Asset Prices and No-Dividend Stocks*

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Abstract

We incorporate stocks that pay no dividends into an otherwise standard, parsimonious dynamic asset pricing framework. We find that such a simple feature leads to profound asset price implications, which are all supported empirically. In particular, we demonstrate that no-dividend stocks command lower mean returns, but also have higher return volatilities and higher market betas than comparable stocks that pay dividends. We also show that the presence of no-dividend stocks in the stock market leads to a lower correlation between the stock market return and aggregate consumption growth rate, a non-monotonic and even a negative relation between the stock market risk premium and its volatility, and a downward sloping term structure of equity risk premia. We provide straightforward intuition for all these results and the underlying economic mechanisms at play.

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Keywords: No-dividend stocks, dynamic asset pricing, incomplete information, risk premium, volatility, market beta, stock market correlation with consumption, market risk premium-volatility relation, term structure of equity premia.

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1 Introduction

There are many stocks do not pay dividends in financial markets. For example, in the US, the proportion of stocks not paying dividends (dividends or share repurchases) is reported to be around 65% (42%) in 2012 (Farre-Mensa, Michaely, and Schmalz (2014)). Much empirical evidence (highlighted below) reveals that there are considerable differences between the behavior of dividend-paying and non-dividend-paying stock returns: stocks that pay no dividends have lower average returns, but also have higher return volatilities and higher market betas than comparable stocks that pay dividends. Existing theoretical works (discussed below), however, do not reconcile with all this evidence. In this paper, we provide the first comprehensive analysis of the asset pricing effects of no-dividend stocks within a familiar consumption-based general equilibrium framework. The model supports all of the empirical evidence above and provides simple intuition for the underlying economic mechanisms at play. Since the stock market consists of both the dividend-paying and non-dividend-paying stocks, our model also generates notable implications for the aggregate stock market returns which are supported by empirical regularities, while offering novel straightforward intuition for them. In particular, we show that the presence of no-dividend stocks in the stock market leads to a lower correlation between the stock market return and aggregate consumption growth rate, possibly a negative relation between the stock market risk premium and its volatility, and a downward sloping term structure of equity risk premia.

When developing the model, we account for several noteworthy features of no-dividend stocks that matter for investors while pricing stocks. First, the absence of dividends introduces information incompleteness, and hence necessities the estimation of future dividends using other relevant fundamental information. Second, it also leads to additional uncertainty about the initiation date of their dividends. Third, no-dividend stocks do not contribute directly to aggregate consumption, and hence to the stochastic discount factor. We accordingly adopt a standard, workhorse, dynamic pure-exchange economy with two types of stocks, a normal stock, which pays dividends at all times, and a no-dividend stock, which pays dividends only after some random time in the future. Our focus is on the period prior to this random time. In the absence of dividends, we employ standard Bayesian filtering theory to estimate the future dividend distribution of the no-dividend stock using other relevant fundamental information and obtain an estimated pseudo-dividend process. This necessary

filtering process induces additional variation in the estimated pseudo-dividend by making it more volatile than the corresponding underlying process, which would have been used under complete information. Our model is parsimonious in the sense that there is a single investor with standard constant relative risk aversion (CRRA) preferences and the aggregate consumption growth rate has a constant mean and volatility. We obtain closed-form solutions for all quantities of interest.

Our model generates rich equilibrium implications. We first show that while the normal stock price is driven by its dividend, the no-dividend stock price by its estimated pseudo-dividend. More importantly, the presence of no-dividend stocks generates a novel spillover effect in that the expected dividend initiation time of no-dividend stocks affects the prices of dividend paying stocks, in addition to affecting their own prices. This is because the expected dividend initiation time is also the time when the stochastic discount factor shocks are anticipated to change, and what portion of the normal stock future dividends are expected to be discounted under the current stochastic discount factor matters for its price. Even though this spillover in equilibrium is due to a simple economic mechanism, to our best knowledge it is a new insight that has not been demonstrated previously in the literature.

Turning to stock price dynamics, we find that the mean return of the no-dividend stock is lower than that of a dividend-paying stock with the same underlying risk, consistent with the empirical evidence (Christie (1990), Naranjo, Nimalendran, and Ryngaert (1998), Fuller and Goldstein (2011)). This is because in the absence of its dividends the no-dividend stock price is driven by its estimated pseudo-dividend, which does not contribute directly to aggregate consumption, and hence comoves less with the aggregate consumption growth rate as compared to a dividend-paying stock with the same underlying risk. Therefore the investor requires a lower risk premium to hold the no-dividend stock in equilibrium, since in our model, as also in standard consumption-based asset pricing models, the stock risk premia are proportional to the covariance of stock returns with the aggregate consumption growth rate.

Furthermore, we demonstrate that the no-dividend stock commanding a lower mean return does not necessarily imply that its returns are less volatile or it has a lower market beta than the normal stock. On the contrary, we show that the no-dividend stock return is more volatile and has a higher market beta than that of a comparable dividend-paying

stock. This is due to the no-dividend stock price being driven by its estimated pseudo-dividend, and the estimation process, necessitated by the absence of dividends, inducing additional variability. This additional variation in the no-dividend stock return also makes its return contribute to and comove with the aggregate stock market return more as compared to a dividend-paying stock with the same underlying risk and relative size, leading to a higher market beta for it. These results are also consistent with the empirical evidence, which documents that stocks that pay no dividends have higher return volatility (Naranjo, Nimalendran, and Ryngaert (1998), Pástor and Veronesi (2003)), and higher market beta (Boudoukh, Michaely, Richardson, and Roberts (2007), Fuller and Goldstein (2011)) than comparable stocks that pay dividends. Moreover, we also offer an alternative interpretation of the no-dividend (normal) stocks in our model as the growth (value) stocks. This is because a typical growth stock is one with a low fundamental to price ratio, while also sharing the three key features of no-dividend stocks in our model. With this interpretation our findings are also consistent with the documented empirical regularities for growth and value stocks, since growth stocks have lower mean returns, higher return volatilities and higher market betas as compared to value stocks (Lettau and Wachter (2007)).

We further demonstrate the usefulness of our insights, by looking at the model implications for the aggregate stock market behavior. One notable implication is that the presence of no-dividend stocks in the stock market leads to a lower correlation between the aggregate stock market return and the aggregate consumption growth rate. This occurs because of the simple reason that the stock market consists of stocks that currently do not pay dividends and hence do not contribute to the current aggregate consumption, while contributing to the fluctuations in the aggregate stock market returns. Our simple numerical illustration also shows that the magnitude of this effect can be quite large, resulting in a very low correlation. This result may help reconcile the observed low correlation in the data (Cochrane and Hansen (1992), Campbell and Cochrane (1999), Cochrane (2005), Albuquerque, Eichenbaum, Luo, and Rebelo (2016), Heyerdahl-Larsen and Illeditsch (2017)).

Moreover, we show that the presence of no-dividend stocks in the stock market leads to a non-monotonic and even a negative relation between the conditional risk premium and volatility of the stock market. This is because the stock market risk premium is a weighted-average of the risk premia of stocks that make up the stock market. With no-dividend stocks,

which command low risk premia but high volatility, being part of the stock market, the stock market risk premium is non-monotonically related to, and in particular is decreasing in its volatility for sufficiently high relative-size of the no-dividend stocks. This result sheds light on the decidedly mixed vast empirical findings on this relation. For example, numerous works find the relation between the stock market conditional risk premium and volatility to be negative (Campbell (1987), Glosten, Jagannathan, and Runkle (1993), Whitelaw (2000), Harvey (2001), Brandt and Kang (2004)), while many others, consistent with the basic intuition, find it to be positive (French, Schwert, and Stambaugh (1987), Scruggs (1998), Ghysels, Santa-Clara, and Valkanov (2005), Bali and Peng (2006), Guo and Whitelaw (2006), Ludvigson and Ng (2007)). Rossi and Timmermann (2010), on the other hand, find a non-monotonic relation between the conditional risk premium and volatility by showing a negative relation for high levels of volatility and a positive relation for low and medium levels of volatility. They argue that the lack of consensus in the earlier empirical literature may be due to this non-monotonic relation. In line with the evidence in Rossi and Timmermann (2010), our model also predicts a negative relation for high levels of volatility, and either a positive or negative relation (depending on the relative-size of the no-dividend stocks) for low levels of volatility.

Finally, we show that the presence of no-dividend stocks can lead to a downward sloping term structure of equity risk premia by showing that short-term assets, claims to short-term aggregate dividends, tend to command a higher mean return than the stock market. This is because a short-term asset is more like a normal stock since the no-dividend stock begins paying out dividends only after some time (which may even be after the short-term asset maturity). Since the mean return of the normal stock is higher than that of a no-dividend stock with the same underlying risk, this leads to a higher mean return for the short-term asset as compared to the stock market. This result is consistent with the findings of van Binsbergen, Brandt, and Koijen (2012), who study a claim on the dividends of the S&P 500 index in the near future, i.e., the short-term asset, and find that the short-term asset commands a higher average return than the underlying index.

Related works that study no-dividends stocks from an asset pricing perspective are Pástor and Veronesi (2003) and Choi, Johnson, Kim, and Nam (2013).[1] Our methodology and

[1]The corporate finance literature, on the other hand, primarily focuses on the firms' dividend policies and

modeling of no-dividend stocks differ considerably from both these works, and hence do many of our results, even though each paper contains one result similar to one of our main results. In particular, Pástor and Veronesi study the effects of parameter uncertainty about a firm's average profitability and primarily focus on its market-to-book ratio, and find that firms that pay no dividends have more volatile returns due to learning effects, a finding similar to ours. However, differently from our setting, in their framework the stochastic discount factor is specified exogenously (hence whether a firm pays dividends or not has no effect on it) and there is no consideration for the additional uncertainty about the dividend initiation date. Therefore, in their framework, it is not possible to obtain our implications for the stock price spillovers, stock mean returns, market beta, and the aggregate stock market returns. On the other hand, Choi, Johnson, Kim, and Nam consider a production economy in which managers choose the firm payout policy while facing non-convex costs in adjusting dividends and investments. They solve their model numerically and show that firms with a low probability of paying dividends in the near term command risk premia close to zero, a result similar to our finding that the no-dividend stock mean return is lower than that of a dividend-paying stock. Even though our framework differs from theirs in several major aspects, one key difference is that we explore the information incompleteness necessitated by the absence of dividends and show how it leads to higher return volatility and market betas for no-dividend stocks, as well as providing implications of these for the aggregate stock market returns.

Our work is also related to the literature on the correlation between the stock market return and the aggregate consumption growth rate. As discussed earlier, this correlation appears to be weak in the data. On the theory side, leading consumption-based asset pricing models, such as habit-formation model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), and rare disaster models of Rietz (1988), Barro (2006) all have difficulty in reconciling this evidence. Recently, Albuquerque, Eichenbaum, Luo, and Rebelo (2016) and Heyerdahl-Larsen and Illeditsch (2017) reconcile with this finding by developing consumption-based models with a single stock and demand shocks that arise from the time variation in investors' rate of time preference, as opposed to having supply

consider issues related to the role of taxes, life-cycle of firms, catering to investor demands and asymmetric information (e.g., signaling and agency problems) (see Farre-Mensa, Michaely, and Schmalz (2014) for a recent survey).

shocks in standard models. We complement this literature by offering an alternative, simple, but novel, explanation that may play a role in explaining this apparent low correlation.

This paper is also related to the vast literature studying the relation between the conditional risk premium and volatility of the stock market. As discussed earlier, there are numerous works that empirically study this relation, but the conclusions on the sign of the relation are mixed. On the theory side, a number of works, using a single stock setup, demonstrate that a non-monotonic and a negative relation can arise in equilibrium if there is time-variation in state variables or investment opportunities (Abel (1988), Backus and Gregory (1993), Veronesi (2000), Whitelaw (2000)). Our contribution here is to illustrate that, using a simple multiple-stocks setup, a non-monotonic and even a negative relation can arise in equilibrium for an alternative, simple reason, that the stock market also consists of no-dividend stocks, which have relatively low mean return but high return volatility.

Finally, this paper is related to the recently growing literature on the shape of the term structure of equity risk premia. In this literature, van Binsbergen, Brandt, and Koijen (2012) show that this term structure is downward sloping, which is somewhat puzzling since it goes against the implications of numerous leading asset pricing models. Indeed, van Binsbergen, Brandt, and Koijen show that the term structure of equity risk premia is upward sloping in the habit-formation model of Campbell and Cochrane (1999) and the long-run risk model of Bansal and Yaron (2004), and it is flat in the rare disaster model of Gabaix (2012). They also show that in the model of Lettau and Wachter (2007) this term structure is downward sloping, however they argue that since the stochastic discount factor is exogenously specified in Lettau and Wachter it lacks the necessary micro foundations to understand the exact nature of the economic shocks.[2] Several recent theoretical works generate a downward sloping term structure of equity risk premia via alternative mechanisms (Belo, Collin-Dufresne, and Goldstein (2015), Croce, Lettau, and Ludvigson (2015), Eisenbach and Schmalz (2016), Hasler and Marfè (2016)). We complement this literature by demonstrating that a downward sloping term structure can easily arise when the stock market consists of stocks that currently

[2]In particular, Lettau and Wachter primarily focus on the quantitative implications of their model for the value premium, and considerably differs from our model in terms of focus, economic mechanisms, and consequently results. Moreover, our framework allows us to consider specific issues related to no-dividend stocks, such as information incompleteness, uncertainty about the future dividend initiation date, which are not possible to study under their setting.

do not pay dividends.

The remainder of the paper is organized as follows. Section 2 presents our model with no-dividend stocks. Section 3 provides our results on the stock prices and their dynamics, while Section 4 on the aggregate stock market behavior. Section 5 concludes. Appendix contains the proofs.

2 Economy with No-Dividend Stock

In this Section, we incorporate no-dividend stocks into a familiar dynamic asset pricing environment. From the viewpoint of investors there are several noteworthy features of no-dividend stocks which ought to be incorporated while pricing stocks. First, the absence of dividends introduces information incompleteness, and hence necessities the estimation of future dividends using other relevant fundamental information. Second, it also leads to additional uncertainty about the initiation date of their dividends. Third, no-dividend stocks do not contribute directly to aggregate consumption, and hence to the stochastic discount factor. In the following, we provide the details of the model we develop with above features.

2.1 Securities Market

We consider a continuous-time pure-exchange economy with infinite horizon. Available for trading are two types of risky stocks, each in positive net supply of one unit, and a riskless bond that is in zero net supply. The first type, which we refer to as the *normal stock*, pays out dividends D_1 at all times with dynamics

$$\frac{dD_{1t}}{D_{1t}} = \mu_1 dt + \sigma_1 d\omega_{1t}, \tag{1}$$

where μ_1 and σ_1 are constants representing the mean and volatility of the stock dividend growth rate, and ω_1 is a Brownian motion. The normal stock price S_1 is to be determined endogenously in equilibrium.

The second stock type, which we refer to as the *no-dividend stock*, pays out dividends D_2

only after a random time τ with dynamics

$$\frac{dD_{2t}}{D_{2t}} = \mu_2 dt + \sigma_2 d\omega_{2t}, \tag{2}$$

where μ_2 and σ_2 are constants representing the mean and volatility of the stock dividend growth rate. The Brownian motion ω_2 is possibly correlated with ω_1 with the correlation coefficient $\rho_{12} \in (-1, 1)$. The dividend initiation time has an independent exponential distribution, $\tau \sim \text{Exp}(\lambda_2)$, where the parameter $1/\lambda_2$ represents the expected dividend initiation time. Since the no-dividend stock does not pay dividends prior to τ, we refer to the unobservable process D_2 with dynamics (2) during this period $t < \tau$ as *pseudo-dividends* since its value at time τ is the initial dividend. The no-dividend stock price S_2 is to be determined endogenously in equilibrium. We refer to the period prior to τ when only the normal stock pays dividends as the *main period* and the period after τ when all the stocks pay dividends as the *benchmark period.* In what follows, we denote the benchmark period quantities with an upper bar ($\bar{}$).

2.2 Absence of Dividends, Incomplete Information and Learning

The absence of dividends on the no-dividend stock during the main period $t < \tau$ introduces information incompleteness while estimating the distribution of the future dividends, an issue that does not exist for the normal stock.[3] This necessities using other relevant observable (albeit noisier) information for estimating the future dividends. Towards that, we consider a fundamental news process F_2 that contains valuable information about the future dividends of the no-dividend stock.[4] Since the no-dividend stock dividends D_2 could in principle start very far in the future and the fundamental news process F_2 needs to contain valuable information

[3]To see this note that the dividend level of the no-dividend stock at a future time $u \geq \tau$ can be written as $\ln D_{2u} = \ln D_{2t} + (\mu_2 - \frac{1}{2}\sigma_2^2)(u - t) + \sigma_2(\omega_{2u} - \omega_{2t})$. Hence while forming a rational estimate of future dividends, in addition to the known parameters μ_2 and σ_2, the investor needs to know the current level D_{2t}, which is not available during the main period. The future estimates of D_2 are then used in the determination of the stock price in equilibrium via $S_{2t} = \text{E}_t\left[\int_\tau^\infty \xi_{t,u} D_{2u} du\right]$, where $\xi_{t,u}$ is the stochastic discount factor.

