Financial Intermediation, Capital Accumulation and Crisis Recovery*

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Abstract

We integrate bank and bond financing into a two-sector neoclassical growth model. Besides providing an analytically tractable macro-banking module, we make three contributions to the literature: First, although the banks' leverage amplifies shocks, the endogenous response of leverage to a sharp decline in bank equity capital is an automatic stabilizer that improves the resilience of the economy. Second, the automatic stabilizer together with a mix of publicly financed bank re-capitalization and dividend payout restrictions engineers a rapid build-up of bank equity, accelerates economic recovery and improves worker welfare after a slump in the banking sector. Third, the model replicates typical patterns of financing over the business cycle: pro-cyclical bank leverage, pro-cyclical bank lending, and counter-cyclical bond financing. In addition, we provide a quantitative analysis of the Great Recession in the US to illustrate both the amplification and automatic stabilization role of bank leverage.

JEL: E21, E32, F44, G21, G28

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1 Introduction

Financial frictions affect the propagation of economic shocks and are an essential factor for understanding short-run dynamics and long-run macroeconomic performance. Typically, financial frictions can be traced back to either contract enforceability problems or asymmetric information and – on this ground – give rise to leverage limits to align the interest of borrowers and lenders.[1]

Since the seminal contributions of Bernanke and Gertler (1989), Bernanke et al. (1996) and Kiyotaki and Moore (1997), it is well-understood that in an economy with financial frictions, even small temporary shocks can have large and persistent effects on economic activity by impacting the net worth of levered agents. In this literature, firms need net worth to credibly commit to the contractual obligations of the credit contract. Deteriorating conditions reduce firm profits, net worth and, thus, the capacity to obtain credit. The propagation of shocks through net worth and credit may have a large and persistent impact on economic activity – a mechanism referred to as the *credit channel*.[2]

Although Holmström and Tirole (1997) extended the analysis to financial intermediaries, it was not until the 2008-2009 financial and banking crisis that macroeconomists took up their proposal. Financial intermediaries channel funds from investors to entrepreneurs, cope with the underlying financial frictions and are, at the same time, subject to frictions themselves. Banks therefore have to hold equity capital to credibly commit to the contractual obligations of the deposit contract: When financial conditions deteriorate, bank profits decline, which negatively affects future bank equity holdings and, thus, the future capacity to attract loanable funds and to supply loans to entrepreneurs. The propagation of shocks through the bank balance sheets has large and persistent effects on economic activity – a mechanism referred to as the *bank lending channel*.[3] In essence, the *bank lending channel* is a propagation mechanism similar to the *credit channel*, but it impacts different borrowers.[4]

In this paper, we develop an analytically tractable two-sector neoclassical growth model where production sectors differ with respect to their access to capital markets. We adopt a medium- to long-run perspective in the sense that output reacts smoothly to adverse shocks and that economic dynamics are essentially driven by capital re-allocation and accumulation instead of abrupt changes in prices. We contribute to the literature in four respects.

First, we provide novel insights into the *bank lending channel*. We show that although the level of leverage is an amplification mechanism of shocks, the endogenous response of leverage to a decline in bank equity capital is an automatic stabilizer that improves the resilience of the

[1] See Quadrini (2011) for an overview of the extensive literature on financial frictions and macroeconomic performance.

[2] This literature includes Carlstrom and Fuerst (1997) and, more recently, Cooley et al. (2004), Christiano et al. (2007), Jermann and Quadrini (2012), Brumm et al. (2015), and Gomes et al. (2016).

[3] This literature includes Van den Heuvel (2008), Meh and Moran (2010), Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Rampini and Viswanathan (2017), Brunnermeier and Sannikov (2014), and Quadrini (2014).

[4] There are also notable deviations from this approach to model banking systems in macroeconomic context, see e.g. Angeloni and Faia (2013) and Acemoglu et al. (2015).

economy to adverse shocks. Specifically, suppose there is a shock that leads to a decline in bank equity capital. Investors, *ceteris paribus*, reduce their supply of loanable funds to the banks in order to restore the initial bank leverage, and thus loan supply decreases. As a consequence, capital productivity in the loan-financed sector increases, and so does bank profit. The financial friction becomes less tight such that investors can increase their supply of loanable funds to the banks without incentivizing them to defect. The ensuing increase in bank leverage partially neutralizes the initial decline in loan supply.[5]

Second, we derive macro-prudential policies consisting of investor-financed re-capitalization of banks and dividend payout restrictions to speed up the economic recovery after a banking crisis, without encouraging banks to take excessive leverage in the expectation of future bailouts. Acharya et al. (2017) derive a parallel result within a different framework emphasising a different economic mechanism. They show that bank equity capital has the characteristics of a public good which justifies dividend payout restrictions, in order to internalize the impact of dividend payments on social welfare and output. In fact, bank recapitalization and dividend payout restrictions have been used during the 2008 − 2009 financial and banking crisis in the United States and during the 2008 − 2014 financial and banking crisis in Europe (see Shin (2016)).

Third, the model replicates typical patterns of financing over the business cycle: procyclical bank leverage, procyclical bank lending and countercyclical bond financing – see Adrian and Shin (2014), Adrian and Boyarchenko (2012), Adrian and Boyarchenko (2013) and Nuño and Thomas (2012) for empirical evidence. These patterns are outcomes of the model if downturns are associated with negative productivity, bank equity or trust shocks – or any combination thereof. Moreover, when recessions are accompanied by a sharp temporary decline in bank equity, they are deeper and more persistent than regular recessions – a result that is consistent with the findings in Bordo et al. (2001), Allen and Gale (2009), and Schularick and Taylor (2012).

Fourth, we provide a quantitative analysis of the Great Recession to illustrate the main properties and results of the model. We calibrate the model to the US economy and show that the output and welfare costs are significantly increased by the bank lending channel beyond what would be implied by the drop of total factor productivity. However, the endogenous response of bank leverage – and thus its role as an automatic stabilizer – prevents the economy from substantially higher output and welfare losses.

Financial frictions are at the core of our macro-banking model: they provide a micro-foundation for the existence of banks and play an essential role for the propagation of adverse shocks.[6] In our model, there are two production sectors. Firms in sector I (*intermediary-financed*) are subject to financial frictions, which prevents them from obtaining financing directly on the financial market. Banks alleviate the moral hazard problem that ensues from these financial frictions

[5]These insights complement earlier insights from the macro-finance literature (Carlstrom and Fuerst (1997) and the survey of Brunnermeier et al. (2012)) when the impact of shocks to entrepreneurs' net worth is dampened by a corresponding increase of the price of capital.

[6]Gersbach and Rochet (2017) study a static version of the same banking model in which bank equity capital cannot be accumulated. Gersbach et al. (2016) integrate banks into the Solow growth model.

and provide loans to firms in sector I. However, bank lending itself is limited, as bankers can only pledge a fraction of their revenues to depositors and are thus subject to a financial friction themselves, that gives rise to an endogenous limit on leverage, which depends on equilibrium returns in sector I and interest rates on deposit. The need for bank lending and the incomplete revenue pledgeability, are the two financial frictions in our model.[7] Firms in sector M (*market-financed*) are not subject to financial frictions and issue corporate bonds, instead. In the baseline model, there are three types of agents: investors, bankers and workers. The latter are immobile across production sectors, as their skills are sector-specific. Workers do not save and consume their entire labor income. Investors and bankers have standard intertemporal preferences and decide in each period how much to save and to consume.[8] Their utility maximization problems yield two accumulation rules for investor wealth and bank equity, respectively. These rules are coupled in the sense that the investor's saving and investment policies depend on how bankers fare and vice versa. Both types of lending – *informed lending* by banks and *uninformed lending* through capital markets – enable capital accumulation in the respective sectors.

The paper is organized as follows. Section 2 relates our paper to the existing literature. Section 3 introduces the model, Section 4 defines and characterizes sequential market equilibria, and Section 5 analyzes long-run economic dynamics and global stability. Section 6 characterizes short-run dynamics and discusses the propagation of adverse shocks when bank leverage is sensitive to equilibrium conditions. Section 7 derives public policies and financial regulation to speed up recoveries when the economy is hit by a negative shock to bank equity capital. In Section 8, we calibrate the model to the US economy and provide a quantitative analysis of the Great Recession to quantify and illustrate the static and dynamic effects derived in the preceding sections. Section 9 provides some extensions to the framework and Section 10 concludes.

2 Relation to the Literature

Our paper is closely related to three recent strands of the literature that integrate financial intermediation into macroeconomic models to analyze the propagation of shocks through bank balance sheets and to derive policies to prevent and manage financial crises.

Our paper is most closely related to recent research that integrates financial intermediation into the neoclassical growth model, e.g. Van den Heuvel (2008), Gertler and Kiyotaki (2010), Rampini and Viswanathan (2017), Brunnermeier and Sannikov (2014), Quadrini (2014), and Acemoglu et al. (2015). Brunnermeier and Sannikov (2014) show that the economy's reaction to adverse shocks can be highly non-linear. Specifically, if the economy is sufficiently far away from its steady state, even small shocks can generate substantial amplification and endogenous fluctuations. In contrast, near the steady state, the economy is resilient to most shocks. He

[7] As we discuss in Section 3.3, the foundation of these frictions can be moral hazard problems à la Holmström and Tirole (1997), asset diversion (as in Gertler and Karadi (2011) and Gertler and Kiyotaki (2010)) or non-alienability of human capital (as in Hart and Moore (1994) and Diamond and Rajan (2000)).

[8] In the extensions, we consider a version of the model in which there are only two types of agents: households, acting as investors and workers, and banks. To preserve clarity in exposition, we solely use the term *household* for that case.

and Krishnamurthy (2013) find similar non-linear effects when risk premia on equity increase sharply as financial constraints become binding. Rampini and Viswanathan (2017) develop a dynamic theory of financial intermediaries that act as collateralization specialists, in which credit crunches are persistent and can delay or stall economic recoveries. They consider a one-sector economy with risk-neutral agents and show that there are – under certain conditions – large reactions to small changes in interest rates. In contrast to Rampini and Viswanathan (2017), we develop a two-sector neoclassical growth model with levered financial intermediation where saving, investment, interest rates, and bank capital accumulation react more smoothly to shocks for three reasons: First, with an alternative investment opportunity that does not rely on levered finance, investors re-optimize their portfolio which already attenuates the immediate impact of an adverse shock. Second, as investors are risk-averse, they adjust their consumption-saving decision in response to an adverse shock to smooth their consumption path. This adjustment, in turn, has far reaching implications for capital accumulation and economic dynamics. Third, leverage itself reacts endogenously and immediately accommodates the banks' lending capacity to smooth out adverse shocks to the bank balance sheet. Nevertheless, the special role of banks in the capital accumulation process with binding leverage constraints as well as the potentially divergent reactions of investor wealth and bank equity capital generate sizeable and persistent output reactions, as bank profits and thus future lending capacities are affected. In this sense, our approach adopts a medium- to long-run perspective on how economies with a large bank-financed sector react to shocks, because economic dynamics are driven by adjustments in capital accumulation instead of abrupt changes in price levels. In contrast to Rampini and Viswanathan (2017), the relative capital productivity of financially constrained and unconstrained firms in our model is endogenously determined by the joint evolution of bank equity and investor capital. The mix of bond and loan finance evolves endogenously and replicates typical financing patterns over the business cycle: countercyclical bond-to-loan financing ratios (see De Fiore and Uhlig (2011)) and procyclical bank leverage (see Adrian and Shin (2014)).

Moreover, our paper is closely related to a recent strand in the literature that integrates banks into New-Keynesian DSGE models, e.g. Meh and Moran (2010), Gertler and Karadi (2011), and Angeloni and Faia (2013). Meh and Moran (2010) and Angeloni and Faia (2013) have provided valuable insights about the bank capital transmission channel. Meh and Moran (2010) find that this channel amplifies the impact of technology shocks on inflation and output, and delays economic recovery. Angeloni and Faia (2013) introduce a fragile banking system, in which banks are subject to runs, into a new-Keynesian DSGE model. They show that a combination of countercyclical capital requirements and monetary policies responding to asset prices or bank leverage is optimal in the sense that it maximizes the *ex-ante* expected value of total payments to depositors and bank capitalists. In contrast to this strand of literature, we abstract from price rigidities and develop a parsimonious neoclassical macro-banking model that exhibits smooth reactions to adverse shocks. In contrast to Angeloni and Faia (2013), we focus on incentive compatible *ex-post* policies to manage financial and banking crises instead of *ex-ante* policies to prevent them.

Finally, in terms of policy implications, our paper is closely related to Martinez-Miera and Suarez

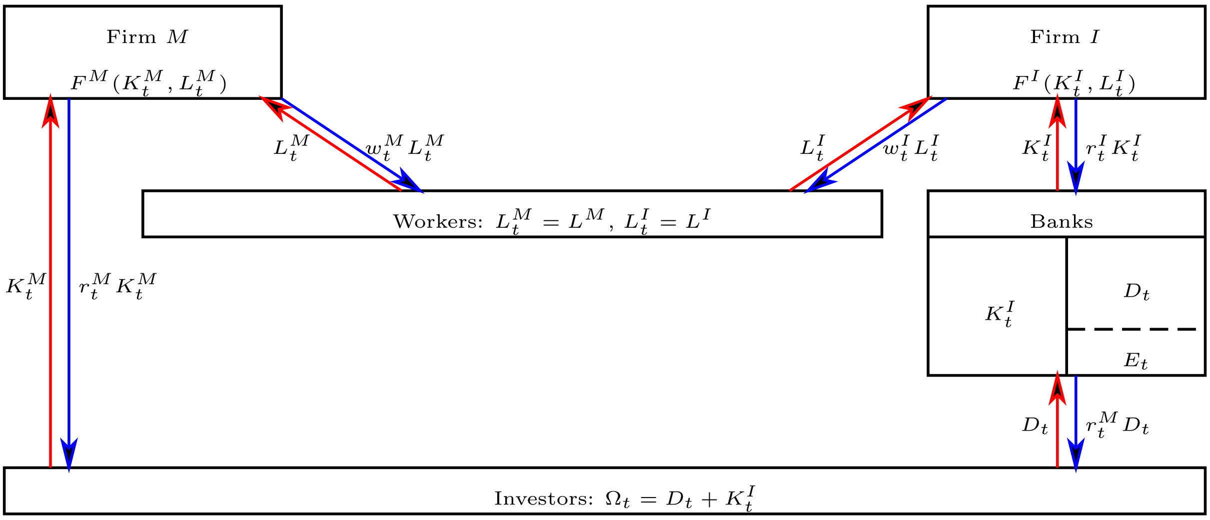
(2012), who study a dynamic general equilibrium model in which banks decide *inter alia* on their exposure to systemic shocks. Capital requirements reduce the direct impact of negative systemic shocks, but they also lower credit supply and output in normal times: optimal capital requirements balance these cost and benefits. Our model is complementary to Martinez-Miera and Suarez (2012) and considers the simultaneous build-up of bank equity and investor wealth after both anticipated and unanticipated shocks to productivity, wealth, and financial frictions. In contrast to Martinez-Miera and Suarez (2012) and Mendicino et al. (2018), who focus on capital requirements and crisis prevention, we focus on crisis management and show that a revenue-neutral combination of investor-financed bank re-capitalization and publicly enforced dividend payout restrictions can speed up the recovery after a banking crisis while leaving the welfare of bankers unaffected. In a similar vein, Itskhoki and Moll (2014) study how taxes or subsidies may favorably impact the transition dynamics in a standard growth model with financial frictions. Our study is complementary, as we focus on two policies that are typically implemented in banking crises: re-capitalization of banks and dividend payout restrictions. Acharya et al. (2011) study dividend payments of banks in the $2008-2009$ financial crisis and argue that early suspension of dividend payments can prevent the erosion of bank capital. Acharya et al. (2017) and Onali (2014) suggest that because dividend payments exert externalities on other banks, dividend payout restrictions can adjust for the negative external effect.

