The interdependence of bank capital and liquidity*

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Abstract

Bank runs may be associated with inefficient liquidation of good investment projects and costly fire sales. We analyze the interdependent effects of bank capital and liquidity on financial stability in a global game model, where banks' failure probabilities are endogenously determined and depend on their balance sheet choices. The main insight of our analysis is that regulation should be designed in a way to consider both sides of the balance sheet. Capital and liquidity regulation are perfect substitutes in dealing with individual bank stability (micro-prudential perspective), while both are needed to deal with system-wide crises (macro-prudential perspective).

Keywords: illiquidity, insolvency, capital and liquidity regulation

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1 Introduction

The 2007-2009 financial crisis was a milestone for financial regulation, leading to significant reforms to the existing capital regulation and the introduction of a new set of liquidity requirements. In particular, banks

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have been required to hold higher capital buffers to reduce their exposure to solvency-driven crises and, at the same time, to increase their liquidity holdings to reduce liquidity mismatch and the consequent risk of liquidity-driven crises. The introduction of a new set of liquidity requirements, namely the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR), as complements to the existing and improved capital-based regulation, has led to a debate in the academic and policy arena on the effective need of all these regulatory tools, their interaction, as well as their potential contrasting effects for financial stability.

Bank (il)liquidity and (in)solvency are closely intertwined concepts and often difficult to tell apart when a crisis manifests. On the one hand, liquidity-driven crises can spur solvency issues; on the other hand, fears about bank solvency may precipitate liquidity problems. Furthermore, when a crisis is underway and a bank faces a large outflow of funds, it becomes very difficult to assess the ultimate source of these withdrawals, which, in turn, may limit policymakers' ability to intervene effectively. It is precisely this close link between solvency and liquidity crises that motivates the discussion about the joint effects that capital and liquidity may have on financial stability.

To visualize the issue, consider a simple bank balance as in Table 1. Bank stability depends negatively on both its leverage (i.e., $\frac{D}{L+I}$) and the proportion of illiquid assets (i.e., $\frac{I}{I+L}$): A bank with a larger share of short-term funding and a larger proportion of illiquid assets is more exposed to roll over risk than a bank with more equity and more liquid assets. It follows that increasing equity (E), while keeping constant the asset side, has a similar effect on stability as increasing the proportion of liquid assets (L), while keeping the liability side constant. This means that, in this example, capital and liquidity are substitutes in terms of their effects on stability.

Assets	Liabilities
Liquid assets (L)	Short-term debt (D)
Illiquid assets (I)	Equity (E)

Table 1: A simplified bank balance sheet

This simple example raises a number of important questions: What are the effects of changes in the level of bank capitalization and portfolio liquidity on bank stability? Do capital and liquidity interact in affecting bank stability? Are capital and liquidity requirements substitutes or complements?

To tackle these questions, we first build a simple two-period global game model with one bank issuing short term debt and equity, and investing in a risky portfolio consisting of liquid and illiquid assets, whose final return depends on the fundamentals of the economy. The portfolio composition determines the trade-off between intermediate and final date portfolio returns, whereby a higher proportion of liquid assets in the portfolio leads to a higher (safe) return at the interim date, but to a lower (risky) return at the final date. This determines bank available resources and, together with the capital structure, affects depositors' decision to roll over their debt and, consequently, the likelihood of a bank failure.

In this setting, the bank fails as a consequence of a massive withdrawal of funds by debt holders at the interim date (i.e., a run). A debt holder's withdrawal decision is based on an imperfect signal regarding date 2 bank portfolio return that each debt holder receives at date 1, as the signal provides information about the fundamental of the economy and the action of the other debt holders.

As standard in the global game literature (see e.g., Morris and Shin, 1998, 2003; Rochet and Vives, 2004; Goldstein and Pauzner, 2005), the equilibrium outcome is that a run occurs when the fundamentals of the economy are below a unique threshold. Within the range of fundamentals where they occur, crises can be classified into either solvency- or liquidity-driven crises. The former happen at the lower part of the crisis region where the signal on the fundamentals is so low that not rolling over the debt claim at the interim date is a dominant strategy for debt holders. The latter occur for an intermediate region of fundamental and are due to the presence of strategic complementarity among debt holders, in that each of them does not roll over out the self-fulfilling belief that others will do the same.

Importantly, in the model the crisis threshold crucially depends on the level of bank capitalization and its portfolio liquidity. Thus, our model delivers a first set of results about the differential effects that a change in the levels of bank capitalization and portfolio liquidity have on the crisis threshold. In particular, we show that the effect of capital and liquidity on the likelihood of crises depends on whether the bank has low, intermediate or high initial levels of capitalization and portfolio liquidity. For a bank with very little capital and/or very illiquid portfolios, an increase in capital or liquidity worsens the probability of liquidity crises. As this bank already faces a high risk of failure at the interim date, higher levels of capitalization or portfolio liquidity raise debt holders' repayment at the interim date, thus increasing further their incentives to run on the bank. By contrast, for a bank with intermediate levels of capital or liquidity, higher capitalization or increased portfolio liquidity reduce the occurrence of liquidity crises. As the risk of failing at the interim date is limited for this bank, a capital or liquidity injection increases further their ability to withstand debt holders' withdrawals and, in turn, debt holders' repayment from rolling over the debt claim until the final date. Finally, for a bank with high initial levels of capital or liquidity, higher capitalization is beneficial for stability, while an increase in portfolio liquidity is detrimental. The reason is that this bank is very little exposed to strategic complementarities among debt holders and mainly faces the risk of solvency crises, for which higher capital is beneficial while liquidity is detrimental.

The comparative statics exercise delivers some initial implications for the design of regulation. First, a one-size-fits-all approach, where all banks are subject to the same requirements, may have undesirable consequences for some banks, especially for those who would need to strengthen their stability the most. Second, capital and liquidity requirements should be designed considering both sides of banks' balance sheets. In this respect, our analysis supports regulatory instruments like the risk-weighted capital ratio, the liquidity coverage ratio and the net stable funding ratio that essentially specify a ratio between banks' assets and liabilities (see, Cecchetti and Kashyap, 2018).

Building on the comparative statics exercise, we then analyze the bank's choice of capital structure, portfolio liquidity and debt holders' repayment in the market equilibrium and show that the allocation is inefficient. The reason is that the bank chooses intermediate levels of capitalization and portfolio liquidity that expose it to runs and so to the premature liquidation of good investment projects. Turning to the analysis of regulation, we show that in this scenario capital and liquidity requirements are both fully effective in reducing instability and so remove the associated inefficiency. To achieve this, the regulator could constrain either bank capital or liquidity choices. The important detail is that, as for the RWC, LCR and NSFR, regulation should link asset and liability side of bank balance sheet. This result also suggests that capital and liquidity regulation are perfect substitutes from a micro-prudential perspective, that is in addressing inefficient individual bank failures.

The optimal regulatory mix changes when we extend the model to embed the possibility of fire sales. To do this, we extend the model by considering the presence of multiple symmetric banks sharing the same fundamental. The key difference relative to the baseline model is that banks sell shares of their portfolios in a secondary asset market to outside investors to meet early withdrawals. Outside investors are endowed with limited resources and may be less able than banks in managing the portfolios they acquire. As a result, the (per unit) amount that bank can raise from the market does no longer correspond to their portfolio liquidity choice, but it depends also on aggregate market conditions. Transferring assets outside the banking sector then may entail a loss of resources, which increases with the size and illiquidity of the pool of assets on sale in the market.

This specification modifies the analysis in two important dimensions. First, it affects debt holders' rollover decision by introducing another source of strategic complementarity. This implies that banks are strategically connected and their failures may spur from contagion: Banks fail because their debt holders are concerned about the health of other banks in the system. Second, the extended framework features an additional inefficiency in banks' decisions since banks do not internalize the effect that their individual choices have on the secondary asset market and so on the other banks.

Overall, in the economy with multiple banks, crises are more likely and also more costly in that good projects are liquidated more often and their premature liquidation entails a larger cost than in the baseline model. In this scenario, both liquidity and capital requirements are needed. This means that from a macro-prudential perspective, liquidity and capital requirements are complements: The former are used to prevent the occurrence of fire sales; the latter are needed to prevent the premature liquidation of profitable investment projects.

Our analysis of the impact of capital and liquidity on bank stability is conducted in a framework where the inefficiencies of the unregulated market equilibrium are all associated with the premature liquidation of bank portfolio. In doing this, we disregard other possible sources of inefficiencies that may motivate the use of capital and liquidity regulation, such as, for example, a moral hazard problem on the side of bank managers.

A number of recent papers has looked at the role and implications of the newly introduced liquidity regulation, also in connection with capital requirements (see, e.g., Walther, 2015; Calomiris, Heider and Hoerova, 2015; and Diamond and Kashyap, 2016). Among those papers, Vives (2014) and König (2015) use global games to study the implications of capital and liquidity on the probability of banking crises. Both papers build on the bank run model developed by Rochet and Vives (2004) and perform a comparative statics exercise on the run threshold. Vives (2014) finds that both capital and liquidity are beneficial for stability, while, as in our paper, König (2015) shows that the effect of liquidity is more mixed since liquid assets are less profitable than illiquid ones in the long run. Although sharing the global game approach, our framework features a richer structure for debt holders' payoff similar to Goldstein and Pauzner (2005). This introduces additional effects of capital and liquidity on bank stability. Similarly to our paper, Schilling (2016) builds her analysis on Goldstein and Pauzner (2005) and shows that an increase in the level of bank capital may harm stability when the liquidation value of banks' assets is low. Besides capturing additional effects of bank capital and portfolio liquidity on stability, our paper differs from Vives (2014), König (2015) and Schilling (2016) in that we endogenize bank capital structure and portfolio liquidity, as well as the remuneration to debt holders. This allows us to highlight several inefficiencies of the market equilibrium and derive implications for optimal regulation.

In this sense, our paper is closely related to the recent contribution by Kashyap, Tsomocos and Vardoulakis (2017), who also use global game techniques to pin down bank default probabilities and characterize the optimal combination of capital and liquidity regulation. Yet, there are two important differences between our framework and theirs. First, the two papers differ in terms of the sources of inefficiencies that regulation is meant to address. In Kashyap, Tsomocos and Vardoulakis (2017), the focus is on the interaction between run and credit risk, while in our paper is on the run risk and fire sales externalities. Second, the two models differ in bank debt holders' payoffs structure, thus leading to different results in terms of the effect that both capital and liquidity have on the crises thresholds. Specifically, in contrast to our analysis, in Kashyap et al. (2017) both capital and liquidity regulation always reduce the probability that a run occurs and improves welfare, while in our framework results are more nuanced

The key aspect of our study is the ability to derive an endogenous probability of a bank crisis and study how it is affected by changes in bank capitalization and portfolio liquidity. To do this, we rely on the global game techniques as developed in the literature originating with Carlsson and van Damme (1993) (see Morris and Shin, 2003 for a survey on the theory and applications of global games). Our paper is close to two contributions in this literature. First, it shares the idea of rollover game with Eisenbach (2017), although in a framework where banks also raise equity and choose the liquidity-return trade-off of their portfolio. Second, it faces the same technical challenge of characterizing the existence of a unique equilibrium in the absence of global strategic complementarities as in Goldstein and Pauzner (2005).

The paper proceeds as follows. Section 2 presents the baseline model. Section 3 characterizes the equilibrium with one bank, while Section 4 that with multiple banks. In both sections, we first derive debt holders' rollover decision. Then, we characterize banks' choices and identify the inefficiencies of the unregulated equilibrium. Finally, we analyzes the effectiveness of regulation in addressing them. Section 5 contains concluding remarks. All proofs are contained in the appendix.

2 The baseline model with one bank

We start by considering a simple three date ($t = 0, 1, 2$) economy, with a representative bank and a continuum $[0, 1]$ of risk neutral investors, each endowed with one unit of resources at date 0 and nothing thereafter.

At date 0, the bank raises one unit of funds from investors and invests in a risky portfolio consisting of both liquid and illiquid assets. For each unit invested at date 0, the portfolio returns a per unit liquidation value $\ell \in [0,1]$ at date 1 and a stochastic return $R(\theta)(1-\alpha\ell)$ at date 2. The liquidation value at date 1 is chosen by the bank at date 0 and captures the degree of portfolio liquidity, while the variable θ represents the aggregate state of the economy and is uniformly distributed over $[0,1]$ with $R'(\theta) > 0$ and $E_\theta[R(\theta)(1-\alpha\ell)] > 1$. Our specification entails a standard liquidity-return trade-off: A more liquid portfolio (i.e., with a higher ℓ) yields a higher date 1 return, but entails a lower expected return at date 2, so that through ℓ the bank chooses a point on a liquidity-return frontier.[1]

At date 0, the bank chooses to raise a fraction k of funds as equity and the remaining fraction $1-k$ as short-term debt.[2] Equity finance entails a cost $\rho > 1$ for the bank, representing the opportunity cost of funds for equity holders. By contrast, the debt contract specifies a promised (gross) interest rate $r_1 = 1$ at date 1 and $r_2 \geq r_1$ date 2. The debt market is perfectly competitive so the bank will always set r_2 at the level required for debt holders to recover their opportunity cost of funds, which we normalize to 1, and be willing to participate.

The assumption $\rho > 1$ implies that bank capital is a more expensive form of financing than debt. This assumption, which is standard in the literature (see e.g., Hellmann, Murdock and Stiglitz, 2000; Repullo, 2004; Allen, Carletti and Marquez, 2011), has been recently endogenized on the basis of market segmentation (see e.g., Allen, Carletti and Marquez, 2015) or the existence of costs associated with the issuance of outside equity (see Harris, Opp and Opp, 2017) and empirically validated based on a different tax treatment between equity and debt (see e.g., Schepens, 2016).[3]

The bank satisfies early withdrawals by liquidating shares of its portfolio. The bank fails at either date when it does not have enough resources to repay the promised due debt repayments. In this case, debt holders receive a share of the bank's available resources while equity holders obtain nothing.

The aggregate state of the economy θ is realized at the beginning of date 1, but is not publicly observed

[1] In this sense, our approach is a reduced form of a framework in which the bank chooses how to allocate its funds between liquid and illiquid assets. It has the advantage of improving tractability by allowing us to abstract from issues like the pecking order of assets to liquidate at date 1 and, relatedly, information available to banks about θ.

[2] We abstract from explaining the optimality of short-term debt. This has been justified in the literature in the presence of asymmetric information problems in credit markets (see e.g., Flannery, 1986 and Diamond, 1991), conflicts between banks' managers and shareholders (see e.g., Calomiris and Kahn, 1991; Diamond and Rajan, 2001 and Eisenbach, 2017) and idiosyncratic liquidity shocks to bank depositors (e.g., Diamond and Dybvig, 1983). These explanations also support the result that (short-term) debt is a cheaper form of bank financing. In the context of our model, taking the use of short-term debt as given helps us focus on the associated coordination problems leading to bank fragility and, in turn, on the need of regulatory intervention.

[3] In most jurisdictions, the cost of debt is tax-deductible, while dividends are not. Schepens (2016) shows that a reduction in the tax discrimination between debt and equity financing leads to a significant increase in bank capital ratios.

until date 2. After θ is realized, at date 1 each debt holder receives a private signal s_i of the form

$$s_i = \theta + \varepsilon_i, \tag{1}$$

where ε_i are small error terms, which are independently and uniformly distributed over $[-\varepsilon, +\varepsilon]$. Based on this signal, debt holders decide whether to withdraw at date 1 or roll over the debt until date 2.

The timing of the model is as follows. At date 0 the bank chooses the capital structure $\{k, 1-k\}$, the debt repayment r_2 and the level of portfolio liquidity ℓ so to maximize its expected profit. At date 1, after receiving the private signal about the state of the fundamentals θ, each debt holder decides whether to roll the debt over. At date 2, the bank portfolio return realizes and all claims are paid, if the bank is solvent. The model is solved backwards.