[4]For simplicity, we assume there is only one fundamental news process for the stock as this is sufficient for our purposes. In reality, there are numerous financial and accounting news series, such as cash-flows and earnings news/announcements, which contain valuable information about a stock's future prospects in the absence of its dividends. Such series would be good candidates for our fundamental process.

about dividends, we assume a long-run dependency between D_2 and F_2 by imposing simple mean-reverting (stationary) dynamics for their logarithmic difference as follows

$$d\left(\ln F_{2t} - \ln D_{2t}\right) = \kappa_2 \left[\zeta_2 - \left(\ln F_{2t} - \ln D_{2t}\right)\right] dt + \nu_2 d\omega_{2t}^*, \tag{3}$$

where $\kappa_2 > 0$, ζ_2, and ν_2 are constants representing the mean-reversion, long-run mean, and the volatility of $\ln F_{2t} - \ln D_{2t}$, respectively, and ω_2^* is a Brownian motion independent of all other Brownian motions introduced earlier. In economic terms, the long-run dependency (the cointegration between $\ln F_{2t}$ and $\ln D_{2t}$) is equivalent to assuming neither the fundamental news process F_2 nor the dividend D_2 grow to be infinitely larger than the other in the long-run.[5] Note that the dynamics of the fundamental news process F_2 itself is readily deduced from the dynamics (2)–(3) as

$$d \ln F_{2t} = (\mu_2 - \tfrac{1}{2}\sigma_2^2 + \kappa_2\zeta_2 + \kappa_2 \ln D_{2t} - \kappa_2 \ln F_{2t})dt + \sigma_2 d\omega_{2t} + \nu_2 d\omega_{2t}^*. \tag{4}$$

As (4) reveals, the mean growth rate of the observable fundamental news process F_2 contains information about the unobserved pseudo-dividend D_2 during the main period $t < \tau$.[6]

In the absence of dividends on the no-dividend stock, we employ the standard Bayesian filtering theory to estimate the unobserved pseudo-dividend D_2 during the main period $t < \tau$. We assume a normally distributed prior for the (logarithmic) pseudo-dividend with mean $\widehat{\ln D}_{20}$ and variance V_{20}. The Bayesian updating rule then implies that the time-t posterior

[5]For instance, in the simpler special case of $\zeta_2 = 0$, the expected long-term (logarithmic) fundamental news gives the expected long-term (logarithmic) dividend, that is, $\lim_{u\to\infty} \mathrm{E}_t\left[\ln F_{2u}\right] = \lim_{u\to\infty} \mathrm{E}_t\left[\ln D_{2u}\right]$. In general, the long-term relation between the growth rates of the fundamental news process F_2 and the dividend D_2 in our model is in line with the behavior of the steady-state of the Gordon growth model in which dividends, earnings, and book equities all grow at the same rate under the so-called clean-surplus accounting (Campbell (2017, p. 131)).

[6]Since the fundamental news process is assumed to exist irrespective of whether the stock currently pays dividends or not, for symmetry, we may also consider a corresponding fundamental news process F_1 for the normal stock having a similar structure with dynamics

$$d \ln F_{1t} = (\mu_1 - \tfrac{1}{2}\sigma_1^2 + \kappa_1\zeta_1 + \kappa_1 \ln D_{1t} - \kappa_1 \ln F_{1t})dt + \sigma_1 d\omega_{1t} + \nu_1 d\omega_{1t}^*,$$

where $\kappa_1 > 0$, ζ_1, and ν_1 are constants representing the mean-reversion, long-run mean, and the volatility of $\ln F_{1t} - \ln D_{1t}$, respectively, and ω_1^* is a Brownian motion independent of all other Brownian motions introduced earlier. However, as there is no information incompleteness about the normal stock future dividend distribution, the information contained in the fundamental news F_1 is redundant in our analysis.

distribution conditional on the information set $\mathcal{G}_t = \sigma\{(D_{1s}, F_{2s}) : 0 \leq s \leq t\}$ is also normally distributed as presented in the following Lemma 1.

Lemma 1. *Let the prior of the (logarithmic) pseudo-dividend* $\ln D_2$ *at time* 0 *be normally distributed with mean* $\widehat{\ln D}_{20}$ *and variance* V_{20}*. Then the posterior of* $\ln D_2$ *at time* $t > 0$ *conditional on the information* $\mathcal{G}_t = \sigma\{(D_{1s}, F_{2s}) : 0 \leq s \leq t\}$ *is also normally distributed with mean* $\widehat{\ln D}_{2t}$ *and variance* V_{2t} *such that the mean estimate of the pseudo-dividend* $\widehat{D}_{2t} = E[D_{2t}|\mathcal{G}_t] = \exp(\widehat{\ln D}_{2t} + \frac{1}{2}V_{2t})$*, henceforth the estimated pseudo-dividend, satisfies the dynamics*

$$\frac{d\widehat{D}_{2t}}{\widehat{D}_{2t}} = \mu_2 dt + \rho_{12}\sigma_2 d\omega_{1t} + \frac{(1-\rho_{12}^2)\sigma_2^2 + \kappa_2 V_{2t}}{\sqrt{(1-\rho_{12}^2)\sigma_2^2 + \nu_2^2}} d\widehat{\omega}_{2t}, \tag{5}$$

$$dV_{2t} = -\left[\frac{((1-\rho_{12}^2)\sigma_2^2 + \kappa_2 V_{2t})^2}{(1-\rho_{12}^2)\sigma_2^2 + \nu_2^2} - (1-\rho_{12}^2)\sigma_2^2\right] dt, \tag{6}$$

where $\widehat{\omega}_2$ *is a* $\mathcal{G}_t$*-Brownian motion independent of the Brownian motion* ω_1*.*

The posterior variance V_{2t} is deterministic and converges to its constant non-zero steady-state value, denoted by $V_{2\infty}$, in the long-run (see (A.7) in the Appendix). To ensure that learning is optimal in the sense that it leads to more precise estimates over time, we set the exogenous prior variance V_{20} to be greater than the steady-state posterior variance, that is, $V_{20} > V_{2\infty}$.[7] One notable implication of Lemma 1 is that the estimation, necessitated by the absence of dividends, induces additional variability in the estimated pseudo-dividend $\widehat{D}_2$ dynamics (5), making it more volatile than the underlying pseudo-dividend D_2 (see also (A.9)–(A.10) in the Appendix). This is because there is an additional uncertainty about the mean estimate of the pseudo-dividend $\widehat{D}_2$ captured by the posterior variance V_{2t}, which would not be present had the dividends been observable. This additional uncertainty amplifies the shocks to the fundamental news process during the estimation and leads to a more volatile estimate of the pseudo-dividend.

[7]The steady-state posterior variance being less than the prior variance not only makes sense economically, but is also consistent with models of learning about a constant parameter. In these models, the posterior variance declines over time and converges to zero in the steady-state since the investor eventually learns about the true parameter value (e.g., Brennan (1998), Pástor and Veronesi (2003), Cvitanić, Lazrak, Martellini, and Zapatero (2006), Collin-Dufresne, Johannes, and Lochstoer (2016)). However, differently from these models of parameter uncertainty, in our setting the investor learns about a stochastic process, and moreover stops learning at a (random) time τ once the dividends are initiated for the first time, since this leads to complete information.

Remark 1 (Alternative fundamental news process). We specify the fundamental news process (4) so that there is a long-run dependency between between D_2 and F_2 for the economic reasons discussed above. However, this specification is not necessary for our mechanism and results, which also obtain under an alternative familiar, but somewhat less plausible in our setting, specification of fundamental news that takes the form of a "signal plus noise" process

$$d \ln F_{2t} = \ln D_{2t} dt + \nu_2 d\omega^*_{2t}. \tag{7}$$

This is because, under this alternative formulation, the investor again uses noisier information when estimating the distribution of future dividends, and this procedure induces additional variability in the estimated pseudo-dividend for the same reasons as discussed above after Lemma 1. Therefore all our subsequent results remain valid with this specification. We provide the details of the analysis with this alternative formulation in the Appendix.

2.3 Preferences and Endowments

There is a single investor in the economy who chooses a non-negative consumption C and a portfolio strategy in the two risky stocks and riskless bond so as to maximize her CRRA preferences from intertemporal consumption

$$u(C_t, t) = e^{-\beta t} \frac{C_t^{1-\gamma}}{1-\gamma}, \tag{8}$$

where β is her rate of time preference, γ is the relative risk-aversion coefficient, subject to the appropriate budget constraint. The single investor is endowed with all the wealth in the economy, which is a claim against the exogenously specified aggregate consumption (endowment) Y with dynamics at all times given by

$$\frac{dY_t}{Y_t} = \mu_Y dt + \sum_n \alpha_n \left(\frac{dD_{nt}}{D_{nt}} - \mu_n dt \right), \tag{9}$$

where $\mu_Y = \sum_n \alpha_n \mu_n$ with the summation $\sum_n$ taken only over the stocks that currently pay dividends, and α_n are the appropriate constants in each period representing the sensitivities of the aggregate consumption growth rate to each dividend shock. Economically these

sensitivities can be thought of as the average relative share of dividends in the aggregate consumption in each period.

As (9) illustrates, the fluctuations in aggregate consumption are driven by current dividend shocks. In particular, during the benchmark period $t \geq \tau$, a positive (negative) shock to any dividend D_1 or D_2 increases (decreases) the aggregate consumption. The magnitude of the increase/decrease is determined by the benchmark period sensitivity parameters, α_1 and α_2, respectively. In contrast, during the main period $t < \tau$, the no-dividend stock pays no dividends, hence its associated sensitivity is zero, and the shocks in aggregate consumption only arise from the shocks to the normal stock dividends with the sensitivity $\alpha_1 + \alpha_2 \leq 1$.[8] In sum, we can rewrite the aggregate consumption dynamics explicitly in terms of the constant sensitivities and the dividend dynamics (1)–(2) as

$$\frac{dY_t}{Y_t} = \begin{cases} (\alpha_1 + \alpha_2)\mu_1 dt + (\alpha_1 + \alpha_2)\sigma_1 d\omega_{1t}, & t < \tau, \\ (\alpha_1\mu_1 + \alpha_2\mu_2)dt + \alpha_1\sigma_1 d\omega_{1t} + \alpha_2\sigma_2 d\omega_{2t}, & t \geq \tau. \end{cases} \tag{10}$$

As a final note, in our specification the aggregate consumption Y_t does not necessarily coincide with the aggregate dividends $D_{1t} + D_{2t}\mathbf{1}_{\{t\geq\tau\}}$, where their difference can be thought of as the investor's implicit non-financial income (such as labor and government transfers). This specification is not only consistent with the data, since the aggregate dividend is only a fraction of the aggregate consumption (Santos and Veronesi (2006)), but is also present in numerous asset pricing models, including Campbell and Cochrane (1999), Brennan and Xia (2001), Bansal and Yaron (2004), Barberis, Greenwood, Jin, and Shleifer (2015).

Remark 2 (Stock dividends and output). Our specification of the aggregate consumption dynamics is in the spirit of Lucas (1978), in which stocks are claims to the trees whose output (dividends) are perishable and must be consumed in that period. This way if a stock

[8] We equate the sum of the sensitivities across periods, so as to capture the feature that the sensitivity of the aggregate consumption growth rate to the aggregate dividend is the same in both periods. This assumption is also motivated by and is consistent with models in which aggregate consumption equals to the sum of multiple dividends in which introducing an additional dividend reduces the relative shares (sensitivities) of existing dividends while their total sum remains 1 (see, for example, Martin (2013)). Moreover, this way we also ensure that when the mean dividend growth rates are the same, $\mu_1 = \mu_2$, the aggregate consumption mean growth rate remains the same for both periods. We also note the slight abuse of notation that rather than introducing a new notation for the main period normal stock sensitivity, throughout the paper we use the sum of the benchmark period sensitivities $\alpha_1 + \alpha_2$ as discussed above.

currently does not pay dividends, it does not contribute to the current aggregate consumption. We capture this economic mechanism, which is key to our analysis, in a tractable way through the constant sensitivities, which in turn lead to constant mean and volatility of the aggregate consumption growth rate in each period, as (10) illustrates. This simplifies the analysis leading to the stock prices being as in the Gordon growth model during the benchmark period, as discussed in Section 3.

In a Lucas-type framework such as ours one cannot distinguish between a firm's output and dividend since all the firm's output is paid out as dividends. We here provide an alternative interpretation of the no-dividend stock in terms of its output process containing a single regime switch. That is, one can also interpret the no-dividend stock dividends during the benchmark period (new-regime) as the firm's additional output over and above its output in the main period (old-regime), which is normalized to zero in our model. With this interpretation all the three key features we discuss for no-dividend stocks are still relevant since the additional output would still alter the shocks of aggregate consumption, and this would occur at a random time, and the information incompleteness about the new-regime dynamics again necessities the estimation of future additional output using other relevant fundamental information.

3 Equilibrium Stock Prices and Dynamics

In this Section, we investigate how the presence of no-dividend stocks affect equilibrium stock prices and their dynamics. In particular, we first demonstrate that their presence generates a novel spillover effect in that the expected dividend initiation time of the no-dividend stocks affects the prices of normal stocks. We then show that the mean return of the no-dividend stock is lower than that of a normal stock with the same underlying risk, consistent with the empirical evidence. We demonstrate that the no-dividend stock commanding a lower mean return does not necessarily imply that its returns are less volatile nor it has a lower market beta than the normal stock. On the contrary, we show that the no-dividend stock return is more volatile and has a higher market beta than that of a comparable normal stock, also consistent with the empirical evidence.

Equilibrium in our economy is defined in a standard way. The economy is said to be in equilibrium if the equilibrium consumption, portfolio strategy, stock and bond prices are such that the investor chooses her optimal consumption and portfolio strategy, and the good, stocks and bond markets clear. The tractability of our model leads to closed-form solutions for stock prices and their dynamics for both periods, as presented in Propositions 1–4.[9] The normal and no-dividend (instantaneous) stock mean returns r_1 and r_2 are defined as $r_{1t} = \mathrm{E}_t\left[(dS_{1t} + D_{1t}dt)/S_{1t}dt\right]$ and $r_{2t} = \mathrm{E}_t\left[(dS_{2t} + D_{2t}\mathbf{1}_{\{t\geq\tau\}}dt)/S_{2t}dt\right]$.

3.1 Stock Prices

Proposition 1 (**Equilibrium stock prices**). *The equilibrium normal and no-dividend stock prices during the benchmark period $t \geq \tau$ are given by*

$$\bar{S}_{1t} = \frac{1}{\bar{r}_1 - \mu_1} D_{1t}, \tag{11}$$

$$\bar{S}_{2t} = \frac{1}{\bar{r}_2 - \mu_2} D_{2t}, \tag{12}$$

and during the main period $t < \tau$ by

$$S_{1t} = \frac{1}{\bar{r}_1 - \mu_1} \frac{\bar{r}_1 - \mu_1 + \lambda_2}{r_1 - \mu_1 + \lambda_2} D_{1t}, \tag{13}$$

$$S_{2t} = \frac{1}{\bar{r}_2 - \mu_2} \frac{\lambda_2}{r_2 - \mu_2 + \lambda_2} \widehat{D}_{2t}, \tag{14}$$

where the equilibrium stock mean returns $\bar{r}_n$ and r_n for $n = 1, 2$, are as in Proposition 2 and the estimated pseudo-dividend $\widehat{D}_{2t}$ is as in Lemma 1.

Consequently, during the main period, all else being fixed,

i) The normal stock price is decreasing in the expected dividend initiation time $1/\lambda_2$ of the no-dividend stock when $r_1 > \bar{r}_1$, and is increasing otherwise.

ii) The no-dividend stock price is decreasing in its expected dividend initiation time $1/\lambda_2$.

[9] The usual parameter restrictions that are necessary to ensure that the stock prices are well defined and finite in our model are provided in the proof of Proposition 1 in the Appendix.

During the benchmark period $t \geq \tau$ when all stocks pay dividends, each equilibrium stock price is driven by its current dividends D_{nt}, as in standard asset pricing models. In our setup, these prices follow the simple Gordon growth model with the constant discount terms given by the stock mean returns net of dividend growth rates. During the main period $t < \tau$ when only the normal stock pays dividends, the equilibrium stock prices still have simple structures, though differ in two major ways. First, while the normal stock price is still driven by its current dividend D_{1t}, the no-dividend stock price is now driven by the estimated pseudo-dividend $\widehat{D}_{2t}$ in the absence of its dividends. Second, both stock prices now have additional terms adjusting for the change in the equilibrium stochastic discount factor shocks at the random time τ, due to the change in aggregate consumption shocks.

