A Life-Cycle Model with Unemployment Traps*

Fabio C. Bagliano^ Carolina Fugazza^
Giovanna Nicodano^^

^Università di Torino and CeRP (Collegio Carlo Alberto)

^^Università di Torino, CeRP (Collegio Carlo Alberto) and Netspar

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Abstract

This paper extends the life-cycle model allowing for a personal disaster risk (PDR) during working age, such as long-term unemployment risk. This non-linear income risk reduces both early consumption and early risk taking, shifting them to later in life, compensating the stimulus due to the longer expected working years of the young. Despite higher wealth, the implied cross-sectional distribution of consumption growth displays negative skewness. Effects are stronger when the rare reduction of labor income is potentially larger, albeit with lower expected value. PDR is a robust, first-order determinant of life-cycle choices, amplifying the welfare losses of suboptimal default investments rules, as well.

Keywords: disaster risk, life-cycle, long-term unemployment risk, wage cut, Target Date Fund.

JEL classification: D15, E21, G11

Address: Dipartimento ESOMAS, Università di Torino, Corso Unione Sovietica 218bis, 10134, Torino (Italy).

E-mails: fabio.bagliano@unito.it; carolina.fugazza@unito.it; giovanna.nicodano@unito.it

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1 Introduction

It is well known that macroeconomic disasters, such as stock market crashes, display sizable life-cycle effects despite their low occurrence (Fagereng and Guiso, 2017). We however know little about the life-cycle implications of personal disasters. This paper allows for a small probability of large and persistent earnings losses, that are not covered by social welfare systems, in an otherwise traditional life-cycle model.

While the model may apply to several kinds of personal disasters, our interpretation and calibration focus on long-term unemployment for the following reasons. First, unemployment benefits usually expire after a short unemployment spell, leaving this personal disaster risk largely uninsured. On the contrary, welfare systems provide insurance coverage of other low-probability disasters, that do not originate moral hazard, such as disability. Second, there is ample evidence on its consequences, in comparison to other uninsured ruins such as personal bankruptcy. Long-term unemployment leads to both skill deterioration and large persistent earnings losses across different industries and demographic groups (Rhum, 1991; Jacobson, Lalond and Sullivan, 1993a; Davis and von Wachter, 2011) as well as countries (Machin and Manning, 1999). Third, such losses are uniform across all education groups (e.g., see Kroft, Lange, Notowidigdo and Katz 2016).[1] Overall, these findings indicate that most workers face a small risk of falling in an unemployment trap (UT).

We model this possibility as follows. A 20-year old individual, who initially has a job, may either remain employed or become unemployed, in which case she receives an unemployment benefit. Subsequently, if she remains unemployed, her unemployment benefit disappears and she suffers from a cut in the permanent component of current labor income. This not only reflects the scarring effects of long-term unemployment but also includes the extreme losses due to exit from the labor force, which is a very likely outcome of persistent unemployment (Neal, 1995; Arulampalam, 2001; Edin and Gustavsson, 2008; Schmieder, von Wachter and Bender, 2016). Next period, she may find a job or remain trapped in long-term unemployment, in which case her income will keep reducing.

Our results show that PDR increases optimal precautionary savings for young workers. Optimal early consumption consequently falls, becoming higher during both late working

[1] For example, in 2013, the share of US unemployed workers with a high school (college) education who had been looking for work for two or more years is 12.8% (13.5%) (see Mayer, 2014).

and retirement years for most workers, who do not incur into any disaster. PDR also considerably lowers financial risk taking early in life. Thus, the presence of PDR sharply changes all life-cycle choices with respect to the cases of no unemployment risk (Bodie, Merton and Samuelson, 1992; Viceira 2001; Cocco, Gomes and Maenhout 2005) and of unemployment risk without earnings cuts (Bremus and Kuzin, 2014). In essence, young workers should follow grandmothers' advice to save and be cautious, since you never know what could happen to you in the future. These implications emerge when the (expected) proportional earnings reduction exceeds a cutoff. This is smaller when the proportion is stochastic instead of constant, even if the worker experiences very large losses only with extremely low probability.

Unemployment traps (UT) lead to negative skewness in the returns to human capital, a feature uncovered by Guvenen, Karahan, Ozkan and Song (2015). Thus, while there is mounting evidence that such skewness explains life-cycle patterns in the data (Catherine, 2018; Galvez, 2017; Shen, 2018), we contribute the first normative analysis of a plausible economic rationale. Importantly, the average implied skewness of the consumption growth distribution becomes negative, only with PDR, despite the higher precautionary savings buffer. This outcome obtains in partial equilibrium, with no endogenous change in the labor income process reinforcing the individual shock. Thus, UT leads to consumption skewness, that is known to improve on asset pricing (in Constantinides and Ghosh, 2014, and Schmidt, 2016).

This analysis widens our knowledge of portfolio choice with non-Gaussian returns, that so far focuses on non-Gaussian stock returns (see Guidolin and Timmerman, 2008, among others). First, the optimal investment profile responds to changes in the disaster state (such as the likelihood of entering and exiting it; the proportional earnings reduction; any welfare buffer or training program affecting them) that in turn command the degree of labor income skewness. Second, the latter changes the welfare assessment of sub-optimal default investment rules that are routinely offered by pension plans, when UT-induced depreciation exceeds the cut-off. Welfare losses range from 3 to 10 times as large, in terms of annual consumption, than the ones in standard calibrations without disaster (Cocco, Gomes and Maenhout, 2005; Love, 2013). Additional losses stem from excess consumption and risk taking when the young confront uncertainty about their future and insufficient consumption and risk taking when this uncertainty is resolved. Third, PDR reduces differences in optimal portfolios despite unequal employment histories, because of higher financial wealth accumulation when young.[2] Embedding optimal investment profiles in pension plans does not therefore require individual-tailor made products, at least until age 45 according to our calibrations.

Finally, PDR dampens the sensitivity of results, confirming itself as a first-order determinant of life-cycle choices. Shifting from Power utility to Epstein-Zin preferences does not affect working-years choices, only inducing slower wealth decumulation and less risk taking during retirement years. Similarly, increasing the correlation between stock returns and labor income shocks leaves the equity investment profile unchanged. In other words, correlation - often emphasized in prior literature- loses its relevance in the presence of skewness-inducing personal disaster.

A three-state Markov chain, as in Bremus and Kuzin (2014), drives the transitions between employment and short- and long-term unemployment states. On top, we let the earnings proportional reduction due to long-term unemployment follow a Beta distribution. Calibrations match both unconditional probabilities of short- and long-run unemployment and the average unemployment benefit in the U.S., borrowing the benchmark scenario from Cocco, Gomes and Menhout (2005) otherwise. Adjusting the shape parameters of the Beta distribution, we concentrate most of the probability mass towards low realizations of the earnings loss due to persistent unemployment. The resulting life cycle profiles depart strongly from the ones without PDR when the expected earnings loss conditional on long-term unemployment is in the range 0.1-0.2. This cutoff will turn out to correspond to a non-stochastic loss equal to 0.6.

This paper is not the first to explicitly connect life cycle precautionary savings to social insurance in general (Hubbard, Skinner and Zeldes, 1995) and to insurance against employment risk in particular (Low, Meghir and Pistaferri, 2010). Our analysis explicitly connects precautionary savings to a personal disaster, such as long-term unemployment or personal bankruptcy, inducing skewness in labor income and consumption.

The insight concerning the resolution of uncertainty over working years is common to several previous papers. Chang, Hong and Karabarbounis (2017) model workers' learning

[2] Such profiles are closer to actual portfolios of US households (Ameriks and Zeldes, 2004).

about their earnings ability over the life cycle. Bagliano, Fugazza and Nicodano (2014) allow for a small, positive correlation between one of the risky asset return and permanent income shocks. Hubener, Maurer and Mitchell (2016) focus on the possibility of changing family status during working age (i.e., marriage, fertility, divorce). On top of this insight, we contribute the personal disaster dimension.

This disaster differs from both the individual stock market disaster modeled in Fagereng, Gottlieb and Guiso (2017) and the aggregate economic collapse explaining asset pricing puzzles in Barro (2006). Both of these circumstances concern financial wealth and may occur during retirement as well.

The rest of the paper is organized as follows. Section 2 introduces to the life-cycle model and outlines the numerical solution procedure. We present the calibration in Section 3 and discuss our main results in Section 4, including the welfare analysis. Robustness checks are presented in Section 5. Section 5 concludes the paper.

2 The life-cycle model

We model an investor who maximizes the expected discounted utility of consumption over her entire life and wishes to leave a bequest, as well. The investor starts working at t_0, retires at t_0+K, dies at the latest at time t_0+T and, before then, has a conditional survival probability, p_t, of being alive in the following period. Investors have a time-separable power utility function:

$$\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0}\left[\sum_{j=1}^{T}\beta^j\left(\prod_{k=0}^{j-2}p_{t_0+k}\right)\left(p_{t_0+j-1}\frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + (1-p_{t_0+j-1})\,b\frac{(X_{it_0+j}/b)^{1-\gamma}}{1-\gamma}\right)\right] \tag{1}$$

where C_{it} is the level of consumption at time t, X_{it} is the amount of wealth left as bequest, $b \geq 0$ is a parameter capturing the strength of the bequest motive, $\beta < 1$ is a utility discount factor, and γ is the constant relative risk aversion parameter.

2.1 Labor and retirement income

During working life individuals receive exogenous stochastic earnings. Individual careers evolve across a three-state Markov chain, considering employment (e) along with short-term (u_1) and long-term (u_2) unemployment. Thus, there is no exit from and return into the labor force, for simplicity.

The following transition matrix, similar to the one in Bremus and Kuzin (2014), describes the career evolution:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} \pi_{ee} & \pi_{eu_1} & \pi_{eu_2} \\ \pi_{u_1e} & \pi_{u_1u_1} & \pi_{u_1u_2} \\ \pi_{u_2e} & \pi_{u_2u_1} & \pi_{u_2u_2} \end{pmatrix} = \begin{pmatrix} \pi_{ee} & 1-\pi_{ee} & 0 \\ \pi_{u_1e} & 0 & 1-\pi_{u_1e} \\ \pi_{u_2e} & 0 & 1-\pi_{u_2e} \end{pmatrix} \tag{2}$$

where $\pi_{nm} = \text{Prob}\,(s_{t+1} = n|s_t = m)$ with $n, m = e, u_1, u_2$. If the worker is employed at t ($s_t = e$), she continues the employment spell at $t+1$ ($s_{t+1} = e$) with probability π_{ee}, otherwise she enters short-term unemployment ($s_{t+1} = u_1$) with probability $\pi_{eu_1} = 1 - \pi_{ee}$. Since she must experience short-term unemployment prior to becoming long-term unemployed, we set the probability of directly entering long-term unemployment at zero, $\pi_{eu_2} = 0$. Conditional on being short-term unemployed at t ($s_t = u_1$), she exits unemployment ($s_{t+1} = e$) with probability π_{u_1e} or becomes long-term unemployed ($s_{t+1} = u_2$) with probability $\pi_{u_1u_2} = 1 - \pi_{u_1e}$; consequently, we set $\pi_{u_1u_1} = 0$. Finally, if she is long-term unemployed at t ($s_t = u_2$), she is re-employed in the following period ($s_{t+1} = e$) with probability π_{u_2e} and remains unemployed with probability $\pi_{u_2u_2} = 1 - \pi_{u_2e}$.

