THE OPPORTUNITY COST OF COLLATERAL*

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PRELIMINARY

Abstract

We develop a dynamic model of borrowing and lending in the interbank market in which banks fund investments through short-term collateralized debt, like repos. This debt is not a perfect substitute for cash: lending banks may not be able to convert their loans to cash to fund their own investments. Hence, lending comes with an opportunity cost that generates positive spreads even absent any credit risk. These spreads enter banks' collateral constraints, generating a two-way feedback between the opportunity cost in the credit market and the price of collateral in the asset market. This feedback results in instability in the form of multiple equilibria, casting light on repo runs. It highlights the unique fragility present in the bilateral repo market, in which banks borrow from one another, but not in the tri-party repo market, in which banks borrow from passive cash investors, who do not suffer the opportunity cost. We show that high-leverage equilibria are inefficient in booms; hence, the model suggests a new rationale for counter-cyclical capital regulation: to select the efficient equilibrium.

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1 Introduction

Banks use short-term collateralized debt to fund diverse investments, spanning prop trades, securitization, and client loans. In particular, they borrow in the tri-party and bilateral repo markets that now total \$12 trillion (CGFS (2017)). Tri-party repos are much like deposits; banks borrow from long-term savers who are looking to park their cash for safe returns (Agueci et al. (2014)). And, as with deposits, credit is stable (Copeland, Martin, and Walker (2014) and Krishnamurthy, Nagel, and Orlov (2014)), unless a shortage of collateral exposes an individual institution to a classical bank run (Martin, Skeie, and von Thadden (2014a) and (2014b)). Bilateral repos are different; banks borrow from similar banks who are lending their temporary excess liquidity to get short-term interest (Baklanova et al. (2015) and Baklanova, Copeland, and McCaughrin (2017)). Credit can be unstable. Notably, in 2008 haircuts spiked and credit tightened dramatically, resulting in the so-called "run on repo" (Gorton and Metrick (2012)), even though over-collateralization kept the risks of default and classical bank runs at a minimum (Gorton (2012)). What, then, makes bilateral repo markets unstable? Can a regulator intervene to improve social welfare?

In this paper, we develop a dynamic model of interbank borrowing and lending to address these questions. In the model, like in the bilateral repo market, but unlike in most models, lenders are not passive investors parking cash. Rather, they are other banks that may need cash to fund their own investment opportunities in the future. But converting their loans to cash is not frictionless—although repos are money-like, they are not perfect substitutes for cash. Hence, lending comes with the possibility of forgone investment opportunities.

This opportunity cost generates positive spreads even absent any credit risk. These spreads enter banks' collateral constraints, generating a two-way feedback between the opportunity cost in the credit market and the price of collateral in the asset market. This feedback results in multiple equilibria, highlighting the unique instability in the bilateral repo market, and casting light on repo runs. We show that these equilibria are welfare ranked, and that high-leverage equilibria are efficient in booms; hence, the model suggests a new rationale for counter-cyclical capital regulation: equilibrium selection—limiting leverage in booms can force banks into the efficient equilibrium.

Model preview. A continuum of ex ante identical banks exist in continuous time. At each time, some banks get investment opportunities, and borrow from the other banks to fund them. The model is based on two key assumptions. First, the pledgeability of cash flows is limited as in, e.g., Holmstrom and Tirole (1997): if a bank gets an investment opportunity, it needs to post collateral to borrow and fund it. Second, the moneyness of loans is limited as in, e.g., Kiyotaki and Moore (2000)[1]: if a bank makes a loan, it cannot frictionlessly convert it to cash to fund its own investments.

Results preview. Our first main result is a characterization of how much investing banks borrow and at what spread. The spread is positive even though, given loans are fully collateralized in the optimal lending contract, there is no credit risk. Thus, the spread is not a risk premium. Rather, it is a result of limited moneyness: it is purely compensation for the forgone investment opportunities that lending banks cannot undertake if they are unable to convert their loans to cash. Hence, the spread is an increasing function of the ratio of the value of the investments they must forgo to the value of the cash they get as interest. This spread determines how much banks have to repay against their collateral ex post, and hence how much they can borrow ex ante. This opportunity-cost channel determines the volume of credit, which leads to our next two results.

Our second main result is that if the return on banks' investments increases, the spread increases and leverage decreases—in line with empirical facts, we find that credit is tight when returns are high, namely in crises, when prices are depressed (e.g., Muir (2017) and Krishnamurthy and Muir (2017)). The reason is that the better investment opportunities are, the more compensation lending banks need for forgoing them. Hence, the higher is the spread. This high spread leads to tight credit, because banks have to promise higher repayments against the same collateral.

Our third main result is that if banks expect the returns on their investments to increase in the future, the spread decreases and leverage increases today—in line with empirical facts, credit is loose when future returns are high, namely in the build up to crises, when banks anticipate low prices in the future. The reason is that the better future investment opportunities are, the more valuable cash is today, since you can save it to invest profitably. Thus, each dollar of interest is worth more today, and lending banks require less total interest as compensation for forgone investment opportunities—it is not the absolute value of forgone investments that matters; it is their value relative to cash. Hence, if cash becomes more valuable, the spread goes down. This low spread leads to loose credit, because banks have to promise lower repayments against the same collateral.

For our fourth main result, we include a market for capital assets to model endogenous asset prices, which we abstracted from so far to keep things as simple as possible. Now, there is a two-way interaction between asset prices and the opportunity-cost channel: tight credit depresses demand, leading to low prices and high returns; these high returns feed back to tighten credit further, via our opportunity cost channel. This feedback loop is powerful enough to generate multiple equilibria due to self-fulfilling beliefs. There is a high-leverage

[1] See also Donaldson and Micheler (2018) and Kiyotaki and Moore (2001a, 2001b, 2005, 2012).

equilibrium and a low-leverage equilibrium: if banks believe returns are high, credit is tight, demand is low, and returns are indeed high, and vice versa. We show that these equilibria are Pareto ranked, in the sense that all banks are better off in one than in the other. Hence, there is a case for regulation that can rule out the inefficient equilibrium.

For our fifth main result, we characterize which equilibrium is better for banks as a function of frictions in the lending market. We find that banks are better off in the low-leverage equilibrium if frictions are low—e.g., loans are easy to sell. Since low frictions seem to correspond to booms, this suggests that bank capital regulation is a good idea in booms—a regulator can improve welfare by capping bank leverage, and thereby forcing the market into the "good" equilibrium. Hence, we provide a new rationale for the idea that capital regulation "measures have to be counter-cyclical, i.e. tough during a credit boom and more relaxed during a crisis" (Brunnermeier et al. (2009), p. 31).

Repos and rehypothecation. In our model, as in practice, money-like debt is debt that can be converted to cash easily, either by selling it in the market or, in the case of repos, rehypothecating collateral, a transaction that resembles "spending a repo."[2] Still, it is not as frictionless as spending cash. In fact, in the 2008 financial crisis, the amount of collateral available for rehypothecation dropped by half (Singh and Aitken (2009, 2010)). This "immobility of collateral" led repo spreads, which had been loose in the build up to the crisis, to shoot up, even though high haircuts kept default risk at a minimum (Gorton and Muir (2015))—in line with our opportunity-cost channel, spreads are not compensation for risk.

Related literature. Our credit cycles, based on the opportunity cost of collateral, complement those in the literature, based on assets being used as collateral. In Kiyotaki and Moore (1997) and Brunnermeier and Pedersen (2009), falling asset prices lead collateral constraints to tighten, which depresses asset demand and make asset prices fall further. Credit fluctuates as a result. Although distinct, our cycles can amplify these, exacerbating financial fragility (Subsection 5).[3] This interaction is specific to markets in which lenders

[2] As Gorton and Metrick (2010) put it,

> [An] important feature of repos is that the...collateral can be "spent"...used as collateral in another, unrelated, transaction.... This...means that there is a money velocity associated with the collateral. In other words, the same collateral can support multiple transactions, just as one dollar of cash can.... The collateral is functioning like cash (p. 510).

[3] Other theories of credit cycle include that of Fostel and Geanakoplos (2008) and Geanakoplos (2010) based on collateral constraints under heterogeneous beliefs, Myerson (2012) on moral hazard, Gu, Mattesini, Monnet, and Wright (2013) on the beliefs about future credit conditions, Gorton and Ordoñez (2014) on the time-varying information sensitivity of debt, and Kurlat (2016) on adverse selection. He and Krishnamurthy (2012, 2013) focus on the role of asset prices in banks' equity constraints.

value cash to fund their own investments—i.e. to markets, like the interbank market, in which lenders can also be borrowers. Thus, unlike other theories, our analysis can help explain why credit in the interbank market is especially prone to drying up.

Our finding that the high-leverage equilibrium is inefficient in booms complements papers on constrained inefficient credit booms. Many of these models, such as Bianchi (2016), Gersbach and Rochet (2012), Korinek and Simsek (2016), and Lorenzoni (2008), are based on pecuniary externalities induced by asset prices in collateral constraints. In our model, both spreads and asset prices enter collateral constraints, and they feed back on each other to generate multiple equilibria. Thus, unlike in these papers, leverage regulation serves as equilibrium selection; Donaldson, Piacentino, and Thakor (2018) propose a similar strategy for the household credit market, arguing it could mitigate labor-search externalities.

Our policy analysis fits into the literature on regulating the leverage cycle. Davydiuk (2018) and Malherbe and Bahaj (2018) argue that countercyclical regulation can mitigate cyclical inefficiencies of bank credit extension in the corporate loan market. We argue, in contrast, that it can mitigate inefficiencies in the interbank market.

Our paper also fits into the literature on the private creation of money-like securities that are imperfect substitutes for cash.[4] With our focus on limited moneyness in the repo market, we relate to the theory literature on repos,[5] especially those that study rehypothecation (e.g., Gottardi, Maurin, and Monnet (2017) and Maurin (2017)). None of the papers in these literatures studies the opportunity cost, which is at the heart of our analysis. Some corporate finance papers, such as Bolton, Chen, and Wang (2011), do; we complement them, focusing on how it is determined in market equilibrium.

[4]E.g., Gorton and Pennacchi (1990), Dang, Gorton, and Hölmstrom (2015a, 2015b), Dang, Gorton, Hölmstrom, and Ordoñez (2015), and Gorton and Ordoñez (2014) all study how banks should design securities to circulate in secondary markets in the presence of asymmetric information. In contrast He and Milbradt (2014) focus on search frictions in the secondary market, and study how they interact with default decisions in the primary market. Stein (2012) focuses on debt maturity, rather than default, and shows that a premium for moneyness leads to inefficient shortening of maturity. Rather than focus on fixed-maturity debt, Donaldson and Piacentino (2018) examine the option to redeem on demand, and argue that the banks' ability to provide this option creates a rationale for traditional banking. Sunderam (2015) moves the focus to shadow banks, studying how they create near-money substitutes.