3 The equilibrium with one bank

In this section, we derive the equilibrium in an economy with one bank. We first derive the unregulated equilibrium by characterizing debt holders' rollover decisions at date 1 for given levels of capital k, portfolio liquidity ℓ and debt repayment r_2. This allows us to pin down the probability of a bank failure. Then, we characterize the bank's choice of capitalization, portfolio liquidity and debt repayment. Finally, we derive the optimal regulatory intervention that addresses the inefficiency plaguing the unregulated economy.

3.1 Debt holders' rollover decisions

Debt holders decide whether to withdraw at date 1 based on the signal s_i they receive since this provides information on both θ and other debt holders' actions. When the signal is high, a debt holder attributes a high posterior probability to the event that the bank portfolio yields a high return and, at the same time, he infers that the other debt holders have also received a high signal. This overall lowers his belief about the likelihood of a bank failure and, as a result, also his own incentive to withdraw at date 1. Conversely, when the signal is low, a debt holder has a high incentive not to roll over the debt, as he attributes a high likelihood to the possibility that the return of the bank portfolio is low and that the other investors withdraw their debt claim at date 1. This suggests that debt holders withdraw at date 1 when the signal is low enough, and roll their debt claims over until date 2 when the signal is sufficiently high.

To show this formally, we first examine two regions corresponding to extremely bad and extremely good realization of the aggregate state variable θ, where each debt holder's action is based only on the realization of θ irrespective of his beliefs about the others' behavior. We start with the lower region.

Lower dominance region. When θ is very low, not rolling over the debt at date 1 is a dominant strategy for every debt holder. This is the case when, upon receiving his signal, a debt holder expects to receive at date 2 a lower payoff than the return 1 he would obtain by withdrawing at date 1, even if all other debt holders wait until date 2. Since $r_2 \geq 1$, this occurs when the bank fails at date 2 and the debt holder obtains the pro-rata share $\frac{R(\theta)(1-\alpha\ell)}{(1-k)}$. We then denote as $\underline{\theta}\,(k,\ell)$ the value of θ that solves

$$\frac{R\,(\theta)\,(1-\alpha\ell)}{(1-k)} = 1 \tag{2}$$

so that the interval $[0, \underline{\theta}\,(k,\ell))$ identifies the range of values of θ where banking crises are only driven by low fundamentals.[4]

Upper dominance region. The upper dominance region of θ corresponds to the range $\left(\overline{\theta}, 1\right]$ in which the state of the economy is so good that rolling over is a dominant strategy. As in Goldstein and Pauzner (2005), we construct this region by assuming that in the range $(\overline{\theta}, 1]$ the bank investment is safe, i.e., it yields $R(1)\,(1-\alpha\ell) > 1$ both at dates 1 and 2. Given that $1 \leq r_2 < R\,(1)\,(1-\alpha\ell)$, this ensures that repaying 1 to the withdrawing debt holders does not affect bank's ability to repay r_2 to the debt holders rolling over the debt until date 2. Then, when an investor receives a signal such that he believes that the fundamental θ is in the upper dominance region, he is certain to receive the promised payment r_2, irrespective of his beliefs on other debt holders' action. As a consequence, he does not have any incentive to withdraw early. In what follows, we assume that $\overline{\theta} \to 1$.

The Intermediate Region. When the signal indicates that θ is in the intermediate range $\left[\underline{\theta}(k,\ell), \overline{\theta}\right]$, a debt holder's rollover decision depends on the realization of θ as well as on his beliefs regarding other debt holders' actions. Debt holders may have the incentive not to rollover their debt claims at the interim date as they fear that others would do the same. Their concern is that a large number of withdrawals at date 1 would force a massive liquidation of the bank portfolio, thus depleting bank's available resources at date 2 and, in turn, their expected payoff. To see this, we first calculate a debt holder's payoff differential between

[4] For the lower dominance region to exist, it must be the case that there are feasible values of θ for which all debt holders receive a signal below $\underline{\theta}\,(k,\ell)$. Since the noise contained in the signal is at most ε, when $s_i < \underline{\theta}\,(k,\ell) - \varepsilon$ all debt holders receive a signal below $\underline{\theta}\,(k,\ell)$. This holds when $\theta < \underline{\theta}\,(k,\ell) - 2\varepsilon$.

rolling over until date 2 and withdrawing at date 1 as given by

$$v(\theta, n) = \begin{cases} r_2 - 1 & \text{if } 0 \leq n < \widehat{n}(\theta) \\ \frac{R(\theta)(1-\alpha\ell)\left[1-\frac{(1-k)n}{\ell}\right]}{(1-k)(1-n)} - 1 & \text{if } \widehat{n}(\theta) \leq n \leq \overline{n} \\ 0 - \frac{\ell}{(1-k)n} & \overline{n} \leq n \leq 1 \end{cases} \tag{3}$$

where n represents the proportion of debt holders withdrawing at date 1,

$$\widehat{n}(\theta) = \frac{R(\theta)(1-\alpha\ell) - (1-k)\, r_2}{(1-k)\left[\frac{R(\theta)(1-\alpha\ell)}{\ell} - r_2\right]} \tag{4}$$

denotes the proportion of investors not rolling over the debt at date 1 pushing the bank at the brick of default at date 2, while

$$\bar{n} = \frac{\ell}{(1-k)} \tag{5}$$

corresponds to the proportion of debt holders running at date 1 forcing the bank to liquidate the entire portfolio at date 1.

A debt holder's payoffs at date 1 and 2 are illustrated in Figure 1. At date 2, a debt holder obtains the promised repayment r_2 as long as bank resources suffice (i.e., for $n < \widehat{n}(\theta)$), while he obtains the pro-rata share $\frac{R(\theta)(1-\alpha\ell)\left[1-\frac{(1-k)n}{\ell}\right]}{(1-k)(1-n)}$ for $n > \widehat{n}(\theta)$. Similarly, at date 1, a debt holder receives the promised repayment $r_1 = 1$ as long as the bank can raise enough resources by prematurely liquidating its portfolio (i.e., for $n < \overline{n}$), while he receives the pro-rata share $\frac{\ell}{(1-k)n}$ for $n > \overline{n}$.

Insert Figure 1

In the range $[0, \overline{n}]$, a debt holder's payoff differential $v(\theta, n)$ is weakly decreasing in n as long as

$$\ell < (1-k) \tag{6}$$

The condition (6) guarantees that $\widehat{n}(\theta) < 1$ and is obtained by differentiating $\frac{R(\theta)(1-\alpha\ell)\left[1-\frac{(1-k)n}{\ell}\right]}{(1-k)(1-n)}$ with respect to n. In words, it states that the value of a bank's portfolio ℓ is not enough to repay $r_1 = 1$ if all debt holders were to withdraw at date 1. When condition (6) holds, debt holders' withdrawal decisions are strategic complements.

As in Goldstein and Pauzner (2005), our model only exhibits the property of one-sided strategic complementarity since in the range $(\overline{n}, 1]$, a debt holder's incentive to roll over his debt claim until date 2 increases with n. This occurs because, when n is very large (i.e., $n > \bar{n}$), the more debt holders withdraw at date 1, the lower a debt holder's payoff from withdrawing at date 1, while the payoff at date 2 is zero. As in their framework, there exists a unique threshold equilibrium in which a debt holder withdraws if and only if his signal is below the threshold $s^*(k, \ell, r_2)$. At this signal value, the debt holder is indifferent between withdrawing at date 1 and rolling over his debt claim until date 2. The following result holds.

Proposition 1 *The model has a unique threshold equilibrium in which debt holders withdraw their debt claims at date* 1 *if they observe a signal below the threshold* $s^*(k, \ell, r_2)$ *and roll them over above. At the limit, when* $\varepsilon \to 0$, $s^*(k, \ell, r_2) \to \theta^*(k, \ell, r_2)$ *and corresponds to the solution to*

$$\int_0^{\widehat{n}(\theta)} r_2 dn + \int_{\widehat{n}(\theta)}^{\bar{n}} \frac{R(\theta)(1-\alpha\ell)\left[1 - \frac{(1-k)n}{\ell}\right]}{(1-k)(1-n)} dn - \int_0^{\bar{n}} 1 dn - \int_{\bar{n}}^1 \frac{\ell}{(1-k)n} dn = 0. \tag{7}$$

Thus, the bank is solvent and always repay the promised amounts for any $\theta > \theta^*(k, \ell, r_2)$.

The proposition states that in the range of fundamentals $\theta \leq \theta^*(k, \ell, r_2)$, a bank fails as debt holders choose not to roll over their debt claims. Such runs are driven exclusively by a bad realization of the aggregate state of the economy θ in the range $\theta < \underline{\theta}(k, \ell)$ and by the fear that other debt holders will not roll over in the range $\theta \in [\underline{\theta}(k, \ell), \theta^*(k, \ell, r_2))$. Thus, in what follows, we refer to the former as solvency-driven crises and to the latter as liquidity-driven ones. For any $\theta > \theta^*(k, \ell, r_2)$, all debt holders choose to roll over their debt claims and the bank is solvent in that it can repay the promised payment $\{r_1, r_2\}$ at date 1 and 2.[5]

Importantly, bank exposure to solvency and to liquidity crises crucially depends on the level of bank capitalization k and its portfolio liquidity ℓ, as illustrated in Figure 2. The key condition determining the relevant run threshold is whether the bank holds a combination of k and ℓ in the region above or below the curve $k^{\max}(\ell)$, where $k^{\max}(\ell)$ solves condition (6) with equality. In the region above $k^{\max}(\ell)$, $\ell \geq (1-k)$ holds and only solvency crises occur. Thus, the relevant crisis threshold is $\underline{\theta}$. The reason is that when $\ell \geq (1-k)$, there is no strategic complementarity among debt holders as the bank needs to liquidate at most one unit for each withdrawing debt holders. By contrast, in the region below $k^{\max}(\ell)$, $\ell < (1-k)$

[5] To keep the notation simple, in what follows, we denote the thresholds $\underline{\theta}(k, \ell)$ and $\theta^*(k, \ell, r_2)$ as $\underline{\theta}$ and θ^*, respectively.

holds and the relevant crisis threshold is θ^*.

Insert Figure 2

The following corollary illustrates how the levels of bank capitalization and portfolio liquidity affect the crisis thresholds for a given repayment r_2.

Corollary 1 *The threshold* $\underline{\theta}$ *decreases with the level of capital* k *and increases with portfolio liquidity* ℓ $\left(\text{i.e., } \frac{\partial \underline{\theta}}{\partial k} < 0 \text{ and } \frac{\partial \underline{\theta}}{\partial \ell} > 0\right)$. *The threshold* θ^* *i) decreases with* k *for any* $k \in \left[\widetilde{k}(\ell), 1\right]$, *and increases otherwise* $\left(\text{i.e., } \frac{\partial \theta^*}{\partial k} < 0 \text{ if } k \geq \widetilde{k}(\ell) \text{ and } \frac{\partial \theta^*}{\partial k} > 0 \text{ otherwise}\right)$; *ii) decreases with* ℓ *for any* $k \in \left(\underline{k}(\ell), \overline{k}(\ell)\right)$, *and increases otherwise* $\left(\text{i.e., } \frac{\partial \theta^*}{\partial \ell} < 0 \text{ if } \underline{k}(\ell) < k < \overline{k}(\ell) \text{ and } \frac{\partial \theta^*}{\partial \ell} > 0 \text{ otherwise}\right)$. *The boundaries* $\widetilde{k}(\ell)$, $\underline{k}(\ell)$ *and* $\overline{k}(\ell)$ *are defined in the Appendix.*

The corollary, which is also illustrated in Figure 3a and 3b, shows that changes in bank capital and portfolio liquidity affect the crisis threshold differently depending on the nature of the crisis (i.e., whether it is solvency- or liquidity-driven) and on the bank initial balance sheet composition (k, ℓ).

Insert Figure 3a and 3b

The corollary shows that capital is beneficial for solvency crises, while liquidity is detrimental. The beneficial effect of capital hinges on the fact that higher capital reduces leverage, thus leaving more resources to repay debt holders at date 2. By contrast, liquidity is detrimental because it reduces bank profitability at date 2.

The effect of capital and liquidity on liquidity crises is more involved as capital and liquidity affect debt holders' payoffs at both dates 1 and 2, as it emerges from (7). While higher capital increases debt holders' expected repayment both at date 1 and 2, higher liquidity increases a debt holder's repayment at date 1, but has an ambiguous effect on the date 2 payoff. On the one hand, an increase in liquidity is associated with a lower (per unit) portfolio return at date 2; on the other hand, more liquidity translates into lower liquidation needs and so more resources at date 2 to pay debt holders. The sign of the overall effect of changes in bank capital structure and portfolio liquidity on θ^* depends on which of these effects dominates. As shown in the corollary, this is determined by the initial bank balance sheet composition.

We start with capital. In the region below the curve $\widetilde{k}(\ell)$, banks are poorly capitalized and/or hold illiquid portfolios and thus are very exposed to a failure at date 1. As a consequence, debt holders value the positive effect of capital on their date 1 payoff more than that at date 2. The opposite is true for banks in the region above the curve $\widetilde{k}(\ell)$ since they are likely to withstand early withdrawals and survive until date 2.

Regarding liquidity, the corollary shows that a marginal increase in liquidity is only beneficial in the region between the curves $\underline{k}(\ell)$ and $\overline{k}(\ell)$. Here, banks are characterized by an intermediate level of capitalization and/or portfolio liquidity. As a consequence, they are less exposed to the risk of failure at date 1 than banks in the region below $\underline{k}(\ell)$ but are still confronted with significant strategic complementarities among debt holders' actions and, in turn, with liquidity problems. Thus, holding a more liquid portfolio allows these banks to liquidate fewer units at date 1 to repay withdrawing debt holders, thus increasing the expected payoff for waiting investors.

By contrast, liquidity is detrimental on the threshold θ^* in the region below $\underline{k}(\ell)$ and in the region between $\overline{k}(\ell)$ and $k^{\max}(\ell)$. In the former, liquidity has the same negative effect as capital in the region below $\widetilde{k}(\ell)$. In the latter, the detrimental effect of liquidity on θ^* is due to the negative effect it has on the (per unit) return of banks' portfolio at date 2 and thus on the payoff that investors can obtain at date 2. Since in this region banks are not much exposed to failure at date 1 as they hold high levels of capital and/or portfolio liquidity, the prospect of lower date 2 profitability reduces debt holders' incentives to roll over, thus increasing θ^*.

Corollary 1 has a number of interesting implications on the effects of capital and liquidity on the probability of liquidity crises. First, increasing capital or liquidity helps the banks that need it the least. A higher level of bank capitalization or portfolio liquidity improves stability for banks facing a lower θ^* thanks to their greater capitalization or portfolio liquidity, while it has a destabilizing effect for poorly capitalized banks and those holding very illiquid portfolios.

Second, the result that a marginal increase in liquidity undermines debt holders' incentives to roll over for banks in the region below the curve $\underline{k}(\ell)$, thus increasing θ^*, would also hold in the case of injection of emergency liquidity by a LOLR, which does not reduce bank's portfolio returns and thus profitability. In other words, the detrimental effect of liquidity for poorly capitalized banks with illiquid portfolios does not depend on the fact that more liquid portfolios tend to be less profitable, but it is rather due to the fact that liquidity injections are also used to repay debt holders in the event of a bank failure.

Third, the results in the corollary suggest that the timing of a regulatory/supervisory intervention is key:

If banks are asked to recapitalize and/or hold more liquidity when a crisis is already underway (or is likely), this may precipitate rather than contain bank distress.