As in Cocco, Gomes and Maenhout (2005), each employed agent receives a stochastic labor income driven by permanent and transitory shocks. In each working period, labor income Y_{it} is generated by the following process:

$$Y_{it} = H_{it}U_{it} \qquad\qquad t_0 \leq t \leq t_0 + K \tag{3}$$

where $H_{it} = F\,(t, \mathbf{Z}_{it})\,P_{it}$ represents the permanent income component. In particular, $F\,(t, \mathbf{Z}_{it}) \equiv F_{it}$ denotes the deterministic trend component that depends on age (t) and a vector of individual characteristics ($\mathbf{Z}_{it}$) such as gender, marital status, household composition and education. The logarithm of the stochastic permanent component is assumed to follow a random walk process:

$$N_{it} = \log P_{it} = \log P_{it-1} + \omega_{it} \tag{4}$$

where ω_{it} is distributed as $N(0, \sigma_\omega^2)$. U_{it} denotes the transitory stochastic component and $\varepsilon_{it} = \log(U_{it})$ is distributed as $N(0, \sigma_\varepsilon^2)$ and uncorrelated with ω_{it}.

We now depart from previous papers, letting unemployment duration affect the permanent component of current labor income, H_{it}. Short-term and long-term unemployment erode such component by a fraction Ψ_1 and Ψ_2, respectively, where

$$\Psi_1 \sim Beta(a_1, b_1) \text{ and } \Psi_2 \sim Beta(a_2, b_2) \tag{5}$$

with $\mathrm{E}\Psi_2 > \mathrm{E}\Psi_1$.[3] This introduces a non-linearity into the expected permanent labor income. In compact form, the permanent component of labor income evolves according to

$$H_{it} = \begin{cases} F\left(t, \mathbf{Z}_{it}\right) P_{it} & \text{if } s_t = e \text{ and } s_{t-1} = e \\ (1 - \Psi_1) H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_1 \\ (1 - \Psi_2) H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_2 \end{cases} \qquad t = t_0, ..., t_0 + K \tag{6}$$

The beta distribution is useful in modeling random variables that take values in finite intervals, such as $[0, 1]$ in the current case. Alternatively, we will also allow these parameters to be non-stochastic, as if the reduction in the permanent component of labor income conditional on unemployment were known with certainty.

In the short-term unemployment state, ($s_t = u_1$), we let each unemployed agent receive an unemployment benefit as a fixed proportion ξ_1 of the previous permanent income $H_{it-1} = F_{it-1} P_{it-1}$, whereas in the long-term unemployment state ($s_t = u_2$) no benefits are available, $\xi_2 = 0$, leaving the unemployment trap uninsured. Thus, the income received during unemployment is

$$Y_{it} = \begin{cases} \xi_1 H_{it-1} & \text{if } \mathrm{s}_t = u_1 \\ 0 & \text{if } \mathrm{s}_t = u_2 \end{cases} \qquad t = t_0, ..., t_0 + K \tag{7}$$

.

Finally, during retirement, income is certain and equal to a fixed proportion λ of the

[3] The longer the unemployment spell the larger is human capital depreciation in the data (Schmieder, von Wachter and Bender, 2016).

permanent component of labor income in the last working year:

$$Y_{it} = \lambda F\left(t, \mathbf{Z}_{it_{0+l}}\right) P_{it_{0+l}} \qquad t_0 + K < t \leq T \tag{8}$$

where retirement age is $t_0 + K$, $t_0 + l$ is the last working period and λ is level of the replacement rate.

2.2 Investment opportunities

We allow investments in a short-term risk-less asset, yielding a constant gross real return R^f, and one risky asset, characterized as "stocks" yielding stochastic gross real returns R_t^s, for each period. The excess returns of stocks over the risk-less asset follows

$$R_t^s - R^f = \mu^s + \nu_t^s \tag{9}$$

where μ^s is the equity premium and ν_t^s is a normally distributed innovation, with mean zero and variance σ_s^2. We do not allow for excess return predictability and other forms of changing investment opportunities over time, as in Michaelides and Zhang (2017), since there is sill no consensus on ex-post, out-of-sample predictability.

At the beginning of each period, financial resources available for consumption and saving are equal to the sum of accumulated financial wealth W_{it} and current labor income Y_{it}, which we call cash on hand $X_{it} = W_{it} + Y_{it}$. Given the level of current consumption, C_{it}, next period cash on hand is equal to

$$X_{it+1} = (X_{it} - C_{it})R_{it}^P + Y_{it+1} \tag{10}$$

where R_{it}^P is the investor's portfolio return:

$$R_{it}^P = \alpha_{it}^s R_t^s + (1 - \alpha_{it}^s)\, R^f \tag{11}$$

with α_{it}^s and $(1 - \alpha_{it}^s)$ denoting the shares of the investor's portfolio in stocks and in the risk-less asset, respectively. As customary in this model, we do not allow for short sales and we assume that the investor is liquidity constrained.

2.3 Solving the life-cycle problem

We can write the optimization problem as:

$$\max_{\{C_{it}\}_{t_0}^{T-1},\{\alpha_{it}^s\}_{t_0}^{T-1}} \left(\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0} \left[\sum_{j=1}^{T} \beta^j \left(\prod_{k=0}^{j-2} p_{t_0+k} \right) \left(p_{t_0+j-1} \frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + \right.\right.\right.$$
$$\left.\left.\left. + \left(1 - p_{t_0+j-1}\right) b \frac{\left(X_{it_0+j}/b\right)^{1-\gamma}}{1-\gamma} \right) \right] \right) \tag{12}$$

$$s.t. \quad X_{it+1} = \left(X_{it} - C_{it}\right) \left(\alpha_{it}^s R_t^s + \left(1 - \alpha_{it}^s\right) R^f \right) + Y_{it+1} \tag{13}$$

with the labor income and retirement processes specified above and the no-short-sales and borrowing constraints imposed. We can now restate the problem in a recursive form, rewriting the value of the optimization problem at the beginning of period t as a function of the maximized current utility and of the value of the problem at $t+1$ (Bellman equation):

$$V_{it}\left(X_{it,}P_{it}, s_{it}\right) = \max_{\{C_{it}\}_{t_0}^{T-1},\{\alpha_{it}^s\}_{t_0}^{T-1}} \left(\frac{C_{it}^{1-\gamma}}{1-\gamma} + \beta E_t \left[p_t V_{it+1}\left(X_{it+1,}P_{it+1}, s_{it+1}\right) \right.\right.$$
$$\left.\left. + \left(1 - p_t\right) b \frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma} \right] \right) \tag{14}$$

At each time t the value function V_{it} describes the maximized value of the problem as a function of three state variables: cash on hand at the beginning of time t (X_{it}), the stochastic permanent component of income at beginning of t (P_{it}), and the labor market state $s_{it}(= e,\ u_1,\ u_2)$.

Let $\widetilde{E_t V}_{it+1}$ denote the expectation operator taken with respect to the stochastic variables ω_{it+1}, ϵ_{it+1}, ν_{t+1}^s, Ψ_1 and Ψ_2. The following Bellman equation highlights the expectation over the employment state at $t+1$:

$$
\begin{aligned}
V_{it}\left(X_{it,}P_{it},s_{it}\right) = \max_{\{C_{it}\}_{t_0}^{T-1},\{\alpha_{it}^{s}\}_{t_0}^{T-1}} & \left(\frac{C_{it}^{1-\gamma}}{1-\gamma}\right. \\
& + \beta\left[p_t \sum_{s_{it+1}=e,u_1,u_2} \pi\left(s_{it+1}|s_{it}\right) \widetilde{E_t V}_{it+1}\left(X_{it+1,}P_{it+1},s_{it+1}\right)\right. \\
& \left.\left. + \left(1-p_t\right) b \sum_{s_{it+1}=e,u_1,u_2} \pi\left(s_{it+1}|s_{it}\right) \frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma}\right]\right)
\end{aligned}
\tag{15}
$$

The history dependence, due to unemployment affecting subsequent labor income, prevents from relying on the standard normalization with respect to the level of P_t. The following value function, at t in each possible state, highlights how the evolution of the permanent component of labor income depends on previous labor market dynamics (dropping the term involving the bequest motive):

$$
V_{it}(X_{it},P_{it},e) \;=u(C_{it})\;+\beta p_t \begin{cases}
\begin{cases}
\widetilde{E_t V}_{it+1}(X_{it+1},P_{it+1},e) \qquad \text{with prob. } \pi_{e,e} \\
\text{with } \; P_{it+1}=P_{it}e^{\omega_{it+1}} \quad \text{and} \\
X_{it+1}=(X_{it}-C_{it})R_{it}^{p}+F_{it+1}P_{it+1}e^{\varepsilon_{it+1}}
\end{cases} \\
\\
\begin{cases}
\widetilde{E_t V}_{it+1}(X_{it+1},P_{it+1},u_1) \qquad \text{with prob. } 1-\pi_{e,e} \\
\text{with } \; P_{it+1}=(1-\Psi_1)P_{it} \quad \text{and} \\
X_{it+1}=(X_{it}-C_{it})R_{it}^{p}+\xi_1 F_{it}P_{it}
\end{cases}
\end{cases}
$$

$$
V_{it}(X_{it},P_{it},u_1)=u(C_{it})+\beta p_t \begin{cases}
\begin{cases}
\widetilde{E_t V}_{it+1}(X_{it+1},P_{it+1},e) \qquad \text{with prob. } \pi_{u_1,e} \\
\text{with } \; P_{it+1}=(1-\Psi_1)P_{it-1}\,e^{\omega_{it+1}}=P_{it}\;e^{\omega_{it+1}} \quad \text{and} \\
X_{it+1}=(X_{it}-C_{it})R_{it}^{p}+F_{it-1}P_{it+1}e^{\varepsilon_{it+1}}
\end{cases} \\
\\
\begin{cases}
\widetilde{E_t V}_{it+1}(X_{it+1},P_{it+1},u_2) \qquad \text{with prob. } 1-\pi_{u_1,e} \\
\text{with } \; P_{it+1}=(1-\Psi_2)(1-\Psi_1)P_{it-1}=(1-\Psi_2)P_{it} \text{ and} \\
X_{it+1}=(X_{it}-C_{it})R_{it}^{p}
\end{cases}
\end{cases}
$$

$$
\widetilde{E_t V}_{it}(X_{it}, P_{it}, u_2) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} \widetilde{E_t V}_{it+1}(X_{it+1}, P_{it+1}, e) & \text{with prob. } \pi_{u_2,e} \\ \text{with } \; P_{it+1} = P_{it} e^{\omega_{it+1}} & \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R_{it}^p + F_{it-2} P_{it+1} e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} \widetilde{E_t V}_{it+1}(X_{it+1}, P_{it+1}, u_2) & \text{with prob. } 1 - \pi_{u_2,e} \\ \text{with } \; P_{it+1} = (1 - \Psi_2) P_{it} \text{ and} \\ X_{it+1} = (X_{it} - C_{it}) R_{it}^p \end{cases} \end{cases} \tag{16}
$$

This problem has no closed form solution. We obtain the optimal values for consumption and portfolio shares, depending on the values of each state variable at each point in time, by means of numerical techniques. To this aim, we apply a backward induction procedure starting from the last possible period of life T and computing optimal consumption and portfolio share policy rules for each possible value of the continuous state variables (X_{it} and P_{it}) by means of the standard grid search method.[4] Going backwards, for every period $t = T-1, T-2, ..., t_0$, we use the Bellman equation (15) to obtain optimal rules for consumption and portfolio shares.

3 Calibration

Calibration of investor's preferences, the labor and pension processes and the moments of the risky asset returns, in the upper panel of Table 1, follow familiar ones in this literature. We just note that the benchmark value for the coefficient of relative risk aversion, $\gamma = 5$, captures an intermediate degree of risk aversion.