[5]Notably, Martin, Skeie, and von Thadden (2014a, 2014b) study coordination-based runs in the tri-party repo market.

2 Model

2.1 Environment, Agents, and Technologies

There is a unit of ex ante identical risk-neutral agents, called banks, which discount the future at rate $\rho > 0$ in continuous time, $t \geq 0$. Each bank starts with initial endowment W_0 of a numeraire consumption good, called cash. Cash is storable at the risk-free rate, which we normalize to zero. There is also an investment good, called capital, which banks can use to do investment opportunities, which arrive with Poisson intensity α. Investments are (very) short-term, riskless, and constant returns to scale:[6] investing capital k at time t yields cash Ak at time $t + \mathrm{d}t$, where $\mathrm{d}t$ is the time differential.[7] After a bank completes its investment, it consumes and dies.

2.2 Budget, Collateral, and Leverage Constraints

A bank with an investment opportunity buys capital k at price p subject to its budget constraint that

$$pk = w + bw, \tag{1}$$

where w denotes the bank's initial wealth and bw the amount it borrows (so b is its leverage). Its borrowing is limited by the collateral constraint that

$$(1 + \sigma)bw \leq pk, \tag{2}$$

where σ denotes the spread (over the risk-free rate of zero).[8]

Just combining the constraints above gives a simple leverage constraint:

$$b \leq \frac{1}{\sigma}. \tag{3}$$

[6]Focusing on short-term riskless investments helps keep the model tractable. Notably, it collapses the contracting space, giving us optimal contracts essentially for free. Still, we consider longer-maturity investments in Appendix C to show that our results are robust.

[7]Note that we assume that capital fully depreciates in production. This rules out capital accumulation, which simplifies the analysis by ensuring that the aggregate capital stock is not a state variable.

[8]In Appendix B, we give one microfoundation for the collateral costraint based on repudiation with the threat of liquidation, as in Hart and Moore (1994). Notably, the constraint can arise through dynamic incentives even if capital fully depreciate in production.

2.3 Lending and Asset Markets

Lending market. A bank can choose to hold cash, in which case it gets investment opportunities with intensity α. Alternatively, it can choose to make loans. We assume that there are frictions in the lending market, so if it chooses to lend, a bank does not make a loan immediately, but only with Poisson intensity β;[9] if it makes a loan, it gets the (endogenous) spread σ. A bank that chooses to lend may still get investment opportunities, but they do not arrive with the same intensity as they do if it holds cash, but with a lower intensity $\phi\alpha < \alpha$. Thus, choosing to lend comes with the benefit of expected interest $\beta \mathrm{d}t \sigma$, but the cost of forgone investment opportunities $\alpha \mathrm{d}t(1-\phi)$. These forgone investment opportunities could result from frictions in reselling loans/rehypothecating collateral that make it hard for a bank to convert its loans to cash and invest, as we formalize in Appendix C.

Asset market. Given we are interested mainly in the interbank lending market, we model the asset market in the simplest way we can. We assume that investing banks face the reduced-form supply curve $S(p)$.[10] We assume it is generated by competitive suppliers, but do not model them explicitly in the baseline model (although we do in Appendix D). Below, we first analyze perfectly elastic supply (exogenous p) and then imperfectly elastic supply.

2.4 Aggregate State

The exogenous parameters α, β, ϕ, and A, and the supply curve S can depend on an aggregate state at time t, denoted by s_t.[11] We assume that s_t changes only once, at random time τ that arrives with Poisson intensity π. (The assumption of a single shock allows us to solve the model in closed form.)

[9] Aside from reflecting reality, this assumption is necessary to keep output bounded, i.e. to ensure the aggregate output over $[t, t+\mathrm{d}t)$ is $o(\mathrm{d}t)$. To see why, observe that aggregate output is the product of output per unit capital and the amount of capital invested. In our model, with short-term investments, the flow output per unit capital is $o(t)$ and lending frictions ensure the amount invested is $o(\mathrm{d}t)$ to ensure the product is too. In Appendix C, we consider a variant of the model with longer-term investments, in which the flow output per unit capital is $o(\mathrm{d}t)$ and lending frictions are no longer necessary; the results are similar (although the set-up is ultimately less tractable).

[10] Note that in many models, capital stays in the hands of investing agents, whereas credit comes from outside—think of the farmers and gatherers in Kiyotaki and Moore (1997). In our set-up, in contrast, credit comes from within the banking system, but capital comes from outside. This is realistic for banks, which are likely to go to other banks to get funding for their investments, but are likely to go to outside markets to acquire assets to invest in. For example, to create mortgage-backed securities, banks buy pools of mortgages from non-bank mortgage originators.

[11] We do not index the discount rate ρ by the state just because we think it is more natural to assume that preference parameters are constant; this does not affect the main results.

2.5 Equilibrium Definition

An equilibrium is an allocation of quantities—those invested, lent, and held in cash—and prices—the credit spread and capital price—at each time t such that the following hold:

(i) Banks optimize: investing banks optimally choose how much to invest and other banks optimally choose how much to hold in cash and how much to lend (with all banks taking σ and p as given).

(ii) The credit and asset markets clear.

Markov equilibrium with binding leverage constraints. We focus on Markov equilibria with binding leverage constraints, i.e. in which the state s_t is a sufficient statistic for the entire history. Throughout, we consider a candidate equilibrium in which the following hold:

(i) **Binding constraints.** The leverage constraints in equation (3) bind.

(ii) **"Interior equilibrium."** Cash holdings are "interior" in the sense that some banks hold cash and some lend at each t.

(iii) **Banks save.** Banks hold cash until they invest rather than consume.

We translate these into conditions on primitives after solving for a candidate equilibrium assuming they hold.

Like the exogenous parameters, the equilibrium leverage b, capital invested k, spread σ, and capital price p are implicitly indexed by s_t. We omit this index when we can, and often use the following shorthand when we cannot: recalling that the state changes only once—there is one "shock" at the random time τ—we use the subscript τ for values after the shock. E.g., for v_{s_t}, which denotes the value of cash below, we write

$$v_{s_t} =: \begin{cases} v & \text{if } t < \tau, \\ \\ v_\tau & \text{if } t \geq \tau. \end{cases} \tag{4}$$

3 Value Functions and Model Solution

In this section, we first write down the value functions corresponding to investing, holding cash, and lending and then find the dynamics of aggregate wealth and its proportions that are invested, lent, and held in cash from market clearing. This sets us up to solve for the equilibrium allocations and prices in the next sections.

3.1 Value Functions

Value of investing V. We begin with the value of investing, which we denote by V. Taking the spread σ as given, a bank with an investment opportunity and wealth w chooses its leverage b to maximize its final output net of the loan repayment. Thus, it solves the program to

$$\text{maximize } Ak - (1+\sigma)bw \tag{5}$$

over $k, b \geq 0$ subject to its budget and leverage constraints (equations (1) and (3)). Substituting $k = (1+b)w/p$ from the budget constraint and denoting the gross return by $R \equiv A/p$, we have

$$V(w) = \max\left\{ \Big(R + (R-1-\sigma)b\Big)w \;\middle|\; b \in \left[0\,,\,\frac{1}{\sigma}\right]\right\}. \tag{6}$$

Observe that V is linear in w, so we define $V := V(1)$ and write $V(w) \equiv Vw$.

Since we focus on equilibria in which the leverage constraints bind, we can express V in terms of σ:

Lemma 1. **(Value of investing.)** *The value of investing against w dollars is Vw, where*

$$V = (R-1)\left(1 + \frac{1}{\sigma}\right). \tag{7}$$

Value of cash v. Next we turn to the value of holding cash. We use v to denote the value of cash and $\bar{v}$ to denote the expected value of cash a differential amount of time dt in the future, $\bar{v}_{s_t} := \mathbb{E}_t\left[v_{s_{t+\mathrm{dt}}}\right]$. Since v inherits linearity from V,[12] we can write it recursively as

$$v(w) = vw = \frac{1}{1+\rho\,\mathrm{dt}}\Bigg(\alpha\mathrm{dt}Vw + (1-\alpha\mathrm{dt})\,\bar{v}w\Bigg), \tag{8}$$

i.e. with probability αdt the bank gets an investment opportunity worth Vw and with probability $1-\alpha$dt it keeps its cash worth $\bar{v}$; everything is discounted at rate ρ.

Value of lending v^ℓ. Finally, we turn to the value of lending, which we denote by v^ℓ. Again v^ℓ inherits linearity from V, so we can write it recursively as

$$v^\ell w = \frac{1}{1+\rho\,\mathrm{dt}}\Bigg(\phi\alpha\mathrm{dt}Vw + (1-\phi\alpha\mathrm{dt})(1+\beta\sigma\mathrm{dt})\bar{v}w\Bigg), \tag{9}$$

i.e. with probability $\phi\alpha$dt the lending bank gets an investment opportunity worth Vw and with probability $1-\phi\alpha$dt it does not, and gets the expected interest $\beta\sigma$dt in cash worth $\bar{v}$;

[12] You can verify this formally by checking equations (21) and (25) below.

everything is discounted at rate ρ.

3.2 Market Clearing and Wealth Shares

We now impose market clearing in the lending market and calculate how aggregate wealth is distributed among investing, lending, and holding cash. Because the value functions are linear, it does not matter how wealth is distributed among individual banks, but only how much of it is allocated to each activity.

The total wealth held by all agents at time t, denoted by W_t, constitutes the wealth held by (i) investors/borrowers, denoted by w_t^I, (ii) lending banks, denoted by w_t^ℓ, and (iii) cash holders, denoted by $w_t^\$$. These quantities must satisfy three constraints:

(i) **Investment arrivals.** New investments arrive at rate α for cash holders and rate $\phi\alpha$ for lenders:

$$w_t^I = \alpha w_t^\$ \mathrm{dt} + \phi\alpha w_t^\ell \mathrm{dt}. \tag{10}$$

(ii) **Market clearing.** The amount borrowed equals the amount lent:

$$b w_t^I = \beta w_t^\ell \mathrm{dt}. \tag{11}$$

(iii) **Adding up.** The wealth of all types of agents is the total wealth:

$$w_t^\$ + w_t^\ell = W_t, \tag{12}$$

where we have used that the measure of investing banks at any single time is vanishingly small compared to the measures of lenders and cash holders, i.e. $w_t^I = o(\mathrm{dt})$ from equation (10).

Combining these three, we can find the wealth shares (assuming they are positive).