3 Model

We integrate a simple model of banks into a two-sector neoclassical growth model. Time is discrete and denoted by $t \in \{0, 1, 2, \dots\}$. There are two production sectors with constant returns to scale technologies using capital and labor to produce a homogeneous good that can be consumed or invested. Sectors differ with respect to their access to capital markets: while firms in sector M (*market-financed* or *bond-financed*) can borrow on a frictionless capital market, firms in sector I (*intermediary-financed* or *loan-financed*) have no direct access to financial markets and rely on bank loans instead. Banks monitor entrepreneurs in sector I and enforce the contractual obligation from the loan contract. However, banks themselves are subject to financial frictions that limit the amount of loanable funds they can attract. Consumption is the numéraire: its price is normalized to 1. There are three types of agents: workers, investors, and bankers.[9] Workers are hand-to-mouth consumers who consume their entire labor income instantaneously. In contrast, investors and bankers choose consumption and investment to maximize lifetime utility. The general structure of the model is depicted in Figure 1 and the details are set out in this section.

[9]Splitting the household sector into workers and investors can be justified on empirical grounds as will be discussed in Section 3.2. It preserves the analytical tractability of the model. We also consider a variation of the model in Appendix E in which there is only one type of households that supplies labor and acts as investor. We show that this variation yields model dynamics that are qualitatively and quantitatively similar to the baseline model.

Figure 1: General Structure of the Model



3.1 Production

Production takes place in two different sectors, labeled sector M and sector I. Both sectors consist of a continuum of identical firms. The production technologies exhibit constant returns to scale in the production factors capital and labor, have positive and diminishing marginal returns regarding each production factor taken separately, and satisfy the Inada conditions. Because of constant returns to scale and competitive markets, we can consider a price-taking representative producer in each sector, without loss of generality. Specifically, the aggregate production technologies are Cobb-Douglas and given by

$$Y_t^j = z^j A(K_t^j)^{\alpha}(L_t^j)^{1-\alpha}, \quad j \in \{M, I\},$$

where A is an index of economy-wide productivity, z^j is an index of sectoral productivity, α $(0 < \alpha < 1)$ is the output elasticity of capital, and K_t^j and L_t^j denote capital and labor input in sector $j \in \{M, I\}$, respectively.

Firms in sector M can borrow frictionlessly on capital markets by issuing corporate bonds. Firms in sector I have neither the reputation nor the transparency to access capital markets. These firms, however, can obtain loans from financial intermediaries that monitor them and enforce the contractual obligation.[10]

Taking interest rates and wage rates as given, the representative firm in each sector $j \in \{M, I\}$

[10] In this paper, market access is restricted for some firms by assumption and is motivated by the empirical pattern. Morellec et al. (2015) provide empirical evidence that bank financing is concentrated in particular sectors. De Fiore and Uhlig (2015) shows that firms relying on bank credit are typically younger and smaller, so that they lack the transparency and reputation to gain direct access to capital markets.

chooses capital and labor to maximize its period profit

$$\max_{\{K_t^j, L_t^j\}} \left\{ z^j A (K_t^j)^\alpha (L_t^j)^{1-\alpha} - r_t^j K_t^j - w_t^j L_t^j \right\}, \quad j \in \{M, I\}, \tag{1}$$

where w_t^j is the wage rate and r_t^j is the rental rate of capital in sector j and period t, respectively. We further define $K_t := K_t^M + K_t^I$ and $L_t := L_t^M + L_t^I$ as total capital and total labor used in production.

3.2 Workers and Investors

There is a continuum of workers with mass L ($L > 0$). Each worker is endowed with one unit of labor, of which he inelastically supplies l and $1-l$ units to firms in sectors M and I, respectively. Workers are hand-to-mouth consumers, i.e. they consume their entire labor income and do not save.[11] As workers are homogeneous, we can consider a representative worker who takes wages as given and earns $w_t^M L^M + w_t^I L^I$, where $L^M = lL$ and $L^I = (1-l)L$, without loss of generality.

The assumption of sector-specific inelastic labor supply is strong but can be traced back to several reasons, e.g. to spatial frictions that substantially reduce labor mobility between sectors or to a lack of skill transferability between sectors.[12] As a consequence, wage differentials between sectors can be persistent and are driven by the joint accumulation of bank equity capital and investor wealth. Note that labor immobility in combination with the Inada conditions ensures that there will be no concentration in either of the two production sectors in the long-run, even when sector-specific productivities z^j differ.

There is a continuum of investors with unit mass. Each investor is endowed with some units of the capital good which can be used for investment in bonds or deposits and for consumption. In the absence of labor income, disposable income is linear homogeneous in wealth and because the period-utility is logarithmic, consumption and saving decisions are linear homogeneous in wealth, too. This implies that the distribution of capital among investors has no impact on aggregate consumption, saving, and investment, such that we can restrict the analysis to a representative investor without loss of generality.[13] At the beginning of period 0, the representative investor is endowed with Ω_0 units of capital. He chooses a sequence of investment into bonds and deposits $\{B_t, D_t\}_{t=0}^\infty$, consumption $\{C_t^H\}_{t=0}^\infty$, and savings $\{\Omega_{t+1}\}_{t=0}^\infty$ to maximize his lifetime utility subject to the sequential budget constraint.

We next observe that competition in the banking sector and Inada conditions in the production sectors imply that in any equilibrium, the return on bonds and deposits coincide. As a result,

[11] There are several well-understood reasons why workers may not want to save and behave like hand-to-mouth consumers, e.g. lower discount factors or borrowing constraints. As reported in Challe and Ragot (2016), estimates of the share of hand-to-mouth households in the United States vary a lot and range from 15% to 60%. A recent study by Kaplan et al. (2014) finds that more than one-third of the population in the Unites States saves little or nothing.

[12] There is empirical evidence to support our assumption, e.g. Acemoglu and Autor (2011) and Bárány and Siegel (2017) assess the impact of sector- or task-specific skills on labor mobility, wages, employment, and structural change.

[13] See Alvarez and Stokey (1998), Krebs (2003a), and Krebs (2003b) for a general derivation of this result.

given $B_t+D_t = \Omega_t$, the disposable income of the representative investor, $r_t^M B_t + r_t^M D_t + (1-\delta)\Omega_t$, simplifies to $(1 + r_t^M - \delta)\Omega_t$. The utility maximization problem of the representative investor is given by

$$\max_{\{C_t^H, \Omega_{t+1}, D_t, B_t\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta_H^t \ln(C_t^H) \right\} \tag{2}$$

subject to

$$\begin{aligned} &C_t^H + \Omega_{t+1} = (1 + r_t^M - \delta)\Omega_t, \text{ for all } t \geq 0 \\ &\Omega_0 \text{ given,} \end{aligned}$$

where r_t^M denotes the return to bonds and deposits, δ is the capital depreciation rate, and $\beta_H = \frac{1}{1+\rho_H}$ $(0 < \beta_H < 1)$ denotes the discount factor and ρ_H the discount rate.

3.3 Bankers

There is a continuum of bankers and each banker owns and runs a financial intermediary. Bankers can alleviate the moral hazard problem of the entrepreneurs in sector I, as they monitor entrepreneurs and enforce contractual obligations. The cost of these activities is neglected.[14] Bankers themselves raise funds from investors at the deposit rate but cannot pledge the entire amount of repayments from entrepreneurs to investors, i.e. bankers are subject to a moral hazard problem themselves. Specifically, if a banker has granted a loan of size k_t^I to entrepreneurs, we assume that an amount θk_t^I of the revenues of the loan are non-pledgeable to outside investors.[15] Note that parameter $\theta \in (0, 1)$ provides a concise measure of the financial friction between bankers and depositors.

At the beginning of period t, a typical banker owns e_t which he uses as equity funding for his bank. He attracts additional funds d_t from investors, lends k_t^I to entrepreneurs in sector I, and purchases $e_t + d_t - k_t^I$ corporate bonds from sector M. Equity e_t is inside equity only, i.e. banks cannot raise equity on the market to improve their lending capacity.[16] In order to attract

[14] We discuss the impact of intermediation costs on the steady state allocation in Section 9 and show that while bank leverage and return on equity are unaffected by intermediation cost, this cost nevertheless reduces steady state investor wealth, bank equity capital and production.

[15] The partial non-pledgeability of revenues leads to moral hazard between bankers and investors, as in Holmström and Tirole (1997), and can alternatively be traced back to the possibility of asset diversion (as in Gertler and Karadi (2011) and Gertler and Kiyotaki (2010)) or to non-alienability of human capital (as in Hart and Moore (1994) and Diamond and Rajan (2000)). See Gersbach and Rochet (2013) for an extensive discussion of different mechanisms that micro-found moral hazard in the banker-depositor relationship. Furthermore, assume that when bankers shirk in the current period, they cannot be excluded from seeking new funds from investors in the next period. This rules out that bankers can pledge revenues from future periods in order to attract more funds today. For example, consider the case of asset diversion. Suppose that a banker attempts to pledge $(1 - \theta')k_t^B$ in the current period, with $\theta' < \theta$ in a long-term contract with more than one period, in which he invests k_t^B more than once. He can divert θk_t^B in period t and seeks new funds in period $t + 1$. This is profitable and thus $(1 - \theta')k_t^B$ cannot be pledged.

[16] This assumption simplifies our analysis without interfering with our main insights, as we mainly focus on financial and banking crises, i.e. times in which banks are under distress, and raising new equity is expensive on the ground of a standard pecking order argument. Our approach is common in the literature, which often follows

loanable funds d_t from investors, a banker has to be able to promise at least $(1+r_t^M)d_t$ to investors, as they would otherwise solely invest into bonds. The banker's profit from investing into sector M and I is given by $(1+r_t^I)k_t^I + (1+r_t^M)(e_t + d_t - k_t^I) - (1+r_t^M)d_t$. Because the amount θk_t^I of revenues is non-pledgeable, incentive compatibility of the deposit contract requires

$$
\begin{aligned}
& (1+r_t^I)k_t^I - (1+r_t^M)(k_t^I - e_t) \geq \theta k_t^I \\
\Leftrightarrow \quad & k_t^I \leq \frac{1+r_t^M}{r_t^M - r_t^I + \theta} e_t := \lambda_t e_t. \qquad (3)
\end{aligned}
$$

Condition (3) is the market-imposed leverage constraint and follows from the investors' decision to limit the supply of loanable funds in order to incentivize the banker to comply with the contractual obligations of the deposit contract. For convenience, we define λ_t as the market-imposed (upper) limit of bank leverage.

The optimal choice of k_t^I by the banker will result from the profit maximization program under the incentive compatibility constraint:

$$
\max_{k_t^I} \left\{ (1+r_t^I)k_t^I - (1+r_t^M)(k_t^I - e_t) \right\}
$$

subject to

$$
(1+r_t^I)k_t^I - (1+r_t^M)(k_t^I - e_t) \geq \theta k_t^I.
$$

There are two different cases, as the incentive compatibility constraint is either binding or non-binding. First, suppose that total bank equity E_t is relatively scarce. In this case, loan supply is limited by low bank equity capital and incentive compatibility constraints are binding. There is under-investment in sector I such that the returns on bonds and deposits satisfy $r_t^I > r_t^M$. Therefore, each individual banker solely invests into sector I, i.e. $k_t^I = e_t + d_t$, and levers up to the leverage limit, i.e. $k_t^I = \lambda_t e_t$ As a result, the banker's return on equity is equal to $r_t^B = \theta\lambda_t - 1$. Note that while establishing the formal condition for scarcity of bank equity in Section 4.1, we conveniently define $\Gamma \subseteq \mathbb{R}_+^2$ as the partition of the state space (E_t, Ω_t) for which the market-imposed leverage constraint is binding.

Second, suppose that total bank equity E_t is relatively abundant, such that the market-imposed leverage constraint is non-binding, i.e. $(E_t, \Omega_t) \in \mathbb{R}_+^2 \setminus \Gamma$. In this case, loan supply is not limited by the level of bank equity, such that competitive capital markets push down the returns in sector I until interest rates in both sectors coincide, $r_t^I = r_t^M$. As a result, the banker's return on equity satisfies $r_t^B = r_t^M$.

Note that in either of the two cases, the return on equity is independent of indiosyncratic banker

the same line of argument, e.g. Meh and Moran (2010) or Gertler and Kiyotaki (2010). A notable extension is Ellingsen and Kristiansen (2011) who develop a static banking model with inside equity, outside equity and deposits.

characteristics. As bankers only derive income from investment, their budget constraint is linear homogeneous in bank equity capital and aggregate loan supply is independent of the equity distribution among bankers. Therefore, we can restrict our analysis to a price taking representative banker who owns E_t in period t. The representative banker has logarithmic period-utility and chooses a sequence of consumption $\{C_t^B\}_{t=0}^{\infty}$ and savings $\{E_{t+1}\}_{t=0}^{\infty}$ to maximize his lifetime utility

$$\max_{\{C_t^B, E_{t+1}\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta_B^t \ln(C_t^B) \right\} \tag{4}$$

subject to

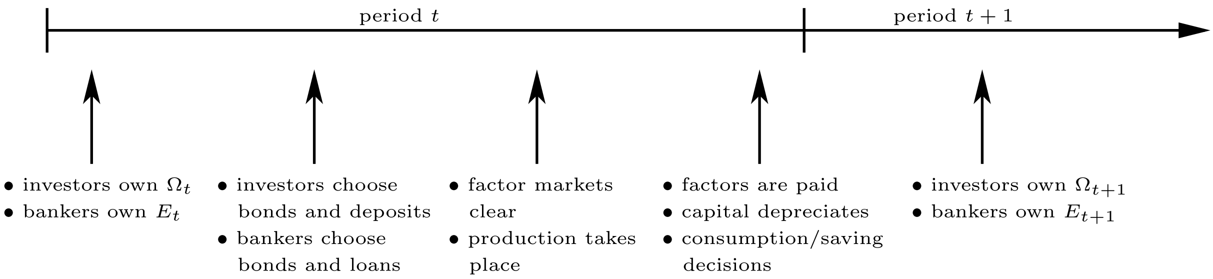
$$\begin{aligned} & C_t^B + E_{t+1} = (1 + r_t^B - \delta^E) E_t \\ & r_t^B = \begin{cases} \theta \lambda_t - 1, & \text{for } (E_t, \Omega_t) \in \Gamma \\ r_t^M, & \text{for } (E_t, \Omega_t) \in \mathbb{R}_+^2 \setminus \Gamma \end{cases}, \quad \text{for all } t \geq 0 \\ & E_0 \text{ given,} \end{aligned}$$

where δ^E denotes the depreciation rate of bank equity capital that can in principle differ from the overall capital depreciation rate δ, where $\beta_B = \frac{1}{1+\rho_B}$ $(0 < \beta_B < 1)$ denotes the discount factor and ρ_B the discount rate.

3.4 Sequence of Events

The sequence of events within a specific period is depicted in Figure 2. At the beginning of period t, the representative investor and banker own Ω_t and E_t units of wealth, respectively. After the investor has chosen his portfolio of bonds B_t and deposits D_t, the banker decides about his investment, taking the amount of loanable funds $E_t + D_t$ as given. Factor markets clear and production takes place. After production factors and depositors have been paid, capital depreciates and agents make their consumption-saving decision, which governs the evolution of investor wealth and bank equity capital, $\Omega_{t+1}(E_t, \Omega_t)$ and $E_{t+1}(E_t, \Omega_t)$.

Figure 2: Sequence of Events



4 Sequential Markets Equilibrium

In this section, we characterize the sequential markets equilibrium defined as follows:

Definition 1. *For any given* $(E_0, \Omega_0) \in \mathbb{R}^2_+$ *a sequential markets equilibrium is a sequence of factor allocations* $\{K_t^M, K_t^I, L_t^M, L_t^I\}_{t=0}^{\infty}$*, factor prices* $\{w_t^M, w_t^I, r_t^M, r_t^I\}_{t=0}^{\infty}$*, consumption choices* $\{C_t^H, C_t^B\}_{t=0}^{\infty}$*, and wealth allocations* $\{E_{t+1}, \Omega_{t+1}\}_{t=0}^{\infty}$ *such that*

(i) $\{C_t^H, \Omega_{t+1}\}_{t=0}^{\infty}$ *solves the representative investor's utility maximization problem* (2)*,*

(ii) $\{C_t^B, E_{t+1}\}_{t=0}^{\infty}$ *solves the representative banker's utility maximization problem* (4)*,*

(iii) $\{K_t^M, K_t^I, L_t^M, L_t^I\}_{t=0}^{\infty}$ *solves the representative firms' profit maximization problem* (1)*, and*

(iv) factor markets and good markets clear.

