The Optimal Inflation Target and the Natural Rate of Interest

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Motivation

- Evidence of a decline in the natural rate of interest
- Implications for monetary policy $\Rightarrow \uparrow$ incidence of the ZLB
- Calls for a higher inflation target (Ball, Blanchard et al., Williams,...)

$\Rightarrow$ *Is a higher inflation target warranted? How much higher?*

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- This paper:

- quantitative analysis of the optimal inflation target (π^*) as a function of the steady state real rate (r^*)
- based on an estimated medium-scale NK model (US and euro area)
- focus on the role of parameter uncertainty

Main Findings

- The relation between r^* and π^* is downward sloping, but not necessarily *one-for-one*
- For a plausible range of r^* the slope of the (r^*, π^*) locus is about -0.9
- That finding is robust to:

  - parameter uncertainty
  - source of variation in r^*
  - alternative assumptions

Related literature

- Quantitative analysis of the optimal inflation target: Khan et al. (2003), Schmitt-Grohé and Uribe (2010), Amano et al. (2009), Carlsson and Westermark (2016), Bilbiie et al. (2014), Ascari et al. (2015), Adam and Weber (2017), Lepetit (2018),...
- Quantitative analysis of the optimal inflation target in the presence of the ZLB: Coibion et al. (2012), Dordal-i-Carreras et al. (2016), Kiley and Roberts (2017), Blanco (2016),...
- Our contribution:

- explicit analysis of the relation between r^* and π^*
- joint modelling of (i) price and wage stickiness (with partial indexation) and (ii) a ZLB constraint
- optimization under parameter uncertainty

The Model

- Representative household with preferences:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ e^{\zeta_{g,t+s}} \log(C_{t+s} - \eta C_{t+s-1}) - \frac{\chi}{1+\nu} \int_0^1 N_{t+s}(h)^{1+\nu} dh \right\}$$

and budget constraint

$$P_t C_t + e^{\zeta_{q,t}} Q_t B_t \leq \int_0^1 W_t(h) N_t(h) dh + B_{t-1} - T_t + D_t$$

- Final goods: perfect competition with technology

$$Y_t = \left(\int_0^1 Y_t(f)^{(\theta_p - 1)/\theta_p} df \right)^{\theta_p/(\theta_p - 1)}$$

- Intermediate goods: monopolistic competition with technology

$$Y_t(f) = Z_t L_t(f)^{1/\phi}$$

where $Z_t = Z_{t-1} e^{\mu_z + \zeta_{z,t}}$

The Model

- Price setting à la Calvo, with stochastic subsidies $\zeta_{u,t}$, and partial indexation

$$P_t(f) = \Pi_{t-1}^{\gamma_p} P_{t-1}(f)$$

- Wage setting à la Calvo, with partial indexation

$$W_t(h) = e^{\gamma_z \mu_z} \Pi_{t-1}^{\gamma_w} W_{t-1}(h)$$

- Interest rate rule:

$$i_t = \max\{i_t^n, 0\}$$

where

$$i_t^n - i = \rho_i (i_{t-1}^n - i) + (1 - \rho_i) \left[a_\pi (\pi_t - \pi^*) + a_y (y_t - y_t^n) \right] + \zeta_{r,t}$$

with $i = \rho + \mu_z + \pi^*$ and where π^* defines the *inflation target*.

Solution Method

1. Detrending by Z_t
2. Log-linearization around deterministic steady state
3. Solution under the ZLB as in Bodenstein et al. (2009) and Guerrieri and Iacoviello (2015)

Calibration and Estimation

- Calibrated parameters: $1/\phi = 0.7$; $\theta_p = 6$; $\theta_w = 3$
- Remaining parameters estimated using Bayesian approach (without ZLB)
- Gaussian priors for (ρ, μ_z, π^*) with means consistent with average inflation, GDP growth and real rate in each economy
- Sample period: 1985Q2-2008Q3
- Vector of observables:

$$x_t = [\Delta \log GDP_t, \Delta \log GDP\ \ Deflator_t, \Delta \log Wage_t, Short\ \ term\ \ rate_t]$$

- Parameter estimates

(a) $r^{ea} > r^{us} \Rightarrow$ larger π cushion needed in the US
(b) greater indexation in the US $\Rightarrow$ more tolerance of higher inflation

Optimal Inflation Target

- Second order approximation to household expected utility: $\mathcal{W}(\pi;\theta)$
- The case of no parameter uncertainty:

$$\pi^*(\theta) = \arg\max_{\pi} \mathcal{W}(\pi;\theta)$$

with solution obtained via numerical simulations allowing for occasionally binding ZLB, and with θ taken to be the *mean*, the *median* or the *mode* of the posterior distribution of parameter estimates:

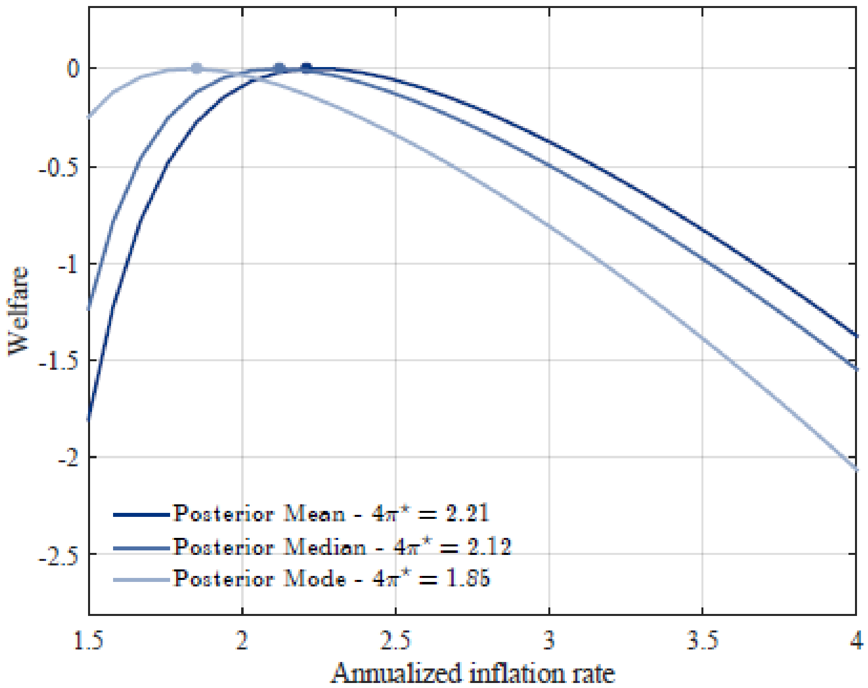
$$\Rightarrow \pi^*_{US} \in [2.21\%,\ 2.12\%,\ 1.85\%]$$