Lemma 2. **(Wealth shares.)**

$$\frac{w_t^I}{W_t} = \frac{\alpha\beta}{\beta + \alpha(1-\phi)b}\mathrm{dt}, \tag{13}$$

$$\frac{w_t^\ell}{W_t} = \frac{\alpha b}{\beta + \alpha(1-\phi)b}, \tag{14}$$

$$\frac{w_t^\$}{W_t} = \frac{\beta - \phi\alpha b}{\beta + \alpha(1-\phi)b}. \tag{15}$$

Flow of funds. Given the wealth shares, we can calculate the growth rate of the economy via the flow of funds condition: a borrower eats his wealth and dies; a lender gets expected interest $\beta\sigma$dt; and a cash holder gets nothing. Thus, the change in total wealth is the interest income of all lenders minus the wealth investing banks consume:

$$\begin{aligned}
\mathrm{d}W_t &= \beta\sigma w_t^\ell \mathrm{dt} - w_t^I && (16)\\
&= \left(\sigma - \frac{1}{b}\right)\beta w_t^\ell \mathrm{dt}, && (17)\\
&= \left(\sigma - \frac{1}{b}\right)\frac{\alpha\beta b}{\beta + \alpha(1-\phi)b} W_t \mathrm{dt} && (18)
\end{aligned}$$

having used the market clearing condition (equation (11)).

Since the leverage constraint in equation (3) binds in equilibrium, $b = 1/\sigma$ and the lenders' interest income is exactly the wealth of investing banks. Thus, the total wealth remains constant:

$$\mathrm{d}W_t = 0. \tag{19}$$

3.3 Equilibrium Spread and Leverage

Given that value functions are linear, non-investing banks must be indifferent between lending and holding cash; otherwise, they would all prefer to do one thing or the other, and the market could not clear.[13] Equating $v = v^\ell$ from equations (8) and (9) and taking the limit $\mathrm{dt} \to 0$, we can solve for the spread σ.

Proposition 1. **(Spread.)** *The spread σ solves*

$$\sigma = \frac{(1-\phi)\alpha}{\beta}\left(\frac{V}{v} - 1\right). \tag{20}$$

This expression for the spread captures the trade-off at the heart of our model. Expected interest income must compensate lending banks for their forgone investment opportunities: the spread σ makes up for lowering the chance of getting the investment value V from α to $\phi\alpha$. Indeed, if $\phi \to 1$, so loans become completely money-like, then spreads go to zero. But for $\phi < 1$, spreads are positive even though there is no credit risk whatsoever.

We can read other implications directly off the expression for σ in equation (20): the spread goes up when forgone investment opportunities are more likely (α is high); when lending frictions are severe (β is low); and when investments are valuable relative to cash (V/v is high). We stress that it is this relative value of investments that matters, not the absolute value V. As a result, the spread can change just because the value of cash changes.

[13]For some parameters, there are equilibria in which all banks choose to lend. However, they do not satisfy our restriction to equilibria in which leverage constraints bind, as described Subsection 2.5.

Now, since the value of cash reflects future investment opportunities, spreads can fluctuate even if current investments and economic conditions are constant: even though loans are short-term, spreads reflect long-term economic conditions.

$t \geq \tau$**: Steady state.** We now solve for the value of cash, starting after the shock ($t \geq \tau$) and working backward. Since we are focusing on Markov equilibria, the economy is in steady state after the single shock: the expected value of cash is the value today ($\bar{v} = v$).

Solving equation (8) for v, we find that the value of cash is a discounted value of future investments:

$$v_\tau = \frac{\alpha_\tau V_\tau}{\rho + \alpha_\tau}, \tag{21}$$

where we use the τ subscripts introduced in equation (4) to emphasize that the expression holds only after the shock.

From here, we can use the expression for the spread (Proposition 1) to solve for the equilibrium for $t \geq \tau$:

PROPOSITION 2. **(Steady state.)** *In steady state (i.e. if $t \geq \tau$), the spread and leverage are given by*

$$\sigma_\tau = \frac{(1 - \phi_\tau)\rho}{\beta_\tau} \tag{22}$$

and

$$b_\tau = \frac{1}{\sigma_\tau} = \frac{\beta_\tau}{(1 - \phi_\tau)\rho}. \tag{23}$$

Observe that the steady-state spread and leverage do not depend on the investment return R_τ or its arrival rate α_τ. This was unexpected to us, but has a straightforward explanation, based on the fact that what matters is not the absolute value of investment opportunities V, but only their value relative to the value of cash v (cf. equation (20)). Indeed, increasing R_τ or α_τ has the direct effect of increasing the opportunity cost of lending—the investment opportunities forgone by lending are more valuable/more likely. But it also has the indirect effect of increasing the value of cash—future investment opportunities available from holding cash are more valuable/more likely (cf. equation (21) for the value of cash). In steady state, these two effects exactly cancel out.

Rather than being determined by the value of forgone investment opportunities, prices and allocations are determined by frictions: spreads go down and leverage goes up as frictions decrease, whether by increasing the intensity of matching with lenders β_τ or the moneyness of loans ϕ_τ. This last finding that increasing moneyness leads to higher leverage is in line with Gorton and Muir's (2015) empirical finding that keeping collateral mobile helps to keep credit loose.

$t < \tau$**: "Dynamics."** Moving onward (but solving backward), we consider what happens before the shock. In this case, the expected value of cash is the average of the value v today and its value v_τ after the shock:

$$\bar{v} = (1 - \pi \mathrm{d}t)v + \pi \mathrm{d}t\, v_\tau. \tag{24}$$

This allows us to use the recursive equation for v (equation (8)) to find v in terms of v_τ: with $\mathrm{d}t^2 = 0$,

$$v = \frac{\alpha V + \pi v_\tau}{\rho + \alpha + \pi}, \tag{25}$$

where v_τ is given by equation (21).

With the expression for the value of cash (25), we can use our results above on the investing banks' program (6) and the spread (Proposition 1) to express the equilibrium allocation as the solution of a quadratic equation.

Lemma 3. **(Equilibrium leverage.)** *Before the shock (i.e. if $t < \tau$), the equilibrium leverage b solves*

$$a_0 + a_1 b + a_2 b^2 = 0 \tag{26}$$

where

$$a_2 = (1 - \phi)(\rho + \pi), \tag{27}$$

$$a_1 = (1 - \phi)(\rho + \pi) - \beta - \frac{(1 - \phi)\pi v_\tau}{R - 1}, \tag{28}$$

$$a_0 = -\beta - \frac{\beta \pi v_\tau}{\alpha(R - 1)}. \tag{29}$$

This expression for b is essential for our analysis of the equilibrium below. Among other things, you can already see that the value of cash after the shock v_τ is a sufficient statistic for all variables after the shock. In other words, we will not have to look at the separate effects of different future parameters on the equilibrium, but just at how they affect v_τ.

Recall, though, that for the expression in equation (26) to be an equilibrium, our assumptions on the candidate equilibrium (Section 2.5) must be satisfied:

(i) **Binding constraints.** From equation (6) leverage constraints bind if

$$R \geq 1 + \sigma, \tag{30}$$

which says that the return on investment is sufficiently high to compensate for the cost of borrowing.

(ii) **"Interior equilibrium."** This is tantamount to all wealth shares being positive, or from equation (15), to

$$b \leq \frac{\beta}{\phi\alpha}. \tag{31}$$

(iii) **Banks save.** This is tantamount to the value of cash being greater than one, the value of consumption, or $v_\tau \geq 1$ and $v \geq 1$: from equations (21) and (25),

$$V_\tau \geq 1 + \frac{\rho}{\alpha_\tau}. \tag{32}$$

and

$$V \geq 1 + \frac{\rho - \pi\left(v_\tau - 1\right)}{\alpha}. \tag{33}$$

4 Equilibrium and Analysis with Elastic Asset Supply

In this section, we first summarize the equilibrium characterization with elastic supply of capital (exogenous $p = A/R$) and give conditions for existence and uniqueness. Then we explore the implications of our opportunity cost channel for credit and business cycles. The model provides explanations for why credit is loose in booms and tight in crises that contrast with received theories but resonate with empirical evidence. It also generates procyclical fluctuations in capital allocation, which can be even more important to aggregate output than productivity shocks, in line with recent evidence on the drivers of business cycles.

4.1 Equilibrium Characterization with Perfectly Elastic Supply

The results so far put us in a position to characterize the equilibrium fully if R is exogenous.

Proposition 3. **(Equilibrium characterization.)** *Leverage b is given by the unique positive solution to equation (26) in Lemma 3 for $t < \tau$ and by Proposition 2 for $t \geq \tau$; the spread is equal to $\sigma = 1/b$; the aggregate wealth is constant and equal to W_0; and the wealth shares are given by Lemma 2.*

The equilibrium exists and is unique whenever the assumptions on the candidate equilibrium (equations (30), (31), (32), and (33)) are satisfied:

Proposition 4. **(Existence and uniqueness.)** *There exists a Markov equilibrium with binding leverage constraints if and only if the following conditions hold:*

$$R_\tau - 1 \geq \frac{(1-\phi_\tau)\rho}{\beta_\tau} \geq \frac{\alpha_\tau \rho}{\left(\alpha_\tau + \rho\right)\beta_\tau}, \tag{34}$$

$$\max\left\{\frac{1}{R-1},\frac{\rho-\alpha\left(R-2\right)-\pi\left(v_{\tau}-1\right)}{\alpha\left(R-1\right)}\right\}\leq b\leq\frac{\beta}{\phi\alpha},\tag{35}$$

where b is given by (26)*, and*

$$v_{\tau}=\frac{\alpha_{\tau}\left(R_{\tau}-1\right)\left(\beta_{\tau}+(1-\phi_{\tau})\rho\right)}{\left(\rho+\alpha_{\tau}\right)(1-\phi_{\tau})\rho}\geq 1.\tag{36}$$

If an equilibrium exists, it is unique.

4.2 The Credit Cycle

Having characterized the equilibrium, we ask what happens when the return R goes up—a state we view as a crisis, in which returns are high, e.g., due to depressed prices (as we endogenize in Subsection 5.1 below). Then we ask what happens when the return R is expected to go up—a state we view as the build up to a crises, in which banks anticipate high expect returns in the future, when the crisis hits.

What happens if there is a crisis today? I.e. if R goes up for $t<\tau$?

PROPOSITION 5. **(Tight credit for high R.)** *Increasing the return on investment R increases the spread σ and decreases the leverage b.*

Intuitively, high R means that investing is valuable. Hence, there is a high opportunity cost of lending. Thus, the spread σ must increase to compensate lenders for this opportunity cost. And increasing the spread tightens the collateral constraint, since what investing banks need to repay goes up, but what they can pledge stays same—the same collateral does not go as far when spreads are high. Thus, high spreads lead credit to tighten when it is needed most.

This result resonates with empirical facts: crises are associated with high returns, high spreads, and tight credit. To date, the literature has stressed the explanation based on decreased risk-bearing capacity.[14] We point out that limited moneyness delivers the same patterns, even with no change in risk-bearing capacity—with no risk whatsoever, in fact. In our model, the chain of causality runs in the opposite direction of the usual story. Tight financial constraints do not lead to high returns—it is not that demand gets depressed, keeping prices down (and hence returns up). Rather, tight financial constraints result from high returns—it is that opportunity costs go up, driving up the spread (which feeds back into the collateral constraint).