The new additional terms during the main period reveal that the no-dividend stock's expected dividend initiation time $1/\lambda_2$ not only affects its own price S_{2t} but also spills over to the normal stock price S_{1t}. This is because the expected dividend initiation time is also the time when the aggregate consumption, and hence the stochastic discount factor, shocks are anticipated to change. Since stock prices are the total expected discounted future dividends, what portion of the normal stock future dividends are expected to be discounted under the current stochastic discount factor matters for its price. This spillover effect is noteworthy since it is not present during the benchmark period, in which each stock price depends only on its own parameters, apart from the obvious indirect dependence through its endogenous equilibrium mean return (Proposition 2). Moreover, even though the no-dividend stock's expected dividend initiation time spilling over to the other stock prices is due to a simple economic mechanism, to the best of our knowledge this is a novel result and has not been demonstrated previously in the literature.

Consequently, Property (i) reveals that the normal stock price decreases in the no-dividend stock's expected dividend initiation time $(1/\lambda_2)$ when its mean return is higher than that in the benchmark period, and increases otherwise. This condition arises because an increase in the expected dividend initiation time increases (decreases) the portion of the normal stock price arising from the value of dividends during the main (benchmark) period with the discounting at r_1 $(\bar{r}_1)$. Therefore, if the main period mean return is higher than that in the benchmark period, that increased portion is discounted at a higher rate and this leads to a lower normal stock price. This result can also be seen in the two polar cases of the

expected dividend initiation time. In the polar case of $1/\lambda_2 \to \infty$, the no-dividend stock is never expected to pay any dividends. In this case, we essentially have a single dividend-paying stock economy at all times, and our stock price expression (13) for the main period simply becomes $S_{1t} = D_{1t}/(r_1 - \mu_1)$, which again follows the simple Gordon growth model. In the opposite polar case of $1/\lambda_2 \to 0$, the no-dividend stock is expected to start paying dividends at this instant. In this case, we essentially have a two dividend-paying stock economy at all times, and our stock price expression (13) for the main period becomes $S_{1t} = D_{1t}/(\bar{r}_1 - \mu_1)$, as in the benchmark period. Hence, depending on whether the mean return in the single dividend-paying stock economy r_1 is higher or lower than that of in the two dividend-paying stock economy $\bar{r}_1$, the normal stock price in one polar case can be higher or lower than the other.

Property (ii), on the other hand, illustrates that the no-dividend stock price always decreases in its expected dividend initiation time, all else being fixed. This is to be expected since, all else being fixed, an increase in its expected dividend initiation time means that the stock is expected to pay dividends for a shorter period of time and this leads to a lower price. Again, this result can be seen immediately in the two polar cases. In the polar case of $1/\lambda_2 \to \infty$, the no-dividend stock is never expected to pay any dividends, and our stock price expression (14) for the main period simply becomes 0 as expected. In the opposite polar case of $1/\lambda_2 \to 0$, the no-dividend stock is expected to start paying dividends at this instant, and our stock price expression (14) for the main period gives the price just prior to observing the first dividend rate and hence becomes $S_{2t} = \widehat{D}_{2t}/(\bar{r}_2 - \mu_2)$.[10]

[10]At this point, we believe it is helpful to highlight that Proposition 1 properties (i)–(ii) are ceteris paribus (all else being fixed) results and do not necessarily imply that one stock price is higher than the other. In particular, for suitable parameter choices of the dividend processes (1)–(2), either equilibrium price ratio, S_{1t}/D_{1t} or $S_{2t}/\widehat{D}_{2t}$, can be greater, less, or equal to other. Indeed, it is straightforward to show that when both stocks have the same underlying risk, $\sigma_1 = \sigma_2$, there exist a unique $\mu_2^* > \mu_1$ as a function of the expected dividend initiation time $1/\lambda_2$ such that both stocks have the same price ratios during the main period. This also illustrates that our model does not necessarily contradict the classic Miller and Modigliani (1961) finding that a firm's dividend policy does not affect its value.

3.2 Stock Mean Returns

Proposition 2 (**Equilibrium stock mean returns**). *The equilibrium mean returns of the normal and no-dividend stocks during the benchmark period $t \geq \tau$ are given by*

$$\bar{r}_1 = \bar{r} + \gamma(\alpha_1\sigma_1^2 + \alpha_2\rho_{12}\sigma_1\sigma_2), \tag{15}$$

$$\bar{r}_2 = \bar{r} + \gamma(\alpha_1\rho_{12}\sigma_1\sigma_2 + \alpha_2\sigma_2^2), \tag{16}$$

and during the main period $t < \tau$ by

$$r_1 = r + \gamma(\alpha_1 + \alpha_2)\sigma_1^2, \tag{17}$$

$$r_2 = r + \gamma(\alpha_1 + \alpha_2)\rho_{12}\sigma_1\sigma_2, \tag{18}$$

where the equilibrium interest rates in the benchmark period $\bar{r}$ and the main period r are

$$\bar{r} = \beta + \gamma(\alpha_1\mu_1 + \alpha_2\mu_2) - \tfrac{1}{2}\gamma\,(\gamma+1)\,(\alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2 + 2\alpha_1\alpha_2\rho_{12}\sigma_1\sigma_2), \tag{19}$$

$$r = \beta + \gamma(\alpha_1 + \alpha_2)\mu_1 - \tfrac{1}{2}\gamma\,(\gamma+1)\,(\alpha_1+\alpha_2)^2\sigma_1^2. \tag{20}$$

Consequently, during the main period, the mean return of the no-dividend stock is lower than that of a normal stock with the same underlying risk $\sigma_1 = \sigma_2$.

Equilibrium stock mean returns consist of the interest rate (the first terms) and the risk premium (the second terms).[11] The stock risk premia in our model are proportional to the covariance of stock returns with the aggregate consumption growth rates, as in standard

[11] We do not focus on the interest rate behavior in our analysis since it is a common component across stocks and our main focus is on the differing behavior of mean returns across the two types of stocks. Moreover, as we show in the Appendix, the equilibrium mean returns for the main period (17)–(18) only take into account of the continuous covariance between the stock returns and the state price density, without taking into account of their possible discrete covariance at time τ since it is zero. This is because even though there are discrete changes in stock prices at the random time τ, there are no associated discrete changes in the aggregate consumption levels, and hence no discrete changes in the state price density, that is $\xi_{\tau-} \equiv \lim_{t\to\tau}\xi_t = \xi_\tau$ leading to a zero risk premium for the discrete covariance: $\lambda_2\,(\xi_\tau/\xi_{\tau-} - 1)\,(S_{n\tau}/S_{n\tau-} - 1) = 0$ for $n = 1, 2$. Moreover, by not taking into account of discrete change components, the main period mean returns we report in Proposition 2, as well as the main period volatilities in the following Proposition 3, are the quantities comparable to those reported in empirical studies, which are based on no-dividend stock portfolios constructed and rebalanced frequently to ensure they do not contain any dividend paying stocks recently.

consumption-based asset pricing models. During the benchmark period when all stocks pay dividends, each stock risk premium is made up of a variance component and a covariance component. The variance component is due to the fact that each stock dividend, which drives the stock price, contributes directly to the current aggregate consumption with the sensitivity α_n, thereby requiring the risk premium $\gamma\alpha_n\sigma_n^2$. The covariance component is due to the fact that each stock dividend (potentially) comoves with the other, thereby requiring the risk premium $\gamma\alpha_n\rho_{12}\sigma_1\sigma_2$. However, during the main period when only the normal stock pays dividends, each stock risk premium has only one component. For the normal stock this is the variance component since its dividend, which drives its price, is the sole contributor to the aggregate consumption. For the no-dividend stock it is the covariance component since its estimated pseudo-dividend, which drives its price, does not directly contribute to the aggregate consumption, but only (potentially) comoves with it.

A notable implication is that the no-dividend stock mean return is lower than that of a normal stock with the same underlying risk, $\sigma_1 = \sigma_2$, during the main period. This is intuitive because as discussed earlier the no-dividend stock price is driven by its estimated pseudo-dividend, which does not contribute directly to the aggregate consumption, and hence comoves less with the aggregate consumption growth rate as compared to a normal stock with the same underlying risk. Therefore the investor requires a lower risk premium to hold the no-dividend stock in equilibrium.[12] This result is consistent with the empirical evidence, which documents that stocks that pay no dividends have lower average returns than comparable stocks that pay dividends (Christie (1990), Naranjo, Nimalendran, and Ryngaert (1998), Fuller and Goldstein (2011)).

3.3 Stock Return Volatilities and Comovements

Proposition 3 presents the equilibrium stock return volatilities σ_{S_n} for each stock $n = 1, 2$ defined as $\sigma_{Snt}^2 = \text{Var}_t\left[dS_{nt}/S_{nt}dt\right]$.

Proposition 3 (**Equilibrium stock return volatilities**). *The equilibrium volatility of the*

[12] In the very special case of no-correlation, $\rho_{12} = 0$, the investor in fact does not require any risk premium to hold the no-dividend stock in equilibrium.

normal and no-dividend stock returns during the benchmark period $t \geq \tau$ are given by

$$\bar{\sigma}_{S_1 t} = \sigma_1, \tag{21}$$

$$\bar{\sigma}_{S_2 t} = \sigma_2, \tag{22}$$

and during the main period $t < \tau$ by

$$\sigma_{S_1 t} = \sigma_1, \tag{23}$$

$$\sigma_{S_2 t} = \sqrt{\rho_{12}^2 \sigma_2^2 + \frac{((1-\rho_{12}^2)\sigma_2^2 + \kappa_2 V_{2t})^2}{(1-\rho_{12}^2)\sigma_2^2 + \nu_2^2}}, \tag{24}$$

where the posterior variance V_{2t} is as in Lemma 1.

Consequently, during the main period, the volatility of the no-dividend stock return is higher than that of a normal stock with the same underlying risk $\sigma_1 = \sigma_2$.

During the benchmark period when all stocks pay dividends, the volatility of each stock return is constant and equals to the volatility of its dividend growth rate, σ_n. During the main period when only the normal stock pays dividends, the return volatility of the normal stock is still as in the benchmark period, while the return volatility of the no-dividend stock is the volatility of its estimated pseudo-dividend growth rate. Therefore, the posterior variance V_{2t} along with the parameters of the fundamental news process κ_2, υ_2 all affect the no-dividend stock return volatility.

Consequently, we show that the no-dividend stock return volatility is higher than that of a normal stock with the same underlying risk, $\sigma_1 = \sigma_2$, during the main period. This is intuitive because as discussed earlier, the no-dividend stock price is driven by its estimated pseudo-dividend, and the estimation process, necessitated by the absence of dividends, induces additional variability, which is reflected in the stock returns. This result is also consistent with the empirical evidence, which documents that stocks that pay no dividends have higher return volatility than comparable stocks that pay dividends (Naranjo, Nimalendran, and Ryngaert (1998), Pástor and Veronesi (2003)).

Proposition 4 presents the equilibrium market betas for stocks β_{S_n} for each stock $n = 1, 2$ defined as $\beta_{S_n t} = \mathrm{Cov}_t\left[(dS_{nt}/S_{nt}), (dS_t/S_t)\right] / \mathrm{Var}_t\left[dS_t/S_t\right]$, where S_t is the aggregate stock

market price given by $S_t = S_{1t} + S_{2t}$.

Proposition 4 (**Equilibrium market betas**). *The equilibrium market beta of the normal and no-dividend stocks during the benchmark period $t \geq \tau$ are given by*

$$\bar{\beta}_{S_1 t} = \frac{\sigma_1^2 \bar{S}_{1t}/\bar{S}_t + \rho_{12}\sigma_1\sigma_2 \bar{S}_{2t}/\bar{S}_t}{\sigma_1^2 \bar{S}_{1t}^2/\bar{S}_t^2 + \sigma_2^2 \bar{S}_{2t}^2/\bar{S}_t^2 + 2\rho_{12}\sigma_1\sigma_2 \bar{S}_{1t}\bar{S}_{2t}/\bar{S}_t^2}, \tag{25}$$

$$\bar{\beta}_{S_2 t} = \frac{\rho_{12}\sigma_1\sigma_2 \bar{S}_{1t}/\bar{S}_t + \sigma_2^2 \bar{S}_{2t}/\bar{S}_t}{\sigma_1^2 \bar{S}_{1t}^2/\bar{S}_t^2 + \sigma_2^2 \bar{S}_{2t}^2/\bar{S}_t^2 + 2\rho_{12}\sigma_1\sigma_2 \bar{S}_{1t}\bar{S}_{2t}/\bar{S}_t^2}, \tag{26}$$

and during the main period $t < \tau$ by

$$\beta_{S_1 t} = \frac{\sigma_1^2 S_{1t}/S_t + \rho_{12}\sigma_1\sigma_2 S_{2t}/S_t}{\sigma_1^2 S_{1t}^2/S_t^2 + \left(\rho_{12}^2\sigma_2^2 + \frac{((1-\rho_{12}^2)\sigma_2^2 + \kappa_2 V_{2t})^2}{(1-\rho_{12}^2)\sigma_2^2 + \nu_2^2}\right) S_{2t}^2/S_t^2 + 2\rho_{12}\sigma_1\sigma_2 S_{1t}S_{2t}/S_t^2}, \tag{27}$$

$$\beta_{S_2 t} = \frac{\rho_{12}\sigma_1\sigma_2 S_{1t}/S_t + \left(\rho_{12}^2\sigma_2^2 + \frac{((1-\rho_{12}^2)\sigma_2^2 + \kappa_2 V_{2t})^2}{(1-\rho_{12}^2)\sigma_2^2 + \nu_2^2}\right) S_{2t}/S_t}{\sigma_1^2 S_{1t}^2/S_t^2 + \left(\rho_{12}^2\sigma_2^2 + \frac{((1-\rho_{12}^2)\sigma_2^2 + \kappa_2 V_{2t})^2}{(1-\rho_{12}^2)\sigma_2^2 + \nu_2^2}\right) S_{2t}^2/S_t^2 + 2\rho_{12}\sigma_1\sigma_2 S_{1t}S_{2t}/S_t^2}, \tag{28}$$

where the posterior variance V_{2t} is as in Lemma 1, the stock prices $\bar{S}_{nt}$ and S_{nt} for $n = 1, 2$ are as in Proposition 1, and $\bar{S}_t$ and S_t denote the stock market price during the benchmark and main periods, respectively.

Consequently, during the main period, the market beta of the no-dividend stock is higher than that of a normal stock with the same underlying risk $\sigma_1 = \sigma_2$ and relative size $S_{1t}/S_t = S_{2t}/S_t$.

During the benchmark period, the equilibrium market betas are in terms of the underlying risks σ_n and relative sizes S_{nt}/S_t. When both stocks have the same underlying risk and relative size, they have the same market beta. During the main period, the market betas are additionally affected by the posterior variance V_{2t} along with the parameters of the fundamental news process κ_2, v_2. This is because the no-dividend stock is part of the aggregate stock market, and hence the no-dividend stock volatility not only affects its own market beta but also the market beta of the normal stock through the stock market volatility.[13]

[13]Note that, since the no-dividend stock's expected dividend initiation time $1/\lambda_2$ affects both stock prices during the main period (Proposition 1), in general, it also affects the market beta of both stocks through the relative sizes.

Consequently, the no-dividend stock market beta is higher than that of a normal stock with the same underlying risk and relative size during the main period. This is because the no-dividend stock return is more volatile (Proposition 3), and hence it contributes to and comoves with the aggregate stock market return more as compared to a normal stock with the same underlying risk and relative size. This result is also consistent with the empirical evidence, which documents that stocks that pay no dividends have higher market beta than comparable stocks that pay dividends (Boudoukh, Michaely, Richardson, and Roberts (2007), Fuller and Goldstein (2011)).

Remark 3 (Our model's relation to value vs growth stocks). In our model, we refer to the second stock type as the no-dividend stock since it does not pay dividends prior to some future random time τ and has zero dividend yield during this main period. Therefore, one could also think of the no-dividend (normal) stocks in our model as the *growth (value)* stocks, since in the literature a typical growth (value) stock is one with a low (high) fundamental to price ratio, where this ratio typically is the book-to-market, earnings yield, dividend yield, or the ratio of cash flows to price (Lettau and Wachter (2007)). Moreover, the three key features of no-dividend stocks in our model are also valid for growth stocks. For example, growth stocks too share the element of estimation of their true future dividends, since their current low fundamentals are not representative of their eventual significant future dividends. Second, growth stocks also share the element of having additional uncertainty about the initiation date of their main dividend paying period. Third, due to their low current fundamentals, the growth stocks currently contribute little to aggregate consumption, and hence to the stochastic discount factor. With this interpretation our findings are also consistent with the documented empirical regularities for growth and value stocks, since as summarized in Lettau and Wachter (2007), in the data, growth stocks have lower mean returns, and yet they have higher return volatilities and higher market betas as compared to value stocks.

4 Equilibrium Stock Market Implications

As we have shown in the previous Section, there are considerable differences between the normal and no-dividend stock return characteristics. Since these stocks make up the aggregate

stock market, these differences in their return characteristics have notable implications for the stock market returns. In this Section, we show that the presence of no-dividend stocks in the stock market leads to a lower correlation between the stock market return and the consumption growth rate, a non-monotonic and even a negative relation between the stock market risk premium and its volatility, and a downward sloping term structure of equity risk premia.

