The Pricing of Market and Idiosyncratic Jump and Volatility Risks

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1. Market and Idiosyncratic Risks

Summary

Extending the method of Cremers, Halling, Weinbaum (2015), I use **option portfolio returns** that have a **constant exposure** to either **jump** or **diffusive risk** to decompose the total volatility risk into four components: **market volatility risk, idiosyncratic volatility risk, market jump risk, and idiosyncratic jump risk**. Returns on these option portfolios load directly on **changes in the corresponding risk premium**. The decomposition helps in explaining **contemporaneous** and **future returns**. While all four components are at play when stocks earn contemporaneously **negative returns**, the **idiosyncratic components** are able to explain most of the cross-sectional variation in **positive returns**. In addition, stocks that have **higher idiosyncratic jump risk** earn **higher subsequent returns**. This relation is robust to various stock characteristics and cannot be explained by the low beta anomaly.

Relation of Jump and Volatility Risk

Total volatility may stem from **continuous** (volatility) or **discontinuous** (jump) stock price movements

- **Market jump** and **volatility risks** are two important **risk channels** (e.g., Bates (1991)):

$$\frac{dS_t^M}{S_t^M} = \alpha_t^M\,dt + \underbrace{\sqrt{V_t^M}dW_t^M}_{\text{volatility risk}} + \underbrace{k_t^M dq^M}_{\text{jump risk}}$$

- On **single stock** level stock prices might be subject to **jump** and **volatility risk** stemming from the **market** or being purely **idiosyncratic**:

$$\frac{dS_t^i}{S_t^i} = \alpha_t^i\,dt + \underbrace{\overbrace{\beta_{VOL}^i\sqrt{V_t^M}dW_t^M}^{\text{market}} + \overbrace{\sqrt{V_t^{\epsilon^i}}dW_t^{\epsilon^i}}^{\text{idiosyncratic}}}_{\text{volatility risk}} + \underbrace{\overbrace{\beta_{JUMP}^i k_t^M dq^M}^{\text{market}} + \overbrace{k_t^{\epsilon^i}dq^{\epsilon^i}}^{\text{idiosyncratic}}}_{\text{jump risk}}$$

→ If risks are priced, stocks that have a **higher exposure** towards these risks should **earn higher expected returns**

⇒ **How are these different risks priced in the cross-section of stock returns?**

Measure for Jump and Volatility Risk

Options embed **forward looking information**, thus natural proxy for measuring **expected jump and volatility risk** (e.g., Martin (2017))

Liquidity of Option Contracts

		Moneyness												
		OTM						ITM						
		≥ 0.5	≥ 0.4	≥ 0.3	≥ 0.2	≥ 0.1	≥ 0	> 0	> 0.1	> 0.2	> 0.3	> 0.4	> 0.5	
Maturity	≤ 30	0.01	0.03	0.14	0.66	3.40	17.66	12.25	1.66	0.47	0.16	0.07	0.10	36.59
	≤ 60	0.03	0.09	0.29	1.00	3.73	11.05	5.78	1.29	0.45	0.19	0.09	0.16	24.17
	≤ 90	0.04	0.07	0.20	0.52	1.39	2.89	1.50	0.45	0.19	0.11	0.05	0.14	7.55
	≤ 120	0.05	0.09	0.22	0.54	1.32	2.43	1.22	0.40	0.17	0.09	0.05	0.12	6.72
	≤ 150	0.06	0.09	0.22	0.51	1.18	1.93	0.92	0.31	0.14	0.08	0.04	0.09	5.58
	≤ 180	0.05	0.09	0.20	0.46	0.98	1.49	0.72	0.25	0.11	0.06	0.04	0.10	4.57
	≤ 210	0.04	0.06	0.14	0.32	0.65	0.97	0.49	0.18	0.10	0.05	0.03	0.06	3.08
	≤ 240	0.03	0.06	0.12	0.26	0.52	0.72	0.36	0.13	0.06	0.04	0.02	0.05	2.37
	≤ 270	0.03	0.03	0.06	0.13	0.22	0.29	0.15	0.06	0.03	0.03	0.02	0.05	1.11
	≤ 300	0.03	0.03	0.05	0.10	0.15	0.20	0.11	0.05	0.04	0.02	0.01	0.04	0.82
	≤ 330	0.03	0.03	0.06	0.10	0.16	0.20	0.11	0.04	0.03	0.02	0.02	0.04	0.85
	≤ 360	0.03	0.03	0.06	0.11	0.17	0.20	0.11	0.05	0.03	0.02	0.01	0.05	0.86
	$\sum$	0.43	0.71	1.77	4.70	13.88	40.04	23.72	4.86	1.82	0.87	0.46	1.00	94.26