$$\Rightarrow \pi^*_{EA} \in [1.58\%,\ 1.49\%,\ 1.31\%]$$

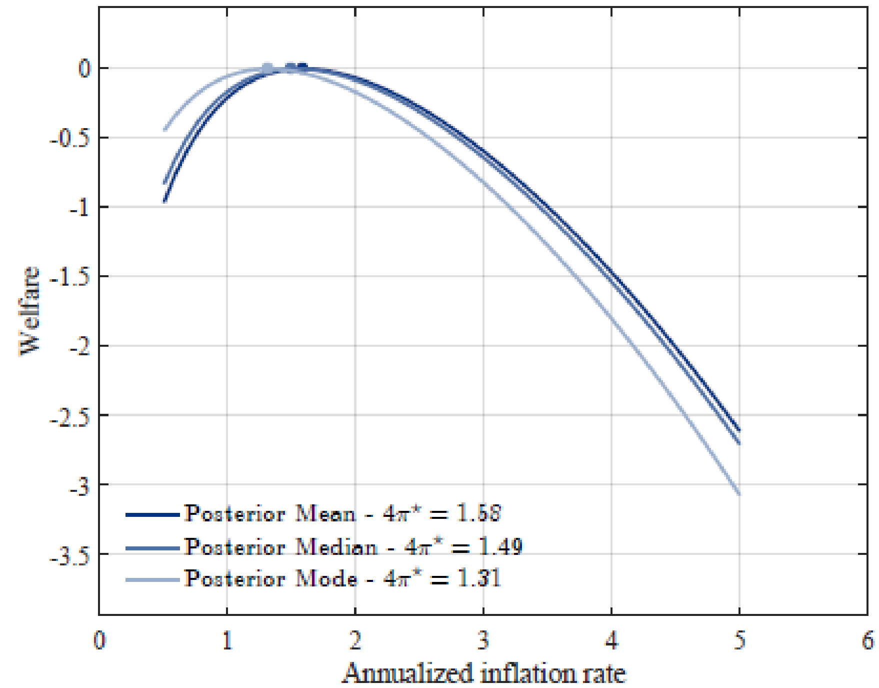
Welfare Losses and the Inflation Target

No Parameter Uncertainty

(a) US

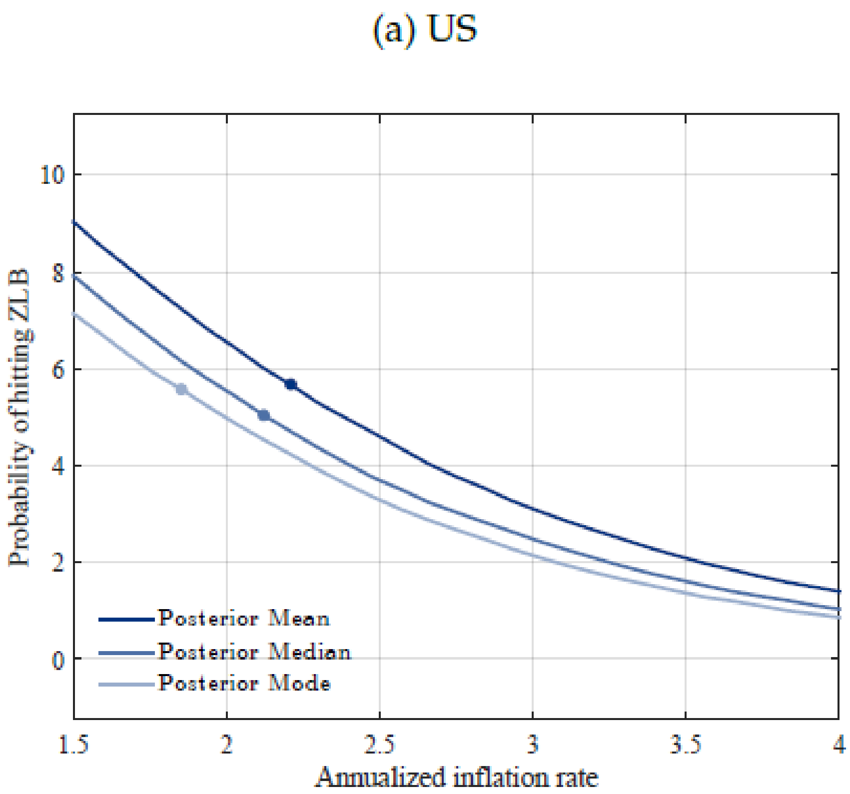


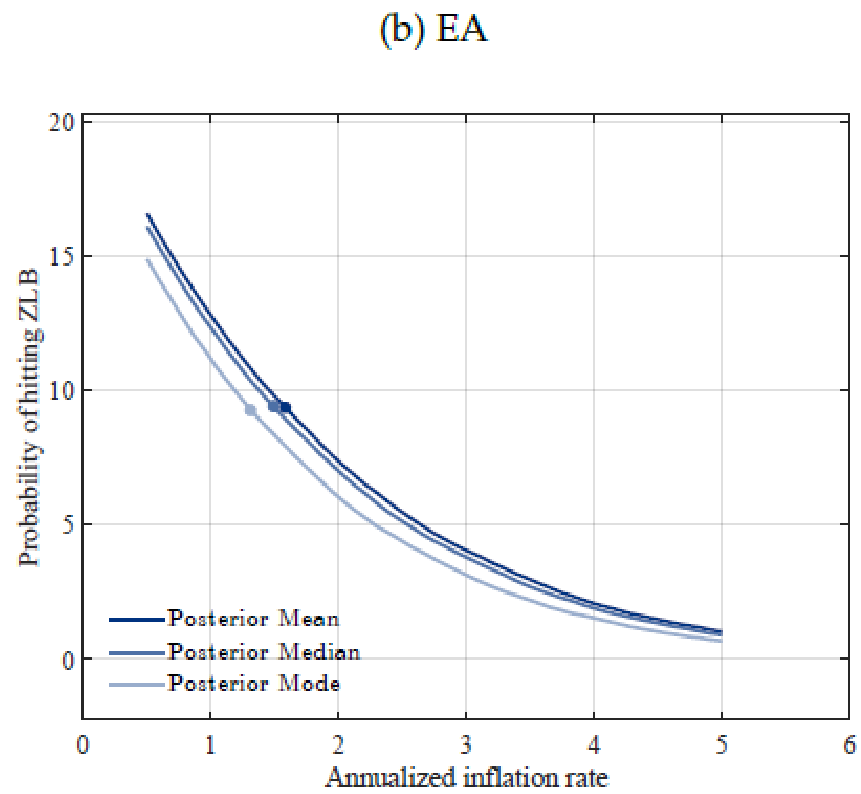
(b) EA



ZLB Incidence and the Inflation Target

No Parameter Uncertainty





Optimal Inflation Target

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with solution obtained via numerical simulations allowing for occasionally binding ZLB, and with θ taken to be the mean, the median or the mode of the posterior distribution of parameter estimates:

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- Allowing for parameter uncertainty:

$$\pi^{**} = \arg\max_{\pi} \int_{\theta} \mathcal{W}(\pi;\theta) p(\theta | X_T) d\theta$$

$$\Rightarrow \pi^{**}_{US} = 2.4\%$$

$$\Rightarrow \pi^{**}_{EA} = 2.2\%$$

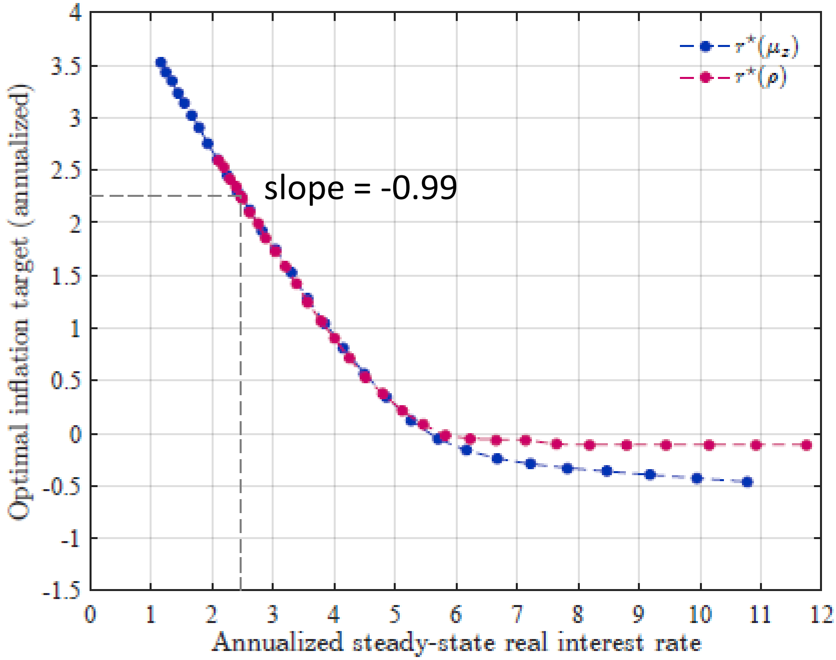
The Optimal Inflation Target and the Natural Rate of Interest

