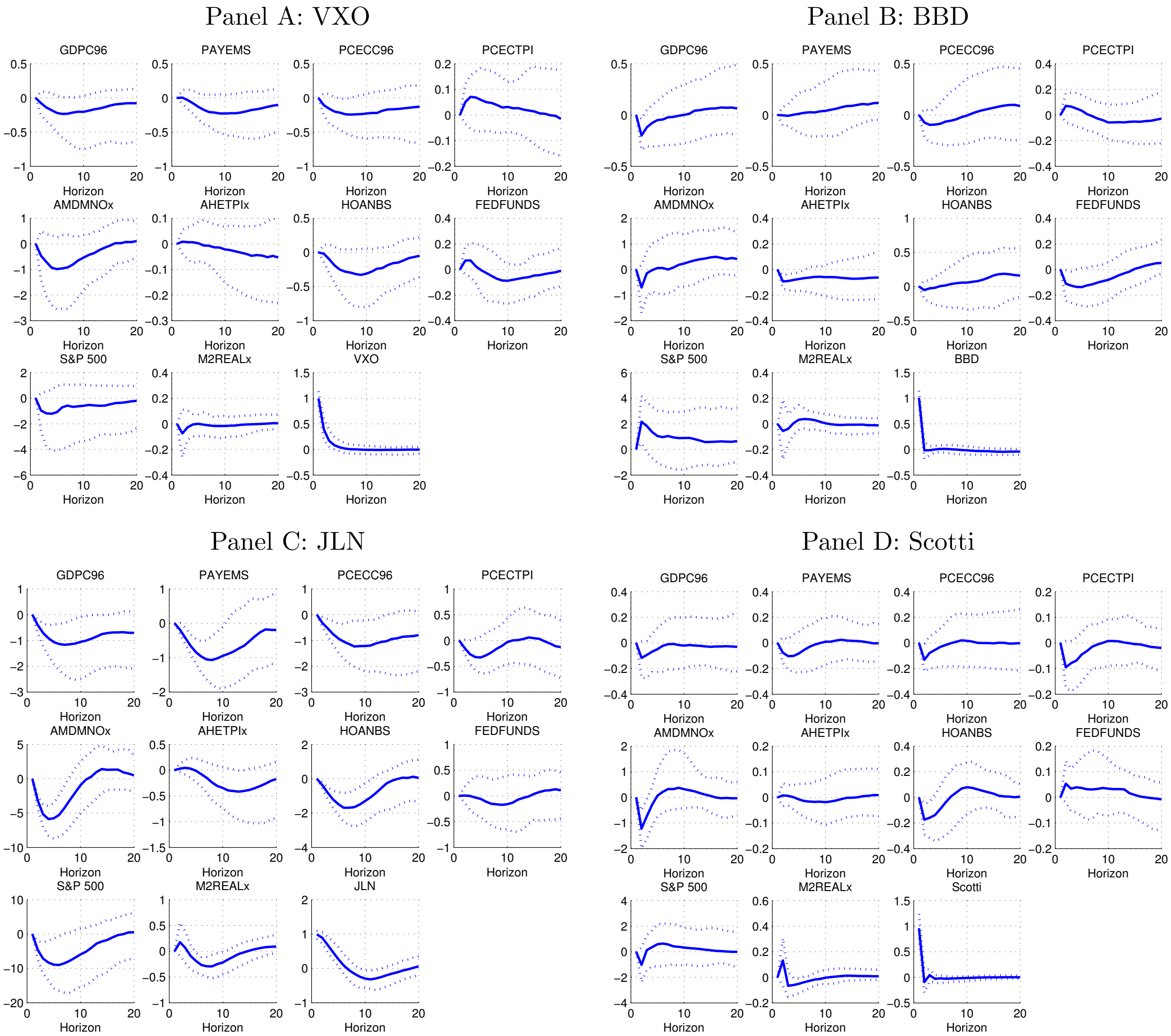
Figure VI: Macroeconomic Impact of Uncertainty

Panel B: Ex-Ante Uncertainty　　　　Panel C: Ex-Post Uncertainty



Note: The figure shows the impulse responses of the ex-ante and ex-post measures of uncertainty based on eq. (13). The uncertainty measures have been standardized.

Figure VII: Macroeconomic Effect of Uncertainty - Alternative Measures



Note: The figure shows the impulse responses for the following uncertainty measures: VXO, JLN, BBD and Scotti. The uncertainty measures have been standardized.

