

Mechanism Design with Limited Commitment

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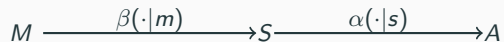
- Full commitment is the standard assumption in dynamic mechanism design
 - Useful: upper bound on the designer's payoff.
 - Convenient: **revelation principle** turns the mechanism selection game into a constrained optimization program.
- This *tractability* is lost when the designer has **limited commitment**.
- Limited commitment looms large in many applications of interest:
 - Bargaining, principal - agent (ratchet effect), fiscal policy, social insurance, international relations.
- Trade - off:
 - Optimal mechanism w/finite horizon (Hart & Tirole (1988), Laffont & Tirole (1990), Skreta (2006,2015), Deb and Said (2015)).
 - Infinite horizon with restrictions (Maestri (2015), Gerardi & Maestri (2017), Strulovici (2017), Acharya and Ortner (2017)).

Revelation principle for mechanism design with limited commitment.

- We study a game between an uninformed designer and an informed agent with persistent private information.
- The designer can commit to today's contract, but not to the continuation ones.

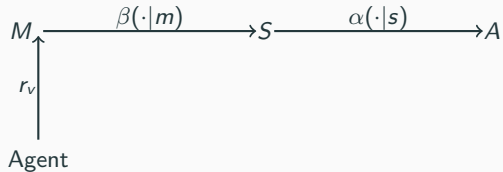
Result

1. Characterize the minimal class of mechanisms that is sufficient to replicate *all* equilibrium payoffs of the mechanism selection game.
2. Transform the designer's problem into a *constrained optimization one*
 - Usual truthtelling and participation constraints,
 - + designer's sequential rationality constraint.



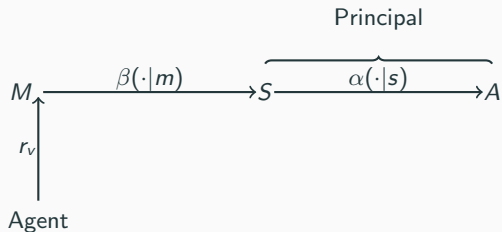
Mechanisms (Myerson '82, Forges '85)

- M is a set of input messages,
- S is a set of output messages,
- β is a communication device,
- α is a (randomized) allocation rule.



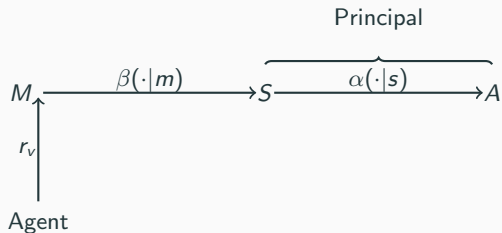
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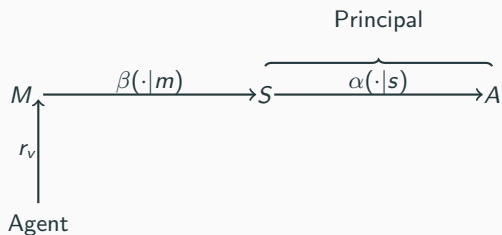
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Full Commitment

Without loss of generality,

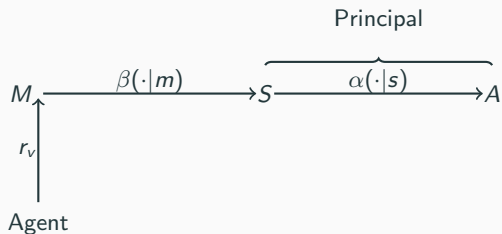
- $M = V$,
- $|S| = |M|$,
- β is "invertible",
- Truth-telling.



Limited Commitment 1: Bester & Strausz (ECMA, 2001)

Assume:

- $|M| = |S|$
- β is "invertible",
- α deterministic.



