Money Markets, Collateral and Monetary Policy[*]

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Abstract

Interbank money markets have been subject to substantial impairments in the recent decade, such as a decline in unsecured lending and substantial increases in haircuts on posted collateral. This paper seeks to understand the implications of these developments for the broader economy and monetary policy. To that end, we develop a novel general equilibrium model featuring heterogeneous banks, interbank markets for both secured and unsecured credit, and a central bank. The model features a number of occasionally binding constraints. The interactions between these constraints - in particular leverage and liquidity constraints - are key in determining macroeconomic outcomes. We find that both secured and unsecured money market frictions force banks to either divert resources into unproductive but liquid assets or to de-lever, which leads to less lending and output. If the liquidity constraint is very tight, the leverage constraint may turn slack. In this case, there are large declines in lending and output. We show how central bank policies which increase the size of the central bank balance sheet can attenuate this decline.

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1 Introduction

Interbank money markets are essential to the liquidity management of banks. They are also important for monetary policy implementation as interbank rates are often central banks' target rates. Money market trade is subject to a number of frictions, which displayed themselves forcefully during the Global Financial Crisis, with the unsecured segment "freezing"(see, e.g., Heider et al. (2015)) and the secured segment facing "runs"due to haircut increases on riskier collateral (see, e.g., Gorton and Metrick (2012)). Yet, the question of what impact the frictions in bank liquidity management have on the broader economy is largely unaddressed.

In this paper, we take a step towards understanding the impact of frictions in money markets on bank lending, real activity and monetary policy. We develop a novel general equilibrium model featuring heterogeneous banks, interbank money markets for both secured and unsecured credit, and a central bank that can conduct open market operations as well as lend to banks against collateral. As a particular advance compared to the existing literature, banks may both face leverage constraints and liquidity constraints: the interaction of these constraints is at the heart of our analysis. Each period in the model is sub-divided into a morning and an afternoon. In the morning, banks choose their assets (loans, bonds and money) and liabilities (central bank funding and deposits), subject to a leverage constraint as proposed by Gertler and Karadi (2011) and Gertler and Kiyotaki (2011). On the liability side, central bank funding must be backed by bond collateral. Deposit funding is uncollateralized but it exposes banks to idiosyncratic withdrawal shocks in the afternoon, as formulated by Bianchi and Bigio (2013). These withdrawal shocks can be managed by borrowing or lending in interbank money markets. Banks face an exogenous probability of being "connected," defined as the ability to borrow in the unsecured market in the afternoon. Those banks that are unable to borrow in the unsecured market, the "unconnected" banks, can satisfy withdrawals either by acquiring bonds in the morning to obtain collateralized funding in the private market in the afternoon and/or by bringing money into the afternoon (self-insurance). All collateralized borrowing is subject to a haircut, with haircuts in the private market being potentially different from haircuts set by the central bank.

Five inequality constraints on banks emerge as crucial: the "morning" leverage constraint, a collateral constraint vis-a-vis the central bank and three short-sale constraints. We show that one cannot a priori impose any of these constraints to bind or to be slack: on the contrary, each of these may turn on or off and each can be crucial for the macroeconomic outcomes, as we traverse the parameter space and for different monetary policies. We view this as a novel, intriguing and central contribution of our paper. Usually, a single inequality is studied and equality is often imposed. By contrast, our five-dimensional inequality space offers a rich set of interactions. Different parameter values then generate different types of bottlenecks, which an astute central bank all needs to take into account and which we argue to be key to understanding the financial system. We investigate the role of these constraints per conducting a steady-state comparative static analysis, when varying the severity of a particular money market friction and imposing a particular monetary policy. Given the high dimensionality of the constraints space, we deliberately chose the steady-state comparative statics as the more illuminating mode of analysis compared to a fully dynamic and stochastic, but likely opaque approach. Additionally, with persistent money market frictions, a steady-state comparative statics appears to be more appropriate in any case.

Indeed, our modelling framework is motivated by two major and persistent money market developments that occurred over the past fifteen years: a decline in the unsecured interbank market and a corresponding increased reliance on the secured market, which consequently exposes banks more substantially to the concurrent increase in collateral haircuts. We document these developments using data for the euro area, but similar changes have been observed in the US (see footnote 2 on the next page).

The first development is documented in Figure 1. While the total turnover was split about equally between unsecured and secured market segments in 2003, the turnover in the unsecured market declined five-fold and was just five percent of total by 2017.[1] The decline in the relative importance of the unsecured market started several years before the global financial crisis of 2008, and further steepened with the onset of the financial and sovereign debt crisis in the euro area.

The second development is the declining value of assets used as collateral in the secured market, which had two sources in recent years. First, there were large and abrupt increases in haircuts on some asset classes. In the euro area, haircuts on government bonds of some euro area countries increased to 80 percent or higher during the sovereign debt crisis (Table 1). Even outside the period of the sovereign crisis, in a relatively calm year such as 2017, private market haircuts remained heterogeneous across countries and did not return to the pre-crisis levels. At the same time, haircuts applied by the European Central Bank (ECB) on the same collateral were much lower than private market haircuts and remained largely

[1] By contrast, turnover levels in the secured market actually increased between 2003 and 2017.

stable throughout this period. Second, the stock of safe (AAA-rated) assets fell due to rating downgrades, which reduced the availability of high-quality collateral that could be pledged in the secured market. In the euro area, the downgrades also affected sovereign bonds, with the proportion of AAA-rated government debt falling from 60 percent of total debt outstanding in 2003 to 20 percent in 2017 (Figure 2).[2]

Different observers may attribute these two developments to different underlying causes. For example, perhaps the private sector haircuts and high yields on certain sovereign bonds reflect a dysfunctional system or a bad equilibrium, which the ECB appropriately seeks to correct, see e.g. Roch and Uhlig (2018). Conversely, perhaps these haircuts are due to the the appropriate rational assessment of default risks of the underlying bonds, while the ECB haircuts are too small. These varying points of view are parts of a heated and contentious debate in Europe, to which we do not wish to contribute in this paper. Instead, our focus is on the response of the system, if these private haircuts increase compared to those charged by the central bank, focusing on the benign branch of events, where no defaults occur. For that reason, we do not explicitly model how these haircuts arise, but instead treat them as an exogenous parameter. We view our results as a positive rather than normative analysis, providing an important piece of an all-encompassing view. We likewise treat the fraction of "unconnected" banks, which can only use the secured interbank market, as an exogenous parameter in our analysis.

We therefore provide two sets of steady state comparative statics scenarios, varying either the fraction of unconnected banks or varying the private sector haircut on government bonds. Both types of money market frictions force banks to either divert resources into unproductive but liquid assets (bonds or money rather than productive capital) or to de-lever (raise fewer deposits as it is deposit funding that exposes banks to liquidity shocks). This leads to less lending and output in the economy. If the liquidity constraint is very tight, the leverage constraint may turn slack. In this case, there are large declines in lending and output, in the absence of central bank intervention. Policies that increase the size of the central bank balance sheet (outright purchases or collateralized lending) alleviate the bank liquidity constraint by expanding the money supply and attenuate the decline in lending and output. They may, of

[2] In the US, the size of the interbank money market declined from the estimated $100 billion before the financial crisis to less than $5 billion today (see Kim et al. (2018)). For the US secured market, Gorton and Metrick (2012) provide evidence of increases in average haircuts on risky collateral from around zero in early 2007 to 50% in late 2008, contributing to the emergence of "repo runs"during the financial crisis.

course, unduly increase the risk exposure by the central bank, but this is outside of our analysis for the reasons stated.

A key contribution of our paper is to allow for three different avenues for central bank liquidity provision. We consider three instruments in particular: central bank holdings of government bonds, the interest rate on central bank loans, and haircuts on accepted collateral. We relate these instruments to the stylized versions of monetary policies pursued by central banks around the world in recent years: i) a pre-crisis policy characterized by a constant balance sheet; ii) a policy where the balance sheet is expanded via *collateralized credit operations* ("CO" henceforth), whereby the central bank stands ready to provide the liquidity demanded by banks at a given interest rate and haircut level; and iii) a policy of *outright asset purchases* ("OP" henceforth), whereby the central bank changes the stock of bonds on its balance sheet to achieve a certain inflation goal.[3]

We calibrate the model to the euro area data and use it to analyze the macroeconomic impact and central bank policies under the two alternative scenarios.

In the first scenario, i.e. when the share of connected banks is varied, a constant-balance sheet policy or collateralized credit operations make no difference, as there is no advantage to borrow from the central bank compared to borrowing on the private market in the afternoon. By contrast, open market asset purchases inject much needed liquidity generally, and can substantially alleviate the negative output effects that would otherwise materialize. In our benchmark calibration, the difference in output between a steady-state with 0.58 share of unconnected banks and that with 0.95 share (average pre-2008 versus 2017 share of secured turnover in total) is around 1.5 percent in the CO case, and 1 percent in the OP case.

In the second scenario of varying private-sector haircuts and under a constant central bank balance sheet policy, the difference in output between a steady-state with 3 percent haircuts and one with 40 percent haircuts is 5 percent. The key to mitigating the reduction in capital and output is to provide liquidity to the unconnected banks and to prevent their leverage constraint from turning slack. This can now be achieved both with the CO policy by lending to banks against collateral at favorable haircuts or per the OP policy of open market purchases of goverment bonds.

The paper proceeds as follows. In section 2, we relate our paper to the existing literature. In section 3, we describe the model. In section 4, we define the equilibrium. In section 5,

[3]The CO and OP policies are reminiscent of the ECB's fixed-rate full allotment policy implemented since 2008 and the Public Sector Purchase Programme implemented since 2015, respectively.

we characterize the system of equilibrium conditions. In section 6, we describe the steady state and present some analytical results. Section 7 illustrates the model predictions through a numerical analysis. Section 8 concludes.

2 Related literature

Our paper is related to the broad literature that investigates the implications of financial frictions for the macroeconomy and for monetary policy as well as to the literature which focuses on frictions in secured and unsecured interbank trade. We now discuss how various elements in our analysis relate to these literatures.

Bank balance sheet constraints and monetary policy

A number of recent papers emphasize the role of banks' balance sheet and leverage constraints for the provision of credit to the real economy and for the transmission of standard and non-standard monetary policies (see e.g. Gertler and Karadi (2011) and Gertler and Kiyotaki (2011)). As in those papers, banks in our model face an enforcement problem and endogenous balance sheet constraints. Additionally, they solve a liquidity management problem that further constrains their actions. Another novel feature of our framework is that we do not impose the various constraints to be binding at all times (as in Brunnermeier and Sannikov (2014); He and Krishnamurthy (2016); Mendoza (2010); Bocola (2016); Justiniano et al. (2017)). Typically, however, only one or few occasionally binding constraints are considered. In our calibrated model, five key constraints can switch from binding to slack and vice versa, interacting in complex ways and determining the effectiveness of monetary policy.

Interbank markets and bank liquidity management

The role of interbank markets in banks' liquidity management is explored by a large literature in banking, starting with Bhattacharya and Gale (1987). Several papers analyse frictions in interbank markets that prevent an efficient distribution of liquidity within the banking system (Flannery (1996); Freixas and Jorge (2008); Freixas and Holthausen (2005); Repullo (2005); Freixas et al. (2011); Afonso and Lagos (2015); Atkeson et al. (2015)). Some of these frictions have played a particularly important role during the Global Financial Crisis. Heider et al. (2015) build a model where asymmetric information about banks' assets and counterparty risk induce banks to hoard liquidity and contribute to generate a "freeze" of the unsecured money market segment. Martin et al. (2014) characterize when expectations-driven runs in the secured market are possible.

Macroeconomic impact of money market frictions

Some recent papers explore the macroeconomic consequences of the money market frictions that featured prominently during the Global Financial Crisis. Altavilla et al. (2018) provide evidence that increases in interbank rate uncertainty, as observed during 2007-2009 and again during the European sovereign crisis, generate a significant deterioration in economic activity. Using a general equilibrium model, Bruche and Suarez (2010) show that freezes in the unsecured money market segment can cause large reallocation of capital across regions, with significant impact on output and welfare. Gertler et al. (2016) point to runs on wholesale banks as a major source of the breakdown of the financial system in 2007-2009, and show in a general equilibrium framework that this can have devastating effects on the real economy. Our paper contributes to this literature by considering both unsecured and secured funding. In our setup, frictions in the unsecured money market segment may in principle be offset by an increased recourse to private secured markets or to central bank funding.

Bank liquidity management and monetary policy

Frictions in the unsecured or secured money markets interact with the effectiveness of monetary policy. Bianchi and Bigio (2013) build a model where banks are exposed to liquidity risk and manage it by borrowing unsecured or by holding a precautionary buffer of reserves. Monetary policy affects lending and the real economy by supplying reserves and thus by changing banks' trade-off between profiting from lending and incurring greater liquidity risk. In a general equilibrium model that features the same search frictions in the interbank market as in Bianchi and Bigio (2013), Arce et al. (2017) show that a policy of large central bank balance sheet that uses interest rate policy to react to shocks achieves similar stabilization properties to a policy of lean balance sheet, where QE is occasionally used when the interest rate hits the zero-lower bound. Piazzesi and Schneider (2017) build a model in which the use of inside money by agents for transaction purposes requires banks to handle payments instructions. Banks thus lend or borrow secured in the interbank market, or use central bank reserves. In this framework, key to the efficiency of the payment system is the provision and allocation of collateral. Policies that exchange reserves for lower quality collateral can be beneficial when high quality collateral is scarce. In our model with both secured and unsecured money markets and a central bank providing collateralized loans or purchasing assets outright, it is the interplay between the bank liquidity and leverage constraints that is key in determining the macroeconomic impact of money market frictions and the effectiveness of central bank policies.

Scarcity of safe assets and the size of central bank balance sheet

The emergence of a shortage of safe assets has been documented and analyzed in a number of recent works (see e.g. Caballero et al. (2017), Andolfatto and Williamson (2015) and Gorton and Laarits (2018)). Some papers discuss the implications of scarcity for monetary policy. Caballero and Farhi (2017) analyze a situation of a deflationary safety trap and point to policies of "helicopter drops" of money, safe public debt issuances, or swaps of private risky assets for safe public debt as possible ways to mitigate the negative impact of safe asset scarcity. Carlson et al. (2016) argue that the central bank could react to safe asset scarcity by maintaining a large balance sheet and a floor system, as large holdings of long-term assets are financed by large amounts of reserves that are safe and liquid assets. Our model enables to compare alternative policies - outright purchases and collateralized credit operations - that can accommodate the increased demand for reserves through an expansion of the balance sheet.

3 The model

The economy is inhabited by a continuum of households, firms and banks. There is a government and a central bank.

Time is discrete, $t = 0, 1, 2, \ldots$. We think of a period as composed of two sub-periods, "morning" and "afternoon". Let us describe each in turn.

At the beginning of each period (in the morning), aggregate shocks occur. Households receive payments from financial assets and allocate their nominal wealth among money and deposits at banks. Households also supply labor to firms, receiving wages in return. The government taxes the labor income of the households, makes payments on its debt and may change the stock of outstanding debt. Banks accept deposits from households and the central bank and make dividend payments to households. After accepting deposits, banks learn their afternoon type in the morning. This latter can be either "connected," in which case banks can borrow in the unsecured interbank market, or "not connected," in which case they cannot, and the only possibility is to borrow by pledging assets in the secured interbank market. Banks then lend to firms (more precisely, finance their capital) and they hold government bonds and reserves ("cash"). The central bank provides funding to banks that wish to borrow against collateral. As an additional policy tool, the central bank can choose "haircuts" on the collateral pledged to access those funds.

During the afternoon, firms use labor and capital to produce a homogeneous output good which is consumed by households. Banks experience idiosyncratic deposit withdrawal shocks which average out to zero across all banks. Conceptually, these relate to random idiosyncratic consumption needs, additional economic activity and immediate payment for these services, which we shall refrain from modelling.[4] Banks can accommodate those shocks by using their existing reserves, by borrowing from other banks in the unsecured market, or by pledging bonds and borrowing in the secured market. They can only access the unsecured market, however, if they are "connected". Banks are assumed to always position themselves so as to meet these liquidity withdrawals, i.e., bank failures are considered too costly and not an option. All banks meet as "one big banker family" at the end of the period. One can think of it as follows. First, the same bank-individual liquidity shock happens "in reverse", so that banks enter the banker-family meeting in the same state they were in at the beginning of the afternoon. However, there would then still be bank heterogeneity because of different portfolio decisions by "connected" and "unconnected" banks. We therefore assume that, at the end of the period, banks all equate their positions and restart the next period with the same portfolio. Alternatively, and equivalently, one can think that there are securities markets which open at the end of the period and allow banks to equate their portfolios. Banks during the period therefore are only concerned with the marginal value of an additional unit of net worth they can produce for the next period.

Firms and banks are owned by households. Similar to Gertler and Kiyotaki (2011) and Gertler and Karadi (2011), banks are operated by bank managers who run a bank on behalf of their owning households. We deviate from those papers in that we assume that banks pay a fixed fraction of their net worth to households as a dividend in the morning of every period.

3.1 The household

There is a continuum of identical households. At the beginning of time t, the representative household holds an amount of cash, $\widetilde{M}^h_{t-1}$, brought from period $t-1$, and receives repayment from banks of deposits opened in the previous period gross of the due interest, $R^D_{t-1}D_{t-1}$. The household allocates the nominal funds at hand among existing nominal assets, namely money,

[4] We follow a long tradition in the banking literature of focusing on the role of interbank money markets in smoothing out idiosyncratic liquidity shocks, as in Bhattacharya and Gale (1987) and Allen and Gale (2000). While analytically convenient, in reality interbank relationships may exhibit more persistent patterns, with some banks being structural borrowers and others structural lenders (Craig and Ma (2018)).