Now, what happens if there is likely to be a crisis in the future? I.e. if R_{τ} goes up for $t\geq\tau$? Or, more generally, given v_{τ} is a sufficient statistic for everything after the shock,

[14]Think of the literature on intermediary asset pricing (e.g., He and Krishnamurthy (2012, 2013)) and on limits to arbitrage (e.g., Gromb and Vayanos (2002) and Shleifer and Vishny (1997)).

including R_τ, what happens as v_τ goes up?

PROPOSITION 6. **(Loose credit in boom.)** *Increasing the future value of cash v_τ decreases the spread σ and increases leverage b.*

Intuitively, high v_τ means that investing in the future is valuable, and hence cash is valuable today—i.e. the value v of cash today is high whenever the value v_τ of cash in the future is; cf. equation (21). And valuable cash means valuable interest: for a fixed spread σ, the interest paid (in cash) is worth more when the value of cash v is high: it is the product σv that matters. In other words, when the value of cash goes up, lenders get the same value at a lower spread. Thus, an increase in v_τ leads to a decrease in σ in equilibrium. And decreasing the spread loosens the collateral constraint, since what investing banks need to repay goes down, but what they can pledge stays the same—the same collateral goes further when spreads are low. The pecuniary externality of the spread on the collateral constraint leads credit to loosen. Banks can borrow freely, even though—indeed because—the investments they need to fund have relatively low returns.

This result also resonates with empirical facts: booms are associated with low returns, low spreads, and loose credit—indeed, as Krishnamurthy and Muir (2017) put it, "spreads fall pre-crisis and appear too low, even as credit grows ahead of a crisis." To date, the literature has stressed the explanation based on how the build-up of leverage in booms can lead to costly deleveraging in recessions.[15] We point out that limited moneyness delivers the same patterns. In our model, the chain of causality again runs in the opposite direction of the usual story. Loose credit does not lead to low returns—it is not that demand gets inflated, keeping prices high (and hence returns low). Rather, loose credit results from low returns—it is the anticipation of high returns in the future that increases the value of cash, lowering the equilibrium spread today (which feeds back into the collateral constraint).

4.3 Aggregate Output

So far, we have studied the credit cycle under the interpretation that high-return times are recessions and low-return times are booms. This interpretation captures the idea that asset prices fall in recessions (making returns high) and rise back up in booms (making returns low): recall that the return on an investment in a unit of capital is its output A divided by the price of its input price p. With perfectly elastic supply, we can normalize the price of capital, $p \equiv 1$, so the investment return R is effectively just the productivity of individual

[15]Think of the literature on debt-induced fire sales (e.g., Lorenzoni (2008)), on neglected risk (e.g., Gennaioli, Shleifer, and Vishny (2012)), on the financial accelerator (e.g., Bernanke and Gertler (1989) and Bernanke, Gertler, and Gilchrist (1996)), and on balance sheet recessions (e.g., Di Tella (2017)).

investments A. Thus, by describing low-R times as booms and high-R times as recessions, we are implying that individual productivity is countercyclical. This may seem at odds with the "[r]ecent macroeconomic literature [which] views [the] stylized fact of procyclical [aggregate] productivity as an essential feature of business cycles" (Basu and Fernald (2001), p. 225). But we show here that there is actually no contradiction: an increase in individual productivity can actually undermine aggregate productivity growth, even in our baseline set-up with exogenous R.

Here, we ask what happens to aggregate output in our model when R goes up. Aggregate output, which we denote by Y, is the product of investing banks' total investment (i.e. their wealth times their gross leverage) with their return on investment:

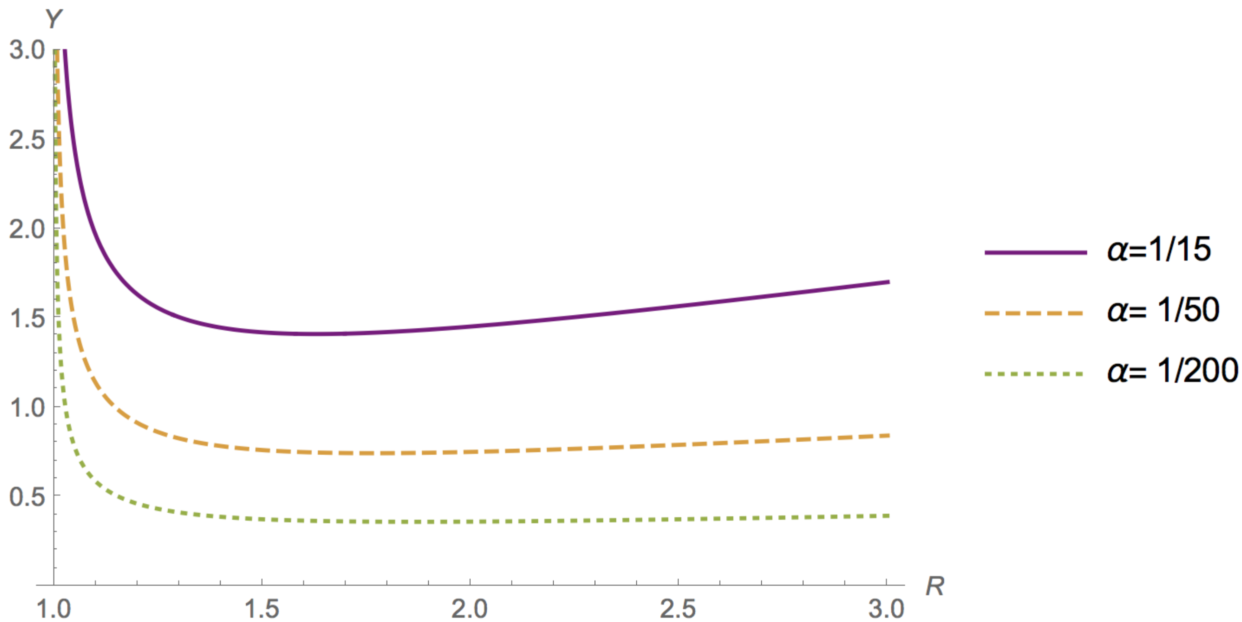
$$Y = (1 + b)w^I R. \tag{37}$$

This expression captures the usual effect of increasing R: it increases the output of each dollar invested. However, there is also a new effect of increasing R: it tightens credit (Proposition 5), and hence decreases the number of dollars invested. As illustrated in Figure 1, when leverage is high (due to low opportunity costs/low α), this new effect can actually overpower the usual one:

PROPOSITION 7. **(Low output for high R.)** *If α is sufficiently small, increasing the return on investment R decreases aggregate output Y.*

This result suggests that capital allocation could be as important as productivity for aggregate output fluctuations. This resonates with evidence in Eisfeldt and Rampini (2006) and Hsieh and Klenow (2009) on the importance of procyclical capital allocation for the business cycle. And it goes a step further. It suggests that individual productivity and aggregate productivity could be two sides of the same coin. Since individual productivity shocks drive up the opportunity cost of collateral, they can lead to negative aggregate productivity shocks.

Figure 1: Aggregate output is decreasing in R for α sufficiently small. In the plot, $\beta = 3/2$, $\pi = \rho = 1$, $v_\tau = 2$, and $\phi = 3/4$.



4.4 Welfare

Since all agents are risk-neutral and ex ante identical, the total welfare is just the expected value of consumption of all banks (recall suppliers are competitive and hence get zero payoff, and do not affect welfare). Since they consume only after they invest, or Vw^I at each time. Integrating up gives the total welfare for each path, which we denote by U^τ:

$$U^\tau \equiv e^{-\rho t} \int_0^\infty V_t w_t^I \tag{38}$$

(there is no dt in the integral since it is already inside $w_t^I = o(dt)$). Since we know that τ is exponentially distributed, we can compute expected welfare $U := \mathbb{E}[U^\tau]$ directly:

Proposition 8. **(Welfare characterization.)** *Welfare is given by the value of cash times the initial amount of cash:*

$$U = vW_0. \tag{39}$$

This expression, although somewhat involved to derive, is easy to understand: given time is continuous and investments arrive with Poisson intensity, effectively no one is investing exactly at $t = 0$; everyone is either holding cash or lending. The values of cash and lending being equal, $v^\ell = v$, we can calculate welfare as if everyone is holding cash, i.e. as the initial value of cash times the initial amount of cash: $U = vW_0$.

The expression for welfare holds for arbitrary leverage b, not just equilibrium b. Since $V = (R-1)(1+b)$ by Lemma 1, this implies that welfare is increasing in leverage. Since leverage is maximal in equilibrium—leverage constraints bind—this implies that the equilibrium outcome is constrained efficient:

Proposition 9. **(Constrained efficiency.)** *The equilibrium is constrained efficient in the sense that it achieves the maximum welfare among all levels of leverage b for $t < \tau$ and b_τ for $t \geq \tau$ that satisfy investing banks' leverage constraints (given the spread σ is determined by market clearing).*

5 Equilibrium and Analysis with Inelastic Asset Supply

So far we have assumed that assets were in perfectly elastic supply, which meant that the asset price p was effectively an exogenous parameter. This approach allowed us to study our opportunity cost channel in isolation. In this section, we explore how it interacts with the asset market. We first show that there is a feedback loop between the opportunity cost in the credit market and the price of collateral in the asset market. This feedback can be strong enough to generate multiple equilibria. These equilibria are welfare ranked, thus leverage regulation can ensure the economy is in the "good" equilibrium in some circumstances. These circumstances resemble booms, supporting the idea that leverage regulation should be counter-cyclical.

5.1 Asset Prices and Multiple Equilibria

We solve for the price p by market clearing: at each time t, the supply $S(p)$ must equal the banks' aggregate demand, which we now calculate. Immediately from a bank's budget constraint (equation (1)), we have that an investing bank's demand for assets is linear in its wealth:

$$k = \frac{(1+b)w}{p}. \tag{40}$$

Hence, we can aggregate up by replacing the individual bank's wealth with the total wealth of all investing banks w^I form equation (13). The market clearing condition is thus

$$\frac{(1+b)w^I}{p} = S(p) \tag{41}$$

or, substituting in for w^I from equation (13):

$$\frac{(1+b)}{p}\frac{\alpha\beta W}{\beta+\alpha(1-\phi)b} = S(p). \tag{42}$$

We can now characterize the equilibrium for a given supply curve S. We focus on fixed supply here and show that similar results hold for imperfectly elastic supply in Appendix E.

PROPOSITION 10. **(Equilibrium characterization with fixed asset supply.)** *Let $S(p) = K\mathrm{d}t$ for each t. Leverage b is given by Proposition 2 for $t \geq \tau$ and a solution to the following equation for $t < \tau$:*

$$\hat{a}_2 b^2 + \hat{a}_1 b + \hat{a}_0 = 0 \tag{43}$$

where

$$\hat{a}_2 := \alpha(\pi+\rho)(1-\phi)\Big((1-\phi)AK - \beta W_0\Big), \tag{44}$$

$$\hat{a}_1 := \beta(1-\phi)\big(\pi+\rho-\alpha\big)AK + \alpha\beta\Big(\beta-(1-\phi)\big(\pi+\rho+\pi v_\tau\big)\Big)W_0, \tag{45}$$

$$\hat{a}_0 := \beta^2\Big((\alpha-\pi v_\tau)W_0 - AK\Big). \tag{46}$$

The spread is equal to $\sigma = 1/b$; the aggregate wealth is constant and equal to W_0; and the wealth shares are given by Lemma 2.