The analysis of the sequential markets equilibrium proceeds in two steps. In the first step, we characterize the intraperiod factor allocation $(K_t^M, K_t^I, L_t^M, L_t^I)$, equilibrium factor prices $(w_t^M, w_t^I, r_t^M, r_t^I)$, and ensuing leverage λ_t for any given beginning-of-period allocation of bank equity capital and investor wealth (E_t, Ω_t). In the second step, we characterize the associated consumption-saving policies which govern the evolution of bank equity $E_{t+1}(E_t, \Omega_t)$ and investor wealth $\Omega_{t+1}(E_t, \Omega_t)$.

4.1 Intraperiod Equilibrium

Consider a period t with beginning-of-period capital allocation (E_t, Ω_t). The firms' profit maximization problems given in (1) yield the standard first-order conditions for interest rates and wages

$$r^j(K_t^j) = \alpha z^j A \left(\frac{K_t^j}{L^j}\right)^{\alpha-1}, \quad j \in \{M, I\} \tag{5}$$

$$w^j(K_t^j) = (1-\alpha) z^j A \left(\frac{K_t^j}{L^j}\right)^{\alpha}, \quad j \in \{M, I\}, \tag{6}$$

where we already imposed market clearing on the labor market, i.e. $L_t^M = L^M$ and $L_t^I = L^I$. We distinguish two cases: first, the case when financial frictions are irrelevant (non-binding) and, second, the case when financial frictions are relevant (binding).

4.1.1 Irrelevant Financial Frictions

Suppose equity is relatively abundant, i.e. $(E_t, \Omega_t) \in \mathbb{R}^2_+ \setminus \Gamma$. Bankers hold sufficient loanable funds, such that production in sector I is not limited by loan supply. In this case, financial frictions are irrelevant and competitive capital markets align the interest rates of both sectors. Defining $z := (\frac{z^I}{z^M})^{\frac{1}{1-\alpha}}$ and $\ell := \frac{L^I}{L^M}$, condition $r^I(K_t^I) = r^M(K_t^M)$ yields $K_t^I = z\ell K_t^M$. In combination with the aggregate resource constraint, we obtain

$$K_t^{M*} = \frac{\Omega_t + E_t}{1 + z\ell} = \frac{1}{1 + z\ell} K_t,$$
$$K_t^{I*} = z\ell \frac{\Omega_t + E_t}{1 + z\ell} = \frac{z\ell}{1 + z\ell} K_t.$$

Incentive compatibility of the deposit contract requires that net earnings $(1 + r_t^M)E_t$ of the banker are at least as large as the non-pledgeable part of revenues θK_t^I. Therefore,

$$E_t \geq \frac{\theta K_t^{I*}}{(1 + r^M(K_t^{M*}))} = \theta \frac{z\ell}{(1 + z\ell)(1 + r^M(K_t^{M*}))} K_t := \overline{E}(K_t), \tag{7}$$

where $\overline{E}(K_t)$ denotes the lower bound of bank equity that makes financial frictions irrelevant given the overall capital $K_t = E_t + \Omega_t$ in the economy. Condition $E_t \geq \overline{E}(K_t)$ is an implicit characterization of the partition $(E_t, \Omega_t) \in \mathbb{R}_+^2 \setminus \Gamma$ of the state space.

4.1.2 Relevant Financial Frictions

Suppose equity is relatively scarce, i.e. $(E_t, \Omega_t) \in \Gamma$. Incentive compatibility of the deposit contract limits the amount of loanable funds, such that production in sector I is limited by a shortage in loan supply. In this case, financial frictions are relevant. Rewriting the market-imposed leverage condition (3) at the aggregate level as $\lambda_t(r^M(K_t^M) - r^I(K_t^I) + \theta)K_t^I - (1 + r^M(K_t^M))E_t = 0$, and defining the left hand side as auxiliary function $\varphi(\lambda_t)$ yields

$$\varphi(\lambda_t) := r^M(\Omega_t + E_t - \lambda_t E_t)(\lambda_t - 1) - r^I(\lambda_t E_t)\lambda_t + \lambda_t \theta - 1 = 0. \tag{8}$$

Note that for any given $(E_t, \Omega_t) \in \Gamma$, condition (8) is one equation in one unknown: λ_t^*.

The function $\varphi(\lambda_t)$ is continuous and strictly monotonically increasing. Because financial frictions are relevant, the interest rate in sector I exceeds the interest rate in sector M, which implies $K_t^I < z\ell K_t^M$ and $K_t^I = \lambda_t E_t$. In combination with the aggregate resource constraint, $\Omega_t + E_t = K_t^M + K_t^I$, these conditions yield an upper bound for market-imposed bank leverage, $\frac{z\ell}{1+z\ell}\frac{K_t}{E_t} \geq \lambda_t$. Thus, $\lambda_t \in [1, \frac{z\ell}{1+z\ell}\frac{K_t}{E_t}]$. Evaluating $\varphi(\lambda_t)$ at $\lambda_t = 1$ gives $\varphi(1) = -(1 + r_t^I - \theta) < 0$. At the upper bound of the interval, financial frictions cease to be binding and interest rates are equal. In this case,

$$\varphi\left(\frac{z\ell}{1 + z\ell}\frac{K_t}{E_t}\right) = \frac{z\ell}{1 + z\ell}\frac{K_t}{E_t}\theta - \left(1 + r^M\left(\frac{K_t}{1 + z\ell}\right)\right).$$

Note that $\varphi(\frac{z\ell}{1+z\ell}\frac{K_t}{E_t})$ is decreasing in E_t and attains zero when $E_t = \overline{E}(K_t)$. Because financial frictions are relevant, i.e. $E_t < \overline{E}(K_t)$, we obtain $\varphi(\frac{z\ell}{1+z\ell}\frac{K_t}{E_t}) > 0$. Therefore, by the intermediate value theorem, there exists a unique $\lambda_t^* \in [1, \frac{z\ell}{1+z\ell}\frac{K_t}{E_t}]$ satisfying $\varphi(\lambda_t^*) = 0$.

For market-imposed bank leverage λ_t^*, equilibrium factor allocations are then given by

$$K_t^{M*} = \Omega_t + E_t - \lambda_t^* E_t$$
$$K_t^{I*} = \lambda_t^* E_t.$$

4.1.3 Existence and Uniqueness of Intraperiod Equilibrium

We summarize both cases in the following proposition.

Proposition 1 (Intraperiod Equilibrium: Factor Allocation)**.**
For all pairs (E_t, K_t), there exists a unique intraperiod equilibrium.

(i) If $E_t \geq \overline{E}(K_t)$, i.e. $(E_t, \Omega_t) \in \mathbb{R}_+^2 \setminus \Gamma$, financial frictions are irrelevant. The capital allocation is given by $K_t^{M*} = \frac{1}{1+z\ell}K_t$ and $K_t^{I*} = \frac{z\ell}{1+z\ell}K_t$.

(ii) If $E_t < \overline{E}(K_t)$, i.e. $(E_t, \Omega_t) \in \Gamma$, financial frictions are relevant. Market-imposed bank leverage λ_t^* is the unique solution of $\varphi(\lambda^*) = 0$ and the capital allocation is given by $K_t^{M*} = \Omega_t + E_t - \lambda_t^* E_t$ and $K_t^{I*} = \lambda_t^* E_t$.

Proof. The proof directly follows from the preceding discussion. □

4.1.4 Comparative Statics when Financial Frictions are Relevant

We now examine the comparative statics of bank leverage, bond finance, loan finance, and output with respect to changes in productivities, investor wealth, bank equity, and financial friction.[17] We summarize the comparative statics in Table 1 and discuss the main insights. A more detailed account of the comparative statics and the underlying economic mechanisms is delegated to Appendix A.1.

Given $K_t^I = \lambda_t E_t$ and $K_t^M = E_t + \Omega_t - \lambda_t E_t$, the main intuition for bank leverage adjustment in response to productivity and equity shocks can be derived from comparing the profits of a single bank if it complies with the deposit contract, $(1 + r_t^I)k_t^I - (1 + r_t^M)d_t$, to the profits of defecting, θk_t^I. First, an increase in common total factor productivity, *ceteris paribus*, raises both, the revenues from investing into sector I, $(1 + r_t^I)k_t^I$, and the repayment obligation that arises from the deposit contract, $(1 + r_t^M)d_t$. As $r_t^I > r_t^M$, the effect on revenues dominates the effect on repayment obligations to depositors and investors can therefore increase their deposits without violating the incentive compatibility of the deposit contract. Hence, market-imposed leverage increases. Second, an increase in aggregate bank equity E_t *ceteris paribus* raises both, bond finance and loan finance. Interest rates fall in both sectors and, thus, the bank's revenues

[17] There is a variety of interpretations why productivities, investor wealth, bank equity, and financial frictions may change. For instance, investor wealth and bank equity may change over time as a result of the accumulation process or savings decisions of investors or bankers may be affected by a preference shock causing a lower capital stocks through higher consumption. Other sources are unexpected shocks to the return on investors' wealth or bank equity.

Table 1: Comparative Statics

	leverage λ	loans K^I	bonds K^M	output Y
common total factor productivity	+	+	−	+
investor wealth	+	+	+	+
bank equity	−	+	−	+
financial friction	−	−	+	−

from investing into sector I and the repayment obligation to depositors decrease. Because loan finance is more elastic to changes in the equity stock than bond finance,[18] the effect on revenues dominates the effect on the repayment obligation, such that profits from complying with the deposit contract decline. As a result, investors have to reduce their deposits in order to restore incentive compatibility – market-imposed bank leverage decreases.

The responses of leverage, bond finance, and loan finance to downturns resulting from a negative shock to common productivity, a decline in bank equity capital, or a worsening of financial frictions – or any combination thereof – are consistent with two empirical facts: First, book leverage in the banking sector is procyclical and, second, loan finance is procyclical and bond finance is countercyclical. Note that procyclicality of bank leverage is also consistent with our empirical findings for the US (see Section 8.2).[19]

4.2 Intertemporal Consumption-Saving Decisions

Because bankers and investors have logarithmic utility and their disposable income is linear homogeneous in wealth, their consumption-saving policies are linear homogeneous in wealth, too. In fact, bankers and investors save a constant fraction of their end-of-period net worth.

Proposition 2 (Intertemporal Equilibrium: Consumption and Saving)**.**
Given $(E_t, \Omega_t) \in \mathbb{R}^2_+$, the representative banker's and representative investor's consumption-saving policies are linear homogenous in end-of-period net worth. Specifically, it holds that:

[18]The elasticity of loan finance with respect to equity is $\frac{\partial K_t^I}{\partial E_t}\frac{E_t}{K_t^I} = 1$, whereas the elasticity of bond finance with respect to equity is $\frac{\partial K_t^M}{\partial E_t}\frac{E_t}{K_t^M} = \frac{(\lambda_t - 1)E_t}{\Omega_t + (\lambda_t - 1)E_t} < 1$.

[19]This is well documented for the US, e.g., Adrian and Shin (2014), Adrian and Boyarchenko (2012), Adrian and Boyarchenko (2013), and Nuño and Thomas (2012). While our empirical findings confirm procyclicality of loan finance for the US, we rely on De Fiore and Uhlig (2011) and De Fiore and Uhlig (2015) who additional provide evidence for countercyclical bond finance and the ensuing countercyclical bond-to-loan finance ratio.

(i) The consumption-saving policy functions

$$\begin{aligned} C_t^B &= (1-\beta_B)(1+r^B(E_t,\Omega_t)-\delta^E)E_t \\ E_{t+1} &= \beta_B(1+r^B(E_t,\Omega_t)-\delta^E)E_t \end{aligned}$$

solve the representative banker's utility maximization problem (4) *where* $r^B(E_t,\Omega_t)$ *is the (net) return on equity in period* t *given by*

$$r_t^B(E_t,\Omega_t) = \begin{cases} \theta\lambda_t(E_t,\Omega_t)-1, & \text{if } (E_t,\Omega_t)\in\Gamma \\ r^M(E_t,\Omega_t), & \text{if } (E_t,\Omega_t)\in\mathbb{R}_+^2\setminus\Gamma. \end{cases}$$

(ii) The consumption-saving policy functions

$$\begin{aligned} C_t^H &= (1-\beta_H)(1+r^M(E_t,\Omega_t)-\delta)\Omega_t \\ \Omega_{t+1} &= \beta_H(1+r^M(E_t,\Omega_t)-\delta)\Omega_t \end{aligned}$$

solve the representative investor's utility maximization problem (2)*.*

Proof. See Appendix A.2. □

Using Condition (8) to rewrite the (net) return on equity for the case in which frictions are binding,

$$\begin{aligned} r^B(E_t,\Omega_t) &= \theta\lambda(E_t,\Omega_t)-1 \\ &= r^M(E_t,\Omega_t)+\lambda(E_t,\Omega_t)(r^I(E_t,\Omega_t)-r^M(E_t,\Omega_t)), \end{aligned}$$

reveals that banks benefit from the interest rate spread and from higher bank leverage.

5 Long-Run Dynamics

This section discuss the long-run economic dynamics that will also underly the calibration of the model for the quantitative analysis in Section 8. For the remainder of this paper, we make the following two assumptions:

Assumption 1 (Time Preferences)**.**
Bankers are more impatient than investors, i.e. $\beta_B<\beta_H$ *or* $\rho_B>\rho_H$*.*

Assumption 2 (Bank Equity Depreciation)**.**
The long-run depreciation rates of investor wealth and bank equity capital are identical, $\delta^E=\delta$*.*

The assumption on preferences guarantees that bank equity capital is sufficiently scarce in the long-run, i.e financial frictions remain relevant. The opposite assumption would be strongly counterfactual given the experience with very low levels of bank equity capital over the last decades.[20]

5.1 Steady State

In a steady state, allocations and prices are constant across time. Suppose that the economy is in a steady state in which financial frictions are relevant. Setting $E_{t+1} = E_t$ and $\Omega_{t+1} = \Omega_t$, the saving policies in Proposition 2 yield

$$\hat{r}^M = \delta + \rho_H \tag{9}$$

$$\hat{\lambda} = \frac{\delta + \rho_B + 1}{\theta}. \tag{10}$$

where $\hat{x}$ denotes the steady state value of variable x. Combining the definition of bank leverage, condition (3), with conditions (9) and (10) yields

$$\hat{r}^I = \hat{r}^M + \frac{\theta(\rho_B - \rho_H)}{1 + \delta + \rho_B} = \delta + \rho_H + \frac{\theta(\rho_B - \rho_H)}{1 + \delta + \rho_B}. \tag{11}$$

Because $\rho_B > \rho_H$, the interest rates satisfy $\hat{r}^I > \hat{r}^M$, which is consistent with the presupposition of binding financial frictions. Given $\hat{r}^I$ and $\hat{r}^M$, the steady state factor and wealth allocations compute as

$$\hat{K}^M = \left(\frac{\alpha z^M A}{\hat{r}^M}\right)^{\frac{1}{1-\alpha}} L^M, \tag{12}$$

$$\hat{K}^I = \left(\frac{\alpha z^I A}{\hat{r}^I}\right)^{\frac{1}{1-\alpha}} L^I, \tag{13}$$

$$\hat{E} = \left(\frac{\alpha z^I A}{\hat{r}^I}\right)^{\frac{1}{1-\alpha}} \frac{\theta}{1 + \delta + \rho_B} L^I, \tag{14}$$

$$\hat{\Omega} = \hat{K}_t^M + \hat{K}_t^I - \hat{E}. \tag{15}$$

So far we have assumed that financial frictions matter at the steady state. We next show that there does not exist a steady state in which financial frictions are irrelevant. Suppose that at the steady state, financial frictions are irrelevant, i.e. $(E_t, \Omega_t) \in \mathbb{R}_+^2 \setminus \Gamma$. According to Proposition 2, capital accumulation is governed by $E_{t+1} = \beta_B(1 + r_t^M - \delta)E_t$ and $\Omega_{t+1} = \beta_H(1 + r_t^M - \delta)\Omega_t$. By Assumption 1, $\beta_B < \beta_H$. We note that first, if $\Omega_{t+1} = \Omega_t$, bank equity decreases and, second, if $E_{t+1} = E_t$, investor wealth increases. Taken together, this contradicts the presupposition that there is a steady state in which financial frictions are irrelevant. We thus obtain

Proposition 3 (Existence and Uniqueness of the Steady State).
There exists a unique steady state $(\hat{E}, \hat{\Omega})$ *in which financial frictions are binding. The steady state allocation is given by conditions* (9) *to* (15).