A.4 Estimation

We estimate the decomposition using its sample counterparts:

$$\widehat{U}_{t+h|t} = \int_{-\infty}^{+\infty} \widehat{u}_{t+h|t}\left(r\right) dr, \ \ t = R, ..., T$$

where R is the size of the rolling window,

$$\widehat{u}_{t+h|t}\left(r\right) = \frac{1}{R} \sum_{j=t-R+1}^{t} \frac{1}{N} \sum_{s=1}^{N} u_{s,j+h|j}\left(r\right) = \frac{1}{R} \sum_{j=t-R+1}^{t} \frac{1}{N} \sum_{s=1}^{N} \left[x_{t+h}\left(r\right) - p_{s,j+h|j}\left(r\right)\right]^2$$

and

$$\begin{aligned} \widehat{U}^{A}_{t+h|t} = & \int_{-\infty}^{+\infty} \left(\overline{p}_{t+h|t}\left(r\right) - \overline{x}_{t+h}\left(r\right)\right)^2 dr + \int_{-\infty}^{+\infty} \widehat{V}(p_{t+h|t}\left(r\right))dr \\ & + \int_{-\infty}^{+\infty} \widehat{V}ol_{t+h|t}\left(r\right) dr - 2 \int_{-\infty}^{+\infty} \widehat{Cov}(x_{t+h}\left(r\right), p_{t+h|t}\left(r\right))dr, \end{aligned} \tag{23}$$

where the terms on the RHS of eq. (23) are as follows:

- $\overline{p}_{t+h|t}\left(r\right), \overline{x}_{t+h}\left(r\right)$ are estimated by $\frac{1}{R} \sum\limits_{j=t-R+1}^{t} p_{j+h|j}\left(r\right), \ \frac{1}{R} \sum\limits_{j=t-R+1}^{t} x_{j+h}\left(r\right);$

- $\widehat{V}ol_{t+h}\left(x_{t+h}\left(r\right)\right)$ is an estimate of the variance of $x_{t+h}\left(r\right)$, which is a binary variable, recursively over time:

$$\widehat{V}ol_{t+h}\left(x_{t+h}\left(r\right)\right) = \overline{x}_{t+h}\left(1 - \overline{x}_{t+h}\right);$$

-$\widehat{V}_{t+h}\left(p_{t+h|t}\left(r\right)\right)$ is an estimate of the variance of $p_{t+h|t}\left(r\right)$ recursively over time:

$$\widehat{V}_{t+h}\left(p_{t+h|t}\left(r\right)\right) = \frac{1}{R} \sum_{j=t-R+1}^{t} \left(p_{j+h|j}\left(r\right) - \overline{p}_{t+h|t}\left(r\right)\right)^2$$

- $\widehat{Cov}(x_{t+h}\left(r\right) p_{t+h|t}\left(r\right))$ is estimated as:

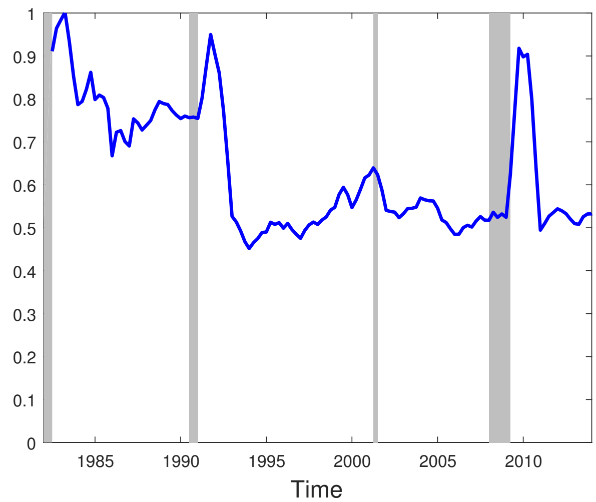
$$\widehat{Cov}(x_{t+h}\left(r\right), p_{t+h|t}\left(r\right)) = \frac{1}{R} \sum_{j=t-R+1}^{t} \left(p_{j+h|j}\left(r\right) - \overline{p}_{t+h|t}\left(r\right)\right) \left(x_{j+h}\left(r\right) - \overline{x}_{t+h}\left(r\right)\right)$$

While we do not need the Normality assumption to calculate the decomposition above, in practice we fit a Gaussian distribution to the predictive density. The main reason is to guarantee that the "Knightian uncertainty/(Realized) Risk" decomposition is consistent with the"Ex-ante"/"Ex-post", since the latter is valid only under Normality. Furthermore, in the empirical implementation we let $R = 4$, which amounts to calculating 4-quarter-moving average of the various components of uncertainty, and we proxy the indefinite integrals with definite ones by treating the extrema of either the realization or the bins as integral bounds.