M_t^h, and deposits, D_t,

$$D_t + M_t^h = R_{t-1}^D D_{t-1} + \widetilde{M}_{t-1}^h. \tag{1}$$

During the day, beginning-of-period money balances are increased by the value of households' revenues and decreased by the value of their expenses. The amount of nominal balances brought by the household into period $t+1$, $\widetilde{M}_t^h$, is thus

$$\widetilde{M}_t^h = M_t^h + (1-\tau_t) W_t h_t + Z_t - P_t c_t, \tag{2}$$

where P_t is the price of the consumption good, c_t is the amount of that good consumed, h_t is hours worked, τ_t is the labor tax rate, W_t is the nominal wage level, and Z_t is the profit payout ("earnings") distributed by banks.

The household then chooses $c_t > 0, h_t > 0, D_t \geq 0, M_t^h \geq 0$ to maximize the objective function

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left[u(c_t, h_t) + v\left(\frac{M_t^h}{P_t}\right)\right] \tag{3}$$

subject to (1) and (2).

3.2 Firms

A representative final-good firm uses capital k_{t-1} and labor h_t to produce a homogeneous final output good y_t according to the production function

$$y_t = \gamma_t k_{t-1}^{\theta} h_t^{1-\theta}$$

where γ_t is a productivity parameter. It receives revenues $P_t y_t$ and pays wages $W_t h_t$. Capital is owned by the firms, which are in turn owned by banks: effectively then, the banks own the capital, renting it out to firms and extracting a nominal "rental rate" $P_t r_t$ per unit of capital.

Capital-producing firms buy old capital k_{t-1} from the banks and combine it with final goods I_t to produce new capital k_t, according to

$$k_t = (1-\delta) k_{t-1} + I_t.$$

New capital is then sold back to banks. Alternatively and equivalently, one may directly assume that the banks undertake the investments.

3.3 The government

The government has some outstanding debt with face value $\overline{B}_{t-1}$. It needs to purchase goods $P_t g_t$ and pays for it by taxing labor income as well as issuing discount bonds with a face value $\Delta\overline{B}_t$ to be added to the outstanding debt next period, obtaining nominal resources $Q_t\Delta\overline{B}_t$ for it in period t. We assume that some suitable no-Ponzi condition holds. The government discount bonds are repaid at a rate κ.

The outstanding debt at the beginning of period $t+1$ will be $\overline{B}_t = (1-\kappa)\,\overline{B}_{t-1} + \Delta\overline{B}_t$. The government budget balance at time t is

$$P_t g_t + \kappa\overline{B}_{t-1} = \tau_t W_t h_t + Q_t\Delta\overline{B}_t + S_t, \tag{4}$$

where S_t are seigniorage payments from the central bank and g_t is an exogenously given process for government expenditures.

The government conducts fiscal policy by adopting a rule for the income tax that stabilizes the real stock of debt, $\overline{b} = \frac{\overline{B}}{P_t}$, at a targeted level $\overline{b}^*$,

$$P_t\left(\tau_t - \tau^*\right) = \alpha\left(B_t - \overline{B}^*\right), \tag{5}$$

where τ_t increases above its target level τ^*, if the real stock of debt is above $\overline{b}^*$. We assume that α is such that the equilibrium is saddle-path stable and that the fiscal rule ensures a gradual convergence to the desired stock of debt, following aggregate disturbances. Notice, however, that in our quantitative section, we provide a comparison of steady state equilibria: in that analysis, the parameter α plays no role. The target value τ^* is the level of the income tax necessary to stabilizes the debt at $\overline{b}^*$.[5]

3.4 The central bank

The central bank chooses the total money supply $\overline{M}_t$ and interacts with banks in the "morning", providing them with funds (F="funds from the central bank"). These funds take the form of one period loans. In period t, banks obtain loans with face value $\overline{F}_t$, getting funding

[5] Notice that τ^* can be obtained by combining $\overline{B}_t = (1-\kappa)\,\overline{B}_{t-1} + \Delta\overline{B}_t$ and equation (4) in steady state, together with the rule $b = \overline{b}^*$, to get

$$\tau^*\left(1-\theta\right)y = g + \kappa\left(1-Q\right)\frac{\overline{b}^*}{\pi} - Q\left(1-\frac{1}{\pi}\right)\overline{b}^* - s.$$

Here $s = \frac{S}{P}$ and π is the steady state inflation rate.

in the amount $Q_t^F \overline{F}_t$, where Q_t^F is the common price or discount factor, which is a policy parameter set by the central bank. Banks also repay previous period liabilities, $\overline{F}_{t-1}$.

The central bank furthermore buys and sells government bonds outright. Let B_{t-1}^C be the stock of government bonds held by the central bank ("C") at the beginning of period t. The government makes payments on a fraction of these bonds, i.e., the central bank receives cash payments κB_{t-1}^C. The remaining government bonds in the hands of the central banks are $(1-\kappa)\, B_{t-1}^C$. The central bank then changes its stock to B_t^C, at current market prices Q_t, using cash. Thus, $B_t^C = (1-\kappa)\, B_{t-1}^C + \Delta B_t^c$

The central bank balance sheet looks as follows at time t:

Assets	Liabilities
$Q_t^F \overline{F}_t$ (loans to banks)	M_t^H (currency held by HH)
$Q_t B_t^C$ (bond holdings)	M_t (bank reserves)
	S_t (seigniorage)

Let

$$\overline{M}_t = M_t^h + M_t$$

be the total money stock before seigniorage is paid. Note that the seigniorage is paid to the government at the end of the period and therefore becomes part of the currency in circulation next period. The flow budget constraint of the central bank is given by:

$$\overline{M}_t - \overline{M}_{t-1} = S_{t-1} + Q_t^F \overline{F}_t + Q_t \left(B_t^C - (1-\kappa)\, B_{t-1}^C \right) - \overline{F}_{t-1} - \kappa B_{t-1}^C. \tag{6}$$

Seigniorage can then be calculated as the residual balance sheet profit,

$$S_t = Q_t^F \overline{F}_t + Q_t B_t^C - \overline{M}_t. \tag{7}$$

3.5 Banks

There is a continuum of banks ("Lenders"), indexed by $l \in (0,1)$, which are owned by the households.

In the morning, banks receive deposits from households and collateralized loans from the central bank, and distribute dividends to households. After accepting deposits, banks learn their afternoon type. With probability ξ_t, they are of the "connected," type, and able to borrow on the unsecured loan market in the afternoon. With probability $1-\xi_t$, they are of the "unconnected," type and can only obtain funding by pledging government bonds as collateral in the secured market. We assume this probability to be iid across banks and time. Banks then lend to firms (more precisely, finance their capital) and hold government bonds and reserves ("cash").

3.5.1 Assets and liabilities

At the end of the morning, after earning income on its assets, paying interest on its liabilities and retrading, but just before paying dividends to share holders, a generic bank l holds four type of assets. It additionally and briefly holds an asset in the afternoon, for a total of five. As an overview, the end-of-morning balance sheet of that bank is

Assets	Liabilities
$P_t k_{t,l}$(capital held)	$D_{t,l}$ (deposits by HH)
$Q_t B_{t,l}$ (bond holdings)	$Q_t^F F_{t,l}$ (secured loans)
$Z_{t,l}$(cash dividends)	$N_{t,l}$ (net worth)
$M_{t,l}$(cash reserves)	

In detail:

1. Capital $k_{t,l}$ of firms, or, equivalently, firms, who in turn own the capital. Capital can only be acquired and traded in the morning. Capital evolves according to $k_{t,l} = (1-\delta)\, k_{t-1,l} + \Delta k_{t,l}$, where $\Delta k_{t,l}$ is the gross investment of bank l in capital.

2. Bonds with a nominal face value $B_{t,l}$. A fraction κ of the government debt will be repaid. The bank changes its government bond position per market purchases or sales ("-") $\Delta B_{t,l}$ in the morning, so that $B_{t,l} = (1-\kappa)\, B_{t-1,l} + \Delta B_{t,l}$ at the end of the morning. If the bank purchases (sells) bonds on the open market, it pays (receives) $Q_t \Delta B_{t,l}$. We allow $\Delta B_{t,l}$ to be negative, indicating a sale.

3. Cash $Z_{t,l}$ earmarked to be distributed to shareholders at the end of the morning. Note that this does not mean that the households end up being forced to hold money, as everything happens "simultaneously" in the morning. If they want to hold those extra earnings as deposits, then D_t would simply already be higher before they receive the earnings from the banks, in "anticipation" of these earning payments.

4. Reserves (M="money") $M_{t,l} \geq 0$. They may add to cash (not earmarked for paying shareholders) in the morning, $M_{t,l} = M_{t-1,l} + \Delta M_{t,l} \geq 0$, as well as in the afternoon, $\widetilde{M}_{t,l} = M_{t,l} + \widetilde{\Delta M}_{t,l} \geq 0$, and after the reverse liquidity shock hits, $M_{t,l} = \widetilde{M}_{t,l} - \widetilde{\Delta M}_{t,l}$.

Bank l has four types of liabilities:

1. Deposits $D_{t,l}$. This is owed to household and subject to aggregate withdrawals and additions $\Delta D_{t,l}$ in the morning, so that $D_{t,l} = R^D_{t-1} D_{t-1,l} + \Delta D_{t,l}$, where R^D_{t-1} is the return on one unit of deposits, agreed at time $t-1$. Additionally, there are idiosyncratic withdrawals and additions in the afternoon, to be described.

2. Secured loans (F="funding") from the central bank at face value $F_{t,l}$. Secured loans require collateral. A bank l with liabilities $F_{t,l}$ to the central bank needs to pledge an amount of government bonds $B^F_{t,l}$, satisfying the collateral constraint

$$F_{t,l} \leq \eta_t \; Q_t B^F_{t,l} \tag{8}$$

where η_t is a haircut parameter and is set by the central bank. Secured loans from the central bank are obtained in the morning. The amount of bonds pledged therefore cannot exceed the holding of those bonds in the morning, $0 \leq B^F_{t,l} \leq B_{t,l}$. The collateral constraints are set in terms of the market value of securities, as is the case in ECB monetary policy operations. Secured loans $F_{t,l}$ provide the banks with liquidity ("cash") $Q^F_t F_{t,l}$. Liquidity is needed in the afternoon. Therefore, the discount rate Q^F_t will not only relate to an intertemporal trade-off, as is common in most models, but importantly also to the intratemporal tradeoff of obtaining potentially costly liquidity in the morning in order to secure sufficient funding in the afternoon.

3. Outstanding unsecured liabilities to other banks issued at the time of the first liquidity shock in the afternoon. Only "connected" banks can issue them. They are repaid at zero interest rate at the time of the reverse liquidity shock.

4. Net worth $N_{t,l}$.

The sum of assets equals the sum of liabilities, at any point in time.

3.5.2 Liquidity needs in the afternoon

At the core of our model there is a bank liquidity management problem. At the beginning of the afternoon, households hold total deposits D_t with banks. We seek to capture the daily churning of deposits at banks, due to cross-household and firm-household payment activities with inside money. We use a modelling device introduced by Bianchi and Bigio (2017). At the start of the afternoon in period t, deposits get reshuffled across banks so that bank l with pre-shuffle end-of-morning deposits $D_{t,l}$ experiences a withdrawal $\omega_{t,l} D_{t,l}$. Negative $\omega_{t,l}$ denote deposits, reflecting random payments from one bank to another. Here, $\omega = \omega_{t,l} \in (-\infty, \omega^{\max}]$, with $0 \leq \omega^{\max} \leq 1$, is a random variable, which is iid across banks l and is distributed according to $F(\omega)$. The remaining post-shuffle beginning-of-afternoon deposits $\widetilde{D}_{t,l}$ are thus

$$\widetilde{D}_{t,l} = (1 - \omega_{t,l}) D_{t,l}$$

In order to meet withdrawals, banks need to have enough reserves at hand to cover them. We assume that banks will always find defaulting on the withdrawals worse than any precautionary measure they can take against it, and thus rule out withdrawal caps and bank runs by assumption. Reserves can be obtained in the morning by various trades, resulting in bank holdings $M_{t,l}$. In the afternoon, additional reserves can be obtained by new unsecured loans from other banks, maturing at the end of the afternoon. New unsecured loans can only be obtained by "connected" banks. Alternatively, "unconnected" banks can get funding by pledging bonds in a secured repo market, vis-a-vis other banks. To that end, it is useful to introduce haircut parameters $0 \leq \widetilde{\eta}_t \leq 1$, imposed by other lending banks. The bank then pledges an amount $\widetilde{B}_{t,l} \leq B_{t,l}$ of bonds and receives in return the cash amount $\widetilde{\eta}_t Q_t \widetilde{B}_{t,l}$ in the first of two transactions, repaying the same amount in the second. The end bond position is therefore the one held in the morning, $B_{t,l}$. Taken literally, there is no risk here that this haircut could reasonably insure against, but this is just due to keeping the model simple. The interest rate on secured private funding is zero. Every bank can lend unsecured, if they so choose.

Implicitly, we are assuming that the discount window of the central bank is not open in the afternoon, i.e., that banks need to obtain central bank funding in the morning in precaution to withdrawal demands in the afternoon. This captures the fact that the discount window is rarely used for funding liquidity needs and that these liquidity transactions happen "fast", compared to central bank liquidity provision.

The withdrawal shock is exactly reversed with a second reverse liquidity shock, so that banks exit the period with the original level of deposits $D_{t,l}$ and can thus repay their unsecured loans or buy back the government securities originally sold. The same holds if the signs are reversed. Thus, the first liquidity shock creates only a very temporary liquidity need that banks must satisfy.

If banks do not have access to the unsecured loan market, they will need to pledge government bonds in the private secured market, in case of liquidity needs. They can only do so with the portion that has not yet been pledged to the central bank. With $\omega^{\max}$ as the maximal withdrawal shock, non-connected banks therefore have to hold government securities satisfying

$$\omega^{\max} D_{t,l} - M_{t,l} \leq \widetilde{\eta}_t Q_t \left(B_{t,l} - B_{t,l}^F \right) \tag{9}$$

where $0 \leq \widetilde{\eta}_t \leq 1$ is the haircut imposed by other lending banks, and where the constraint is in terms of the unpledged portion of the government bond holdings $B_{t,l} - B_{t,l}^F$. We denote constraint (9) as the bank's "afternoon constraint“.

As all the afternoon transactions are reversed at the end of the afternoon and since all within-afternoon interest rates are zero, banks will be entirely indifferent between using any of the available sources of liquidity: what happens in the afternoon stays in the afternoon. The only impact of these choices and restrictions is that banks need to plan ahead of time in the morning to make sure that they have enough funding in the afternoon, in the worst case scenario. If a bank is unconnected, that worse-case scenario is particularly bad, as it needs to have enough of cash reserves plus unpledged bonds to meet the maximally conceivable afternoon deposit withdrawal.

In order to keep heterogeneity tractable, we assume that all banks meet as one big banker family at the end of the period. At that point, they equate their asset and liability positions. They also share their net worth that is then redistributed equally to all banks. As a result, each bank restarts the next period with the same portfolio and allocation of net worth.

3.5.3 Objective function and the leverage constraint

We shall consider only scenarios where bank net worth remains positive and assume that banks repay a portion ϕ of their net worth to households each period,

$$Z_{t,l} = \phi N_{t,l}.$$

The net worth of bank l at the beginning of period t, before payments to shareholders, is equated across banks, i.e. $N_{t,l} = N_t$ and satisfies

$$\begin{aligned} N_t &= \max\{0, P_t \left(r_t + 1 - \delta\right) k_{t-1,l} + M_{t-1,l} + \left(\left(1-\kappa\right) Q_t + \kappa\right) B_{t-1,l} - R_{t-1}^D D_{t-1,l} - F_{t-1,l}\} \\ &= \max\{0, P_t k_{t,l} + \ Q_t B_{t,l} + M_{t,l} - D_{t,l} - Q_t^F F_{t,l} + Z_{t,l}\}. \end{aligned}$$

The first equation is the net worth calculated on the balance of assets and their earnings and payments before the bank makes its portfolio decision, while the second equation exploits the equality of assets to liabilities after the portfolio decision.

The bank's budget constraint is

$$P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} + \phi N_t = D_{t,l} + Q_t^F F_{t,l} + N_t \tag{10}$$

As in Gertler and Kiyotaki (2011) and Gertler and Karadi (2011), we assume that there is a moral hazard constraint in that bank managers may run away with a fraction of their assets in the morning, after their asset trades are completed and after dividends are paid to the household. The constraint is

$$\lambda \left(P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} \right) \leq V_{t,l}$$

where $0 \leq \lambda \leq 1$ is a leverage parameter. Implicitly, we assume that the same leverage parameter holds for all assets, and that bankers can run away with all assets, including government bonds that may have been pledged as collateral vis-a-vis the central bank.[6]

3.6 The rest of the world

We assume that a share of the stock of government bonds is held by the rest of the world and that foreigners have an elastic demand for those bonds.[7] Because unconnected banks can buy or sell bonds to foreigners, they can change their bond holdings independently from the government's outstanding stock of debt.

[6] Alternatively, one may wish to impose that banks cannot run away with assets pledged to the central bank as collateral. In that case, the collateral constraint would be

$$\lambda \left[P_t \left(k_{t,l} - k_{t,l}^F \right) + Q_t \left(B_{t,l} - B_{t,l}^F \right) + M_{t,l} \right] \leq V_{t,l}$$

or a version in between this and the in-text equation. Since collateral pledged to the central bank typically remains in the control of banks, we feel that the assumption used in the text is more appropriate.