Recall that in the steady state ($t \geq \tau$), the equilibrium spread and leverage do not depend on the return R (Proposition 2). Now, since p matters only in so far as $R = A/p$, p does not affect them either. Hence the interaction between credit and asset prices reduces to the standard channel, by which tight credit depresses asset demand, decreasing prices—this is the partial equilibrium effect captured in equation (42).

Away from steady state, however, there is a two-way interaction between this standard channel and our opportunity cost channel. Recall that increasing R tightens credit due to our opportunity cost channel (Proposition 5). This depresses asset demand and lowers prices, as per the standard channel. But now, since $R = A/p$, this feeds back into higher returns, which make our channel kick in again, tightening credit further. This feedback loop between the two channels is powerful enough to generate multiple equilibria:

PROPOSITION 11. **(Multiple equilibria.)** *There can be two equilibria satisfying the description in Proposition 10: a "high-leverage equilibrium" in which b is high for $t < \tau$ and a "low-leverage equilibrium" in which b is lower for $t < \tau$. These equilibria correspond to the two solutions of equation (43), which are both positive whenever β is sufficiently large and α is sufficiently small. These are all Markov equilibria in which the leverage constraints bind.*

As in the equilibrium characterization with elastic asset supply, we need to check that the requirements we imposed on the allocation hold. Although it is intractable to write these conditions in terms of primitives now that R depends on the solution to a quadratic equation, it is still easy to check whether a candidate allocation satisfies them. Hence, to show existence, we just pick one set of parameters, calculate the two candidate equilibria corresponding to the two solutions of equation (43), and show that they both satisfy our requirements: with $\alpha = 1/20$, $A = \beta = 3/2$, $\rho = \pi = K = W_0 = 1$, $v_\tau = 2$, and $\phi = 3/4$, there are two equilibria. In both equilibria our requirements in equations (30), (31), (32), and (33) are satisfied. In the high-leverage equilibrium, $b \approx 31.6$ and prices are high, $p \approx 1.3$. In the low-leverage equilibrium, $b \approx 8.7$ and prices are lower, $p \approx 0.5$.

5.2 Welfare Ranking and Capital Regulation

With elastic supply, we found that more leverage is always good for everyone (Proposition 9). Here, in contrast, leverage can have a downside: by increasing asset demand and driving up prices, it decreases the returns on each dollar invested. Thus, we can ask whether banks are necessarily better off in the high-leverage equilibrium here. The answer is no.

PROPOSITION 12. **(Welfare ranking.)** *The equilibria in Proposition 11 are welfare ranked. The low-leverage equilibrium is better as long as*

$$\frac{1-\phi}{\beta} \leq \frac{W_0}{AK}. \tag{47}$$

The result implies that if a regulator caps leverage, forcing the economy into the low-leverage equilibrium, then welfare is higher if and only if $(1-\phi)/\beta$ is low. $(1-\phi)/\beta$ is a measure of the frictions in the lending market: it is low when moneyness ϕ is high (i.e. if it is easy for lending banks to undertake investment opportunities) and if the lending intensity β is high (i.e. if it is easy for lending banks to find borrowers). Thus, the result says that forcing the economy into the low-leverage equilibrium is good if lending frictions are low.

If we assume that such frictions are countercyclical, so $(1-\phi)/\beta$ is low in booms, we can speak to counter-cyclical capital regulation:

Corollary 1. **(Counter-cyclical regulation.)** *Under the interpretation that low* $(1 - \phi)/\beta$ *is a boom, capping leverage in a boom help force economy into the good equilibrium.*

Thus, our model supports the idea that regulation should be countercyclical, as advocated by, e.g., Brunnermeier, Crockett, Goodhart, Persaud, and Shin (2009) who say "measures have to be counter-cyclical, i.e. tough during a credit boom and more relaxed during a crisis" (p. 31).

6 Conclusion

We present a model of the interbank market based on the limited money of collateralized debt. We find that positive spreads compensate lenders for parting with cash, even absent any credit risk. Thus, the model captures several essential features of the repo market, in which collateral makes debt almost risk-less, but spreads are still positive, and went up in the crisis (Hördahl and King (2008) and Gorton and Metrick (2012)). We find that high returns today cause credit to tighten, whereas the anticipation of high expected returns in the future cause it to loosen, consistent with empirical evidence on the credit cycle (Krishnamurthy and Muir (2017)). This opportunity cost channel creates a two-way feedback between the credit market and the asset market, which generates instability in the form of multiple equilibria, and hence casts light on repo runs. The equilibria are welfare ranked, and a regulator can intervene, using capital requirements to do equilibrium selection, i.e. to force the economy into the low-leverage equilibrium. Since high leverage equilibria are inefficient in booms, but not in crises, this suggests a new rationale for countercyclical capital regulation.

A Proofs

A.1 Proof of Lemma 1

The result follows from the program in the text; see equation (6). □

A.2 Proof of Lemma 2

The result follows immediately from solving the system of equations (10)–(12). □

A.3 Proof of Proposition 1

Using $\mathrm{dt}^2 = 0$, we can re-write equation (8) for the value of cash v as

$$(1 + \rho\mathrm{dt})v = \alpha\mathrm{dt}(V - \bar{v}) + \bar{v} \tag{48}$$

and equation (9) for the value of lending as

$$(1 + \rho\mathrm{dt})v^{\ell} = \alpha\mathrm{dt}\phi(V - \bar{v}) + \bar{v} + \beta\sigma\mathrm{dt}\bar{v}. \tag{49}$$

Equating the expressions in equations (48), using that $\mathrm{dt}\bar{v} = \mathrm{dt}v$,[16] and (49) and solving for σ gives the expression in the proposition. □

A.4 Proof of Proposition 2

From equation (21) we have that

$$\frac{V_\tau - v_\tau}{v_\tau} = \frac{\rho}{\alpha}. \tag{50}$$

We can substitute this into equation (20) to find the spread:

$$\sigma_\tau = \frac{(1 - \phi_\tau)\rho}{\beta_\tau}, \tag{51}$$

which is the expression in the proposition.

Now, given we are looking for equilibria in which the leverage constraint in equation (3) binds, we have $b_\tau = 1/\sigma_\tau$, or

$$b_\tau = \frac{\beta_\tau}{(1 - \phi_\tau)\rho}, \tag{52}$$

[16]For $t \geq \tau$ this immediate—in that case there is no future shock and $\bar{v} \equiv v$—and for $t < \tau$ if follows from $\mathrm{dt}^2 = 0$—in that case $\bar{v} = \pi\mathrm{dt}v_\tau + (1 - \pi\mathrm{dt})v$, implying $\mathrm{dt}\bar{v} = \mathrm{dt}v$ as desired.

which is the expression in the proposition. $\square$

A.5 Proof of Lemma 3

We can use the value of cash (equation (24)),

$$v = \frac{\alpha V + \pi v_\tau}{\rho + \alpha + \pi}, \tag{53}$$

to write

$$\frac{V - v}{v} = \frac{(\rho + \alpha + \pi)V - \alpha V - \pi v_\tau}{\alpha V + \pi v_\tau} \tag{54}$$

$$= \frac{(\rho + \pi)V - \pi v_\tau}{\alpha V + \pi v_\tau}. \tag{55}$$

Now, substituting this into the expression for the spread in Proposition 1, we find

$$\sigma = \frac{\alpha(1-\phi)}{\beta} \cdot \frac{(\rho + \pi)V - \pi v_\tau}{\alpha V + \pi v_\tau}. \tag{56}$$

From the binding constraint from in equation (3), we get

$$\sigma = \frac{1}{b}, \tag{57}$$

which we can equate to the spread σ in equation (56) to find

$$\frac{1}{b} = \frac{\alpha(1-\phi)}{\beta} \cdot \frac{(\rho + \pi)V - \pi v_\tau}{\alpha V + \pi v_\tau}. \tag{58}$$

Rearranging gives

$$\Big(\beta + \alpha(1-\phi)b\Big)\pi v_\tau = \alpha\Big((1-\phi)(\rho+\pi)b - \beta\Big)V. \tag{59}$$

Finally, use $V = (R-1)(1+b)$ from Lemma 1 to get

$$\Big(\beta + \alpha(1-\phi)b\Big)\pi v_\tau = \alpha\Big((1-\phi)(\rho+\pi)b - \beta\Big)(R-1)(1+b). \tag{60}$$

Rearranging gives

$$a_2 b^2 + a_1 b + a_0 = 0, \tag{61}$$

where a_2, a_1 and a_0 are as given in the proposition. This equation has a unique positive solution because $a_0 a_2 < 0$, hence b is given by the expression in the proposition (equation (26)). □

A.6 Proof of Proposition 3

The equilibrium leverage is given by Proposition 2 for $t \geq \tau$ and Lemma 3 for $t < \tau$. The spread $\sigma = 1/b$ because the leverage constraints bind (equation (3)). The wealth shares follow from Lemma 2, where the aggregate wealth is constant since $\mathrm{d}W = 0$ (equation (19)). □

A.7 Proof of Proposition 4

Proposition 3 characterizes a unique candidate equilibrium, so the equilibrium is unique if it exists. For a Markov equilibrium with binding leverage constraints to exist, the candidate equilibrium must satisfy the requirements in equations (30), (31), (32), and (33). Conversely, if the candidate equilibrium satisfies the four assumptions, it is indeed a Markov equilibrium with binding leverage constraints because all banks optimize, markets clear, leverage constraints bind, and it is Markov. Thus, a necessary and sufficient condition for the existence is that the four requirements all hold.

Using that leverage constraints bind, i.e. $b = 1/\sigma$ (equation (3)), we can write equations (30) and (31) as

$$\frac{1}{R-1} \leq b \leq \frac{\beta}{\phi\alpha}. \tag{62}$$

Observe that these must be satisfied both before and after the shock, while equation (32) only applies after the shock and equation (33) only applies before the shock.

We begin with $t < \tau$. Substituting V from Lemma 1 into equation (33) yields

$$b \geq \frac{\rho - \alpha\left(R-2\right) - \pi\left(v_\tau - 1\right)}{\alpha\left(R-1\right)}, \tag{63}$$

where v_τ is given by equation (21). Combining this with (62), we have

$$\max\left\{\frac{1}{R-1}, \frac{\rho - \alpha\left(R-2\right) - \pi\left(v_\tau - 1\right)}{\alpha\left(R-1\right)}\right\} \leq b \leq \frac{\beta}{\phi\alpha}. \tag{64}$$

This is the second condition of the proposition (equation (35)).