[20]The opposite assumption would imply that bankers own the entire wealth of the economy in the long-run.

Proof. The proof directly follows from the preceding discussion. ☐

5.2 Global Stability

In terms of global dynamics, we prove the following proposition:

Proposition 4 (Global Stability of the Steady State with Financial Frictions)**.**
For any initial $(E, \Omega) \in \mathbb{R}^2_+$*, the economy converges to the unique steady state. Financial frictions always matter for* t *sufficiently large.*

Proof. See Appendix B.1. ☐

The proof is delegated to Appendix B.1 and proceeds in two steps. In the first step we show that for any $(E_0, \Omega_0) \in \mathbb{R}^2_+ \setminus \Gamma$, that is for any wealth allocation for which financial frictions are irrelevant, the wealth allocation converges towards the boundary of the partition of the state space $(E_\tau, \Omega_\tau) \in \Gamma$ for which financial frictions matter in finite time τ. In the second step we show that for any $(E_\tau, \Omega_\tau) \in \Gamma$, the wealth allocation converges towards to the unique steady state. Details are delegated to Appendix B.1.

5.3 Permanent Shocks to Productivity and Financial Frictions

Analyzing the long-run impact of permanent shocks to the common total factor productivity and the financial friction yields two insights: First, an increase in total factor productivity shifts steady state factor and wealth allocations proportionally up, but leaves the steady state bond-to-loan finance ratio and the steady state bank leverage unaffected. Hence, the economy will never grow out of financial frictions. For the transition phase, the increase in common total factor productivity is accompanied by an increase in bank leverage, loan finance, and a decrease in bond finance, i.e. short run relative re-allocations from investors and the bond financed sector towards bankers and the bank financed sector.

Second, an increase of the intensity of financial friction θ lowers the steady state level of capital $\hat{K}$ and raises the steady state level of bank equity $\hat{E}$ if bankers are not too impatient. In particular, equity overshoots during the whole transition towards the new steady state. Because of the consumption saving policies characterized in Proposition 2, this effect on equity accumulation paths implies that the banker achieves higher consumption levels throughout the transition as well as in the new steady state such that bankers unambiguously benefit from the increase in the financial friction. The opposite holds for investors such that the increase in the financial friction finally leads to re-allocation of resources from investors to bankers. Details are delegated to Appendices B.2 and B.3.

6 Short-Run Dynamics and the Sensitivity of Bank Leverage

In this section, we provide new insights into the propagation of shocks and show that the elasticity of bank leverage with respect to common total factor productivity and the elasticity of bank leverage with respect to bank equity are essential factors for the resilience of the economy to adverse shocks. We focus on the interesting and plausible case when equity is relatively scarce and, thus, financial frictions are binding, i.e. $(E_t, \Omega_t) \in \Gamma$.

First, consider a temporary negative shock to common total factor productivity A, which we call *regular recession*. The main insights are summarized in Proposition 5:

Proposition 5 (Productivity Shocks and Short-Run Dynamics)**.**
After a negative shock to total factor productivity (regular recession),

(i) bank leverage decreases,

(ii) capital is re-allocated from sector I to sector M, and

(iii) total output declines. The decline in output is stronger, the more sensitive bank leverage reacts to changes in total factor productivity (amplification).

Proof. See Appendix C.1. □

The intuition for these results is as follows: The decrease in common total factor productivity A leads, *ceteris paribus*, to a proportional decrease in the loan and deposit rate. As a result, the banker's profits from the deposit contract, $(1+r_t^I)K_t^I - (1+r_t^M)(K_t^I - E_t)$, decrease which makes defecting and earning θK_t^I, instead, more attractive.[21] Investors therefore reduce their supply of funds to bankers to restore incentive compatibility of the deposit contract: equilibrium bank leverage declines. The ensuing factor re-allocation to the less capital efficient sector M implies, that the output decline is disproportionately higher than the decline in total factor productivity itself. This is called *amplification*. Hence, for *regular recessions*, the endogenous response of bank leverage leads to amplification of productivity shocks and the amplification is stronger, the more sensitive bank leverage is.

Second, consider a negative shock to bank equity E_t, which we call *banking crisis*. The main insights are summarized in Proposition 6:

Proposition 6 (Equity Shocks and Short-Run Dynamics)**.**
After a negative shock to bank equity (banking crisis),

(i) bank leverage increases,

(ii) investment decreases in sector I and increases in sector M, and

[21] Note that with Cobb-Douglas production technologies, the derivative of $(1+r_t^I)K_t^I - (1+r_t^M)(K_t^I - E_t)$ with respect to A can be rewritten as $(r_t^I K_t^I - r_t^M(K_t^I - E_t))/A > 0$. Hence, the decline in revenues always dominates the decline in repayment obligations.

(iii) total output declines. The decline in output is weaker, the more sensitivity leverage reacts to changes in bank equity (stabilization).

Proof. See Appendix C.2. ☐

The intuition for these results is as follows: The decrease in bank equity capital leads, *ceteris paribus*, to a decrease in household's deposits, scaled by bank leverage, and, thus, to a decrease in loan supply to sector I. Due to the decline in loans, the marginal product of capital in sector I increases. As a result, the banker's profits from the deposit contract, $(1+r_t^I)K_t^I - (1+r_t^M)(K_t^I - E_t)$, increase, which makes the financial friction less tight as defecting and earning θK_t^I instead gets less attractive. Therefore, investors partially compensate the initial reduction in deposits and bank leverage increases. The increase in bank leverage, in turn, partially offsets the impact of the initial decline in bank equity on the amount of loans, which buffers the resource re-allocation to the less capital efficient sector M and, thus, buffers the decline in total output. This offsetting effect is called *stabilization*.

Note that the adjustment in bank leverage also increases the bank equity accumulation rate $\frac{E_{t+1}-E_t}{E_t} = \beta_B(\theta\lambda_t - \delta) - 1$ (see Proposition 2) thereby reducing the persistent and potentially costly *bank lending channel*. For *banking crises*, i.e. recessions due to a sudden decline in bank equity capital, the endogenous adjustment of bank leverage leads to an instantaneous and dynamic stabilization of output. The stabilization is stronger, the more sensitive bank leverage responds to the decline in bank equity capital.

In sum, while the level of bank leverage is a key amplifier of adverse shocks, the elasticity of leverage with respect to common total factor productivity is a de-stabilizer in *regular recessions*, and the elasticity of leverage with respect to bank equity capital is a stabilizer in *banking crises*.

7 Managing Recoveries

Despite the stabilization effect of bank leverage, our quantitative exercise in Section 8 indicates that banking crises can be quite costly in terms of welfare. In this section, we thus examine how to manage recoveries from banking crises with two standard policy instruments: dividend payout restrictions and capital injections into banks. Both instruments have been used extensively during and after the financial and banking crisis 2008/2009 to speed up economic recovery and to redistribute the gains of the policy between workers, investors, and bankers.

In this paper, we limit the analysis to shocks that lead to a temporary decline in bank equity capital under *laissez faire*, e.g. an unexpected charge-off on bank loans that is equivalent to an additional depreciation of bank equity capital for certain periods. Hence, the depreciation rate of bank capital is $\delta_t^E > \delta$ for some period t. We focus on a policy that – from an *ex-post* perspective – stimulates economic recovery and consists of a sequence of investor financed capital injections $\{Tr_t\}_{t=0}^{\infty}$ into the banking sector and a sequence of dividend payout restrictions $\{d_t\}_{t=0}^{\infty}$ for banks. While capital injections contribute to weaken the *bank lending channel* by reducing the initial amplification of shocks on bank balance sheets, dividend payout restrictions serve three purposes:

First, they accelerate the accumulation of bank equity capital as bankers are forced to save a higher share of their end-of-period net-worth. Second, dividend payout restrictions contribute to higher utility of workers and investors since the accelerated recovery leads to an increase in the marginal productivity of labor and capital. Third, by choosing the dividend payout restrictions such that the consumption paths of bankers under *laissez faire* and the proposed policy are identical, bankers are indifferent between both consumption paths which excludes excessive consumption in the expectation of future bailouts.[22] Specifically, for $d_t > \beta_B$ set appropriately and given Tr_t, the banker's consumption-saving decisions reads

$$\begin{aligned} E_{t+1} &= d_t((\theta_t\lambda_t - \delta_t^E)E_t + Tr_t) \geq \beta_B(\theta_t\lambda_t - \delta_t^E)E_t, \\ C_{t+1}^B &= (1-d_t)((\theta_t\lambda_t - \delta_t^E)E_t + Tr_t) = \beta_B(\theta_t\lambda_t - \delta_t^E)E_t. \end{aligned}$$

In contrast to *laissez faire*, we refer to this policy package as *balanced bailout* to indicate balanced incentives for banks.

In Proposition 7, we show that the total capital stock, K_t, and capital employed in sector I, K_t^I, under *balanced bailout* exceed their *laissez faire* values in all periods. As a result, total output will be always higher under *balanced bailout*. Furthermore, this implies that total wage income of workers is always higher than under *laissez faire*, as well, such that workers unambiguously benefit from such a scheme. For investors, the result is ambiguous: although benefiting from faster recoveries with higher returns, investors suffer from financing the capital injections to the banking sector. Which effect finally dominates depends on the specific calibration and the strength of the *bank lending channel*. Investors can benefit from *balanced bailout* if bank equity after the shocks is small and, thus, leverage is high. Then, bank equity shocks lead to high reductions in loan supply and high output losses which leads to a strong *bank lending channel* that causes deep recessions with persistent capital re-allocations towards the less efficient production sector. Policies to prevent the *bank lending channel* to unfold – like the investor financed capital injections – may be welfare improving even for investors who finance the capital injection in first place.

Proposition 7 (Dividend Payout Restrictions and Capital Injections)**.**
Suppose there is a shock that leads to a temporary decline in bank equity capital in period 0 *with* $1 - \delta_1^E > \beta_H(1-\delta)$. *Then, there exists a feasible sequence of transfer payments from investors to banks,* $\{Tr_t\}_{t=0}^{\infty}$, *and associated sequence of dividend payout restrictions,* $\{d_t\}_{t=0}^{\infty}$ *with the following properties:*

(i) Total capital K_t *and total output* Y_t *exceed their respective laissez faire values in all periods.*

[22] Policies that would make bankers better off in the accelerated recovery than without policy interventions may introduce moral hazard. Bankers may have an incentive to pay out more dividends, thereby consuming more, in order to cause a negative bank equity shock and a bailout. However, there are no incentives for such a behavior at the individual level as banks can trigger a bailout only collectively.

(ii) Lifetime-utility of bankers is constant by construction and lifetime-utility of workers increases. The impact on lifetime-utility of investors is ambiguous.

Proof. See Appendix D. □

A couple of remarks are at order. First, the condition $1 - \delta_1^E > \beta_H(1 - \delta)$, is a sufficient condition and means that the negative bank equity shock is not too extreme. Even if the condition is is violated, the policy package still may accelerate economic recovery, which can be verified numerically. Second, Proposition 7can be extended to any finite sequence of bank equity shocks. Third, with further policy instruments such as consumption taxes or investment subsidies, the gains from faster recoveries can be distributed in a way that a *balanced bailout* even leads to a Pareto improvement.

8 Quantitative Analysis

This section provides a quantitative assessment of the theoretical results that have been derived in Sections 4, to 7. Specifically, we calibrate the model to the US economy and quantify the impact of shocks to productivity, bank equity capital, and the financial friction on output, welfare, and the speed of recovery. For this purpose, we extend the model along two dimensions. First, we introduce uncertainty about common total factor productivity, the depreciation rate of bank equity capital, and the financial friction. Second, we impose a regulatory limit on bank leverage that may become binding outside the steady state since capital requirements were not binding before the Great Recession. The main insights from the previous sections are, *mutatis mutandis*, unaffected by theses extensions as long as the regulatory limit on leverage does not bind. A binding regulatory limit on leverage would limit the stabilization via endogenous leverage adjustments in a banking crisis.

8.1 Stochastic Process and Regulatory Limit on Leverage

For the quantitative analysis, we assume that common total factor productivity, the depreciation rate of bank equity capital, and the financial friction are stochastic and we denote the period t realizations of the respective parameters by A_t, δ_t^B, and θ_t. The associated stochastic processes are specified as the deviations from mean values $\ln(A)$, δ^E, and $\ln(\theta)$.[23] Specifically, we assume that $\ln(A_t) = \ln(A) + \eta_{A,t}$, $\delta_t^E = \delta^E + \eta_{\delta,t}$, and $\ln(\theta_t) = \ln(\theta) + \eta_{\theta,t}$ where the 3-by-1 vector of deviations, $\eta_t = [\eta_{A,t}, \eta_{\delta,t}, \eta_{\theta,t}]'$ follows a VAR(1) process

$$\eta_t = P_\eta \eta_{t-1} + \epsilon_t. \tag{16}$$

Here, P_η denotes the diagonal 3-by-3 coefficient matrix and $\epsilon_t = [\epsilon_{A,t}, \epsilon_{\delta,t}, \epsilon_{\theta,t}]'$ denotes a 3-by-1 multi-normally distributed random vector with mean 0 and variance covariance matrix Σ_η.

[23] Note that the we specify the stochastic process of depreciation rate of bank equity capital in levels because its relative deviations in the quantitative analysis (Section 8) exceed 200 percent.

Three remarks are in order. First, the coefficients of P_η measure the persistence of shocks and the diagonal elements of P_η are below unity to guarantee stability of the process. Second, shocks to bank equity depreciation represent, for instance, unexpected defaults of loans net of recoveries of previously charged-off loans. Third, shocks to financial frictions reflect shocks to trust in the repayment willingness of bankers or can be associated with unexpected changes in regulatory schemes, uncertainty, and transparency on financial markets.

For the quantitative analysis, we further assume that there is a regulatory upper limit on bank leverage $\lambda^{reg} > \lambda^*$. Hence, the resulting leverage in the economy is now given by $\min\{\lambda_t, \lambda^{reg}\}$ where λ_t is the market-imposed leverage constraint that solves condition (8).

8.2 Calibration

The calibration strategy proceeds in two steps: First, we calibrate the time-invariant parameters – including the means of the time-variant parameters – to match the steady state to long-run stylized facts of the US economy. Second, we estimate the joint stochastic process of the common total factor productivity, the depreciation of bank equity capital, and the financial friction and impose it on the steady state calibration obtained in the first calibration step. The model is calibrated to quarterly frequency based on macroeconomic time series and microeconomic bank level data from 1991Q1 to 2017Q4. Data on real activity is taken from the Federal Reserve Economic Data (FRED), the Penn World Table (PWT), and the online update to Fernald (2012). Data on the financial sector are compiled from the quarterly bank level Reports of Condition and Income (Call) and from De Fiore and Uhlig (2011).[24]

Excluding the parameters that characterize the stochastic process, there are ten model parameters to be calibrated on steady state conditions in the first step: the production parameters α, A, z^M, and z^I, the capital depreciation rate δ, the time preference factors β_H and β_B, the financial friction θ, and the amount of labor L and its allocation across sectors l. To pin down these parameter values, we impose six calibration targets derived from US time series data, three normalizations, and one assumption that comes from long-run restrictions on endogenous variables. Table 2 provides a summary of the calibration targets, the data sources, and the calibrated parameter values.

