The Impact of Equity Tail Risk on Bond Risk Premia: Evidence of Flight-to-Safety in the U.S. Term Structure

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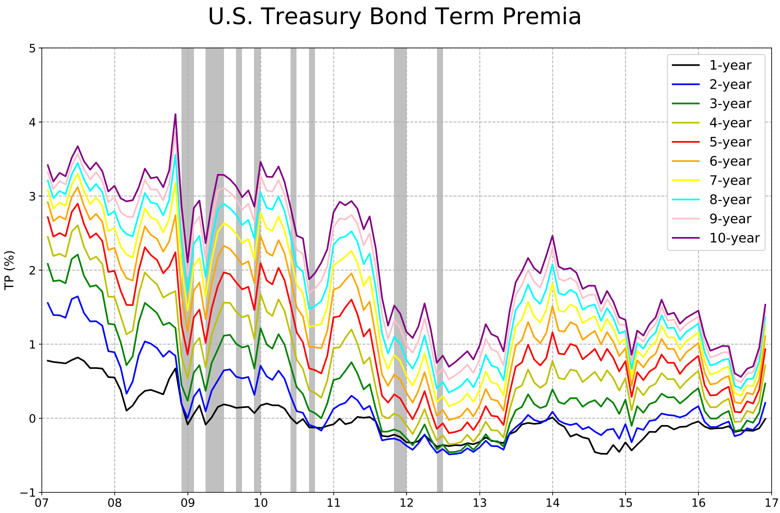
Introduction: Motivation and Contributions

- **Flight to Safety (FTS)**: market stress event with ($\Uparrow$) expected returns for stocks and **($\Downarrow$) expected returns for Treasuries**.
- We study FTS in bond pricing by examining the effects of equity tail risk on the U.S. yield curve dynamics. To do so, we rely on:
  - **Equity Left Tail Factor** for the downside tail risk of the stock market
  - **Gaussian ATSM**[1] in which bond yields are driven both by factors of bond-market origin and by the **equity left tail factor**
- We pick a risk measure able to predict equity returns and we examine its role in a term structure model for U.S. interest rates.
- We find that equity tail risk is priced within the ATSM and short-term Treasuries are more strongly affected by FTS than are long-term ones.

[1] Affine Term Structure Model

Introduction: Empirical Application



Vertical gray bars indicate elevated equity tail risk ($\geq$ 85%-ile)

Equity Tail Risk

- For the U.S., U.K. and Euro-zone stock market index, we estimate the 3-Factor Double Exponential Model by Andersen et al. (2015):

$$\frac{dX_t}{X_{t-}} = (r_t - \delta_t)dt + \sqrt{V_{1,t}}\; dW^{\mathbb{Q}}_{1,t} + \sqrt{V_{2,t}}\; dW^{\mathbb{Q}}_{2,t} + \int_{\mathbb{R}^2} (e^x - 1)\widetilde{\mu}^{\mathbb{Q}}(dt, dx, dy)$$

$$dV_{1,t} = \kappa_1(\bar{v}_1 - V_{1,t})dt + \sigma_1\sqrt{V_{1,t}}\; dB^{\mathbb{Q}}_{1,t} + \mu_1 \int_{\mathbb{R}^2} x^2 1_{\{x<0\}}\mu(dt, dx, dy)$$

$$dV_{2,t} = \kappa_2(\bar{v}_2 - V_{2,t})dt + \sigma_2\sqrt{V_{2,t}}\; dB^{\mathbb{Q}}_{2,t}$$

$$dU_t = -\kappa_u U_t dt + \mu_u \int_{\mathbb{R}^2} [(1-\rho_u)x^2 1_{\{x<0\}} + \rho_u y^2]\mu(dt, dx, dy)$$