We set the utility discount factor $\beta = 0.96$ and the strength of the bequest motive $b = 2.5$. This bears the interpretation of the number of years of her descendants' consumption that the investor intends to save for. The agent begins her working life at the age of 20 and works for (a maximum of) 45 periods (K) before retiring at the age of 65. After retirement, she can live for a maximum of 35 periods until the age of 100. In each period, we take the conditional probability of being alive in the next period p_t from the life expectancy tables of the US National Center for Health Statistics.

The risk-less interest rate and the equity premium, μ^s, are set at 0.02 and 0.04, respectively. The standard deviation of the return innovations is equal to $\sigma_s = 0.157$. Finally, we impose zero correlation between stock return innovations and aggregate permanent labor income disturbances ($\rho_{sY} = 0$).

[4] The problem is solved over a grid of values covering the space of both the state variables and the controls in order to ensure that the obtained solution is a global optimum.

3.1 Calibrating labor income and unemployment traps

We calibrate the labor income process with the parameters for households with high school education (but not a college degree) in Cocco, Gomes and Maenhout (2005), given that the chance of long-term unemployment is similar across education groups. For this type of household, the variances of the permanent and transitory shocks (ω_{it} and ε_{it} respectively) are respectively equal to $\sigma_\omega^2 = 0.0106$ and $\sigma_\varepsilon^2 = 0.0738$. After retirement, income is a constant proportion λ of the final (permanent) labor income, with $\lambda = 0.68$

Table 1: Baseline calibration parameters

Description	Parameter	Value
Working life (max)	T	20 -65
Retirement (max)	$t_0 + K$	65 -100
Discount factor	β	0.96
Risk aversion	γ	5
Replacement ratio	λ	0.68
Variance of permanent shocks to labor income	σ_ω^2	0.0106
Variance of transitory shocks to labor income	σ_ϵ^2	0.0738
Risk-less rate	r	0.02
Excess returns on stocks	μ^s	0.04
Variance of stock returns innovations	σ_s	0.157
Stock ret./permanent lab. income shock correlation	ρ_{sY}	0

	No unemployment risk	**Unemployment with no traps**	**Unemployment traps**
Unemployment benefits			
Short-term unemployed (ξ_1)	-	0.3	0.3
Long-term unemployed (ξ_2)	-	0	0
Short-term unemployed (Ψ_1)	-	-	0
Long-term unemployed (Ψ_2)	-	-	0.6

This table reports benchmark values of relevant parameters.

We will present three main calibrations of the model, corresponding to the three columns in the bottom panel of Table 1: (i) "‘No unemployment risk"’ as in Cocco, Gomes and Maenhout (2005); (ii) "‘Unemployment risk with no traps"’, similar to Bremus and Kuzin (2014); (iii) "‘Unemployment traps"’. Case (i) corresponds to $\pi_{ee} = 1$ and all other entries equal to zero in the transition matrix (2). In case (ii), the matrix will be the same as in case (iii), but unemployment will have no scarring effects, (i.e. $\Psi_1 = \Psi_2 = 0$). These two reference cases will help our understanding of the model results.

For cases (ii) and (iii) we pick transition probabilities, so as to match the observed average unemployment rates at different durations, using results in Kroft et al.(2016). The transition probability from employment to short-term unemployment is thus set to 4%, implying a probability of remaining employed equal to 96% since moving directly to long-term unemployment has no chance. We also set the probability of leaving short-term unemployment at 85%, given the annual outflow rate from the first year of unemployment to employment. This implies a probability of remaining unemployed of 0.15, since it is not possible to stay in short-term unemployment. The matrix of annual transition probabilities between labor market states is therefore:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} 0.96 & 0.04 & 0 \\ 0.85 & 0 & 0.15 \\ 0.85 & 0 & 0.15 \end{pmatrix} \tag{17}$$

Our calibration appears conservative in two dimensions. On the one hand, it implies unconditional probabilities of short-(3.8%) and long-run unemployment (0.6%) that are lower than the 2015 total (5.3%) and long-term (1.7%) unemployment rates. On the other hand, it does not represent the declining employment opportunities of the long-term unemployed over time, and their increasing likelihood to leave the labor force (Krueger, Cramer and Cho, 2014). Indeed, the probability of finding a job after 24 months of unemployment can be as low as 40% (Kroft et al., 2016).

Calibrated unemployment benefits mirror the average U.S. ones, with $\xi_1 = 0.3$ and $\xi_2 = 0$ in the case of short- and long-term unemployment spells, respectively. The U.S. unemployment insurance system provides a low replacement rate of the last labor income for maximum 26 weeks, at the state level. To our knowledge, no additional weeks of federal benefits are available.[5]

Our calibration of earnings losses consider that they are larger the longer is unemployment (Cooper, 2013). Moreover, they average between 43 and 66% of pre-displacement wage, according to recent estimates (Jacobson, LaLonde and Sullivan, 2005). Last but not least, job opportunities decline over time to such an extent that the long-term unemployed are likelier to exit the labor force than to become re-employed (Krueger, Cramer and Cho, 2014; Kroft, Lange, Notowidigdo and Katz, 2016),[6] an event that is not represented in

[5]The temporary Emergency Unemployment Compensation (EUC) program expired at the end of 2013, and no state currently qualifies to offer more weeks under the permanent Extended Benefits (EB) program.

[6]This occurs across all ages, industries and education levels, even when job openings increase (Ghayad

our transition matrix. We thus, conservatively, set to zero the earnings cut conditional on the first year of unemployment (Ψ_1=0). The calibrated distributions for Ψ_2 conservatively imply a median value lower than 1%, while the 75^{th} percentile ranges between 6% and 27%. For instance, with a=0.1 always, the median cut is lower than 0.1% and the 75% percentile is lower than 6.6% with b=0.9. With b=0.7, the median and 75% percentile cut increase but remain lower than 0.17 and 10%. As b further declines to 0.4, the median and 75% cut become lower than 0.55% and 27%, respectively.[7] In all three cases, percentage cuts respectively lower than 93, 97, 99 occur with 0.99 probability, capturing the case of those individuals leaving the labor force. We also translate this reasoning in a non-stochastic Ψ_2=0.6, as follows. We fix the probability of losing earnings by leaving the labor force at 0.3 and the probability of finding a new job with a 40% cut in earnings at 0.7. This is the value reported in Table 1.

4 Results

We simulate life-cycle consumption and investment decisions for 10,000 agents on the basis of the optimal policy functions reported in the Appendix. We initially focus on investment and wealth accumulation profiles, since they synthetically summarize all choices including savings and consumption. In particular we juxtapose non-stochastic and stochastic earnings losses conditional on unemployment. A discussion of optimal savings and consumption growth profiles follows. We then revert back to other aspects of investment profiles, including heterogeneity of optimal ones and welfare losses caused by suboptimal ones.

4.1 Life-Cycle Profiles

Figure 1, panel (a), shows the average optimal stock shares plotted against age. Without unemployment risk (dotted line), the striking, well-known result of downward-sloping equity portfolio profiles appears. The optimal equity share decreases with age from 100% when young to around 80% at retirement, since also the proportion of overall wealth invested in the risk-less asset, through human capital, declines with age. This profile is robust to the presence of unemployment risk, if there are no UT (dashed line). The equity portfolio share still declines during working years, being slightly lower at all ages, with a 100% optimal stock share only for very young investors.

With UT (solid line), the optimal stock investment changes its shape. It becomes almost

and Dickens 2012).

[7] Gupta and Nadarajah (2004) provide properties of beta distributions and its generalizations.

flat and is drastically reduced at any age, around 55-60%. One way to explain this pattern is to recall that persistence and negative skewness of earnings shocks reduce the value of human capital well below the level implied by discounting earnings at the risk-free rate and increase its stock component (Huggett and Kaplan, 2016). Another way is to recognize that this added risk is particularly relevant for younger workers, inducing a lower optimal stock investment conditional on financial wealth especially when young. Since uncertainty on the occurrence of disaster resolves as the worker approaches retirement, the age profile remains flat over working life.[8]

These results highlight that long-term unemployment risk dampens the incentive to invest in stocks under otherwise standard calibrations. On the contrary pure unemployment persistence without disaster, as in Bremus and Kuzin (2014), displays close to no effect on the age profile of optimal portfolio composition.

This pattern in the optimal equity portfolio share owes to higher wealth accumulation, in turn induced by larger optimal precautionary savings.[9] Panel (b) of Figure 1 displays the average financial wealth accumulated over the life cycle for the three scenarios. In the face of a possible, albeit rare, reduction in the permanent component of earnings, individuals accumulate more financial wealth during working life to buffer possible disastrous labor market outcomes.[10]

Optimal average consumption when young consequently falls, but it is much higher during both late working years and retirement years.

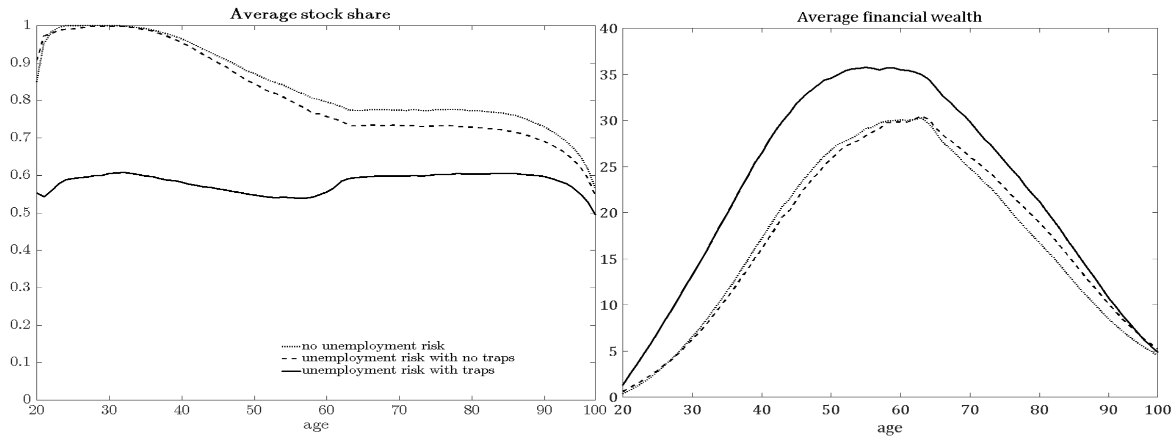
[8]The relatively low investment in stocks during retirement is due to the presence of a positive bequest motive.

[9]Consistent with these predictions, Norwegian households engage in additional saving and in shifting toward safe assets in the years prior to unemployment, as well as in depletion of savings after the job loss (see Basten, Fagereng and Telle, 2016).

[10]Love (2006) uncovers connection between unemployment insurance benefits and calibrated contributions to pension funds by the young, suggesting that precautionary savings when young is due to unemployment risk.