To sum up, the analysis of the properties of the crisis threshold θ^*shows that the same increase in capital or liquidity may have very different effects on stability for weakly, moderately or strongly capitalized banks, as well as for banks with low, moderate or high portfolio liquidity. The results highlight the importance of considering the interaction between capital and liquidity to assess their effects on stability. As we will show in details below, looking at such interaction is crucial for the bank's choice of capital and liquidity, as well as for designing and evaluating capital and liquidity regulation.

3.2 Bank choice

Having computed the crisis threshold θ^*, we can now characterize the bank's decisions concerning capital k, portfolio liquidity ℓ and debt interest rate r_2. A bank chooses these variables simultaneously at date 0 to solve the following problem:

$$\max_{k,\ell,r_2} \Pi^B = \int_{\theta^*}^{1} \left[R(\theta)(1-\alpha\ell) - (1-k)\, r_2\right] d\theta - k\rho \tag{8}$$

subject to

$$IR^D\colon \int_0^{\theta^*} \frac{\ell}{1-k} d\theta + \int_{\theta^*}^{1} r_2 d\theta \geq 1, \tag{9}$$

$$\Pi^B \geq 0, \tag{10}$$

$$0 \leq k \leq 1,\ 0 \leq \ell \leq 1. \tag{11}$$

The bank chooses k, ℓ and r_2 to maximize its expected profit Π^B subject to a number of constraints. The first term in (8) is the revenue of the bank net of the debt repayments at date 2 when, for $\theta \geq \theta^*$, the bank remains solvent; while the second term represents the expected return to equity holders. Condition (9), which represents debt holders' participation constraint, requires their expected payoffs from lending to the bank to be at least equal to the storage return. For $\theta < \theta^*$, debt holders choose not to roll over their debt at date 1 thus forcing the liquidation of the entire bank portfolio for the value ℓ. Each debt holder then receives the pro-rata share of bank's available resources $\frac{\ell}{1-k}$. For $\theta \geq \theta^*$, debt is rolled over and investors receive the promised repayment r_2. The second constraint in (10) is a non-negativity constraint on bank profit, while the last two conditions in (11) are simply physical constraints on the level of capital and portfolio liquidity.

The solution to the bank's maximization problem yields the following result.

Proposition 2 *The market equilibrium features $r_2^B > 1$ and the pair $\left\{k^B, \ell^B\right\}$ as given by the solutions to*

$$-\left[\frac{\partial \theta^*}{\partial k} + \frac{\partial \theta^*}{\partial r_2}\frac{dr_2}{dk}\right]\left[R\left(\theta^*\right)(1-\alpha\ell) - \ell\right] - \rho + 1 = 0, \tag{12}$$

and

$$-\left[\frac{\partial \theta^*}{\partial \ell} + \frac{\partial \theta^*}{\partial r_2}\frac{dr_2}{d\ell}\right]\left[R\left(\theta^*\right)(1-\alpha\ell) - \ell\right] + \int_0^{\theta^*} d\theta - \int_{\theta^*}^1 \alpha R\left(\theta\right) d\theta = 0. \tag{13}$$

The equilibrium pair $\left\{k^B, \ell^B\right\}$ lie in the region bounded by the curves $\underline{k}\left(\ell\right)$ and $\overline{k}\left(\ell\right)$ and satisfies $(1-k) > \ell$ so that liquidity crises occur in equilibrium.

In choosing its capital structure, a bank trades off the marginal benefit of capital with its marginal cost. The former, as represented by the first term in (12), is the gain in expected profits $\left[R\left(\theta^*\right)(1-\alpha\ell) - \ell\right]$ induced by a lower probability of a crisis, as measured by $\frac{\partial \theta^*}{\partial k}$. The latter, as captured by the last two terms in (12), is the increase in funding cost $\rho - 1$ associated with an increased reliance on equity financing.

The interpretation for (13) is analogous, with the only difference that the marginal benefits of an increase in liquidity consists now of two terms. First, more liquidity is beneficial as it leads to a lower crisis probability, as captured by the first term in (13). Second, more liquidity leads to higher portfolio return at date 1, as captured by the second term in (13). Finally, the marginal cost of increasing liquidity corresponds to the third term in (13), which captures the effect that more liquidity has on the date 2 return of bank portfolio.

At the optimum, banks always choose k^B and ℓ^B in the range where both $\frac{\partial \theta^*}{\partial k} < 0$ and $\frac{\partial \theta^*}{\partial \ell} < 0$ as this allows them to be less exposed to crises and, in turn, reduce financing costs, with an overall positive effect on their profits. Furthermore, their choice is consistent with the inequality $(1-k) > \ell$, which implies that they choose to be exposed to liquidity crises. The intuition behind this result is in line with a standard risk-shifting story and hinges on the existence of a wedge between the private (i.e., the bank's) and the social cost of capital. Banks are protected by limited liability and make zero profits when a run occurs. As a consequence, they do not find it optimal to increase capital up to the level satisfying $(1-k) = \ell$ necessary to eliminate liquidity crises. Doing so requires them to bear the higher cost of capital (as represented by $\rho > 1$) while leaving the benefits of the increased investment return to debt holders. In other words, when k^B and ℓ^B are such that $(1-k) = \ell$, the benefit for the bank in terms of reduced crisis probability approaches zero,[6]

[6] When $(1-k) = \ell$ holds, $\theta^* \to \underline{\theta}$ and $r_2 = 1$ from debt holders' participation constraint in (9). Thus, $R\left(\theta^*\right)(1-\alpha\ell) - \ell = 0$.

while the marginal cost in terms of higher funding costs (i.e., $\rho > 1$) is still positive.

3.3 Regulatory intervention

The market equilibrium characterized above is inefficient because the bank's choice of k and ℓ spurs the occurrence of liquidity crises, thus entailing an output loss. To see this, denote as TO the total output generated in the market equilibrium. This corresponds to the sum of bank's profit, debt holders' and equity holders' payoffs and is, thus, equal to:

$$\begin{aligned} TO &= \int_{\theta^*}^{1} \left[R(\theta)(1-\alpha\ell) - (1-k) r_2\right] d\theta - k\rho + (1-k)\int_0^{\theta^*} \frac{\ell}{1-k} d\theta + (1-k)\int_{\theta^*}^{1} r_2 d\theta + k\rho = \\ &= \int_0^{\theta^*} \ell d\theta + \int_{\theta^*}^{1} R(\theta)(1-\alpha\ell)\, d\theta. \end{aligned} \tag{14}$$

When the bank is forced to liquidate its portfolio at date 1 in response to a massive withdrawal of funds by debt holders, the resources produced in the economy correspond to the portfolio liquidation value ℓ. Such liquidation is inefficient for any $\theta > \theta^E$, where θ^E corresponds to the solution to

$$R(\theta)(1-\alpha\ell) = \ell, \tag{15}$$

since, in the absence of liquidation, the bank would generate a higher portfolio return $R(\theta)(1-\alpha\ell)$ at date 2. Importantly, from comparing (2), (7) and (15) it holds $\theta^E < \underline{\theta} < \theta^*$. This implies that in the market equilibrium liquidity crises are always inefficient, while solvency crises are inefficient for $\theta^E < \theta < \underline{\theta}$. Note that only when $\ell = (1-k)$, $\theta^E = \underline{\theta}$ and solvency crises are always efficient.

The total output loss TL spurred by inefficient crises at date 1 is then equal to

$$TL = \int_{\theta^E}^{\theta^*} \left[R(\theta)(1-\alpha\ell) - \ell\right] d\theta. \tag{16}$$

From (16), it is easy to see that the higher θ^*, the larger the loss TL in total output. The question is whether and how the market solution can be improved upon with the help of public intervention. To this end, we consider the possibility for a regulator to constrain bank's balance sheet choices (i.e., capital and portfolio liquidity) with the objective of reducing the inefficiency plaguing the market equilibrium. Regulation is announced at date 0 before the bank takes its choices. Then, given the regulatory constraint(s) in place and the bank choice(s), debt holder take their rollover decisions. The following proposition characterizes the optimal regulatory intervention.

Proposition 3 *The regulator eliminates the inefficiency of the market solution by setting either k^R or ℓ^R such that $(1-k) = \ell$ holds. Thus, capital and liquidity regulation are perfect substitutes in addressing inefficient crises.*

Regulation takes a simple form when the only inefficiency of the market equilibrium is represented by the premature liquidation of a bank portfolio. The regulator needs to enforce that bank's choice satisfies $(1-k) = \ell$. This can be achieved by either imposing a capital or a liquidity requirement and thus letting the bank free to choose the unregulated variable and the debt repayment r_2. The important feature of regulation is that it links the two sides of the bank balance sheet. In this respect, the optimal regulation described in the paper resembles real world regulation such as risk-weighted capital ratios (RWC), the liquidity coverage ratio (LCR) and the net stable funding ratio (NSFR).

4 The economy with G banks

So far, we have focused on an economy with only one bank. We now extend the baseline model and consider the presence of multiple symmetric banks, indexed with $g = 1, 2...G$, which share the same fundamental θ. As in the baseline model, each bank raises 1 unit of funds in the form of a fraction k_g as capital and $1 - k_g$ as debt to be invested in a risky portfolio and chooses the liquidity ℓ_g of the portfolio. However, unlike in the baseline model, ℓ_g may no longer represents the amount that the bank can raise for each unit of their portfolio liquidated at date 1. The reason is that we now assume that to obtain the funds needed to repay withdrawing debt holders $n_g(1 - k_g)$, banks access a secondary market where they sell shares of their portfolio to a continuum of measure 1 of outside investors holding in aggregate an amount $w > 0$, which we refer to as market liquidity.[7] Thus, market conditions determine the amount that each bank can raise from the market.

As common in the literature (see e.g., Shleifer and Vishny, 1992; Acharya and Yorulmazer, 2008; Acharya, Shin and Yorulmazer, 2010; and Eisenbach, 2017), we assume that these investors may be less able than banks in managing the portfolios they acquire from banks so that transferring assets outside the banking sector may entail a loss of resources. In other words, each bank can raise less than the liquidation value ℓ_g

[7] We consider the case where market liquidity w is sufficiently large to absorb the sale of even a very illiquid portfolio by one bank, but not by G banks.

for each unit of the portfolio sold to outside investors. The idea here is that investors bear some costs in managing banks' assets and these costs increase with the aggregate degree of specificity of the assets as well as with the amount on sale. The specificity of bank assets can be traced back to the liquidity of each bank portfolio ℓ_g in that assets that are more liquid are less costly to be managed outside the banking sector, i.e., they are less specific. By contrast, the amount of assets on sale increases with the aggregate number of withdrawing depositors.

To ease the exposition, we denote as $\mathbf{n}$, $\mathbf{k}$ and $\boldsymbol{\ell}$ the vectors of proportions of debt holders running, bank capital and portfolio liquidity, respectively, and specify $Q(\mathbf{n}, \mathbf{k}, \boldsymbol{\ell})$ as a measure of the quantity and specificity of the pool of assets on sale in the market, with $\frac{\partial Q(.)}{\partial n_g} > 0$, $\frac{\partial Q(.)}{\partial k_g} < 0$ and $\frac{\partial Q(.)}{\partial \ell_g} < 0$ for all $g = 1, ...G$.[8] Thus, to capture the idea of fire sales due to asset redeployment, we introduce the variable χ representing the market price so that each bank g raises $\chi \leq \ell_g$ units for each unit of its portfolio sold in the market at date 1, and can be specified as follows:

$$\chi(Q, w) = \begin{cases} \ell_g & \text{if } Q < \widehat{Q}(w) \\ h(Q) & \text{if } Q \geq \widehat{Q}(w) \end{cases}, \tag{17}$$

with $h(Q) < \ell_g$, $\frac{\partial h(.)}{\partial Q} < 0$. As emerges from (17), the amount that each individual bank can raise in the market depends on both its choice of portfolio liquidity ℓ_g and on the aggregate market funding conditions. Market conditions are good when either a few assets are sold in the market or the pool of assets on sale is not too specific relative to investors' resources (i.e., when $Q < \widehat{Q}(w)$). In this case, each individual bank can raise exactly ℓ_g per unit of portfolio liquidated at date 1. On the contrary when there is a large pool of assets on sale and/or they are very specific (i.e., $Q \geq \widehat{Q}(w)$), fire sales emerge in that banks can only raise (per unit) $\chi(Q, w) = h(Q) < \ell_g$.

This specification modifies the analysis in two important dimensions. First, it affects debt holders' rollover decisions, as it introduces another source of strategic complementarity. Second, it leads to an additional inefficiency in banks' decisions, since we consider that G is large enough that banks do not internalize the effect that their individual choices have on the secondary assets market price.

We characterize the equilibrium in an economy with G banks following the same steps as in Section 3. We first characterize debt holders' rollover decisions at date 1 for given levels of capital k_g, portfolio liquidity ℓ_g and debt repayment r_{2g}. Then, we determine k_g, ℓ_g and r_{2g}. Finally, we derive the optimal regulatory

[8] In what follows, the variables in bold always identify vectors.

intervention that addresses the inefficiencies plaguing the unregulated economy.

4.1 Debt holders' rollover decision

As in the baseline model, debt holders decide whether to withdraw at date 1 based on the signal s_i they receive since this provides information on both θ and other debt holders' actions. Unlike the baseline model, a debt holder in bank g is not only concerned about the action taken by the other debt holder in his own bank, but also by that of all other debt holders in the economy.

A key feature of the theoretical framework is the existence of two types of strategic complementarity: *within*- and *between*-banks strategic complementarity (see, Goldstein, 2005). The former, which was already present in the baseline model, refers to the fact that a debt holder's incentive to withdraw at date 1 increases with the proportion of other debt holders in his own bank withdrawing at date 1. The latter, instead, captures the fact that a debt holder's incentive to withdraw at date 1 also increases with the proportion of debt holders in the other banks in the economy taking a similar action. The reason is that the more debt holders choose to withdraw early in other banks, the more those banks need to liquidate, thus leading to a larger drop in the liquidation value, as captured by a lower $\chi(Q, w)$. The lower liquidation value leads to more sales by the bank and so fewer resources left to repay the debt holders. Thus, the *between*-banks strategic complementarity is a direct consequence of the existence of a common asset market, and its strength depends on the aggregate market conditions as emerges in (17).

Despite the existence of two types of strategic complementaries, as in the baseline model, debt holders choose to withdraw early when they receive a low enough signal and roll over otherwise. To see this formally, we follow the same steps as in Section 3.1 and so first characterize two extreme ranges of fundamentals where debt holders in each bank have a dominant action. The upper and dominance regions are the same as the ones in the baseline model, thus we have the same threshold $\underline{\theta}$ and $\overline{\theta}$. The reason is that $\underline{\theta}$ is computed under the assumption that no debt holders withdraw in any bank and $\overline{\theta}$ is obtained assuming that the bank's investment becomes safe and yield $R(1)(1-\alpha\ell)$ also at date 1.

Besides these extreme ranges of θ, a debt holder's action depends crucially on what all the other agents, both within his own bank and in the other banks, do. In this case the existence of between banks strategic complementarity affects the derivations of the crisis threshold as illustrated in the following proposition.