- For each stock market index, we obtain the "pure tail" factor $\widetilde{U}$ as the residual of the regression of U on the spot variance $V = V_1 + V_2$.
- We define the **Equity Left Tail Factor** as the market capitalization weighted average of the $\widetilde{U}$ factor of the three stock market indices:

$$\widetilde{U}^{Equity}_t = \sum_{i=1}^{3} w^i_t\; \widetilde{U}^i_t$$

Term Structure Modeling (I)

- We let the U.S. Term Structure be driven by the following factors:

$$\mathbf{X}_t = \left[\widetilde{U}_t^{Equity},\ PC1_t,\ PC2_t,\ PC3_t,\ PC4_t,\ PC5_t \right]'$$

- The price of the zero-coupon Treasury bond with maturity n is:

$$P_t^{(n)} = \mathrm{E}_t \left[M_{t+1} P_{t+1}^{(n-1)} \right]$$

- The pricing kernel, M_{t+1}, is exponentially affine in the factors:

$$M_{t+1} = \exp\Big(-r_t - \frac{1}{2}\boldsymbol{\lambda}_t^{'}\boldsymbol{\lambda}_t - \boldsymbol{\lambda}_t^{'}\boldsymbol{\Sigma}^{-1/2}\mathbf{v}_{t+1} \Big)$$

- The market prices of risk, $\boldsymbol{\lambda}_t$, are affine in the factors:

$$\boldsymbol{\lambda}_t = \boldsymbol{\Sigma}^{-1/2}(\boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1 \mathbf{X}_t)$$

Term Structure Modeling (II)

- The data generating process for log excess returns is:

$$rx_{t+1}^{(n-1)} = \overbrace{\beta^{(n-1)'}(\boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1 \mathbf{X}_t)}^{\text{Expected Return}} - \overbrace{\frac{1}{2}(\beta^{(n-1)'}\boldsymbol{\Sigma}\beta^{(n-1)} + \sigma^2)}^{\text{Convexity Adjustment}} + \\ + \underbrace{\beta^{(n-1)'}\mathbf{v}_{t+1}}_{\text{Priced Return Innovation}} + \underbrace{e_{t+1}^{(n-1)}}_{\text{Return Pricing Error}}$$

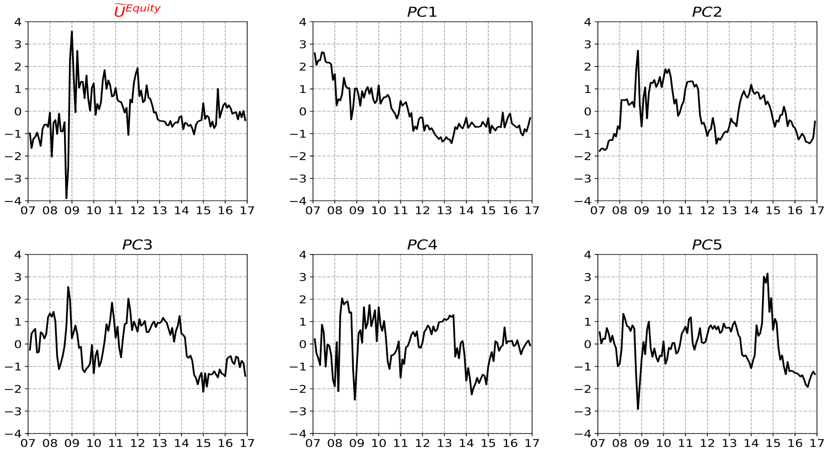
- Zero-coupon bond yields, risk-neutral yields and bond term premia are calculated as follows:

$$y_t^{(n)} = -\frac{1}{n}\left[a_n + \mathbf{b}_n^{'}\mathbf{X}_t\right] + u_t^{(n)}$$

$$y_t^{(n)\ \text{RN}} = \frac{1}{n}\sum_{i=0}^{n-1} \mathrm{E}_t[r_{t+i}] = -\frac{1}{n}[a_n^{\text{RN}} + \mathbf{b}_n^{\text{RN}'}\mathbf{X}_t]$$

$$TP_t^{(n)} = y_t^{(n)} - y_t^{(n)\ \text{RN}}$$

Empirical Application: Pricing Factors of U.S. Treasuries



$\widetilde{U}^{Equity}$: market capitalization weighted average of the "pure tail" factor of U.S., U.K. and Euro-zone stock market indices.