(percentage of total trading volume)

→ mostly **at-the-money** equity options are traded

→ measure for jump and volatility risk should depend on ATM options

Bollerslev, Todorov, Xu (2015) use a semi-parametric approach to estimate tail parameters

→ relies on **OTM** options ⚡

Cremers, Halling, Weinbaum (2015) construct option strategies that hedge JUMP and VOL risks

- **delta-gamma neutral** and **vega positive straddles** to measure **volatility risk**
- **delta-vega neutral** and **gamma positive straddles** to measure **jump risk**

→ only **ATM** options are used ✓

→ **large fluctuations** in **option sensitivities** towards these risks ⚡

⇒ I build on, but **extend the method of Cremers, Halling, Weinbaum (2015)**

2. Data and Methodology

Option Portfolio Returns

Assume the stock price follows a double stochastic volatility model:

$$\begin{aligned} dS_t^i/S_t^i &= \alpha_t^i\,dt + \beta_{VOL}^i\sqrt{V_t^M}dW_t^M + \sqrt{V_t^{\epsilon^i}}dW_t^{\epsilon^i} \\ dV_t^M &= \kappa^M\left(\bar{V}^M - V_t^M\right)dt + \sigma dW_t^{V,M} \\ dV_t^{\epsilon^i} &= \kappa^{\epsilon^i}\left(\bar{V}^{\epsilon^i} - V_t^{\epsilon^i}\right)dt + \sigma dW_t^{V,\epsilon^i} \end{aligned}$$

The instantaneous excess option returns of the volatility portfolios are then given by:

$$\begin{aligned} \frac{dO_t^M}{O_t^M} - rdt &= \frac{\partial O^M}{\partial V_t^M}\frac{1}{O_t^M}\left(dV_t^M - \mathbb{E}^{\mathbb{Q}}\left[dV_t^M\right]\right) \\ \frac{dO_t^i}{O_t^i} - rdt &= \underbrace{\frac{\partial O^i}{\partial V_t^M}\frac{1}{O_t^i}\left(dV_t^M - \mathbb{E}^{\mathbb{Q}}\left[dV_t^M\right]\right)}_{\text{market}} + \underbrace{\frac{\partial O^i}{\partial V_t^{\epsilon^i}}\frac{1}{O_t^i}\left(dV_t^{\epsilon^i} - \mathbb{E}^{\mathbb{Q}}\left[dV_t^{\epsilon^i}\right]\right)}_{\text{idiosyncratic}} \end{aligned}$$

→ Returns on the **market volatility portfolio** are **proportional** to the **vega** of the portfolio and **changes** in the **market variance risk premium**

→ Returns of **individual volatility portfolios** are **proportional** to **changes** in the **market variance risk premium, the idiosyncratic variance risk premium** and the **sensitivities** towards these risks (where $\frac{\partial O^i}{\partial V_t^i} = \frac{\partial O^i}{\partial V_t^M} + \frac{\partial O^i}{\partial V_t^{\epsilon^i}}$)

⇒ I **impose constant vega (gamma)** of 100 (0.01) to construct VOL (JUMP) portfolio

→ Allows to **estimate** the component stemming from market volatility risk in a **linear fashion**

→ *(Same approach for jumps)*

Construction of VOL and JUMP Portfolio

Every day I select **two option pairs** with **different maturities** between **7 and 90 days**