Next, we turn to the steady state. Recall that equation (32) is equivalent to $v_\tau \geq 1$. Substituting the V from Lemma 1 and b from Proposition 2 into the expression for v_τ in equation (21), we get that

$$v_\tau = \frac{\alpha_\tau \left(R_\tau - 1\right)\left(\beta_\tau + (1-\phi_\tau)\rho\right)}{\left(\rho + \alpha_\tau\right)(1-\phi_\tau)\rho} \geq 1, \tag{65}$$

which is the third condition of the proposition (equation (36)).

Lastly, using the steady state equilibrium leverage from Proposition 2, we can write equation (62) as

$$R_\tau - 1 \geq \frac{(1-\phi_\tau)\rho}{\beta_\tau} \geq \frac{\alpha_\tau \rho}{\left(\alpha_\tau + \rho\right)\beta_\tau}, \tag{66}$$

which is the first condition of the proposition (equation (34)). □

A.8 Proofs of Proposition 5 and Proposition 6

The comparative statics results in both propositions can be established by differentiating the expression for b in equation (26). Here, we present an alternative graphical proof to this "brute force" approach instead. To do so, we rewrite the equation for b as follows:

$$\Big(\beta + \alpha(1-\phi)b\Big)\frac{\pi v_\tau}{R-1} = \alpha\Big((1-\phi)(\rho+\pi)b - \beta\Big)(1+b). \tag{67}$$

Observe that the equilibrium leverage b is the point at which the line on the LHS intersects the parabola on the RHS. Then, increases in R and v_τ rotate the line (in opposite directions) without affecting the parabola, delivering the desired comparative statics.

In Figure 2, we start by plotting these two curves, the line on the LHS,

$$\text{LHS} : y = \Big(\beta + \big(\alpha(1-\phi)\big)b\Big)\frac{\pi v_\tau}{R-1} \tag{68}$$

and the parabola on the RHS,

$$\text{RHS} : y = \alpha\Big((1-\phi)(\rho+\pi)b - \beta\Big)(1+b). \tag{69}$$

Comparative statics on equilibrium leverage are comparative statics on the b-coordinate where these curves intersect.

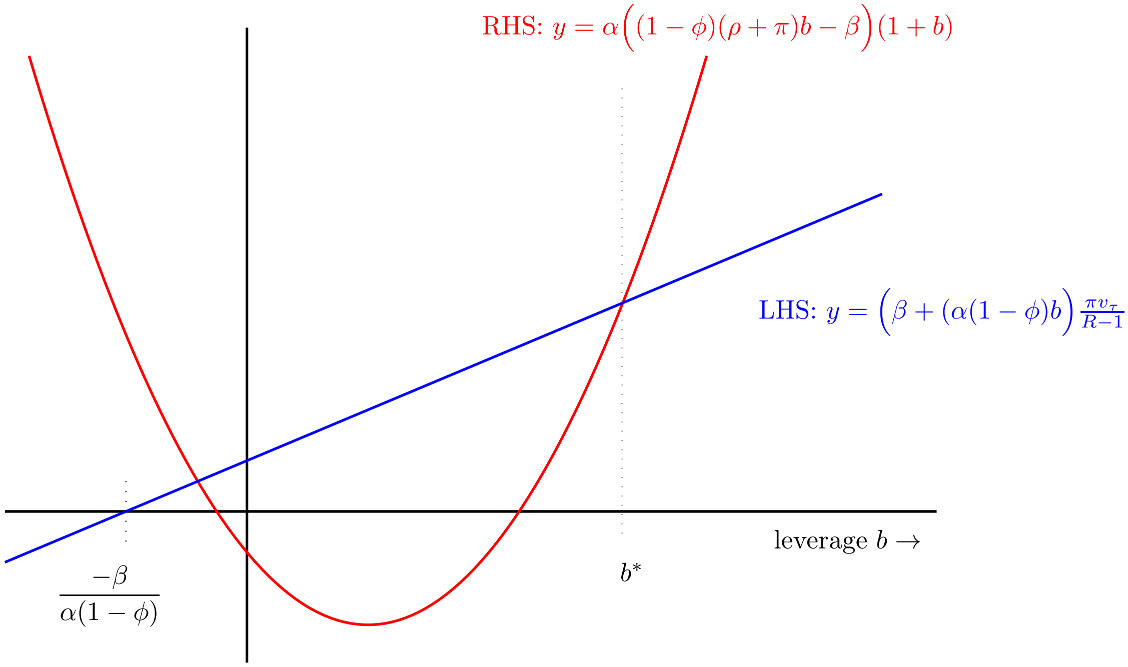


Figure 2: The equilibrium leverage is the b-coordinate of the point at which the line on the LHS and parabola on the RHS intersect.

Observe that the parabola on the RHS is always positive and it is already written in factored form: it has one root at $b = -1 < 0$ and one at $b = \frac{\beta}{(1-\phi)(\rho+\pi)} > 0$. The line on the LHS has root $\frac{-\beta}{\alpha(1-\phi)}$. This can be positive or negative (we drew the figure with this negative), but it does not matter for the argument. What does matter, is that it does not depend on R or v_τ, and neither does the parabola. Hence, changing R and v_τ just rotates the line, leaving its horizontal intercept and the entire parabola unchanged.

An increase in R or a decrease in v_τ corresponds to a clockwise rotation, which means that the line intersects the parabola sooner—i.e. the equilibrium b is lower—as depicted in Figure 3. In other words, as R increases, b decreases, and as v_τ increases, b increases, as desired.

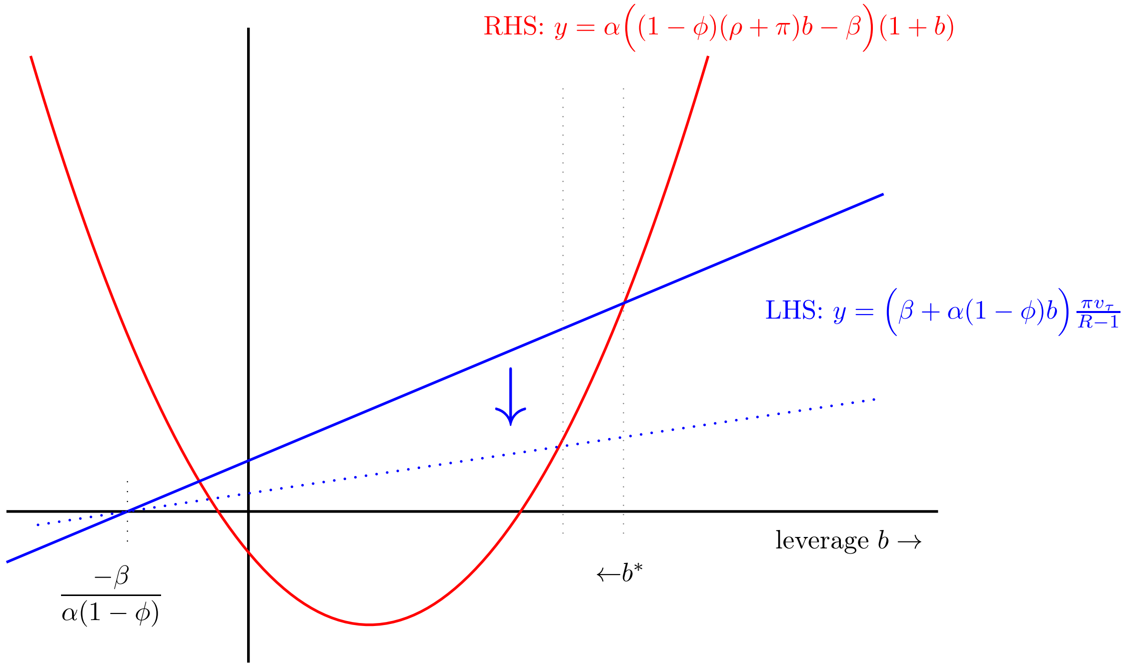


Figure 3: The line on the LHS rotates clockwise as R increases or v_τ decreases. Thus the b-coordinate of intersection with the parabola on the RHS decreases.

A.9 Proof of Proposition 7

Because we have fully characterized the equilibrium in closed form, we can compute the comparative static $\partial Y/\partial R$ directly. There is still a little of work involved only because b is slightly complicated (it is the solution to the quadratic equation (26)) and we have to take the limit $\alpha \to 0$ (since the result is for α small), which requires a little care.

Using the expressions for Y and w^I from equations (37) and (13) and the fact that $W_t \equiv W_0$ from equation 19, we have that

$$Y = (1+b)\frac{\alpha\beta W_0}{\beta + \alpha(1-\phi)b}R, \tag{70}$$

where b is the positive root of the quadratic equation (26). Differentiating we have that

$$\frac{\partial Y}{\partial R} = \frac{\alpha\beta\left(\beta + \Big(\beta + \alpha(1-\phi)\Big)b + \alpha(1-\phi)b^2 + \Big(\beta - \alpha(1-\phi)\Big)R\dfrac{\partial b}{\partial R}\right)}{\Big(\beta + \alpha(1-\phi)b\Big)^2}W_0. \tag{71}$$

Now, from the above we have that $\partial Y/\partial R < 0$ whenever

$$\alpha\left(\beta + \Big(\beta + \alpha(1-\phi)\Big)b + \alpha(1-\phi)b^2 + \Big(\beta - \alpha(1-\phi)\Big)R\frac{\partial b}{\partial R}\right) < 0. \tag{72}$$

By implicitly differentiating equation (26), we find that

$$\frac{\partial b}{\partial R} = \frac{\pi\big(\beta + \alpha(1-\phi)b\big)v_\tau}{(R-1)^2\left(-\alpha(1-\phi)(\pi+\rho)(1+b) + \alpha\big(\beta - (1-\phi)(\pi+\rho)b\big) + \dfrac{\alpha(1-\phi)\pi v_\tau}{R-1}\right)}. \tag{73}$$

Substituting this into equation (72), we get that $\partial Y/\partial R < 0$ whenever

$$\begin{aligned}&\alpha\beta + \alpha\Big(\beta + \alpha(1-\phi)\Big)b + \alpha^2(1-\phi)b^2 + \\ &+ \frac{\Big(\beta - \alpha(1-\phi)\Big)\Big(\beta + \alpha(1-\phi)b\Big)\pi v_\tau R}{\left(\Big(\beta - (\pi+\rho)(1-\phi)\Big)(R-1) - 2(1-\phi)(\pi+\rho)(R-1)b + (1-\phi)\pi v_\tau\right)(R-1)} < 0.\end{aligned} \tag{74}$$

We want to know whether this inequality holds in the limit as $\alpha \to 0$, but we cannot yet take the limit mechanically, because b is a function of α. We need to prove the following lemma first:

LEMMA 4. *As $\alpha \to 0$, $b \to \infty$ and $\alpha b \to 0$.*

Proof. We start by substituting in for a_0 into the equation for b (equation (26)) to get

$$b = \frac{-a_1 + \sqrt{a_1^2 + 4a_2\left(\beta + \dfrac{\beta\pi v_\tau}{\alpha(R-1)}\right)}}{2a_2}. \tag{75}$$

Importantly, a_2 and a_1 do not depend on α (cf. equations (27) and (28)).