$PC1$-$PC5$: first 5 principal components extracted from Treasury yields of maturities $n = 3, 6, \ldots, 120m$, orthogonal to $\widetilde{U}^{Equity}$.

Empirical Application: Factor Exposures and Prices of Risk

- **Test for unspanned factors**: the Wald statistic, under $H_0 : \boldsymbol{\beta}_i = \mathbf{0}_{N\times 1}$, is defined as:

$$W_{\beta_i} = \hat{\boldsymbol{\beta}}_i' \hat{\mathcal{V}}_{\beta_i}^{-1} \hat{\boldsymbol{\beta}}_i \overset{\alpha}{\sim} \chi^2(N)$$

- **Test for priced risk factors**: the Wald statistic, under $H_0 : \boldsymbol{\lambda}_i' = \mathbf{0}_{1\times(K+1)}$, is defined as:

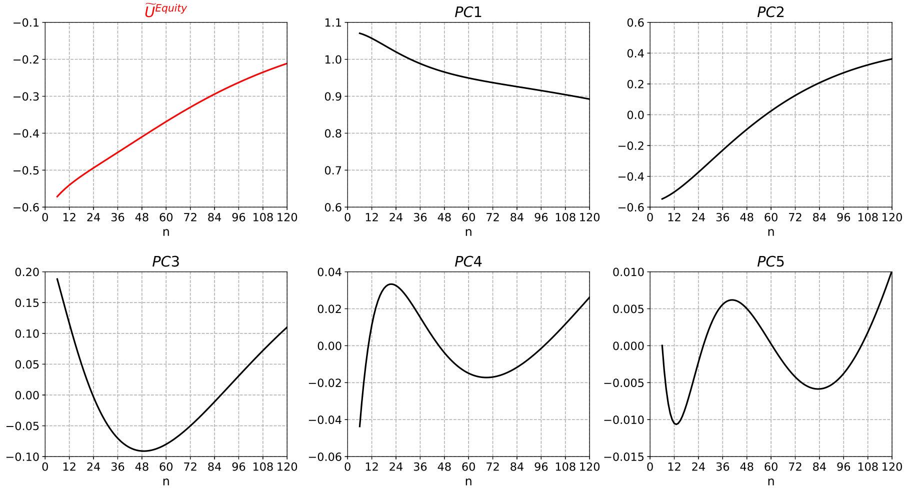
$$W_{\Lambda_i} = \hat{\boldsymbol{\lambda}}_i' \hat{\mathcal{V}}_{\lambda_i}^{-1} \hat{\boldsymbol{\lambda}}_i \overset{\alpha}{\sim} \chi^2(K+1)$$

- **Test for time-varying market prices of risk**: the Wald statistic, under $H_0 : \boldsymbol{\lambda}_{1_i}' = \mathbf{0}_{1\times(K)}$, is defined as:

$$W_{\lambda_{1_i}} = \hat{\boldsymbol{\lambda}}_{1_i}' \hat{\mathcal{V}}_{\lambda_{1_i}}^{-1} \hat{\boldsymbol{\lambda}}_{1_i} \overset{\alpha}{\sim} \chi^2(K)$$