The calibration targets are the mean values of the respective seasonally adjusted time series variables from 1991Q1 to 2017Q4. The first three targets – the national saving rate $\overline{s} = 0.1801$, the capital-to-output ratio $\overline{K/Y} = 12.3763$, and the labor share of income $\overline{wL/Y} = 0.6516$ – are standard in the literature and directly compiled from FRED, PWT, and the online data update to Fernald (2012), respectively. The financial calibration targets are derived from the Call Reports which contain detailed information on bank level balance sheets and income statements on a quarterly basis. In this paper, we restrict ourselves to commercial banks.[25] The data

[24] Call Report data are provided by different sources: 1991Q1 to 2000Q4, Chicago FED, and 2001Q1 to 2017Q4, Federal Financial Institutions Examinations Council (FFIEC). Data prior to 1991 suffer from availability and consistency problems for some of the variables under consideration. Details on the data and the construction of consistent time series and bank balance sheets is deferred to Appendix E.1.

[25] For the sample selection, we follow a similar line of argument as den Haan et al. (2007).

Table 2: Calibration – Targets and Parameters

	variable	description	source	value
calibration targets	$\overline{s}$	aggregate saving rate	FRED	0.1801
	$\overline{K/Y}$	capital-to-output ratio	PWT	12.3763
	$\overline{wL/Y}$	labor share of income	Fernald (2012)	0.6516
	$\overline{\lambda}$	bank leverage	Call	10.7808
	$\overline{r}^B$	(net) return on bank equity	Call	0.0276
	$\overline{K^I/K^M}$	loan-to-bond-finance ratio	De Fiore and Uhlig (2011)	0.6667
parameters	α	output elasticity of capital		0.3484
	A	average total factor productivity (normalization)		1.0000
	z^M	productivity in sector M (normalization)		1.0000
	z^I	productivity in sector I		1.0168
	δ	depreciation rate of investor wealth		0.0146
	β_H	time preferences of investors		0.9871
	β_B	time preferences of bankers		0.9731
	θ	average financial friction		0.0967
	L	total labor endowment (normalization)		1.0000
	ℓ	relative labor allocation to sector M		0.5885
shock process	P_η	AR(1)-coefficient matrix of shocks		$\begin{pmatrix} +0.8850 & 0 & 0 \\ 0 & +0.9527 & 0 \\ 0 & 0 & +0.8815 \end{pmatrix}$
	Σ_η	variance-covariance matrix of shocks		$\begin{pmatrix} +0.0015 & -0.0000 & -0.0000 \\ -0.0000 & +0.0016 & +0.0000 \\ -0.0000 & +0.0000 & +0.0024 \end{pmatrix}$

are cross-sectionally aggregated and seasonally adjusted using the Census X-13ARIMA. The fourth and fifth calibration targets are average bank leverage $\overline{\lambda} = 10.7808$ and average (net) return on bank equity $\overline{r}^B = 0.0276$ computed as the asset-to-equity ratio and the net-income-to-equity ratio, respectively. The sixth calibration target is the average bond-to-loan finance ratio $\overline{K^M/K^I} = 1.5000$ obtained from De Fiore and Uhlig (2011).

The three normalizations can be done without loss of generality, as they neither affect steady state leverage nor returns on bonds, deposits, and loans. Therefore, we set $L = 1.0000$, $z^M = 1.0000$, and $A = 1.0000$. The assumption on long-run variables imposes wage equality in the long-run, i.e. $\hat{w}^M = \hat{w}^I$, which reflects intergenerational mobility between sectors.

In order to complete the calibration, we next derive the time series for A_t, δ_t^E, and θ_t, and estimate the underlying stochastic process (see condition (16)). The time series of total factor productivity is taken from the online data update to Fernald (2012). The time series of the equity depreciation rate is more involved, since it requires additional assumptions on the bank balance sheet. Specifically, we assume that we can split the bank balance sheet into two balance sheets with similar bank leverage. The first balance sheet contains only loans on the asset side and imputes a deposit-to-equity share on the liability side consistent with total bank leverage. The second balance sheet contains all other balance sheet items and reflects all activities of financial intermediaries that are absent from our model. Supposing that the mix of loans and other assets is stable, we can calculate how losses on loans impact bank equity. In particular, net loan charge-off, i.e. loan charge-offs net of recoveries, corresponds to total equity charge-off. As a result, the net loan charge-off rate is the net equity charge-off rate, i.e. the equity depreciation rate, scaled by bank leverage. Finally, with the time series for the return on bank equity, the bank leverage, and the depreciation rate of bank equity capital, we obtain the respective time series of the financial friction using the condition for the (net) return on equity (see Proposition 2)

$$\theta_t = \frac{1 + r_t^B + \delta_t^E}{\tilde{\lambda}_t}$$

where $\tilde{\lambda}_t$ is the HP-filtered trend of bank leverage in period t.

Note that dividing by $\tilde{\lambda}_t$ instead of λ_t removes fluctuations in the financial friction that are generated by sudden changes in bank equity capital. Further details on the calibration are delegated to Appendix E.1.

Table 3: Correlation of Shocks and Leverage

	$\Delta \ln(A)$	$\Delta\delta^E$	$\Delta \ln(\theta)$	$\Delta \ln(Y)$	$\Delta \ln(\lambda)$	$\Delta \ln(K^I)$
$\Delta \ln(A)$	+1.0000	−0.2746 (0.0042)	+0.0295 (0.7630)	+0.6591 (0.0000)	−0.0939 (0.3360)	+0.1688 (0.0822)
$\Delta\delta^E$		+1.0000	+0.7946 (0.0000)	−0.4947 (0.0000)	+0.0436 (0.6556)	+0.2797 (0.0035)
$\Delta \ln(\theta)$			+1.0000	−0.2642 (0.0060)	−0.2491 (0.0097)	−0.1121 (0.2503)
$\Delta \ln(Y)$				+1.0000	+0.2073 (0.0321)	+0.4888 (0.0000)
$\Delta \ln(\lambda)$					+1.0000	+0.3943 (0.0000)

Note: Δx refers to the deviation of x from its HP-trend with smoothing parameter 1600. The numbers are temporary cross-correlations and the associated p-values are in parantheses.

After de-trending the time series using a HP-filter with smoothing parameter 1600, we compute the cross-correlation matrix (see Table 3) of key economic parameters and variables. Note that the empirical cyclical patterns are consistent with our notion of *regular recessions* and *banking* *crises* where the latter is characterized by a decrease in common total factor productivity, an increase in the depreciation rate of bank equity capital, and an increase in the financial friction. In particular, the cross-correlations show that bank leverage is (weakly) pro-cyclical and that loan finance is counter-cyclical which is consistent with the predictions of the model (see Section 4.1.4).

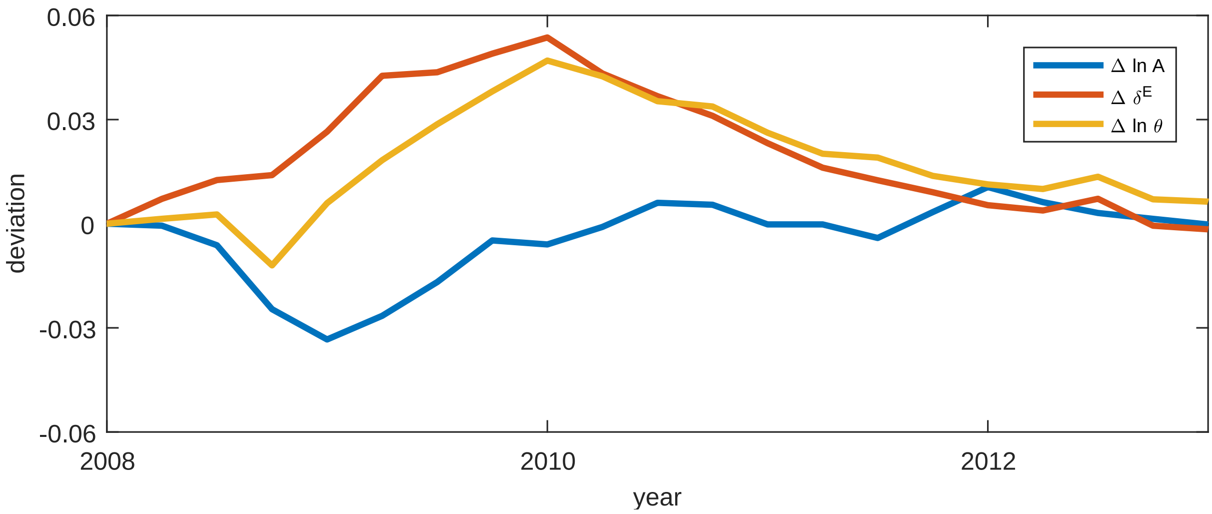
We use global solution methods to compute policy functions on the minimal relevant state space, i.e. bank equity capital, investor wealth, common total factor productivity, the depreciation rate of bank equity capital, and the financial friction. Details on the algorithm are deferred to Appendix E.2.

8.3 Business Cycles and the Great Recession

We now examine the business cycle implications of the model. Specifically, we plot the impulse response functions and compute output and welfare costs of the Great Recession.

In our analysis, we date the Great Recession from 2008Q1 to 2013Q4. The untypical long period of the Great Recession accounts for the fact that although common total factor productivity did recover in 2009, it was not until 2013 that the depreciation rate of bank equity capital and the financial friction returned to their pre-crisis values of 2008Q1. Thus, this dating of the Great Recession is consistent with our notion of *banking crises* as a decline in common total factor productivity, an increase in the depreciation rate of bank equity capital, and an increase in the financial friction. The deviations of these parameters from their pre-crisis levels 2008Q1 are depicted in Figure 3.

Figure 3: Great Recession – Shock Sequences



Note: Δx refers to the deviation of x from its HP-trend with smoothing parameter 1600. The deviations from trend are further normalized by their respective 2008Q1 value.

Figure 4 shows the model-induced impulse response functions for the Great Recession. We include further simulation results by, first, shutting down shocks to the financial friction and, second, additionally neglecting shocks to the depreciation rate of bank equity capital. We obtain three insights: First, short-run output dynamics are mainly driven by shocks to common total factor productivity. Shocks to the depreciation rate of bank equity capital, however, interfere with the long-run recovery of output and lead to persistent long-run output losses, because these shocks trigger the *bank lending cannel* and cause persistent re-allocation of wealth and production factors.[26]

Second, due to persistent re-allocation of wealth, there are significant consequences of shocks to the depreciation rate of bank equity capital for investors in terms of consumption and welfare. Thus, although these shocks originate in the financial sector, there are non-negligible spillovers to the non-financial sector.

Third, the impulse response functions are consistent with empirical patterns of bond and loan financing over the business cycle. At this stage, we limit ourselves to a descriptive account of the cyclicality of the bond-to-loan finance ratio conditional on the underlying shock sequences while a detailed discussion of the underlying economic mechanisms can be found in Section 4.1.4 and Appendix A.1.

We next quantify output and welfare costs of the Great Recession. *Regular recessions* are associated with a temporary decline in common total factor productivity, bank leverage is procyclical and the bond-to-loan finance ratio is countercyclical. If the recession is accompanied by a sharp decline in bank equity capital – our notion of a *banking crisis* – the countercyclicality of the bond-to-loan finance ratio is reinforced, while bank leverage becomes counter-cyclical. Finally, if the *banking crisis* additionally comes with an increase of the financial friction,[27] the countercyclicality of the bond-to-loan finance ratio is further reinforced.

In order to quantify the persistent distributional consequences of recessionary shocks, we compute output and welfare costs of the Great Recession.[28] Specifically, let $\{Y_t\}_{t=0}^{\infty}$ and $\{\overline{Y}_t\}_{t=0}^{\infty}$ denote the output paths in presence and absence of recessionary shocks, respectively. Define Δ_Y as the required adjustment in per period output such that the present discounted value of output paths $\{Y_t\}_{t=0}^{\infty}$ and $\{\overline{Y}_t\}_{t=0}^{\infty}$ are aligned, i.e.

$$\begin{aligned} &\sum_{t=0}^{\infty}(1+\Delta_Y)(1+r_{\infty}^M)^{-t}Y_t = \sum_{t=0}^{\infty}(1+r_{\infty}^M)^{-t}\overline{Y}_t \\ \Leftrightarrow \quad &\Delta_Y = \frac{\sum_{t=0}^{\infty}(1+r_{\infty}^M)^{-t}\overline{Y}_t}{\sum_{t=0}^{\infty}(1+r_{\infty}^M)^{-t}Y_t} - 1. \end{aligned}$$

Note that we used the long-run interest rate denoted by r_{∞}^M to discount future output.[29] Moreover, let $\{C_t^i\}$ and $\{\overline{C}_t^i\}$ denote the consumption path of agent $i \in \{H, W, B\}$ in the presence and absence of recessionary shocks, respectively. Define Δ^i as the required adjustment in per-period

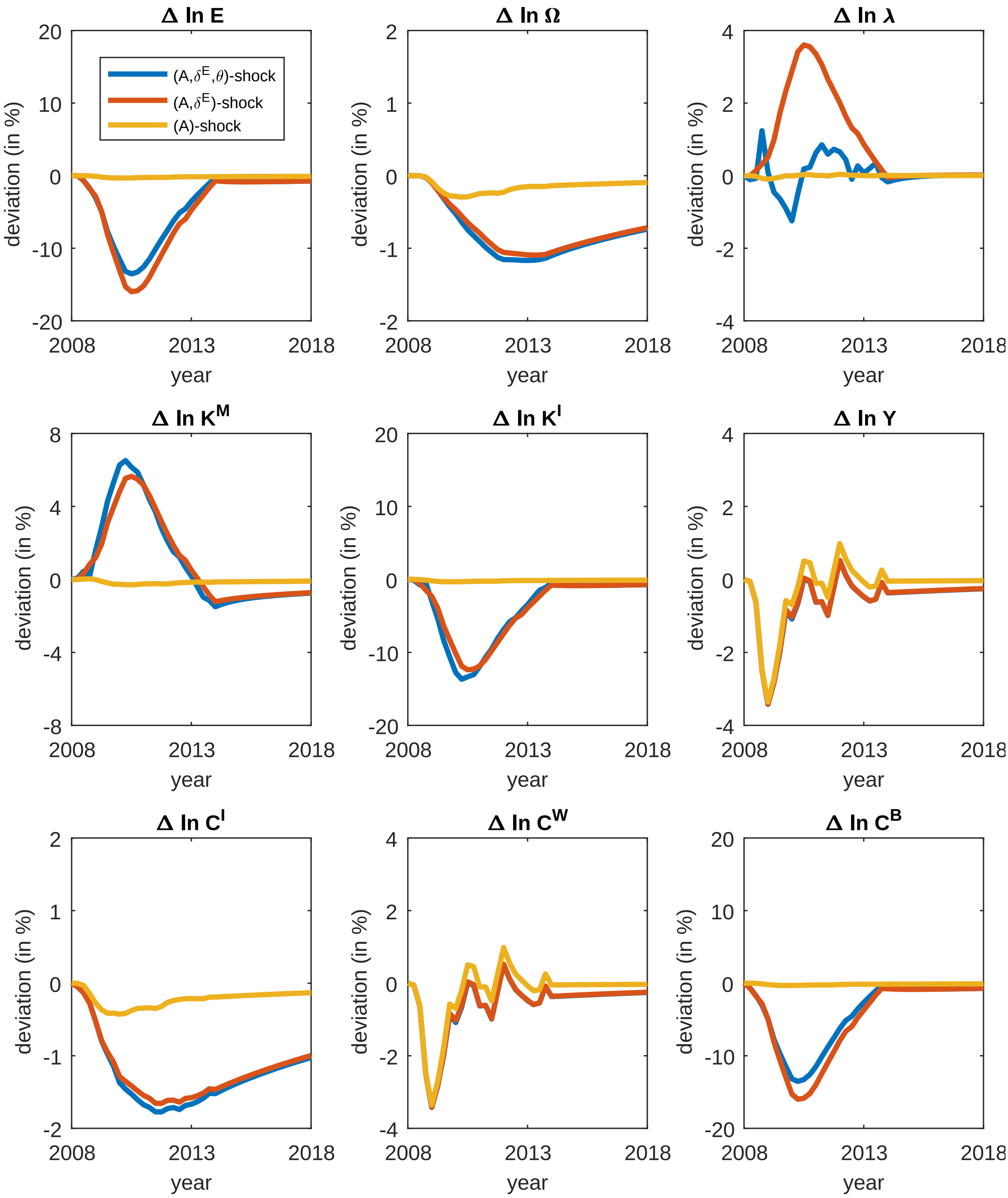
[26] See Section 4.1.4 and Appendix A.1 for a detailed discussion of the underlying economic mechanisms.