Proposition 4 *For given* $\mathbf{k}$, $\boldsymbol{\ell}$, $\mathbf{r}_2$, *the model has a unique threshold equilibrium where all debt holders in* *bank g withdraw at date 1 if they receive a signal below s_g^* and roll over otherwise. The vector $\mathbf{s}_{FS}^*$ of equilibrium threshold signals corresponds to the solution of the system of G indifference conditions*

$$
\begin{aligned}
&\int_0^{\widehat{n}\left(s_b^*,s_{(-g)}^*\right)} r_{2g}dn_g + \int_{\widehat{n}(.)}^{\overline{n}\left(s_{(-g)}^*\right)} \frac{R\left(s_g^*+\varepsilon-2\varepsilon n_g\right)(1-\alpha\ell_g)\left(1-\frac{n_g(1-k_g)}{\chi\left(Q\left(\boldsymbol{\ell},\boldsymbol{k},s_g^*,s_{(-g)}^*\right),w\right)}\right)}{(1-k_g)(1-n_g)}dn_g \\
&\qquad - \int_0^{\overline{n}\left(s_{(-g)}^*\right)} 1dn_g - \int_{\overline{n}\left(s_{(-g)}^*\right)}^1 \frac{\chi\left(Q\left(\boldsymbol{\ell},\boldsymbol{k},s_g^*,s_{(-g)}^*\right),w\right)}{(1-k_g)n_g}dn_g = 0
\end{aligned}
\tag{18}
$$

for all $g = 1, ...G$.

The vector of equilibrium thresholds $\mathbf{s}_{FS}^*$ is the solution of a system of G equations representing a debt holder's indifference condition between rolling over the debt until date 2 and withdrawing date 1. It emerges from the expression (18) in the proposition that debt holders' actions crucially depend on the actions taken by all other debt holders in the economy, both in their own bank and in other banks. In particular, the greater the likelihood of a run in another bank (i.e., $s_{(-g)}^*$), the larger the probability of a run in bank g since the thresholds s_g^* and $s_{(-g)}^*$ are positively related. This implies that there are ranges of θ where debt holders run on their own bank only because they fear that runs are going to occur in other banks in the economy.

The positive correlation between the equilibrium thresholds of all G banks is a direct implication of the *between*-banks strategic complementarity induced by the existence of a common asset market. It captures the role that fire sales pricing play for contagion between banks. In an economy without fire sales, debt holders in different banks would take their withdrawal decisions independently from each other despite the common fundamental θ affecting the date 2 return of their bank's portfolio. In this case, bank failures would also be (strategically) independent that is no contagion would emerge in the economy.

The following proposition describes this feature of the equilibrium with G banks.

Proposition 5 *When $\varepsilon \to 0$, all debt holders withdraw early if $\theta < \theta_{FS}^*$ and roll over otherwise, with $\theta_{FS}^* > \theta^*$ characterized in the baseline model.*

The possibility of fire sales has two important effects on financial stability, as highlighted in Proposition 5. First, banks become more fragile in that they fail in a larger range of fundamentals θ relative to the baseline model where they raise ℓ_g for each unit of portfolio liquidated at date 1 (i.e., $\theta_{FS}^* > \theta^*$). Second, in the region between θ^* and θ_{FS}^* all banks fail not because their share the same fundamental but rather because of the strategic interdependence: Each bank fails because debt holders are concerned about the health of the other banks in the system as this has implications for market funding conditions.

4.2 Banks' choices

Having characterized debt holders' withdrawal decisions and so the likelihood of a bank failure, we can now move to solve banks' choices of capital k_g, liquidity ℓ_g and interest rate on debt r_{2g}. We use the superscript G to denote the equilibrium variables. As banks are symmetric we focus on a representative bank and so remove the subscript g. The maximization problem is similar to that in Section 3.2 with only a few differences. First, the bank makes positive profits only for $\theta > \theta^*_{FS} > \theta^*$. Second, debt holders' participation constraint is now equal to

$$IR^D_{FS}\colon \int_0^{\theta^*_{FS}} \frac{\underline{\chi}(\boldsymbol{\ell}, w)}{1-k_g} d\theta + \int_{\theta^*_{FS}}^1 r_2 d\theta \geq 1, \tag{19}$$

where $\underline{\chi}(\boldsymbol{\ell}, w)$ captures the amount the bank obtains by liquidating its portfolio at date 1. As in the baseline model, debt holders receive the promised repayment r_2 only if no runs occur, while in the event of a run they receive a pro-rate share of bank's available resources. The function $\underline{\chi}(\boldsymbol{\ell}, w)$ only depends on the degree of specificity of the pool of assets on sale, as determined by all banks' liquidity choices $\boldsymbol{\ell}$. It no longer depends on $\mathbf{k}$ and $\mathbf{n}$ as all banks liquidate the entire portfolio (i.e., 1 unit).

The solution to the bank's maximization problem yields the following result.

Proposition 6 *The market equilibrium features $r_2^G > 1$ and the pair $\{k^G, \ell^G\}$ as given by the solutions to*

$$-\left[\frac{\partial \theta^*_{FS}}{\partial k} + \frac{\partial \theta^*_{FS}}{\partial r_2}\frac{dr_2}{dk}\right]\left[R(\theta^*_{FS})(1-\alpha\ell) - \underline{\chi}(\boldsymbol{\ell}, w)\right] - \rho + 1 = 0, \tag{20}$$

and

$$-\left[\frac{\partial \theta^*_{FS}}{\partial \ell} + \frac{\partial \theta^*_{FS}}{\partial r_2}\frac{dr_2}{d\ell}\right]\left[R(\theta^*_{FS})(1-\alpha\ell) - \underline{\chi}(\boldsymbol{\ell}, w)\right] - \int_{\theta^*}^1 \alpha R(\theta)\, d\theta = 0. \tag{21}$$

In equilibrium liquidity crises occur since $\left(1-k^G\right) > \underline{\chi}\left(\boldsymbol{\ell}^G, w\right)$ holds.

In choosing their capital structure k and portfolio liquidity ℓ, banks trade-off their marginal benefits and costs. The former, which are captured by the first term in (20) and (21), refer to the reduction in the run threshold θ^*_{FS} and in the bank financing costs associated with higher capital and liquidity. The latter, which corresponds to the difference $\rho - 1$ in (20) and to the last term in (21), capture the higher financing costs and the lower return, respectively, triggered by a marginal increase in k and ℓ.

As in the baseline model, banks choose to be exposed to liquidity crises. The reason is that when

$1-k=\underline{\chi}(\boldsymbol{\ell},w)$, the gain in terms of profits in the case of a run approaches zero.[9] Furthermore, at $1-k=\underline{\chi}(\boldsymbol{\ell},w)$ the loss in terms of higher financing costs is still equal to $\rho-1$. Thus, reducing k slightly so that $1-k<\underline{\chi}(\boldsymbol{\ell},w)$ holds entails a second-order cost and a first-order benefit and so it is always optimal.

4.3 Regulatory intervention

The market equilibrium characterized above entails two inefficiencies. First, as in the baseline model, banks' choice of k and ℓ spurs the occurrence of liquidity crises, thus leading to the liquidation of potentially profitable portfolios. Second, banks do not internalize the effect that their liquidity choice has on the market funding conditions, thus leading to fire sales. We can then specify the total output loss in the unregulated economy with G banks as follows:

$$TL=\int_{\theta^E}^{\theta^*_{FS}}\left[R(\theta)(1-\alpha\ell)-\ell\right]d\theta+\int_0^{\theta^*_{FS}}\left[\ell-\underline{\chi}(\boldsymbol{\ell},w)\right]d\theta. \tag{22}$$

The first term in (22) is the output loss associated with the premature liquidation of good portfolios, while the second term captures the loss associated with fire sales.

From (22), it is easy to see that the higher θ^*_{FS}, the larger the loss TL in total output. As in the baseline model, the question is whether a regulator could reduce the output loss by constraining bank's balance sheet choices (i.e., capital and liquidity regulation).

The regulator chooses capital and liquidity requirements to maximize total output as specified in (14), while taking as given debt holders' withdrawal decisions as characterized in Proposition 4, the debt repayment r_2 chosen by the bank and the non-negative profit constraint of banks. Thus, formally, the regulator chooses the level of bank capital k^R and portfolio liquidity ℓ^R at date 0 to solve the following problem:

$$\max_{k^R,\ell^R}\int_0^{\theta^*_{FS}}\underline{\chi}(\boldsymbol{\ell},w)\,d\theta+\int_{\theta^*_{FS}}^1 R(\theta)(1-\alpha\ell)\,d\theta$$

subject to

$$\begin{aligned} r_2^B &= \arg\max \Pi^B, \\ \Pi^B &\geq 0,\ 0\leq k^R\leq 1,\ 0\leq \ell^R\leq 1. \end{aligned}$$

[9] When $1-k=\underline{\chi}(\boldsymbol{\ell},w)$, the term $\left[R\left(\theta^*_{FS}\right)(1-\alpha\ell)-\underline{\chi}(\boldsymbol{\ell},w)\right]$ simplifies to $\left[R(\underline{\theta})(1-\alpha\ell)-\underline{\chi}(\boldsymbol{\ell},w)\right]=0$ since $\theta^*_{FS}\to\underline{\theta}$ and $\underline{\theta}$ is given by (2).

Using the fact that banks choose the debt repayment r_2 from debt holders' participation constraint in (9) holding with equality, banks' profits can be rearranged as

$$\Pi^B = \int_0^{\theta^*_{FS}} \underline{\chi}(\boldsymbol{\ell}, w)\, d\theta + \int_{\theta^*_{FS}}^1 R(\theta)(1-\alpha\ell)\, d\theta - (1-k) - k\rho, \tag{23}$$

and the regulator's problem can be re-written as follows:

$$\max_{k^R, \ell^R} \int_0^{\theta^*_{FS}} \underline{\chi}(\boldsymbol{\ell}, w)\, d\theta + \int_{\theta^*_{FS}}^1 R(\theta)(1-\alpha\ell)\, d\theta$$

subject to

$$\int_0^{\theta^*_{FS}} \underline{\chi}(\boldsymbol{\ell}, w)\, d\theta + \int_{\theta^*_{FS}}^1 R(\theta)(1-\alpha\ell)\, d\theta \geq (1-k) + k\rho. \tag{24}$$

Essentially, the regulator's problem boils down to choose ℓ^R and k^R so to eliminate the two inefficiencies plaguing the market solution whenever this is feasible vis-à-vis the constraint of bank's non-negative profits. This is the case when $\chi(\boldsymbol{\ell}, w) = 1$ and $\theta^*_{FS} = \theta^E$ can be enforced by the regulator. The following proposition characterizes the optimal regulatory intervention.

Proposition 7 *The regulator eliminates the inefficiency of the market solution by setting $\ell^R \geq \widehat{\ell}(w)$ such that $\chi(\boldsymbol{\ell}, w) = \ell^R$ and $k^R = 1 - \ell^R$ so that only efficient crises occur. Thus, capital and liquidity regulation are complements.*

Regulation in the presence of fire sales features both liquidity and capital requirements. The former are needed to eliminate fire sales and so enforce that banks' portfolios are acquired at no discount by outside investors (i.e., $\chi(\boldsymbol{\ell}, w) = \ell$). Once fire sales are eliminated, the latter works as in the baseline model: Forcing banks to hold an amount of capital such that $1 - k = \ell$ holds allows to eliminate strategic complementarity among debt holders' actions so that liquidity crises no longer occur. As shown in Section 3.3, when $1 - k = \ell$ holds only efficient solvency-driven crises occur$-$i.e., $\underline{\theta} \to \theta^E$ as emerges comparing (2) and (15).

The proposition also shows that the liquidity requirements ℓ^R vary with the degree of tightness of the market, as captured by outside investors' wealth w. This is captured by the variable $\widehat{\ell}(w)$, which corresponds to the minimum amount of portfolio liquidity that allow investors to acquire and manage banks' assets at no cost. As lower w is associated with a tighter market and more severe fire sales, banks must be require to hold more liquidity to eliminate the fire sale (i.e., $\widehat{\ell}(w)$ and ℓ^R are higher).

The regulatory intervention in the presence of inefficient crises and fire sales constrains banks more relative to the baseline model. This may have important implications for the feasibility of the intervention. In the baseline model, a bank is forced to make its balance sheet choices such that $1-k=\ell$ holds. However, it is then free to choose any combination of capital and portfolio liquidity that satisfies such requirement. In the extended model, instead, banks are more constrained in that their choice of k and ℓ are a subset of those in the baseline model (i.e., both $1-k=\ell$ and $\ell \geq \widehat{\ell}(w)$ must be satisfied). Thus, it may be the case that eliminating both types of inefficiencies is not feasible when the pairs $\{k, \ell\}$ enforcing $\chi(\boldsymbol{\ell}, w) \rightarrow \ell$ and $\theta^*_{FS} \rightarrow \theta^E$ would entail negative profits for the banks. Whether this is the case it depends on the exogenous parameter of the model α, ρ and w through their effect on the threshold θ^E, the severity of fire sales and the financing cost for the bank. Clearly, higher values of α or ρ make the bank profit condition in (24) more binding, while a lower w is associated to more severe fire sales and so higher required portfolio liquidity. In other words, eliminating both inefficiencies may not feasible when market conditions are tight and/or the cost of bank equity and portfolio liquidity is high. In this case, the regulator needs to choose between enforcing the efficient liquidation of the banks' asset and reducing the severity of fire sales.

5 Concluding remarks

In this paper we develop a model where banks' exposure to crises depends on their balance sheet composition and both banks' and debt holders' decisions are endogenously determined. The paper offers a convenient framework to evaluate the implications of bank capital and liquidity on the likelihood of crises, as it allows to endogenize the probability of crises, distinguish their different type, and account for the different effects that changes in bank capital structure and portfolio liquidity have on each of them.

One of the main implications of the analysis is that, in order to be beneficial for stability, regulation should be designed considering both sides of banks' balance sheet. The same (marginal) increase in capital and liquidity may be beneficial for some banks, while detrimental for others. Real world regulatory tools like risk-weighted capital ratio (RWC), liquidity coverage ratio (LCR) or net stable funding ratio (NSFR) seem to fulfil this criterion, as they specify a ratio between banks' assets and liabilities (see Cecchetti and Kashyap, 2018).

The analysis of the impact of capital and liquidity on bank stability is also the starting point to characterize regulation. In our framework, public intervention in the form of capital and liquidity requirements is desirable as the market equilibrium is plagued by two inefficiencies. First, banks choose levels of capitalization and portfolio liquidity that are consistent with the occurrence of liquidity crises and, as such, lead to inefficient portfolio liquidation. Second, in choosing their capital structure and portfolio liquidity banks do not internalize the effect that such choices have on market funding conditions that is on the existence and severity of fire sales.

We show that when market funding conditions for the banks are tight, the cost of capital and liquidity for banks are high, the regulator is not able to eliminate the inefficiencies of the market equilibrium so that the regulator's solution entails either too little liquidation of banks' assets but reduced fire sales or the efficient liquidation of the banks' assets with fire sale.

While our framework endogenizes banks' choices on both asset and liability side of the balance sheet, the analysis of regulation only focuses on the interaction between run risk and fire sales, thus on inefficiencies related to the liability side of bank balance sheet. However, in reality there are important inefficiencies connected to the asset side decision of the bank (e.g., moral hazard associated with the riskiness of bank portfolios) which (capital) regulation is designed to tackle. Incorporating this into our analysis so to study the design of regulation in the presence of interaction between fire sales, run and credit risk is an interesting path for future research.

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7 Appendix

Proof of Proposition 1: The proof follows closely that in Goldstein and Pauzner (2005) since our model also exhibits the property of *one-sided* strategic complementarity.