Figure 1: Life-cycle average profiles

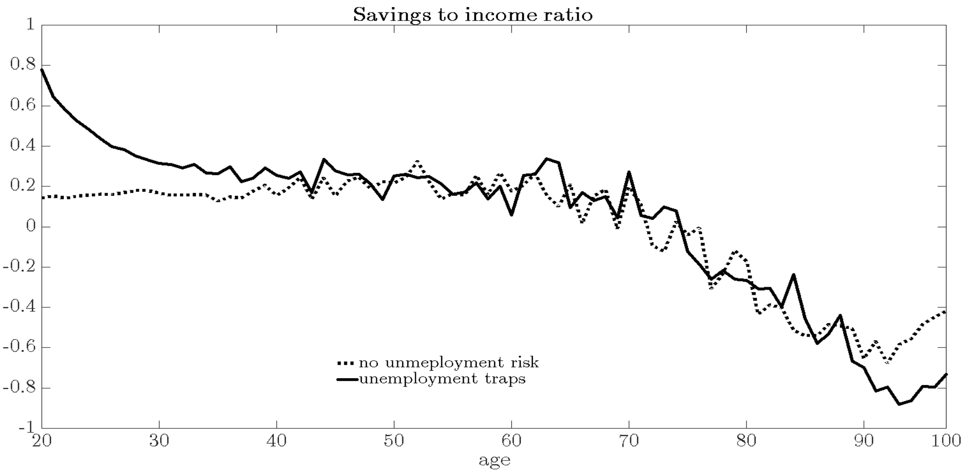
(a) (b)



This figure displays the mean simulated stock investment and financial wealth accumulation life-cycle profiles. Age ranges from 20 to 100. The three cases correspond to no unemployment risk (dotted line); unemployment risk with no traps (dashed line); unemployment risk with traps (solid line). In the latter case, the parameters governing the during short-term and long-term unemployment spells are $\Psi_1 = 0$ and $\Psi_2 = 0.6$. Financial wealth is expressed in ten thousands of US dollars.

Figure 2 displays the life-cycle profile of the ratio between savings and total (financial plus labor) income, comparing the case without unemployment risk to the one with unemployment traps. When the worker is 20 years old, the average propensity to save is especially high in the latter case, reaching 0.8 compared with less than 0.2 when unemployment risk is absent. Such propensity monotonically decreases in age, converging to the known pattern when the worker is in her forties. The figure clearly depicts the impact on savings of the resolution of uncertainty concerning a personal disaster as workers age.

Figure 2: Life-cycle profiles of savings rate

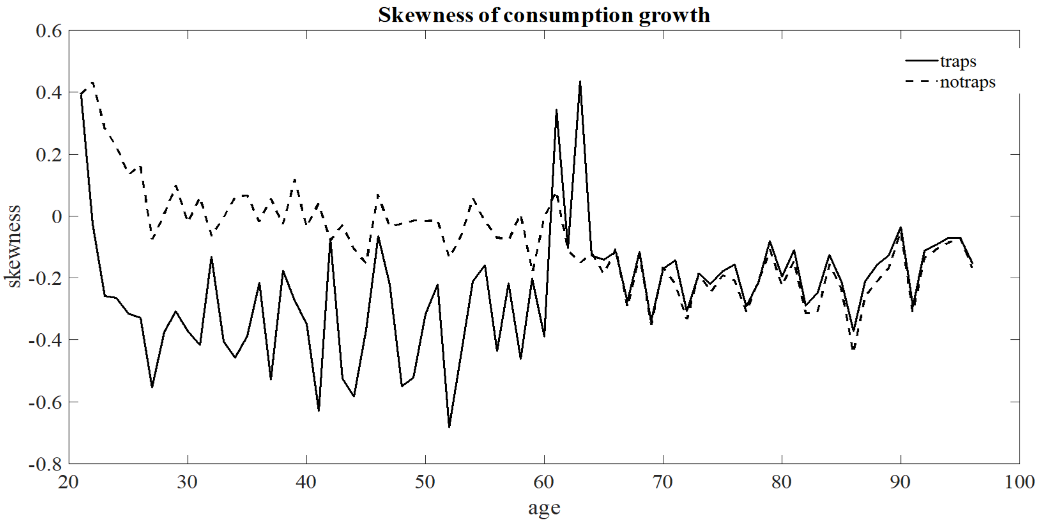


This figure displays the savings dynamics for individuals of age 20 to 100, relative to total income (i.e. labor income plus financial income). The two cases correspond to no unemployment risk (dotted line) and unemployment risk with traps: $\Psi_1 = 0$ and $\Psi_2 = 0.6$ (solid line).

Higher savings when young obviously implies lower consumption when young. What is perhaps less obvious is whether higher wealth shields consumption from the skewness of labor income shocks. Without unemployment risk, average skewness of consumption growth is slightly negative over the life cycle (0.12) but slightly positive over working life (0.019). With UT, average skewness is negative both over the life-cyle (-0.24) and over working life (-0.29).

Figure 3 displays the life-cycle profile of the average skewness of consumption growth. The average skewness without unemployment risk is decreasing but remains above the one with UT during working years. The latter also displays larger peaks and troughs because of a tiny part of the population either exiting long-term unemployment or experiencing its scarring effects.

Figure 3: Life-cycle skewness of consumption growth



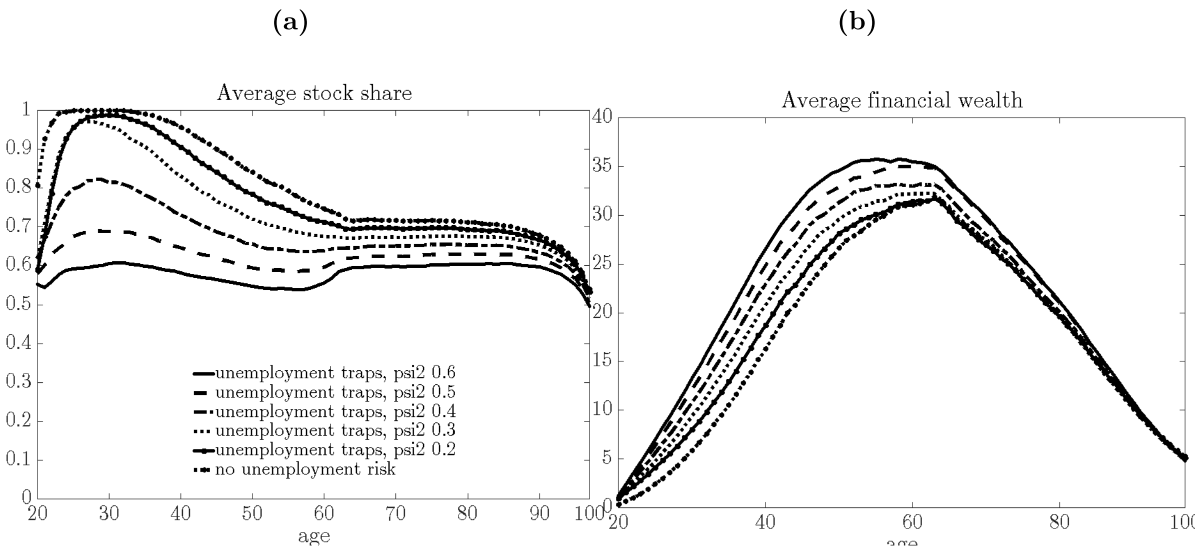
This figure displays the average skewness of consumption growth for individuals of age 20 to 100. The two cases correspond to no unemployment risk (dotted line) and unemployment risk with traps: $\Psi_1 = 0$ and $\Psi_2 = 0.6$ (solid line).

4.1.1 Stochastic earnings reduction

Key aspects of our analysis are both the magnitude and potential variation of the parameter, Ψ_2. Available estimates of earning losses can be as low as 15% (Couch and Placzek, 2010) of their pre-displacement levels, without accounting for exits from the labor force. Figure 4 shows the results of an experiment with the parameter, Ψ_2, in the range $0.2 - 0.6$. The life-cycle profiles of both the optimal risky portfolio and wealth accumulation track closely the no unemployment risk one if the earnings reduction, conditional on long-term unemployment, is equal to 20%. At 40%, a hump shape appears in the portfolio share of stocks; at Ψ_2=0.5, the profile is flat, as in the 0.6 case.

These sensitivity results imply that labor market institutions targeted to long-term unemployment (and other personal disasters) impact life-cycle choices, if they are able to cushion earnings losses thereby reducing PDR. The variation of such institutions across countries may thus explain the different life-cycle patterns in equity investing found in Norwegian (Fagereng, Gottlieb and Guiso, 2017) and US data (Ameriks and Zeldes, 2004).

Figure 4: Life-cycle profiles with unemployment traps: sensitivity to

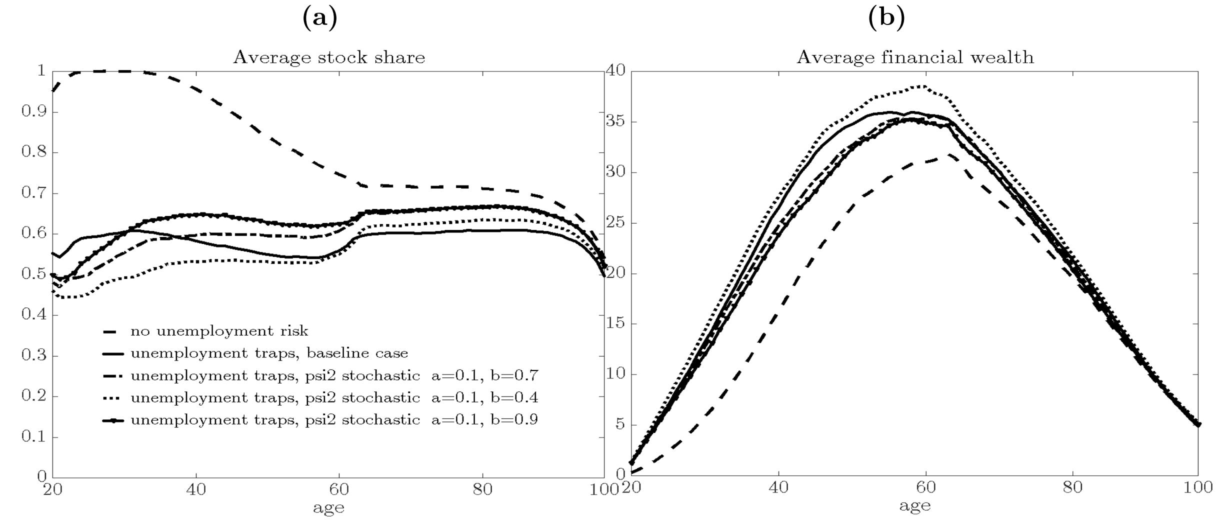


This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Various cases are considered: no unemployment risk; unemployment traps with alternative values of , i.e. with Ψ_2 decreasing from 0.6 to 0.2 (in all cases $\Psi_1 = 0$). Financial wealth is expressed in ten thousands of US dollars.

We now turn to the case of stochastic parameter, where Ψ_2 follows the Beta distribution with expected value ranging between 0.10 and 0.20 and standard deviation between 0.21 and 0.32, implying an expected $10\% - 20\%$ erosion of the individual permanent labor income component after the second year of unemployment. More precisely, it represents a median value for the proportional lower than 1%, and a 75^{th} percentile loss ranging between 6% and 25%.

Figure 5 shows that under all distributional assumptions, life cycle profiles are very similar to the case of the benchmark value of $\Psi_2 = 0.6$. This outcome indicates that extremely rare but potentially disastrous labor income shocks drive optimal cautiousness by young workers and their optimal, limited risk taking in the stock market.

Figure 5: Life-cycle profiles with unemployment traps: stochastic



The figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Various cases are considered: no unemployment risk; unemployment traps with deterministic ($\Psi_1 = 0$ and $\Psi_2 = 0.6$), unemployment traps with stochastic . In the latter case, Ψ_2 follows a beta distribution with shape parameters a and b. In particular, a=0.1 and b=0.4, 0.7 and 0.9, implying an expected value for Ψ_2 equal to 0.20, 0.12 and 0.10, respectively. Financial wealth is expressed in ten thousands of US dollars.

The above results imply that the optimal stock investment is flat in age, even for a moderately risk averse worker. In the face of a very rare but possibly large cut in the permanent component of earnings, workers on average invest about 55% of their financial wealth in stocks. Conservative optimal investments in stocks when young appear in other circumstances, such as the presence of housing wealth (Cocco, 2004; Kraft and Munk, 2011); the sensitivity of the expected labor income growth rate to the real short-term interest rate (Munk and Sorensen, 2010) and the resolution of uncertainty (Chang, Hong and Karabarbounis, 2017; Bagliano, Fugazza and Nicodano, 2014; Hubener, Maurer and Mitchell, 2016). The disaster dimension implies that these profiles are optimal on average even if the disaster materializes only for a tiny portion of the population, in the left tail of the income distribution.