[7] We introduce the elastic foreign sector demand for two reasons. First, a large fraction of euro area sovereign debt is held by non-euro area residents, and these bondholders actively re-balance their bond positions. Koijen et al. (2016) document that during the Public Sector Purchase Programme implemented by the ECB since March 2015, for each unit of sovereign bonds purchased by the ECB, the foreign sector sold 0.64 of it. Second, when solving the model we will focus on the parameter space in which connected banks choose not to hold bonds. In a closed economy, therefore, unconnected banks would have to absorb whatever amount of bonds is issued by the government (after deducting the fixed amount held by the central bank). The price of the bond would have to adjust to clear the market. Such direct link between the bond market and the unconnected banks' decisions would be quantitatively implausible.

We do not wish to model the foreign sector explicitly. We simply assume that international investors have a demand for domestic bonds, B_t^w, that reacts to movements in the real return on these bonds,

$$\frac{B_t^w}{P_t} = f\left(\varkappa - \frac{1}{\varrho} log\widetilde{Q}_t \pi_t\right) \tag{11}$$

where $\varrho > 0$, $\varkappa \geq 0$ and $\widetilde{Q}_t^{-1} = \frac{\kappa}{Q} + (1-\kappa)$ denotes the return from investing one unit of money in bonds. The function $f(\cdot)$ provides a non-linear transformation ensuring that the foreign demand does not become negative when the net return becomes zero.[8] Notice that, if $\varrho = 0$, the bond demand is infinitely elastic. In that case, the real return is fixed and foreign holdings take whatever value is needed to clear the bond market.

The flow budget constraint of the foreign sector is

$$Q_t B_t^w + P_t c_t^w = [\kappa + (1-\kappa) Q] B_{t-1}^w. \tag{12}$$

4 Analysis

In appendix A, we define and characterize the equilibrium. Here, we describe the decision of households, firms and banks in turn.

4.1 Households and firms

The household maximizes his preferences, equation (3), subject to the budget constraints

$$D_t + M_t^h \leq R_{t-1}^D D_{t-1} + M_{t-1}^h + (1-\tau_{t-1}) W_{t-1} h_{t-1} + Z_{t-l} - P_{t-1} c_{t-1} \tag{13}$$

Note that there are further restrictions on the choice variables, i .e. $c_t > 0, h_t > 0$, $M_t^h > 0$ and $D_t \geq 0$. We do not list these constraints separately for the following reasons. For $c_t > 0$, $h_t > 0$, and $M_t^h > 0$, we can assure nonnegativity with appropriate choice for preferences and per the imposition of Inada conditions. We constrain the analysis a priori to $D_t > 0$, despite the possibility in principle that it could be zero or negative when allowing for more generality[9].

[8] More specifically, in our numerical analysis, we use the functional form $f(\cdot) = \left(\varkappa - \frac{1}{\varrho} log\widetilde{Q}_t \pi_t\right) \frac{[arctan(200(1-Q_t)+3.14)]}{3.14}$.

[9] We have not analyzed this matter for the dynamic evolution of the economy. It may well be that net worth of banks temporarily exceeds the funding needed for financing the capital stock, and that therefore deposits ought to be negative, rather than positive. For now, the attention is on the steady state analysis, however, and on returns to capital exceeding the returns on deposits.

Firms choose labor $h_t > 0$, and capital $k_t > 0$ to maximize their profits. The optimality conditions for households and firms are reported in appendix B.

4.2 Banks

The value of the " bank family" V_t, is given by

$$V_t = \xi_t V_{t,c} + (1 - \xi_t) V_{t,u}, \tag{14}$$

where the subscripts c and u denote " connected" and " unconnected" banks, respectively. We need to calculate $V_{t,l}$. In order to do so, we state the following proposition.

Proposition 1 (linearity) *The problem of bank l is linear in net worth and*

$$V_{t,l} = \psi_t N_{t,l} \tag{15}$$

for any bank l and some factor ψ_t. In particular, $V_{t,l} = 0$ if $N_{t,l} = 0$.

Proof: Since there are no fixed costs, a bank with twice as much net worth can invest twice as much in the assets. Furthermore, if a portfolio is optimal at some scale for net worth, then doubling every portion of that portfolio is optimal at twice that net worth. Thus the value of the bank is twice as large, giving the linearity above.

The proposition above implies

$$V_t = \psi_t N_t \tag{16}$$

giving us a valuation of a marginal unit of net worth at the beginning of period t, for a representative bank.

Suppose that, at the end of the period, the bank family has various assets, k_t, B_t, and M_t, brought to it by the various banks as they get together. The end-of-period value $\tilde{V}_t$ is

$$\tilde{V}_t = \beta (1 - \phi) E_t \left[\frac{u_c (c_{t+1}, h_{t+1})}{u_c (c_t, h_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} N_{t+1} \right] \tag{17}$$

We guess a functional form for the end-of-period value of the bank family that is linear in its assets and liabilities,

$$\tilde{V}_t = \tilde{\psi}_{t,k} \ P_t k_t + \tilde{\psi}_{t,B} B_t \ + \tilde{\psi}_{t,M} \ M_t - \tilde{\psi}_{t,D} D_t - \tilde{\psi}_{t,F} F_t \tag{18}$$

Here $\tilde{\psi}_{t,x}$ denote the marginal value (cost) to the bank family of investing (obtaining) one unit of money in asset (liability) x. Combining (10) and (17), and comparing with (18), we obtain expressions for $\tilde{\psi}_{t,k}, \tilde{\psi}_{t,B}, \tilde{\psi}_{t,M}, \tilde{\psi}_{t,D}$, and $\tilde{\psi}_{t,F}$, which we report in appendix C.

For bank of type l, we can write its end of the morning value as

$$V_{t,l} = \phi N_t + \tilde{V}_{t,l} \tag{19}$$

The banks contribute to the end of period value $\tilde{V}_t$ per

$$\tilde{V}_{t,l} = \tilde{\psi}_{t,k} P_t k_{t,l} + \tilde{\psi}_{t,B} B_{t,l} + \tilde{\psi}_{t,M} M_{t,l} - \tilde{\psi}_{t,D} D_{t,l} - \tilde{\psi}_{t,F} F_{t,l} \tag{20}$$

The leverage constraint for bank l can then be rewritten as

$$\phi N_t + \tilde{V}_{t,l} \geq \lambda \left(P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} \right) \tag{21}$$

Banks will pledge just enough collateral to the central bank to make the collateral constraint binding, nothing more (even if indifferent between that and pledging more: then, "binding" is an assumption). For both types of banks,

$$F_{t,l} = \eta_t Q_t B^F_{t,l} \tag{22}$$

with

$$0 \leq B^F_{t,l} \leq B_{t,l} \tag{23}$$

There are also nonnegativity constraints for investing in cash, in bonds, and for financing from the central bank, for both types of banks:

$$0 \leq M_{t,l} \tag{24}$$

$$0 \leq B_{t,l} \tag{25}$$

$$0 \leq F_{t,l}. \tag{26}$$

Note that we are interested in cases where banks choose to raise deposits and to extend loans. The former requirement ensures that banks have liquidity shocks in the afternoon and thus provides a meaningful role for interbank markets. The latter requirement generates an active link between financial intermediation and real activity in our economy.

We can have cases, however, when banks decide not to raise central bank finance, as in the case of connected banks that can always get afternoon zero-interest rate unsecured loans from other banks, if the need arises (this is assuming that $Q_t^F \leq 1$, otherwise there would be arbitrage possibilities for banks!). Similarly, banks can decide not to hold bonds, if their liquidity value is too low and the cost of satisfying the afternoon constraint with cash is sufficiently low. Alternatively, they can decide not to hold cash, if they have access to afternoon unsecured or secured finance, and if the expected return on capital is higher than the expected return on money.

To simplify the analysis, we restrict our attention to regions of the parameter space such that the economy is in an interior equilibrium for $D_{t,l}$ and $k_{t,l}$ in all the cases we consider. In light of the considerations above, we explicitly allow for corner solutions for $M_{t,l}$, $B_{t,l}$ and $F_{t,l}$.

As for the afternoon, there is no need to keep track of trades, except to make sure that the afternoon funding constraints for the unconnected banks, equation (9), holds.

In the morning, banks $l = u$ and $l = c$ maximize (20) subject to the budget constraint (10), the leverage constraint (21), the collateral constraints (22) and (23), as well as the afternoon constraint (9) only for the unconnected banks.

These are linear programming problem, maximizing a linear objective subject to linear constraints. So, the solution is either a corner solution or there will be indifference between certain asset classes, resulting in no-arbitrage conditions.

Let $\mu_{t,l}^{BC}$ denote the Lagrange multiplier on the budget constraint (10), $\mu_{t,l}^{RA}$ the Lagrange multiplier on the leverage constraint (21), $\mu_{t,l}^{CC}$ the Lagrange multiplier on the collateral constraint (22), $\mu_{t,u}$ the Lagrange multiplier on the afternoon funding constraint of the unconnected banks, $\mu_{t,l}^{M} \geq 0$, $\mu_{t,l}^{F} \geq 0$, and $\mu_{t,l}^{B} \geq 0$ the Lagrange multipliers on the constraints $M_{t,l} \geq 0$, $F_{t,l} \geq 0$ and $B_{t,l} \geq 0$, respectively, and $\mu_{t,l}^{C} \geq 0$ the Lagrange multiplier on the collateral constraint at the central bank, $B_{t,l}^F \leq B_{t,l}$ (see 23).

An important role is played in the model by the Lagrange multiplier on the afternoon constraint of the unconnected banks, $\mu_{t,u}$. From the complementary slackness condition,

$$\mu_{t,u}\left[\omega^{\max} D_{t,u} - M_{t,u} - \widetilde{\eta}_t Q_t \left(B_{t,u} - B_{t,u}^F\right)\right] = 0,$$

it follows that $\mu_{t,u}$ is positive whenever the afternoon constraint is binding. This lagrangean multiplier therefore measures the severity of the liquidity problem faced by unconnected banks.

The first-order conditions characterizing the choice of banks $l = u$ and $l = c$ for assets and liabilities, as well as the complementary slackness conditions, are reported in appendix C.

5 Steady state analysis

We characterize a stochastic steady state where prices grow at the rate π and all shocks are zero except for the idiosyncratic liquidity shock ω faced by banks. We denote with small letters all real variables, i.e. the corresponding variables in capital letter divided by the price of the consumption good, P_t. The steady state is characterized by the set of conditions reported in Appendix D. In what follows, we provide some analytical results for the bank problem in the steady state.

We focus our analysis on the set of parameters such that the return to capital (the productive asset) exceeds the cost of deposits so that banks raise outside financing to invest in capital. This ensures a meaningful role for banks in intermediating deposits into investment.[10]

The following proposition provides a sufficient condition for connected and unconnected banks to raise outside funding to invest in capital.

Proposition 2 *If*

$$\tilde{\psi}_k > \tilde{\psi}_D \tag{27}$$

holds, then both connected and unconnected banks raise outside funding to invest in capital.

Proof. See Appendix E.

Banks can raise outside funding in the form of deposits and/or in the form of central bank funding. The next proposition states a necessary and sufficient conditions for connected banks to raise outside funding in the form of deposits only.[11] It also provides a characterization of their bond and money holdings.

Proposition 3 *Suppose condition (27) holds. A connected bank does not borrow from the central bank and relies solely on deposit funding if and only if*

$$\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right)\frac{1}{\eta} + \left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right)Q^F > 0 \tag{28}$$

[10] Our numerical analysis in the next Section shows that this set of parameters is non-empty. Note that for households to deposit with banks, it must be that $R^D > 1$ or, equivalently, $\frac{\pi}{\beta} > 1$.

[11] Indeed, historically, banks funded themselves at the central bank only during crisis times, when the financial markets malfunctioned.

holds. A connected bank does not hold money. In addition, if the afternoon constraint of the unconnected banks binds, then a connected bank does not hold bonds.

Proof. See Appendix E.

Under the conditions in the above Proposition, in the morning, connected banks raise deposits and invest those, together with their net worth, in capital. Condition (28) states that connected banks do not use central bank funding whenever it is more expensive than deposit funding ($\frac{\tilde{\psi}_F}{Q^F} > \tilde{\psi}_D$) or whenever the collateral cost of obtaining central bank funding, given by the collateral premium $\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}$, is high enough (so that (28) holds).

In the afternoon, connected banks have access to the unsecured market in which they can smooth out liquidity shocks. Therefore, they do not hold any precautionary money reserves since holding money carries an opportunity cost. Whenever the afternoon constraint of the unconnected banks binds, the collateral premium is strictly positive and the physical return on bonds is lower than the return on capital. Hence, connected banks prefer to invest solely in capital since they do not value bonds for their collateral value in the afternoon.

Unlike the connected banks, the unconnected banks are subject to the afternoon constraint whenever they raise deposit funding. To cover deposit withdrawals, unconnected banks need to bring into the afternoon either enough money and/or enough bonds to borrow in the secured market. The afternoon constraint makes deposit funding relatively less attractive for the unconnected banks compared to the connected banks. If their afternoon constraint is tight, they may choose to top up deposit funding with some central bank funding in the morning. The next proposition provides a sufficient condition for unconnected banks that are unconstrained in the afternoon to raise outside funding solely in the form of deposits.

Proposition 4 *Suppose condition (27) holds and the afternoon constraint of an unconnected bank is slack. Then an unconnected bank does not borrow from the central bank and relies solely on deposit funding if and only if condition (28) holds. An unconnected bank does not hold money in this case.*

Proof. See Appendix E.

If the afternoon constraint is slack, the decisions of the unconnected banks are similar to those of the connected banks in Proposition 2: they raise deposits, do not borrow from the central bank and do not hold money. They must, however, hold some bonds to satisfy the (slack) afternoon constraint.

The next Proposition characterizes funding decisions of the unconnected banks that are constrained by the afternoon constraint. Furthermore, it characterizes how they manage their afternoon liquidity needs.

Proposition 5 *Suppose condition (27) holds and the afternoon constraint of an unconnected bank binds. Then an unconnected bank raises deposits. Furthermore, if*

$$\widetilde{\eta} > \eta Q^F \left[\omega^{\max} - \frac{1+\mu_u^{RA}}{\mu_u}\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right)\right] \tag{29}$$

holds, then an unconnected bank does not borrow from the central bank. If condition

$$\widetilde{\eta} < \frac{\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}}{\tilde{\psi}_k - \tilde{\psi}_M} \tag{30}$$

holds, then an unconnected bank combines deposits with central bank funding in the morning, and satisfies its afternoon liquidity needs by holding money. If (29) and (30) fail to hold, then an unconnected bank may combine deposits with central bank funding in the morning, and satisfies its afternoon liquidity needs by borrowing in the secured market and/or by holding money.

Proof. See Appendix E.

If the afternoon constraint binds, then unconnected banks may borrow from the central bank in the morning. Whether or not they do so depends on the functioning of the secured market in the afternoon. If haircuts in the secured market are low (i.e., (29) holds), then unconnected banks only raise deposits and do not tap into central bank funding. By contrast, if haircuts in the secured market are high (i.e., (30) holds), then unconnected bank top up deposit funding with central bank funding. They pledge their entire bond portfolio with the central bank in the morning and do not use the secured market in the afternoon. In that case, they must hold money to satisfy their afternoon liquidity needs.

6 Numerical results

In this section, we calibrate the model to euro area data. We then evaluate the macroeconomic impact of the observed money market developments and the effectiveness of alternative central bank policies.

Our results highlight the complex interactions between various occasionally binding constraints, which is a novel feature of our model. There are eleven occasionally binding constraints: equations (21) and (23)-(26), for $l = u, c$, and equation (9) for $l = u$. To simplify the numerical analysis, we restrict our attention to regions of the parameter space where conditions (27)-(28) are satisfied, so that proposition 2 and 3 hold. In those regions, connected banks hold neither bonds nor money, nor do they borrow at the central bank, i.e. $b_c = b_c^F = m_c = f_c = 0$. This effectively limits the number of interacting occasionally binding constraints to seven: for both types of banks, $l = u, c$, the leverage constraint (equations (21)); for $l = u$, the afternoon constraint (equation 9), the non-negativity constraints for bonds pledged at the central bank and the constraint that those bonds cannot exceed the stock of bonds held in the morning (summarized in condition (23)), and the non-negativity constraint for money, bonds and loans from the central bank (conditions (24)-(26)).

When a single parameter changes, constraints can turn from binding to slack, and then to binding again, due to the interaction with other constraints. The particular constraint that binds is typically crucial for determining the effectiveness of policy interventions.

6.1 Calibration

In the model, each period is a quarter. In the numerical analysis, we assume the following functional form of the utility function:

$$u\left(c_t, h_t\right) + v\left(\frac{M_t^h}{P_t}\right) = log(c_t) + \frac{1}{\chi}log(\frac{M_t^h}{P_t}) - \frac{h_t^{1+\epsilon}}{1+\epsilon}.$$

Table 2 summarize the value of all the parameter under the chosen calibration, which we discuss in turn below.

We set the discount factor at $\beta = 0.994$,[12] and the inverse Frisch elasticity at $\epsilon = 0.4$. The depreciation rate is fixed at $\delta = 0.02$, and the capital income share θ at 0.33. The fraction of government bonds repaid each period, κ, is 0.042, corresponding to an average maturity of the outstanding stock of euro area sovereign bonds of 6 years.[13] The parameters determining the value of collateral in the private market and at the central bank reflect the data shown in

[12]The inverse of the discount factor $1/\beta$ determines the real rate on household deposits. This rate has been very low in the euro area (in fact, it was negative for overnight deposits both before and after the onset of the financial crisis). To match this stylized fact, we choose a relatively high discount rate β.

[13]Average maturity is computed as a weighted average of all maturities of euro area government bonds, with weights given by outstanding amounts in year 2011. Source: Bloomberg, ECB and authors' calculations. Bond level data used in Andrade et al. (2016) give a similar average maturity in 2015, pointing to a stable maturity structure of euro area debt over time.