Further, $a_2 > 0$, so it is immediate that $b \to \infty$ as $\alpha \to 0^+$.

To see that $\alpha b \to 0$, multiply by α and carry it under the square root:

$$\alpha b = \frac{-\alpha a_1 + \sqrt{\alpha^2 a_1^2 + 4a_2\left(\alpha^2\beta + \dfrac{\alpha\beta\pi v_\tau}{(R-1)}\right)}}{2a_2}. \tag{76}$$

All the terms above go to zero as $\alpha \to 0^+$, hence so does the whole expression.

□

Given this lemma, we can take the limit of equation (74) by simply deleting the α and αb terms. Now, we have the for α sufficiently small, $\partial Y/\partial R < 0$ as long as

$$\frac{\beta^2 \pi v_\tau \pi}{\Big(\big(\beta - (\pi+\rho)(1-\phi)\big)(R-1) - 2(1-\phi)(\pi+\rho)(R-1)b + (1-\phi)\pi v_\tau\Big)(R-1)} < 0. \tag{77}$$

Observe that the only thing left that depends on α is b in the denominator, which becomes large as α becomes small. Since it has a negative coefficient, the inequality is always satisfied for small α. □

A.10 Proof of Proposition 8

In this proof, we do not suppose banks' leverage b is necessarily the equilibrium leverage b. This makes the proof a bit more computationally cumbersome, but shows the generality of our welfare formulation and also allows us to do welfare analysis of policies that regulate leverage, forcing it away from its equilibrium level.

We start with a lemma that characterizes the welfare for each path. Then we use it to characterize the ex ante welfare as the expectation over all paths. To do this, we use equation (18) to write

$$\mathrm{d}W_t = g_t W_t \mathrm{dt} \tag{78}$$

where the growth rate g_t is

$$g_t := \left(\sigma - \frac{1}{b}\right)\frac{\alpha\beta b}{\beta + \alpha(1-\phi)b}. \tag{79}$$

Lemma 5. **(Welfare of path τ.)**

$$U^\tau = \frac{1 - e^{-(\rho-g)\tau}}{\rho - g}\frac{\alpha\beta V W_0}{\beta + \alpha(1-\phi)b} + \frac{\alpha_\tau \beta_\tau V_\tau W_0}{\beta_\tau + \alpha_\tau(1-\phi_\tau)b_\tau}\frac{e^{(g-\rho)\tau}}{\rho - g_\tau}. \tag{80}$$

Proof. The proof is by direct computation:

$$U^\tau = \int_0^\infty e^{-\rho t} V_{s_t} w_t^I \tag{81}$$

$$= \int_0^\infty e^{-\rho t} V_{s_t} \frac{\alpha_{s_t}\beta_{s_t} W_t}{\beta_{s_t} + \alpha_{s_t}(1-\phi_{s_t})b_{s_t}} \mathrm{dt} \tag{82}$$

$$= \int_0^\tau e^{-\rho t} \frac{\alpha\beta V W_t}{\beta + \alpha(1-\phi)b} \mathrm{dt} + \int_\tau^\infty e^{-\rho t} \frac{\alpha_\tau \beta_\tau V_\tau W_t}{\beta_\tau + \alpha_\tau(1-\phi_\tau)b_\tau} \mathrm{dt} \tag{83}$$

$$= \frac{\alpha\beta V W_0}{\beta + \alpha(1-\phi)b} \int_0^\tau e^{(g-\rho)t} \mathrm{dt} \; + \; \frac{\alpha_\tau \beta_\tau V_\tau e^{(g-g_\tau)\tau} W_0}{\beta_\tau + \alpha_\tau(1-\phi_\tau)b_\tau} \int_\tau^\infty e^{(g_\tau-\rho)t} \mathrm{dt} \tag{84}$$

$$= \frac{1-e^{-(\rho-g)\tau}}{\rho-g} \frac{\alpha\beta V W_0}{\beta + \alpha(1-\phi)b} \; + \; \frac{\alpha_\tau \beta_\tau V_\tau W_0}{\beta_\tau + \alpha_\tau(1-\phi_\tau)b_\tau} \frac{e^{(g-\rho)\tau}}{\rho - g_\tau} \tag{85}$$

□

We start by computing the integral under the expectation $U = \mathbb{E}\left[U^\tau\right]$ using the expression in Lemma 5:

$$\mathbb{E}\left[U^\tau\right] = \frac{1-\mathbb{E}\left[e^{-(\rho-g)\tau}\right]}{\rho-g} \frac{\alpha\beta V W_0}{\beta + \alpha(1-q)b} \; + \; \frac{\alpha_\tau \beta_\tau V_\tau W_0}{\beta_\tau + \alpha_\tau(1-\phi_\tau)b_\tau} \frac{\mathbb{E}\left[e^{(g-\rho)\tau}\right]}{\rho - g_\tau} \tag{86}$$

Now, given that τ is a Poisson variable (it is exponentially distributed), we have that

$$\mathbb{E}\Big[e^{-(\rho-g)\tau}\Big] = \int_0^\infty e^{-(\rho-g)\tau} \pi e^{-\pi\tau} \mathrm{d}\tau = \frac{\pi}{\rho - g + \pi}. \tag{87}$$

Substituting into the above, computing, and substituting v_τ in from equation (21), we get

$$U = \frac{W_0}{\rho - g + \pi}\left(\frac{\alpha\beta V}{\beta + \alpha(1-\phi)b} + \pi v_\tau\right). \tag{88}$$

Now, substituting in for g from equation (79) we get that

$$U = \frac{\alpha\beta V + \big(\beta + \alpha(1-\phi)b\big)\pi v_\tau}{(\rho+\pi)\big(\beta + \alpha(1-\phi)b\big) - \alpha\beta\big(\sigma b - 1\big)} W_0. \tag{89}$$

Finally, we can substitute for σ from Proposition 1 and use the expression for v (equation (8)) to get the expression in the proposition:

$$U = \frac{\alpha V + \pi v_\tau}{\rho + \alpha + \pi} W_0 \equiv v W_0. \tag{90}$$

□

A.11 Proof of Proposition 9

The result follows almost immediately from the fact that

$$U = \frac{\alpha(R-1)(1+b) + \pi v_\tau}{\rho + \alpha + \pi} \tag{91}$$

where

$$v_\tau = \frac{\alpha_\tau (R_\tau - 1)(1 + b_\tau)}{\rho + \alpha_\tau}, \tag{92}$$

by Proposition 8, Lemma 1, and equation (21). This implies that welfare is increasing in b and b_τ: more leverage is always better.

There is just one subtlety to address before we can conclude that making leverage constraints bind is always the best way to max out on leverage: regulating leverage after τ can affect the leverage constraint before τ. I.e. if capping b_τ loosened leverage constraints before τ, allowing you to increase b, it could improve welfare. But this is not the case. Increasing b_τ increases v_τ mechanically. And recall that v_τ is a sufficient statistic for all variables after the shock, and that leverage constraints before the shock actually loosen as v_τ increases by Proposition 6. □

A.12 Proof of Proposition 10

The proposition follows from using the equilibrium price in equation (42) to write R

$$R = R(b) = \frac{AK\big(\beta + \alpha(1-\phi)b\big)}{(1+b)\alpha\beta W}, \tag{93}$$

which we can substitute back into the quadratic equation (61) for b to find the equilibrium allocation.

Specifically, for $t \geq \tau$, the equilibrium spread does not depend on prices, as described in the text. For $t < \tau$, we get the following equation for b:

$$(1+b)\alpha\left(\beta - (1-\phi)(\pi+\rho)b - \frac{\beta\Big(\beta + \alpha(1-\phi)b\Big)\pi v_\tau W}{\alpha\beta(1+b)W - \Big(\beta + \alpha(1-\phi)b\Big)AS}\right) = 0. \tag{94}$$

This can be re-written as a cubic, and factored as follows:

$$(1+b)\left[\hat{a}_2 b^2 + \hat{a}_1 b + \hat{a}_0\right] = 0 \tag{95}$$

where

$$\hat{a}_2 := \alpha(\pi+\rho)(1-\phi)\Big((1-\phi)AK - \beta W\Big), \tag{96}$$

$$\hat{a}_1 := \beta(1-\phi)\big(\pi+\rho-\alpha\big)AK + \alpha\beta\Big(\beta - (1-\phi)\big(\pi+\rho+\pi v_\tau\big)\Big)W, \tag{97}$$

$$\hat{a}_0 := \beta^2\Big(\alpha W - AK - W\pi v_\tau\Big). \tag{98}$$

To focus on the positive solutions, we can factor $1+b$. This gives the quadratic equation in the proposition. □

A.13 Proof of Proposition 11

The quadratic in equation (43) has two positive solutions whenever $\hat{a}_0\hat{a}_2 > 0$ and $-\hat{a}_1/\hat{a}_2 > 0$. These conditions are satisfied whenever β is sufficiently large and α is sufficiently small:

- $\hat{a}_2 < 0$ because β is large and $\hat{a}_0 < 0$ because α is small. Hence $\hat{a}_0\hat{a}_2 > 0$.
- $\hat{a}_1 > 0$ because α is small (first term) and β is large (second term). Hence $-\hat{a}_1/\hat{a}_2 > 0$.
- $\hat{a}_0\hat{a}_2 > 0$ and $-\hat{a}_1/\hat{a}_0 > 0$ implies there are two positive roots. □

A.14 Proof of Proposition 12

Recall that the welfare is

$$U = \frac{\alpha V + \pi v_\tau}{\rho+\alpha+\pi} W_0 \tag{99}$$

by Proposition 8 and the multiple equilibria are identical for $t \geq \tau$. Hence we need to compare only V across the equilibria. We start with the expression for V in Lemma 1,

$$V = \big(R(b)-1\big)(1+b) \tag{100}$$

and use $R = A/p$ and p in equation (42) to write

$$V = \left(\frac{\big(\beta+\alpha(1-\phi)b\big)AK}{\alpha\beta(1+b)W_0} - 1\right)(1+b) \tag{101}$$

$$= \frac{\beta(AK-\alpha W_0) + \alpha\big((1-\phi)AK - \beta W_0\big)b}{\alpha\beta W_0}, \tag{102}$$

which is affine in b. It is increasing if and only if

$$\frac{1-\phi}{\beta} \geq \frac{W_0}{AK}, \tag{103}$$

which is the condition for the high-leverage equilibrium to be better in the proposition. $\square$

A.15 Proof of Corollary 1

The result follows immediately from Proposition 12. $\square$

B Formalization of Collateral Constraints when Capital Depreciates

Here we give a microfoundation of the collateral constraint in equation (2), and show that assets can be useful as collateral even if they fully depreciate in production. We do this by dividing the investment horizon into two "sub-periods," and considering a two-date repayment schedule in which, as in Hart and Moore (1994), a borrower can always repudiate its debt, and hence never repays more than lenders' liquidation value.