4.2 Heterogeneity

The average risk-taking pattern may hide considerable differences across agents, due to different realizations of positive and negative labor income shocks across workers. The present section investigates the distribution of both conditional optimal stock share and accumulated wealth.

The case of no unemployment risk is displayed in panels (a) and (b) of Figure 6, showing the 25^{th}, 50^{th} and 75^{th} percentiles of the distributions. Both the optimal stock share and the stock of accumulated financial wealth are highly heterogeneous across workers as well as retirees. The exception is young workers as they tilt their entire portfolio towards stocks given the relatively risk-less nature of their human capital. Heterogeneity of portfolio shares depends on the shape and movements through age of the policy functions displayed in Figure 12, relating optimal stock shares to the amount of available cash on hand, and on the level of cash on hand itself. Relatively steep policy functions imply that even small differences in the level of accumulated wealth result in remarkably different asset allocation choices. At the early stage of the life cycle, when accumulated financial wealth is modest, it is optimal for everybody to be fully invested in stocks. As investors grow older, different realizations of background risk induce large differences in savings and wealth accumulation. This situation pushes investors on the steeper portion of their policy functions and determines a gradual increase in the heterogeneity of optimal risky portfolio shares during their working life. After retirement, investors decumulate their financial wealth relatively slowly, due to the bequest motive, and still move along the steeper portion of their relevant policy functions; as a consequence, the dispersion of optimal shares tends to persist.

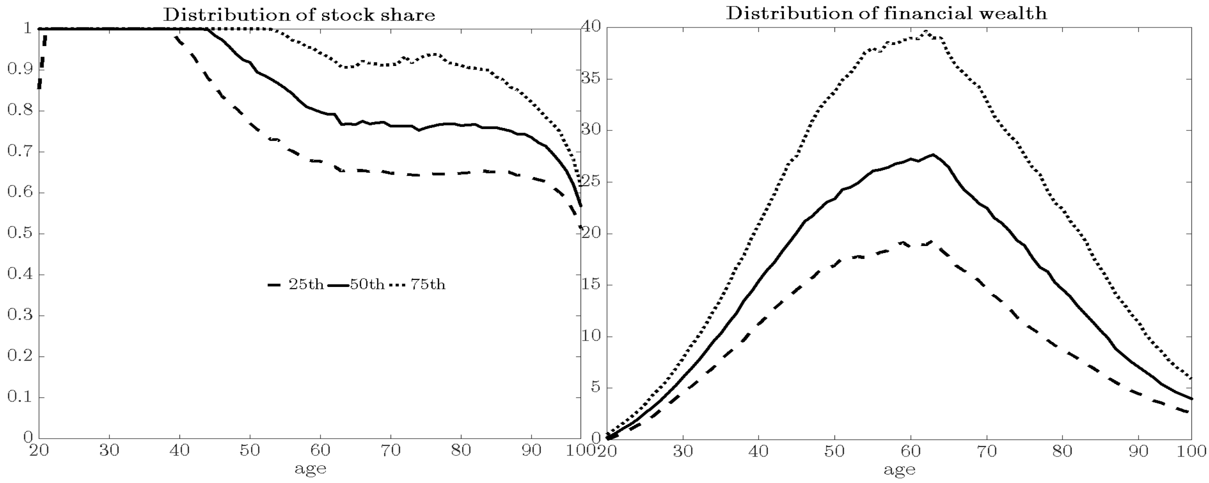
Panels (c) and (d) of Figure 6 display the life-cycle distribution of stock share and financial wealth for the UT case. Compared with the previous case, the distribution of optimal stock shares is much less heterogeneous over the whole life cycle. Heterogeneity shrinks even for young workers, given the rare but potentially high earnings risk they bear throughout their careers. In case of UT, policy functions are relatively flat (see panel (b) of Figure 12) implying that even large differences in the level of accumulated wealth result in homogenous asset allocation choices.

Panels (c) and (d) of Figure 6 display the life-cycle distribution of stock share and financial wealth for UT case. Compared with the case of no unemployment risk, the distribution of optimal stock shares is much less heterogeneous over the whole life cycle. In particular, heterogeneity shrinks during working life even for young workers, given the high earnings risk they bear at the beginning of their careers. In case of UT, policy functions are relatively flat (see panel (b) of Figure 12) implying that even large differences in the level of accumulated wealth result in relatively homogenous asset allocation choices.

Figure 6: Life-cycle percentile profiles

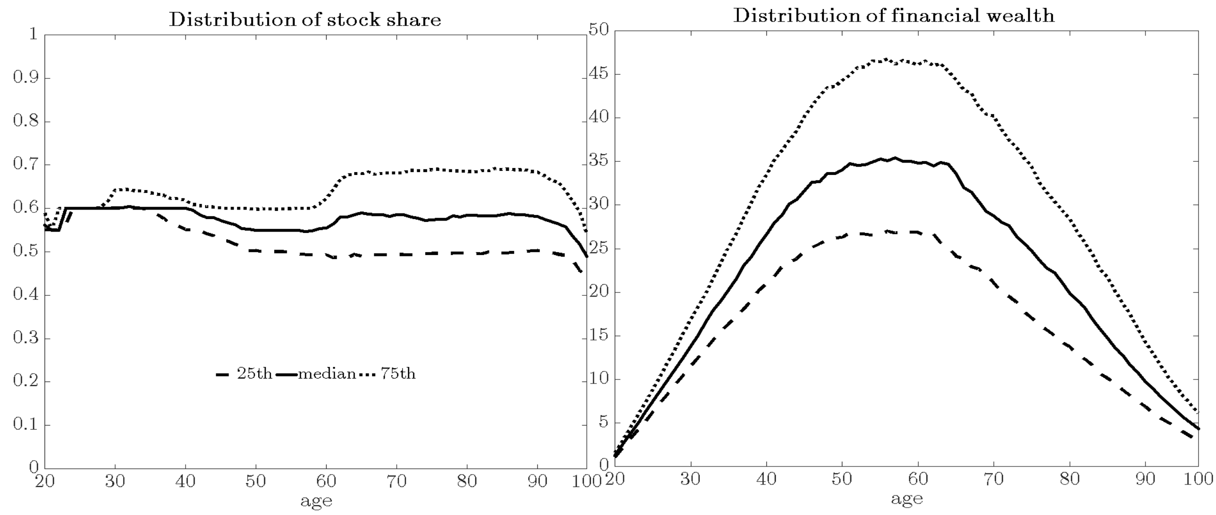
No unemployment risk

(a) **(b)**



Unemployment traps

(c) **(d)**



This figure displays the distribution of simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100 in the case of unemployment risk (panels (a) and (b)) and unemployment traps (panels (c) and (d)). The parameters governing the during short-term and long-term unemployment spells are $\Psi_1 = 0$ and $\Psi_2 = 0.6$. Financial wealth is expressed in ten thousands of US dollars.

Thus, PDR not only affects average savings, consumption and risk taking of the population. It also shrinks differences across young agents with different working histories, as they all self insure.

4.3 Welfare analysis of suboptimal choices

Workers usually delegate the task of managing long-term saving on their behalf to their pension funds. The strategies proposed by portfolio managers often embed the feature of a decreasing age profile of investment in the riskier assets, with a portfolio share in stocks often in excess of 80% when young. These strategies resemble the ones that are optimal in the absence of unemployment risk. In fact, Cocco, Gomes and Maenhout (2005) find that the representative worker should enjoy only a slightly higher (0.64%) consumption level to be compensated for the adoption of a suboptimal "age rule" by her pension fund. Bagliano, Fugazza and Nicodano (2014) find that a compensation of a similar amount is needed when the investor's asset menu also includes bonds, unless stock returns and permanent income shocks are positively correlated.[11] Love (2013) finds even lower welfare losses when optimizing over the parameters of the rule of thumb.

This section provides a quantitative assessment of the welfare loss associated with the adoption of simple portfolio allocation rules of thumb related to age when there are rare unemployment traps. We also explore an alternative suboptimal situation, in which there are unemployment traps but the worker adopts the utility-maximizing consumption and portfolio allocation that ignores them.[12] This case is inspired by the scant discussion of long-term unemployment in the United States prior to the recent crisis, which suggests little awareness of this tail risk before 2007.

In particular, we consider two suboptimal asset allocation patterns related to the investor's age. The first is the typical "age rule" analysed by Cocco, Gomes and Maenhout (2005), with a risky portfolio share set at 100 less the investor's age.[13] The second rule of thumb, denoted as "target-date fund (TDF) rule", comes closer to actual strategic asset allocation patterns adopted by Target-Date Funds. As shown in panel (a) of Figure 7, the stock portfolio share is set at 90% until the age of 40, is gradually decreased over the remaining working life down to 50% at retirement age (65), and further reduced in the early retirement period to reach a low of 30% at the age of 72. This TDF rule echoes the one investigated by Bagliano, Fugazza and Nicodano (2014) with the investor's asset menu also including bonds. In both cases considered above, the worker is aware of unemployment traps and optimally chooses saving and consumption given the rule-of-thumb portfolio allocation. Our welfare analysis concludes with the suboptimal case when the worker maximizes expected

[11] Only in the case of positive correlation is the compensating consumption higher; it may reach 3.9% for the benchmark risk aversion parameter ($\gamma = 5$).

[12] In all the three suboptimal cases, the underlying labor income process is the one implied by the presence of unemployment traps.

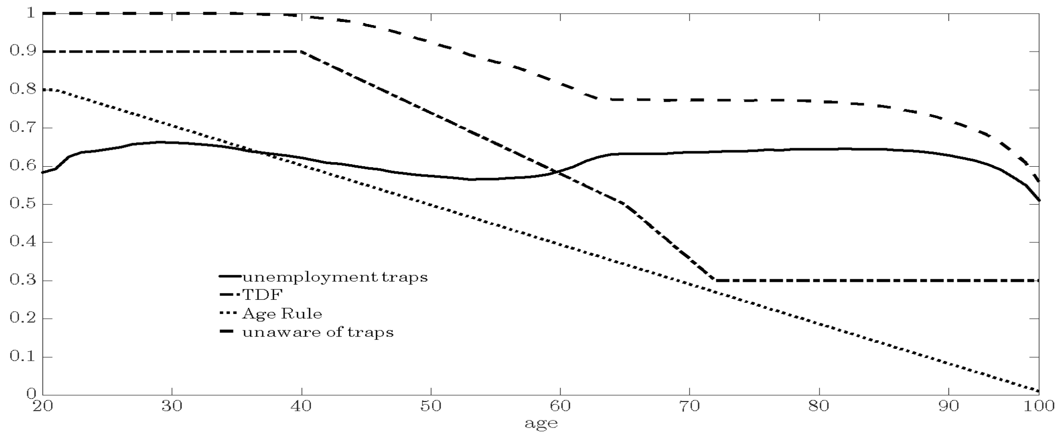
[13] In a variant of this "age rule", the worker starts saving for retirement 40 years before the target retirement date, setting the initial share of stocks at 80% and letting it fall to 40% at retirement (Bodie, Treussard and Willen, 2007).

utility oblivious of rare personal disasters.