4.1 Stock Market Correlation with Consumption

Proposition 5 presents the equilibrium correlation of the stock market return with the aggregate consumption growth rate, $\rho_{SYt} = \text{Cov}_t\left[(dS_t/S_t),(dY_t/Y_t)\right]/\sqrt{\text{Var}_t\left[dS_t/S_t\right]\text{Var}_t\left[dY_t/Y_t\right]}$, where S_t is the aggregate stock market price given by $S_t = S_{1t} + S_{2t}$. In Proposition 5, when making comparisons, the benchmark period sensitivities are evaluated at their relative size, $\alpha_n/(\alpha_1+\alpha_2) = \bar{S}_{nt}/(\bar{S}_{1t}+\bar{S}_{2t})$, which are proportional to the relative dividends, $D_{nt}/(D_{1t}+D_{2t})$. We make this economically sensible adjustment when investigating the correlation behavior during the benchmark period to prevent our model yielding spurious effects due to our simplifying specification of constant sensitivities α_n in Section 2.3. This is because, α_n represents the sensitivity of the aggregate consumption growth rate to the shock in dividend D_n during the benchmark period, and hence a high (low) level of dividend D_n, and hence the stock price $\bar{S}_{nt}$, should be accompanied with a high (low) sensitivity α_n to obtain economically sensible implications of our model. On the other hand, the economic quantities during the main period are insensitive to the values of α_n, and hence our main results for this period, as well as our earlier results in Propositions 1–4, do not require this adjustment.[14]

Proposition 5 (**Equilibrium stock market correlation with consumption**). *The equilibrium correlation of the stock market return with the aggregate consumption growth rate during the benchmark period $t \geq \tau$ is given by*

$$\bar{\rho}_{SYt} = \frac{\left(\alpha_1\sigma_1^2 + \alpha_2\rho_{12}\sigma_1\sigma_2\right)\bar{S}_{1t}/\bar{S}_t + \left(\alpha_2\sigma_2^2 + \alpha_1\rho_{12}\sigma_1\sigma_2\right)\bar{S}_{2t}/\bar{S}_t}{\sqrt{\left(\sigma_1^2\bar{S}_{1t}^2/\bar{S}_t^2 + \sigma_2^2\bar{S}_{2t}^2/\bar{S}_t^2 + 2\rho_{12}\sigma_1\sigma_2\bar{S}_{1t}\bar{S}_{2t}/\bar{S}_t^2\right)\left(\alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2 + 2\alpha_1\alpha_2\rho_{12}\sigma_1\sigma_2\right)}}, \tag{29}$$

[14]Moreover, even though we make this adjustment for the benchmark period correlation, our numerical analysis shows that the implications of Proposition 5 hold more generally for a wide set of parameter values for the benchmark period sensitivities.

and during the main period $t < \tau$ *by*

$$\rho_{SYt} = \frac{(\alpha_1 + \alpha_2)\,\sigma_1^2 S_{1t}/S_t + (\alpha_1 + \alpha_2)\,\rho_{12}\sigma_1\sigma_2 S_{2t}/S_t}{\sqrt{\left(\sigma_1^2 S_{1t}^2/S_t^2 + \left(\rho_{12}^2\sigma_2^2 + \frac{((1-\rho_{12}^2)\sigma_2^2+\kappa_2 V_{2t})^2}{(1-\rho_{12}^2)\sigma_2^2+\nu_2^2}\right) S_{2t}^2/S_t^2 + 2\rho_{12}\sigma_1\sigma_2 S_{1t}S_{2t}/S_t^2\right)(\alpha_1+\alpha_2)^2\,\sigma_1^2}}, \tag{30}$$

where the posterior variance V_{2t} *is as in Lemma 1, the stock prices* $\bar{S}_{nt}$ *and* S_{nt} *for* $n = 1, 2$ *are as in Proposition 1, and* $\bar{S}_t$ *and* S_t *denote the stock market price during the benchmark and main periods, respectively.*

Consequently, the correlation of the stock market return with the aggregate consumption growth rate during the main period is lower than that of during the benchmark period.

During the benchmark period, a shock to any dividend D_1 or D_2 causes fluctuations in the aggregate consumption, with the magnitude of the effect being driven by the sensitivity parameters α_n (Section 2.3). This leads to the covariance, and hence the correlation, of the stock market return with the aggregate consumption growth rate to depend on both dividend growth rate variances and covariances (the numerator of (29)). However, during the main period, the shocks in aggregate consumption arise only from the shocks to the normal stock dividends D_1. Hence the covariance of the stock market return with the aggregate consumption growth rate only depends on the normal stock dividend growth rate variance and covariance (the numerator of (30)). Moreover, as in the case of market betas, the correlation of the stock market return with the aggregate consumption growth rate during the main period is additionally affected by the posterior variance V_{2t} and the parameters of the fundamental news process κ_2, v_2, through the stock market volatility.

The notable implication here is that the correlation of the aggregate stock market return with the aggregate consumption growth rate during the main period is lower than that of during the benchmark period. This result is intuitive as it simply says that when the stocks that do not contribute to the current aggregate consumption are also part of the stock market, the stock market return is less correlated with the aggregate consumption. In our model, this result follows from two effects that reinforce each other. The stock market return covaries less with the aggregate consumption per consumption volatility, but it is also more volatile (since the no-dividend stock return is more volatile) during the main period as

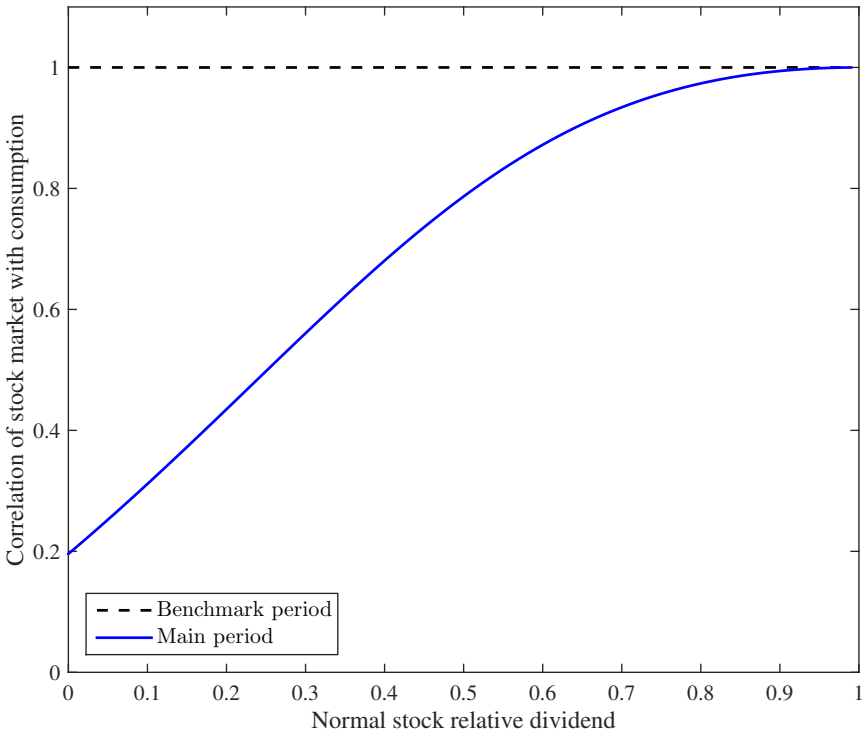

Figure 1: **Correlation of stock market return with consumption growth rate.** This figure plots the equilibrium correlation of the stock market return with the aggregate consumption growth rate against the normal stock relative dividend, $D_1/(D_1 + D_2)$ during the benchmark period and $D_1/(D_1 + \widehat{D}_2)$ during the main period. The parameter values are: $\mu_1 = \mu_2 = 11.6\%$, $\sigma_1 = \sigma_2 = 10.93\%$, $\rho_{12} = 0.25\%$, $\lambda_2 = 0.1$, $\kappa_2 = 5\%$, $\zeta_2 = 0$, $\nu_2 = 27.4\%$, $V_{20} = 0.58$, $\gamma = 3$, $\beta = 0.10$, $t = 0$, $\alpha_1 + \alpha_2 = 15\%$, and $\alpha_1 = (\alpha_1 + \alpha_2)D_1/(D_1 + D_2)$.

compared to the benchmark period.

Figure 1 illustrates our correlation result by plotting the equilibrium correlation of the aggregate stock market return with the aggregate consumption growth rate against the normal stock relative dividend.[15] We see that during the main period, as the normal stock

[15]The parameter values used in all our figures are determined as follows. First, consistent with the findings of Santos and Veronesi (2006), we set the total sensitivity $\alpha_1 + \alpha_2$ to the relative-share of the aggregate dividend in the aggregate consumption during the main period, 15%. We then match the mean and volatility of the aggregate consumption growth rate during the main period to the corresponding ones in the data, 1.74% and 1.64%, respectively, as reported in Campbell (2017). This gives the mean and volatility of the normal stock dividend growth rate as $\mu_1 = 11.6\%$ and $\sigma_1 = 10.93\%$, which we also use for the corresponding quantities for the no-dividend stock, $\mu_2 = 11.6\%$ and $\sigma_2 = 10.93\%$, and also set the

relative size gets smaller and the no-dividend stock becomes more dominant in the stock market, this correlation monotonically decreases. As discussed in the Introduction, this correlation appears to be weak in the data (Cochrane and Hansen (1992), Campbell and Cochrane (1999), Cochrane (2005), Albuquerque, Eichenbaum, Luo, and Rebelo (2016), Heyerdahl-Larsen and Illeditsch (2017)), and leading consumption-based asset pricing models have difficulty in reconciling this evidence. Our contribution here is to demonstrate that this low correlation may be due to a very simple reason that is typically not considered in standard consumption-based asset pricing models. That is, the stock market consists of many stocks that currently do not pay dividends and hence do not contribute to the current aggregate consumption or dividends, while contributing to the fluctuations in the aggregate stock market returns. Therefore, it naturally follows that the stock market returns, which are partially driven by the fluctuations in no-dividend stocks, correlate less with the current aggregate consumption growth rate.

4.2 Risk Premium-Volatility Relation

We next investigate our model implications on the relation between the conditional risk premium and volatility of the aggregate stock market. Although this relation has been empirically investigated extensively, the conclusions on the sign of the relation are mixed. In recent work Rossi and Timmermann (2010) argue that the lack of consensus in the earlier empirical literature may be due to the non-monotonic relation between these quantities. We here demonstrate that the presence of no-dividend stocks in the stock market can indeed generate a non-monotonic relation between these endogenous quantities. Towards that, in Proposition 6 we first present the equilibrium stock market risk premium and volatility in

correlation coefficient to $\rho_{12} = 0.25\%$. We choose $\lambda_2 = 0.1$ so that the expected dividend initiation time is 10 years. We set the mean-reversion κ_2 and the long-run mean ζ_2 of the fundamental news process to 5% and 0, respectively, and match its volatility to the reported volatility of the earnings growth rate (29.5%) in Longstaff and Piazzesi (2004) – this yields $\nu_2 = 27.4\%$. We choose the prior variance V_{20} so that it satisfies the simpler sufficient condition given in footnote 17 in the Appendix with equality, $V_{20} = (\nu_2/\kappa_2)\sqrt{(1-\rho_{12}^2)\sigma_2^2}$, which for our parameter choices implies the prior variance to be $V_{20} = 0.58$ for the level of the (logarithmic) pseudo-dividend. The relative risk aversion coefficient is set to $\gamma = 3$ and the current time $t = 0$. We use a sufficiently high rate of time preference $\beta = 0.10$ so that the stock prices are finite, and hence well-defined. Finally, consistent with our discussion above we evaluate the benchmark period sensitivities at their relative dividend shares, $\alpha_n = (\alpha_1 + \alpha_2)D_n/(D_1 + D_2)$.

our model.

Proposition 6 (**Equilibrium stock market risk premium and volatility**). *The equilibrium risk premium and volatility of the aggregate stock market during the benchmark period $t \geq \tau$ are given by*

$$\bar{r}_{S_t} - \bar{r} = \gamma(\alpha_1\sigma_1^2 + \alpha_2\rho_{12}\sigma_1\sigma_2)(\bar{S}_{1t}/\bar{S}_t) + \gamma(\alpha_1\rho_{12}\sigma_1\sigma_2 + \alpha_2\sigma_2^2)(\bar{S}_{2t}/\bar{S}_t), \tag{31}$$

$$\bar{\sigma}_{S_t} = \sqrt{\sigma_1^2\bar{S}_{1t}^2/\bar{S}_t^2 + \sigma_2^2\bar{S}_{2t}^2/\bar{S}_t^2 + 2\rho_{12}\sigma_1\sigma_2\bar{S}_{1t}\bar{S}_{2t}/\bar{S}_t^2}, \tag{32}$$

and during the main period $t < \tau$ by

$$r_{S_t} - r = \gamma(\alpha_1 + \alpha_2)\sigma_1^2(S_{1t}/S_t) + \gamma(\alpha_1 + \alpha_2)\rho_{12}\sigma_1\sigma_2(S_{2t}/S_t), \tag{33}$$

$$\sigma_{S_t} = \sqrt{\sigma_1^2 S_{1t}^2/S_t^2 + \left(\rho_{12}^2\sigma_2^2 + \frac{((1-\rho_{12}^2)\sigma_2^2 + \kappa_2 V_{2t})^2}{(1-\rho_{12}^2)\sigma_2^2 + \nu_2^2}\right) S_{2t}^2/S_t^2 + 2\rho_{12}\sigma_1\sigma_2 S_{1t}S_{2t}/S_t^2}. \tag{34}$$

where the posterior variance V_{2t} is as in Lemma 1, the stock prices $\bar{S}_{nt}$ and S_{nt} for $n = 1, 2$ are as in Proposition 1, $\bar{S}_t$ and S_t denote the stock market price.

In both periods, the equilibrium stock market risk premia are simple weighted averages of the normal and no-dividend stock risk premia, where the weights are the relative sizes of the stocks. These stock relative sizes also affect the equilibrium stock market volatility as they determine the extent to which each stock's volatility contribute to the stock market volatility. We demonstrate the relation between the stock market risk premium and volatility in our model with a scatter plot in Figure 2, where each point is obtained by varying the normal stock relative dividend ($D_1/(D_1+D_2)$ during the benchmark period and $D_1/(D_1+\widehat{D}_2)$ during the main period).

As Figure 2 illustrates, during the benchmark period when all stocks pay dividends, the relation between the stock market risk premium and volatility is monotonically positive, consistent with the standard intuition. However, during the main period when only the normal stock pays dividends, this relation becomes non-monotonic and even negative. This is because the stock market risk premium is the (relative size) weighted-average of the corresponding risk premia of stocks that make up the stock market. Therefore, when the no-dividend stocks, the stocks that command low risk premia but high volatility, are also

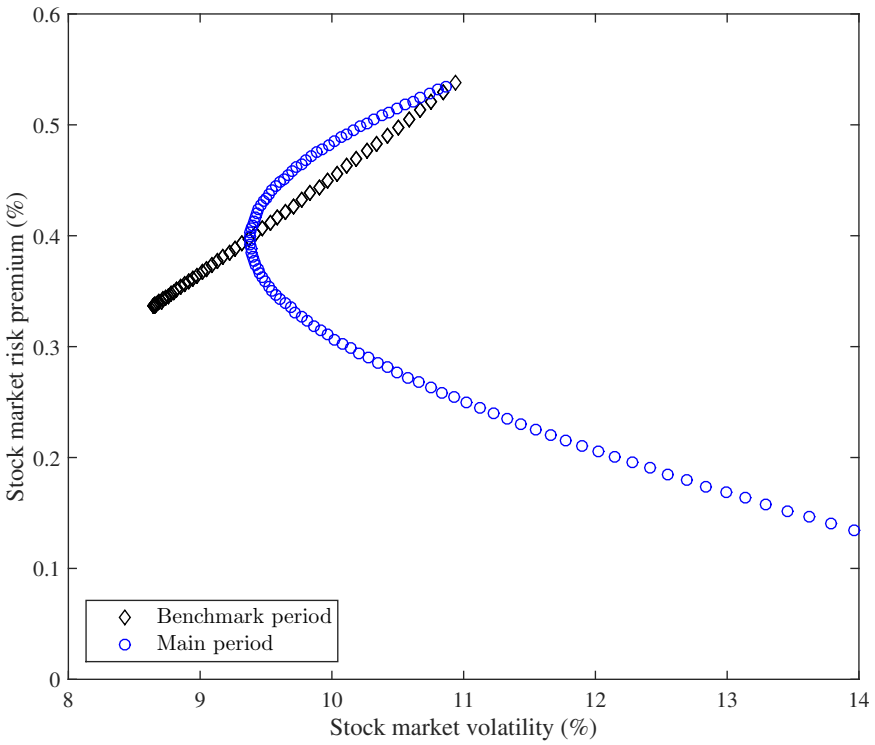

Figure 2: **Stock market risk premium versus volatility.** This figure plots the equilibrium relation between the conditional risk premium and volatility of the aggregate stock market return for varying levels of normal stock relative dividend, $D_1/(D_1 + D_2)$ during the benchmark period and $D_1/(D_1 + \widehat{D}_2)$ during the main period. The parameter values are as in Figure 1.

part of the stock market, the stock market risk premium is non-monotonically related to its volatility. In particular, the risk premium is decreasing in its volatility for high volatility levels, corresponding to high relative-size of the no-dividend stocks, which are represented by the points on the lower right corner in Figure 2.[16] Our prediction is in line with the

[16]We note that as can be seen from the y-axis for Figure 2, our model generates a low risk premium for the stock market for plausible parameter values. This is to be expected given our simplistic setting, e.g., a single investor, standard CRRA preferences, constant mean and volatility for the aggregate consumption growth rate, which is very similar to the settings of the original "equity premium puzzle" literature. It is well-known in this literature that models with these simplistic features yield fairly low risk premium for reasonable parameter values (typically less than 1%) as opposed to what is observed in the data (typically around 6%). In order to preserve simplicity and tractability, in this paper we refrain from introducing other features that are typically employed in the literature to obtain a more realistic equity premium, and leave that for future research.

empirical evidence in Rossi and Timmermann (2010), who find a non-monotonic relation between the conditional risk premium and volatility by showing a negative relation for high levels of volatility and a positive relation for low and medium levels of volatility.