[27] According to Bloom et al. (2012), downturns are associated with a general increase of uncertainty, which could be interpreted as less trust in repayment pledges and, thus, larger financial friction θ in our context.

[28] See Lucas (1987) for detailed discussion of the general approach to measure welfare costs and the interpretation of the output and welfare measures.

[29] Alternatively, we could also use the respective paths of the interest rate to compute the discount kernel, i.e.

Figure 4: Great Recession – Impulse Response Functions



Note: Simulation results for (A, δ^E, θ)-shocks – Great Recession – (A, δ^E)-shocks, and (A)-shocks. The economy is initially in its steady state and regulatory leverage is sufficiently high to be non-binding throught the transition phase.

Δ_Y is the solution to

$$\sum_{t=0}^{\infty}(1+\Delta_Y)Y_t\prod_{\tau=0}^{t}(1+r_\tau^M)^{-1}=\sum_{t=0}^{\infty}\prod_{\tau=0}^{t}(1+\tilde{r}_\tau^M)^{-1}\overline{Y}_t.$$

consumption such that agent i is indifferent between the given consumption paths $\{C_t^i\}_{t=0}^\infty$ and $\{\overline{C}_t^i\}_{t=0}^\infty$, i.e.

$$\sum_{t=0}^\infty \beta_i^t \ln\Big((1+\Delta^i)C_t^i\Big) = \sum_{t=0}^\infty \beta_i^t \ln \overline{C}_t^i$$
$$\Leftrightarrow \quad \Delta^i = \exp\bigg((1-\beta_i)\Big(\sum_{t=0}^\infty \beta_i^t \ln \overline{C}_t^i - \sum_{t=0}^\infty \beta_i^t \ln C_t^i\Big)\bigg) - 1.$$

In contrast to Lucas (1987), we consider the cost of the Great Recession from an *ex-post* perspective, i.e. we use the model-induced output and consumption paths for the specific shock realization of the Great Recession, instead of an *ex-ante* perspective in which we would compute the expected values over complete contingent output and consumption paths.

Table 4 shows that both output and welfare costs of the Great Recession are substantial. The output cost of the Great Recession are equivalent to a permanent decrease in GDP of 0.5323 percent. Albeit short-run dynamics of output are, *inter alia*, driven by common total factor productivity, the persistent factor re-allocation leads to long-run output losses: the output cost of shocks to common total factor productivity only amount to 0.2678 percent, which is approximately half of the output cost of the Great Recession. The welfare cost of the Great Recession exhibit a similar pattern that is the consequence of the *bank lending channel*. For instance, investors value the cost of the Great Recession equal to a permanent decrease in per-period consumption by 0.5640 percent. There are non-negligible spillovers from shocks to the depreciation rate of bank equity capital to the non-financial sector as the induced persistent re-allocation of production factors slows down economic recovery. Specifically, the welfare cost of the Great Recession for investors drops from 0.5640 percent to 0.0976 percent when there are only shocks to common total factor productivity.

8.4 Capital Regulation and Automatic Stabilization

This section provides novel insights into the role of the sensitivity of bank leverage as an automatic stabilizer in *banking crises*. We consider different regulatory regimes to vary the sensitivity of bank leverage. Specifically, we consider three regulatory regimes: first, *laissez faire* ($\lambda^{reg} = \infty$), second, a leverage limit of 5.0 percent above the steady state leverage to which we refer as *weak regulation* ($\lambda^{reg} = 1.05\hat{\lambda}$), and, third, a leverage limit of 1.0 percent above steady state leverage to which we refer as *strong regulation* ($\lambda^{reg} = 1.01\hat{\lambda}$).

For the analysis here, we consider shocks to common total factor productivity and capital depreciation similar to the shock sequence associated wit the Great Recession. The impulse response functions conditional on the regulatory regimes are depicted in Figure 5. It is clear that the stronger the regulation, the more likely a specific shock sequence causes bank leverage to hit the regulatory leverage constraint. This implies that the growth rate of bank equity capital $\frac{E_{t+1}}{E_t} = \beta_B(\theta \min\{\lambda_t, \lambda^{reg}\} - \delta_t^E)$ is bounded from above, which slows down the accumulation

In this case, however, the present discounted value of output is mainly driven by the short-run decline in the interest rate and does not necessarily reflect the temporary decline in output.

Table 4: Welfare and Output Costs of the Great Recession

	shocks to ...	output cost	welfare cost		
			investor	worker	banker
$\lambda^{reg} = \infty$	(A, δ^E, θ)-shock	+0.5323	+0.5640	+0.3408	+3.3963
	(A, δ^E)-shock	+0.5235	+0.5408	+0.3349	+3.9666
	(A)-shock	+0.2678	+0.0976	+0.1434	+0.1402
$\lambda^{reg} = 1.05\hat{\lambda}$	(A, δ^E, θ)-shock	+0.5323	+0.5640	+0.3408	+3.3963
	(A, δ^E)-shock	+0.5235	+0.5408	+0.3349	+3.9666
	(A)-shock	+0.2678	+0.0976	+0.1434	+0.1402
$\lambda^{reg} = 1.01\hat{\lambda}$	(A, δ^E, θ)-shock	+0.6257	+0.7367	+0.4152	+4.4756
	(A, δ^E)-shock	+0.9119	+1.2565	+0.6530	+8.361
	(A)-shock	+0.2678	+0.0976	+0.1434	+0.1402

Note: Simulation results for (A, δ^E, θ)-shocks – Great Recession – (A, δ^E)-shocks, and (A)-shocks for different regulatory regimes: laissez faire refers to $\lambda^{reg} = \infty$, weak regulation refers to $\lambda^{reg} = 1.05\hat{\lambda}$, and strong regulation refers to $\lambda^{reg} = 1.01\hat{\lambda}$. Output costs are denominated in percent of the present discounted value of output. Welfare costs are denominated in percent of consumption equivalent units.

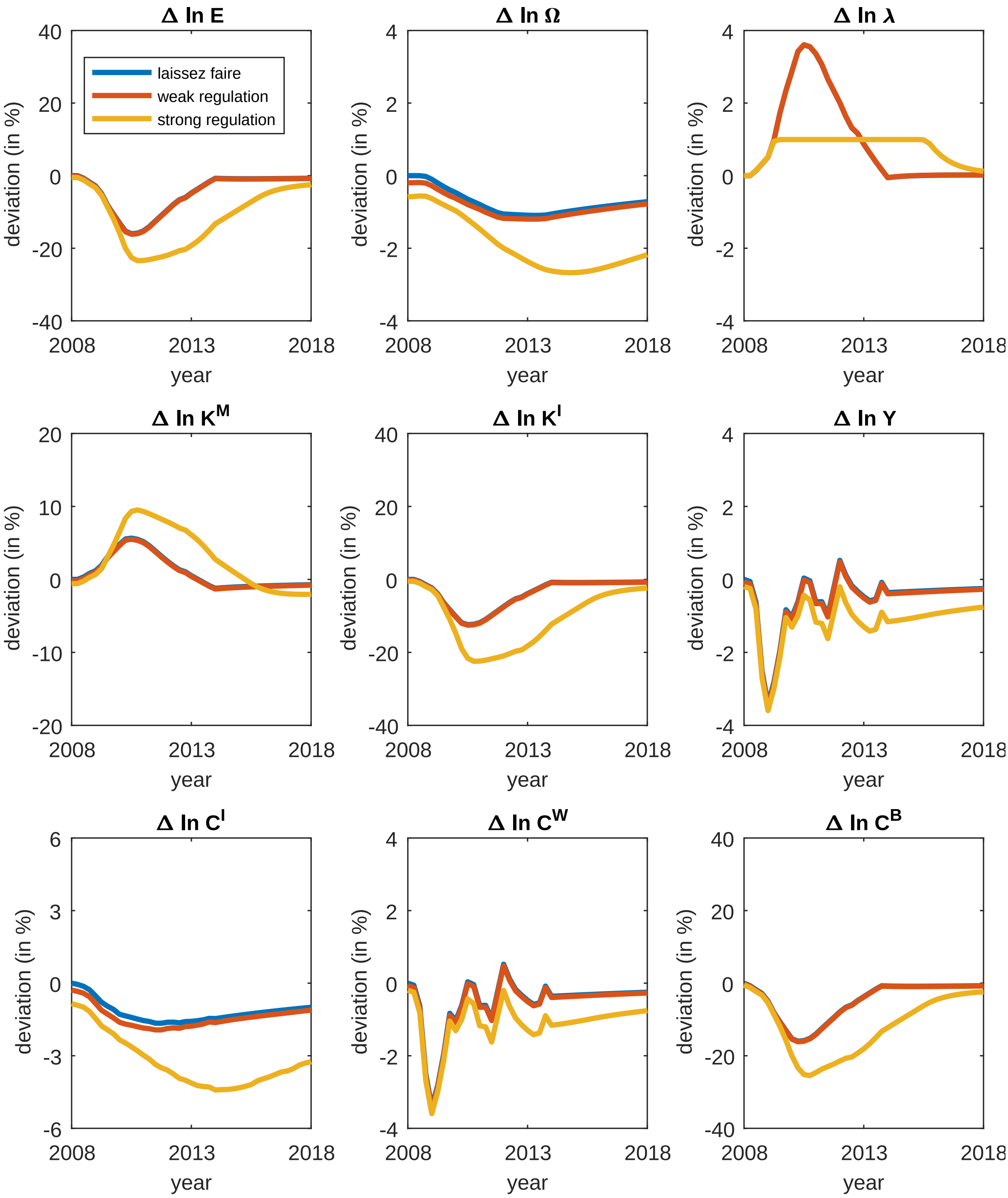
of bank equity capital, thereby intensifying the *bank lending channel* with adverse consequences on output and consumption paths. We find that when regulation is strong, the output cost increases from 0.5235 percent to 0.9119 percent. Similarly, the welfare cost, e.g. for investors, increases from 0.5408 percent to 1.2529 percent (see Table 4). Note that if we consider the Great Recession, i.e. including shocks to the financial friction, output and welfare costs are almost unaffected by *strong regulation*. This is because the shock to the financial friction already buffers the reaction of bank leverage such that the regulatory limit almost never binds.

Of course, bank leverage in itself is a destabilizing element as it multiplies a decline of bank equity into a decline of investment in the sector with the highest marginal productivity of capital. The increase of bank leverage when bank equity capital declines, however, is an automatic stabilizer. In contrast, when there are only shocks to common total factor productivity, the sensitivity of bank leverage destabilizes the economy. For the quantitative analysis of the Great Recession, however, we find that the destabilizing effect of the leverage adjustment in response to the immense drop in total factor productivity is negligible, despite of its magnitude.

8.5 Speeding up Recoveries

We now assess the quantitative effects of a *balanced bailout* (as introduced in Section 7) in response to the Great Recession. Specifically, we assume that the policy-maker implements an investor-financed capital transfer to induce a path for bank equity capital $E_t^{bb} = (1-\zeta)E_t^{lf} + \zeta\hat{E}$

Figure 5: Great Recession – Impulse Response Functions: Laissez Faire vs. Capital Regulation



Note: Simulation results for (A, δ^E)-shocks for different regulatory regimes: laissez faire refers to $\lambda^{reg} = \infty$, weak regulation refers to $\lambda^{reg} = 1.05\hat{\lambda}$, and strong regulation refers to $\lambda^{reg} = 1.01\lambda$. The economy is initially in its steady state.

parameterized by ζ $(0 < \zeta \leq 1)$. Note that $\zeta = 0$ and $\zeta = 1$ correspond to *laissez faire* and complete bailout, respectively. Capital transfers are financed by a sequence of wealth taxes on the investor's end-of-period net-worth. The policy-maker further chooses a sequence of dividend payout restrictions to align the consumption path of the banker under the *balanced budget* regime with *laissez faire*: $C_t^{B,bb} = C_t^{B,lf}$.

Table 5 summarizes the output and welfare costs for different levels of the *balanced bailout* characterized by the choice of ζ. In line with our theoretical analysis in Section 7, we find that the output cost is decreasing in ζ. Specifically, while output cost under *laissez faire* is 0.5323 percent, it decreases to 0.5254 percent when $\zeta = 0.33$ and 0.5229 when $\zeta = 0.66$. Observe that the output gains from *balanced bailout* are, however, limited compared to the output losses. Furthermore, welfare cost of bankers are constant by construction and welfare costs of workers are increasing as accelerated recovery unambiguously leads to an accelerated increase of wages in the economy: The welfare cost of workers decrease from 0.3408 percent under *laissez faire* to 0.3382 percent under *balanced bailout* with $\zeta = 0.66$. The quantitative analysis yields increasing welfare cost for investors, which means that from their perspective, the negative effect of wealth taxes dominates the positive effect of an accelerated economic recovery.

In addition, we report further results for a re-calibrated version of the model in which we choose a calibration target for bank leverage twice as high. In this version, the amplification of shocks is larger which makes it more likely that the costly *bank lending channel* gets triggered.

Table 5: Welfare and Output Costs of the Great Recession: *Balanced Bailout* vs *Laissez Faire*

	shocks to ...	output cost	welfare cost		
			investor	worker	banker
$\lambda = 10.7808$	*laissez faire* ($\zeta = 0.00$)	+0.5323	+0.5640	+0.3408	+3.3963
	balanced bailout ($\zeta = 0.33$)	+0.5254	+0.6059	+0.3384	+3.3963
	balanced bailout ($\zeta = 0.66$)	+0.5220	+0.6492	+0.3382	+3.3963
$\lambda = 21.5616$	*laissez faire* ($\zeta = 0.00$)	+0.4006	+0.3587	+0.2460	+1.9079
	balanced bailout ($\zeta = 0.33$)	+0.3993	+0.3708	+0.2457	+1.9079
	balanced bailout ($\zeta = 0.66$)	+0.3987	+0.3832	+0.2459	+1.9079

Note: Simulation results for (A, δ^E, θ)-shocks – Great Recession – for different policy regimes. The policy regimes are convex combinations between the laissez faire path of bank equity capital and the steady state value of bank equity capital, where parameter ζ is the weight given to the laissez faire. Output costs are denominated in percent of the present discounted value of output. Welfare costs are denominated in percent of consumption equivalent units.

9 Extensions

We briefly discuss two extensions of the model to examine the robustness of our findings.

So far, we have assumed that banks do not incur costs when they monitor entrepreneurs. Typically, however, commercial or universal banks have to spend considerable resources on such activities. Such costs can easily be integrated in our model. For example, suppose that banks incur a cost c ($c > 0$) per unit of loans they monitor. Then, the non-pledgeable part of revenues increases to $(c + \theta)K_t^I$, whereof bankers only obtain θK_t^I from their lending activities. The leverage constraint, condition (3), adjusts to

$$K_t^I = \frac{1 + r_t^M}{r_t^M - r_t^I + \theta + c} E_t.$$

The solution of this extended model is almost similar to the baseline model. Specifically, for the steady state, we obtain

$$\begin{aligned} \hat{r}^M &= \delta + \rho_H, \\ \theta\hat{\lambda} &= \delta + \rho_B + 1, \\ \hat{r}^I &= \hat{r}^M + \frac{\theta(\rho_B - \rho_H)}{1 + \delta + \rho_B} + c. \end{aligned}$$

Hence, steady state leverage $\hat{\lambda}$ and the return on equity $\theta\hat{\lambda}$ are unaffected by financial intermediation costs. However, less capital can be invested in sector I, which reduces both the level of bank equity and investor wealth in the steady state.