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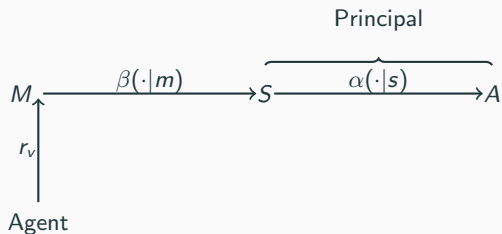
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Then, for outcomes in the Pareto frontier, it is without loss of generality

- $M = V$,

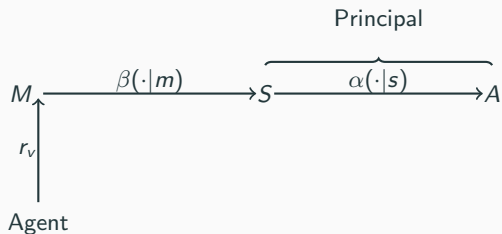
However, *Truth-telling*.



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Assume:

- $X_M \neq X_S$
- β is "invertible"
- α deterministic.



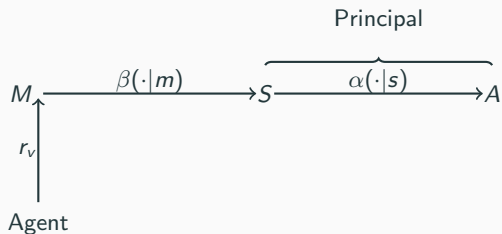
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- $M = V$,
- Truthtelling.



Limited Commitment 1: Bester & Strausz (JET, 2007)

Assume:

- $M \neq S$ ask: when is it without loss of generality to have $|M| = |S|$?
- β is "invertible".
- α deterministic.

Then, without loss of generality

- $M = V$,
- Truthtelling.

Revelation principle:

- $S \approx \Delta(V)$.
- In a general class of games, this language allows us to replicate **any equilibrium payoff of the interaction between the designer and the agent.**
 - *No need to assume transfers/time separability/history independence.*
- Mechanism serves dual role: allocation today & information tomorrow.
- Mechanisms with $M = V$ and $S = \Delta(V)$ are denoted **canonical**.

Parsimonious representation:

- In finite horizon, we can write the designer's problem as a sequence of constrained maximization problems.
- Truth-telling + participation + **designer's sequential rationality.**
- **Constrained Information Design.**

Revelation Principle

Mechanism Selection Game: Model

- Two players, the principal and the agent, interact over T periods.
 - T can be infinity.
- The principal holds the bargaining power.
- The agent has private information: type $v \in V$, $|V| < \infty$.
- Each period an allocation $a \in A$ is determined, where A is a compact space.
- Given a sequence of allocations $a^t = (a_0, \dots, a_{t-1})$, the principal can only choose $a_t \in \mathcal{A}(a^t)$.
- Payoffs: $W(a, v)$ for the principal and $U(a, v)$ for the agent for $a \in A^T, v \in V$.

Mechanism Selection Game: Mechanisms

The action set for the principal at time t is given by:

$$\mathcal{M}_t = \{\mathbf{M}_t = (\langle M^{\mathbf{M}_t}, \beta^{\mathbf{M}_t}, S^{\mathbf{M}_t} \rangle, \alpha^{\mathbf{M}_t})\}$$

where:

- $M^{\mathbf{M}_t}$ is a finite set of **input messages**, $|V| \leq |M^{\mathbf{M}_t}|$,
- $S^{\mathbf{M}_t}$ is a set of **output messages**, $S^{\mathbf{M}_t}$ contains an image of $\Delta(V)$,
- $\beta^{\mathbf{M}_t} : M^{\mathbf{M}_t} \mapsto \Delta^*(S^{\mathbf{M}_t})$ is the **communication device**,
- $\alpha^{\mathbf{M}_t} : S^{\mathbf{M}_t} \mapsto \Delta^*(A)$ is the **allocation rule**.