As discussed in the Introduction, numerous works find a negative relation between the stock market conditional risk premium and volatility (e.g., Campbell (1987), Glosten, Jagannathan, and Runkle (1993), Whitelaw (2000), Harvey (2001), Brandt and Kang (2004)), while many others, consistent with the basic intuition, find this relation to be positive, (e.g., French, Schwert, and Stambaugh (1987), Scruggs (1998), Ghysels, Santa-Clara, and Valkanov (2005), Bali and Peng (2006), Guo and Whitelaw (2006), Ludvigson and Ng (2007)). On the theory side, a number of works, using a single stock setup, demonstrate that a non-monotonic and a negative relation can arise in equilibrium if there is time-variation in state variables or investment opportunities. Our contribution here is to illustrate that, using a simple multiple-stocks setup, a non-monotonic and a negative relation can also arise in equilibrium for a very simple reason, that the stock market also consists of no-dividend stocks, whose mean return-volatility relation goes against the standard intuition (low mean return but high return volatility).

4.3 Term Structure of Equity Risk Premia

Finally, we investigate our model implications for the shape of the term structure of equity risk premia. There has been growing interest in this term structure following the findings of van Binsbergen, Brandt, and Koijen (2012), who study a claim on the dividends of the S&P 500 index in the near future, i.e., the short-term asset, and find that the short-term asset commands a higher average return (and Sharpe ratio) than the underlying index, and conclude that the term structure of equity risk premia is downward sloping. As discussed in the Introduction, this empirical finding is considered somewhat puzzling since it goes against the implications of several leading asset pricing models. We here demonstrate that the presence of no-dividend stocks in the stock market can generate this downward sloping term structure of equity risk premia. Towards that, we define the short-term asset following van Binsbergen, Brandt, and Koijen (2012) as a claim to the aggregate dividends upto maturity T at a time t, and then present its equilibrium mean return along with the corresponding

one for the stock market, denoted by $r_{S_{t,T}}$ and r_{S_t}, respectively, in Proposition 7.

Proposition 7 (**Equilibrium short-term asset and stock market mean returns**). *The equilibrium mean returns of the short-term asset and the stock market during the benchmark period $t \geq \tau$ are given by*

$$\bar{r}_{S_{t,T}} = \frac{\bar{h}_{1t,T}\bar{S}_{1t}}{\bar{h}_{1t,T}\bar{S}_{1t} + \bar{h}_{2t,T}\bar{S}_{2t}}\bar{r}_1 + \frac{\bar{h}_{2t,T}\bar{S}_{2t}}{\bar{h}_{1t,T}\bar{S}_{1t} + \bar{h}_{2t,T}\bar{S}_{2t}}\bar{r}_2, \tag{35}$$

$$\bar{r}_{S_t} = \frac{\bar{S}_{1t}}{\bar{S}_{1t} + \bar{S}_{2t}}\bar{r}_1 + \frac{\bar{S}_{2t}}{\bar{S}_{1t} + \bar{S}_{2t}}\bar{r}_2, \tag{36}$$

and during the main period $t < \tau$ by

$$r_{S_{t,T}} = \frac{h_{1t,T}S_{1t}}{h_{1t,T}S_{1t} + h_{2t,T}S_{2t}}r_1 + \frac{h_{2t,T}S_{2t}}{h_{1t,T}S_{1t} + h_{2t,T}S_{2t}}r_2, \tag{37}$$

$$r_{S_t} = \frac{S_{1t}}{S_{1t} + S_{2t}}r_1 + \frac{S_{2t}}{S_{1t} + S_{2t}}r_2, \tag{38}$$

where

$$\bar{h}_{nt,T} = 1 - e^{-(\bar{r}_n - \mu_n)(T-t)}, \qquad n = 1, 2, \tag{39}$$

$$h_{1t,T} = 1 - e^{-(r_1 - \mu_1 + \lambda_2)(T-t)} - \left(\lambda_2 \frac{r_1 - \mu_1 + \lambda_2}{\bar{r}_1 - \mu_1 + \lambda_2}\right) \frac{e^{-(\bar{r}_1 - \mu_1)(T-t)} - e^{-(r_1 - \mu_1 + \lambda_2)(T-t)}}{r_1 - \bar{r}_1 + \lambda_2}, \tag{40}$$

$$h_{2t,T} = 1 - e^{-(r_2 - \mu_2 + \lambda_2)(T-t)} - (r_2 - \mu_2 + \lambda_2) \frac{e^{-(\bar{r}_2 - \mu_2)(T-t)} - e^{-(r_2 - \mu_2 + \lambda_2)(T-t)}}{r_2 - \bar{r}_2 + \lambda_2}, \tag{41}$$

and the stock prices $\bar{S}_{nt}$ and S_{nt} for $n = 1, 2$ are as in Proposition 1, and the mean returns $\bar{r}_n$ and r_n for $n = 1, 2$ are as in Proposition 2.

Consequently, during the main period, the mean return of the short-term asset is higher than that of the stock market if and only if $h_{1t,T} > h_{2t,T}$ when the stocks have the same underlying risk $\sigma_1 = \sigma_2$.

The short-term asset and the stock market equilibrium mean returns are weighted averages of the mean returns of the normal and no-dividend stocks. For the stock market these weights are simply the relative sizes of the stocks, whereas for the short-term asset the weights also include deterministic terms $h_{nt,T}$, which represent the fraction of each stock in the short-term asset. In particular, during the benchmark period, these fractions have

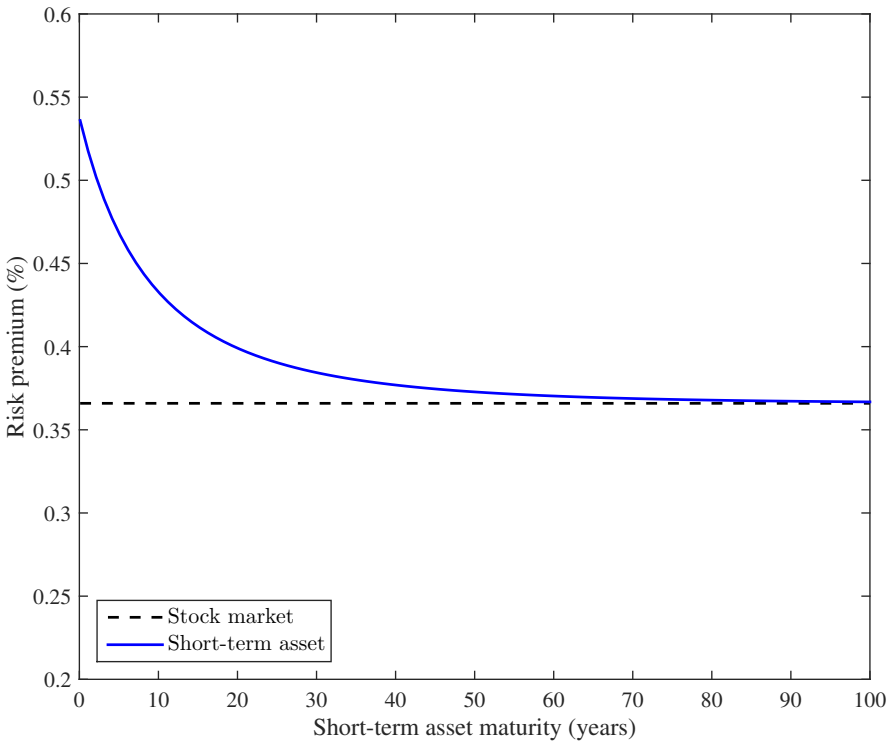

Figure 3: **Term structure of equity risk premia.** This figure plots the main period equilibrium risk premium of the short-term asset and the stock market against the maturity date of the short-term asset $T-t$. The parameter values are as in Figure 1 with $D_1 = \widehat{D}_2 = 1$.

simple forms and are driven by the dividend yield $\bar{r}_n - \mu_n$ and the short-term asset maturity $T-t$. During the main period, as (39)–(41) illustrate, these fractions are more involved and are additionally affected by the no-dividend stock's expected dividend initiation time $1/\lambda_2$. These more complicated forms for the fractions $h_{nt,T}$ arise because the short-term asset is a claim to the aggregate dividends upto $T-t$, during which the aggregate dividends (and the stochastic discount factor) shocks may remain the same or change due to the initiation of the no-dividend stock dividends.

Importantly, we find that during the main period the mean return of the short-term asset is higher than that of the stock market if and only if the fraction of the normal stock in the short-term asset is greater than the corresponding fraction for the no-dividend stock. This condition is satisfied for plausible parameter values since the short-term asset is more like

a normal stock than a no-dividend stock. This is because the value of the short-term asset only depends on the aggregate dividends upto its maturity, during which the no-dividend stock may not start paying dividends, and hence it is represented less in the short-term asset. Moreover, the normal stock mean return is higher than that of a no-dividend stock with the same underlying risk, $\sigma_1 = \sigma_2$, (Proposition 2), and hence by giving a higher weight to the stock with the higher mean return, the short-term asset mean return becomes higher than that of the stock market. In fact, as we show in the proof of this proposition in the Appendix, when the no-dividend stock does not start paying dividends for sure until the maturity of the short-term asset, the short term asset mean return becomes identical to the normal stock mean return, which is always higher than the stock market mean return.

This result also implies a downward sloping term structure of equity risk premia as illustrated in Figure 3, which plots the main period equilibrium risk premium (mean return minus the interest rate) of the short-term asset and the stock market against the maturity date of the short-term asset $T - t$. We see that during the main period, the shorter the maturity of the short-term asset, the higher its risk premium, which converges monotonically to the stock market risk premium as its maturity increases. We note that even though we do not provide it for brevity, the corresponding term structure for the Sharpe ratio is also similar and downward sloping in our model.

5 Conclusion

In this paper, we provide an analysis of stocks that pay no-dividends in an otherwise standard, parsimonious, consumption-based asset pricing framework. Our analysis leads to closed-form solutions for quantities of interest and profound implications that are supported empirically. We first find that the presence of no-dividend stocks generates a novel spillover effect in that the expected dividend initiation time of the no-dividend stocks affects the prices of normal stocks. Consistently with empirical evidence, we also find that no-dividend stocks command lower mean returns while having higher return volatilities and higher market betas than comparable stocks that pay dividends. We also show that the presence of no-dividend stocks in the stock market leads to a lower correlation between the stock market return and the consumption growth rate, a non-monotonic and even a negative relation between

the stock market risk premium and its volatility, and a downward sloping term structure of equity risk premia.

The framework we consider in this paper is parsimonious in the sense that there is a single investor with standard CRRA preferences and the aggregate consumption growth rate has a constant mean and volatility. Therefore, this framework can be extended in several different dimensions to study other potentially important issues such as, heterogeneous investors, more exotic preferences, more general aggregate consumption process. For instance, considering decreasing relative risk aversion (DRRA) preferences rather than CRRA may yield interesting implications in our framework. This is because investor's relative risk aversion would be more sensitive to the shocks for the normal stock than to the shocks for the no-dividend stock, which may help explain the findings of Fuller and Goldstein (2011) that no-dividend stocks command lower mean returns even more in declining markets. We leave these considerations for future research.

Appendix: Proofs

Proof of Lemma 1. We employ the standard Bayesian filtering theory (e.g., Liptser and Shiryaev (2001), Theorem 12.7) to estimate the unobserved pseudo-dividend D_{2t} given the information set $\mathcal{G}_t = \sigma\{(D_{1s}, F_{2s}) : 0 \leq s \leq t\}$, during the main period. We denote the vector of relevant observable processes by $X_t \equiv \left[\begin{array}{cc} \ln D_{1t} & \ln F_{2t} \end{array} \right]^\intercal$ along with the vectors consisting of its drift terms

$$A_0 \equiv \left[\begin{array}{c} \mu_1 - \frac{1}{2}\sigma_1^2 \\ \mu_2 - \frac{1}{2}\sigma_2^2 + \kappa_2\zeta_2 - \kappa_2 \ln F_{2t} \end{array} \right], \qquad A_1 \equiv \left[\begin{array}{c} 0 \\ \kappa_2 \end{array} \right], \tag{A.1}$$

and the variance and covariance matrices of observable and unobservable processes as

$$\Sigma_{oo} \equiv \left[\begin{array}{cc} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 + \nu_2^2 \end{array} \right], \qquad \Sigma_{uo} \equiv \left[\begin{array}{cc} \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 \end{array} \right]. \tag{A.2}$$

The filtering theory then implies that if the prior of the $\ln D_2$ at time 0 is normally distributed with mean $\widehat{\ln D}_{20}$ and variance V_{20}, then the posterior of $\ln D_2$ at time $t > 0$ conditional on the information $\mathcal{G}_t$ is also normally distributed with mean $\widehat{\ln D}_{2t} = \mathrm{E}\left[\ln D_{2t} | \mathcal{G}_t\right]$ and variance $V_{2t} = \mathrm{E}\left[(\ln D_{2t} - \widehat{\ln D}_{2t})^2 | \mathcal{G}_t\right]$ that satisfy the dynamics

$$\begin{aligned} d\widehat{\ln D}_{2t} &= (\mu_2 - \frac{1}{2}\sigma_2^2)dt + (\Sigma_{uo} + V_{2t}A_1^\intercal)\,\Sigma_{oo}^{-1}\left[dX_t - (A_0 + A_1\widehat{\ln D}_{2t})dt\right], & \text{(A.3)} \\ dV_{2t} &= -\left[(\Sigma_{uo} + V_{2t}A_1^\intercal)\,\Sigma_{oo}^{-1}\,(\Sigma_{uo} + V_{2t}A_1^\intercal)^\intercal - \sigma_2^2\right]dt. & \text{(A.4)} \end{aligned}$$

Substituing (A.1)–(A.2) into the posterior mean dynamics (A.3) and rearranging yields

$$d\widehat{\ln D}_{2t} = (\mu_2 - \frac{1}{2}\sigma_2^2)dt + \rho_{12}\sigma_2\frac{\nu_2^2 - \kappa_2 V_{2t}}{(1-\rho_{12}^2)\sigma_2^2 + \nu_2^2}d\omega_{1t} + \sqrt{\sigma_2^2 + \nu_2^2}\frac{(1-\rho_{12}^2)\sigma_2^2 + \kappa_2 V_{2t}}{(1-\rho_{12}^2)\sigma_2^2 + \nu_2^2}d\widehat{\omega}_{2t}^*, \tag{A.5}$$

where the innovation process $\widehat{\omega}_2^*$ is given by

$$d\widehat{\omega}_{2t}^* = \frac{1}{\sqrt{\sigma_2^2 + \nu_2^2}}\left[d\ln F_{2t} - (\mu_2 - \frac{1}{2}\sigma_2^2 + \kappa_2\zeta_2 - \kappa_2\ln F_{2t} + \kappa_2\widehat{\ln D}_{2t})dt\right],$$

with the correlation $d\omega_{1t}d\widehat{\omega}^*_{2t} = \left(\rho_{12}\sigma_2/\sqrt{\sigma_2^2+\nu_2^2}\right)dt$. Since it is typically more convenient to work with independent (uncorrelated) Brownian motions, we define a new Brownian motion $\widehat{\omega}_2$ that is independent of the Brownian motion ω_1 through the relation

$$d\widehat{\omega}^*_{2t} = \sqrt{\frac{\rho_{12}^2\sigma_2^2}{\sigma_2^2+\nu_2^2}}d\omega_{1t} + \sqrt{1-\frac{\rho_{12}^2\sigma_2^2}{\sigma_2^2+\nu_2^2}}d\widehat{\omega}_{2t},$$

which after substituting into (A.5) yields the dynamics

$$d\widehat{\ln D}_{2t} = (\mu_2 - \frac{1}{2}\sigma_2^2)dt + \rho_{12}\sigma_2 d\omega_{1t} + \frac{(1-\rho_{12}^2)\sigma_2^2 + \kappa_2 V_{2t}}{\sqrt{(1-\rho_{12}^2)\sigma_2^2+\nu_2^2}}d\widehat{\omega}_{2t}. \tag{A.6}$$