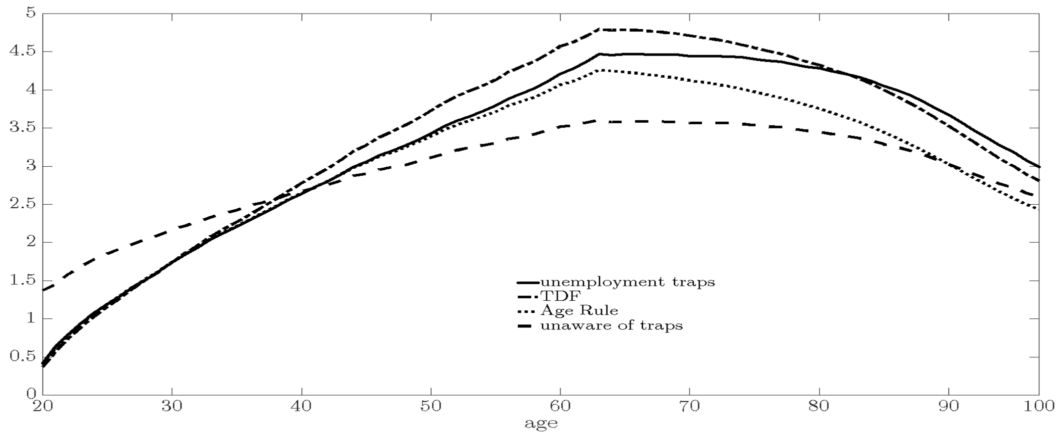
The metric used to perform welfare comparisons is the standard consumption-equivalent variation employed by Cocco, Gomes and Maenhout (2005). The consumption-equivalent variation is obtained by simulating optimal consumption and wealth accumulation choices conditional on following the optimal asset allocation strategy and each of the alternative (suboptimal) investment rules and by deriving the associated expected discounted lifetime utility levels. By inverting the derived expected discounted lifetime utility, we compute the constant consumption stream needed to compensate the investor for the adoption of suboptimal strategies. We then compute the percentage increase in annual consumption required by the investor to obtain the same level of expected utility warranted by the optimal life-cycle strategy for each suboptimal rule. Throughout our comparisons, we adopt the benchmark calibration parameters reported in Table 1.

Figure 7: Optimal and suboptimal life-cycle profiles

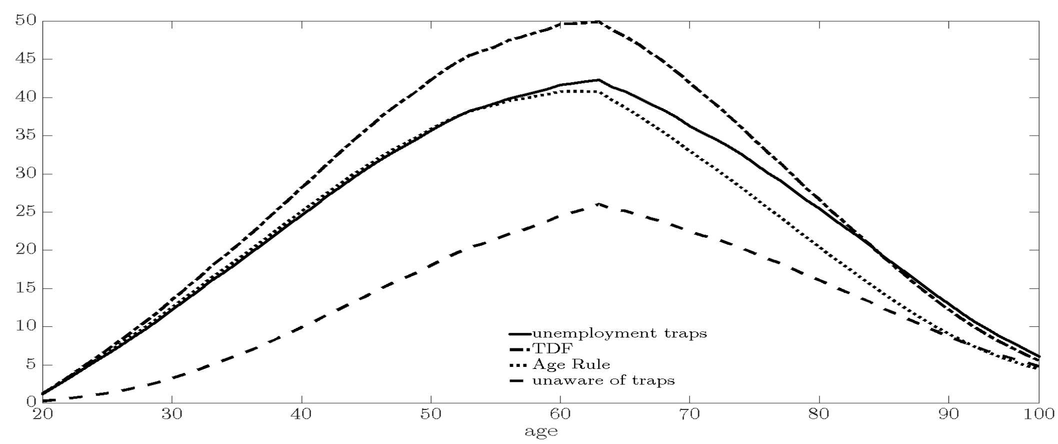
(a) Equity portfolio share



(b) Consumption



(c) Financial wealth



This figure compares the optimal (solid line) and suboptimal life cycle profiles (dotted line: "Age Rule"; dashed-dotted line: "Target Date Fund rule"; dashed line: "unaware of traps", i.e. optimization without taking into account the existence of unemployment traps). Financial wealth and consumption are expressed in ten thousands of US dollars.

4.3.1 Welfare comparison of default investment rules

The left-hand side of Table 2 shows the welfare gains associated with switching from the "age rule" to the optimal portfolio choice. Both the mean and the median increases in welfare-equivalent consumption are equal to 3.3%. Welfare gains are three times larger than prior estimates in the literature. Such gains derive from the fact that consumption and savings are distorted by the higher risk taking when the worker faces a large amount of uncertainty about future labor and pension income, as well as by the lower risk taking when uncertainty is resolved. Average consumption (panel (b) of Figure 7) is close to the optimal level during early working years under the "age rule", but it is much lower during retirement. Moreover, as shown in panel (c), while wealth accumulation until age 55 is close to optimal, average financial wealth at retirement and thereafter turns out to be lower under the "age rule". This pattern is due to agents who, having incurred a trap, save less and ultimately - given the quick reallocation towards the risk-less asset - are also able to consume and bequeath less. Those workers who do not experience personal disasters are able to set aside more wealth, but gradual conversion into the risk-less asset reduces the return on the financial wealth relative to investors adopting optimal portfolio shares. The pattern of welfare gains across income brackets is surprising, however, as revealed by panel (b) of Table 2. Mean welfare gains when income at age 64 is below the 5th percentile of income distribution are lower than for agents with income above the 95th percentile (1.6% versus 2.4%). We tentatively ascribe such a result to the fact that a distorted portfolio rule delivers lower utility losses at the bottom of the income distribution because of lower financial wealth.

Table 2: Welfare Gains

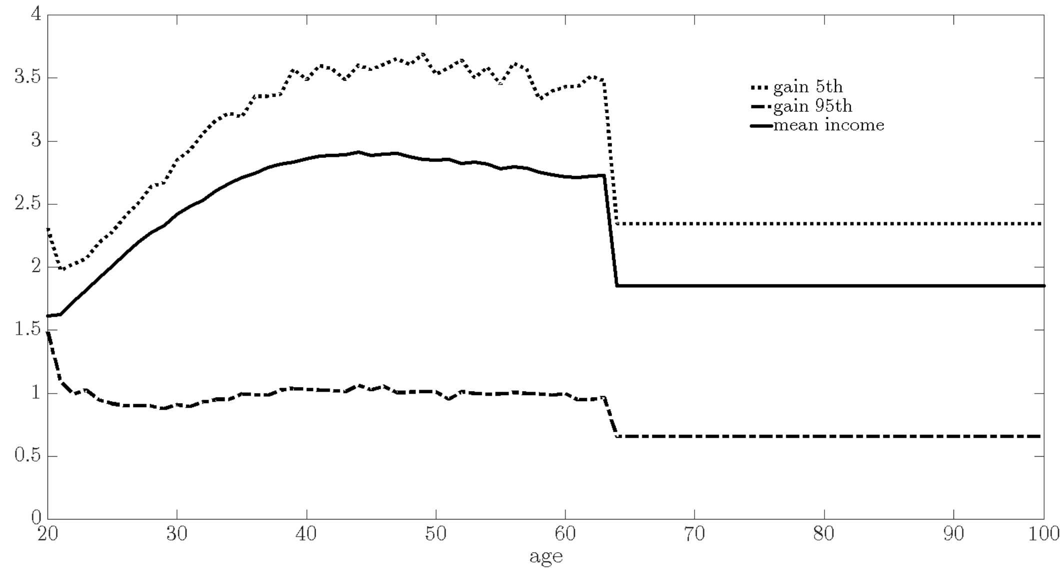
	Age Rule	TDF rule	Unaware of Traps
(a) Distribution of welfare gains (% points)			
Mean	3.3	12.0	642.5
Median	3.3	11.8	215.8
5th	1.5	8.0	-40.5
95th	5.4	17.0	573.6
(b) Welfare gains conditional on income at age 64 (% points)			
Below 5th percentile	1.6	9.5	1024.0
Above 95th percentile	2.4	12.3	218.2

This table reports welfare gains (in percentage points) due to the adoption of optimal choices instead of suboptimal ones. The first and second columns display gains from abandoning suboptimal "Age Rule" and "Target Date Fund Rule " for asset allocation. The third column refers to gains deriving from taking account of unemployment traps in the optimization process with respect to being unaware of traps. Panel (a) displays the distribution of welfare gains. Panel (b) compares mean gains for workers, at age 64, with income below the 5th percentile and above the 95th percentile of income distribution.

The middle column of Table 2 displays welfare gains when the investor adopts the optimal asset allocation pattern instead of the TDF rule, with the stock portfolio share exceeding the one dictated by the "age rule" until age 55 and later falling below it. Given that higher exposure to financial risk is present early in life and lower exposure during retirement, mean and median welfare gains from adopting the optimal portfolio rule are much higher (above 10% of yearly consumption). This pattern emerges despite two seeming improvements, highlighted in Figure 7. The first is that mean consumption under a TDF portfolio allocation is higher from age 30 until age 80 than consumption under the optimal portfolio rule. The second is that mean financial wealth exceeds the optimal one until the investor is past age 80. The increased consumption during working life and early retirement is thus more than offset by the marked reduction during later retirement years, followed by a lower bequest. Once again, the mean welfare gains are lower for those with income (at age 64) below the 5th percentile of income distribution than for those with income above the 95th percentile (9.5% versus 12.3%). Both experiments suggest that distortions in asset allocation produce larger welfare losses for richer households.

The right-hand column of Table 2 delivers an upper bound on median welfare gains from becoming aware of unemployment traps (more than 200%). Mean welfare gains are even higher (642%). This pattern derives from a much higher average consumption until age 40, leading to lower buffer savings, for the unaware worker who chooses consumption and portfolios composition as in Cocco, Gomes and Maenhout (2005) (see panel (b) in Figure 7). If a long-term unemployment spell occurs, consumption and bequest drop dramatically, in some cases almost to zero. This drop implies a large utility loss for a tiny share of workers, causing a sizable increase in mean welfare loss relative to the median value. Since close-to-zero consumption is likelier at the bottom of the income distribution, mean welfare gains from becoming aware of traps are higher for those with low rather than high income, at age 64 (1024% against 218% as shown in panel (b) of Table 2). Figure 8 confirms the above interpretation, displaying mean income profiles for individuals conditional on welfare gains. Workers with lifetime earnings above the mean obtain welfare gains below the 5th percentiles. In turn, workers enjoying the largest welfare gains are those with lifetime labor income well below the average. Clearly, the earlier the traps occur, the lower the buffer wealth and the worse the consumption and welfare consequences. The latter observation hints at a welfare-improving scheme designed to support the fraction of workers hit by long-term unemployment when young, along the lines suggested by Michelacci and Ruffo (2015). More generally, the size of the welfare losses suggests to move towards schemes that protect against longer-term unemployment (e.g., see Setty 2017).

Figure 8: Income profiles conditional on welfare gains



This figure displays mean income profiles conditional on welfare gains from becoming aware of traps. The dotted line represents the mean income profile for individuals with welfare gain below the 5-th percentile. The solid and dash-dotted lines refer to individuals with welfare gains respectively at the average and above the 95th percentile.

5 Robustness and Other Challenges

The life-cycle responses, including the flat age profile in stock investment, are usually highly sensitive to small parametric changes (see e.g. Bagliano, Fugazza and Nicodano, 2014). This section investigates the robustness of our results.

A first robustness check concerns the sensitivity of our results to a lower probability of personal disaster. In performing such analysis, we also allow for an asymmetric reduction in the probability of long-term unemployment with respect to workers' age. Recent data from US labor market statistics indeed show that the composition of long-term unemployment is shifting towards the elderly. In 2015 the overall and the long-term unemployment rates in US were about 5.7% and 1.7%, respectively, with the share of long-term unemployment in the overall unemployment rate differing widely among age groups: from 20% among young workers (16-24 years old), to 35% among prime age workers (25-55), and up to 41% among older workers (over 55).