A.5 Estimation

The "ex-ante" uncertainty, $\sigma_{t+h|t}/\sqrt{\pi}$, can be more generally estimated, for any predictive distribution, as: $\int_{-\infty}^{+\infty}\left[\int\left(x_{t+h}\left(r\right)-P_{s,t+h|t}\left(r\right)\right)^{2}dP_{s,t+h|t}\right]dr=\int P_{s,t+h|t}\left(r\right)\left(1-P_{s,t+h|t}\left(r\right)\right)dr$. Figure VII shows indeed that they are the same.

Figure VII: Alternative Estimation of Ex-Ante Uncertainty



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A.6 Robustness to Final Release

This sub-section evaluates the robustness of the results to using final releases of data instead of real-time vintages of data. By comparing Figure VIII with Figure 3 in the paper, it is clear that the results are robust.[26]

Figure VIII: Decomposing Uncertainty

Panel A: Uncertainty, Aggregate Uncertainty and Disagreement

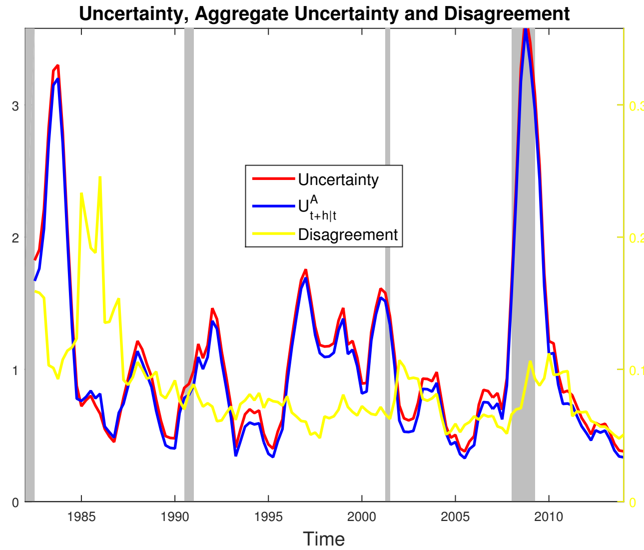
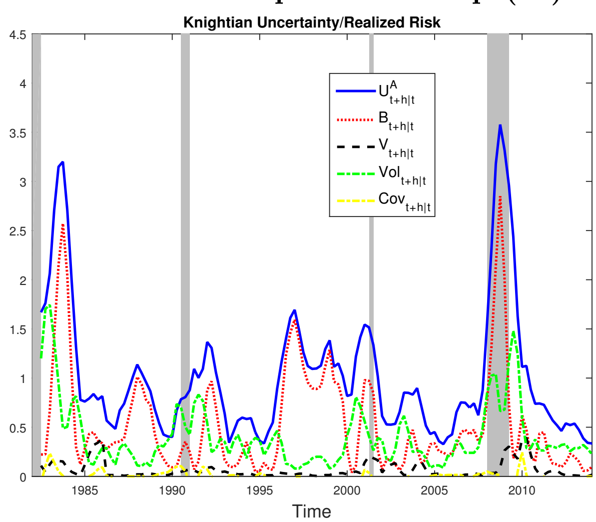


Figure VIII: Decomposing Aggregate Uncertainty

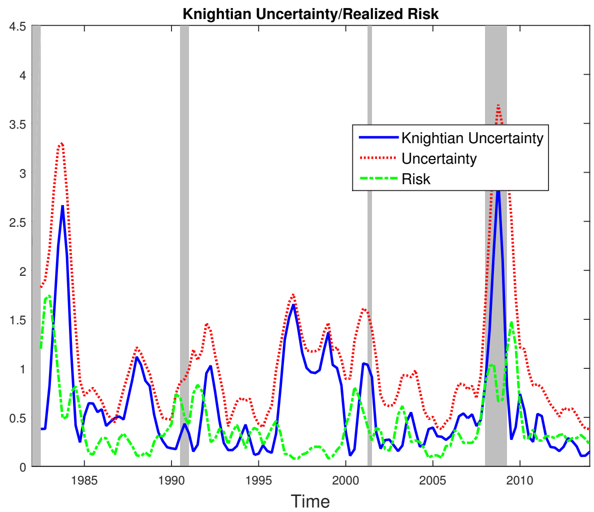
Panel B: Decomposition in Eq. (11)