Substituting (A.1)–(A.2) into the posterior variance dynamics (A.4) yields

$$dV_{2t} = -\left[\frac{\rho_{12}^2\sigma_1^2\sigma_2^2(\nu_2^2-\kappa_2 V_{2t}) + \left(\sigma_1^2(\sigma_2^2+\kappa_2 V_{2t}) - \rho_{12}^2\sigma_1^2\sigma_2^2\right)(\sigma_2^2+\kappa_2 V_{2t})}{\sigma_1^2(\sigma_2^2+\nu_2^2) - \rho_{12}^2\sigma_1^2\sigma_2^2} - \sigma_2^2\right]dt,$$

which after rearranging becomes as reported in (6). The steady-state value of the posterior variance $V_{2\infty}$ is the positive constant which solves the quadratic equation resulting from setting $dV_{2t} = 0$ in (6), and given by

$$V_{2\infty} = \frac{1}{\kappa_2}\left[\sqrt{((1-\rho_{12}^2)\sigma_2^2+\nu_2^2)(1-\rho_{12}^2)\sigma_2^2} - (1-\rho_{12}^2)\sigma_2^2\right]. \tag{A.7}$$

Moreover, the closed-form solution for the posterior variance at all times follows from the well-known solution to the Riccati equation and is given by

$$V_{2t} = V_{2\infty}\frac{1+pe^{-mt}}{1-qe^{-mt}}, \tag{A.8}$$

where we have defined the constants

$$
\begin{aligned}
m &\equiv 2\kappa_2\sqrt{\frac{(1-\rho_{12}^2)\sigma_2^2}{(1-\rho_{12}^2)\sigma_2^2+\nu_2^2}},\\
p &\equiv \frac{\kappa_2 V_{20}\left[\sqrt{((1-\rho_{12}^2)\sigma_2^2+\nu_2^2)(1-\rho_{12}^2)\sigma_2^2}+(1-\rho_{12}^2)\sigma_2^2\right]-(1-\rho_{12}^2)\sigma_2^2\nu_2^2}{\kappa_2 V_{20}\left[\sqrt{((1-\rho_{12}^2)\sigma_2^2+\nu_2^2)(1-\rho_{12}^2)\sigma_2^2}-(1-\rho_{12}^2)\sigma_2^2\right]+(1-\rho_{12}^2)\sigma_2^2\nu_2^2},\\
q &\equiv \frac{\kappa_2 V_{20}-\sqrt{((1-\rho_{12}^2)\sigma_2^2+\nu_2^2)(1-\rho_{12}^2)\sigma_2^2}+(1-\rho_{12}^2)\sigma_2^2}{\kappa_2 V_{20}+\sqrt{((1-\rho_{12}^2)\sigma_2^2+\nu_2^2)(1-\rho_{12}^2)\sigma_2^2}+(1-\rho_{12}^2)\sigma_2^2}.
\end{aligned}
$$

It is also easy to see from (A.8) that the prior variance V_{20} is greater than the steady-state posterior variance, $V_{20} > V_{2\infty}$ if and only if $p+q>0$.[17]

Finally, applying Itô's Lemma to the relation for the estimated pseudo-dividend $\widehat{D}_{2t} = \exp(\widehat{\ln D}_{2t} + \frac{1}{2}V_{2t})$ gives its dynamics as

$$
\frac{d\widehat{D}_{2t}}{\widehat{D}_{2t}} = d\widehat{\ln D}_{2t} + \frac{1}{2}\left(d\widehat{\ln D}_{2t}d\widehat{\ln D}_{2t} + dV_{2t}\right) = d\widehat{\ln D}_{2t} + \frac{1}{2}\sigma_2^2 dt,
$$

where the second equality follows from the posterior variance dynamics of (6) and (A.6), which after substituting into the last expression above yields (5).

The volatility of the estimated pseudo-dividend is readily given by the dynamics (5) as

$$
\sigma_{\widehat{D}_{2t}} = \sqrt{\rho_{12}^2\sigma_2^2 + \frac{((1-\rho_{12}^2)\sigma_2^2+\kappa_2 V_{2t})^2}{(1-\rho_{12}^2)\sigma_2^2+\nu_2^2}}. \tag{A.9}
$$

Since $V_{20} > V_{2t} > V_{2\infty}$ at all times t, (A.9) takes its minimum value, σ_2, when the posterior variance is at its steady-state (A.7) implying

$$
\sigma_{\widehat{D}_{2t}} > \sigma_2, \tag{A.10}
$$

that is, the estimated pseudo-dividend is indeed more volatile than the pseudo-dividend at all times. □

[17] A simpler sufficient condition for $V_{20} > V_{2\infty}$ to hold is given by $V_{20} \geq \frac{\nu_2}{\kappa_2}\sqrt{(1-\rho_{12}^2)\sigma_2^2}$, which is satisfied for an appropriate choice of an initial prior.

Proof of statements in Remark 1. Under the alternative "signal plus noise" specification of the fundamental news (7), the corresponding quantities of (A.1)–(A.2) in the proof of Lemma 1 become

$$A_0 \equiv \begin{bmatrix} \mu_1 - \frac{1}{2}\sigma_1^2 \\ 0 \end{bmatrix}, \qquad A_1 \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \Sigma_{oo} \equiv \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \nu_2^2 \end{bmatrix}, \qquad \Sigma_{uo} \equiv \begin{bmatrix} \rho_{12}\sigma_1\sigma_2 & 0 \end{bmatrix}.$$

Following similar steps to those for the filtering in the proof of Lemma 1 yield the dynamics for the posterior mean and variance as

$$\begin{aligned} d\widehat{\ln D}_{2t} &= (\mu_2 - \frac{1}{2}\sigma_2^2)dt + \rho_{12}\sigma_2 d\omega_{1t} + \frac{V_{2t}}{\nu_2} d\widehat{\omega}_{2t}^*, \\ dV_{2t} &= -\left[\frac{V_{2t}^2}{\nu_2^2} - \sigma_2^2(1-\rho_{12}^2)\right] dt, \end{aligned}$$

where the innovation process $\widehat{\omega}_2^*$ is given by $d\widehat{\omega}_{2t}^* = \frac{1}{\nu_2}\left[d\ln F_{2t} - \widehat{\ln D}_{2t}dt\right]$, with the correlation $d\omega_{1t}d\widehat{\omega}_{2t}^* = 0$. In this case, the steady-state value of the posterior variance $V_{2\infty}$ is simply given by $V_{2\infty} = \nu_2\sigma_2\sqrt{1-\rho_{12}^2}$, and the volatility of the estimated pseudo-dividend by

$$\sigma_{\widehat{D}_{2t}} = \sqrt{\rho_{12}^2\sigma_2^2 + \frac{V_{2t}^2}{\nu_2^2}},$$

from which we again conclude that $\sigma_{\widehat{D}_{2t}} > \sigma_2$, that is, the estimated pseudo-dividend is more volatile than the pseudo-dividend under this alternative specification also. □

Proof of Proposition 1. We proceed by determining the equilibrium state price density process in our economy. We then recover the equilibrium normal and no-dividend stock prices over the benchmark period $t \geq \tau$ and the main period $t < \tau$.

In this economy, the equilibrium state price density process ξ at all times is given by the marginal utility of the representative investor evaluated at the aggregate consumption

$$\xi_t = e^{-\beta t}Y_t^{-\gamma}. \tag{A.11}$$

By no arbitrage, the normal and no-dividend stock prices during the benchmark period $t \geq \tau$

when all stocks pay dividends are given by

$$\bar{S}_{nt} = \frac{1}{\xi_t} \mathrm{E}_t \left[\int_t^\infty \xi_u D_{nu} du \right], \qquad \text{for } n = 1, 2. \tag{A.12}$$

Applying Itô's Lemma to ξD_n, using the dynamics of the dividends (1)–(2) and the state price density for the benchmark period

$$\frac{d\xi_t}{\xi_t} = -\bar{r} dt - \gamma \alpha_1 \sigma_1 d\omega_{1t} - \gamma \alpha_2 \sigma_2 d\omega_{2t}, \tag{A.13}$$

where $\bar{r}$ is the equilibrium interest rate in the benchmark period given by (19), yields

$$\begin{aligned} \frac{d\xi_t D_{1t}}{\xi_t D_{1t}} &= -(\bar{r}_1 - \mu_1) dt + (1 - \gamma \alpha_1) \sigma_1 d\omega_{1t} - \gamma \alpha_2 \sigma_2 d\omega_{2t}, \\ \frac{d\xi_t D_{2t}}{\xi_t D_{2t}} &= -(\bar{r}_2 - \mu_2) dt - \gamma \alpha_1 \sigma_1 d\omega_{1t} + (1 - \gamma \alpha_2) \sigma_2 d\omega_{2t}, \end{aligned}$$

where the constants $\bar{r}_1$ and $\bar{r}_2$ in the drift terms are given by

$$\bar{r}_1 = \bar{r} + \gamma(\alpha_1 \sigma_1^2 + \alpha_2 \rho_{12} \sigma_1 \sigma_2), \tag{A.14}$$

$$\bar{r}_2 = \bar{r} + \gamma(\alpha_1 \rho_{12} \sigma_1 \sigma_2 + \alpha_2 \sigma_2^2), \tag{A.15}$$

Since the process ξD_n, for $n = 1, 2$, has a constant drift of $-(\bar{r}_n - \mu_n)$ during this period, we have

$$\mathrm{E}_t \left[\xi_u D_{nu} \right] = e^{-(\bar{r}_n - \mu_n)(u-t)} \xi_t D_{nt},$$

which after substituting into the stock price expression (A.12) yields

$$\bar{S}_{nt} = \int_t^\infty e^{-(\bar{r}_n - \mu_n)(u-t)} du D_{nt}, \qquad \text{for } n = 1, 2.$$

Evaluating the simple integral (under the parameter restriction of $\bar{r}_n - \mu_n > 0$, so that the stock price is finite, and hence, well-defined) leads to the stock price expressions (11)–(12) over the benchmark period.

We next determine the stock prices during the main period $t < \tau$ when only the normal stock pays dividends. By no arbitrage, the normal stock price during the main period is

given by

$$S_{1t} = \frac{1}{\xi_t} \mathrm{E}_t \left[\int_t^\tau \xi_u D_{1u} du + \xi_\tau \bar{S}_{1\tau} \right], \tag{A.16}$$

where τ is an exponential random variable that is independent from all Brownian motions with its conditional distribution function for all times t due to its memoryless property given by

$$G(u-t) = \mathbb{P}(\tau \leq u | \tau > t) = \mathbb{P}(\tau \leq u - t) = 1 - e^{-\lambda_2 (u-t)},$$

with the corresponding density function

$$g(u-t) = \lambda_2 e^{-\lambda_2 (u-t)}. \tag{A.17}$$

Hence, the first term in (A.16) becomes

$$\mathrm{E}_t \left[\int_t^\tau \xi_u D_{1u} du \right] = \mathrm{E}_t \left[\int_t^\infty \xi_u D_{1u} \mathbf{1}_{\{u<\tau\}} du \right] = \mathrm{E}_t \left[\int_t^\infty \xi_u D_{1u} \mathbb{P}(u < \tau | \tau > t) \, du \right],$$

where the last equality follows from taking the expectation with respect to τ and the property of indicator functions. Substituing the right tail probability $\mathbb{P}(u < \tau | \tau > t) = 1 - G(u-t) = e^{-\lambda_2 (u-t)}$ gives the first term as

$$\mathrm{E}_t \left[\int_t^\tau \xi_u D_{1u} du \right] = \mathrm{E}_t \left[\int_t^\infty \xi_u D_{1u} e^{-\lambda_2 (u-t)} du \right], \tag{A.18}$$

where now the expectation needs to be taken with respect to the Brownian motions only. Applying Itô's Lemma to ξD_n, using the dynamics of the dividends (1) and the state price density for the main period

$$\frac{d\xi_t}{\xi_t} = -r dt - \gamma(\alpha_1 + \alpha_2)\sigma_1 d\omega_{1t}, \tag{A.19}$$

where r is the equilibrium interest rate in the main period given by (20), yields

$$\frac{d\xi_t D_{1t}}{\xi_t D_{1t}} = -(r_1 - \mu_1)dt + (1 - \gamma(\alpha_1 + \alpha_2))\sigma_1 d\omega_{1t},$$

where the constant r_1 in the drift term is given by

$$r_1 = r + \gamma(\alpha_1 + \alpha_2)\sigma_1^2. \tag{A.20}$$

Since the process ξD_1 has a constant drift of $-(r_1 - \mu_1)$ during this period, we have

$$\mathrm{E}_t\left[\xi_u D_{1u}\right] = e^{-(r_1-\mu_1)(u-t)}\xi_t D_{1t}, \tag{A.21}$$

which after substituting into (A.18) yields

$$\mathrm{E}_t\left[\int_t^\tau \xi_u D_{1u} du\right] = \int_t^\infty e^{-(r_1-\mu_1+\lambda_2)(u-t)} du \xi_t D_{1t} = \frac{1}{r_1-\mu_1+\lambda_2}\xi_t D_{1t}, \tag{A.22}$$

where the last equality follows from solving the simple integral (under the parameter restriction of $r_1 - \mu_1 > 0$, so that the normal stock price is finite, and hence well-defined for any value of λ_2). For the second term in (A.16), we substitute the normal stock price at time τ given by (11) to obtain

$$\mathrm{E}_t\left[\xi_\tau \bar{S}_{1\tau}\right] = \frac{1}{\bar{r}_1 - \mu_1}\mathrm{E}_t\left[\xi_\tau D_{1\tau}\right].$$

Taking the expectation with respect to τ gives

$$\mathrm{E}_t\left[\xi_\tau D_{1\tau}\right] = \mathrm{E}_t\left[\int_t^\infty \xi_u D_{1u} g\left(u-t\right) du\right] = \mathrm{E}_t\left[\int_t^\infty \xi_u D_{1u}\lambda_2 e^{-\lambda_2(u-t)} du\right],$$

and using the conditional expectation result (A.21) for this period again, we obtain

$$\mathrm{E}_t\left[\xi_\tau D_{1\tau}\right] = \int_t^\infty e^{-(r_1-\mu_1+\lambda_2)(u-t)} du \lambda_2 \xi_t D_{1t} = \frac{\lambda_2}{r_1-\mu_1+\lambda_2}\xi_t D_{1t}. \tag{A.23}$$

Substituting (A.22)–(A.23) into (A.16), and rearranging gives the normal stock price during the main period as reported in (13). Note that there is a discrete change in the normal stock price at time τ given by

$$\bar{S}_{1\tau} - S_{1\tau-} = \frac{1}{\bar{r}_1 - \mu_1}\left(D_{1\tau} - \frac{\bar{r}_1 - \mu_1 + \lambda_2}{r_1 - \mu_1 + \lambda_2} D_{1\tau}\right), \tag{A.24}$$

where $S_{1\tau-}$ denotes the left-limit of the price just prior to τ, $S_{1\tau-} = \lim_{t\to\tau} S_{1t}$.

Finally, we determine the no-dividend stock price during the main period, whose price, by no arbitrage, is given by

$$S_{2t} = \frac{1}{\xi_t}\mathrm{E}_t\left[\xi_\tau \bar{S}_{2\tau}\right] = \frac{1}{\bar{r}_2 - \mu_2}\frac{1}{\xi_t}\mathrm{E}_t\left[\xi_\tau D_{2\tau}\right], \tag{A.25}$$

where the second equality follows from substituting the time τ price given by (12). Since in the absence of its dividends during the main period $t < \tau$ the investor uses the estimated pseudo-dividend $\widehat{D}_2$ (Lemma 1) to estimate the distribution of future dividends, we simply substitute $\widehat{D}_{2\tau}$ for $D_{2\tau}$ in the above expectation to obtain

$$\mathrm{E}_t\left[\xi_\tau D_{2\tau}\right] = \mathrm{E}_t\left[\xi_\tau \widehat{D}_{2\tau}\right] = \mathrm{E}_t\left[\int_t^\infty \xi_u \widehat{D}_{2u} g\left(u-t\right) du\right] = \mathrm{E}_t\left[\int_t^\infty \xi_u \widehat{D}_{2u} \lambda_2 e^{-\lambda_2(u-t)} du\right], \quad \text{(A.26)}$$

where again the second equality follows from taking the expectation with respect to τ.[18] Applying Itô's Lemma to $\xi\widehat{D}_2$, and using the dynamics of the estimated pseudo-dividend (5) and the state price density for the main period (A.19) yields

$$\frac{d\xi_t \widehat{D}_{2t}}{\xi_t \widehat{D}_{2t}} = -(r_2-\mu_2)dt + (\rho_{12}\sigma_2 - \gamma(\alpha_1+\alpha_2)\sigma_1)d\omega_{1t} + \frac{(1-\rho_{12}^2)\sigma_2^2 + \kappa_2 V_{2t}}{\sqrt{(1-\rho_{12}^2)\sigma_2^2+\nu_2^2}} d\widehat{\omega}_{2t},$$

where the constant r_2 in the drift term is given by

$$r_2 = r + \gamma(\alpha_1+\alpha_2)\rho_{12}\sigma_1\sigma_2. \quad \text{(A.27)}$$

Since $\xi\widehat{D}_2$ has a constant drift of $-(r_2-\mu_2)$ during this period, we have

$$\mathrm{E}_t\left[\xi_u \widehat{D}_{2u}\right] = e^{-(r_2-\mu_2)(u-t)} \xi_t \widehat{D}_{2t},$$

which after substituting into (A.26) yields

$$\mathrm{E}_t\left[\xi_\tau D_{2\tau}\right] = \int_t^\infty e^{-(r_2-\mu_2+\lambda_2)(u-t)} du \lambda_2 \xi_t \widehat{D}_{2t} = \frac{\lambda_2}{r_2-\mu_2+\lambda_2} \xi_t \widehat{D}_{2t}, \quad \text{(A.28)}$$

where the last equality follows from solving the simple integral (under the parameter restriction of $r_2 - \mu_2 > 0$, so that the no-dividend stock price is finite, and hence well-defined for any value of λ_2). Substituting (A.28) into (A.25) gives the no-dividend stock price during the main period as reported in (14). Similarly, the discrete change in the no-dividend stock

[18] Note that during the main period $t < \tau$, the estimation of D_2 does not affect the aggregate consumption, and hence the state price density ξ, since the fluctuations in aggregate consumption are driven by current dividend shocks, D_1.

at time τ is given by

$$\bar{S}_{2\tau} - S_{2\tau-} = \frac{1}{\bar{r}_2 - \mu_2} \left(D_{2\tau} - \frac{\lambda_2}{r_2 - \mu_2 + \lambda_2} \widehat{D}_{2\tau} \right), \tag{A.29}$$

where again $S_{2\tau-}$ denotes the left-limit of the price just prior to τ, $S_{2\tau-} = \lim_{t \to \tau} S_{2t}$.