- The baseline (r^*, π^*) relation

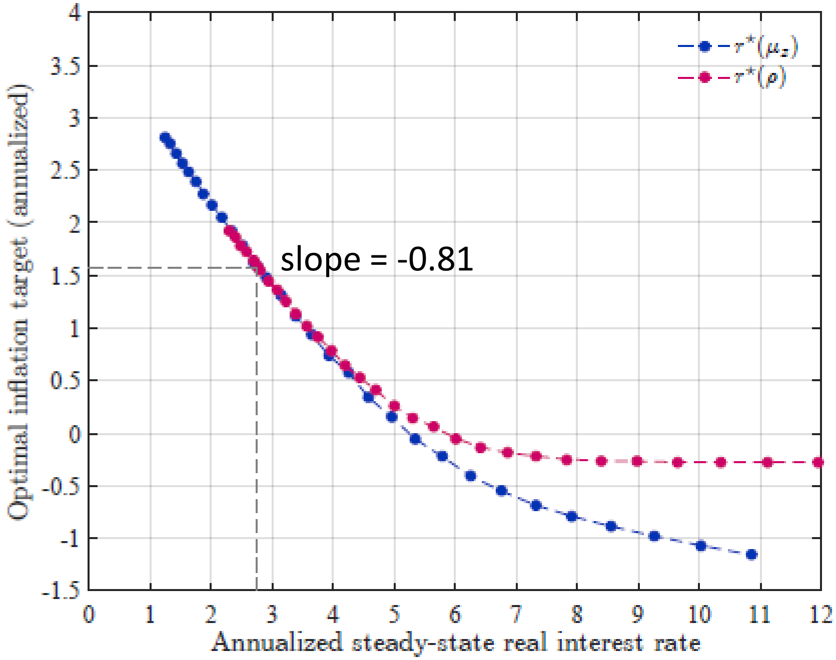
(a) varying μ_z
(b) varying ρ

The (r*, π*) Locus
(at the posterior mean)

(a) US



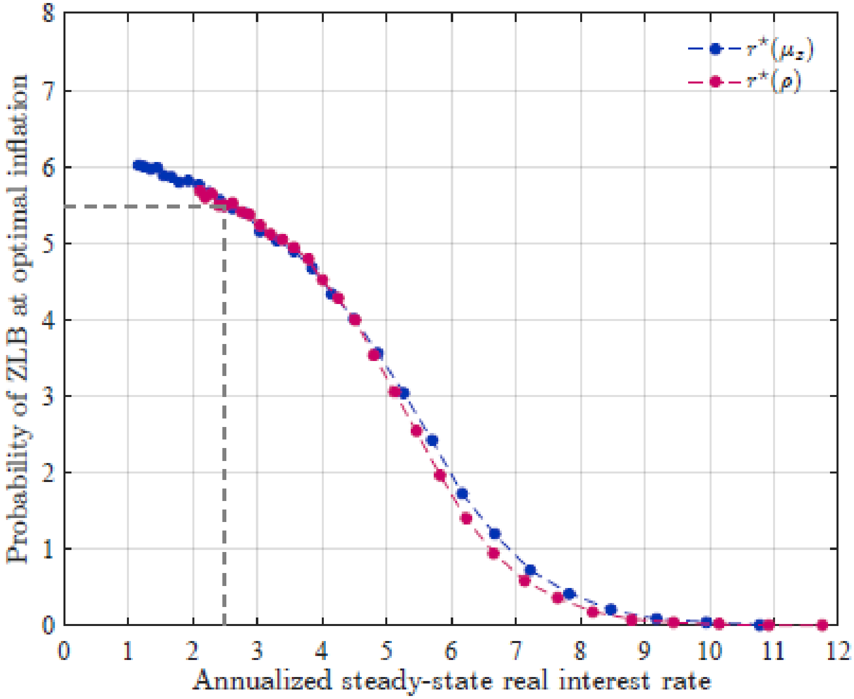
(b) EA



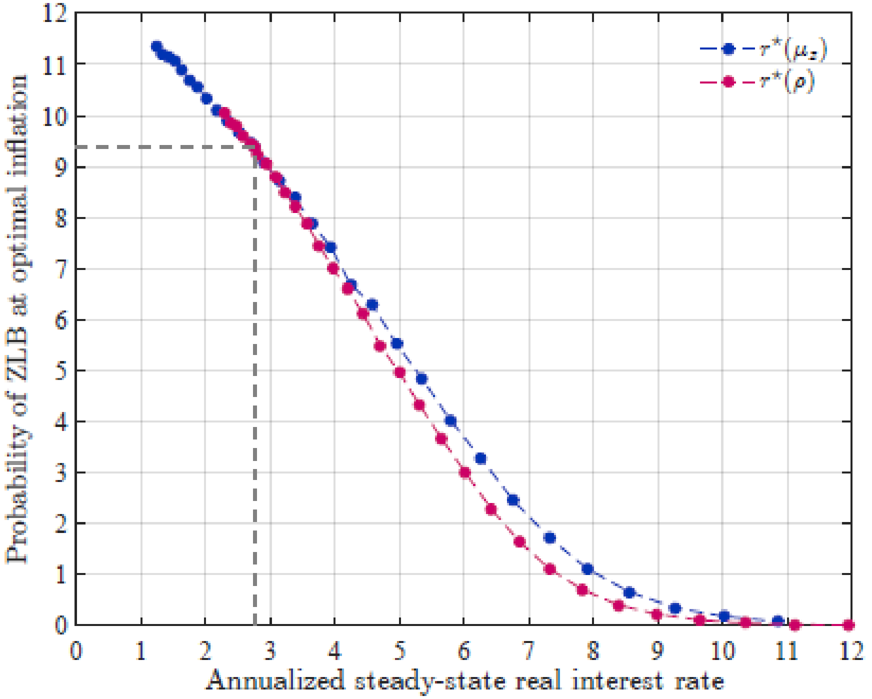
ZLB Incidence and the Steady State Real Rate

At the optimal inflation target

(a) US



(b) EA



The Optimal Inflation Target and the Natural Rate of Interest

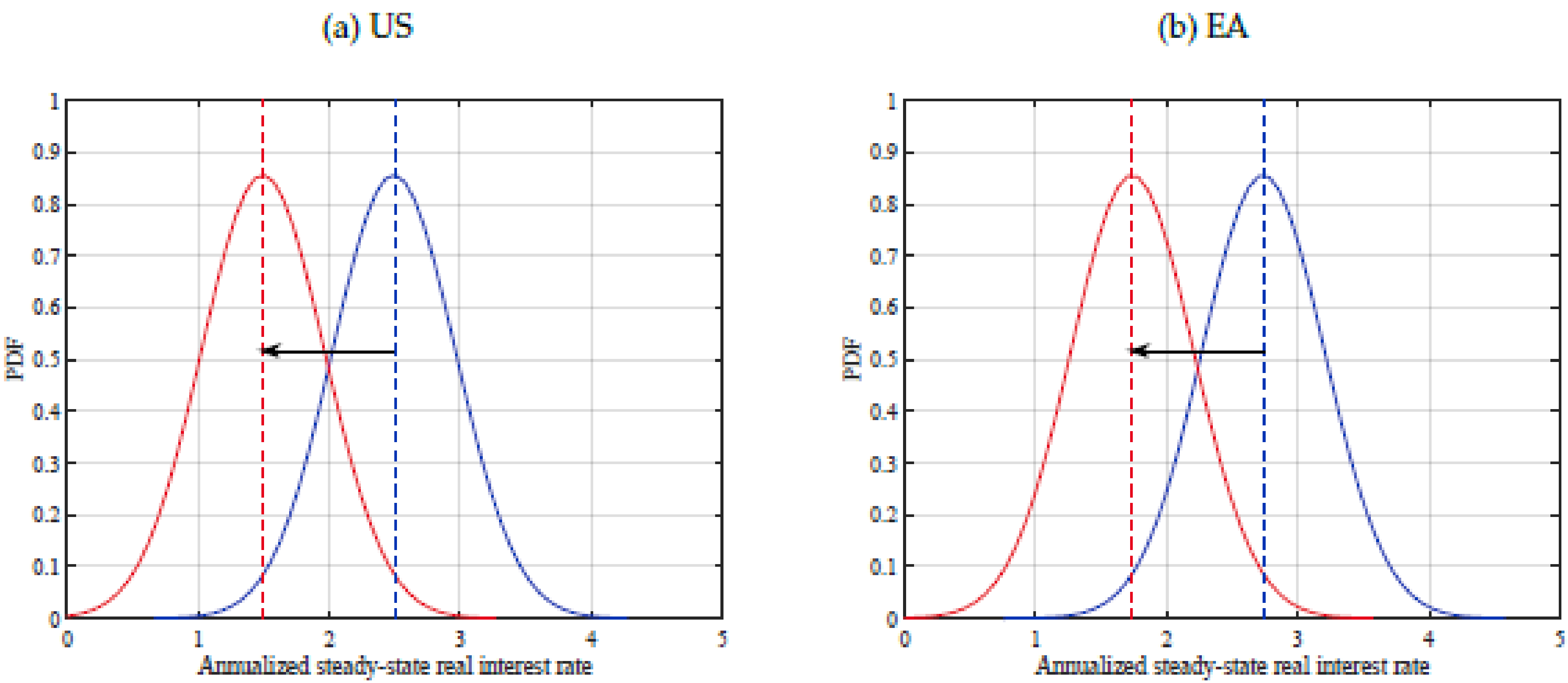
- The baseline (r^*, π^*) relation

(a) varying μ_z
(b) varying ρ

- The (r^*, π^*) relation *under uncertainty*: shift of -1% in the distribution of $r^*(\theta)$ due to a -1% shift in the mean of μ_z