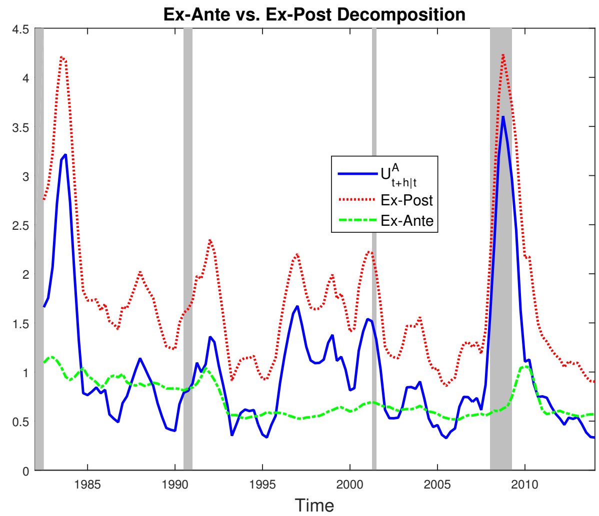
[26]The robustness is conducted for output-growth based uncertainty measures only, since revisions to inflation are typically very small and have been shown in the literature not to make much difference for the empirical results.

Figure VIII: Decomposing Aggregate Uncertainty

Panel C: Decomposition in Eq. (11)



Panel D: Ex-Ante Vs. Ex-Post



Note: Panel A of Figure 3 depicts the evolution of uncertainty, aggregate uncertainty and disagreement (eq. 9) over time. Panel B shows the evolution of the components of aggregate uncertainty based on eq. (11). Panels C and D show the evolution of the components of aggregate uncertainty based on eq. (12) and eq. (13), respectively. Results are based on quarterly data based on fixed horizon forecasts obtained by Dovern et al.'s (2012) procedure using fully revised data.

A.7 A Detailed Analysis of Uncertainty Across Forecast Horizons

Each plot in Figure IX contains eight density forecasts made in a given year: 1 quarter-ahead, 2 quarter-ahead, etc. Each density is then compared to the corresponding realization of GDP growth, depicted as a vertical line. Two things can be noted from those graphs. First, densities tend to get narrower at shorter horizons. That's what one would expect based on our analysis: the shorter the horizon, the more concentrated the forecast will be. This illustrates why ex-ante uncertainty is lower at short horizons than at a longer horizons, which is what we found with our uncertainty measure. Second, since densities at longer horizons are less concentrated, the actual realizations may still end up well inside the predictive distribution and hence the ex post error (in terms of likelihood) need not be greater than that of a concentrated, short-term forecast.

To see this in detail, consider the examples for the following years:

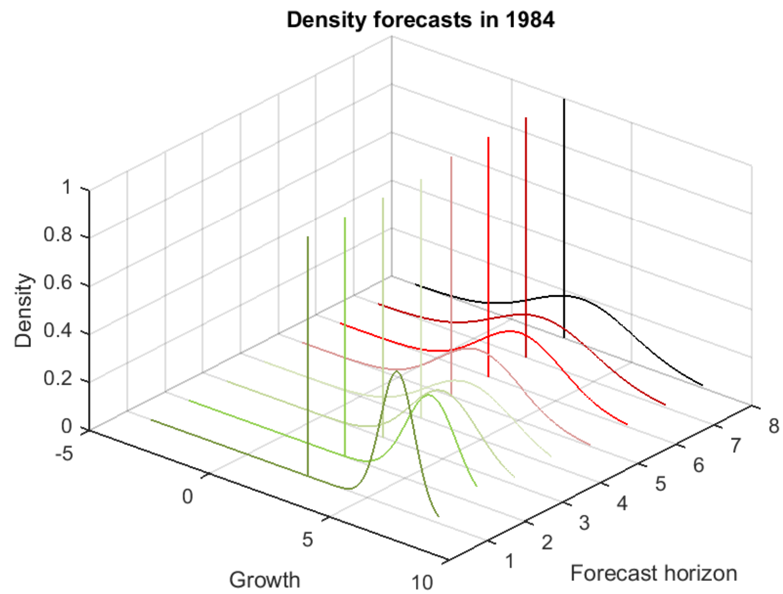
- 1984: Long horizon forecasts were quite flat and in the end, the realization fell quite close to the center of the curve. On the other hand, the short term forecasts were concentrated and missed the realization substantially. Ex post error is higher for short term horizons than for long term.