- Take those two pairs that are **closest to the money**
- If **multiple option pairs** are equally close to the money, I use the ones with **shortest** and **longest maturity** *(largest spread in vega and gamma)*
- I construct the option portfolios using one **long** and one **short position** in two **straddles** (for VOL (JUMP) short (long) in short maturity and long (short) in long maturity):

$$\begin{aligned} &\arg\min_{\omega} \sum_{i=1}^{4}\left(\frac{\omega^i O^i}{abs(\omega^\top)O}\right)^2 \\ &s.t. \\ &\qquad \omega^\top\Delta = 0 \\ &\qquad \omega^\top\mathcal{V} = 0 \\ &\qquad \omega^\top\Gamma = 0.01 \end{aligned}$$

- 4 options to hedge 3 greeks

→ I run a numerical optimization and **minimize** the **relative portfolio weights** for all options *(minimize potential data noise)*

- for VOL portfolio: set $\omega^\top\Gamma = 0$ and $\omega^\top\mathcal{V} = 100$

I measure the **portfolio return** over **one trading day**

- JUMP and VOL portfolios could be **zero cost portfolios**
  - → Calculate returns relative to the **absolute amount** invested (Tosi and Ziegler (2018)) *(Portfolio return is always well defined)*
- If options are not observable the next day I **interpolate** using a **kernel smoother**
  - Smooth over maturity, moneyness and put-call identifier
  - Same bandwidth as OptionMetrics

⇒ **Same approach for SPX options and equity options**

Data

I use **full cross-section of stock option data**:

Sample period:	01/1996 - 04/2016, daily frequency
Stock data:	CRSP all stocks on NYSE, AMEX and NASDAQ
Option data:	OptionMetrics

I **filter the data** similar to **Goyal and Saretto (2009)** and exclude options with:

- zero open interest
- no available implied volatility
- violations against standard no-arbitrage bounds

Quoted options are **American styled**. Given an implied volatility I calculate **synthetic European option prices**, to account for the early exercise premium:

- reprice all American options with a Cox et al. (1979) (CRR) tree, using 1000 steps and incorporating expected discrete dividends
- discard all options with a pricing error of more than 1%
- use the same CRR-tree to price the corresponding European option

3. Empirical Results

Market JUMP and VOL Portfolio

Summary Statistics for Market Volatility and Jump Risk Factors

Panel A: Descriptive Statistics

	Mean	SD	Median	Skewness	Kurtosis	Sharpe Ratio
VOL^M	-0.0005	0.0149	-0.0011	0.9749	11.4895	-0.5744
$JUMP^M$	-0.0022	0.0397	-0.0063	3.9010	40.2054	-0.8715
ΔVIX^{2^M}	0.0073	0.1356	-0.0072	1.9314	16.0358	-