$$\pi_\Delta^{**} = \arg\max_\pi \int_{\theta_\Delta} \mathcal{W}(\pi; \theta_\Delta) p(\theta_\Delta | X_T) d\theta_\Delta$$

Impact on Welfare of a Downward Shift in $r^*(\theta)$ under Parameter Uncertainty



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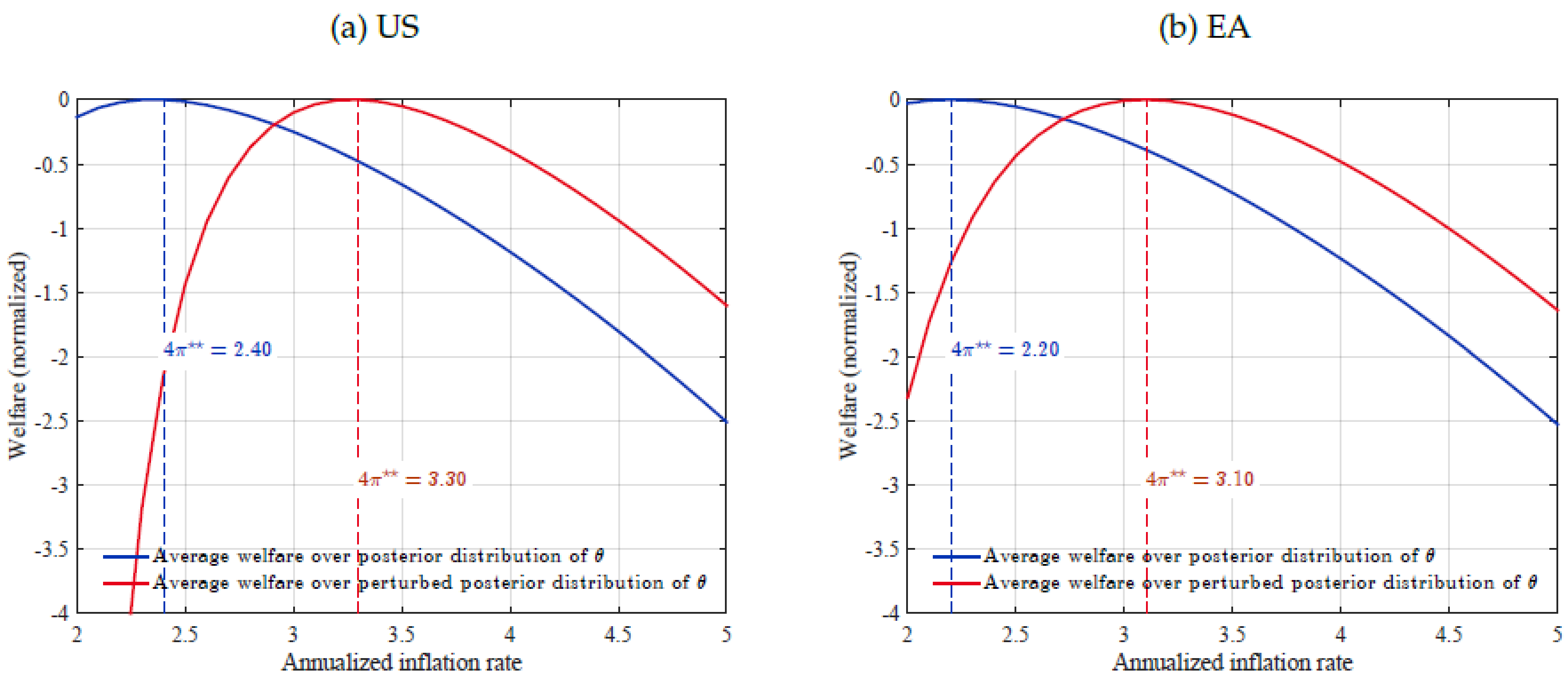
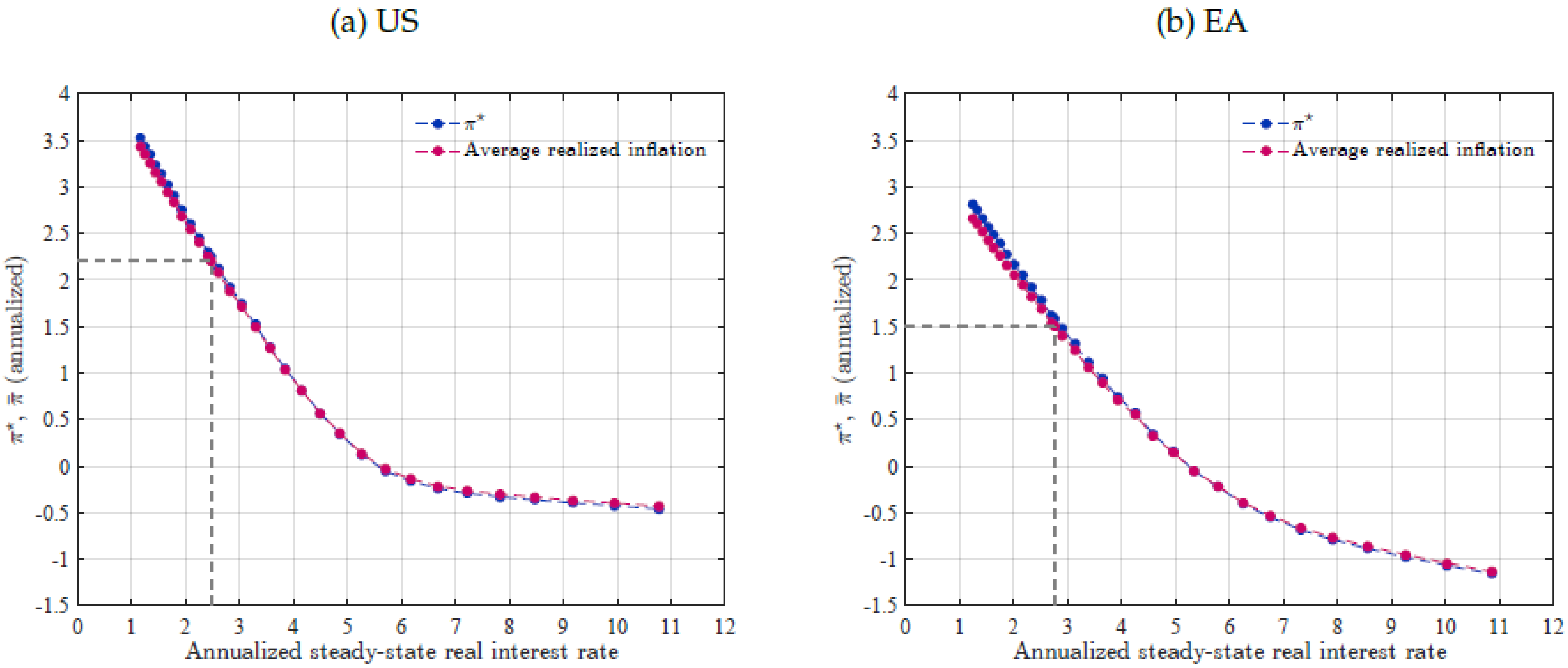