Table 1. The haircuts on government bonds in private markets and at the central bank are set equal to each other, at $1 - \widetilde{\eta} = 1 - \eta = 0.03$ (corresponding to a 3% haircut). The private haircut value is taken from LCH Clearnet, a large European-based multi-asset clearing house, and refers to an average haircut on French, German and Dutch bonds across all maturities in 2010. The value for the central bank haircut matches the haircut imposed by the ECB on sovereign bonds with credit quality 1 and 2 (corresponding to a rating AAA to A-) in 2010.

Two novel parameters of our model, which capture frictions in the funding markets and are key to determining banks' choices, are the share of "unconnected" banks, $1 - \xi$, and the maximum fraction of deposits that households can withdraw in the afternoon, $\omega^{\max}$.

We compute the average pre-crisis value of $1 - \xi$ using data from the Euro Money Market Survey, which underlie Figure 1. We set $1 - \xi = 0.58$, corresponding to the 2003-2007 average share of cumulative quarterly turnover in the secured market in the total turnover, which sums up the turnover in the secured and in the unsecured segments (where 2003 is the first available observation in the survey while 2007 is the last year before the Global Financial Crisis). To assess the impact of the observed decline in unsecured market access, we compute the same average over 2008-2017 (where 2017 is the last available observation). The average value for that period is 0.85.

We determine $\omega^{\max}$ using the information embedded in the liquidity coverage ratio (LCR) - a prudential instrument that requires banks to hold high-quality liquid assets (HQLA) in an amount that allows them to meet 30-days liquidity outflows under stress. We implicitly assume here that regulators can estimate with high precision the 30-days outflows in a period of stress. We therefore calibrate $\omega^{\max}$ so that the maximum amount of deposit withdrawal in the model, $\omega^{\max} D_{t,u}$, equates the observed holdings of HQLA.[14] More specifically, we use the European Banking Authority report from December 2013, which provides LCR data for 2012Q4 and covers 357 EU banks from 21 EU countries. Their total assets sum to EUR 33000 billion, and the aggregate HQLA to EUR 3739 billion. We take $\omega^{\max}$ to be the ratio of aggregate HQLA over total assets so that $\omega^{\max} = 0.1$.

[14] In our model, whenever the afternoon constraint binds, banks hold liquid assets in the amount of $M_u + \widetilde{\eta} Q \left(B_u - B_u^F\right)$ to cover afternoon withdrawals $\omega^{\max} D$. Since $F = 0$ in our calibrated steady state, and net worth is a small fraction of total liabilities, we approximate D with total assets. Alternatively, we can approximate $\omega^{\max}$ using the run-off rates on deposits, as specified in the LCR regulation (e.g., run-off rate of 10% means that 10% of the deposits are assumed to possibly leave the bank in 30 days). Run-off rates for deposits range between 5% for the most stable, fully insured deposits to 15% for less stable deposit funding. Our calibration of $\omega^{\max}$ at 0.1 is consistent with these rates.

We choose the parameter of the foreign demand for bonds, $\varkappa$, to ensure that, if foreign bond holdings take a value consistent with their observed share in total debt, then $\widetilde{Q}$ and π also take their average value at that steady state (.955 and 1.005, respectively). The steady state calibration cannot inform us about the elasticity of foreign bond demand ϱ, so we pick a value that produces an elasticity which is in line with available empirical evidence. We take data reported by Koijen et al. (2016) on average foreign holdings of euro area government bonds over the periods 2013Q4-2014Q4 and 2015Q2 to 2015Q4. We compute the percentage change in foreign holdings between the two periods to be -3.3%. We then calculate the percentage change between the same periods in the average real return on euro area government bonds to be -38%.[15] We then set ϱ to replicate the observed elasticity of foreign bond holdings with respect to changes in the real return on bonds, i.e. $\varrho = 1.76$. We check robustness to alternative values (not reported) and find little impact on our quantitative analysis.

We are left with six parameters that we calibrate to match the model-based predictions on some key variables from their empirical counterparts: the share of net worth distributed by banks as dividends, ϕ, the share of assets bankers can run away with, λ, the coefficient determining the utility from money holdings for households, χ, the expenditure on public goods, g, the real stock of government bonds purchased by the central bank, b^C, and the targeted stock of real debt in the economy, $\overline{b}^*$. The targeted variables are: i) average debt to GDP; ii) bank leverage; iii) share of banks' bond holdings in total debt; iv) share of foreign sector's bond holdings in total debt; v) government bond spread; and vi) average inflation. Table 3 reports the value taken by the six variables in the data (computed over the pre-crisis period, 1999-2006, unless otherwise indicated) and the model prediction under the chosen parameterization.[16]

[15] Notice that the period 2015Q2-2015Q4 coincides with the introduction of the Public Sector Purchase Programme, which was implemented by the ECB in March 2015.

[16] The average debt to GDP is computed using data on debt securities issued by euro area (EU12) governments from Eurostat (Annual Financial Accounts for General Government). The value of bank leverage is taken from Andrade et al. (2016). The share of banks' bond holdings in total debt is set at the value reported in Koijen et al. (2016) for 2015, 23%. To compute the share of the foreign sector's bond holdings, we first use data from SDW (the ECB database) to calculate the share of central bank's holdings in total government debt. We impute to this item not only outright purchases of government bonds but also collateralized loans extended in refinancing operations (the main instrument through which the ECB injects liquidity in normal times). The ratio to total sovereign debt is 10%. Koijen et al. (2016) report that households hold 3% of government bonds. We then impute to the foreign sector the remaining share, which amounts to 64%. The government bond spread is computed using data from SDW. We build average government bond yields by weighting yields of all euro area government bonds, for all maturities, with the respective amounts in 2011. We then build the spread relative to the overnight rate, the Eonia. Average inflation is computed using quarterly changes of the HICP index taken from SDW.

6.2 Macroeconomic impact and central bank policies

We assess the implications of the observed changes in the money market landscape for the macroeconomy and for central bank policies by means of a comparative statics analysis.

We consider the following monetary policy instruments: the interest rate on central bank loans, Q^F (which we fix at 0.997), the haircut on collateral charged by the central bank, $1-\eta$, and the stock of government bonds on its balance sheet, b^C. We map these central bank instruments into three types of monetary policies implemented by the ECB in recent years: i) a pre-financial crisis policy characterized by a constant balance sheet (b^C is held constant, banks do not borrow from the central bank as we set $\eta = 0$; inflation is determined endogenously); ii) a CO policy whereby the size of the balance sheet is determined by the demand for funding of the banking sector at a given policy rate (b^C is held constant, $\eta = 0.97$ so that banks may borrow from the central bank; inflation is determined endogenously); and iii) a OP policy whereby the central bank changes the stock of bonds on its balance sheet to achieve an inflation goal of 2% (inflation is fixed in this exercise while b^C is endogenous).

Our benchmark central bank policy is the constant balance sheet policy. We compare outcomes under the benchmark policy to outcomes under a CO policy and to a OP policy of maintaining constant inflation.

6.2.1 Reduced access to the unsecured market

The first exercise we conduct aims at analyzing the macroeconomic effects of a shrinking unsecured money market segment. In this comparative statics exercise, the share of unconnected banks, $1 - \xi$, varies between 0.58 and 0.95. Figures 3 and 4 show the results for the constant balance sheet policy and for the OP policy, respectively.

In both figures, the solid red line denotes the share of unconnected banks under our benchmark calibration ($1 - \xi = 0.58$). In Figure 3, the green dashed lines indicate the level of $1 - \xi$ at which unconnected banks start holding money so that the multiplier μ_u^M becomes zero. In addition, in Figure 4, the orange dashed lines indicate the level of $1 - \xi$ at which unconnected banks stop holding bonds. As we shall see, these two constraints will play a major role in this exercise.

In the calibrated steady-state (at the solid red line), the collateral premium on bonds is positive and the afternoon constraint binds for unconnected banks. The amount of deposits raised by connected and unconnected banks is of a broadly comparable magnitude. Unconnected banks, however, invest less in capital than connected banks, as they need to invest part of the funds in bonds to be pledged in the secured market in the afternoon. At this point, the return on bonds is higher than the return on money (not shown), and unconnected banks choose not to hold money to satisfy their afternoon liquidity needs.

If more banks in the economy are unconnected (moving rightward in both figures), a larger number of banks faces an afternoon withdrawal constraint, which raises the aggregate demand for bonds and the bond price. In the region where $1 - \xi < 0.79$, the real return on bonds falls for foreign investors, inducing them to sell part of their bond holdings to domestic banks. The amount of bonds held by each unconnected bank, b_u, nonetheless mildly declines, as more banks need to hold bonds as collateral, and the supply of bonds is fixed. When the share of unconnected banks increases further, i.e., when $1 - \xi$ exceeds 0.79, the high price of bonds lowers the return on bonds to the point when it is equalized with the return on money. From this point onward (indicated by the green dashed lines), unconnected banks also use money to self-insure against afternoon withdrawals. That is, their demand for money increases.

Under the constant balance sheet policy (Figure 3), the supply of money is fixed. Higher demand for money by unconnected banks is accommodated by an increase in the nominal interest rate (the deposit rate), which induces households to reduce their money holdings. Scarce money balances are therefore reallocated from households to unconnected banks. A higher nominal rate requires an increase in inflation,[17] which raises the opportunity cost of holding money for unconnected banks and further tightens their afternoon constraint. Unconnected banks respond by reducing their deposit intake and, therefore, investment in capital. This puts downward pressure on aggregate capital and, correspondingly, upward pressure on the return on capital. As the net worth of unconnected banks decline, and there is an increasing share of those, the aggregate net worth which is equally distributed to all banks in the morning declines. This results in a tightening of the run-away constraint of connected banks which induces them to also reduce their investment in capital and their deposit intake. Therefore, aggregate deposits and capital fall and so does output. Quantitatively, the decline in output between a steady-state with 0.58 share of unconnected banks and that with 0.85 share (pre- to post-2008 average share of secured turnover in total) is around 0.84 percent.

[17] This is an artefact of our steady-state analysis in which the Fisher equation holds. An alternative way to think about the adjustment in response to a higher demand for real money balances when the nominal money supply is fixed is that the price level must decrease so that the real money supply increases. That is, increased demand for scarce money balances necessitates deflation in the short-run.

Under the CO policy, the outcome is the same as under the constant balance sheet policy. This is because central bank funding is not used in this case (and therefore the central bank balance sheet remains constant) as deposit funding is less expensive than - and therefore preferred to - central bank funding.

Under the OP policy (Figure 4), the central bank can expand its balance sheet by purchasing bonds and thus increase the supply of money to help relax the afternoon constraint of unconnected banks. When $1 - \xi$ exceeds 0.82 (indicated by the orange dashed lines) and the price of bonds is high, unconnected banks sell off their entire bond holdings to the central bank and choose to hold money instead to satisfy the afternoon constraint. As inflation is kept constant, the opportunity cost of holding money is constant (and low) as well. However, aggregate capital and output still fall simply because the share of unconnected banks - who invest less in capital - increases in the economy. As this effect is driven by the change in the relative share of banks in the economy, it is not something that the central bank can affect. Quantitatively, the decline in output between a steady-state with 0.58 share of unconnected banks and that with 0.85 share (pre- to post-2008 average share of secured turnover in total) is 0.76 percent.

In sum, reduced access to the unsecured market can reduce investment and output via two channels. First, since unconnected banks need to satisfy withdrawal shocks by holding bonds and/or by holding money, they can invest less in capital. Therefore, as the share of unconnected banks in the economy increases, capital and output decrease. Central bank policy cannot do anything about this channel. Second, as more banks become unconnected, bonds and money become more scarce, tightening the withdrawal constraint, reducing aggregate deposits, investment in capital and, consequently, output. Central bank policy can mitigate the second channel if it provides money to banks at a low opportunity cost by maintaining a constant low inflation (OP policy). In the comparison between steady-states with 0.58 versus 0.85 shares of unconnected banks, the first channel dominates and therefore there is no difference between policies. However, if we compared steady-states with 0.58 share of unconnected banks to that with 0.95 (share of secured turnover in total in 2017), then the contraction in output would be 1.48 percent in the constant balance sheet or CO case, and only 1.05 percent in the OP case.

6.2.2 Reductions in collateral value

In this subsection, we analyze the macroeconomic effects of changing collateral value by comparing different private haircuts in the secured market. In this comparative statics exercise, the private haircut moves from the benchmark pre-crisis value of 3 percent to 70 percent. Figures 5, 6 and 7 show the results under the policies of constant balance sheet, CO and OP, respectively.

In these figures, the solid red line denotes the secured market haircut under our benchmark calibration ($1 - \widetilde{\eta} = 0.03$). The green dashed lines indicate the level of $1 - \widetilde{\eta}$ at which unconnected banks start holding money so that the multiplier μ_u^M becomes zero. The blue dashed lines indicate the level of $1 - \widetilde{\eta}$ at which the leverage constraint of unconnected banks turns slack and the multiplier μ_u^{RA} becomes zero. The cyan dashed lines indicate the level of $1 - \widetilde{\eta}$ at which unconnected banks start borrowing from the central bank so that the multiplier μ_u^F becomes zero. The magenta dashed lines indicate the level of $1 - \widetilde{\eta}$ at which unconnected banks pledge their entire bond holdings at the central bank and no longer use secured market ($b_u = b_u^F$ and the collateral constraint binds). The orange dashed lines indicate the level of $1 - \widetilde{\eta}$ at which unconnected banks no longer hold bonds. As we shall see, these five constraints will play a major role in this exercise.

In the calibrated steady-state (at the solid red line), the collateral premium on bonds is positive and the afternoon constraint binds for unconnected banks. At higher haircut levels (moving rightward in all figures), it becomes more difficult for unconnected banks to satisfy their liquidity needs in the secured market.

Under the constant balance sheet policy (Figure 5), at higher haircut levels, bond collateral value in the private market decreases and unconnected banks start demanding money to self-insure against afternoon withdrawal shocks (as of $1 - \widetilde{\eta} = 0.09$, indicated by the green dashed lines). As the supply of money is fixed under this policy, higher demand for money by unconnected banks is accommodated by the decrease of money holdings by households. This is facilitated by the increase in the deposit rate, which increases the opportunity cost of holding money for unconnected banks and further tightens their afternoon constraint. Unconnected banks respond by reducing their deposit intake and, therefore, investment in capital. The reduction in net worth of unconnected banks induces a reduction in the net worth allocated also to connected banks. This tightens the run away constraint of connected banks, which therefore also reduce their investment in capital and deposit intake. When the haircut reaches 0.27, unconnected banks are very constrained in the secured market but they cannot increase their money holdings any further as households' money holdings are at a minimum and the central bank is not ready to increase the supply of money. At this point, unconnected banks become so constrained in the afternoon that they dramatically reduce their deposit intake. Their leverage constraint turns slack. Bond prices collapse. From here onwards unconnected banks' deposit intake and therefore investment in capital continues to fall. Connected banks are able to pick up some of the deposits from unconnected banks but only up to a limit as they face a tight leverage constraint. As a result, aggregate deposits, capital and output decline. Quantitatively, the decline in output between a steady-state with 3% private haircut and one with 40% haircut is 4.93%.

Both the CO and OP policies are able to substantially mitigate output contractions by preventing the leverage constraint of unconnected banks from turning slack.

Under the CO policy (Figure 6), this is achieved by unconnected banks accessing central bank funding as haircut in the secured market reaches 0.23 (indicated by the cyan dashed lines). Unconnected banks reduce their deposit funding (as their afternoon constraint is tight due to the high secured market haircut) and substitute it with the central bank funding (which is subject to a much more favorable haircut of 0.03). As the central bank provides funding to banks, its balance sheet expands and so does the money supply. Therefore, unconnected bank can further increase their money holdings, without the need for a reallocation of money holdings from households (indeed, households increase their money holdings again as the nominal interest rate declines). As the private haircut increases above 0.38 (indicated by the magenta dashed lines), unconnected banks pledge all their bond collateral at the central bank and stop using the secured market to manage their afternoon liquidity needs, relying solely on money holdings instead. From this point onwards, the economy is insulated from further increases in the secured market haircut. Deposits, capital, and output stabilize. Quantitatively, the decline in output between a steady-state with 3% private haircut and one with 40% haircut is just 0.52%.

Under the OP policy (Figure 7), the central bank prevents the leverage constraint of unconnected banks from turning slack by purchasing bonds and thus increasing the supply of money which helps relax the afternoon constraint of unconnected banks. When private haircut reaches 0.09, unconnected banks start selling bonds to the central banks and - to a much smaller extent - to foreigners. The bond price decreases. When the private haircut reaches 0.14 (indicated by the orange dashed lines), unconnected banks sell off their entire bond holdings to the central bank and choose to hold money instead to satisfy the afternoon constraint. From this point onwards, the economy is insulated from further increases in the secured market haircut. Deposits, capital, and output stabilize. Quantitatively, the decline in output between a steady-state with 3% private haircut and one with 40% haircut is just 0.06%.

In sum, the key to stabilizing output when haircuts in the private market increase is to expand the central bank balance sheet either through a provision of collateralized loans to banks (using more favorable haircuts and the CO policy) or through bond purchases which replace bonds that become less valuable as collateral in the private market with money so that banks can self-insure against liquidity shocks (OP policy). The differential effectiveness of the OP and CO policies under our calibration in terms of mitigating output reductions is driven by the fact that these policies operate on different bank constraints. OP work directly towards relaxing the afternoon liquidity constraint: OP make deposit funding more attractive at the margin, by providing money to banks at a low opportunity cost. This prevents large declines in deposits and capital of the unconnected banks which could otherwise occur for high haircut levels. By contrast, CO offer banks additional (central bank) funding, once the deposits are too expensive at the margin. In that case, unconnected banks top up deposit funding with central bank funding, which prevents large declines in capital of the unconnected banks. As central bank funding is more costly than deposit funding since it requires holding costly collateral, lending and output decline more under CO policy than under the OP policy.