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- A mechanism is **canonical** if $(V, \Delta(V))$ are its sets of input and output messages.
- Let \mathcal{M}^C denote the set of canonical mechanisms.
- Assume that $\mathcal{M}^C \subseteq \mathcal{M}_t$.

Mechanism Selection Game: Timing

In each period t ,

- Both players observe a draw from a correlating device $\omega \sim U[0, 1]$.
- The principal offers the agent a mechanism \mathbf{M}_t .
- The agent observes the mechanism and accepts/rejects:
 - If she rejects, an allocation $a^* \in A$ gets implemented. Move to next period.
(Assume $a^* \in \mathcal{A}(a^t)$ for all $t, a^t \in A^t$).
- If she accepts, sends report $m \in M^{\mathbf{M}_t}$, unobserved to the principal.
- $s \in S^{\mathbf{M}_t}$ is drawn according to $\beta^{\mathbf{M}_t}(\cdot|m)$, observed by the principal.
- $a \in A$ is drawn according to $\alpha^{\mathbf{M}_t}(\cdot|s)$, observed by the principal.

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Equilibrium

A *Perfect Bayesian Equilibrium* is a tuple $\langle \Gamma^*, (\pi_v^*, r_v^*)_{v \in V}, \mu^* \rangle$ such that:

1. Strategies are sequentially rational,
2. Beliefs are obtained via Bayes' rule whenever possible.

Alternatively, consider the following **canonical game** where, for all t , $\mathcal{M}_t \equiv \mathcal{M}^C$, i.e.,

- $M^{\mathcal{M}_t} = V$,
- $S^{\mathcal{M}_t} = \Delta(V)$.

That is, the principal only chooses β and α .

Theorem

Fix any PBE of the mechanism-selection game, $\langle \Gamma^*, (\pi_v^*, r_v^*)_{v \in V}, \mu^* \rangle$.

Then there exists a payoff-equivalent PBE of the canonical game, $\langle \Gamma', (\pi'_v, r'_v)_{v \in V}, \mu' \rangle$, such that

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$$\mu'(\mathbf{M}_t^C, 1, \mu)(v) = \frac{\mu'(v)\beta^{\mathbf{M}_t^C}(\mu|v)}{\sum_{v' \in V} \mu'(v')\beta^{\mathbf{M}_t^C}(\mu|v')}$$

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Canonical input messages: $M = V$

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There is a payoff equivalent PBE s.t. the agent conditions her strategy on v and the public history alone.

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- It implies that the principal cannot peek into his past devices.
- It follows from:
 - If the agent conditions on past input messages, then she is indifferent.
 - It is possible to construct a strategy for the agent that gives the principal the same payoff.

Canonical output messages: $S = \Delta(V)$

$$S = \Delta(V)$$

- Let M_t be a mechanism on the support of Γ^* and $s \in S^{M_t}$
- Upon observing s , two things happen:
 - The allocation is drawn from $\alpha^{M_t}(\cdot|s)$.
 - Principal updates his beliefs about V and past inputs using β^{M_t} and r_V^* :
 $\mu_s^*(V, \cdot)$.
- Lemma 1 implies that $\mu_s^*(V, \cdot)$ is constant.
 - \Rightarrow relevant part of beliefs are about the agent's type!
- Natural conjecture: relabel $s \simeq \mu_s^*$.