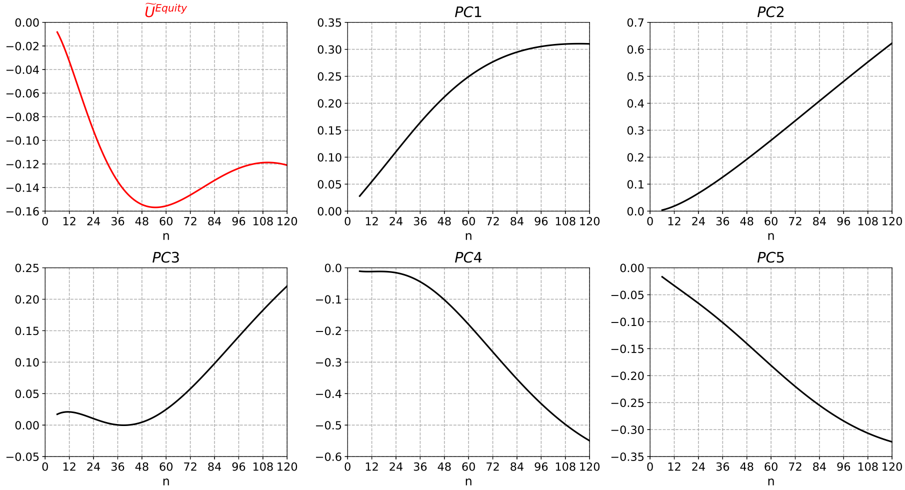
p-value	$\widetilde{U}^{Equity}$	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
W_{β_i}	0.000	0.000	0.000	0.000	0.000	0.000
W_{Λ_i}	0.057	0.022	0.028	0.001	0.103	0.000
$W_{\lambda_{1_i}}$	0.036	0.012	0.021	0.002	0.349	0.000

Empirical Application: Yield Loadings



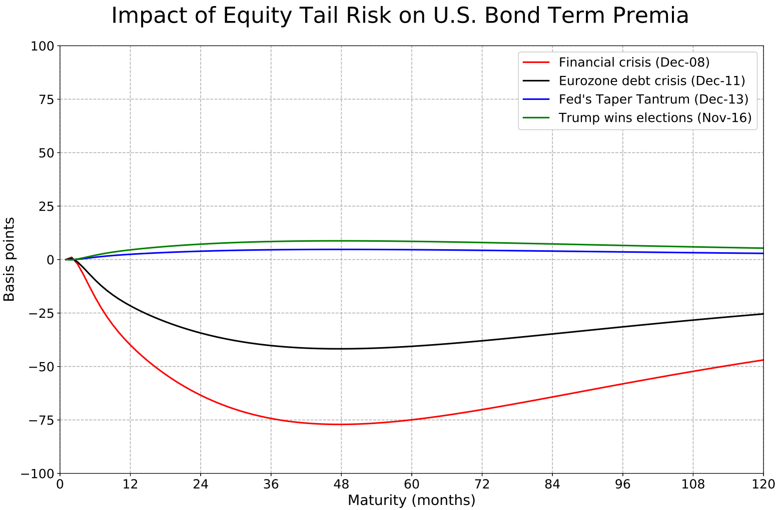
Yield loadings on $\widetilde{U}^{Equity}$ are $(-)$ across all maturities: Equity Tail Risk $(\Uparrow) \Rightarrow$ Bond Prices $(\Uparrow) \Rightarrow$ **FTS**

Empirical Application: Expected Return Loadings



Expected return loadings on $\widetilde{U}^{Equity}$ are (−) across all maturities: Equity Tail Risk (⇑) ⇒ Bond Risk Premia (⇓) ⇒ **FTS**

Empirical Application: Equity Tail Risk and Term Premia



Concluding Remarks

- We study Flight to Safety in the context of bond pricing with equity tail risk driving the U.S. Treasury yield curve.
- Equity left tail factor is extracted from options on international stock market indices and is used as a pricing factor in a Gaussian ATSM.
- Application to our dataset of U.S. zero-coupon yields and S&P 500, FTSE 100 and EURO STOXX 50 equity-index options shows:
  - Equity tail risk is significantly priced within the term structure model.
  - Consistent with the theory of FTS, bond prices increase and future excess returns shrink in response to a shock to the equity left tail factor.
  - The equity left tail factor has significant explanatory power for future returns on Treasuries with maturities up to four years.
  - The short end of the U.S. yield curve has strongly been affected by equity tail risk since the outburst of the recent financial crisis.