7 Conclusions

We developed a general equilibrium model featuring heterogeneous banks, interbank markets for both secured and unsecured credit, and a central bank that can conduct open market operations as well as lend to banks against collateral. The model features a number of occasionally binding constraints. The interactions between these constraints - in particular leverage and liquidity constraints - are key in determining macroeconomic outcomes.

We use the model to answer three questions: How do money market frictions affect the macroeconomy? How do bank leverage and liquidity constraints interact? What does this imply for central bank policies?

We find that both secured and unsecured money market frictions force banks to either divert resources into unproductive but liquid assets (bonds or money) or to de-lever (raise fewer deposits as it is deposit funding that exposes banks to liquidity shocks). This leads to less lending and output in the economy. If the liquidity constraint is very tight, the leverage constraint may turn slack. In this case, there are large declines in lending and output (up to 5% in our calibrated example), in the absence of central bank intervention. Central bank policies that increase the size of the central bank balance sheet (via outright purchases or credit operations) can prevent the leverage constraint from turning slack and significantly attenuate the decline in lending and output. The outright purchases are more effective than credit operations in terms of mitigating output reductions - with the output decline under the outright purchase policy as small as 0.1% - as they work directly towards relaxing the afternoon liquidity constraint.

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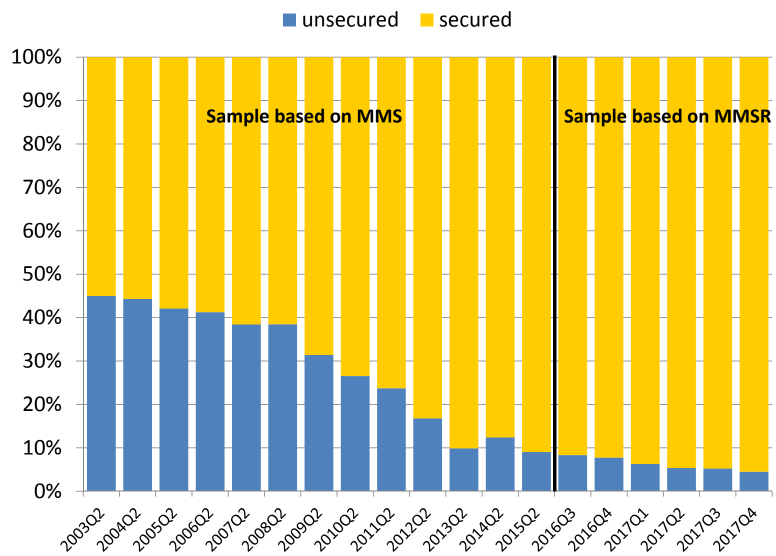


Figure 1: Quarterly turnover in unsecured and secured interbank money markets

Notes: Cumulative quarterly turnover in the euro area unsecured and secured interbank money market segments (EUR billion). Source: Euro Area Money Market Survey (MMS) until 2015; Money Market Statistical Reporting (MMSR) transactions-based data thereafter. The MMS was conducted once a year, with each data point corresponding to the second quarter of the respective year; the panel comprised 98 euro area credit institutions. The survey was discontinued in 2015. Sample from the Money Market Statistical Reporting refers to 38 banks, all of which also participated in the MMS.

Table 1: ECB vs private haircuts on sovereign bonds

	ECB		Private	
	CQS1-2	CQS3	Germany	Portugal
2010	2.8	7.8	2.8	7.9
2011	2.8	7.8	3.3	38.4
2012	2.8	7.8	3.4	79.7
2013	2.7	8.2	3.4	80.0
2014	2.2	9.4	3.4	80.0
2017	2.2	9.4	2.8	28.4

ECB haircuts: CQS1-2 refers to sovereign bonds with credit quality 1 and 2, corresponding to a rating AAA to A-; CQS3 refers to bonds with credit quality 3, corresponding to a rating BBB+ to BBB-. Private haircuts: the column 'Germany' refers to an average haircut on bonds from Germany, France, and the Netherlands. Source: ECB and LCH Clearnet.

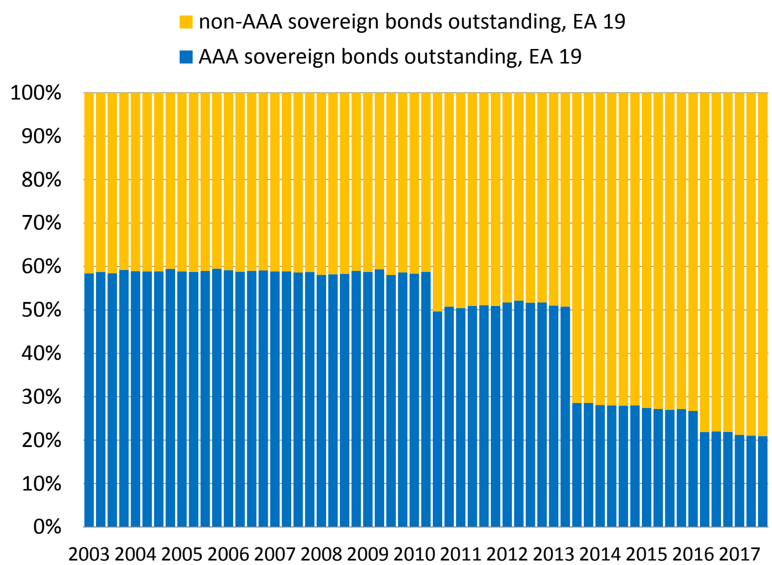


Figure 2: Share of safe (AAA-rated) euro area government debt in total

Breakdown of euro area government debt outstanding according to the credit rating (percentages of total). Country is taken as AAA-rated if the country is AAA-rated by at least one of the following three rating agencies: Moody's, Fitch, S&P. The kinks in the chart correspond to dates when specific countries moved from "at least one AAA" to "no AAA". This happened in 2009 Q3 for Ireland, in 2010 Q3 for Spain, in 2013 Q3 for France, and in 2016 Q2 for Austria. Source: ECB.

Table 2: Parameter values

Parameter	**Description**	**Value**
θ	Capital share in income	0.330
δ	Capital depreciation rate	0.020
β	Discount rate households	0.994
ϵ	Inverse Frisch elasticity	0.400
χ^{-1}	Coefficient in households' utility	0.006
g	Government spending	0.181
κ^{-1}	Average maturity bonds (years)	6.000
ϕ	Fraction net worth paid as dividends	0.038
ξ	Fraction banks with access to unsecured market	0.420
$\widetilde{\eta}$	Haircut on bonds set by banks	0.970
η	Haircut on bonds set by central bank	0.970
λ	Share of assets bankers can run away with	0.149
$\omega^{\max}$	Max possible withdrawal as share of deposits	0.100
$\varkappa$	Intercept foreign demand function	10.122
B_C	Bonds held by central bank	1.200
B^*	Stock of debt	7.500
ϱ	Parameter foreign bond demand	1.757
Q^F	Price central bank loans	0.997

Table 3: Calibration targets

Variable	**Data**	**Model**
Debt/GDP	0.68	0.68
Bank leverage	6.00	6.06
Govt bond spread (annual)	0.002	0.002
Share bonds unconnected banks	0.23	0.21
Share bonds foreign sector	0.64	0.63
Inflation (annual)	0.02	0.02

Figure 3: Comparative statics: changing the share of banks with no access to unsecured funding, $1-\xi$, constant balance sheet policy

Red solid lines denote the calibrated steady state. Green dashed lines denote the share of unconnected banks at which the non-negativity conditions on their cash holdings become slack. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, net deposit rate and percent deviation of output from steady-state.

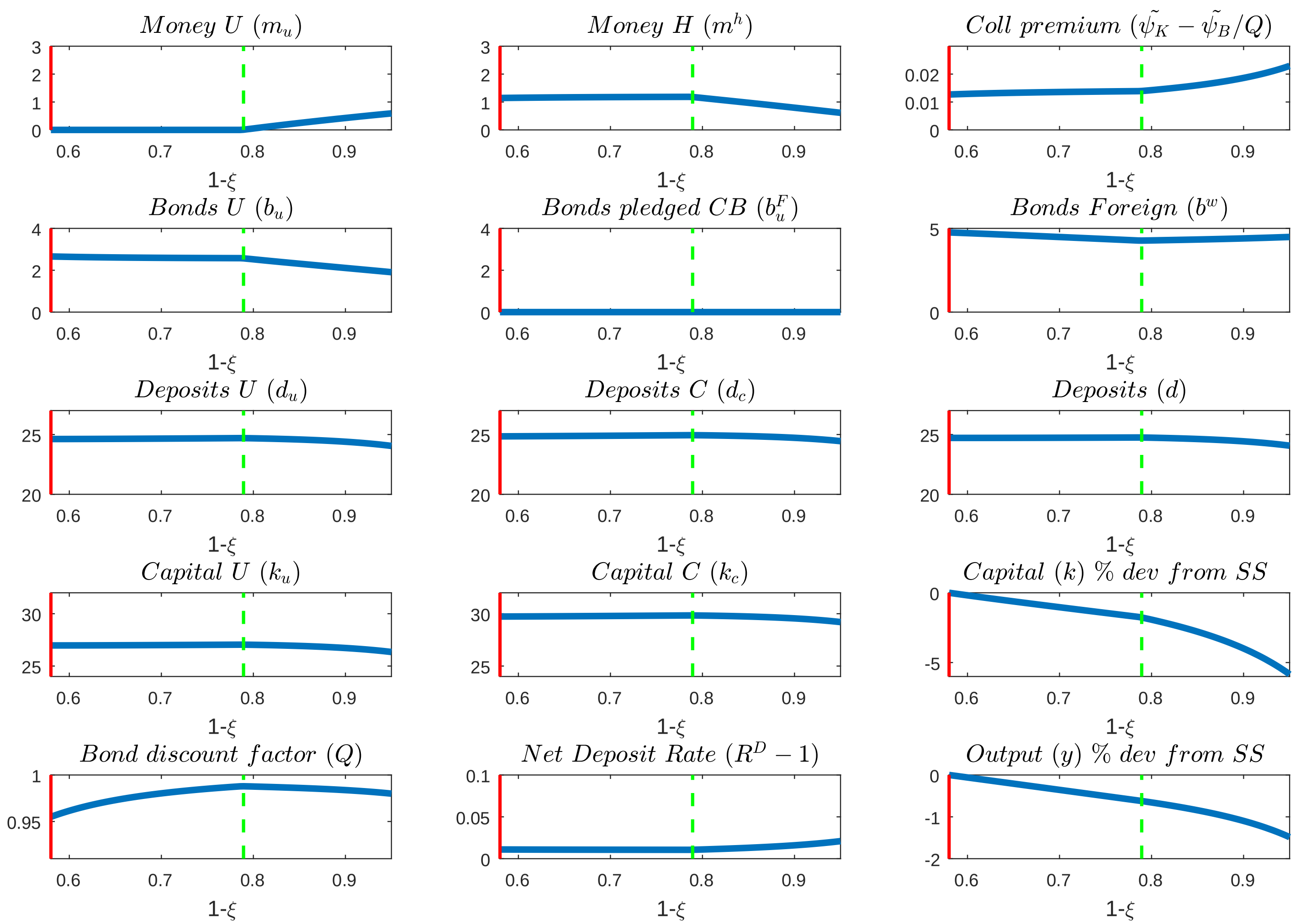


Figure 4: Comparative statics: changing the share of banks with no access to unsecured funding, $1-\xi$, OP policy

Red solid lines denote the calibrated steady state. Green and orange dashed lines denote the stock of government bonds at which the non-negativity condition on cash holdings of unconnected banks becomes slack and the one on their bond holdings becomes binding, respectively. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, central bank bond holdings, and percent deviation of output from steady-state.

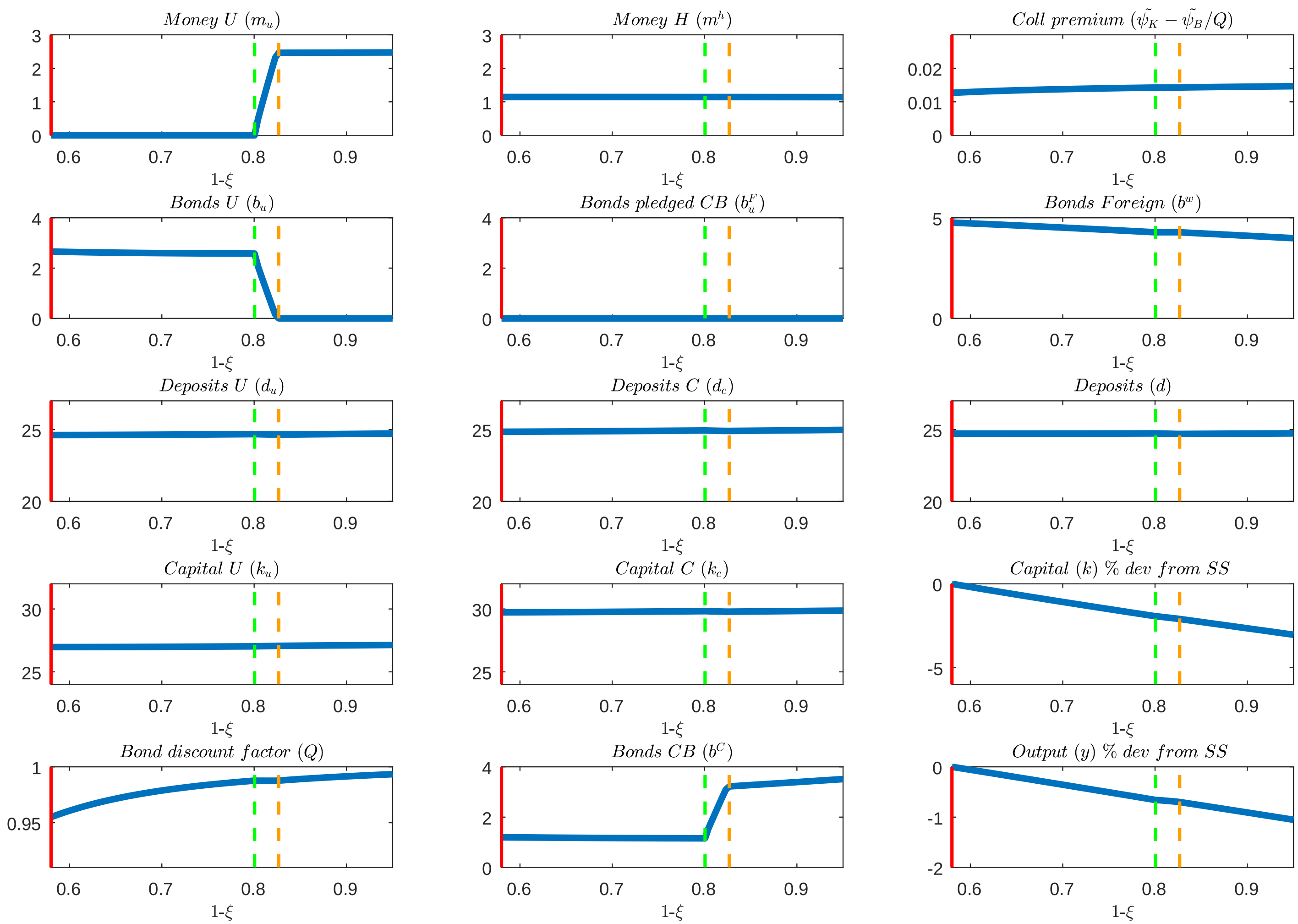


Figure 5: Comparative statics: changing the value of the private haircut, $1-\widetilde{\eta}$, constant balance sheet policy.

Red solid lines denote the calibrated steady state. Green and blue dashed lines denote the level of private haircuts at which the non-negativity conditions on cash holdings and the run-away constraint of unconnected banks become slack, respectively. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, net deposit rate and percent deviation of output from steady-state.