Assume that debt holders behave accordingly to a threshold strategy, that is each debt holder withdraw at date 1 if he receives a signal below s^* and rolls over otherwise. Then, the fraction of debt holders not rolling over the debt claim n is equal to the probability of receiving a signal below s^*. Given that debt holders' signals are independent and uniformly distributed in the range $[\theta-\varepsilon,\theta+\varepsilon]$, $n(s^*,\theta)$ is equal to

$$n(s^*,\theta)=\begin{cases} 1 & \text{if } \theta\leq s^*-\varepsilon \\ \frac{s^*-\theta+\varepsilon}{2\varepsilon} & \text{if } s^*-\varepsilon\leq\theta\leq s^*+\varepsilon \\ 0 & \text{if } \theta\geq s^*+\varepsilon \end{cases}. \tag{25}$$

When θ is lower than $s^*-\varepsilon$, $n=1$ and all $(1-k)$ debt holders receive a signal below s^* and they withdraw at date 1. On the contrary, when θ is higher than $s^*+\varepsilon$, $n=0$ and all $(1-k)$ debt holders receive a signal above s^* and, as a result, decide to roll over their debt claim. In the intermediate range of fundamental, when $s^*-\varepsilon\leq\theta\leq s^*+\varepsilon$, there is a partial run, in that only some debt holders withdraw at date 1. The proportion of those not rolling over their debt claim decreases linearly with θ, as fewer investors observe a signal below the threshold s^*.

Denote as $\Delta(s_i,\dot{n}(\theta))$ an agent's expected difference in utility between withdrawing at date 2 and at date 1 when he holds beliefs $\dot{n}(\theta)$ regarding the number of depositors running. The function $\Delta(s_i,\dot{n}(\theta))$ is given by

$$\Delta(s_i,\dot{n}(\theta))=\frac{1}{2\varepsilon}\int_{s_i-\varepsilon}^{s_i+\varepsilon}E_n\left[v(\theta,\dot{n}(\theta))\right]d\theta.$$

Since for any realization of θ, the proportion of depositors running is deterministic, we can write $n(\theta)$ instead of $\dot{n}(\theta)$ and the function $\Delta(s_i,n(\theta))$ simplifies to

$$\Delta(s_i,\dot{n}(\theta))=\frac{1}{2\varepsilon}\int_{s_i-\varepsilon}^{s_i+\varepsilon}v(\theta,n(\theta))d\theta.$$

Notice that when all depositors behave according to the same threshold strategy x^*, $\dot{n}(\theta)=n(\theta,x^*)$ defined in (25). The following lemma states a few properties of the function $\Delta(s_i,\dot{n}(\theta))$.

Lemma 1 *i) The function $\Delta(s_i,\dot{n}(\theta))$ is continuos in s_i; ii) for any $a>0$, $\Delta(s_i+a,\dot{n}(\theta)+a)$ is non-decreasing in a, iii) $\Delta(s_i+a,\dot{n}(\theta)+a)$is strictly increasing in a if there is a positive probability that $n<\overline{n}$ and $\theta<\overline{\theta}$.*

Proof of Lemma 1: The proof follows Goldstein and Pauzner (2005). The function $\Delta(.)$ is continuous in s_i, as s_i only changes the limits of integration in the formula for $\Delta(s_i,n(s^*,\theta))$. To show that the function $\Delta(s_i,n(s^*,\theta))$ is non-decreasing in a, we need first to show that $v(\theta,n)$ is non-decreasing in θ. As θ increases, we have two effects. First, a higher θ implies that $R(\theta)$ is higher, thus increasing the date 2 payoff in the range $\widehat{n}(\theta) < n \leq \overline{n}$. Second, a change in θ affects the threshold $\widehat{n}(\theta)$ as follows:

$$\begin{aligned}\frac{\partial \widehat{n}(\theta)}{\partial \theta} &= R'(\theta)(1-\alpha\ell)\frac{\left[\frac{R(\theta)(1-\alpha\ell)}{\ell} - r_2\right] - \left[R(\theta)(1-\alpha\ell) - (1-k)r_2\right]\frac{1}{\ell}}{(1-k)\left[\frac{R(\theta)(1-\alpha\ell)r_1}{\ell} - r_2\right]^2} = \\ &= \frac{R'(\theta)(1-\alpha\ell)}{(1-k)\left[\frac{R(\theta)(1-\alpha\ell)}{\ell} - r_2\right]^2} r_2 \left[\frac{(1-k)}{\ell} - 1\right] > 0,\end{aligned}$$

since $R'(\theta) > 0$ and $(1-k) > \ell$. Thus, since the interval $[0, \widehat{n}(\theta)]$ where the utility differential $v(\theta, n) = r_2 - 1 > 0$ becomes larger, while the range $(\overline{n}, 1]$ is unaffected by a change in θ, the date 2 payoff also increases so that the utility differential $v(\theta, n)$ is non-decreasing in θ. This also implies that $\Delta(s_i + a, \dot{n}(\theta) + a)$ is non-decreasing in a, as when a increases, debt holders see the same distribution of n but expects θ to be larger. In order for $\Delta(s_i + a, \dot{n}(\theta) + a)$ to be strictly increasing in a, we need that $\theta < \overline{\theta}$ and that there is a positive probability that $n < \overline{n}$. This is the case because, when $n < \overline{n}$ and $\theta < \overline{\theta}$, $v(\theta, n)$ is strictly increasing in θ, and, thus, $\Delta(s_i + a, \dot{n}(\theta) + a)$ is strictly increasing in a. ■

Since the rest of the proof follows closely that in Goldstein and Pauzner (2005) we omit it here and only specify the condition pinning down the threshold s^*. A debt holder who receives the signal s^* is indifferent between rolling over the debt claim until date 2 and withdrawing it at date 1. The threshold s^* can be computed as the solution to

$$f(\theta, k, \ell) = \int_0^{\widehat{n}(s^*)} r_2 dn + \int_{\widehat{n}(s^*)}^{\bar{n}} \frac{R(\theta(n))(1-\alpha\ell)\left[1 - \frac{(1-k)n}{\ell}\right]}{(1-k)(1-n)} dn - \int_0^{\bar{n}} 1 dn - \int_{\bar{n}}^1 \frac{\ell}{(1-k)n} dn = 0, \quad (26)$$

where from (25), we obtain $\theta(n) = s^* + \varepsilon - 2\varepsilon n$ and $\widehat{n}(s^*)$ solves $R(\theta(n))(1-\alpha\ell)\left[1 - \frac{(1-k)n}{\ell}\right] - (1-k)(1-n)r_2 = 0$. At the limit, when $\varepsilon \to 0$, $\theta(n) \to s^*$ and we denote it as θ^*, which corresponds to the solution to the condition (7) in the proposition.

To complete the proof, we need to show that the bank is solvent for any $\theta > \theta^*$. To do that we need to exclude the possibility that the bank fails at date 2 despite all debt holders rolling over the debt until date 2. Denote as $\widehat{\theta}$ the level of fundamental at which the bank fails at date 2 even when all debt holders roll over the debt claim (i.e., when $n = 0$). The threshold $\widehat{\theta}$ solves

$$R(\theta)(1-\alpha\ell) - (1-k)r_2 = 0.$$

In order to show that the bank is always solvent for any $\theta > \theta^*$, we need to show that the threshold θ^* characterized in (26) larger than $\widehat{\theta}$. To see this, denote as $\widetilde{\theta}$ the level of θ at which the bank is at the margin between failing and being solvent at date 2 when n debt holders withdraw early. Then, $\widetilde{\theta}$ is the solution to

$$R(\theta)(1-\alpha\ell)\left[1 - \frac{(1-k)n(s^*, \theta)}{\ell}\right] - (1-k)(1-n(s^*, \theta))r_2 = 0, \quad (27)$$

where $n\left(s^{*},\widetilde{\theta}\right)$ is given in (25). Rearranging (27) as

$$R(\theta)(1-\alpha\ell)-(1-k)r_2-n(s^{*},\theta)\left[\frac{R(\theta)(1-\alpha\ell)(1-k)}{\ell}-(1-k)r_2\right]=0,$$

it is easy to see that (27) is negative when evaluated at $\theta=\widehat{\theta}$ when $(1-k)>\ell$ holds. Thus, since (27) is increasing in θ, it follows that $\widetilde{\theta}>\widehat{\theta}$.

The equilibrium in debt holders' withdrawal decision characterized in the proposition corresponds to the pair $\{s^{*},\theta^{*}\}$ solving (27) and the indifference condition as given by $v(\theta,n)=0$. Thus, it is the case that, when $\varepsilon\to 0$, $s^{*}\to\theta^{*}=\widetilde{\theta}>\widehat{\theta}$ and the proposition follows. $\square$

Proof of Corollary 1: The proof proceeds in steps. First, we compute the effect of k and ℓ on the crisis threshold $\underline{\theta}$ and then their effect on the threshold θ^{*}.

Denote as $f(\theta,k,\ell)=0$ the condition pinning down the threshold $\underline{\theta}(k,\ell)$ as given in (2). By using the implicit function theorem, we have that

$$\frac{\partial\underline{\theta}(k,\ell)}{\partial k}=-\frac{\frac{\partial f(\theta,k,\ell)}{\partial k}}{\frac{\partial f(\theta,k,\ell)}{\partial\theta}}\text{ and }\frac{\partial\underline{\theta}(k,\ell)}{\partial\ell}=-\frac{\frac{\partial f(\theta,k,\ell)}{\partial k}}{\frac{\partial f(\theta,k,\ell)}{\partial\theta}}.$$

The denominator $\frac{\partial f(\theta,k,\ell)}{\partial\theta}=\frac{R'(\theta)(1-\alpha\ell)}{1-k}>0$ as $R'(\theta)>0$. Thus, the sign of $\frac{\partial\underline{\theta}(k,\ell)}{\partial k}$ and $\frac{\partial\underline{\theta}(k,\ell)}{\partial\ell}$ are equal to the opposite sign of the respective numerators. Deriving (2) with respect to k and ℓ we obtain

$$\frac{\partial f(\theta,k,\ell)}{\partial k}=\frac{R(\underline{\theta})(1-\alpha\ell)}{(1-k)^{2}}>0,\ \frac{\partial f(\theta,k,\ell)}{\partial\ell}=-\frac{R(\underline{\theta})\alpha}{(1-k)}<0,$$

which imply $\frac{\partial\underline{\theta}(k,\ell)}{\partial k}<0$ and $\frac{\partial\underline{\theta}(k,\ell)}{\partial\ell}>0$.

Consider now the effect of capital and liquidity on the threshold θ^{*}. Denote as $g(\theta,k,\ell)=0$ the equation pinning down θ^{*}, as defined in (26). Using the implicit function theorem we obtain:

$$\frac{\partial\theta^{*}}{\partial k}=-\frac{\frac{\partial g(\theta,k,\ell)}{\partial k}}{\frac{\partial g(\theta,k,\ell)}{\partial\theta}}\text{ and }\frac{\partial\theta^{*}}{\partial\ell}=-\frac{\frac{\partial g(\theta,k,\ell)}{\partial\ell}}{\frac{\partial g(\theta,k,\ell)}{\partial\theta}},$$

The denominator $\frac{\partial g(\theta,k,\ell)}{\partial\theta}$ is given by

$$\frac{\partial g(\theta,k,\ell)}{\partial\theta}=\int_{\widehat{n}(\theta)}^{\bar{n}}\frac{R'(\theta^{*})(1-\alpha\ell)\left[1-\frac{(1-k)n}{\ell}\right]}{(1-k)(1-n)}dn>0$$

since the derivatives of the extremes of the integrals cancel out. Thus, the sign of $\frac{\partial\theta^{*}}{\partial k}$ and $\frac{\partial\theta^{*}}{\partial\ell}$ are equal to the opposite sign of $\frac{\partial g(\theta,k,\ell)}{\partial k}$ and $\frac{\partial g(\theta,k,\ell)}{\partial\ell}$, respectively.

We start from $\frac{\partial g(\theta,k,\ell)}{\partial k}$. Deriving (26) with respect to k and multiplying it by -1, we obtain

$$\frac{1}{(1-k)^2}\left[-\int_{\widehat{n}(\theta^*)}^{\overline{n}} \frac{R(\theta^*)(1-\alpha\ell)}{(1-n)}dn + \ell\int_{\overline{n}}^{1}\frac{1}{n}dn\right], \tag{28}$$

since the derivatives of the extremes of the integrals cancel out. Similarly, differentiating (26) with respect to ℓ, after a few manipulation and multiplying it by -1, we obtain

$$\frac{1}{(1-k)\ell}\left[\int_{\widehat{n}(\theta^*)}^{\overline{n}} \alpha\ell\frac{R(\theta^*)}{1-n}dn + \ell\int_{\overline{n}}^{1}\frac{1}{n}dn - \int_{\widehat{n}(\theta^*)}^{\overline{n}}\frac{R(\theta^*)(1-k)n}{(1-n)\ell}dn\right]. \tag{29}$$

as the derivatives of the extremes of the integrals cancel out.

Consider first the effect of k on θ^*. The expression in (28) can be rearranged as follows:

$$R(\theta^*)(1-\alpha\ell)\,Log\left[\frac{1-\overline{n}}{1-\widehat{n}(\theta^*)}\right] - \ell Log[\overline{n}]. \tag{30}$$

The first term is negative since $\overline{n} > \widehat{n}(\theta^*)$ and so $\frac{1-\overline{n}}{1-\widehat{n}(\theta^*)} < 1$, while the second one is positive since $\overline{n} < 1$. Using $\overline{n} = \frac{\ell}{(1-k)}$ and $\widehat{n}(\theta^*) = \frac{R(\theta^*)(1-\alpha\ell)-(1-k)r_2}{(1-k)\left[\frac{R(\theta^*)(1-\alpha\ell)}{\ell}-r_2\right]}$, after a few manipulations, the expression above can be rewritten as follows:

$$Log\left[\left(1-\frac{\ell r_2}{R(\theta^*)(1-\alpha\ell)}\right)^{\frac{R(\theta^*)(1-\alpha\ell)}{\ell}}\right] - Log\left[\left(\frac{\ell}{(1-k)}\right)\right]. \tag{31}$$

Denote as $\widetilde{k}(\ell)$ the solution to $Log\left[\left(1-\frac{\ell r_2}{R(\theta^*)(1-\alpha\ell)}\right)^{\frac{R(\theta^*)(1-\alpha\ell)}{\ell}}\right] - Log\left[\left(\frac{\ell}{(1-k)}\right)\right] = 0$. The expression in (31) can be rearranged as

$$\widetilde{k}(\ell) = 1-\ell(\Lambda)^{-\frac{R(\theta^*)(1-\alpha\ell)}{\ell}}, \tag{32}$$

where $\Lambda = \left(1-\frac{\ell r_2}{R(\theta^*)(1-\alpha\ell)}\right)$. Since for any pair $\{k,\ell\}$, θ^* varies between $\underline{\theta}$ and $\overline{\theta} \to 1$, it holds that $\widetilde{k}(\ell) < k^{\max}(\ell)$ for any $\ell \in (0,1)$, since $k^{\max}(\ell) = 1-\ell$ and $(\Lambda)^{-\frac{R(\theta^*)(1-\alpha\ell)}{\ell}} > 1$. Furthermore, from (32), it follows that $\widetilde{k}(\ell) \to 1$, when $\ell \to 0$ and that $\widetilde{k}(\ell) = 0$ requires $\ell > 0$.