Proof Sketch

$$S = \Delta(V)$$

However, the principal can be using s to:

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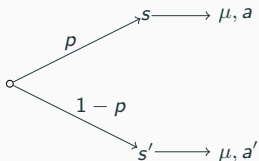
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- Offer the agent a richer set of allocations

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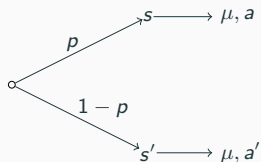
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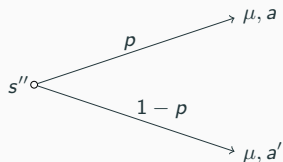
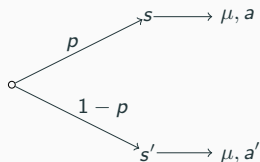
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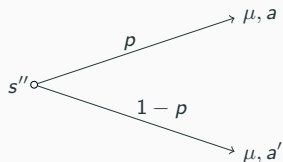
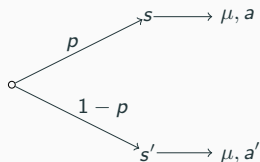
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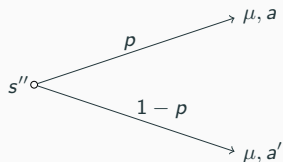
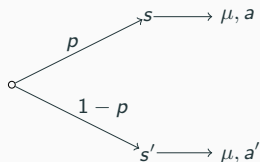


- Coordinate continuation play

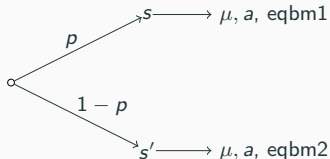
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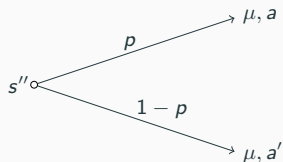
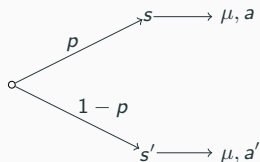
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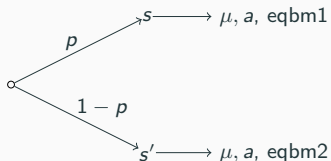
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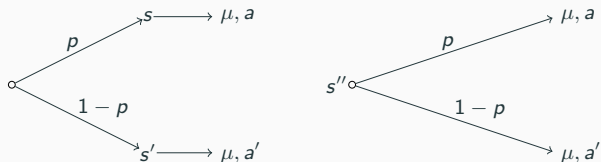
- Coordinate continuation play (**correlating device**)



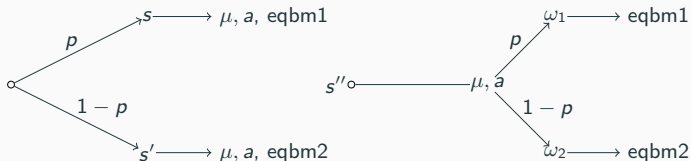
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Lemma 2

There is a one-to-one mapping between output messages and equilibrium beliefs.

Truth-telling and participation with probability 1

Truthtelling and participation with probability 1

Fix a history and a $\mathbf{M}_t \in \text{supp } \Gamma^*$. Let

$$\begin{aligned} \sigma(\mathbf{M}_t) : S^{\mathbf{M}_t} &\mapsto \Delta(V) \\ \sigma(\mathbf{M}_t)(s) &= \sum_{h_A^t, m \in M^{\mathbf{M}_t}} \mu^*(h^t, \mathbf{M}_t, 1, s)(\cdot, m), \end{aligned}$$

we can define for each $\mu \in \Delta(V)$,

$$\begin{aligned} \alpha^{\mathbf{M}_t^C}(a|\mu) &= \alpha^{\mathbf{M}_t}(a|\sigma^{-1}(\mathbf{M}_t)(\mu)) \\ \beta^{\mathbf{M}_t^C}(\mu|v) &= \sum_{m \in M^{\mathbf{M}_t}} \beta^{\mathbf{M}_t}(\sigma^{-1}(\mathbf{M}_t)(\mu)|m)r_v^*(\mathbf{M}_t, 1)(m), \end{aligned}$$

Participation with probability 1:

- As usual, we can have the agent participate, but
 - not only need to guarantee she receives the same allocation, but also,
 - make sure that this can be done without altering the continuation mechanism for the agent.