Panel B: Pairwise Correlations

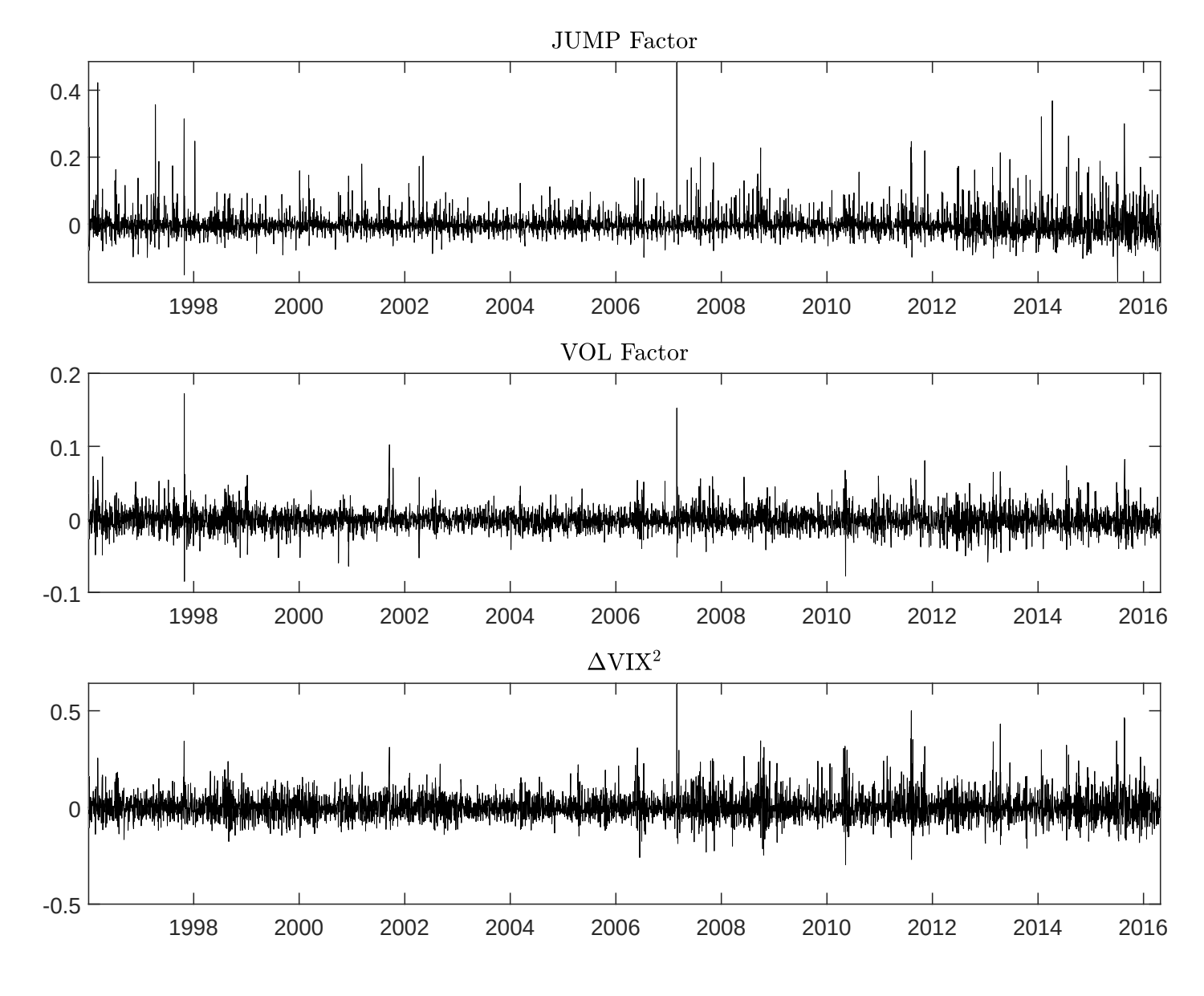
	VOL^M	$JUMP^M$	ΔVIX^{2^M}
VOL^M	1		
$JUMP^M$	0.1803	1	
ΔVIX^{2^M}	0.5649	0.4125	1

⇒ Investors willing to **pay a premium** to **hedge market volatility** and **jump risk**

⇒ **Positive correlation** of **VOL** and **JUMP** portfolio returns

⇒ **VOL** and **JUMP highly correlated** with **changes** in expected risk-neutral **total variance**

Time Series of JUMP and VOL Portfolio Returns



Contemporaneous Market Regressions

	$\Delta VIX_t^{2^M}$	r_t^M	$r_t^{M^+}$	$r_t^{M^-}$
Intercept	0.0040^{***} (0.0007)	0.0001 (0.0002)	0.0090^{***} (0.0002)	-0.0079^{***} (0.0002)
VOL	2.3582^{***} (0.0484)	-0.2637^{***} (0.0112)	-0.0171 (0.0113)	-0.1064^{***} (0.0108)
JUMP	0.6221^{***} (0.0195)	-0.0518^{***} (0.0045)	0.1471^{***} (0.0054)	-0.1117^{***} (0.0037)
adj. R^2	0.4292	0.1310	0.2226	0.3479

⇒ **Large changes** in expected risk-neutral **total variance** go along with **large returns** of the **VOL** or **JUMP** portfolio

⇒ Changes in expected risk-neutral total variance load positive on both measures

⇒ Market returns load negative on both, JUMP and VOL, returns

⇒ **Only market jumps** are able to **explain positive market returns**

Cross-Sectional JUMP and VOL Portfolios

Summary Statistics for Individual Volatility and Jump Risk Factors

Panel A: Descriptive Statistics

	Mean	SD	Median	Skewness	Kurtosis	Sharpe Ratio
VOL^i	0.0008	0.0305	-0.0001	4.7852	406.35	0.0000
$JUMP^i$	-0.0029	0.0498	-0.0056	4.0309	138.25	-0.0001
ΔVIX^{2^i}	0.1031	14.7282	-0.0143	367.95	146125.52	-