Further Experiments

- Average vs target inflation

Average vs Target Inflation

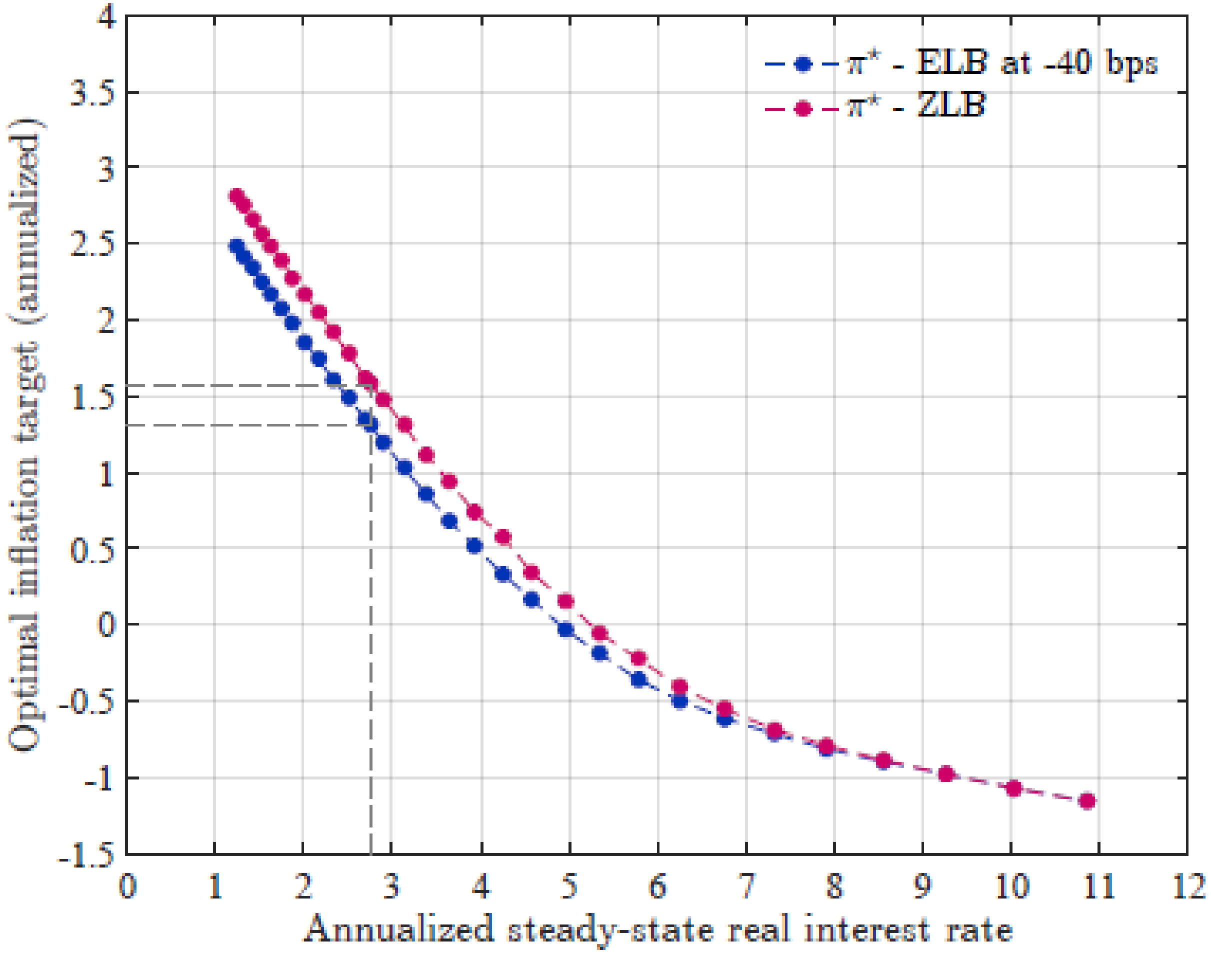


Further Experiments

- Average vs target inflation
- A negative effective lower bound (ELB)

$$i_t \geq -0.40\%$$

A Negative Effective Lower Bound in the Euro Area



Further Experiments

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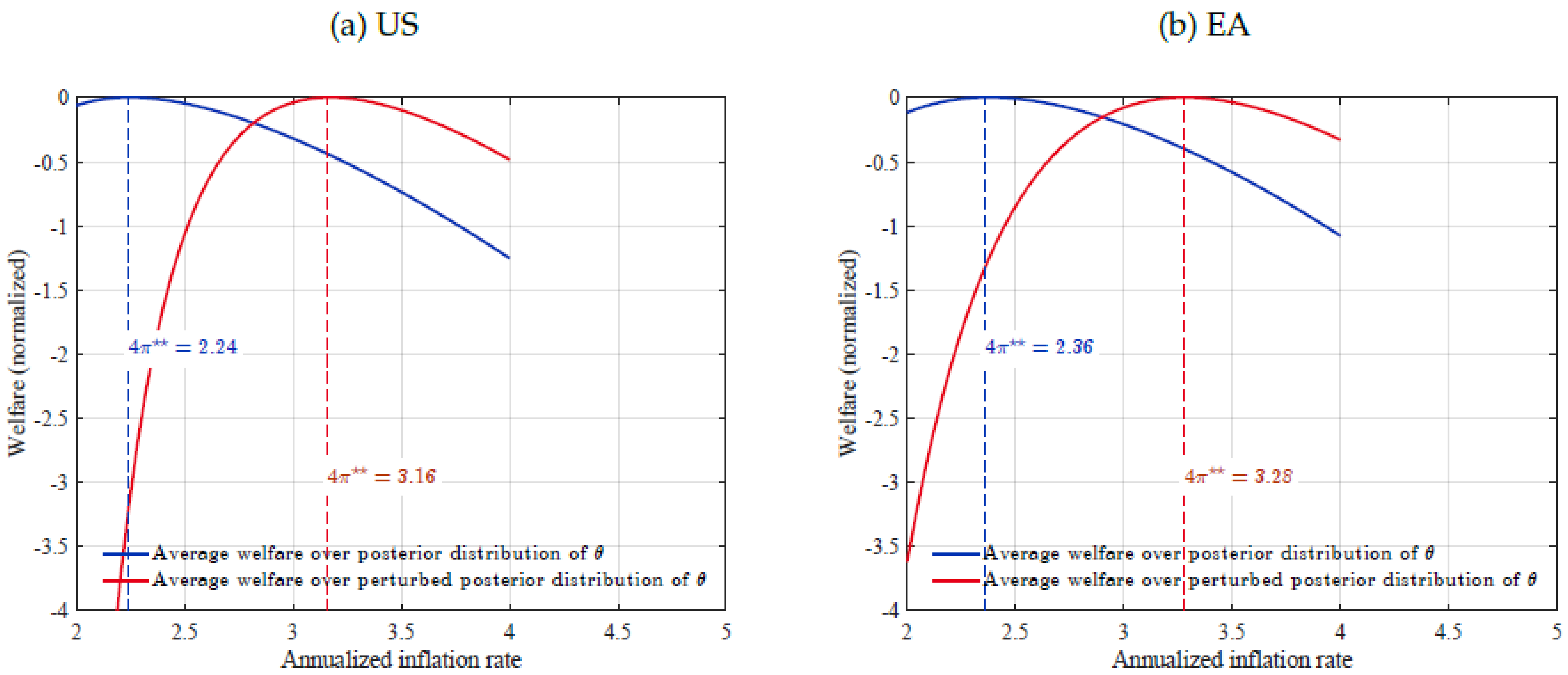
- Known reaction function: (ρ_i, a_π, a_y) fixed at posterior means