Specifically, suppose that a borrower finances an investment in capital k from a lender at time t. The investment does not mature till $t + \mathrm{d}t$, but the borrower can make a repayment after one sub-period, at an interim date we refer to as $t + \mathrm{d}t/2$. At this point, k is intact; the lender can seize it and sell it at p. In this case, the investment is terminated. Otherwise, the borrower can (re)negotiate a repayment with the lender and avoid seizure. In this case, the investment is continued, and capital is fully used up in production.[17]

We assume that the borrower can promise repayments at the end of each sub-period, but is free to repudiate them. Hence, it always renegotiates the lender down to its seizure value of the collateral. The lender thus gets nothing at $t + \mathrm{d}t$ (since its seizure value is zero) and at most pk at $t + \mathrm{d}t/2$. Thus, its maximum total repayment is pk, and the most it is willing to lend is likewise pk (assuming no discounting). Denoting the repayment by $(1 + \sigma)bw$ as above, this gives rise to the collateral constraint in equation (2):

$$(1 + \sigma)bw \leq pk. \tag{104}$$

[17]Notice that capital does not depreciate at all if it is not used in production in the second sub-period, but can still be redeployed later if it is seized in the first sub-period (and hence not yet used in production). This bears some resemblance to "one-hoss shay depreciation," in which new and used capital are prefect substitutes, but new capital has a longer remaining useful life (see, e.g., Rampini (2016)). (To make this mapping exact, we would have to model the price of capital assuming that new and used capital were indistinguishable, hence priced the same.) Finally, we should point out that our results do not change much if we use the more standard "straight line depreciation," i.e. if capital depreciates linearly to zero over the life of the investment. In this case, the collateral constraint just has an added parameter $\theta = 1/2$, i.e. it reads $(1 + \sigma)bw \leq \frac{1}{2}pk$ (cf. equation (2)).

C Formalizations of Limited Moneyness

We refer to ϕ as the moneyness of loans, and with $\phi < 1$, we want to capture frictions that inhibit reselling loans or rehypothecating collateral. Hence, some banks that choose to make loans instead of hold cash cannot undertake their investment opportunities. Note, however, that in our baseline model, limited moneyness affects all banks that choose to lend, even though they only successfully make loans with probability $\beta \mathrm{d}t$. This could seem undesirable, since the banks that don't successfully make loans should have cash on hand to undertake their investment opportunities. But it is a purely technical distinction that we need only to ensure that the expected flow payoffs from choosing to lend and choosing to hold cash are both $o(\mathrm{d}t)$. Here, we show that it does not arise in discrete time or if banks investments have random maturity (instead of maturity dt).

Discrete time. Like in the baseline model, if a bank holds cash, it gets an investment opportunity with probability α. Hence, the value of holding \$1 in cash is

$$v = \frac{1}{1+\rho}\Big(\alpha V + (1-\alpha)v\Big), \tag{105}$$

as in equation (8).

Unlike from the baseline model, if a bank chooses to lend, it lends for sure. Then it gets an investment opportunity with probability α. In this case it can sell the loan for \$1 to invest and get V with probability ϕ.[18] (So ϕ is literally the resaleability of the loan.) Otherwise, it earns the (endogenous) spread σ. Hence, the value of lending \$1 is

$$v^{\ell} = \frac{1}{1+\rho}\Big(\alpha\phi V + (1-\alpha\phi)(1+\sigma)v\Big). \tag{106}$$

Lemma 6. **(Spread in discrete time.)** *In the discrete-time version of the model described above, the equilibrium spread solves*

$$\sigma = \frac{\alpha(1-\phi)}{1-\alpha\phi}\left(\frac{V}{v} - 1\right). \tag{107}$$

Combined with Proposition 1, this implies that this model is effectively nested by our baseline model:

Corollary 2. **(Nesting with discrete time.)** *If $\beta = 1 - \alpha\phi$, the equation for the*

[18] By assuming that the resale value is \$1 we are implicitly assuming that resale takes place at the beginning of the period, when the buyer discounts the spread on the loan due to his own opportunity cost of forgone investments.

spread in the baseline model (Proposition 1) is the same as the equation for the spread in the discrete-time version (Lemma 6).

Note that the only reason that we do not use this discrete-time set-up as our baseline model is that the differential calculus ($\mathrm{dt}^2 = 0$) makes the continuous-time analysis simple.

Random maturity. Now we consider a variation of the baseline model, in which the investment maturity is random: investing w at time t yields Rw at the maturity τ_γ, which arrives independently with Poisson intensity γ.

Like in the baseline model, if a bank holds cash, it gets an investment opportunity with probability $\alpha\mathrm{dt}$. Hence, the value of holding \$1 in cash is

$$v = \frac{1}{1+\rho}\Big(\alpha V + (1-\alpha)v\Big), \tag{108}$$

as in equation (8).

Unlike in the baseline model, but like in the discrete-time model above, if a bank chooses to lend, it lends for sure. Now, however, we assume that the loan matures when the investment matures. Thus, over each time increment, a lending bank's loan matures with probability $\gamma\mathrm{dt}$ in which case it gets its interest in cash. At any time before that, it can resell the loan and undertake its investment opportunity with probability ϕ. Hence, the value of lending \$1 is

$$v^\ell = \frac{1}{1+\rho\mathrm{dt}}\left(\gamma\mathrm{dt}(1+\sigma)v + (1-\gamma\mathrm{dt})\Big(\alpha\mathrm{dt}\phi V + (1-\alpha\mathrm{dt}\phi)v^\ell\Big)\right). \tag{109}$$

Lemma 7. **(Spread with random maturity.)** *With the investment maturity arriving at rate γ, the spread solves*

$$\sigma = \frac{(1-\phi)\,\alpha}{\gamma}\left(\frac{V}{v} - 1\right). \tag{110}$$

Combined with Proposition 1, this implies that this model is effectively nested by our baseline model:

Corollary 3. **(Nesting with random maturity.)** *If $\beta = \gamma$, the equation for the spread in the baseline model (Proposition 1) is the same as the equation for the spread in the discrete-time version (Lemma 7).*

Note that there are two reasons that we use the short-maturity set-up in our baseline model, rather they do not use this random-maturity one. First, we do not need to solve for optimal debt maturity—since investments are short-term in the baseline model, the debt they back must be too. Second, we do not need to consider the possibility that the aggregate state changes in the course of an investment.

That said, under the assumption that debts mature when investments do, the stationary model (with no aggregate shock) with random maturity is easy to solve. It has almost the same solution as the baseline, as we now show briefly.

Like in the baseline model, a bank with an investment opportunity and wealth w and leverage b gets payoff

$$\mathbb{E}\left[e^{-\rho\tau_\gamma}\Big(R+(R-1-\sigma)\,b\Big)w\right]=\frac{\gamma}{\rho+\gamma}\big(R+(R-1-\sigma)\,b\big)w. \tag{111}$$

and the value of investing \$1 is thus

$$V=\frac{\gamma}{\rho+\gamma}\left(R-1\right)\left(1+\frac{1}{\sigma}\right), \tag{112}$$

which is a scaled version of the value of investing in the baseline model (equation (6)).

Similarly, the wealth shares evolve almost as in the baseline model. They are determined by the following three equations (cf. equations 10–12):

(i) **New investments arrive at the same rate as old investments mature.**

$$\gamma w_t^I \mathrm{dt}=\alpha w_t^{\$}\mathrm{dt}+\phi\alpha w_t^{\ell}\mathrm{dt} \tag{113}$$

(ii) **Market clearing.**

$$w_t^{\ell}=bw_t^I \tag{114}$$

(iii) **Adding up.**

$$w_t^{\ell}+w_t^I+w_t^{\$}=W_t. \tag{115}$$

Combining these, we have the analogy of Lemma 2 (and equation (17)).

LEMMA 8. **(Wealth shares with random maturity.)**

$$\frac{w_t^I}{W_t}=\frac{\alpha}{\alpha+\gamma+\alpha\left(1-\phi\right)b} \tag{116}$$

$$\frac{w_t^{\ell}}{W_t}=\frac{\alpha b}{\alpha+\gamma+\alpha\left(1-\phi\right)b} \tag{117}$$

$$\frac{w_t^{\$}}{W_t}=\frac{\gamma-\alpha\phi b}{\alpha+\gamma+\alpha\left(1-\phi\right)b} \tag{118}$$

and

$$\begin{aligned} dW_t &= \gamma\sigma w_t^\ell \, dt - \gamma w_t^I \, dt \\ &= \left(\sigma - \frac{1}{b}\right) \gamma w_t^\ell \, dt. \end{aligned} \tag{119}$$

D Competitive Suppliers

Here we sketch a simple model in which suppliers produce the supply $S(p)$. Importantly, suppliers make zero profit, and hence do not feature in our welfare analysis.

Consider a unit continuum suppliers. We assume that each supplier i has a linear production technology with constant marginal cost of producing k_i, but that this marginal cost depends on the aggregate quantity of of capital produced $K = \sum k_i$. This could reflect any kind of congestion externality; for example, all suppliers use the same input goods, but input goods are scarce, so the more that others use them the harder they are for each to find.

Denoting the cost function by c, each supplier i maximizes

$$pk_i - c'(K)k_i. \tag{120}$$

Linearity and market clearing imply that it must be that

$$k_i = (c')^{-1}(p). \tag{121}$$

Since there is a unit mass of suppliers, this defines the supply curve—$k_i = k_i \times 1 \equiv S(p)$—which we took in reduced form above. Hence, choosing different functional forms for the supply curve S is tantamount to specifying a cost function c. For example, in Appendix E below, we suppose that

$$S(p) = \frac{C_0}{C_1 - p}, \tag{122}$$

which corresponds to

$$c'(K) = C_1 - \frac{C_0}{K} \tag{123}$$

and, integrating,

$$c(K) = C_1 K - \log \frac{C_0}{K}. \tag{124}$$

(Note that we need to ensure that $K > C_0/C_1$ for c to be increasing.)

E Imperfectly Elastic Supply

Suppose the supply curve is given by

$$S(p) = \frac{C_0}{C_1 - p}. \tag{125}$$

From market clearing (equation (42)), we have that p solves

$$\frac{(1+b)}{p} \frac{\alpha\beta W}{\beta + \alpha(1-\phi)b} = \frac{C_0}{C_1 - p} \tag{126}$$

so

$$p = \frac{\alpha\beta C_1(1+b)}{\alpha\beta(1+b) + C_0\big(\beta + \alpha(1-\phi)\big)b/W}. \tag{127}$$

Substituting $R = A/p$ into the equation for b in Lemma 3 gives a quadratic equation that can have two positive roots corresponding to multiple equilibria.

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