Panel B: Pairwise Correlations

	VOL^i	$JUMP^i$	ΔVIX^{2^i}	VOL^M	$JUMP^M$
VOL^i	1				
$JUMP^i$	-0.3236^{***} (0.0032)	1			
ΔVIX^{2^i}	0.1554^{***} (0.0024)	0.1960^{***} (0.0018)	1		
VOL^M	0.0775^{***} (0.0018)	0.0357^{***} (0.0015)	0.0922^{***} (0.0017)	1	
$JUMP^M$	0.0526^{***} (0.0013)	0.1123^{***} (0.0018)	0.0578^{***} (0.0016)	0.0003	1

Panel C: Descriptive Statistics of Decomposed Measures

	Mean	SD	Median	Skewness	Kurtosis	Sharp Ratio
$VOL^{i,M}$	-0.0075	5.1704	-0.0040	-0.0379	774.2500	-0.0228
$VOL^{i,\epsilon}$	0.1105	6.0085	-0.0007	0.4764	472.2220	0.2907
$JUMP^{i,M}$	-0.0004	0.0066	-0.0006	1.3226	25.4584	-1.0402
$JUMP^{i,\epsilon}$	-0.0014	0.0421	-0.0045	4.5905	13.1580	-0.5425

⇒ Investors willing to **pay a premium** to hedge risks (negative median)

⇒ Negative correlation of VOL and JUMP portfolio returns

⇒ **VOL** and **JUMP highly correlated** with **changes** in **stock's** expected risk-neutral **total variance**

Estimates the market and idiosyncratic part:

$$r_t^{i,x} = \alpha + \underbrace{\beta^i r_t^{M,x}}_{VOL^{i,M}/JUMP^{i,M}} + \epsilon_t^i$$

⇒ Investors willing to pay a premium to hedge idiosyncratic and market risks

Contemporaneous Time-Series Regressions

	$\Delta VIX_t^{2^i}$	r_t^i	$r_t^{i^+}$	$r_t^{i^-}$
Intercept	0.0148^{***} (0.0067)	0.0007 (0.0013)	0.0250^{***} (0.0012)	-0.0221^{***} (0.0010)
$VOL^{i,M}$	0.1649^{***} (0.1388)	-0.0553^{***} (0.0279)	-0.0239 (0.0272)	-0.0060^{***} (0.0246)
$VOL^{i,\epsilon}$	0.0130^{***} (0.0027)	-0.0005 (0.0004)	0.0017^{***} (0.0004)	-0.0022^{***} (0.0004)
$JUMP^{i,M}$	1.5870^{***} (4.7951)	-0.4716^{***} (1.0824)	0.4147^{***} (1.2251)	-0.4409^{***} (0.7554)
$JUMP^{i,\epsilon}$	0.8455^{***} (0.1469)	0.0158 (0.0280)	0.2887^{***} (0.0257)	-0.2660^{***} (0.0223)
adj. R^2	0.1081	0.0862	0.3119	0.3797

⇒ Changes in risk-neutral total variance of single stocks load positive on all four measures

⇒ Stock returns load negative on market JUMP and VOL, only

⇒ Diverse effect of idiosyncratic components:

→ **Positive returns** load **positive** on **idiosyncratic VOL** and **JUMP**

→ **Negative returns** load **negative** on **idiosyncratic VOL** and **JUMP**

Cross-Sectional Predictive Analysis

In order to infer the pricing, I run a **cross-sectional predictive analysis**. Every month I use the **daily observation over the last month** to **estimate the market and idiosyncratic** part of the portfolio returns. I sort stocks into **quintile portfolios** and hold them for **one month**:

Cross-Sectional Single Sorts

	1	2	3	4	5	1-5
Panel A: Sort on $VOL^{i,M}$						
$VOL^{i,M}$	-0.0294^{***} (0.0001)	-0.0091^{***} (0.0001)	0.0001 (0.0001)	0.0094^{***} (0.0001)	0.0310^{***} (0.0002)	-0.0606^{***} (0.0002)
r_t	0.0085^{***} (0.0036)	0.0104^{***} (0.0037)	0.0118^{***} (0.0038)	0.0140^{***} (0.0038)	0.0139^{***} (0.0040)	-0.0060^{***} (0.0028)
r_{t+1}	0.0069^{**} (0.0034)	0.0043 (0.0038)	0.0070^{*} (0.0039)	0.0084^{**} (0.0040)	0.0053 (0.0042)	0.0007 (0.0023)
α_{t+1}	0.0014 (0.0012)	-0.0016 (0.0015)	0.0012 (0.0018)	0.0031 (0.0019)	-0.0011 (0.0027)	0.0017 (0.0027)
Panel B: Sort on $VOL^{i,\epsilon}$						
$VOL^{i,\epsilon}$	-0.0963^{***} (0.0001)	-0.0300^{***} (0.0001)	0.0049^{***} (0.0001)	0.0427^{***} (0.0001)	0.1313^{***} (0.0002)	-0.2278^{***} (0.0002)
r_t	0.0083^{**} (0.0037)	0.0098^{***} (0.0033)	0.0116^{***} (0.0036)	0.0133^{***} (0.0039)	0.0188^{***} (0.0062)	-0.0111^{*} (0.0060)
r_{t+1}	0.0079^{*} (0.0044)	0.0060 (0.0040)	0.0067^{*} (0.0035)	0.0041 (0.0037)	0.0041 (0.0038)	0.0030 (0.0029)
α_{t+1}	0.0019 (0.0019)	0.0001 (0.0015)	0.0011 (0.0014)	-0.0015 (0.0015)	-0.0016 (0.0025)	0.0027 (0.0029)
Panel C: Sort on $JUMP^{i,M}$						
$JUMP^{i,M}$	-0.0637^{***} (0.0002)	-0.0239^{***} (0.0001)	-0.0057^{***} (0.0001)	0.0131^{***} (0.0001)	0.0558^{***} (0.0003)	-0.1200^{***} (0.0004)
r_t	0.0098^{***} (0.0040)	0.0114^{***} (0.0036)	0.0135^{***} (0.0036)	0.0116^{***} (0.0035)	0.0132^{***} (0.0038)	-0.0039 (0.0027)
r_{t+1}	0.0045 (0.0052)	0.0080^{***} (0.0033)	0.0056 (0.0035)	0.0089^{***} (0.0035)	0.0051 (0.0040)	-0.0014 (0.0036)
α_{t+1}	-0.0025 (0.0030)	0.0032^{***} (0.0011)	0.0003 (0.0016)	0.0030^{***} (0.0012)	-0.0008 (0.0020)	-0.0025 (0.0039)
Panel D: Sort on $JUMP^{i,\epsilon}$						
$JUMP^{i,\epsilon}$	-0.2404^{***} (0.0002)	-0.1358^{***} (0.0002)	-0.0654^{***} (0.0002)	0.0156^{***} (0.0002)	0.1993^{***} (0.0003)	-0.4413^{***} (0.0003)
r_t	0.0064^{**} (0.0031)	0.0070^{***} (0.0028)	0.0096^{***} (0.0035)	0.0137^{***} (0.0038)	0.0171^{***} (0.0050)	-0.0114^{***} (0.0039)
r_{t+1}	0.0091^{**} (0.0042)	0.0077^{**} (0.0034)	0.0071^{**} (0.0036)	0.0059 (0.0040)	0.0028 (0.0047)	0.0054^{**} (0.0033)
α_{t+1}	0.0032 (0.0023)	0.0024^{*} (0.0015)	0.0014 (0.0014)	0.0000 (0.0016)	-0.0043^{*} (0.0025)	0.0067^{***} (0.0035)

⇒ contemporaneous returns are monotonically increasing

⇒ 1-5 except for market returns statistically significant

⇒ **next month returns/alphas** are **monotonically decreasing** for portfolios sorted on idiosyncratic jumps

→ for low portfolios, **changes** in (expected) **physical jumps smaller** than **changes** in expected **risk-neutral jumps** → investors **dislike jumps** and **demand a premium** for holding these stocks

⇒ **Positive price of idiosyncratic jump risk** ✓

⇒ Results are robust to stock characteristics, risk-neutral moments and the low beta anomaly