A Known Reaction Function



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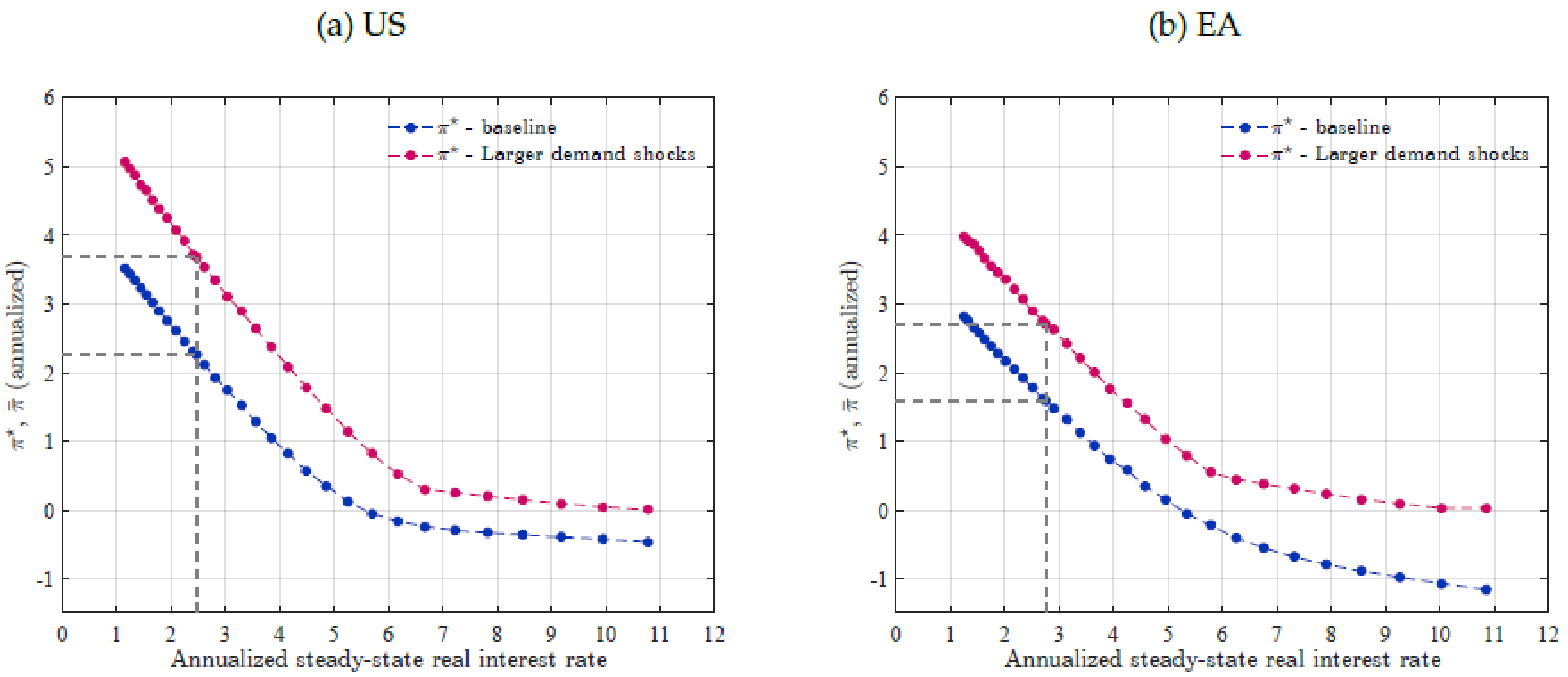
- Known reaction function: (ρ_i, a_π, a_y) fixed at posterior means
- Larger shocks: $+30\%$ increase in σ_q and σ_g

Larger Shocks



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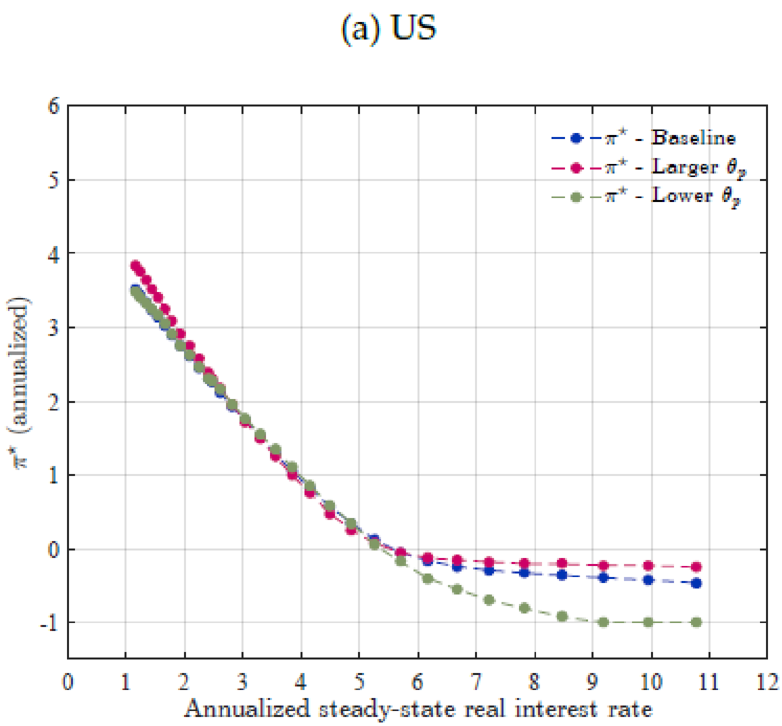
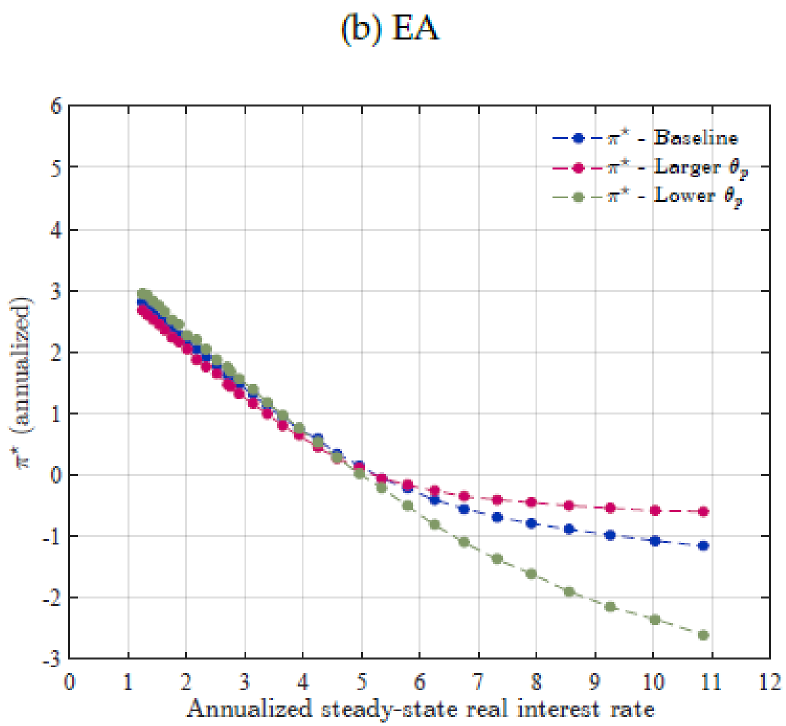
- Alternative steady state markups