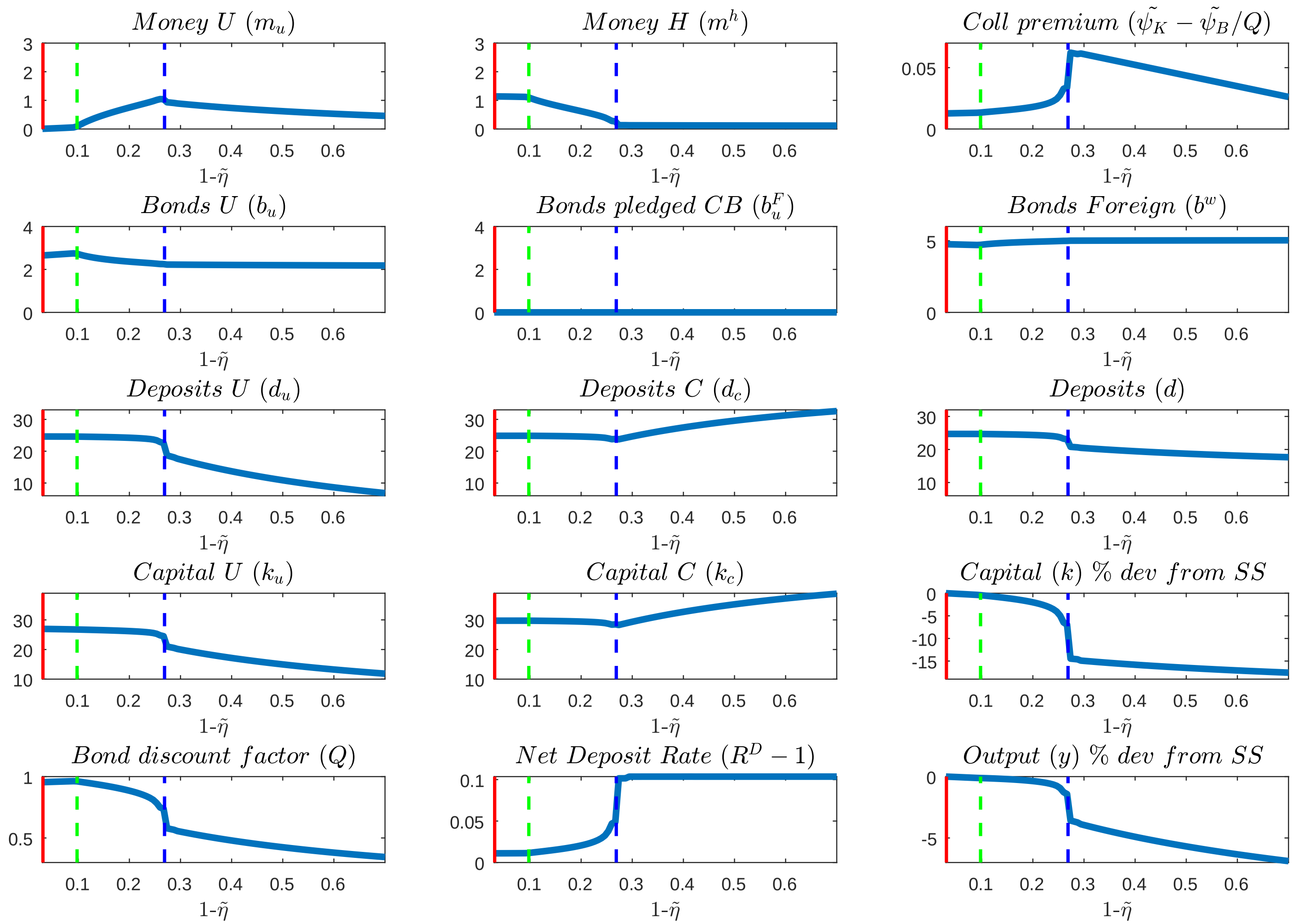


Figure 6: Comparative statics: changing the value of the private haircut, $1-\tilde{\eta}$, CO policy

Red solid lines denote the calibrated steady state. Green, cyan and magenta dashed lines denote the level of private haircuts at which the non-negativity conditions on cash holdings and on bonds pledged at the central bank become slack, and the constraint on bonds pledged at the central bank not exceeding bond holdings becomes binding, respectively. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, net deposit rate and percent deviation of output from steady-state.

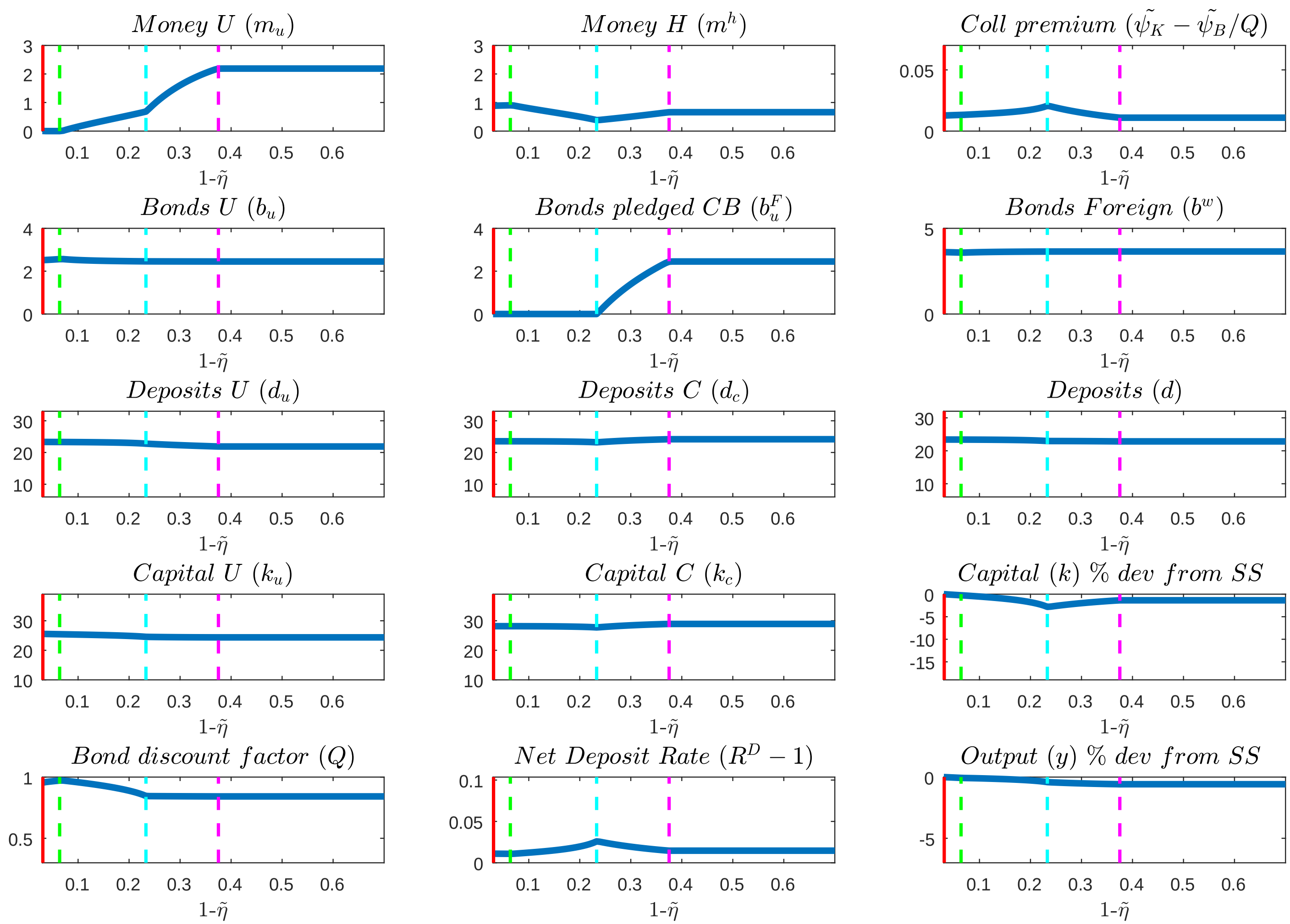
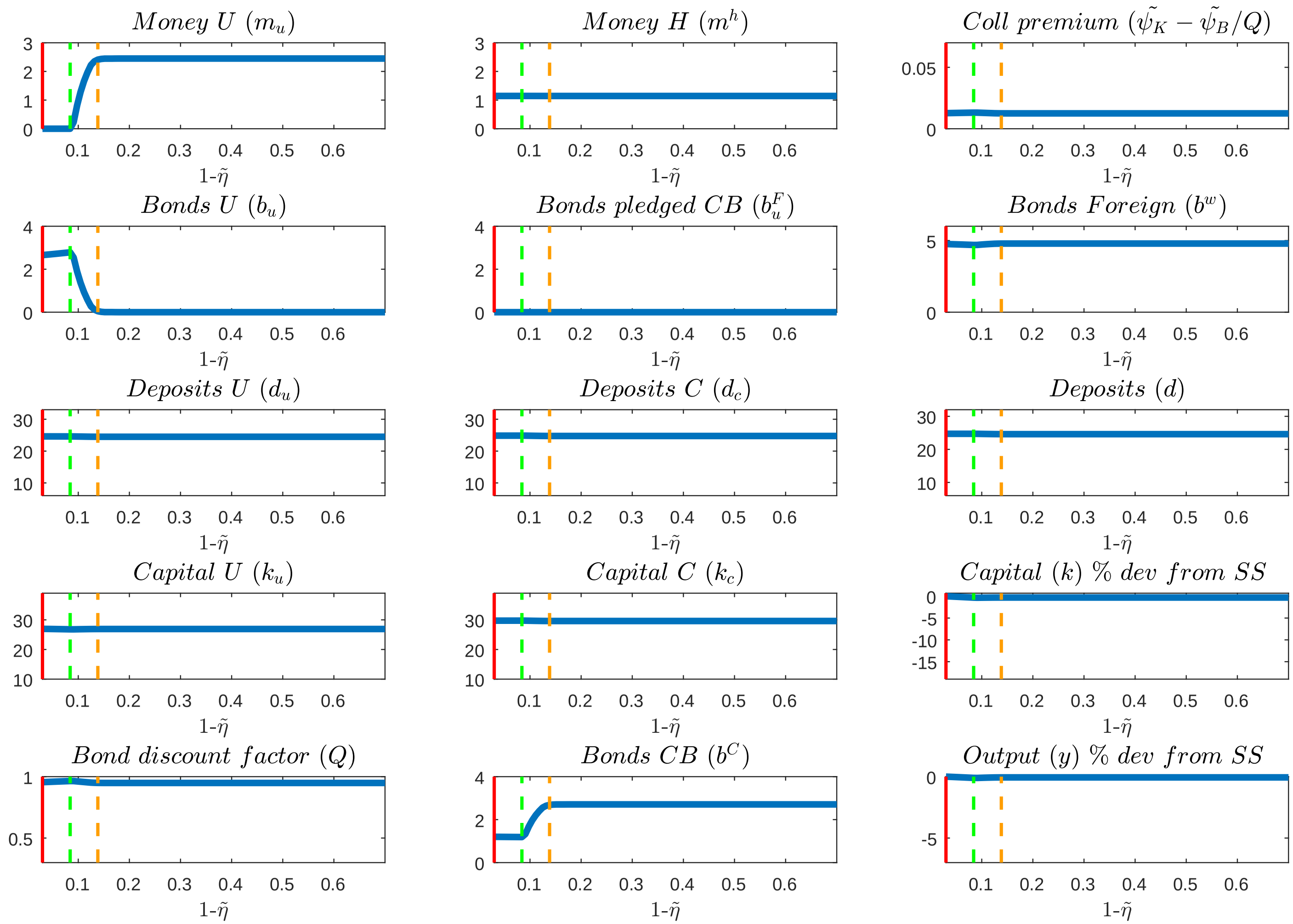


Figure 7: Comparative statics: changing the value of the private haircut, $1-\tilde{\eta}$, OP policy

Red solid lines denote the calibrated steady state. Green and orange dashed lines denote the stock of government bonds at which the non-negativity condition on cash holdings of unconnected banks becomes slack and the one on their bond holdings becomes binding, respectively. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, central bank bond holdings, and percent deviation of output from steady-state.



Online Appendix

A Equilibrium

An equilibrium is a vector of sequences such that:

1. Given $P_t, \tau_t, W_t, R^D_{t-1}, Z_t$, the representative household chooses $c_t > 0, h_t > 0, D_t \geq 0, M^h_t \geq 0$ to maximize their objective function

$$\max E_t \left[\sum_{t=0}^{\infty} \beta^t \left[u(c_t, h_t) + v\left(\frac{M^h_t}{P_t} \right) \right] \right]$$

   subject to

$$D_t + M^h_t \leq R^D_{t-1} D_{t-1} + M^h_{t-1} + (1 - \tau_{t-1}) W_{t-1} h_{t-1} + Z_{t-l} - P_{t-1} c_{t-1}.$$

2. Final good firms choose capital and labor to maximize their expected profits from production, which makes use of the technology

$$y_t = \gamma_t k^{\theta}_{t-1} h^{1-\theta}_t.$$

3. Capital-producing firms choose how much old capital k_{t-1} to buy from banks and to combine with final goods I_t to produce new capital k_t, according to the technology

$$k_t = (1 - \delta) k_{t-1} + I_t.$$

4. Bank families aggregate the assets and liabilities of the individual family members:

$$V_t = \xi_t V_{t,c} + (1 - \xi_t) V_{t,u} \tag{31}$$

$$k_t = \xi_t k_{t,c} + (1 - \xi_t) k_{t,u} \tag{32}$$

$$D_t = \xi_t D_{t,c} + (1 - \xi_t) D_{t,u} \tag{33}$$

$$B_t = \xi_t B_{t,c} + (1 - \xi_t) B_{t,u} \tag{34}$$

$$F_t = \xi_t F_{t,c} + (1 - \xi_t) F_{t,u} \tag{35}$$

$$M_t = \xi_t M_{t,c} + (1 - \xi_t) M_{t,u} \tag{36}$$

5. Given the stochastic paths for the endogenous variables c_t, h_t, r_t, P_t, Q_t, Q_t^F, η_t, and stochastic exogenous sequence for $\widetilde{\eta}_t$ and the draw of the type according to ξ_t, the representative date-t connected bank chooses $k_{t,c}$, $B_{t,c}$, $B_{t,c}^F, F_{t,c}$, $D_{t,c}$, $M_{t,c}$ and the representative date-t unconnected bank chooses $k_{t,u}$, $B_{t,u}$, $B_{t,u}^F$, $F_{t,u}$, $D_{t,u}$, $M_{t,u}$ to maximize the banks' objective function, i.e. to maximize

$$V_{t,l} = P_t E\left[\phi \sum_{s=0}^{\infty} \left(\beta\left(1-\phi\right)\right)^s \frac{u_c\left(c_{t+s}, h_{t+s}\right)}{u_c\left(c_t, h_t\right)} \frac{N_{t+s}}{P_{t+s}}\right] \tag{37}$$

where

$$\begin{aligned} N_t = \max\{0, \quad & P_t\left(r+1-\delta\right)\left(\xi_{t-1}k_{t-1,c} + \left(1-\xi_{t-1}\right)k_{t-1,u}\right) \\ & + \left(\xi_{t-1}M_{t-1,c} + \left(1-\xi_{t-1}\right)M_{t-1,u}\right) \\ & + \left(\left(1-\kappa\right)Q_t + \kappa\right)\left(\xi_{t-1}B_{t-1,c} + \left(1-\xi_{t-1}\right)B_{t-1,u}\right) \\ & - \left(\xi_{t-1}F_{t-1,c} + \left(1-\xi_{t-1}\right)F_{t-1,u}\right) \\ & - R_{t-1}^D\left(\xi_{t-1}D_{t-1,c} + \left(1-\xi_{t-1}\right)D_{t-1,u}\right)\} \end{aligned} \tag{38}$$

s.t. for $l = c, u$,

$$\begin{aligned} V_{t,l} &\geq \lambda\left(P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}\right) \\ 0 &\leq B_{t,l} - B_{t,l}^F \\ P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} + \phi N_t &= D_{t,l} + Q_t^F F_{t,l} + N_t \\ F_{t,l} &\leq \eta_t Q_t B_{t,l}^F \end{aligned}$$

as well as

$$\omega^{\max} D_{t,u} - M_{t,u} \leq \widetilde{\eta}_t Q_t\left(B_{t,u} - B_{t,u}^F\right)$$

for the unconnected banks.

6. The central bank chooses the total amount of money supply $\overline{M}_t$, the haircut parameter η_t, the discount factor on central bank funds Q_t^F, the bond purchases B_t^C as well as the seigniorage payment S_t. It satisfies the balance sheet constraint

$$S_t = Q_t^F \overline{F}_t + Q_t B_t^C - \overline{M}_t \tag{39}$$

and the budget constraint

$$\overline{M}_t = Q^F_{t-1}\overline{F}_{t-1} + Q_{t-1}B^C_{t-1} + Q^F_t\overline{F}_t \tag{40}$$
$$-\overline{F}_{t-1} + Q_t\left(B^C_t - (1-\kappa)\,B^C_{t-1}\right) - \kappa B^C_{t-1}$$

7. The government satisfies the debt evolution constraint, the budget constraint and the tax rule

$$\overline{B}_t = (1-\kappa)\,\overline{B}_{t-1} + \Delta\overline{B}_t \tag{41}$$
$$P_t\, g_t + \kappa\overline{B}_{t-1} = \tau_t W_t h_t + Q_t\Delta\overline{B}_t + S_t \tag{42}$$
$$\tau_t\, W_t h_t = \alpha\overline{B}_{t-1}. \tag{43}$$

8. The foreign sector chooses the amount of domestic bonds to hold

$$\frac{B^w_t}{P_t} = f\left(\varkappa - \frac{1}{\varrho} log\widetilde{Q}_t\pi_t\right), \tag{44}$$

and satisfies the budget constraint

$$Q_t B^w_t + P_t c^w_t = \left[\kappa + (1-\kappa)\,Q\right] B^w_{t-1}. \tag{45}$$

9. Markets clear:

$$c_t + g_t + I_t + c^w_t = y_t \tag{46}$$
$$\overline{B}_t = B_t + B^C_t + B^w_t \tag{47}$$
$$\overline{F}_t = F_t \tag{48}$$
$$\overline{M}_t = M_t + M^h_t \tag{49}$$

B Optimality conditions of households and firms

The optimality conditions of the households are given by:

$$-\frac{u_l\left(c_t, h_t\right)}{u_c\left(c_t, h_t\right)} = (1-\tau_t)\,\frac{W_t}{P_t}$$
$$v_M\left(m^h_t\right) = u_c\left(c_t, h_t\right)\left(R^D_t - 1\right)$$
$$\frac{u_c\left(c_{t-1}, h_{t-1}\right)}{P_{t-1}} = \beta R^D_t\left[\frac{u_c\left(c_t, h_t\right)}{P_t}\right].$$

First-order conditions arising from the problem of the firms are

$$\begin{aligned} y_t &= \gamma_t k_{t-1}^{\theta} h_t^{1-\theta}, \\ W_t h_t &= (1-\theta) P_t \; y_t, \\ r_t k_{t-1} &= \theta y_t, \\ k_t &= (1-\delta) k_{t-1} + I_t. \end{aligned}$$