We now consider a variant of the model in which households are not divided into investors and workers, so that there are only two types of agents: households and bankers. The former own the capital stock Ω and supply labor inelastically. The problem of the household is *mutatis mutandis* to the investor's problem in Section 3.2 with the budget constraint augmented by labor income:

$$C_t^H + \Omega_{t+1} = w_t^I L^I + w_t^M L^M + \Omega_t(1 + r_t^M - \delta).$$

Optimization yields the standard Euler equation for the household problem

$$\frac{1}{C_t^H} = \beta_H \frac{1 + r_{t+1}^M - \delta}{C_{t+1}^H}. \tag{17}$$

Comparing the steady state of the two-agent economy to the steady state of the three-agent economy (Section 5) reveals that the steady state conditions are the same and, thus, the steady state allocation is unaffected by this variant of the model. The transitional dynamics in the two-agent economy, however, cannot be made explicit anymore, as there are no closed form solutions for the consumption-saving policies.

In Table 6, we show that the adjustment in the transition dynamics of the two-agent economy yields quantitative results of a similar order of magnitude as for the three-agent economy.

Table 6: Welfare and Output Cost of the Great Recession: Three-Agent vs Two-Agent Economy

	shocks to ...	output cost	welfare cost		
			investor	worker	banker
three-agent	(A, δ^E, θ)-shock	+0.5323	+0.5640	+0.3408	+3.3963
	(A, δ^E)-shock	+0.5232	+0.5408	+0.3349	+3.9666
	(A)-shock	+0.2678	+0.0976	+0.1434	+0.1402
two-agent	(A, δ^E, θ)-shock	+0.5053	+0.3850		+3.3457
	(A, δ^E)-shock	+0.5001	+0.3741		+3.9107
	(A)-shock	+0.2988	+0.1331		+0.2149

Note: Simulation results for (A, δ^E, θ)-shocks – Great Recession – (A, δ^E)-shocks, and (A)-shocks for different household structures. The three-agent economy refers to a model in which households are split between investors and workers (baseline model). The two agent-economy refers to a model in which households derive capital and labor income. Output costs are denominated in percent of the present discounted value of output. Welfare costs are denominated in percent of consumption equivalent units.

10 Conclusion and Outlook

We have presented a simple model of capital accumulation in which financial intermediaries are essential for some firms. The model delivers a set of insights into the underlying shock propagation mechanism and replicates various stylized facts, and allows to study policy responses to downturns associated with a decline of bank equity. Because our model preserves the analytical tractability, it can serve as a macro banking module that can be conveniently embedded in more complex economic models.

Numerous further generalizations and extensions are possible. We briefly outline three promising directions for further research starting from our framework. First, as the Eurozone and a great part of Asia rely heavily on bank loans, while corporate bonds are much more dominant in the US,[30] our framework can help to investigate which type of economic structure is more resilient to adverse shocks. Second, apart from monitoring firms, banks also perform risk sharing and maturity transformation. Including these functions into our banking model with capital accumulation is challenging but could provide further valuable insights. Third, introducing frictional labor markets with imperfect labor transition between production sectors – thereby affecting the sensitivity of capital returns and bank leverage – can explain on how labor market institutions and financial frictions interact and how they jointly affect amplification and persistence of adverse shocks.

[30] See e.g. De Fiore and Uhlig (2011) and Ghosh (2006).

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A Appendix – Sequential Markets Equilibrium

A.1 Comparative Statics

Corollary 1 summarizes the impact of shocks to productivities, investor wealth, bank equity, and financial frictions on market-imposed bank leverage.

Corollary 1. *Suppose that financial frictions matter, i.e.* $(E_t, \Omega_t) \in \Gamma$. *Then,*

(i) $\frac{\partial \lambda_t}{\partial A} > 0$, $\frac{\partial \lambda_t}{\partial z^I} > 0$, *and* $\frac{\partial \lambda_t}{\partial z^M} < 0$.

(ii) $\frac{\partial \lambda_t}{\partial \Omega_t} > 0$ *and* $\frac{\partial \lambda_t}{\partial E_t} < 0$.

(iii) $\frac{\partial \lambda_t}{\partial \theta} < 0$.

Proof. The partial derivatives of $\varphi(\lambda_t)$ are

$$\begin{aligned}
\frac{\partial \varphi(\lambda_t)}{\partial \lambda_t} &= -\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta > 0 \\
\frac{\partial \varphi(\lambda_t)}{\partial \Omega_t} &= \frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1) < 0 \\
\frac{\partial \varphi(\lambda_t)}{\partial E_t} &= -\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)^2 - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t^2 > 0 \\
\frac{\partial \varphi(\lambda_t)}{\partial z^M} &= \frac{r_t^M(K_t^M)}{z^M}(\lambda_t - 1) > 0 \\
\frac{\partial \varphi(\lambda_t)}{\partial z^I} &= -\frac{r_t^I(K_t^I)}{z^I}\lambda_t < 0 \\
\frac{\partial \varphi(\lambda_t)}{\partial A} &= \frac{(r_t^M(K_t^M) - r_t^I(K_t^I))\lambda_t - r_t^M(K_t^M)}{A} < 0 \\
\frac{\partial \varphi(\lambda_t)}{\partial \theta} &= \lambda_t > 0.
\end{aligned}$$

Because of inelastic labor supply and the Inada conditions, there will always be production in both sectors, such that $K_t^I > 0$. Therefore, the market-imposed leverage constraint implies $r_t^M(K_t^M) - r_t^I(K_t^I) + \theta > 0$. The inequalities in the equations above follow from $r_t^M(K_t^M) - r_t^I(K_t^I) + \theta > 0$, $\lambda_t > 1$, and $\frac{\partial r_t^j(K_t^j)}{\partial K_t^j} < 0$. Total differentiation of $\varphi(\lambda_t)$ and application of the implicit function theorem yield $\frac{\partial \lambda_t}{\partial A_t} > 0$, $\frac{\partial \lambda_t}{\partial z^I} > 0$, $\frac{\partial \lambda_t}{\partial z^M} < 0$, $\frac{\partial \lambda_t}{\partial \Omega_t} > 0$, $\frac{\partial \lambda_t}{\partial E_t} < 0$, and $\frac{\partial \lambda_t}{\partial \theta} < 0$. □

The main intuition for the results can be derived by comparing the profits of a single bank if it complies with the deposit contract, $(1 + r_t^I)k_t^I - (1 + r_t^M)d_t$, to the profits of defecting, θk_t^I. The intuitive argument neglects some equilibrium adjustments which, however, only partially off-set the direct effect.

First, a productivity increase in sector M *ceteris paribus* increases the deposit rate and thus the repayment obligation that arises from the deposit contract, $(1 + r_t^M)d_t$. Since the profit margin of bankers declines, investors have to cut down their investment into deposits to preserve the incentive compatibility of the deposit contract. Thus, market-imposed bank leverage decreases.

A productivity increase in sector I *ceteris paribus* increases the revenues of providing loans to sector I, $(1+r_t^I)k_t^I$, and thus profits from complying with the deposit contract. Investors can thus increase their deposits without violating the incentive compatibility of the deposit contract. As a result, market-imposed bank leverage increases. The effect of a common productivity shock is more involved, as it *ceteris paribus* increases the bank's revenues from investing into sector I as well as the repayment obligation to depositors. However, because $r_t^I > r_t^M$, the revenue increase dominates the rise of the repayment obligation, such that – similar to the productivity shock in sector I – market-imposed bank leverage increases.

Second, an increase in aggregate investor wealth Ω_t *ceteris paribus* increases investment in sector M and thus decreases r_t^M. Therefore, the bank's repayment obligation from complying with the deposit contract declines and profit increases. Hence, investors are willing to increase deposits until the incentive constraint becomes binding and, thus, market-imposed bank leverage increases. An increase in aggregate bank equity E_t *ceteris paribus* increases both bond finance K_t^M and loan finance K_t^I. Interest rates fall in both sectors, such that the bank's revenues from investing into sector I and the repayment obligation to depositors decrease. Because loan finance is more elastic to changes in the equity stock than bond finance,[31] the revenue effect dominates the effect on the repayment obligation, such that profits from complying with the deposit contract fall. As a result, investors have to reduce their deposits in order to restore incentive-compatibility: market-imposed bank leverage decreases.

Finally, when financial frictions between depositors and banks become more severe, the value for bankers from defecting increases. Investors cut down their investment in deposits to incentivize banks to comply with the deposit contract. As a result, market-imposed bank leverage declines.

The following two corollaries establish the impact of shocks to common productivity, investor wealth, bank equity, and financial frictions on bond and loan finance, respectively.

Corollary 2. *Suppose that financial frictions matter, i.e.* $(E_t, \Omega_t) \in \Gamma$. *Then,*

(i) $\frac{\partial K_t^M}{\partial A} < 0$,

(ii.1) $\frac{\partial K_t^M}{\partial \Omega_t} > 0$,

(ii.2) $\frac{\partial K_t^M}{\partial E_t} < 0$,

(iii) $\frac{\partial K_t^M}{\partial \theta_t} > 0$.

[31] The elasticity of loan finance with respect to equity is $\frac{\partial K_t^I}{\partial E_t}\frac{E_t}{K_t^I} = 1$ whereas the elasticity of bond finance with respect to equity is $\frac{\partial K_t^M}{\partial E_t}\frac{E_t}{K_t^M} = \frac{(\lambda_t - 1)E_t}{\Omega_t + (\lambda_t - 1)E_t} < 1$.

Proof. When financial frictions are relevant, $K_t^M = E_t + \Omega_t - \lambda_t E_t$. Thus,

$$\begin{aligned}
\frac{\partial K_t^M}{\partial A} &= -\frac{\partial \lambda_t}{\partial A} E_t < 0, \\
\frac{\partial K_t^M}{\partial \Omega_t} &= 1 - \frac{\partial \lambda_t}{\partial \Omega_t} E_t > 0, \\
\frac{\partial K_t^M}{\partial E_t} &= 1 - \frac{\partial \lambda_t}{\partial E_t} E_t - \lambda_t < 0, \\
\frac{\partial K_t^M}{\partial \theta} &= -\frac{\partial \lambda_t}{\partial \theta} E_t > 0.
\end{aligned}$$

The results for the partial derivatives with respect to A and θ follow directly from Corollary 1. The result for the derivatives with respect to Ω_t and E_t come from the following considerations using condition (8):

$$\begin{aligned}
\frac{\partial K_t^M}{\partial \Omega_t} &= 1 - \frac{\partial \lambda_t}{\partial \Omega_t} E_t \\
&= 1 + \frac{\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t}{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta} \\
&= \frac{-\frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta}{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta} > 0 \\
\frac{\partial K_t^M}{\partial E_t} &= 1 - \left(\frac{\partial \lambda_t}{\partial E_t} E_t + \lambda_t\right) \\
&= 1 + \frac{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)^2 E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t^2 E_t}{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta} - \lambda_t \\
&= -\frac{\frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + (\lambda_t - 1)(r_t^M(K_t^M) - r_t^I(K_t^I) + \theta)}{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t - \frac{\partial r_t^I}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta} \\
&= -\frac{1 + \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}K_t^I + r_t^I(K_t^I) - \theta}{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta} < 0,
\end{aligned}$$

where the third line uses the definition of leverage to rewrite $(\lambda_t - 1)(r_t^M(K_t^M) - r_t^I(K_t^I) + \theta)$ as $1 + r_t^I(K_t^I) - \theta$. The inequalities follow from $r_t^M(K_t^M) - r_t^I(K_t^I) + \theta > 0$, $\lambda_t > 1$, $\frac{\partial r_t^j(K_t^j)}{\partial K_t^j} < 0$, and $\frac{\partial r_t^I(K_t^I)}{\partial K_t^I}K_t^I + r_t^I(K_t^I) = \alpha r_t^I(K_t^I) > 0$. □

Corollary 3. *Suppose that financial frictions matter, i.e.* $(E_t, \Omega_t) \in \Gamma$. *Then,*

(i) $\frac{\partial K_t^I}{\partial A} > 0$,

(ii.1) $\frac{\partial K_t^I}{\partial \Omega_t} > 0$,

(ii.2) $\frac{\partial K_t^I}{\partial E_t} > 0$,

(iii) $\frac{\partial K_t^I}{\partial \theta} < 0$.

Proof. When financial frictions are relevant, $K_t^I = \lambda_t E_t$. Thus,

$$\frac{\partial K_t^I}{\partial A} = \frac{\partial \lambda_t}{\partial A} E_t > 0,$$
$$\frac{\partial K_t^I}{\partial \Omega_t} = \frac{\partial \lambda_t}{\partial \Omega_t} E_t > 0,$$
$$\frac{\partial K_t^I}{\partial E_t} = \frac{\partial \lambda_t}{\partial E_t} E_t + \lambda_t > 0,$$
$$\frac{\partial K_t^I}{\partial \theta} = \frac{\partial \lambda_t}{\partial \theta} E_t < 0.$$

Where the results for the partial derivatives with respect to A_t, Ω_t, and θ directly follow from Corollary 1. The result for the derivative with respect to E_t come from the following consideration, using condition (8):

$$\begin{aligned}
\frac{\partial K_t^I}{\partial E_t} &= \frac{\partial \lambda_t}{\partial E_t} E_t + \lambda_t \\
&= -\frac{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)^2 E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t^2 E_t}{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta} + \lambda_t \\
&= \frac{\frac{-\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t + \lambda_t(r_t^M(K_t^M) - r_t^I(K_t^I) + \theta)}{-\frac{\partial r_t^M(K_t^M)}{\partial K_t^M}(\lambda_t - 1)E_t - \frac{\partial r_t^I(K_t^I)}{\partial K_t^I}\lambda_t E_t + r_t^M(K_t^M) - r_t^I(K_t^I) + \theta} > 0.
\end{aligned}$$

where the inequality follows from $r_t^M(K_t^M) - r_t^I(K_t^I) + \theta > 0$, $\lambda_t > 1$, and $\frac{\partial r_t^j(K_t^j)}{\partial K_t^j} < 0$. □

Finally, we establish the impact of shocks to common productivity, investor wealth, and bank equity on total output.

Corollary 4. *Suppose that financial frictions matter, i.e.* $(E_t, \Omega_t) \in \Gamma$. *Then,*

(i) $\frac{\partial Y_t}{\partial A} > 0$,

(ii.1) $\frac{\partial Y_t}{\partial \Omega_t} > 0$,

(ii.2) $\frac{\partial Y_t}{\partial E_t} > 0$,

(iii) $\frac{\partial Y_t}{\partial \theta} < 0$.