The condition for property (i) that during the main period, all else being fixed, the normal stock price is decreasing in the expected dividend initiation time $1/\lambda_2$ of the no-dividend stock follows from taking the partial derivative of (13) with respect to $1/\lambda_2$. Note that $\frac{\partial}{\partial(1/\lambda_2)} S_{1t} < 0$ if and only if $\frac{\partial}{\partial \lambda_2} S_{1t} > 0$, and we have

$$\frac{\partial}{\partial \lambda_2} S_{1t} = \frac{1}{\bar{r}_1 - \mu_1} \frac{r_1 - \bar{r}_1}{(r_1 - \mu_1 + \lambda_2)^2} D_{1t},$$

which is positive if and only if $r_1 > \bar{r}_1$. The condition for property (ii) that during the main period, all else being fixed, the no-dividend stock price is decreasing in its expected dividend initiation time $1/\lambda_2$ follows from taking the partial derivative of (14) with respect to $1/\lambda_2$. Note that $\frac{\partial}{\partial(1/\lambda_2)} S_{2t} < 0$ if and only if $\frac{\partial}{\partial \lambda_2} S_{2t} > 0$, and we have

$$\frac{\partial}{\partial \lambda_2} S_{2t} = \frac{1}{\bar{r}_2 - \mu_2} \frac{r_2 - \mu_2}{(r_2 - \mu_2 + \lambda_2)^2} \widehat{D}_{2t},$$

which is always positive since $r_2 - \mu_2 > 0$. □

Proof of Proposition 2. The benchmark period equilibrium mean returns for the normal and no-dividend stocks are given by adding the interest rate to their risk premia:

$$\bar{r} - \frac{d\xi_t}{\xi_t} \frac{d\bar{S}_{nt}}{\bar{S}_{nt}}, \qquad \text{for } n = 1, 2. \tag{A.30}$$

Applying Itô's Lemma to the benchmark period stock prices (11)–(12) leads to the continuous dynamics for the stock prices as

$$\frac{d\bar{S}_{nt}}{\bar{S}_{nt}} = \frac{dD_{nt}}{D_{nt}} = \mu_n dt + \sigma_n d\omega_{nt}, \qquad \text{for } n = 1, 2. \tag{A.31}$$

which after substituting into (A.30) along with the benchmark period state price density dynamics (A.13) gives the benchmark period equilibrium mean returns as $\bar{r}_1$ and $\bar{r}_2$ in (A.14)

and (A.15).

The main period equilibrium mean returns for the normal and no-dividend stocks are given by adding the interest rate to their risk premia:

$$r - \frac{d\xi_t}{\xi_t}\frac{dS_{nt}}{S_{nt}}, \qquad \text{for } n = 1, 2. \tag{A.32}$$

Applying Itô's Lemma to the main period stock prices (13)–(14) leads to the continuous dynamics for the stock prices as

$$\frac{dS_{1t}}{S_{1t}} = \frac{dD_{1t}}{D_{1t}} = \mu_1 dt + \sigma_1 d\omega_{1t}, \tag{A.33}$$

$$\frac{dS_{2t}}{S_{2t}} = \frac{d\widehat{D}_{2t}}{\widehat{D}_{2t}} = \mu_2 dt + \rho_{12}\sigma_2 d\omega_{1t} + \frac{(1-\rho_{12}^2)\sigma_2^2 + \kappa_2 V_{2t}}{\sqrt{(1-\rho_{12}^2)\sigma_2^2 + \nu_2^2}} d\widehat{\omega}_{2t}, \tag{A.34}$$

which after substituting into (A.32) along with the main period state price density dynamics (A.19) gives the main period equilibrium mean returns as r_1 and r_2 in (A.20) and (A.27).

The property during the main period that the mean return of the no-dividend stock is lower than that of a normal stock with the same underlying risk follows immediately by comparing (17) and (18). □

Proof of Proposition 3. The equilibrium volatilities of the normal and no-dividend stock returns during the benchmark period $t \geq \tau$ are given by the diffusion terms in (A.31), and during the main period $t < \tau$ are given by the diffusion terms in (A.33) and (A.34), respectively. The property during the main period that the volatility of the no-dividend stock return is higher than that of a normal stock with the same underlying risk follows immediately by comparing (23) and (24) using the relation (A.9)–(A.10). □

Proof of Proposition 4. During the benchmark period $t \geq \tau$, the aggregate stock market price is given by $\bar{S}_t = \bar{S}_{1t} + \bar{S}_{2t}$, with dynamics

$$\begin{aligned} \frac{d\bar{S}_t}{\bar{S}_t} + \frac{D_{1t} + D_{2t}}{\bar{S}_t} dt &= \left(\frac{d\bar{S}_{1t}}{\bar{S}_{1t}} + \frac{D_{1t}}{\bar{S}_{1t}} dt\right)\frac{\bar{S}_{1t}}{\bar{S}_t} + \left(\frac{d\bar{S}_{1t}}{\bar{S}_{1t}} + \frac{D_{2t}}{\bar{S}_{2t}} dt\right)\frac{\bar{S}_{2t}}{\bar{S}_t} \\ &= \ldots dt + \sigma_1 \frac{\bar{S}_{1t}}{\bar{S}_t} d\omega_{1t} + \sigma_2 \frac{\bar{S}_{2t}}{\bar{S}_t} d\omega_{2t}, \end{aligned} \tag{A.35}$$

where the second equality follows from substituting the benchmark period stock price dynamics (A.31). The dynamics (A.31) and (A.35) readily yield the covariance between the individual stocks and the aggregate stock market returns as

$$\begin{aligned} \mathrm{Cov}_t\left[\frac{d\bar{S}_{1t}}{\bar{S}_{1t}}, \frac{d\bar{S}_t}{\bar{S}_t}\right]\frac{1}{dt} &= \sigma_1^2\frac{\bar{S}_{1t}}{\bar{S}_t} + \rho_{12}\sigma_1\sigma_2\frac{\bar{S}_{2t}}{\bar{S}_t}, \qquad \text{(A.36)} \\ \mathrm{Cov}_t\left[\frac{d\bar{S}_{2t}}{\bar{S}_{2t}}, \frac{d\bar{S}_t}{\bar{S}_t}\right]\frac{1}{dt} &= \rho_{12}\sigma_1\sigma_2\frac{\bar{S}_{1t}}{\bar{S}_t} + \sigma_2^2\frac{\bar{S}_{2t}}{\bar{S}_t}. \qquad \text{(A.37)} \end{aligned}$$

Using (A.35), we obtain the aggregate stock market return variance for this period as

$$\mathrm{Var}_t\left[\frac{d\bar{S}_t}{\bar{S}_t}\right]\frac{1}{dt} = \sigma_1^2\left(\frac{\bar{S}_{1t}}{\bar{S}_t}\right)^2 + \sigma_2^2\left(\frac{\bar{S}_{2t}}{\bar{S}_t}\right)^2 + 2\rho_{12}\sigma_1\sigma_2\frac{\bar{S}_{1t}}{\bar{S}_t}\frac{\bar{S}_{2t}}{\bar{S}_t}. \qquad \text{(A.38)}$$

Substituing (A.36)–(A.38) into the definition of the market beta

$$\bar{\beta}_{S_nt} = \frac{\mathrm{Cov}_t\left[\frac{d\bar{S}_{nt}}{\bar{S}_{nt}}, \frac{d\bar{S}_t}{\bar{S}_t}\right]}{\mathrm{Var}_t\left[\frac{d\bar{S}_t}{\bar{S}_t}\right]},$$

gives the equilibrium market beta of the normal and no-dividend stocks for the benchmark period as reported in (25)–(26).

During the main period $t < \tau$, the aggregate stock market price is given by $S_t = S_{1t} + S_{2t}$, with dynamics

$$\begin{aligned} \frac{dS_t}{S_t} + \frac{D_{1t}}{S_t}dt &= \left(\frac{dS_{1t}}{S_{1t}} + \frac{D_{1t}}{S_{1t}}dt\right)\frac{S_{1t}}{S_t} + \frac{dS_{2t}}{S_{2t}}\frac{S_{2t}}{S_t} \\ &= \ldots dt + \left(\sigma_1\frac{S_{1t}}{S_t} + \sigma_2\rho_{12}\frac{S_{2t}}{S_t}\right)d\omega_{1t} + \frac{(1-\rho_{12}^2)\sigma_2^2 + \kappa_2 V_{2t}}{\sqrt{(1-\rho_{12}^2)\sigma_2^2 + \nu_2^2}}\frac{S_{2t}}{S_t}d\widehat{\omega}_{2t}, \qquad \text{(A.39)} \end{aligned}$$

where the second equality follows from substituting the main period stock price dynamics (A.33) and (A.34), which also yield the covariance between the individual stocks and the

aggregate stock market returns as

$$\text{Cov}_t\left[\frac{dS_{1t}}{S_{1t}},\frac{dS_t}{S_t}\right]\frac{1}{dt} = \sigma_1^2\frac{S_{1t}}{S_t}+\rho_{12}\sigma_1\sigma_2\frac{S_{2t}}{S_t}, \tag{A.40}$$

$$\text{Cov}_t\left[\frac{dS_{2t}}{S_{2t}},\frac{dS_t}{S_t}\right]\frac{1}{dt} = \rho_{12}\sigma_1\sigma_2\frac{S_{1t}}{S_t}+\left(\rho_{12}^2\sigma_2^2+\frac{((1-\rho_{12}^2)\sigma_2^2+\kappa_2 V_{2t})^2}{(1-\rho_{12}^2)\sigma_2^2+\nu_2^2}\right)\frac{S_{2t}}{S_t}. \tag{A.41}$$

Using (A.39), we obtain the aggregate stock market return variance for this period as

$$\text{Var}_t\left[\frac{dS_t}{S_t}\right]\frac{1}{dt} = \sigma_1^2\left(\frac{S_{1t}}{S_t}\right)^2+\left(\rho_{12}^2\sigma_2^2+\frac{((1-\rho_{12}^2)\sigma_2^2+\kappa_2 V_{2t})^2}{(1-\rho_{12}^2)\sigma_2^2+\nu_2^2}\right)\left(\frac{S_{2t}}{S_t}\right)^2+2\rho_{12}\sigma_1\sigma_2\frac{S_{1t}}{S_t}\frac{S_{2t}}{S_t}. \tag{A.42}$$

Substituing (A.40)–(A.42) into the definition of the market beta

$$\beta_{S_nt} = \frac{\text{Cov}_t\left[\frac{dS_{nt}}{S_{nt}},\frac{dS_t}{S_t}\right]}{\text{Var}_t\left[\frac{dS_t}{S_t}\right]},$$

gives the equilibrium market beta of the normal and no-dividend stocks for the main period as reported in (27)–(28).

The property during the main period that the market beta of no-dividend stock is higher than that of a normal stock with the same underlying risk and relative size follows by comparing (27) and (28). This property holds when

$$\sigma_1^2\frac{S_{1t}}{S_t}+\rho_{12}\sigma_1\sigma_2\frac{S_{2t}}{S_t} < \rho_{12}\sigma_1\sigma_2\frac{S_{1t}}{S_t}+\left(\rho_{12}^2\sigma_2^2+\frac{((1-\rho_{12}^2)\sigma_2^2+\kappa_2 V_{2t})^2}{(1-\rho_{12}^2)\sigma_2^2+\nu_2^2}\right)\frac{S_{2t}}{S_t},$$

which after substituting $S_{1t}/S_t = S_{2t}/S_t$ simplifies to

$$\sigma_1^2 < \left(\rho_{12}^2\sigma_2^2+\frac{((1-\rho_{12}^2)\sigma_2^2+\kappa_2 V_{2t})^2}{(1-\rho_{12}^2)\sigma_2^2+\nu_2^2}\right).$$

For the same underlying risk $\sigma_1 = \sigma_2$, the above condition always holds due to the relation (A.9)–(A.10). □

Proof of Proposition 5. Using the dynamics (10) and (A.35), we obtain the equilibrium covariance of the stock market return with the aggregate consumption growth rate during

the benchmark period $t \geq \tau$ as

$$\text{Cov}_t \left[\frac{d\bar{S}_t}{\bar{S}_t}, \frac{dY_t}{Y_t} \right] \frac{1}{dt} = \left(\alpha_1 \sigma_1^2 + \alpha_2 \rho_{12} \sigma_1 \sigma_2 \right) \frac{\bar{S}_{1t}}{\bar{S}_t} + \left(\alpha_2 \sigma_2^2 + \alpha_1 \rho_{12} \sigma_1 \sigma_2 \right) \frac{\bar{S}_{2t}}{\bar{S}_t}. \tag{A.43}$$

Substituing (A.38) and (A.43) along with the variance of the consumption growth rate during this period

$$\text{Var}_t \left[\frac{dY_t}{Y_t} \right] \frac{1}{dt} = \alpha_1^2 \sigma_1^2 + \alpha_2^2 \sigma_2^2 + 2\alpha_1 \alpha_2 \rho_{12} \sigma_1 \sigma_2,$$

into the definition of the correlation

$$\bar{\rho}_{SYt} = \frac{\text{Cov}_t \left[\frac{d\bar{S}_t}{\bar{S}_t}, \frac{dY_t}{Y_t} \right]}{\sqrt{\text{Var}_t \left[\frac{d\bar{S}_t}{\bar{S}_t} \right] \text{Var}_t \left[\frac{dY_t}{Y_t} \right]}},$$

gives the equilibrium correlation of the stock market return with the aggregate consumption growth rate during the benchmark period $t \geq \tau$ as reported in (29). Similarly, using the dynamics (10) and (A.39), we obtain the equilibrium covariance of the stock market return with the aggregate consumption growth rate during the main period $t < \tau$ as

$$\text{Cov}_t \left[\frac{dS_t}{S_t}, \frac{dY_t}{Y_t} \right] \frac{1}{dt} = (\alpha_1 + \alpha_2) \sigma_1^2 \frac{S_{1t}}{S_t} + (\alpha_1 + \alpha_2) \rho_{12} \sigma_1 \sigma_2 \frac{S_{2t}}{S_t}. \tag{A.44}$$

Substituing (A.42) and (A.44) along with the variance of the consumption growth rate during this period

$$\text{Var}_t \left[\frac{dY_t}{Y_t} \right] \frac{1}{dt} = (\alpha_1 + \alpha_2)^2 \sigma_1^2,$$

into the definition of the correlation

$$\rho_{SYt} = \frac{\text{Cov}_t \left[\frac{dS_t}{S_t}, \frac{dY_t}{Y_t} \right]}{\sqrt{\text{Var}_t \left[\frac{dS_t}{S_t} \right] \text{Var}_t \left[\frac{dY_t}{Y_t} \right]}},$$

gives the equilibrium correlation of the stock market return with the aggregate consumption growth rate during the main period $t < \tau$ as reported in (30).