- The theorem says that all equilibrium payoffs of the mechanism selection game are also equilibrium payoffs of the canonical game.
- However, canonical game has a smaller set of deviations.
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Proposition

Any equilibrium payoff of the canonical game can be attained in an equilibrium of the mechanism selection game.

- In the canonical game, not all deviations are to mechanisms that induce truthtelling and participation.
- It follows from the proof of the proposition that these are all the deviations that matter.
- Hence, in finite horizon, can write the principal's problem as selecting between mechanisms such that
 - Agent participates with probability 1.
 - Agent tells the truth.
 - Recommended beliefs are *realized* beliefs.
 - Continuation mechanisms satisfy sequential rationality.

Constrained Information Design

Indeed, once $S \simeq \Delta(V)$, we can think of

- Principal in period t : Sender,
- Principal in period $t + 1$: Receiver.

with some special features:

- Sender also takes actions: designs allocation,
- Not all information structures are available: only those that satisfy the PC and IC constraints \Rightarrow **Constrained Information Design**.

We exploit the connection to ID to provide a program in the finite horizon case that solves for the principal's optimal mechanism:

- Extend the one-inequality constraint result in Le Trest and Tomala (2017) to allow for any number of equality and inequality constraints.
- Characterize the number of posteriors the principal induces.
- Available in a short paper.

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- Revelation principle for mechanism design with limited commitment:
 - **Canonical outputs: beliefs.**
 - Single agent.
 - Finite types. (continuum in Appendix)
- Separate allocation from information revelation.
- Beliefs: non self-referential language.
- Parsimonious representation of the equilibrium payoffs of the mechanism selection game.

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Not in the talk:

- Application to **infinite horizon sale of a durable good:**
 - Foundation for dynamic bargaining with one-sided offers and one-sided incomplete information.

Thank you!

Mechanisms

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We endow the principal with a collection $(M_i, S_i)_{i \in \mathcal{I}}$ such that

- M_i is finite and $|V| \leq |M_i|$ for all $i \in \mathcal{I}$,
- S_i contains an image of $\Delta(V)$ for all $i \in \mathcal{I}$,
- $(V, \Delta(V))$ is an element of the collection.

Denote by \mathcal{M} the set of all mechanisms with message sets $(M_i, S_i)_{i \in \mathcal{I}}$.

Hence, the **action set** for the principal at time t is given by:

$$\mathcal{M} = \{\mathbf{M}_t = (\langle M^{\mathbf{M}_t}, \beta^{\mathbf{M}_t}, S^{\mathbf{M}_t} \rangle, \alpha^{\mathbf{M}_t})\}$$

where:

- $(M^{\mathbf{M}_t}, S^{\mathbf{M}_t}) = (M_i, S_i)$ for some $i \in \mathcal{I}$,
- $\beta^{\mathbf{M}_t} : M^{\mathbf{M}_t} \mapsto \Delta^*(S^{\mathbf{M}_t})$ is the **communication device**,
- $\alpha^{\mathbf{M}_t} : S^{\mathbf{M}_t} \mapsto \Delta^*(A)$ is the **allocation rule**.

Participation

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- We can guarantee all types of the agent participate with probability 1,
- This may require using messages m^* , s^* that are only sent by the 0-probability types, v^* .
- PBE (and SE) do not impose any restrictions on the principal's belief at s^*
in the original equilibrium when v^ did not participate, the principal could have believed it was $v' \neq v^*$!*
- Potentially, the belief at s^* coincides with the belief after s' , for some s' that shows up on path.

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⇒ This endangers the one-to-one map between used outputs and beliefs

How do we deal with this?

- We “remove” input messages m^* that lead to output messages that are used only by 0-probability types.
- This removes deviations for the positive probability types, but may violate participation for the 0-probability types.
- Consequently, the only output messages that have positive probability under some device are those that have positive probability under the agent’s reporting strategy and the principal’s beliefs.