Alternative Steady State Price Markups

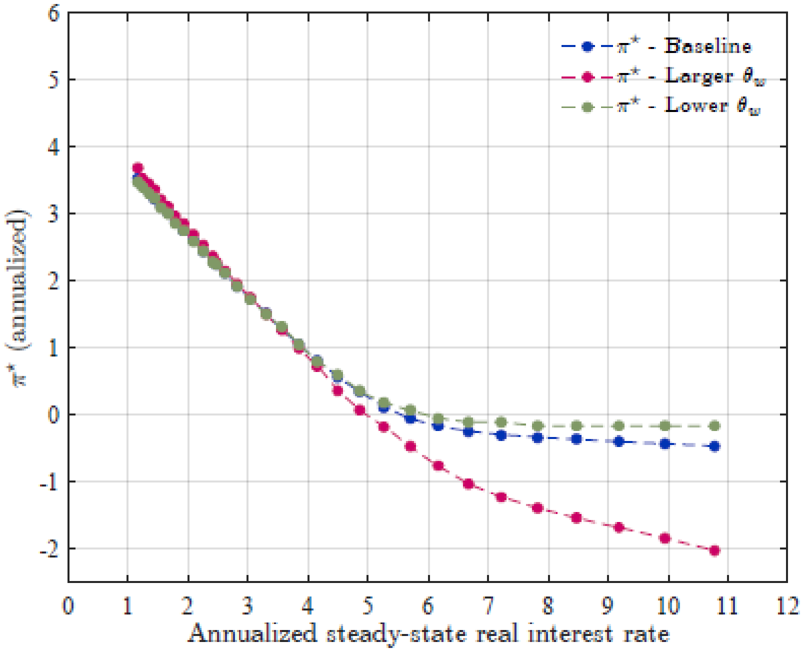




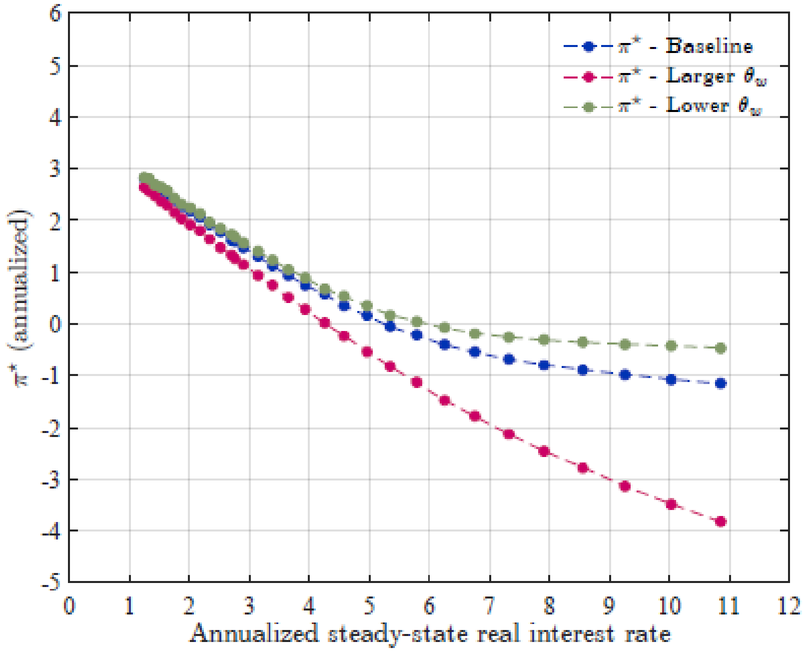
Note: the blues dots correspond to the baseline scenario wherein all the structural parameters are set at their posterior mean $\bar{\theta}$. The red dots correspond to the counterfactual simulation with θ_p set to 10. The green dots correspond to the counterfactual simulation with θ_p set to 3.

Alternative Steady State Wage Markups

(a) US



(b) EA



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Further Experiments

- Average vs target inflation
- A negative effective lower bound (ELB)

$$i_t \geq -0.40\%$$

- Known reaction function: (ρ_i, a_π, a_y) fixed at posterior means
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- Alternative steady state markups
- Alternative interest rate smoothness parameter

Alternative Interest Rate Smoothing Parameters

Figure: $(r^{\star}, \pi^{\star})$ – US

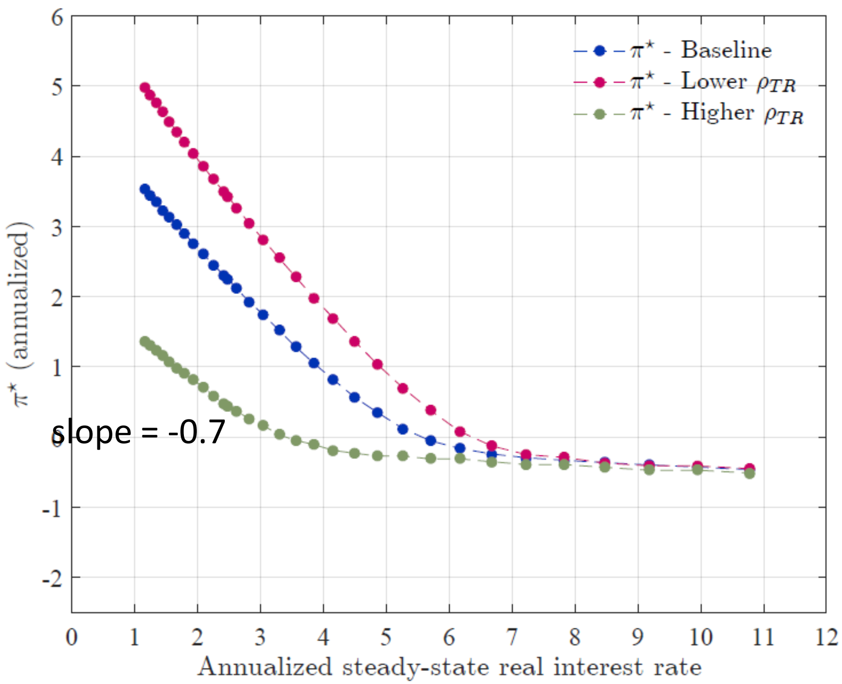
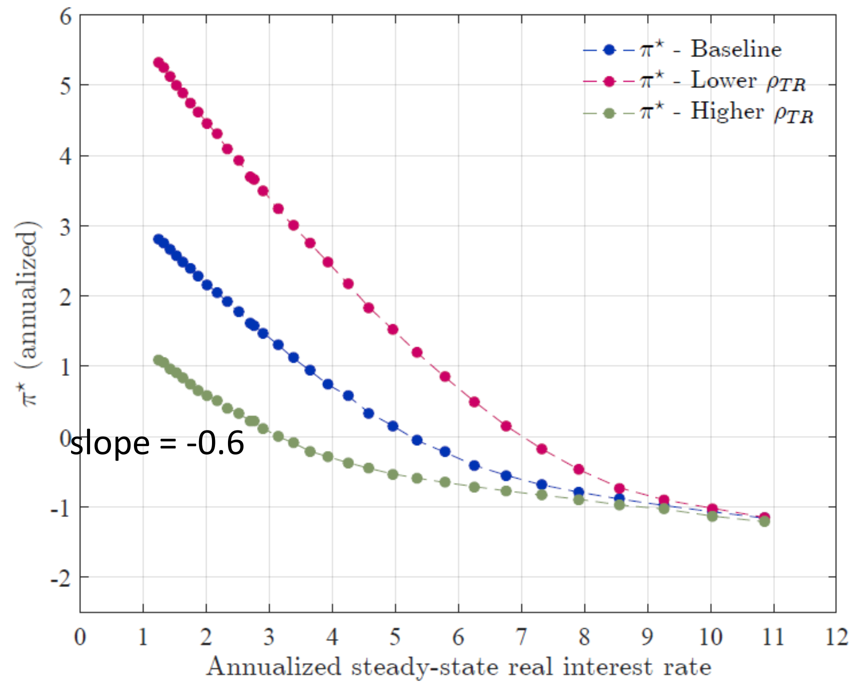


Figure: $(r^{\star}, \pi^{\star})$ – EA



Conclusions

- Analysis of the (r^*, π^*) relation
- Robust finding: a 1% decline in r^* call for an increase of about 0.9% in π^*

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- Robust finding: a 1% decline in r^* call for an increase of about 0.9% in π^*
- Alternatives to an increase in π^*:

  - unconventional monetary policies when ZLB becomes binding
  - adoption of price level targeting
  - countercyclical fiscal policies