Consider a pair $\{k,\ell\}$ in the region below $k^{\max}(\ell)$. When we approach the curve $k^{\max}(\ell)$, the threshold $\theta^* \to \underline{\theta}$. To see this, we can rearrange the expression in (26) as follows:

$$\int_0^{\widetilde{n}(\theta)}\left[\min\left\{r_2, \frac{R(\theta)(1-\alpha\ell)\left[1-\frac{(1-k)n}{\ell}\right]}{(1-k)(1-n)}\right\}-1\right]dn + \tag{33}$$
$$+\int_{\widetilde{n}(\theta)}^{\bar{n}}\left[\frac{R(\theta)(1-\alpha\ell)\left[1-\frac{(1-k)n}{\ell}\right]}{(1-k)(1-n)}-1\right]dn - \int_{\bar{n}}^{1}\frac{\ell}{(1-k)n}dn,$$

with $\overline{n} = \frac{\ell}{(1-k)}$ and $\widetilde{n}(\theta) = \frac{R(\theta)(1-\alpha\ell)-(1-k)}{(1-k)\left[\frac{R(\theta)(1-\alpha\ell)}{\ell}-1\right]}$ denoting the proportion of debt holders withdrawing at date 1 at which the bank's resources at date 2 are exactly enough to pay 1 to debt holders rolling over the debt claim until date 2. When $k \to k^{\max}(\ell)$, $\overline{n} \to \widetilde{n}(\theta) \to 1$ and the expression above simplifies to

$$\int_0^1 \left[\min\left\{r_2, \frac{R(\theta)(1-\alpha\ell)\left[1-\frac{(1-k)n}{\ell}\right]}{(1-k)(1-n)}\right\} - 1\right] = 0.$$

Since $r_2 > r_1$, θ^* solves $\frac{R(\theta)(1-\alpha\ell)}{(1-k)} - 1 = 0$, which is equivalent to the equation pinning down $\underline{\theta}$, as given in (2).

This implies that for pairs $\{k, \ell\}$ very close to the curve $k^{\max}(\ell)$, $\frac{\partial\theta^*}{\partial k} < 0$. Thus, since $\frac{\partial\theta^*}{\partial k} < 0$ is zero on the curve $\widetilde{k}(\ell)$, it must be the case that $\frac{\partial\theta^*}{\partial k} < 0$ in the region between $\widetilde{k}(\ell)$ and $k^{\max}(\ell)$.

Consider now a pair $\{k, \ell\}$ below the curve $\widetilde{k}(\ell)$ and close to the axes origin. For $k << 1$ and $\ell \to 0$, the expression in (31) is positive since the second term approaches to $-\infty$, while the is equal to

$$Lim_{\ell\to 0}\frac{R(\theta^*)(1-\alpha\ell)}{\ell}Log[\Lambda] = Lim_{\ell\to 0}\frac{Log[\Lambda]}{\frac{\ell}{R(\theta^*)(1-\alpha\ell)}},$$

and using l' Hopital's rule, after a few manipulations, we obtain

$$Lim_{\ell\to 0}\frac{R(\theta^*)(1-\alpha\ell)}{\ell}Log[\Lambda] = -\frac{r_2\frac{1}{Lim_{\ell\to 0}[R(\theta^*)(1-\alpha\ell)]}}{\frac{1}{Lim_{\ell\to 0}[R(\theta^*)(1-\alpha\ell)]}} = -r_2 < 0,$$

where $Lim_{\ell\to 0}[R(\theta^*)(1-\alpha\ell)]$ is equal to a finite number. This implies that $\frac{\partial\theta^*}{\partial k} > 0$ for $k << 1$ and $\ell \to 0$. Since the derivative $\frac{\partial\theta^*}{\partial k}$ is zero on the curve $\widetilde{k}(\ell)$, by continuity it stays positive below $\widetilde{k}(\ell)$.

Consider now the effect of liquidity ℓ on θ^*. The expression (29) determining the sign of $\frac{\partial\theta^*}{\partial\ell}$ can be rearranged as follows, after adding and subtracting $\frac{1}{(1-k)\ell}\int_{\widehat{n}(\theta^*)}^{\bar{n}}\frac{R(\theta^*)}{1-n}dn$:

$$\frac{\partial g(\theta,k,\ell)}{\partial\ell} = \frac{1}{(1-k)\ell}\left[\int_{\widehat{n}(\theta^*)}^{\bar{n}}\frac{R(\theta^*)(1-\alpha\ell)}{1-n}dn - \ell\int_{\bar{n}}^{1}\frac{1}{n}dn - \int_{\widehat{n}(\theta^*)}^{\bar{n}}\frac{R(\theta^*)}{(1-n)}\left(1-\frac{(1-k)n}{\ell}\right)dn\right].$$

Since, from (28), we have that $\frac{\partial g(\theta,k,\ell)}{\partial k} = \frac{1}{(1-k)^2}\left[\int_{\widehat{n}(\theta^*)}^{\bar{n}}\frac{R(\theta^*)(1-\alpha\ell)}{1-n}dn - \ell\int_{\bar{n}}^{1}\frac{1}{n}dn\right]$, we can write

$$\begin{aligned}\frac{\partial g(\theta,k,\ell)}{\partial\ell} &= \frac{(1-k)}{\ell}\frac{\partial g(\theta,k,\ell)}{\partial k} - \frac{1}{(1-k)\ell}\int_{\widehat{n}(\theta^*)}^{\bar{n}}\frac{R(\theta^*)}{(1-n)}\left(1-\frac{(1-k)n}{\ell}\right)dn = \\ &= \frac{1}{\ell}\left[(1-k)\frac{\partial g(\theta,k,\ell)}{\partial k} - \frac{1}{(1-k)}\int_{\widehat{n}(\theta^*)}^{\bar{n}}\frac{R(\theta^*)}{(1-n)}\left(1-\frac{(1-k)n}{\ell}\right)dn\right].\end{aligned} \quad (34)$$

From (34), then, it is easy to see that when $k \leq \widetilde{k}(\ell)$ $\frac{\partial g(\theta,k,\ell)}{\partial\ell} < 0$, as $\frac{\partial g(\theta,k,\ell)}{\partial k} \leq 0$. This implies that $\frac{\partial\theta^*}{\partial\ell} > 0$ in the region below the curve $\widetilde{k}(\ell)$.

Consider now the range $\left(\widetilde{k}(\ell),\ k^{\max}(\ell)\right)$. We want to show that in this range there are levels of bank capitalization k for which increasing liquidity leads to a lower probability of panic-driven runs, i.e., $\frac{\partial\theta^*}{\partial\ell} < 0$

for some $k \in \left(\widetilde{k}(\ell),\ k^{\max}(\ell)\right)$. To do this, we need to show that there exist a region of k and ℓ, where the expression in the bracket in (29) is negative. After a few manipulations, we can rearrange it as follows:

$$R(\theta^*)(1-\alpha\ell)\,Log(\Lambda)\,\overline{n} - R(\theta^*)\,Log(\Lambda)(1-\overline{n}) + R(\theta^*)\,\overline{n} - R(\theta^*)\,\widehat{n}(\theta^*) - \ell Log(\overline{n})\,\overline{n},$$

where $Log(\Lambda) = -\int_{\widehat{n}(\theta^*)}^{\overline{n}} \frac{1}{1-n} dn$ and $\overline{n} = \frac{\ell}{(1-k)}$. Adding and subtracting $R(\theta^*)\,\alpha\ell Log(\Lambda)$ to the expression above, it can be further rearranged as follows:

$$\begin{aligned}&-R(\theta^*)(1-\alpha\ell)\,Log(\Lambda)\,\overline{n} - R(\theta^*)\,Log(\Lambda) + R(\theta^*)\,\overline{n} - R(\theta^*)\,\widehat{n}(\theta^*) + R(\theta^*)\,Log(\Lambda)\,\overline{n} \\ &+[R(\theta^*)\,Log(\Lambda) - \ell Log(\overline{n})]\overline{n}.\end{aligned} \tag{35}$$

The term in the square bracket in (35) is negative for $k \geq \widetilde{k}(\ell)$ because $R(\theta^*)\,|Log(\Lambda)| > R(\theta^*)(1-\alpha\ell)\,|Log(\Lambda)|$ and $R(\theta^*)(1-\alpha\ell)\,|Log(\Lambda)| = \ell\,|Log(\overline{n})|$ at $k = \widetilde{k}(\ell)$. Denote as $k^T(\ell)$ the level of capital and liquidity at which the terms in the first four terms sum up to zero. The curve $k^T(\ell)$ lies below $k^{\max}(\ell)$ and above $\widetilde{k}(\ell)$. Too see this, we can rearranged the first four terms in (35) as follows:

$$\int_{\widehat{n}(\theta^*)}^{\overline{n}} \frac{R(\theta^*)}{1-n}\left[-(1-\alpha\ell)\frac{\ell}{(1-k)} + 1 - \frac{n(1-k)}{\ell}\right] dn$$

It is easy to see that the expression in the square bracket is increasing in k. When $k = \widetilde{k}(\ell)$, the terms in the bracket sum up to $-(1-\alpha\ell)(\Lambda)^{\frac{R(\theta^*)(1-\alpha\ell)}{\ell}} + 1 - n(\Lambda)^{-\frac{R(\theta^*)(1-\alpha\ell)}{\ell}}$. Evaluating this at $n = \widehat{n}(\theta^*)$, we obtain $-(1-\alpha\ell)(\Lambda)^{\frac{R(\theta^*)(1-\alpha\ell)}{\ell}} < 0$. Then, when $k = k^T(\ell)$, it follows that $\frac{\partial\theta^*}{\partial\ell} < 0$ because $[R(\theta^*)(1-\alpha\ell)\,Log(\Lambda) - \ell Log(\overline{n})] < 0$ for any $k > \widetilde{k}(\ell)$.

Given that $\widetilde{k}(\ell) < k^T(\ell) < k^{\max}(\ell)$ and $\frac{\partial\theta^*}{\partial\ell} > 0$ for pairs $\{k,\ell\}$ below the curve $\widetilde{k}(\ell)$ and above the curve $k^{\max}(\ell)$, by continuity, there must exist two thresholds $\underline{k}(\ell) \in \left(\widetilde{k}(\ell), k^T(\ell)\right)$ and $\overline{k}(\ell) \in \left(k^T(\ell), k^{\max}(\ell)\right)$, such that $\frac{\partial\theta^*}{\partial\ell} > 0$ for pairs $\{k,\ell\}$ between the curves $\widetilde{k}(\ell)$ and $\underline{k}(\ell)$ and $\frac{\partial\theta^*}{\partial\ell} < 0$ for pairs $\{k,\ell\}$ between the curves $\underline{k}(\ell)$ and $\overline{k}(\ell)$. Thus, the proposition follows. □

Proof of Proposition 2: The proof proceeds in steps. First, we characterize the equilibrium choice of k, ℓ and r_2. Second, we show that the equilibrium k and ℓ are consistent with $\frac{\partial\theta^*}{\partial k} < 0$ and $\frac{\partial\theta^*}{\partial\ell} < 0$. Finally, we show that in equilibrium banks choose k and ℓ in such a way that $(1-k) > \ell$ holds in equilibrium so that liquidity crises occur.

Before starting solving the bank's problem, it is important to notice that the interest rate r_2 affects the threshold θ^* and it is chosen at date 0 from the debt holder's participation constraint, thus anticipating the withdrawal threshold θ^*. Differentiating the LHS of (9) with respect to θ^*, we obtain

$$-\left[r_2 - \frac{\ell}{1-k}\right] + \int_{\theta^*}^{1} \frac{dr_2}{d\theta^*} d\theta, \tag{36}$$

where $\frac{dr_2}{d\theta^*}$ can be computed using the implicit function theorem from (26) and it is then equal to

$$-\frac{\int_{\widehat{n}(\theta)}^{\bar{n}} \frac{R'(\theta^*)(1-\alpha\ell)\left[1-\frac{(1-k)nr_1}{\ell}\right]}{(1-k)(1-n)} dn}{\int_0^{\widehat{n}(\theta)} dn} < 0.$$

This implies that the expression in (36) is negative and so each pair $\{k, \ell\}$ implements only one θ^*.

Now we move on to solve bank's optimal choice. The conditions (12) and (13) in the proposition are obtained by substituting r_2 from (9) into (8) and differentiating it with respect to k and ℓ.

To prove that the bank's choice is always consistent with $\frac{\partial\theta^*}{\partial k} < 0$ and $\frac{\partial\theta^*}{\partial\ell} < 0$, we show that the effect of a change in k and ℓ on the threshold θ^* is positive when $\frac{\partial\theta^*}{\partial k} > 0$ and $\frac{\partial\theta^*}{\partial\ell} > 0$, even accounting for the indirect effect of k and ℓ on θ^* via r_2.

We can compute the total effect of k on θ^* $\frac{d\theta^*}{dk}$ as follows. Implicitly differentiating (9) with respect to k, we obtain

$$\frac{d\theta^*}{dk} = -\frac{\int_0^{\theta^*} \frac{\ell\chi}{(1-k)^2} d\theta + \int_{\theta^*}^1 \frac{dr_2}{dk} d\theta}{-\left[r_2 - \frac{\ell x}{(1-k)}\right] + \int_{\theta^*}^1 \frac{dr_2}{d\theta^*} d\theta},$$

where $\frac{\partial r_2}{\partial\theta^*} < 0$ and $\frac{\partial r_2}{\partial k}$ is obtained by implicitly differentiating (26) and is equal to

$$\frac{dr_2}{dk} = -\frac{\frac{\partial f(\theta^*,k,\ell)}{\partial k}}{\frac{\partial f(\theta^*,k,\ell)}{\partial r_2}}.$$

Given that $\frac{\partial f(\theta^*,k,\ell)}{\partial r_2} > 0$, as long as $\frac{\partial f(\theta^*,k,\ell)}{\partial k} < 0$, $\frac{dr_2}{dk} > 0$ and $\frac{d\theta^*}{dk} > 0$ since $-\left[r_2 - \frac{\ell}{(1-k)}\right] + \int_{\theta^*}^1 \frac{\partial r_2}{\partial\theta^*} d\theta < 0$. As shown in the proof of Proposition **??**, $\frac{\partial f(\theta^*,k,\ell)}{\partial k} < 0$ for pairs $\{k, \ell\}$ below the curve $\widetilde{k}(\ell)$. Following the same steps to compute $\frac{d\theta^*}{d\ell}$, we have that

$$\frac{d\theta^*}{d\ell} = -\frac{\int_0^{\theta^*} \frac{\ell}{(1-k)} d\theta + \int_{\theta^*}^1 \frac{dr_2}{d\ell} d\theta}{-\left[r_2 - \frac{\ell}{(1-k)}\right] + \int_{\theta^*}^1 \frac{dr_2}{d\theta^*} d\theta},$$

with

$$\frac{\partial r_2}{\partial\ell} = -\frac{\frac{\partial f(\theta^*,k,\ell)}{\partial\ell}}{\frac{\partial f(\theta^*,k,\ell)}{\partial r_2}}.$$

The derivative $\frac{dr_2}{d\ell}$ and, in turn, $\frac{d\theta^*}{d\ell}$ are positive when $\frac{\partial f(\theta^*,k,\ell)}{\partial\ell} < 0$. As shown in the proof of Proposition **??**, this is the case for any pair $\{k, \ell\}$ below the curve $\underline{k}(\ell)$ and above the curve $\overline{k}(\ell)$. Thus, the bank would only choose a pair $\{k, \ell\}$ in the region bounded by the curves $\underline{k}(\ell)$ and $\overline{k}(\ell)$.