A second check regards the modeling of the link between unemployment risk during working life and retirement income, so as to make sure that our results do not originate exclusively from long-term unemployment occurring during the very last working years, which heavily reduces retirement income.

Further, since the power utility function implies that the worker is indifferent to intertemporal correlation of consumption shocks (e.g., see Bommier, 2007), we adopt Epstein-Zin preferences to investigate whether positive correlation aversion impacts the response to UT. A similar motivation leads us to analyze the sensitivity of the equity-investment profile to positive correlation between stock returns and labor income shocks.

Finally, we discuss further aspects relating to our study.

5.1 Age-dependent unemployment risk

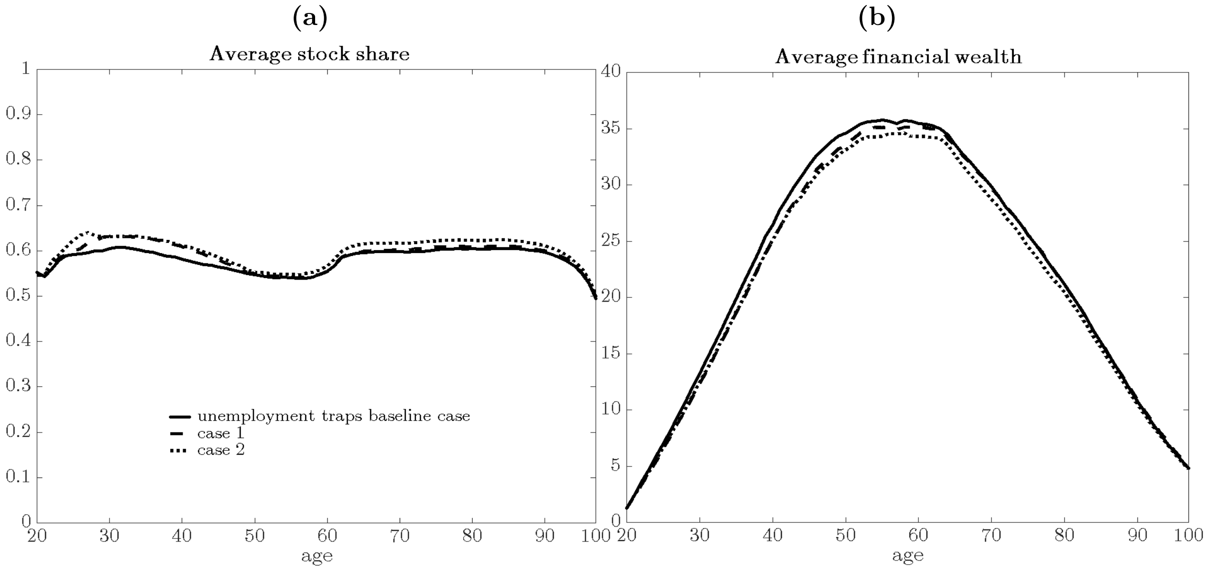
In this section, we calibrate our model with unemployment traps, allowing for both a smaller and age-dependent long-term unemployment risk. We change the transition probability from short-term to long-term unemployment, denoted as $\pi_{u_1u_2}$ in the following transition probabilities matrix:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} 0.96 & 0.04 & 0 \\ 1-\pi_{u_1u_2} & 0 & \pi_{u_1u_2} \\ 0.85 & 0 & 0.15 \end{pmatrix}$$

with respect to the baseline calibration in (17) where $\pi_{u_1u_2} = 0.15$ irrespective of the worker's age. In "*case 1*", the probability of entering long-term unemployment is reduced by one third (from 0.15 to 0.10) only for workers younger than 50 years old. In "*case 2*", we further reduce the probability of entering long-term unemployment for very young workers, setting $\pi_{u_1u_2} = 0.075$ for workers less than 30 years old. In all scenarios, transition probabilities are rather conservative implying steady-state long-term unemployment rates lower than the actual one. For reference, in the baseline case, the steady-state long-term unemployment rate is 0.6%, while it is 0.5% and 0.4%, in cases 1 and 2, respectively.

Figure 9 reports the life-cycle profiles for the optimal conditional stock holding and financial wealth accumulation when the long-term unemployment risk is age-dependent. Compared with the baseline case, the age profile of stock investment is only slightly modified. A lower long-term unemployment risk at young ages implies a moderately higher stock share during prime age but it does not significantly alter investors' behaviour later over the working life and during retirement. In addition, it has virtually no effect on wealth accumulation.

Figure 9: Life-cycle profiles with unemployment traps: age-dependent long-term unemployment risk



This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. The probability of entering long-term unemployment for an unemployed worker is set to 0.15 in the baseline case, to 0.10 only for workers younger than 50 in Case 1, to 0.10 for all workers in Case 2. : $\Psi_1 = 0$ and $\Psi_2 = 0.6$. Financial wealth is expressed in ten thousands of US dollars.

5.2 Unemployment risk and retirement income

In our model, pension benefits are a fixed proportion of the last labor income earned prior to retirement age. Such income is especially sensitive to earnings losses due to the occurrence of long-term unemployment in the years just before retirement. Thus, we analyze whether our results are robust to changes in modeling the link between long-term unemployment at old ages and subsequent pension provisions.

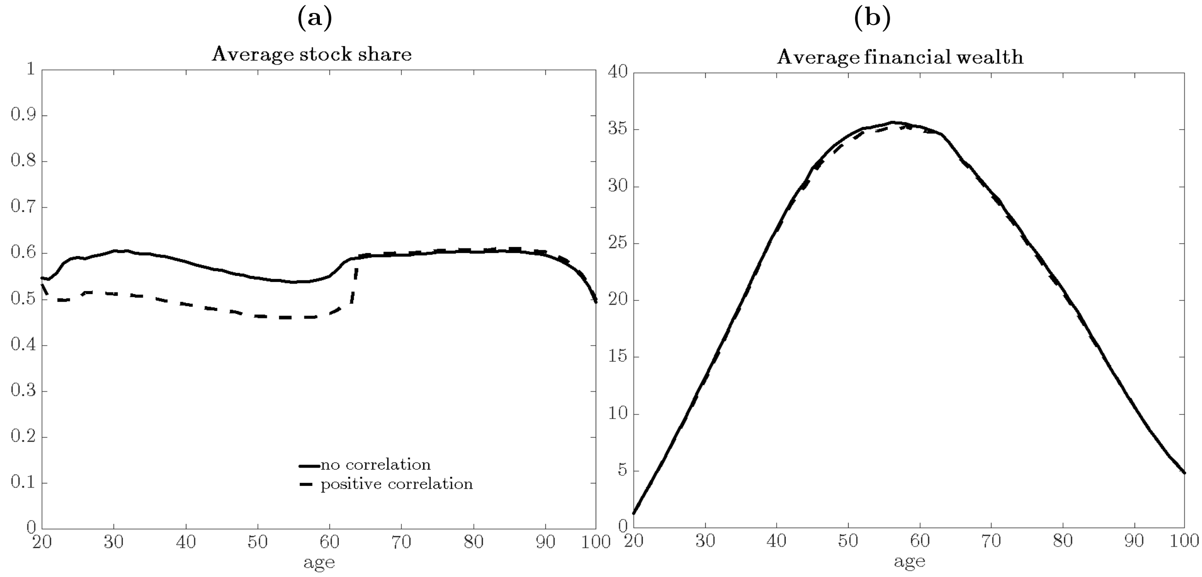
To begin with, we assume no earnings cut loss in the event that unemployment occurs in the years immediately before retirement. Our simulation results show that the flattening of the optimal stock share profile carries over to this setting, suggesting that it is not an artifact of how we model pension income. In a second check, we take the solution of our original model (calibrated in the case of UT) and focus on simulated life-cycle profiles for two selected groups of agents. The first group includes workers who have experienced just one long-term unemployment spell of 5 years over the entire working life at the beginning of their job career (i.e., before the age of 35), whereas the other group contains workers who have experienced just one long-term unemployment spell of 5 years over their entire working life but at the end of their career (i.e., after the age of 60). We find that in both cases, average life-cycle stock share profiles exhibit the flattening property. This experiment confirms that the flattening is due to PDR, and is not sensitive to specific assumptions on pension income.

5.3 Correlation versus Skewness

In this section, we consider two possible avenues that might reinforce our results. The first one is to allow for a positive correlation between stock return innovations and the innovations in permanent labor income ($\rho_{sY} > 0$). Results in Bagliano, Fugazza and Nicodano (2014) show that a realistically small correlation has large effects on life-cycle choices when it interacts with a higher variance of the permanent component of labor income shocks. One may therefore expect a similar effect in the presence of unemployment traps. Empirical estimates of the stock return-labor income correlation differ widely, even when we restrict the scope to the US economy. Cocco, Gomes and Maenhout (2005) report estimated values not significantly different from zero across various education groups, in line with Heaton and Lucas (2000), whose estimates range from -0.07 to 0.14 . However, Campbell, Cocco, Gomes and Maenhout (2001) find higher values, ranging from 0.33 for households with no high school education to 0.52 for college graduates. In the simulations below, we adopt an intermediate positive value of $\rho_{sY} = 0.2$.

Figure 10 shows optimal portfolio shares of stocks and the pattern of financial wealth accumulation with no correlation and with a positive correlation between labor income shocks and stock returns. While the shape of life-cycle profiles is relatively unaffected, the average stock share is lower at all ages. In case of positive correlation, labor income is closer to an implicit holding of stocks, reducing the incentive to invest in stocks at all ages. More specifically, in comparison with the case of no correlation, such investors are relatively more exposed to stock market risk and will prefer to offset such risk by holding a lower fraction of their financial portfolio in stocks. The stock share remains substantially flat over the whole working life, displaying limited variability around a level of about 50%. At the retirement age of 65, human capital becomes risk-less since pension income is certain and therefore uncorrelated with stock return innovations. Thus investors rebalance their portfolio towards stocks: during retirement, the level and time profile of the stock share are very close to the case of no correlation. Further, the relative increase in human capital risk due to a positive correlation does not substantially alter the pattern of financial wealth accumulation.

Figure 10: Life-cycle profiles with unemployment traps: positive correlation between labor income and stock returns

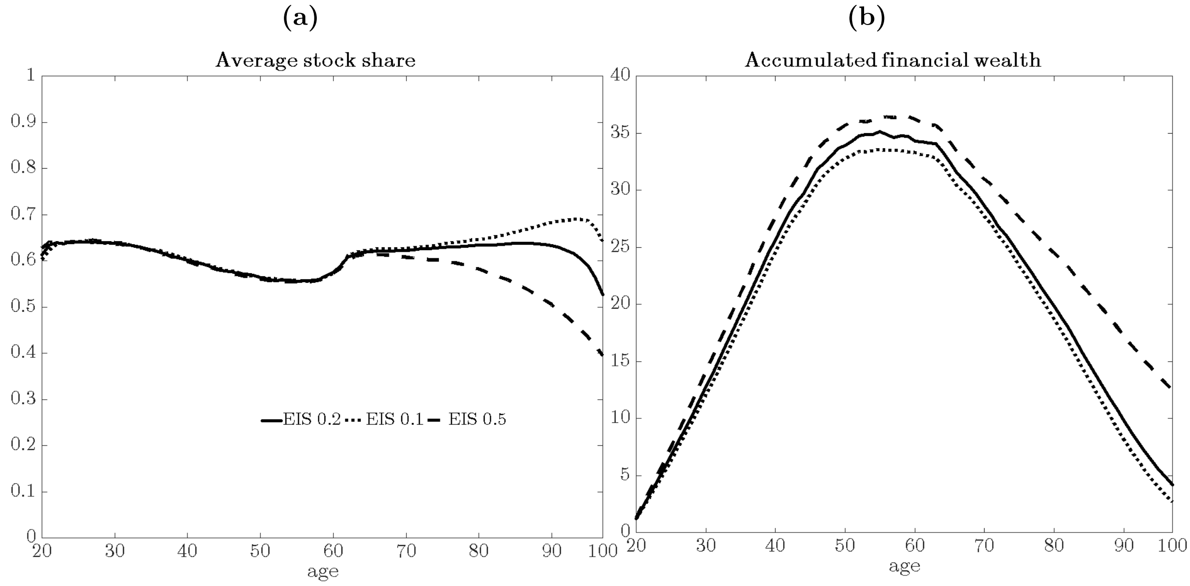


This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Positive correlation between labor income shocks and innovation to stock returns: $\rho_{sY} = 0.2.$: $\Psi_1 = 0$ and $\Psi_2 = 0.6$. Financial wealth is expressed in ten thousands of US dollars.