C The problem of the banks

The marginal value associated to each unit of asset and marginal cost associated to each unit of liabilities for the bank family are given by

$$\tilde{\psi}_{t,k} = \beta(1-\phi) E_t \left[\frac{u_c(c_{t+1}, h_{t+1})}{u_c(c_t, h_t)} \psi_{t+1} (r_{t+1} + 1 - \delta) \right] \tag{50}$$

$$\tilde{\psi}_{t,B} = \beta(1-\phi) E_t \left[\frac{u_c(c_{t+1}, h_{t+1})}{u_c(c_t, h_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} \; ((1-\kappa) Q_{t+1} + \kappa) \right] \tag{51}$$

$$\tilde{\psi}_{t,D} = \beta(1-\phi) E_t \left[\frac{u_c(c_{t+1}, h_{t+1})}{u_c(c_t, h_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} \; R_t^D \right] \tag{52}$$

$$\tilde{\psi}_{t,F} = \beta(1-\phi) E_t \left[\frac{u_c(c_{t+1}, h_{t+1})}{u_c(c_t, h_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} \right] \tag{53}$$

$$\tilde{\psi}_{t,M} = \beta(1-\phi) E_t \left[\frac{u_c(c_{t+1}, h_{t+1})}{u_c(c_t, h_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} \right] \tag{54}$$

The first-order conditions characterizing the choice of banks $l = u$ and $l = c$ for capital, bonds, money, are given by

$$\left(1 + \mu_{t,l}^{RA}\right) \frac{\tilde{\psi}_{t,k}}{P_t} = \mu_{t,l}^{BC} + \lambda \mu_{t,l}^{RA} \text{for } l = c, u$$

$$\begin{aligned} \left(1 + \mu_{t,l}^{RA}\right) \tfrac{\tilde{\psi}_{t,B}}{Q_t} &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda - \mu_{t,l}^{C} \quad \text{for } l = c \\ \left(1 + \mu_{t,l}^{RA}\right) \tfrac{\tilde{\psi}_{t,B}}{Q_t} &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda - \mu_{t,l}^{C} - \mu_{t,u} \widetilde{\eta}_t \quad \text{for } l = u \end{aligned}$$

$$\begin{aligned} \left(1 + \mu_{t,l}^{RA}\right) \tilde{\psi}_{t,M} + \mu_{t,c}^{M} &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda \quad \text{for } l = c \\ \left(1 + \mu_{t,l}^{RA}\right) \tilde{\psi}_{t,M} + \mu_{t,c}^{M} &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda - \mu_{t,u} \quad \text{for } l = u \end{aligned}$$

Those characterizing banks' choices for deposits, central bank funding, and bonds to be pledged at the central bank, are

$$\begin{aligned} \left(1 + \mu_{t,l}^{RA}\right) \tilde{\psi}_{t,D} &= \mu_{t,l}^{BC} \quad \text{for } l = c \\ \left(1 + \mu_{t,l}^{RA}\right) \tilde{\psi}_{t,D} &= \mu_{t,l}^{BC} - \omega^{\max} \mu_{t,u} \quad \text{for } l = u \end{aligned} \tag{55}$$

$$\left(1+\mu_{t,l}^{RA}\right)\tilde{\psi}_{t,F}=\mu_{t,l}^{BC}Q_t^F-\mu_{t,l}^{CC}+\mu_{t,l}^{F}\text{for } l=c,u \tag{56}$$

$$\mu_{t,l}^{CC}\eta_t=\mu_{t,l}^{C} \quad \text{for } l=c$$
$$\mu_{t,l}^{CC}\eta_t=\mu_{t,u}\widetilde{\eta}_t+\mu_{t,l}^{C} \quad \text{for } l=u$$

The complementary slackness conditions are

$$\mu_{t,l}^{F}F_{t,l}=0 \tag{57}$$

$$\mu_{t,l}^{M}M_{t,l}=0 \tag{58}$$

$$\mu_{t,l}^{C}\left(B_{t,l}-B_{t,l}^{F}\right)=0 \tag{59}$$

$$\mu_{t,l}^{RA}\left[\phi N_{t,A}+\tilde{V}_{t,l}-\lambda\left(P_tk_{t,l}+Q_tB_{t,l}+M_{t,l}\right)\right]=0 \tag{60}$$

$$\mu_{t,l}^{B}B_{t,l}=0 \tag{61}$$

for $l=u,c$, and

$$\mu_{t,u}\left[\omega^{\max}D_{t,u}-M_{t,u}-\widetilde{\eta}_tQ_t\left(B_{t,u}-B_{t,u}^{F}\right)\right]=0$$

for unconnected banks only.

D The equations characterizing the steady state

We characterize the steady state of the model.

Define a generic variable as the corresponding capital letter variable, divided by the contemporaneous price level, i.e. $x_t=\frac{X_t}{P_t}$. The steady state is characterized by the following conditions:

1. 4 household equations:

$$R^D=\frac{\pi}{\beta}$$

$$-\frac{u_l\left(c,l\right)}{u_c\left(c,l\right)}=\left(1-\tau\right)w$$

$$v_M\left(m^h\right)=u_c\left(c,n\right)\left(R^D-1\right)$$

$$c=\left(1-\tau\right)wl+\left(\frac{1}{\beta}-1\right)\pi d+\left(1-\pi\right)m^h+\phi n$$

2. 3 firms' equations:

$$y = \gamma k^{\theta} l^{1-\theta}$$

$$wl = (1-\theta)\, y$$

$$rk = \theta y$$

and

$$I = \delta k.$$

3. 5 central bank equations: 2 equations

$$s = Q^F \overline{f} + Q b^C - \overline{m}$$

$$\overline{m} = \left[Q^F - \frac{1}{\pi}\left(1 - Q^F\right)\right] \overline{f} + \left[Q - \kappa \frac{1}{\pi}(1-Q)\right] b^C$$

plus the value of 3 variables (policy instruments): η, Q^F, b^C.

Note that the seigniorage revenue of the central bank is given by the interest rate payments on its assets:

$$s = \frac{1}{\pi}\left(1 - Q^F\right)\overline{f} + \kappa \frac{1}{\pi}(1-Q)\, b^C.$$

4. 2 government equations:

$$\overline{b} = \overline{b}^*$$

$$\tau^*(1-\theta)\, y = g + \kappa (1-Q) \frac{\overline{b}^*}{\pi} - Q\left(1 - \frac{1}{\pi}\right)\overline{b}^* - s.$$

where g is exogenous.

5. 4 market clearing equations:

$$\overline{f} = f$$

$$\overline{m} = m + m^h$$

$$\overline{b} = b + b^C + b^w$$

$$y = c + c^w + g + I$$

where the market clearing condition for the goods market (last equation above) is redundant due to the Walras law.

6. 45 bank equations:

8 equations common to c and u banks,

$$\nu = \psi n$$

$$\begin{aligned} n = \max\{0,\ & (r+1-\delta)(\xi k_c + (1-\xi) k_u) \\ & + (\xi m_c + (1-\xi) m_u) \frac{1}{\pi} \\ & + ((1-\kappa) Q + \kappa)(\xi b_c + (1-\xi) b_u) \frac{1}{\pi} \\ & - (\xi f_c + (1-\xi) f_u) \frac{1}{\pi} \\ & - \frac{1}{\beta} (\xi d_c + (1-\xi) d_u) \Big\}, \end{aligned}$$

$$\widetilde{v} = \tilde{\psi}_k k + \tilde{\psi}_B b + \tilde{\psi}_M m - \tilde{\psi}_D d - \tilde{\psi}_F f$$

$$\begin{aligned} \tilde{\psi}_k &= \beta (1-\phi) \psi (r+1-\delta) \\ \tilde{\psi}_B &= \beta (1-\phi) \frac{1}{\pi} \psi [(1-\kappa) Q + \kappa] \\ \tilde{\psi}_D &= \beta (1-\phi) \frac{1}{\beta} \psi \\ \tilde{\psi}_F &= \beta (1-\phi) \frac{1}{\pi} \psi \\ \tilde{\psi}_M &= \beta (1-\phi) \frac{1}{\pi} \psi \end{aligned}$$

18 equations for $l = c, u$:

$$k_l + Qb_l + m_l + \phi n = d_l + Q^F f_l + n$$

$$\phi n + \widetilde{v}_l = \lambda (k_l + Qb_l + m_l)$$

$$v_l = \phi n + \widetilde{v}_l$$

$$\widetilde{v}_l = \tilde{\psi}_k k_l + \tilde{\psi}_B b_l + \tilde{\psi}_M m_l - \tilde{\psi}_D d_l - \tilde{\psi}_F f_l$$

$$f_l = \eta Q b_l^F$$

$$\mu_l^F f_l = 0$$

$$\mu_l^M m_l = 0$$

$$\mu_l^C \left(b_l - b_l^F\right) = 0$$

$$\mu_l^B b_l = 0$$

7 equations for unconnected banks:

$$\left(1+\mu_u^{RA}\right) \tilde{\psi}_k = \mu_u^{BC} + \lambda \mu_u^{RA} \tag{62}$$

$$\left(1+\mu_u^{RA}\right) \frac{\tilde{\psi}_B}{Q} + \mu_u^B = \mu_u^{BC} + \lambda \mu_u^{RA} - \mu_u^C - \mu_u \widetilde{\eta} \tag{63}$$

$$\left(1+\mu_u^{RA}\right) \tilde{\psi}_M + \mu_u^M = \mu_u^{BC} + \lambda \mu_u^{RA} - \mu_u \tag{64}$$

$$\left(1+\mu_u^{RA}\right) \tilde{\psi}_D = \mu_u^{BC} - \omega^{\max} \mu_u \tag{65}$$

$$\left(1+\mu_u^{RA}\right) \frac{\tilde{\psi}_F}{Q^F} = \mu_u^{BC} - \mu_u^C \frac{1}{Q^F} + \mu_u^F \frac{1}{Q^F} \tag{66}$$

$$\mu_u^C = \mu_u^{CC} \eta - \mu_u \widetilde{\eta} \tag{67}$$

$$\mu_u \left[\omega^{\max} d_u - m_u - \widetilde{\eta} Q \left(b_u - b_u^F\right)\right] = 0 \tag{68}$$

6 equations for connected banks:

$$\left(1+\mu_c^{RA}\right) \tilde{\psi}_k = \mu_c^{BC} + \lambda \mu_c^{RA} \tag{69}$$

$$\left(1+\mu_c^{RA}\right) \frac{\tilde{\psi}_B}{Q} + \mu_c^B = \mu_c^{BC} + \lambda \mu_c^{RA} - \mu_c^C \tag{70}$$

$$\left(1+\mu_c^{RA}\right) \tilde{\psi}_M + \mu_c^M = \mu_c^{BC} + \lambda \mu_c^{RA} \tag{71}$$

$$\left(1+\mu_c^{RA}\right) \tilde{\psi}_D = \mu_c^{BC} \tag{72}$$

$$\left(1+\mu_c^{RA}\right) \frac{\tilde{\psi}_F}{Q^F} = \mu_c^{BC} - \mu_c^C \frac{1}{Q^F} + \mu_c^F \frac{1}{Q^F} \tag{73}$$

$$\mu_c^C = \mu_c^{CC} \eta \tag{74}$$

6 bank aggregation equations:

$$v = \xi v_c + (1-\xi) v_u$$

$$k = \xi k_c + (1-\xi) k_u$$

$$d = \xi d_c + (1-\xi) d_u$$

$$b = \xi b_c + (1-\xi) b_u$$

$$f = \xi f_c + (1-\xi) f_u$$

$$m = \xi m_c + (1-\xi) m_u.$$

7. 2 rest of the world equations

$$b^w = f\left(\varkappa - \frac{1}{\varrho}\log \widetilde{Q_t}\pi_t\right)$$

$$Qb^w + c^w = [\kappa + (1-\kappa)Q]\frac{b^w}{\pi}.$$

These are 66 equations (one redundant by the Walras law) in 65 endogenous variables:

$$\left\{\begin{array}{c} y, k, c, c^w, l, d, n, m^h, b, b^w, f, m, v, \widetilde{v}, \overline{b}, \tau^*, \\ \psi, \tilde{\psi}_k, \tilde{\psi}_B, \tilde{\psi}_M, \tilde{\psi}_D, \tilde{\psi}_F, \mu_u, w, r, Q, R^D, \pi, I, s, \overline{f}, \overline{m} \end{array}\right\}$$

plus

$$\left\{k_l, m_l, f_l, b_l, b_l^F, d_l, v_l, \widetilde{v}_l, \mu_l^F, \mu_l^M, \mu_l^{RA}, \mu_l^{BC}, \mu_l^{CC}, \mu_l^C, \mu_l^B\right\},$$

plus the value of the three policy instruments

$$\eta^A, Q^F, b^C,$$

and of the following exogenous variables: $g, \xi, \widetilde{\eta}$.

The bank first-order conditions can be further simplified as follows. For the unconnected banks, conditions (62)-(68) can be simplified to:

$$\mu_u\left[\omega^{\max}d_u - m_u - \widetilde{\eta}Q\left(b_u - b_u^F\right)\right] = 0 \tag{75}$$

$$\omega^{\max}\mu_u = \left(1+\mu_u^{RA}\right)\left(\tilde{\psi}_k - \tilde{\psi}_D\right) - \lambda\mu_u^{RA} \tag{76}$$

$$\mu_u^{CC}\eta = \left(1+\mu_u^{RA}\right)\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) - \mu_u^B \tag{77}$$

$$\mu_u^M = \left(1+\mu_u^{RA}\right)\left(\tilde{\psi}_k - \tilde{\psi}_M\right) - \mu_u \tag{78}$$

$$\mu_u^F\frac{1}{Q^F} = \left(1+\mu_u^{RA}\right)\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right) - \omega^{\max}\mu_u + \mu_u^{CC}\frac{1}{Q^F} \tag{79}$$

For the connected banks, conditions (69)-(74) can be simplified to,

$$\mu_c^{RA} = \frac{\tilde{\psi}_k - \tilde{\psi}_D}{\lambda - \left(\tilde{\psi}_k - \tilde{\psi}_D\right)} \tag{80}$$

$$\mu_c^B = \left(1 + \mu_c^{RA}\right)\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) - \mu_c^{CC}\eta \tag{81}$$

$$\mu_c^M = \left(1 + \mu_c^{RA}\right)\left(\tilde{\psi}_k - \tilde{\psi}_M\right) \tag{82}$$

$$\mu_c^F \frac{1}{Q^F} = \left(1 + \mu_c^{RA}\right)\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right) + \mu_c^{CC}\frac{1}{Q^F} \tag{83}$$

E Proofs

Proof of Proposition 2

For a connected bank, equations (69) and (72) write as:

$$\left(1 + \mu_c^{RA}\right)\tilde{\psi}_k + \mu_c^k = \mu_c^{BC} + \lambda\mu_c^{RA}$$
$$\left(1 + \mu_c^{RA}\right)\tilde{\psi}_D = \mu_c^{BC} + \mu_c^D$$

where $\mu_c^k \geq 0$ and $\mu_c^D \geq 0$ are Lagrange multipliers on the capital and deposits, respectively, of a connected bank.

Combining the above equations and re-arranging yields

$$\left(1 + \mu_c^{RA}\right)\left(\tilde{\psi}_k - \tilde{\psi}_D\right) = \lambda\mu_c^{RA} - \mu_c^k - \mu_c^D.$$

Since $\tilde{\psi}_k > \tilde{\psi}_D$, $\mu_c^k \geq 0$, and $\mu_c^D \geq 0$, we must have $\mu_c^{RA} > 0$, implying that the run-away constraint of a connected bank binds. Given that the run-away constraint binds, it follows that connected banks raise outside funding to top up their net worth to invest in capital.

For an unconnected bank, equations (62) and (65) write as:

$$\left(1 + \mu_u^{RA}\right)\tilde{\psi}_k + \mu_u^k = \mu_u^{BC} + \lambda\mu_u^{RA}$$
$$\left(1 + \mu_u^{RA}\right)\tilde{\psi}_D = \mu_u^{BC} - \omega^{\max}\mu_u + \mu_c^D$$

where $\mu_u^k \geq 0$ and $\mu_u^D \geq 0$ are Lagrange multipliers on the capital and deposits, respectively, of an unconnected bank.

Combining the above equations and re-arranging yields

$$\left(1+\mu_u^{RA}\right)\left(\tilde{\psi}_k-\tilde{\psi}_D\right)=\lambda\mu_u^{RA}+\omega^{\max}\mu_u-\mu_u^k-\mu_u^D.$$

Since $\tilde{\psi}_k > \tilde{\psi}_D$, $\mu_u^k \geq 0$, and $\mu_u^D \geq 0$, we must have $\mu_u^{RA} > 0$, $\mu_u > 0$, or both. For $\mu_u^{RA} > 0$, the run-away constraint of a unconnected bank binds, which means it raises outside funding to top up their net worth to invest in capital. For $\mu_u > 0$, the afternoon constraint of an unconnected bank binds, which means it raises deposits. This completes the proof.

Proof of Proposition 3

By equation (81),

$$\mu_c^{CC}=\left(1+\mu_c^{RA}\right)\frac{1}{\eta}\left(\tilde{\psi}_k-\frac{\tilde{\psi}_B}{Q}\right)-\frac{1}{\eta}\mu_c^B\geq 0$$

since $\mu_c^{CC} \geq 0$. It also follows that

$$\tilde{\psi}_k-\frac{\tilde{\psi}_B}{Q}\geq 0 \tag{84}$$

since $\mu_c^B \geq 0$. Substitute for μ_c^{CC} in equation (83) to obtain:

$$\mu_c^F=\left(1+\mu_c^{RA}\right)\left[\left(\frac{\tilde{\psi}_F}{Q^F}-\tilde{\psi}_D\right)Q^F+\frac{1}{\eta}\left(\tilde{\psi}_k-\frac{\tilde{\psi}_B}{Q}\right)\right]-\frac{1}{\eta}\mu_c^B. \tag{85}$$

We first show that if condition (28) holds, then $\mu_c^F > 0$ and $f_c = 0$ (connected banks do not borrow from the central bank). We prove the claim by contradiction, i.e., suppose that condition (28) holds and yet $\mu_c^F = 0$ (and $f_c > 0$). There are two cases: either $\mu_c^B > 0$ or $\mu_c^B = 0$. Consider $\mu_c^B > 0$. Then, $b_c = 0$ (connected banks do not hold any bonds), implying that $b_c^F = 0$ (connected banks have no bonds to pledge to the central bank) and, therefore, $f_c = 0$. A contradiction.