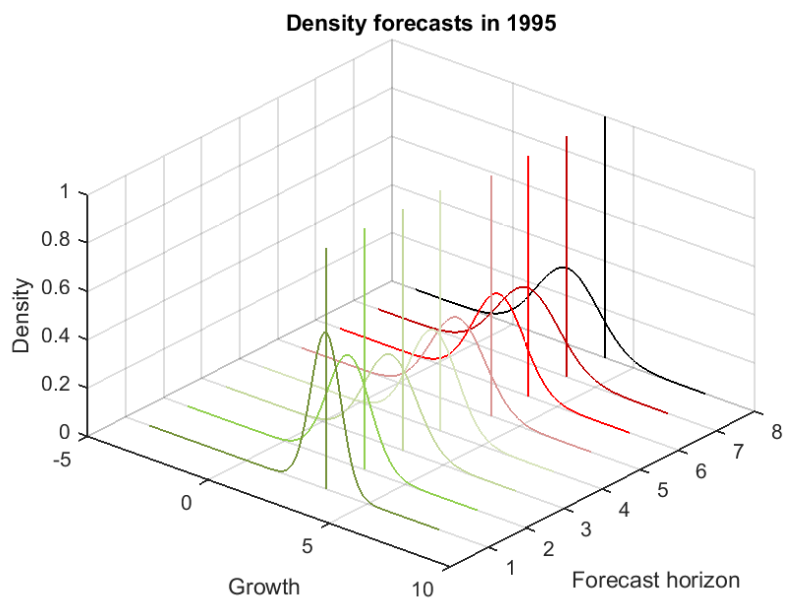
- 1995: This picture shows the opposite situation. Long-horizon forecasts missed the realization, but short-term forecasts hit the nail on the head. Ex post error is lower at short horizons than at long horizons.

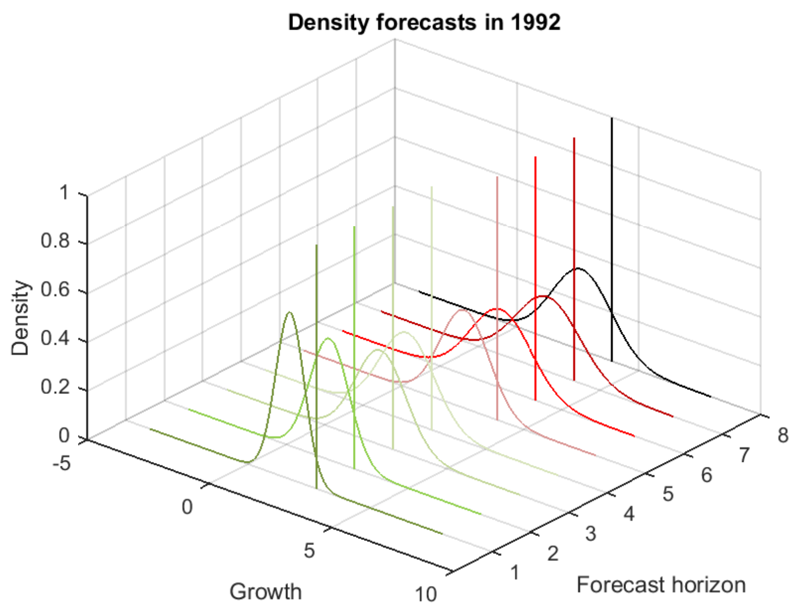
- 1992: Both long and short term horizon failed in predicting. Ex post error should be about the same in both cases.

As one looks across different points in time, there are many more cases where the pictures look like the situation in 1984 than in 1995, which explains why, on average, our results show that ex-ante uncertainty decreases as the horizon decreases, but ex-post uncertainty increases.

Figure IX: Predictive Densities and Realizations Across Horizons for a Given Year






Density forecasts in 1992
Density
1
0.8
0.6
0.4
0.2
0
-5
0
5
10
Growth
1
2
3
4
5
6
7
8
Forecast horizon

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