To complete the proof, we need to show that the equilibrium k and ℓ satisfy $(1-k) > \ell$. To see this, we rearrange the first order conditions for k and ℓ:

$$\begin{array}{c} -\frac{\partial\theta^*}{\partial k}\left[R(\theta^*)(1-\alpha\ell) - (1-k)r_2\right] + \int_{\theta^*}^1 r_2 d\theta - \rho \\ -\frac{dr_2}{dk}\left\{\int_{\theta^*}^1 (1-k)\, d\theta + \frac{\partial\theta^*}{\partial r_2}\left[R(\theta^*)(1-\alpha\ell) - (1-k)r_2\right]\right\} = 0 \end{array}, \tag{37}$$

and

$$\begin{array}{c} -\frac{\partial \theta^*}{\partial \ell}\left[R\left(\theta^*\right)(1-\alpha \ell)-(1-k)\, r_2\right]-\int_{\theta^*}^1 \alpha R(\theta)\, d\theta \\ -\frac{dr_2}{d\ell}\left\{\int_{\theta^*}^1 (1-k)\, d\theta+\frac{\partial \theta^*}{\partial r_2}\left[R\left(\theta^*\right)(1-\alpha \ell)-(1-k)\, r_2\right]\right\}=0 \end{array} . \tag{38}$$

Assume that a bank sets $\ell = (1-k)$. From (9), it follows immediately that $r_2^* = 1$. Then, the expression (37) simplifies to

$$\int_{\underline{\theta}}^1 r_2 d\theta - \rho - \left.\frac{dr_2}{dk}\right|_{(1-k)=\ell} \int_{\underline{\theta}}^1 (1-k)\, d\theta < 0,$$

since $\rho > 1$ and $\frac{dr_2}{dk}$ can be computed using the implicit function theorem on (9) and is, then equal to

$$\left.\frac{dr_2}{dk}\right|_{(1-k)=\ell} = -\frac{\int_0^{\underline{\theta}} \frac{\ell}{(1-k)^2} d\theta}{\int_{\underline{\theta}}^1 d\theta} < 0.$$

Similarly, the expression (38) simplifies to

$$-\int_{\underline{\theta}}^1 \alpha R(\theta)\, d\theta - \left.\frac{dr_2}{d\ell}\right|_{(1-k)=\ell} \int_{\underline{\theta}}^1 (1-k)\, d\theta < 0,$$

with

$$\left.\frac{dr_2}{d\ell}\right|_{(1-k)=\ell} = -\frac{\int_0^{\underline{\theta}} \frac{1}{(1-k)} d\theta}{\int_{\underline{\theta}}^1 d\theta} < 0.$$

The fact that (37) is negative when evaluated at $(1-k) = \ell$ implies that the bank will always choose a level of k so that the inequality $(1-k) > \ell$ holds in equilibrium. This, in turn, implies that $r_2^* > 1$ for the (9) to be satisfied. Thus, the proposition follows. □

Proof of Proposition 3: When $(1-k) = \ell$ holds, $\theta^* \to \underline{\theta}$, as shown in the proof of Corollary 1. Then, since when $(1-k) = \ell$ holds $\underline{\theta} = \theta^E$, it follows that the total output loss as given in (16) is eliminated. The regulator can achieve this outcome by either setting $k^R = 1 - \ell^B$ or $\ell^R = 1 - k^B$, where k^B and ℓ^B are chosen by the bank, given the regulatory requirement. On the curve $k^{\max}(\ell)$, the bank chooses the pair $\{k, \ell\}$ by solving the problem specified in (8)-(11). Thus, the proposition follows. □

Proof of Proposition 4: The proof proceeds in steps. First, we prove that in any bank debt holders behave according to a threshold strategy when assuming that debt holders in other banks also behave according to a threshold strategy. Second, we characterize the equilibrium thresholds. Finally, we show that they are unique.

When debt holders at any bank behave according to a threshold strategy s_g^*, the proportion of debt holders in bank g withdrawing at date 1 is equal to the probability of receiving a signal below s_g^*. We denote it as $n_g = n_g(\theta, s_g^*)$ and it is still given by (25).

A debt holder's utility differential is similar to the one in (3) with the difference that the bank liquidates $\frac{(1-k_g)n_g}{\chi(Q,w)}$ units of the portfolio to meet the $(1-k_g)\, n_g$ early withdrawals rather than $\frac{(1-k_g)n_g}{\ell_g}$ as in the baseline model. This implies that a debt holder's utility differential is also a function of $n_{(-g)}$ that is of the proportion of other debt holders withdrawing at date 1 in all other banks.

The proof that a debt holder in bank g behave according to a threshold strategy when all other debt holders in the economy also behave according to a threshold strategy follows the same steps as in the proof of Proposition. Thus, denote as $\Delta(s_i, \dot{n}(\theta), \dot{\mathbf{n}}_{(-g)}(\theta))$ the expected difference in utility between withdrawing at date 2 and at date 1 of debt holder i in bank g when he holds beliefs $\dot{n}(\theta)$ and $\dot{\mathbf{n}}_{(-g)}(\theta)$ regarding the number of depositors running in his own bank and in all the other banks in the economy. The function $\Delta(s_i, \dot{n}(\theta), \dot{\mathbf{n}}_{(-g)}(\theta))$ is given by

$$\Delta(s_{ig}, \dot{n}(\theta), \dot{n}_{(-g)}(\theta)) = \frac{1}{2\varepsilon} \int_{s_i-\varepsilon}^{s_i+\varepsilon} E_n \left[v(\theta, \dot{n}(\theta), \dot{\mathbf{n}}_{(-g)}(\theta)) \right] d\theta.$$

Since for any realization of θ, the proportion of depositors running is deterministic, we can write $n_g(\theta)$ and $\mathbf{n}_{(-g)}(\theta)$ instead of $\dot{n}_g(\theta)$ and $\dot{\mathbf{n}}_{(-g)}(\theta)$ and the function $\Delta(s_i, \dot{n}(\theta), \dot{\mathbf{n}}_{(-g)}(\theta))$ simplifies to

$$\Delta(s_i, \dot{n}(\theta), \dot{\mathbf{n}}_{(-g)}(\theta)) = \frac{1}{2\varepsilon} \int_{s_i-\varepsilon}^{s_i+\varepsilon} v(\theta, n(\theta), \mathbf{n}_{(-g)}(\theta)) d\theta.$$

Notice that when all debt holders behave according to the same threshold strategy s^*, $\dot{n}(\theta) = n(s^*, \theta)$ as defined in (25). As in the proof of Proposition 1, for a unique threshold signal to exist we need to show that the utility differential of a debt holder in bank g is decreasing in n and increasing in θ. Regarding the former, the $v(.)$ is as in the baseline model, thus still exhibiting the property of one-sided strategic complementarity. Regarding the latter, unlike the baseline model, we also need to account the effect that θ has on $v(\theta, n(\theta), \mathbf{n}_{(-g)}(\theta))$ through its effect on the proportion of debt holders running in other banks $\mathbf{n}_{(-g)}(\theta)$. Specifically, the effect of θ on the $v(.)$ is given by

$$\begin{aligned} &\int_{\widehat{n}(\theta)}^{\overline{n}} \frac{R'(\theta)(1-\alpha\ell_g)\left[1 - \frac{n_g(1-k_g)}{\chi(Q,w)}\right]}{(1-k_g)(1-n_g)} dn_g \\ &+ \left[\int_{\widehat{n}(\theta)}^{\overline{n}} \frac{R(\theta)(1-\alpha\ell_g)\frac{n(1-k_g)}{\chi(Q,w)}}{(1-k_g)(1-n_g)} dn_g - \int_{\overline{n}}^{1} \frac{\chi(Q,w)}{(1-k_g)n_g} dn_g \right] \chi'(Q,w), \end{aligned} \tag{39}$$

since the derivatives of the extreme of the integrals cancel out and with $\chi'(Q,w) > 0$ since as θ increases the proportion of debt holders running in all other banks decreases from (25) and $\frac{\partial Q}{\partial n_g} > 0$ for all $g = 1, ...G$. The expression in (39) is positive for $R'(\theta)$ sufficiently large.

Having shown that, even accounting for the effect of θ on the proportion of debt holders running in all other banks, the function $v(.)$ is increasing in θ, it exhibits the same properties as in the baseline model. Thus, the rest of the proof goes through and all debt holders in bank g withdraw at date 1 if they receive a signal below s_g^* and roll over otherwise when they expect debt holders in the other banks also to behave according to a threshold strategy. The condition (18) in the proposition represents a debt holder's indifference condition between rolling over the debt and withdrawing at date 1 and it is obtained by substituting $\theta = s_g^* + \varepsilon - 2\varepsilon n_g$ into the expression for the proportion of early withdrawing debt holders $n_{(-g)}$ for all banks other than g as given in (25). The equilibrium corresponds to the vector of threshold signals $\mathbf{s}^*$ solving the system of G indifference condition as the one given in (18).

To complete the proof we need to show that the system of G indifference conditions has a unique solution. Denote as $f_g\left(s_g^*, \mathbf{s}_{(-g)}^*\right) = 0$ each indifference condition. We can rearrange the system in a matrix form as $\mathbf{A}\mathbf{s}^* = \mathbf{b}$, with $\mathbf{b} \neq \mathbf{0}$.

The matrix of the coefficients $\mathbf{A}$ is equal to

$$\mathbf{A} = \begin{bmatrix} \frac{\partial f_1(.)}{\partial s_1^*} & \frac{\partial f_1(.)}{\partial s_2^*} & ... & \frac{\partial f_1(.)}{\partial s_G^*} \\ . & & & . \\ . & \frac{\partial f_2(.)}{\partial s_2^*} & ... & . \\ . & & & . \\ \frac{\partial f_G(.)}{\partial s_1^*} & ... & ... & \frac{\partial f_G(.)}{\partial s_G^*} \end{bmatrix},$$

where the terms on the diagonal capture the effect of the threshold signal s_g^* on the indifference condition of a debt holder in bank g (i.e., $\frac{\partial f_g(.)}{\partial s_g^*}$), while all other terms are the effect of the threshold signal of debt holders in a bank other than g on the indifference condition of a debt holder in bank g (i.e., $\frac{\partial f_g(.)}{\partial s_{(-g)}^*}$). From (18), it is easy to see that $\frac{\partial f_g(.)}{\partial s_g^*} > 0$ while $\frac{\partial f_g(.)}{\partial s_{(-g)}^*} < 0$ and $\left|\frac{\partial f_g(.)}{\partial s_g^*}\right| > \left|\frac{\partial f_g(.)}{\partial s_{(-g)}^*}\right|$. Furthermore, given the bank are symmetric, in equilibrium they choose the same k_g, ℓ_g and r_{2g}. This implies that $\frac{\partial f_g(.)}{\partial s_{(-g)}^*}$ is the same for all g and $\frac{\partial f_g(.)}{\partial s_g^*} = \frac{\partial f_{(-g)}(.)}{\partial s_{(-g)}^*}$. Then, it follows that the determinant of matrix $\mathbf{A}$ is equal to

$$\left(\frac{\partial f_g(.)}{\partial s_g^*} - \frac{\partial f_g(.)}{\partial s_{(-g)}^*}\right)^{(G-1)} \left(\frac{\partial f_g(.)}{\partial s_g^*} + (G-1)\frac{\partial f_{(-g)}(.)}{\partial s_g^*}\right) \neq 0$$

and the system of G indifference conditions has a unique solution, which we denote as the vector $\mathbf{s}_{FS}^*$. Thus, the proposition follows. $\square$

Proof of Proposition 5: The proof follows Goldstein (2005) and it is done for $\varepsilon \to 0$, so that $s_g^* \to \theta_g^*$ given that $\theta = s_g^* + \varepsilon - 2\varepsilon n_g$. The arguments in his proof establish that there is a unique threshold of fundamental θ, which we denote as θ_{FS}^*, below which debt holders at all bank withdraw at date 1 and roll over otherwise.

The proof hinges on the characterization of the equilibrium thresholds in the case where debt holders in a bank g have extreme beliefs about the actions of debt holders in the other banks. Denote as $\theta_g^*\left(\mathbf{n}_{(-g)} = \mathbf{1}\right)$ and $\theta_g^*\left(\mathbf{n}_{(-g)} = \mathbf{0}\right)$ debt holders' equilibrium threshold in the case they expect that no investors roll over and all investors roll over, respectively, in all other $-g$ banks in the economy. As banks are symmetric, $\theta_g^*\left(\mathbf{n}_{(-g)} = \mathbf{1}\right)$ and $\theta_g^*\left(\mathbf{n}_{(-g)} = \mathbf{0}\right)$ are the same for all banks. These thresholds under extreme beliefs can be computed following the same steps illustrated in Proposition 4 but fixing the proportion of debt holders running in other banks. Notice that the threshold $\theta_g^*\left(\mathbf{n}_{(-g)} = \mathbf{0}\right)$ is the same as θ^* characterized in Proposition 1 since if only one bank sells asset in the market no fire sale occurs and $\chi = \ell_g$. Since threshold characterized in Proposition 4 are computed for $0 \leq n_{(-g)} \leq 1$ and the actions of debt holders in different banks are strategic complements, it follows that equilibrium thresholds θ_g^* lies in the range $(\theta_g^*\left(\mathbf{n}_{(-g)} = \mathbf{0}\right), \theta_g^*\left(\mathbf{n}_{(-g)} = \mathbf{1}\right)]$ that is it is strictly larger than $\theta_g^*\left(\mathbf{n}_{(-g)} = \mathbf{0}\right) \equiv \theta^*$.

To complete the proof we need to show that all θ_g^* converges to the same value θ_{FS}^*. Assume by contradiction that $\theta_g^* < \theta_{(-g)}^*$. Then, a debt holder i in bank g receiving the signal $s_i = \theta_g^*$ is indifferent between running and rolling over and believes that debt holders in all other banks in the economy withdraw at date 1. Thus, θ_g^* would converge to $\theta_g^* \left(\mathbf{n}_{(-g)} = \mathbf{1}\right) > \theta_{(-g)}^*$. A similar argument rules out the possibility that $\theta_g^* > \theta_{(-g)}^*$. Thus, in equilibrium it must be that $\theta_g^* = \theta_{(-g)}^*$ and we denote it as θ_{FS}^* and the proposition follows. $\square$

Proof of Proposition 6: The proof is analogous to that of Proposition 2. The conditions (20) and (21) in the proposition are obtained by substituting r_2 from (19) into (8) and differentiating it with respect to k and ℓ. Following the same steps as in the proof of Proposition 2, it can be shown that when banks choose $k = \underline{\chi}(\boldsymbol{\ell}, w)$, $\theta_{FS}^* \to \underline{\theta}$, which is still given by (2). Furthermore, from (19), $r_2 = 1$ and the bracket $[R(\theta_{FS}^*)(1 - \alpha\ell) - (1 - k)] = 0$ and so both (20) and (21) become negative. This implies that $k^G < \underline{\chi}(\boldsymbol{\ell}, w)$ holds and so the proposition follows. $\square$

Proof of Proposition 7: Denote as $\widehat{\ell}(w)$ the minimum level of portfolio liquidity that each bank should hold such that no fire sales occur in equilibrium. Thus, by setting $\ell^R \geq \widehat{\ell}(w)$, $\underline{\chi}(\boldsymbol{\ell}, w) = \ell$. Once $\underline{\chi}(\boldsymbol{\ell}, w) = \ell$ holds, efficient liquidation can be enforced by setting k^R such that $1 - k = \ell$ holds so that $\theta_{FS}^* \to \underline{\theta} = \theta^E$ and the proposition follows. $\square$