The property that the correlation of the stock market return with the aggregate consumption growth rate during the main period is lower than the corresponding correlation

during the benchmark period follows by first noting that the main period correlation (after canceling out the sensitivity parameters from numerator and denominator) is given by

$$\rho_{SYt} = \frac{\sigma_1 S_{1t} + \rho_{12}\sigma_2 S_{2t}}{\sqrt{\sigma_1^2 S_{1t}^2 + \left(\rho_{12}^2\sigma_2^2 + \frac{((1-\rho_{12}^2)\sigma_2^2+\kappa_2 V_{2t})^2}{(1-\rho_{12}^2)\sigma_2^2+\nu_2^2}\right) S_{2t}^2 + 2\rho_{12}\sigma_1\sigma_2 S_{1t}S_{2t}}},$$

and the benchmark period correlation when the sensitivities are evaluated at their relative size $\alpha_n/(\alpha_1+\alpha_2) = \bar{S}_{nt}/(\bar{S}_{1t}+\bar{S}_{2t})$ becomes

$$\bar{\rho}_{SYt} = \frac{\sqrt{\sigma_1^2\bar{S}_{1t}^2 + \sigma_2^2\bar{S}_{2t}^2 + 2\rho_{12}\sigma_1\sigma_2\bar{S}_{1t}\bar{S}_{2t}}}{\sqrt{\sigma_1^2\bar{S}_{1t}^2 + \sigma_2^2\bar{S}_{2t}^2 + 2\rho_{12}\sigma_1\sigma_2\bar{S}_{1t}\bar{S}_{2t}}} = 1.$$

Rearranging yields the result $\rho_{SYt} < \bar{\rho}_{SYt}$ if and only if

$$\sigma_1^2 S_{1t}^2 + \left(\rho_{12}^2\sigma_2^2 + \frac{((1-\rho_{12}^2)\sigma_2^2+\kappa_2 V_{2t})^2}{(1-\rho_{12}^2)\sigma_2^2+\nu_2^2}\right) S_{2t}^2 + 2\rho_{12}\sigma_1\sigma_2 S_{1t}S_{2t} > (\sigma_1 S_{1t} + \rho_{12}\sigma_2 S_{2t})^2,$$

and this strict inequality always holds since $((1-\rho_{12}^2)\sigma_2^2+\kappa_2 V_{2t})^2 > 0$. □

Proof of Proposition 6. During the benchmark period $t \geq \tau$, the aggregate stock market price is given by $\bar{S}_t = \bar{S}_{1t} + \bar{S}_{2t}$ with the dynamics as in (A.35), which immediately gives the stock market risk premium as the (relative size) weighted-average of the normal and no-dividend stock risk premiums as in (31). The benchmark period stock market volatility (32) is given by the square root of (A.38).

During the main period $t < \tau$, the aggregate stock market price is given by $S_t = S_{1t} + S_{2t}$ with the dynamics as in (A.39), which immediately gives the stock market risk premium as the (relative size) weighted-average of the normal and no-dividend stock risk premiums as in (33). The main period stock market volatility (34) is given by the square root of (A.42). □

Proof of Proposition 7. During the benchmark period $t \geq \tau$, by no-arbitrage, the short-term asset price is given by

$$\bar{S}_{t,T} = \frac{1}{\xi_t}\mathrm{E}_t\left[\int_t^T \xi_u D_u du\right],$$

where the aggregate dividend is $D_u = D_{1u} + D_{2u}$ for all $u \geq t$, and the state price density is as in (A.11) with the benchmark period aggregate consumption dynamics in (10). Using the

individual stock results in the proof of Proposition 1

$$\begin{aligned}\frac{1}{\xi_t}\mathrm{E}_t\left[\int_t^T \xi_u D_{1u}du\right] &= \int_t^T e^{-(\bar{r}_1-\mu_1)(u-t)}duD_{1t} = \frac{1-e^{-(\bar{r}_1-\mu_1)(T-t)}}{\bar{r}_1-\mu_1}D_{1t},\\ \frac{1}{\xi_t}\mathrm{E}_t\left[\int_t^T \xi_u D_{2u}du\right] &= \int_t^T e^{-(\bar{r}_2-\mu_2)(u-t)}duD_{2t} = \frac{1-e^{-(\bar{r}_2-\mu_2)(T-t)}}{\bar{r}_2-\mu_2}D_{2t},\end{aligned}$$

where $\bar{r}_1$, $\bar{r}_2$ are as in (15)–(16), we obtain the benchmark period short-term asset price as

$$\bar{S}_{t,T} = \bar{h}_{1t,T}\bar{S}_{1t} + \bar{h}_{2t,T}\bar{S}_{2t}, \tag{A.45}$$

where $\bar{S}_{1t}$, $\bar{S}_{2t}$ are as in (11)–(12) and the deterministic processes $\bar{h}_{nt,T}$, for $n = 1, 2$, are as in (39).

The benchmark period mean return of the short-term asset $\bar{r}_{S_{t,T}}$ is given by adding the interest rate to its risk premium:

$$\bar{r}_{S_{t,T}} = \bar{r} - \frac{d\xi_t}{\xi_t}\frac{d\bar{S}_{t,T}}{\bar{S}_{t,T}}. \tag{A.46}$$

Applying Itô's Lemma to the benchmark period short-term asset price (A.45) leads to the continuous dynamics for the short-term asset price as

$$\frac{d\bar{S}_{t,T}}{\bar{S}_{t,T}} = \ldots dt + \frac{\bar{h}_{1t,T}\bar{S}_{1t}}{\bar{h}_{1t,T}\bar{S}_{1t}+\bar{h}_{2t,T}\bar{S}_{2t}}\frac{d\bar{S}_{1t}}{\bar{S}_{1t}} + \frac{\bar{h}_{2t,T}\bar{S}_{2t}}{\bar{h}_{1t,T}\bar{S}_{1t}+\bar{h}_{2t,T}\bar{S}_{2t}}\frac{d\bar{S}_{2t}}{\bar{S}_{2t}},$$

which after substituting into (A.46) along with the mean return relation for stocks (A.30) leads to

$$\bar{r}_{S_{t,T}} = \bar{r} + \frac{\bar{h}_{1t,T}\bar{S}_{1t}}{\bar{h}_{1t,T}\bar{S}_{1t}+\bar{h}_{2t,T}\bar{S}_{2t}}(\bar{r}_1-\bar{r}) + \frac{\bar{h}_{2t,T}\bar{S}_{2t}}{\bar{h}_{1t,T}\bar{S}_{1t}+\bar{h}_{2t,T}\bar{S}_{2t}}(\bar{r}_2-\bar{r}).$$

After canceling out the interest rates gives the benchmark period equilibrium short-term asset mean return as in (35).

The equilibrium mean return of the stock market (36) is simply given by the short-term asset mean return evaluated at $\bar{h}_{nt,T} = 1$, since $\lim_{T\to\infty}\bar{S}_{t,T} = \bar{S}_t$ and $\lim_{T\to\infty}\bar{h}_{nt,T} = 1$ for $n = 1, 2$.

During the main period $t < \tau$, by no-arbitrage, the short-term asset price is given by

$$S_{t,T} = \frac{1}{\xi_t} \mathrm{E}_t \left[\int_t^T \xi_u D_u du \right],$$

where the aggregate dividend is $D_u = D_{1u} + D_{2u} \mathbf{1}_{\{u>\tau\}}$ for all $u \geq t$. However, the relevant stochastic discount factor dynamics is not immediately clear since the no-dividend stock may start paying dividends before the short-term asset maturity, hence altering the stochastic discount factor shocks. To determine the short-term asset price, we first find the short-term asset price for a fixed τ denoted by $S^{\tau}_{t,T}$, which is decomposed into two cases, $T \leq \tau$ and $T > \tau$, as

$$S^{\tau}_{t,T} = S^{\tau}_{t,T} \mathbf{1}_{\{T \leq \tau\}} + S^{\tau}_{t,T} \mathbf{1}_{\{T>\tau\}}. \tag{A.47}$$

Then by taking the expectation of $S^{\tau}_{t,T}$ with respect to $\tau - t$, we determine the main period short-term asset price $S_{t,T}$. In the first case $T \leq \tau$, the aggregate dividend is $D_u = D_{1u}$ for all $t \leq u < T$, and the state price density is as in (A.11) with the main period aggregate consumption dynamics in (10). In this case, we have

$$S^{\tau}_{t,T} = \frac{1}{\xi_t} \mathrm{E}_t \left[\int_t^T \xi_u D_{1u} du \right],$$

and using the individual stock results in the proof of Proposition 1

$$\frac{1}{\xi_t} \mathrm{E}_t \left[\int_t^T \xi_u D_{1u} du \right] = \int_t^T e^{-(r_1 - \mu_1)(u-t)} du D_{1t} = \frac{1 - e^{-(r_1-\mu_1)(T-t)}}{r_1 - \mu_1} D_{1t},$$

where r_1 is as in (17), we obtain the short-term asset price for a fixed τ in the first case as

$$S^{\tau}_{t,T} = \frac{1 - e^{-(r_1-\mu_1)(T-t)}}{r_1 - \mu_1} D_{1t}. \tag{A.48}$$

We note that in this case when the no-dividend stock does not start paying dividends for sure until the maturity of the short-term asset, the short-term asset price becomes proportional to the normal stock price (13), and hence its mean return becomes identical to the normal stock mean return. In the second case $T > \tau$, the aggregate dividend is $D_u = D_{1u}$ and the state price density is as in (A.11) with the main period aggregate consumption dynamics in (10) for all $t \leq u < \tau$, but the aggregate dividend is $D_u = D_{1u} + D_{2u}$ and the state price

density is as in (A.11) with the benchmark period aggregate consumption dynamics in (10) for all $\tau \leq u < T$. In this case, we have

$$S_{t,T}^{\tau} = \frac{1}{\xi_t} \mathrm{E}_t \left[\int_t^{\tau} \xi_u D_{1u} du + \xi_\tau \bar{S}_{\tau,T} \right],$$

where the time-τ short-term asset price is as in the benchmark period and is given by

$$\bar{S}_{\tau,T} = \frac{1 - e^{-(\bar{r}_1 - \mu_1)(T-\tau)}}{\bar{r}_1 - \mu_1} D_{1\tau} + \frac{1 - e^{-(\bar{r}_2 - \mu_2)(T-\tau)}}{\bar{r}_2 - \mu_2} D_{2\tau}.$$

Using the individual stock results in the proof of Proposition 1

$$\frac{1}{\xi_t} \mathrm{E}_t \left[\int_t^{\tau} \xi_u D_{1u} du \right] = \int_t^{\tau} e^{-(r_1 - \mu_1)(u-t)} du D_{1t} = \frac{1 - e^{-(r_1 - \mu_1)(\tau - t)}}{r_1 - \mu_1} D_{1t},$$

and

$$\frac{1}{\xi_t} \mathrm{E}_t \left[\xi_\tau \bar{S}_{\tau,T} \right] = \frac{1 - e^{-(\bar{r}_1 - \mu_1)(T-\tau)}}{\bar{r}_1 - \mu_1} \frac{1}{\xi_t} \mathrm{E}_t \left[\xi_\tau D_{1\tau} \right] + \frac{1 - e^{-(\bar{r}_2 - \mu_2)(T-\tau)}}{\bar{r}_2 - \mu_2} \frac{1}{\xi_t} \mathrm{E}_t \left[\xi_\tau D_{2\tau} \right],$$

with

$$\begin{aligned} \frac{1}{\xi_t} \mathrm{E}_t \left[\xi_\tau D_{1\tau} \right] &= e^{-(r_1 - \mu_1)(\tau - t)} D_{1t}, \\ \frac{1}{\xi_t} \mathrm{E}_t \left[\xi_\tau D_{2\tau} \right] &= e^{-(r_2 - \mu_2)(\tau - t)} \widehat{D}_{2t}, \end{aligned}$$

we obtain the short-term asset price for a fixed τ in the second case as

$$S_{t,T}^{\tau} = \frac{1 - e^{-(r_1 - \mu_1)(\tau - t)}}{r_1 - \mu_1} D_{1t} + \frac{1 - e^{-(\bar{r}_1 - \mu_1)(T-\tau)}}{\bar{r}_1 - \mu_1} e^{-(r_1 - \mu_1)(\tau - t)} D_{1t} + \frac{1 - e^{-(\bar{r}_2 - \mu_2)(T-\tau)}}{\bar{r}_2 - \mu_2} e^{-(r_2 - \mu_2)(\tau - t)} \widehat{D}_{2t}. \tag{A.49}$$

Finally, substituting (A.48)–(A.49) into (A.47), and taking the expectation with respect to $\tau - t$, using the independent exponential distribution for its density given by (A.17), we

determine the main period short-term asset price $S_{t,T}$ as

$$\begin{aligned} S_{t,T} &= \lambda_2 \int_0^\infty \frac{1-e^{-(r_1-\mu_1)(T-t)}}{r_1-\mu_1} D_{1t} \mathbf{1}_{\{u \geq T-t\}} e^{-\lambda_2 u} du \\ &+\lambda_2 \int_0^\infty \left[\frac{1-e^{-(r_1-\mu_1)u}}{r_1-\mu_1} + \frac{1-e^{-(\bar{r}_1-\mu_1)(T-t-u)}}{\bar{r}_1-\mu_1} e^{-(r_1-\mu_1)u} \right] D_{1t} \mathbf{1}_{\{T-t>u\}} e^{-\lambda_2 u} du \\ &+\lambda_2 \int_0^\infty \frac{1-e^{-(\bar{r}_2-\mu_2)(T-t-u)}}{\bar{r}_2-\mu_2} e^{-(r_2-\mu_2)u} \widehat{D}_{2t} \mathbf{1}_{\{T-t>u\}} e^{-\lambda_2 u} du, \end{aligned}$$

which after removing the indicator functions becomes

$$\begin{aligned} S_{t,T} &= \frac{1-e^{-(r_1-\mu_1)(T-t)}}{r_1-\mu_1} D_{1t} \lambda_2 \int_{T-t}^\infty e^{-\lambda_2 u} du \\ &+\frac{1}{r_1-\mu_1} D_{1t} \lambda_2 \int_0^{T-t} \left(1-e^{-(r_1-\mu_1)u}\right) e^{-\lambda_2 u} du \\ &+\frac{1}{\bar{r}_1-\mu_1} D_{1t} \lambda_2 \int_0^{T-t} \left(1-e^{-(\bar{r}_1-\mu_1)(T-t-u)}\right) e^{-(r_1-\mu_1)u} e^{-\lambda_2 u} du \\ &+\frac{1}{\bar{r}_2-\mu_2} \widehat{D}_{2t} \lambda_2 \int_0^{T-t} \left(1-e^{-(\bar{r}_2-\mu_2)(T-t-u)}\right) e^{-(r_2-\mu_2)u} e^{-\lambda_2 u} du. \end{aligned}$$

Evaluating the simple exponential integrals and rearranging yield

$$S_{t,T} = h_{1t,T} S_{1t} + h_{2t,T} S_{2t}, \tag{A.50}$$

where S_{1t}, S_{2t} are as in (13)–(14) and the deterministic processes $h_{1t,T}$, $h_{2t,T}$ are as in (40)–(41).

The main period mean return of the short-term asset $r_{S_{t,T}}$ is given by adding the interest rate to its risk premia:

$$r_{S_{t,T}} = r - \frac{d\xi_t}{\xi_t} \frac{dS_{t,T}}{S_{t,T}}. \tag{A.51}$$

Applying Itô's Lemma to the main period short-term asset price (A.50) leads to the continuous dynamics for the short-term asset price as

$$\frac{dS_{t,T}}{S_{t,T}} = \ldots dt + \frac{h_{1t,T} S_{1t}}{h_{1t,T} S_{1t} + h_{2t,T} S_{2t}} \frac{dS_{1t}}{S_{1t}} + \frac{h_{2t,T} S_{2t}}{h_{1t,T} S_{1t} + h_{2t,T} S_{2t}} \frac{dS_{2t}}{S_{2t}},$$

which after substituting into (A.51) along with the mean return relation for stocks (A.32)

leads to

$$r_{S_{t,T}} = r + \frac{h_{1t,T}S_{1t}}{h_{1t,T}S_{1t} + h_{2t,T}S_{2t}}(r_1 - r) + \frac{h_{2t,T}S_{2t}}{h_{1t,T}S_{1t} + h_{2t,T}S_{2t}}(r_2 - r).$$

After canceling out the interest rates gives the main period equilibrium short-term asset mean return as in (37).

The equilibrium mean return of the stock market (38) is simply given by the short-term asset mean return evaluated at $h_{nt,T} = 1$, since $\lim_{T\to\infty} S_{t,T} = S_t$ and $\lim_{T\to\infty} h_{nt,T} = 1$ for $n = 1, 2$.

The property during the main period that the mean return of the short-term asset is higher than that of the stock market if and only if $h_{1t,T} > h_{2t,T}$ when the stocks have the same underlying risk $\sigma_1 = \sigma_2$ follows from comparing (37) and (38). Since the mean returns are weighted averages of the individual stock mean returns and the normal stock has a higher mean return $r_1 > r_2$ when stocks have the same underlying risk $\sigma_1 = \sigma_2$ (Proposition 2), this property holds if the weight that is given to the normal stock in the short-term asset mean return is greater than the corresponding weight in the stock market mean return, that is

$$\frac{h_{1t,T}S_{1t}}{h_{1t,T}S_{1t} + h_{2t,T}S_{2t}} > \frac{S_{1t}}{S_{1t} + S_{2t}},$$

and this inequality holds if and only if $h_{1t,T} > h_{2t,T}$. □

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