Consider $\mu_c^B = 0$. Then equation (85) re-writes as

$$\mu_c^F=\left(1+\mu_c^{RA}\right)\left[\left(\frac{\tilde{\psi}_F}{Q^F}-\tilde{\psi}_D\right)Q^F+\frac{1}{\eta}\left(\tilde{\psi}_k-\frac{\tilde{\psi}_B}{Q}\right)\right].$$

By condition (28),

$$\left(\tilde{\psi}_k-\frac{\tilde{\psi}_B}{Q}\right)\frac{1}{\eta}+\left(\frac{\tilde{\psi}_F}{Q^F}-\tilde{\psi}_D\right)Q^F>0$$

implying that $\mu_c^F > 0$ and $f_c = 0$. A contradiction.

We now show that if $\mu_c^F > 0$, then condition (28) holds. We prove the claim by contradiction, i.e., suppose that $\mu_c^F > 0$ and yet condition (28) does not hold. There are again two cases, either $b_c = 0$ (connected banks do not hold any bonds) or $b_c > 0$. Consider $b_c = 0$. Then, $\mu_c^B \geq 0$. Since (28) does not hold, we have that

$$\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right) Q^F + \frac{1}{\eta}\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) \leq 0$$

and, by equation (85), $\mu_c^F \leq 0$. A contradiction.

Consider $b_c > 0$. Then, $\mu_c^B = 0$. Since (28) does not hold, we have that

$$\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right) Q^F + \frac{1}{\eta}\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) \leq 0$$

and, by equation (85), $\mu_c^F \leq 0$. A contradiction.

In sum, condition (28) is a necessary and sufficient condition for connected banks not to borrow from the central bank. A simple sufficient condition for connected banks not to borrow from the central bank is

$$\frac{\tilde{\psi}_F}{Q^F} > \tilde{\psi}_D.$$

This is because by (84), $\tilde{\psi}_k \geq \frac{\tilde{\psi}_B}{Q}$, so that $\frac{\tilde{\psi}_F}{Q^F} > \tilde{\psi}_D$ implies that $\mu_c^F > 0$ and $f_c = 0$. This condition is intuitive: if the interest rate on central bank funding is higher than the rate on deposits, central bank funding will not be used. It is both more expensive in terms of the interest rate and it requires collateral.

Next, we show that if condition (28) holds and if the afternoon constraint of unconnected banks binds, $\mu_u > 0$, then a connected bank does not hold any bonds, i.e., $b_c = 0$. Combining (62) with (63), we get

$$\mu_u^B = \left(1 + \mu_u^{RA}\right)\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) - \mu_u^C - \mu_u \widetilde{\eta}.$$

Since $\mu_u > 0$, $\mu_u^C \geq 0$ and $\mu_u^B \geq 0$, it follows that

$$\tilde{\psi}_k > \frac{\tilde{\psi}_B}{Q} \tag{86}$$

must hold.

Now turning to the connected banks, combine (69) and (70), to get

$$\mu_c^B = \left(1 + \mu_c^{RA}\right)\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) - \mu_c^C. \tag{87}$$

Recall that (28) holds so that $f_c = 0$. We now show that $\mu_c^C = 0$ must hold. Intuitively, if a bank does not borrow from the central bank, it cannot be collateral-constrained at the central bank. Formally, consider the following complementary slackness condition in a connected bank problem:

$$\mu_c^C \left(b_c - b_c^F\right) = 0.$$

Since a connected bank does not borrow from the central, $f_c = 0$, we have that $b_c^F = 0$ since $f_c = \eta Q b_c^F$ and $b_c^F \geq 0$. Therefore, the above complementary slackness condition simplifies to

$$\mu_c^C b_c = 0.$$

There are two possibilities: either the bond holdings are positive, $b_c > 0$ or they are zero, $b_c = 0$. In the former case, it follows that $\mu_c^C = 0$, which proves the claim. In the latter case, we have $b_c = 0$, and a bank does not hold any bonds, does not pledge any bonds, and thus is not constrained by the bond collateral constraint, implying $\mu_c^C = 0$.

Having establsihed that $\mu_c^C = 0$ for a bank that does not borrow from the central bank, (87) simplifies to

$$\mu_c^B = \left(1 + \mu_c^{RA}\right)\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right).$$

Since $\tilde{\psi}_k > \frac{\tilde{\psi}_B}{Q}$, we have that $\mu_c^B > 0$ implying that $b_c = 0$.

Finally, we show that connected banks do not hold money. Combining (80) and (82) for connected banks implies that if

$$\tilde{\psi}_k > \tilde{\psi}_M$$

holds, we have $\mu_{A,c}^M > 0$ and thus $m_{A,c} = 0$. The above condition is equivalent to

$$\theta \frac{y}{\left(\xi k_c + (1 - \xi)\, k_u\right)} + 1 - \delta > \frac{1}{\pi}. \tag{88}$$

Condition (27) is equivalent to

$$\theta \frac{y}{\left(\xi k_c + (1 - \xi)\, k_u\right)} + 1 - \delta > \frac{1}{\beta},$$

and given that for households to deposit with banks, it must be that $\frac{\pi}{\beta} > 1$, the condition (88) is always satisfied.

Proof of Propositon 4

Since the afternoon constraint is slack, $\mu_u = 0$, it follows from Proposition 2 that $\mu_u^{RA} > 0$ and unconnected banks raise either deposit or central bank funding, or both. To show that condition (28) is a necessary and sufficient condition for an unconnected banks unconstrained by the afternoon constraint to not borrow from the central bank, $\mu_u^F > 0$ and $f_u = 0$, the proof follows the same lines as the proof of Proposition 3 for connected banks since the relevant first-order conditions of the unconnected banks simplify to those of the connected banks.

Proof of Propositon 5

Since the afternoon constraint binds, it must be that unconnected banks raise deposit funding, $d_u > 0$.

We first show that when (29) holds, unconnected banks do not borrow from the central bank. We prove the claim by contradiction: suppose (29) holds and yet unconnected banks borrow from the central bank so that $f_u > 0$ and $\mu_u^F = 0$. Using (79), we get

$$\omega^{\max}\mu_u = \left(1 + \mu_u^{RA}\right)\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right) + \mu_u^{CC}\frac{1}{Q^F}.$$

Since $\mu_u^{CA} \geq 0$ we have $\mu_u^{CC} \geq \mu_u \frac{\widetilde{\eta}}{\eta}$ by (67). Therefore,

$$\omega^{\max}\mu_u = \left(1 + \mu_u^{RA}\right)\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right) + \mu_u^{CC}\frac{1}{Q^F} \geq \left(1 + \mu_u^{RA}\right)\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right) + \mu_u\frac{\widetilde{\eta}}{\eta}\frac{1}{Q^F}$$

and we have that

$$\mu_u\left(\omega^{\max} - \frac{\widetilde{\eta}}{\eta}\frac{1}{Q^F}\right) - \left(1 + \mu_u^{RA}\right)\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right) \geq 0.$$

But condition (29) is equivalent to

$$\mu_u\left(\omega^{\max} - \frac{\widetilde{\eta}}{\eta}\frac{1}{Q^F}\right) - \left(1 + \mu_u^{RA}\right)\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right) < 0.$$

A contradiction.

Note that whenever deposit funding is cheaper than central bank funding so that $\frac{\tilde{\psi}_F}{Q^F} > \tilde{\psi}_D$, then a simple sufficient condition boils down to $\widetilde{\eta} > \eta Q^F \omega^{\max}$.

We now show that if condition (30) holds, then unconnected banks top up deposit funding with central bank funding in the morning by pledging their entire bond portfolio, $b_u^F = b_u$. We prove the claim by contradiction: Suppose that $\widetilde{\eta} < \frac{\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}}{\tilde{\psi}_k - \tilde{\psi}_M}$ and yet unconnected banks use bonds to borrow from the secured market so that $b_u^F < b_u$ (and, consequently, $\mu_u^C = 0$). Since $b_u > 0$, we have $\mu_u^B = 0$. Since $\mu_u^C = 0$, we have by (67) that $\mu_u^{CC} = \mu_u \frac{\widetilde{\eta}}{\eta}$. Using this to substitute out μ_u^{CC} in (77), we have:

$$\mu_u = \left(1 + \mu_u^{RA}\right) \left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) \frac{1}{\widetilde{\eta}}.$$

By (78), we have

$$\mu_u^M = \left(1 + \mu_u^{RA}\right) \left(\tilde{\psi}_k - \tilde{\psi}_M\right) - \mu_u \geq 0$$

so that

$$\left(1 + \mu_u^{RA}\right) \left(\tilde{\psi}_k - \tilde{\psi}_M\right) \geq \mu_u.$$

Then, we have that

$$\left(1 + \mu_u^{RA}\right) \left(\tilde{\psi}_k - \tilde{\psi}_M\right) \geq \left(1 + \mu_u^{RA}\right) \left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) \frac{1}{\widetilde{\eta}}$$

or, equivalently,

$$\widetilde{\eta} \geq \frac{\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}}{\tilde{\psi}_k - \tilde{\psi}_M}.$$

A contradiction.

Since $b_u^F = b_u$, unconnected banks cannot borrow in the secured market. The binding afternoon constraint then implies that

$$m_u = \omega^{\max} d_u > 0.$$

This completes the proof.

F Additional comparative statics

In this Appendix, we present results of a comparative statics exercise which aims to capture the effects of safe asset scarcity, a concern which became particularly pronounced in the aftermath of the Global Financial Crisis. In the euro area, the share of AAA-rated sovereign bonds in GDP declined from 30% pre-crisis to just 14% in 2017 (a country is taken as AAA-rated if the country is AAA-rated by at least one of the following three rating agencies: Moody's, Fitch, S&P). To analyze the macroeconomic effects of this development, the supply of government bonds $\bar{b}$ in our model varies between 7.50 units (the steady-state level) and 3.75 units. Figures 8 and 9 show the results for the constant balance sheet and the OP policy, respectively.

In both figures, the solid red line denotes the supply of government bonds under our benchmark calibration ($\bar{b} = 7.50$). The green dashed lines indicate the level of $\bar{b}$ at which unconnected banks start holding money so that the multiplier μ_u^M becomes zero. The blue dashed lines indicate the level of $\bar{b}$ at which the leverage constraint of unconnected banks turns slack and the multiplier μ_u^{RA} becomes zero. The orange dashed lines indicate the level of $\bar{b}$ at which unconnected banks stop holding bonds. As we shall see, these three constraints will play a major role in this exercise.

In the calibrated steady-state (at the solid red line), the collateral premium on bonds is positive and the afternoon constraint binds for unconnected banks. If the stock of government bonds is lower (moving rightward in both figures), it becomes more difficult for unconnected banks to obtain collateralized funding of any kind.

Under the constant balance sheet policy (Figure 8), the figures resemble what happens as private haircuts increase. In particular, if bonds are more scarce (as of $\bar{b} = 6.93$, indicated by the green dashed lines), unconnected banks demand money to self-insure against afternoon liquidity shocks. As the supply of money is fixed under this policy, higher demand for money by unconnected banks is accommodated by the decrease of money holdings by households. This is facilitated by the increase in the nominal rate (the deposit rate), which is proportional to inflation. Higher inflation increases the opportunity cost of holding money for unconnected banks and further tightens their afternoon constraint. Unconnected banks respond by reducing their deposit intake and, therefore, investment in capital. This puts a downward pressure on aggregate capital and, correspondingly, an upward pressure on the return on capital. For the connected banks, this tightens their leverage constraint and, therefore, they reduce their investment in capital and their deposit intake. When the supply of bonds is 6.33 units, unconnected banks are very constrained in the secured market but they cannot increase money holdings any further as households reduced their money holdings to a minimum. At this point, unconnected banks become so constrained in the afternoon that they must reduce their deposit intake. Their leverage constraint turns slack. Connected banks are able to pick up some of the deposits from unconnected banks but only up to a limit as they are constrained by the leverage constraint. As a result, aggregate deposits, capital and output decline. Quantitatively, if the stock of bonds is halved, output contracts by 3.3 percent.

Under the CO policy, the outcome is the same as under the constant balance sheet policy. This is because providing collateralized central bank funding through CO when bonds are scarce cannot mitigate output contractions.

By contrast, OP policy is very effective in stabilizing output in this case. It achieves this by substituting scarce bonds with another liquid asset - money - while maintaining the opportunity cost of holding money low. Specifically, for the stock of government bonds is at 6.86, unconnected banks sell their entire bond holdings to the central bank. For steady-states with a lower stock of government bonds, a lower stock of government bonds is reflected only in lower foreign bond holdings. Quantitatively, if the stock of bonds is halved, output contracts only by 0.1 percent under the OP policy.

Figure 8: Comparative statics: changing the stock of government bonds, $\bar{b}$, constant balance sheet policy

Red solid lines denote the calibrated steady state. Green, blue and orange dashed lines denote the stock of government bonds at which the non-negativity condition on cash holdings of unconnected banks becomes slack, their leverage constraint becomes slack, and the non-negativity condition on their bond holdings becomes binding, respectively. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, net deposit rate, and percent deviation of output from steady-state.

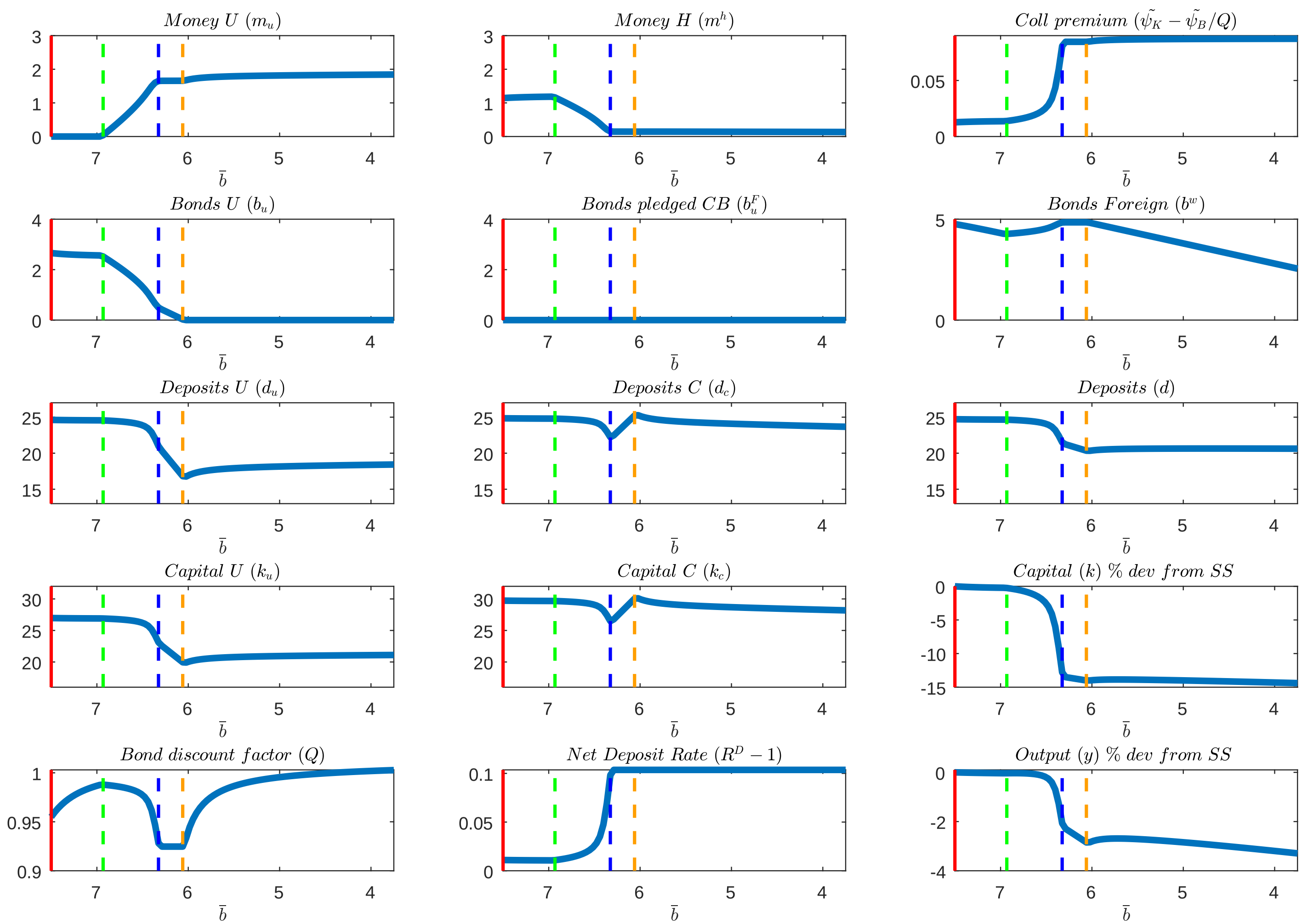


Figure 9: Comparative statics: changing the stock of government bonds, $\bar{b}$, OP policy

Red solid lines denote the calibrated steady state. Green and orange dashed lines denote the stock of government bonds at which the non-negativity condition on cash holdings of unconnected banks becomes slack and the one on their bond holdings becomes binding, respectively. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, central bank bond holdings, and percent deviation of output from steady-state.