- Transition and credibility

Table 1: Estimation Results - US

Parameter	Prior Shape	Prior Mean	Priod std	Post. Mean	Post. std	Low	High
ρ	Normal	0.20	0.05	0.19	0.05	0.11	0.27
μ_z	Normal	0.44	0.05	0.43	0.04	0.36	0.50
π^*	Normal	0.61	0.05	0.62	0.05	0.54	0.69
α_p	Beta	0.66	0.05	0.67	0.03	0.61	0.73
α_w	Beta	0.66	0.05	0.50	0.05	0.43	0.58
γ_p	Beta	0.50	0.15	0.20	0.07	0.08	0.32
γ_w	Beta	0.50	0.15	0.44	0.16	0.21	0.68
γ_z	Beta	0.50	0.15	0.50	0.18	0.26	0.75
η	Beta	0.70	0.15	0.80	0.03	0.75	0.85
ν	Gamma	1.00	0.20	0.73	0.15	0.47	0.97
a_π	Gamma	2.00	0.15	2.13	0.15	1.89	2.38
a_y	Gamma	0.50	0.05	0.50	0.05	0.42	0.58
ρ_{TR}	Beta	0.85	0.10	0.85	0.02	0.82	0.89
σ_z	Inverse Gamma	0.25	1.00	1.06	0.22	0.74	1.38
σ_R	Inverse Gamma	0.25	1.00	0.10	0.01	0.09	0.11
σ_q	Inverse Gamma	0.25	1.00	0.39	0.11	0.16	0.61
σ_g	Inverse Gamma	0.25	1.00	0.23	0.04	0.16	0.29
σ_u	Inverse Gamma	0.25	1.00	0.24	0.05	0.06	0.46
ρ_R	Beta	0.25	0.10	0.51	0.06	0.41	0.61
ρ_z	Beta	0.25	0.10	0.27	0.13	0.09	0.45
ρ_g	Beta	0.85	0.10	0.98	0.01	0.97	1.00
ρ_q	Beta	0.85	0.10	0.88	0.04	0.80	0.95
ρ_u	Beta	0.80	0.10	0.80	0.10	0.65	0.96

Note: 'std' stands for Standard Deviation, 'Post.' stands for Posterior, and 'Low' and 'High' denote the bounds of the 90% probability interval for the posterior distribution.

Table 2: Estimation Results - EA

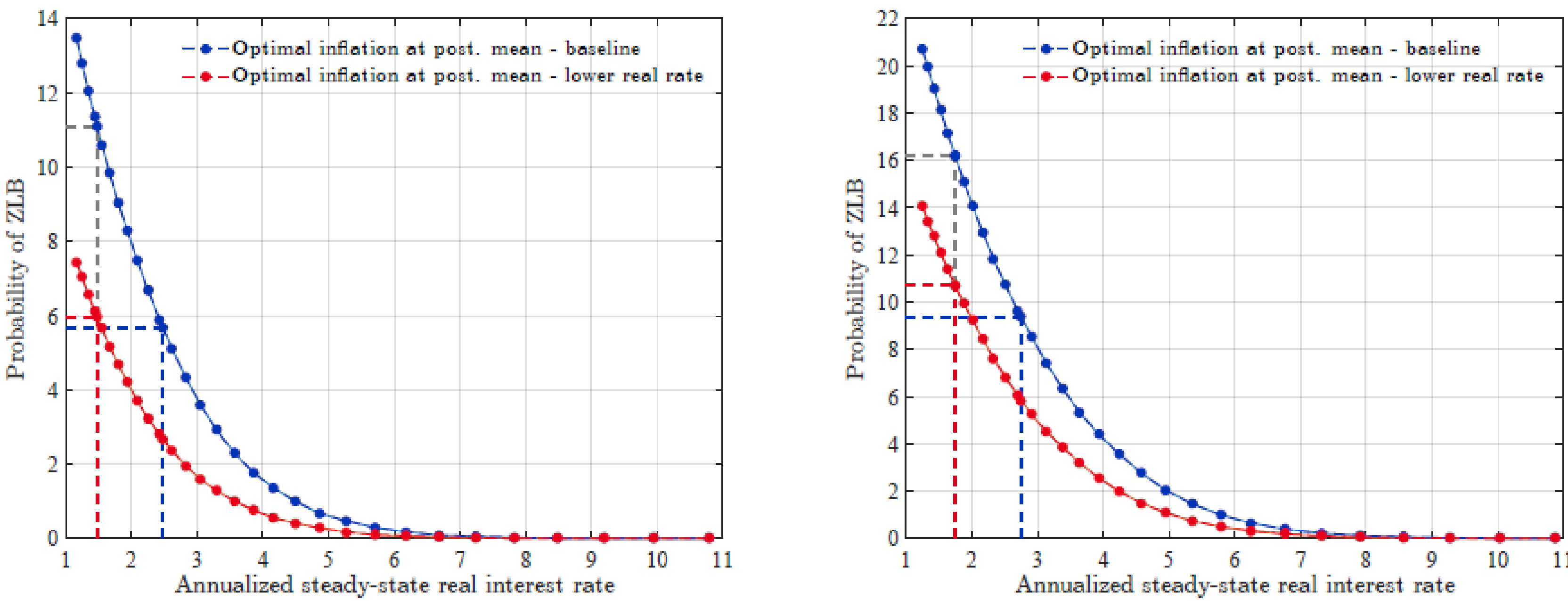
Parameter	Prior Shape	Prior Mean	Priod std	Post. Mean	Post. std	Low	High
ρ	Normal	0.20	0.05	0.21	0.05	0.13	0.29
μ_z	Normal	0.50	0.05	0.47	0.05	0.40	0.55
$\pi^\star$	Normal	0.80	0.05	0.79	0.05	0.71	0.86
α_p	Beta	0.66	0.05	0.62	0.05	0.55	0.68
α_w	Beta	0.66	0.05	0.59	0.04	0.52	0.65
γ_p	Beta	0.50	0.15	0.12	0.04	0.04	0.19
γ_w	Beta	0.50	0.15	0.34	0.12	0.15	0.53
γ_z	Beta	0.50	0.15	0.51	0.18	0.26	0.76
η	Beta	0.70	0.15	0.74	0.04	0.69	0.80
ν	Gamma	1.00	0.20	0.96	0.18	0.65	1.25
a_π	Gamma	2.00	0.15	2.02	0.14	1.80	2.25
a_y	Gamma	0.50	0.05	0.50	0.05	0.42	0.58
ρ_{TR}	Beta	0.85	0.10	0.87	0.02	0.84	0.90
σ_z	Inverse Gamma	0.25	1.00	0.86	0.16	0.63	1.10
σ_R	Inverse Gamma	0.25	1.00	0.11	0.01	0.10	0.12
σ_q	Inverse Gamma	0.25	1.00	0.23	0.05	0.13	0.32
σ_g	Inverse Gamma	0.25	1.00	0.21	0.04	0.15	0.27
σ_u	Inverse Gamma	0.25	1.00	0.23	0.05	0.06	0.43
ρ_R	Beta	0.25	0.10	0.39	0.07	0.27	0.50
ρ_z	Beta	0.25	0.10	0.24	0.10	0.09	0.39
ρ_g	Beta	0.85	0.10	1.00	0.01	0.99	1.00
ρ_q	Beta	0.85	0.10	0.94	0.03	0.90	0.98
ρ_u	Beta	0.80	0.10	0.79	0.10	0.64	0.96

Note: 'std' stands for Standard Deviation, 'Post.' stands for Posterior, and 'Low' and 'High' denote the bounds of the 90% probability interval for the posterior distribution.

ZLB Incidence and the Steady State Real Rate

Understanding the Mechanism

(a) US (b) EA



Note: The blue dots correspond to the relation linking $r^\star$ and the probability of ZLB, holding the optimal inflation target $\pi^\star$ at the baseline value. The red dots correspond the same relation when the optimal inflation target $\pi^\star$ is set at the value consistent with a steady-state real interest rate one percentage point lower.

The Slope of the (r^*, π^*) Locus: Summary Table

Table 3: Effect of a decline in $r^\star$ under alternative notions of optimal inflation

	US		EA	
	Baseline	Lower $r^\star$	Baseline	Lower $r^\star$
Mean of $\pi^\star$	2.00	3.00	1.79	2.60
Median of $\pi^\star$	1.96	2.90	1.47	2.28
$\pi^\star$ at post. mean	2.21	3.20	1.58	2.39
$\pi^\star$ at post. median	2.12	3.11	1.49	2.30
$\pi^{\star\star}$	2.40	3.30	2.20	3.10
$\pi^{\star\star}$, frozen MP	2.24	3.16	2.36	3.28
$\pi^\star$ at post. mean, ELB -40 bp	—	—	1.31	2.08
Average realized inflation at post. mean	2.20	3.19	1.56	2.36
Average realized inflation at post. mean, ELB -40 bp	—	—	1.24	1.97

Note: all figures are in annualized percentage rate.