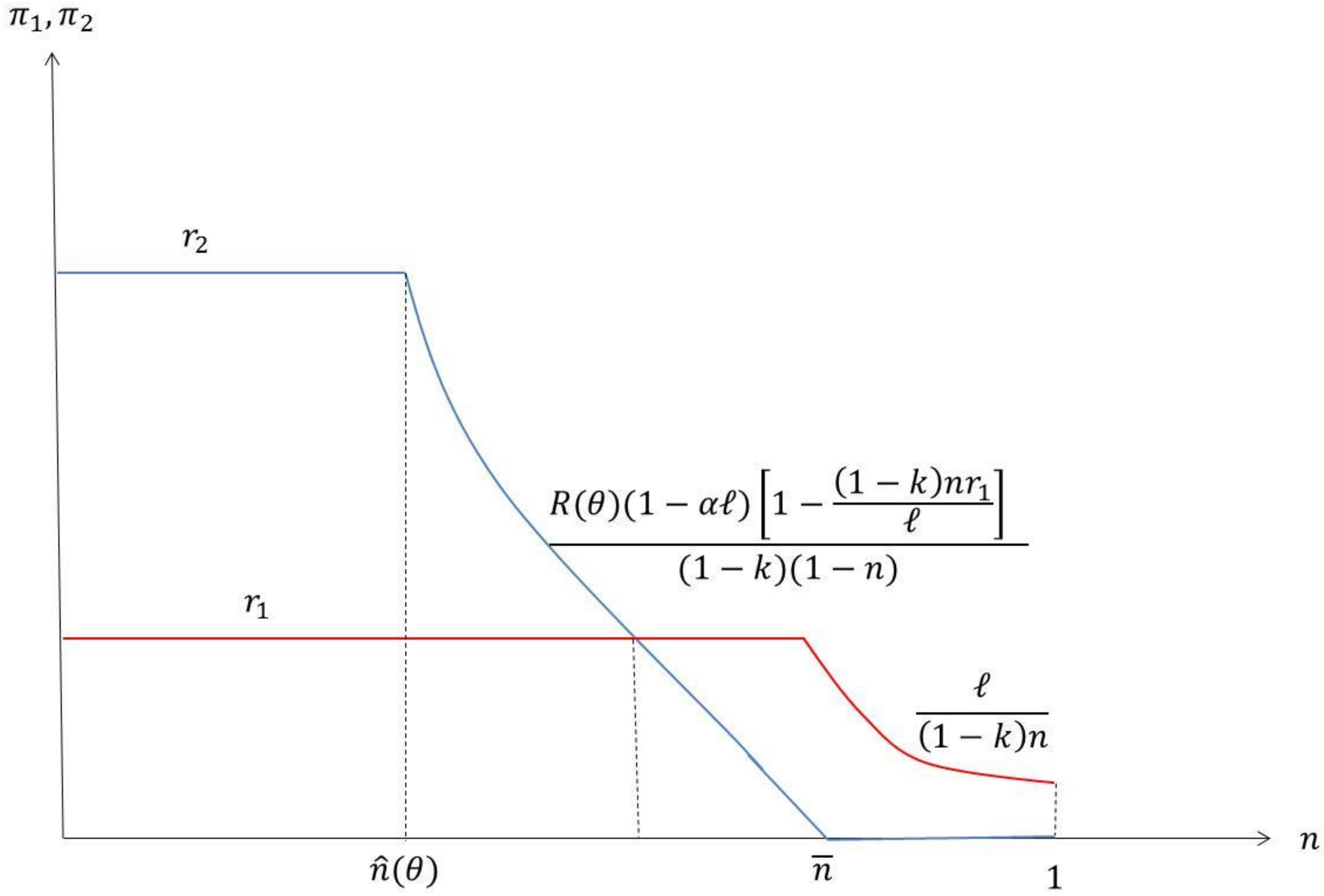

Figure 1: Debt holders' payoffs. The figure illustrates how a debt holder's payoffs changes with the proportion of debt holders withdrawing their funds at date 1 n for a given $\theta > \underline{\theta}(k, \ell)$. The blue line represents a debt holder's payoff at date 2. A debt holder receives the promised repayment r_2 as long as the bank has enough funds to repay it (i.e., when $n < \hat{n}(\theta)$); otherwise he obtains a pro-rata share of bank's available resources. Such pro-rata is equal to zero when the bank liquidates the entire portfolio at date 1 (i.e., $n \geq \overline{n}$). The red line represents a debt holder's payoff at date 1. A debt holder receives the promised repayment r_1 as long as the bank has enough resources (i.e., $n < \overline{n}$); otherwise he obtains a pro-rata share.

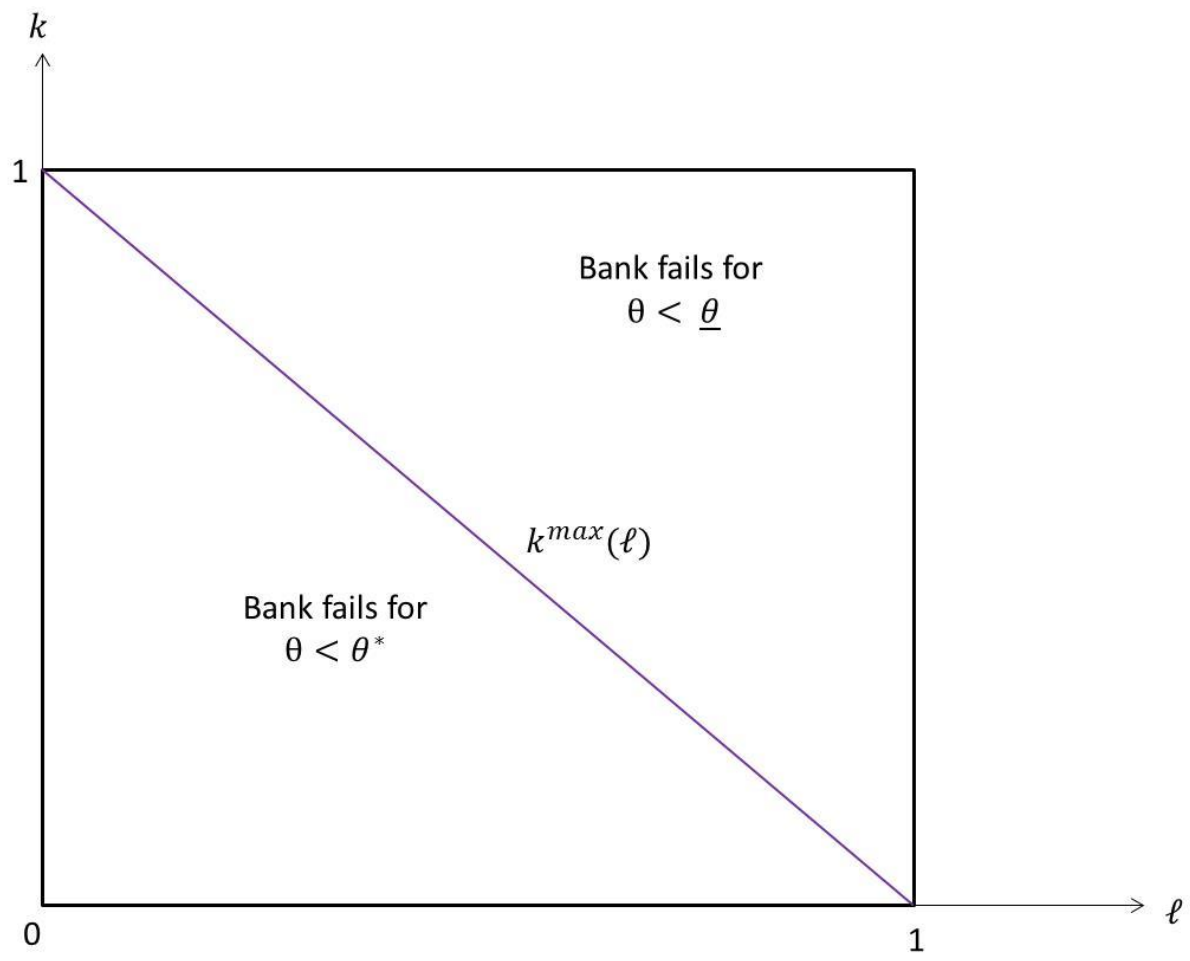

Figure 2: Capital, liquidity and type of crises. The figure illustrates how bank's exposure to crises depends on its capital structure k and portfolio liquidity ℓ. A bank characterized by high capital and/or portfolio liquidity (i.e., one falling in the region above the curve $k^{max}(\ell)$) is only exposed to solvency-driven crises so that the relevant crisis threshold is $\underline{\theta}$. A bank characterized by low capital and/or portfolio liquidity (i.e., one falling in the region below the curve $k^{max}(\ell)$) is also exposed to liquidity-driven crises so that the relevant crisis threshold is θ^*. The curve $k^{max}(\ell)$ corresponds to $(1-k)=\ell$ and pins down the pairs $\{k,\ell\}$ for which there is no strategic complementarity among debt holders' withdrawal decisions.

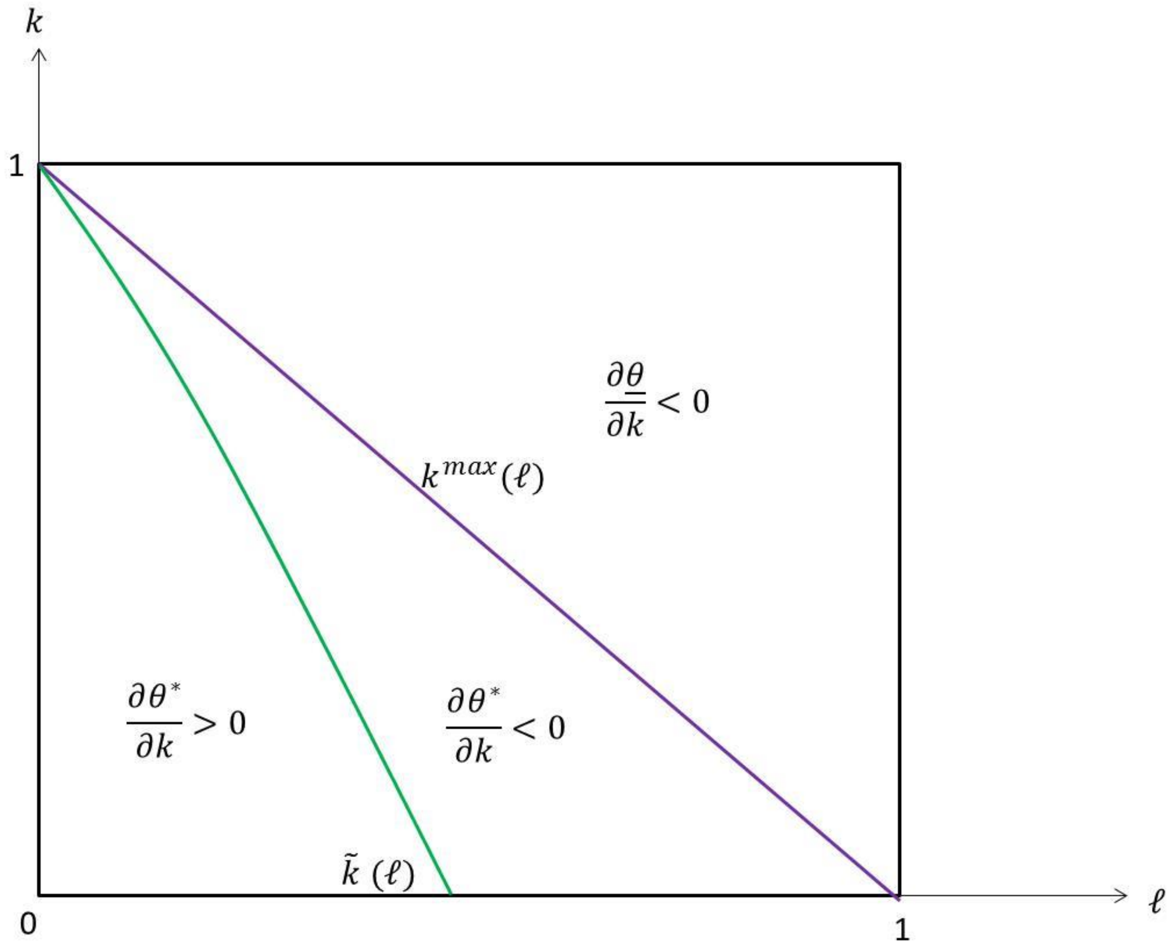

Figure 3a: Capital and Stability. The figure illustrates the effect of a marginal increase in bank capital on stability. Capital has a beneficial effect on stability for a bank characterized by intermediate and high values of capital and/or portfolio liquidity (i.e., $\frac{\partial\theta^*}{\partial k} < 0$ in the region above the curve $\tilde{k}(\ell)$), while it is detrimental otherwise.

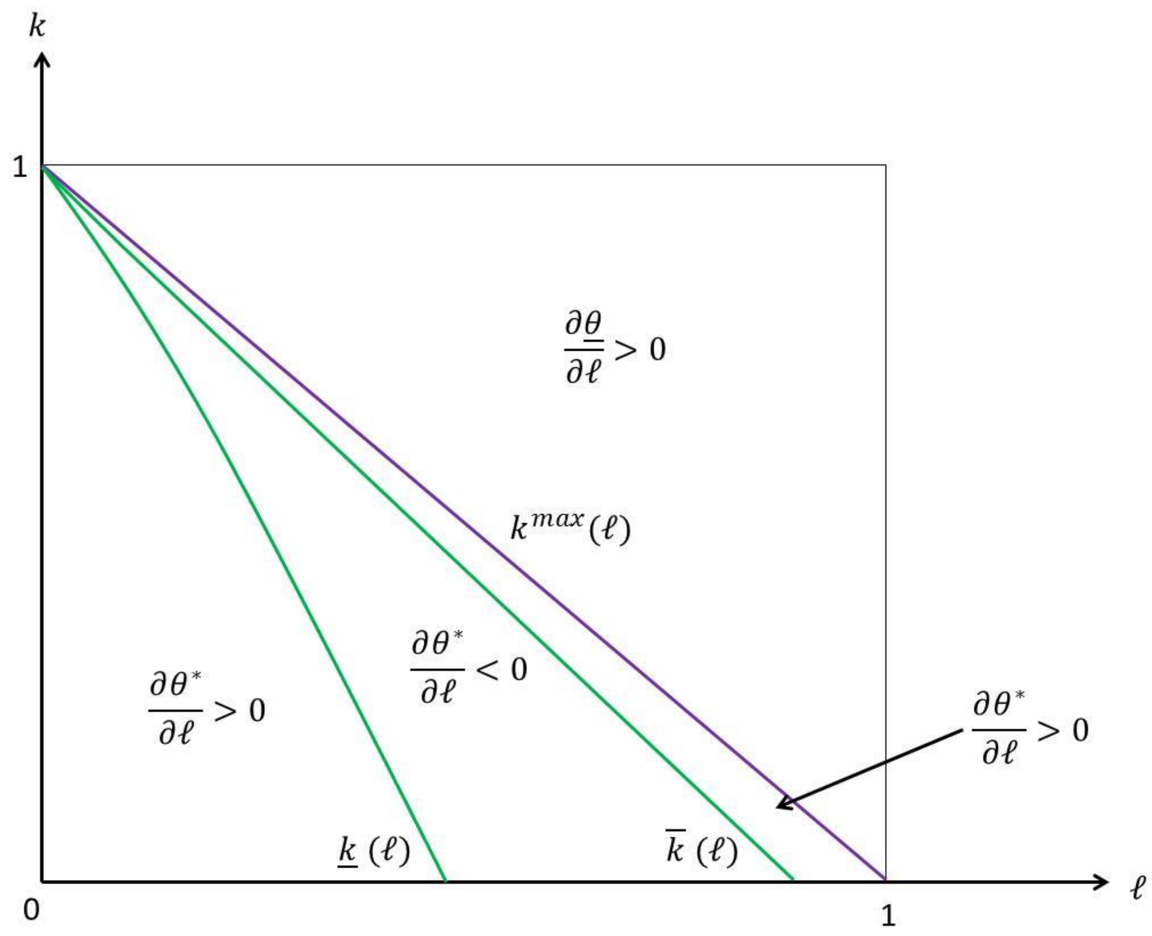

Figure 3b: Portfolio liquidity and stability. The figure illustrates the effect of a marginal increase in portfolio liquidity on stability. Liquidity has a beneficial effect on stability only for a bank characterized by intermediate values of capital and/or portfolio liquidity (i.e., $\frac{\partial \theta^*}{\partial \ell} < 0$ in the region bounded by the curves $\underline{k}(\ell)$ and $\overline{k}(\ell)$), while it is detrimental otherwise.