The second experiment implements a change in preferences that allows for inter-temporal correlation aversion (Bommier, 2007). With a power utility function, the worker is indifferent to positive or negative inter-temporal correlation of consumption (shocks). With Epstein-Zin preferences, the worker is averse to positive correlation when the coefficient of relative risk aversion is greater than the inverse of the elasticity of inter-temporal substitution (EIS). Adopting a recursive (Epstein-Zin) formulation for preferences and keeping the risk aversion parameter constant ($\rho_{sY} = 0.2$), we simulate the model with positive (EIS=0.5) and negative (EIS=0.1) correlation aversion, comparing the results with our baseline case of indifference (i.e., power utility, EIS=0.2). Figure 11 shows that aversion to positive correlation has a negligible effect during working years, while it causes a slower wealth decumulation and less risk taking during the retirement period, especially as death approaches. This finding is consistent with the known property that higher mortality risk magnifies the effects of inter-temporal correlation aversion (Bommier, 2013).

Overall, the preceding experiments confirm the robustness of the flattening of the life-cycle profile to changes in both hedging opportunities in the stock market and to the inter-temporal elasticity of substitution, pointing to the dominance of the personal disaster risk over second-order-moment effects.

Figure 11: Life-cycle profiles with unemployment traps: recursive preferences



This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Preferences over consumption are recursive, represented by an Epstein Zin utility function. Elasticity of intertemporal substitution varies from 0.1 to 0.5. : $\Psi_1 = 0$ and $\Psi_2 = 0.6$. Financial wealth is expressed in ten thousands of US dollars.

5.4 Discussion of further aspects

Positive work on life-cycle choices takes up the challenge of matching stylized empirical facts by appropriately calibrating parameters. For instance, a sufficiently high subjective discount factor prevents households from accumulating a counter-factually high level of wealth. It also reduces the stock market participation cost needed to match the observed non-participation patterns (e.g., see Fagereng, Gottlieb and Guiso, 2017). Similarly, allowing for subsistence consumption levels contains high saving rates, especially early in life when liquidity constraints are likelier to bind (see e.g. Hubbard, Skinner and Zeldes, 1995; DeNardi, French and Jones, 2010). While these patterns obtain also in our model in unreported calibrations, this paper focuses on the normative implications of personal disaster risk, in general, and long-term unemployment risk, in particular.

Let us note that while our calibrations consider income profiles of workers with high school education, long-term unemployment risk uniformly affects individuals with all education levels (e.g., see Mayer, 2014). Our model may however nest other types of uninsured personal disasters during working age, such as personal bankruptcy, that may impact more college-educated workers. The logic of our results indicate that types of bankruptcy provisions, such as Chapter 11 versus Chapter 7, should affect the proportional loss parameters, thereby impacting the skewness of labor income distributions and life-cycle choices.

Finally, our model shows that PDR changes the optimal consumption, savings and risk taking patterns during working life in partial equilibrium, without considering endogenous feed-backs deriving from aggregate patterns such demography (Poterba, 2001), the business cycle (Constantinides and Ghosh, 2014) and concurrent macroeconomics disasters (Barro, 2006). The logic of the model suggests that the possibility of states with lower future stock returns realizations corresponding to the rare unemployment trap, implying negative co-skewness in the joint distribution of labor income and stock returns, may add to reduced equity risk taking by the young - unless compensated by a highly attractive equity premium.[14]

6 Conclusions

This paper investigates the effects of PDR on life-cycle choices, introducing higher moments and non-linearities in the labor income process.

This methodological innovation enables new insights. Because of a small probability of experiencing persistent wage cuts earlier in their career, young workers face higher uncertainty concerning future income and social security pension levels than older workers. At the same time, young workers with continuous careers have larger human capital than older workers. When persistent wage cuts may be considerable, even if so unlikely to have a modest expected value, the first effect offsets the second and the optimal equity investment profile is relatively flat over working life. This result departs from the implications of previous models with linear income shocks highlighting the importance of unemployment traps, and individual disaster risk in general, for optimal portfolios.

Our calibrations indicate that a modified design for Target Date Funds implies an average 3% -10% increase in welfare-equivalent annual consumption, depending on the benchmark "age rule". Such more balanced design fits different kinds of workers, given the limited heterogeneity in life-cycle investments induced by the threat of personal disasters. More generally, our analysis indicates that the pattern of risk taking at different ages in Target Date Funds should be related to the share of uninsured long-term employment risk.

Our analysis implies that observed life-cycle profiles of household portfolios respond to the coverage of long-term unemployment risk both across cohorts and across states or countries. A similar pattern should also be visible in response to legal and regulatory cushioning of other personal ruins, such as personal bankruptcy, occurring during working years.

[14] In the US, the share of unemployed workers who are jobless for more than one year doubled during the recent crisis, reaching 24% of total unemployment in 2014.

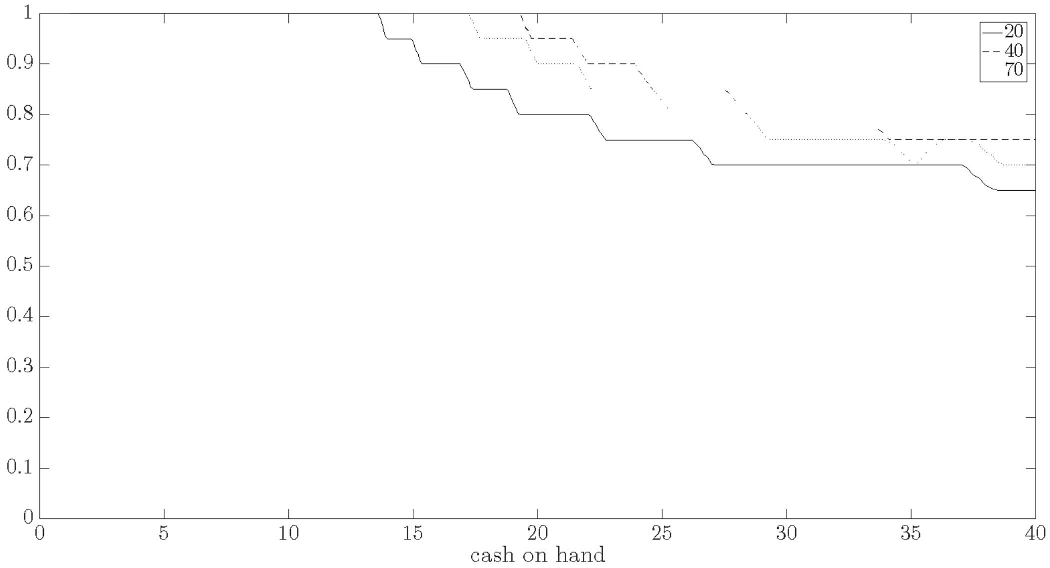
7 Appendix: Optimal policies

Figure 12 compares investors' optimal stock shares in the standard case of "*no unemployment risk*" (panel (a)) and in our preferred scenario with "*unemployment traps*" (panel (b)). In particular, the figure plots the optimal stock share as a function of cash on hand for an average level of the permanent labor income component of investors at three different ages (20, 40, and 70). In the case with no unemployment risk, standard life-cycle results are obtained. labor income acts as an implicit risk-free asset and affects the optimal portfolio composition depending on an investor's age and wealth. For example, at age 20 the sizable implicit holding of the risk-free asset (through human capital) makes it optimal for less-wealthy investors to tilt their portfolio towards the risky financial asset. Indeed, for a wide range of wealth levels, agents optimally choose to be fully invested in stocks. The optimal stock holding decreases with financial wealth because of the relatively lower implicit investment in (risk-free) human capital.

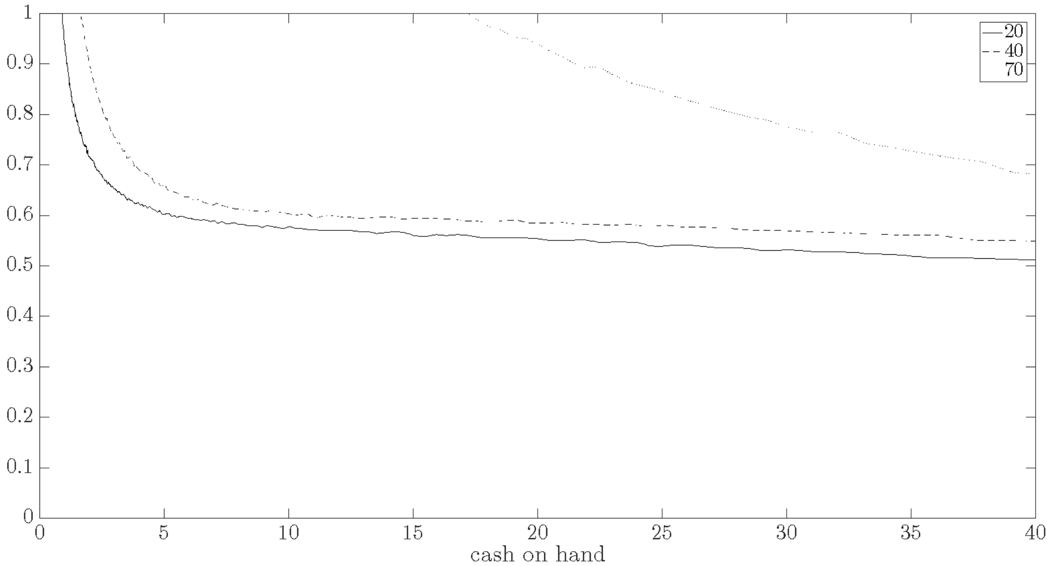
When the model is extended to allow for permanent effects of unemployment spells on labor income prospects at re-employment ("*unemployment traps*"), with the parameters governing the proportional erosion of permanent labor income set at $\Psi_1 = 0$ after one year of unemployment and at $\Psi_2 = 0.6$ after 2 years, the resulting policy functions are shifted abruptly leftward. The optimal stock share still declines with financial wealth but a 100% share of investment in stocks is optimal only at very low levels of wealth. In this PDR case, stock investment is considerably lower than in the case of no unemployment risk for almost all levels of financial wealth.

Figure 12: Policy functions

(a) No unemployment risk



(b) Unemployment traps



This figure shows the portfolio rules for stocks as a function of cash on hand for an average level of the stochastic permanent labor income component. The policies refer to selected ages: 20, 40, and 70. Panel (a) and (b) refer respectively to the cases with no unemployment risk and with unemployment traps. In the latter case, the parameters governing the during short-term and long-term unemployment spells are $\Psi_1 = 0$ and $\Psi_2 = 0.6$. Cash on hand is expressed in ten thousands of US dollars.

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