Proof. Total output is defined as $Y_t = Y_t^M + Y_t^I$. Then, the derivative with respect to common total factor productivity reads

$$\begin{aligned}
\frac{\partial Y_t}{\partial A} &= \frac{\partial Y_t^M}{\partial A} + \frac{\partial Y_t^M}{\partial K_t^M}\frac{\partial K_t^M}{\partial A} + \frac{\partial Y_t^I}{\partial A} + \frac{\partial Y_t^I}{\partial K_t^I}\frac{\partial K_t^I}{\partial A} \\
&= \frac{Y_t}{A} + (r_t^I(K_t^I) - r_t^M(K_t^M))\frac{\partial K_t^I}{\partial A} > 0,
\end{aligned}$$

where we used $\frac{\partial K_t^M}{\partial A} = -\frac{\partial K_t^I}{\partial A}$. The derivative with respect to Ω_t is

$$\begin{aligned}\frac{\partial Y_t}{\partial \Omega_t} &= \frac{\partial Y_t^M}{\partial K_t^M}\frac{\partial K_t^M}{\partial \Omega_t} + \frac{\partial Y_t^I}{\partial K_t^I}\frac{\partial K_t^I}{\partial \Omega_t} \\ &= r_t^M(K_t^M) + (r_t^I(K_t^I) - r_t^M(K_t^M))\frac{\partial K_t^I}{\partial \Omega_t} > 0,\end{aligned}$$

where we used $\frac{\partial K_t^M}{\partial \Omega_t} = 1 - \frac{\partial K_t^I}{\partial \Omega_t}$. Similarly, the derivative with respect to E_t is

$$\begin{aligned}\frac{\partial Y_t}{\partial E_t} &= \frac{\partial Y_t^M}{\partial K_t^M}\frac{\partial K_t^M}{\partial E_t} + \frac{\partial Y_t^I}{\partial K_t^I}\frac{\partial K_t^I}{\partial E_t} \\ &= r_t^M(K_t^M) + (r_t^I(K_t^I) - r_t^M(K_t^M))\frac{\partial K_t^I}{\partial E_t} > 0,\end{aligned}$$

where we used $\frac{\partial K_t^M}{\partial E_t} = 1 - \frac{\partial K_t^I}{\partial E_t}$. Finally, the derivative with respect to θ yields

$$\begin{aligned}\frac{\partial Y_t}{\partial \theta} &= \frac{\partial Y_t^M}{\partial K_t^M}\frac{\partial K_t^M}{\partial \theta} + \frac{\partial Y_t^I}{\partial K_t^I}\frac{\partial K_t^I}{\partial \theta} \\ &= (r_t^I(K_t^I) - r_t^M(K_t^M))\frac{\partial K_t^I}{\partial \theta} < 0,\end{aligned}$$

where we used $\frac{\partial K_t^M}{\partial \theta} = -\frac{\partial K_t^I}{\partial \theta}$. □

An increase in productivity or total capital, i.e. either investor wealth or bank equity capital, directly rises total output. For an increase in the financial friction, we note that this leads to a more inefficient allocation of capital and, thus, has a negative impact on total output.

A.2 Proof of Proposition 2

We consider a general structure that applies to both, the banks' and the investors' optimization problem. Consider the following optimization problem

$$\max_{\{C_t, X_{t+1}\}} \mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t \ln C_t\right]$$

subject to

$$(1+\tau_{C,t})C_t + (1+\sigma_{X,t})X_{t+1} = (1-\tau_{X,t})R_t X_t$$

where C_t is consumption, X_t is the agent's net worth, $\tau_{C,t}$ is a consumption tax rate, $\sigma_{X,t}$ is an investment subsidy,[32] $\tau_{X,t}$ is a wealth tax, and β is the discount factor. The gross-return R_t is known in period t but evolves according to a stochastic process. Clearly, the problem also nests

[32] An investment subsidy benefits investors for their total investment into both sectors by a factor $1+\sigma_{X,t}$.

the case $\tau_{C,t}=0$, $\sigma_{X,t}=0$, and $\tau_{X,t}$ which underlies Proposition 2. The Euler equation reads

$$\frac{1+\sigma_{X,t}}{1+\tau_{C,t}}\frac{1}{C_t}=\beta\mathbb{E}_t\Big[\frac{(1-\tau_{X,t+1})R_{t+1}}{(1+\tau_{C,t+1})C_{t+1}}\Big].$$

Suppose the consumption policy is $C_t=\frac{\tilde{c}}{1+\tau_{C,t}}(1-\tau_{X,t})R_tX_t$ where $\tilde{c}$ is an unknown coefficient that is to be determined. Given the budget constraint, the guess for the consumption policy yields a respective condition for the saving policy: $X_{t+1}=\frac{1}{1+\sigma_{X,t}}(1-\tilde{c})(1-\tau_{X,t})R_tX_t$. The Euler equation can thus be rewritten as follows

$$\begin{aligned}
&\frac{1+\sigma_{X,t}}{1+\tau_{C,t}}\frac{1}{C_t}=\beta\mathbb{E}_t\Big[\frac{(1-\tau_{X,t+1})R_{t+1}}{(1+\tau_{C,t+1})C_{t+1}}\Big]\\
\Leftrightarrow\quad&\frac{1+\sigma_{X,t}}{\tilde{c}(1-\tau_{X,t})R_tX_t}=\beta\mathbb{E}_t\Big[\frac{(1-\tau_{X,t+1})R_{t+1}}{\tilde{c}(1-\tau_{X,t+1})R_{t+1}X_{t+1}}\Big]\\
\Leftrightarrow\quad&\frac{1+\sigma_{X,t}}{(1-\tau_{X,t})R_tX_t}=\beta\frac{1}{X_{t+1}}\\
\Leftrightarrow\quad&\frac{1+\sigma_{X,t}}{(1-\tau_{X,t})R_tX_t}=\beta\frac{1+\sigma_{X,t}}{(1-\tilde{c})(1-\tau_{X,t})R_tX_t}\\
\Leftrightarrow\quad&\tilde{c}=1-\beta.
\end{aligned}$$

Thus, $C_t=\frac{1-\beta}{1+\tau_{C,t}}(1-\tau_{X,t})R_tX_t$ and $X_{t+1}=\frac{\beta}{1+\sigma_{X,t}}(1-\tau_{X,t})R_tX_t$.

B Appendix – Long-run Dynamics

B.1 Proof of Proposition 4

To establish global stability, our analysis proceeds in two steps. In the first step, we show that for any $(E_0,\Omega_0)\in\mathbb{R}^2_+\setminus\Gamma$, i.e. for any initial capital allocation for which financial frictions are irrelevant (non-binding), the economy moves to the partition in the state space in which frictions become binding in finite time $\tau>0$. In the second step, we show that for any $(E_\tau,\Omega_\tau)\in\Gamma$, i.e. for any capital allocation for which financial frictions are relevant (binding), the economy converges to its unique steady state.

Hence, let us consider an equity-wealth allocation for which financial frictions are irrelevant, i.e. $(E_0,\Omega_0)\in\mathbb{R}^2_+\setminus\Gamma$ or, equivalently, $\frac{E_0}{K_0}\geq\theta\frac{z\ell}{1+z\ell}\frac{1}{1+r^M(K_0)}$. In this case, equity is relatively abundant. Suppose now that financial frictions remain irrelevant in all future periods. Then, the law of motions for bank equity and investor wealth (see Proposition 2) implies that the equity-to-wealth ratio $\frac{E_t}{\Omega_t}$ declines at a constant rate $\frac{\beta_H-\beta_B}{\beta_H}>0$. Moreover, $\frac{E_t}{K_t}$ declines at an accelerating rate $\frac{\beta_H-\beta_B}{\beta_BE_t/\Omega_t+\beta_H}\geq\frac{\beta_H-\beta_B}{\beta_BE_0/\Omega_0+\beta_H}>0$, such that $\lim_{t\to\infty}\frac{E_t}{K_t}=0$. We note that because the production technologies satisfy the Inada conditions, there is a strictly positive lower bound for the sequence of total capital $\{K_t\}_{t=0}^{\infty}$ for any (E_0,Ω_0). Specifically, for K_t sufficiently low, the capital return $r^M_t(K_t)$ is sufficiently high to spur the accumulation of investor wealth and bank equity capital. As a result, there exists a τ such that

$$\frac{E_\tau}{K_\tau} < \theta \frac{z\ell}{1+z\ell} \frac{1}{1+r^M(K_\tau)},$$

which contradicts the presupposition that financial frictions remain irrelevant in all future periods. Therefore, financial frictions become binding in finite time.

Second, consider an allocation $(E_\tau, \Omega_\tau) \in \Gamma$, i.e. financial frictions are relevant. In this case, equity is relatively scarce. The "$\Delta E = 0$"-locus is the combination of all E and Ω such that $E_{t+1} = E_t$. According to Proposition 2, $E_{t+1} = E_t$ corresponds to $1 = \beta_B(1 + r^B(E_t, \Omega_t))$. Implicit differentiation of the "$\Delta E = 0$"-locus condition yields $\frac{\partial \Omega}{\partial E}|_{\Delta E=0} = -\frac{\partial \lambda}{\partial E} / \frac{\partial \lambda}{\partial \Omega} > 0$, i.e. the "$\Delta E = 0$"-locus has a positive slope in the (E, Ω)-space. On the left side of the "$\Delta E = 0$"-locus , equity increases, and on the right side, equity decreases. Similarly, the "$\Delta\Omega = 0$"-locus is the combination of all E and Ω such that $\Omega_{t+1} = \Omega_t$. According to Proposition 2, $\Omega_{t+1} = \Omega_t$ corresponds to $1 = \beta_H(1 + r^M(E_t, \Omega_t) - \delta)$. Implicit differentiation of the "$\Delta\Omega = 0$"-locus condition yields $\frac{\partial \Omega}{\partial E}|_{\Delta\Omega=0} = -(1 - \lambda - \frac{\partial \lambda}{\partial E} E)/(1 - \frac{\partial \lambda}{\partial \Omega} E) = -\frac{\partial K^M}{\partial E} / \frac{\partial K^M}{\partial \Omega} > 0$. Above the locus, investor wealth decreases and below the locus, investor wealth increases. We further note that for $(E, \Omega) \in \Gamma$,

$$\begin{aligned}
\frac{\partial \Omega}{\partial E}|_{\Delta E=0} - \frac{\partial \Omega}{\partial E}|_{\Delta\Omega=0} &= -\frac{\frac{\partial \lambda}{\partial E}}{\frac{\partial \lambda}{\partial \Omega}} + \frac{1 - \lambda - \frac{\partial \lambda}{\partial E} E}{1 - \frac{\partial \lambda}{\partial \Omega} E} = \frac{-\frac{\partial \lambda}{\partial E} + (1-\lambda)\frac{\partial \lambda}{\partial \Omega}}{\frac{\partial \lambda}{\partial \Omega}(1 - \frac{\partial \lambda}{\partial \Omega} E)} \\
&= \frac{\frac{\partial \varphi}{\partial E} - (1-\lambda)\frac{\partial \varphi}{\partial \Omega}}{\frac{\partial \varphi}{\partial \lambda}\frac{\partial \lambda}{\partial \Omega}(1 - \frac{\partial \lambda}{\partial \Omega} E)} = \frac{-\frac{\partial r^M}{\partial K^M}(\lambda-1)^2 - \frac{\partial r^I}{\partial K^I}\lambda^2 + \frac{\partial r^M}{\partial K^M}(\lambda-1)^2}{\frac{\partial \varphi}{\partial \lambda}\frac{\partial \lambda}{\partial \Omega}(1 - \frac{\partial \lambda}{\partial \Omega} E)} \\
&= \frac{-\frac{\partial r^I}{\partial K^I}\lambda^2}{\frac{\partial \varphi}{\partial \lambda}\frac{\partial \lambda}{\partial \Omega}(1 - \frac{\partial \lambda}{\partial \Omega} E)} > 0,
\end{aligned} \tag{B1}$$

i.e. the "$\Delta E = 0$"-locus is steeper than the "$\Delta\Omega = 0$"-locus. Inspecting the relative location of the loci and the dynamics of bank equity and investor wealth relative to the loci reveals stability of the economic system for any $(E_\tau, \Omega_\tau) \in \Gamma$.

B.2 Permanent Shocks to Productivity

Consider a permanent increase in the common factor productivity. Conditions (9) to (11) directly reveal that the steady state interest rates and leverage are independent of the technology level, and conditions (12) to (15) show that the capital allocation and wealth distribution are proportional to $A^{1/(1-\alpha)}$. The following corollary summarizes these considerations:

Corollary 5. *An increase in common total factor productivity by a factor* $(1 + \Delta_A)$ *yields an increase of the steady state capital allocation and wealth distribution by factor* $(1 + \Delta_A)^{1/(1-\alpha)}$. *The steady state bond-to-loan finance ratio,* K^M/K^I, *is independent of changes in common total factor productivity.*

Proof. The proof directly follows from Proposition 3. □

In terms of transitional dynamics, Corollaries 1 to 3 imply that a decline in common factor productivity is instantaneously accompanied by a decrease in bank leverage, a decrease in loan finance and an increase in bond finance. In other word, there is a shift towards the less capital-efficient production sector, which amplifies the decline of output. The returns in both sectors decrease, which leads to a decline in the growth rate of investor wealth. However, bank leverage decreases as well, which leads to a decline in bank equity holdings and therefore can trigger the costly bank capital transmission channel. In the long-run, bond and loan finance decline, and so does bank equity capital and investor wealth. However, bank leverage, is unaffected in the long-run.

B.3 Permanent Shocks to Financial Frictions

There are several examples of permanent shocks to the financial friction between depositors and bankers that could materialize in an increase in θ. For instance, it can become more difficult to enforce contractual obligations, thereby worsening the underlying moral hazard problem. Another example is decreasing trust in the banking sector as a result of shifted beliefs about the repayment behavior of bankers.

A permanent increase in financial frictions, i.e. a permanent shift in the belief in bankers' ability to repay, has several implications for the steady state allocation, as the inefficiency of the allocation increases. We directly derive the following corollary from Proposition 3:

Corollary 6. *An increase of the intensity of financial frictions, i.e. an increase of θ,*

(i) lowers the steady-state level of capital $\hat{K}$, and

(ii) increases bank equity $\hat{E}$ if bankers are not too impatient.

Proof. The statement for $\hat{K}$ follows immediately from the fact that a higher value of θ increases $\hat{r}^I$, which leads to a reduction of $\hat{K}^I$. At the same time, $\hat{r}^M$ is unaffected by the degree of the financial friction, such that $\hat{K}^M$ is unaffected. Therefore, $\hat{K} = \hat{K}^I + \hat{K}^M$ falls. The impact on $\hat{E}$ is more involved. Differentiation yields

$$\frac{\partial \hat{E}}{\partial \theta} = \frac{1}{1+\delta+\rho_B}\left\{\frac{1}{\hat{r}^I}\left(\frac{\alpha A z^I}{\hat{r}^I}\right)^{\frac{1}{1-\alpha}}\left(\hat{r}^I - \frac{1}{1-\alpha}\frac{\theta(\rho_B-\rho_H)}{1+\delta+\rho_B}\right)\right\}L^I. \tag{B2}$$

When ρ_B is sufficiently close to ρ_H, we obtain $\frac{\partial \hat{E}}{\partial \theta} > 0$. □

An important consequence of Corollary 6 is that in the steady state, more severe financial frictions lower the total amount of capital and the share owned by investors, but not the wealth of bankers if bankers are not too impatient. The reason is subtle. A higher value of θ lowers leverage. However, when ρ_B is close to ρ_H, $\hat{r}^I$ is close to $\hat{r}^M$ and, thus, $\hat{K}^I$ is close to $\frac{\hat{K}^M}{z\ell}$. In the steady state, the latter is independent of θ, variations of θ have little effect on $\hat{K}^I$ for ρ_B close to ρ_H. Because $\hat{K}^I = \hat{\lambda}\hat{E}$, a higher value of θ is associated with a higher value of $\hat{E}$.

The distributional implication from an increase of the financial friction carries over to the transitional dynamics.

Proposition 8. *Suppose that ρ_B is sufficiently close to ρ_H and the economy is hit by a negative permanent shock to financial frictions. Then, the steady state intertemporal utility of bankers increases.*

Proof. As a direct consequence of Corollary 6, steady state bank equity increases from $\hat{E}(\theta)$ to $\hat{E}(\theta')$.[33] This means that during the transition phase, $\theta'\lambda_t$ has to be larger than $\delta + \rho_B + 1$ (see consition (10)) and thus consumption of bankers during the transition phase is higher than in the steady state associated with θ. As the steady state return on equity is independent of financial frictions, bankers will have higher utility in each period when the economy is hit by an adverse shock to financial frictions. $\square$

In contrast to bankers, investors and workers are hurt by an increase in financial frictions. Workers are also hurt in the long-run, as aggregate wages decline towards the new steady state associated with $\theta' > \theta$. For investors, however, the intraperiod utility losses vanish over time, as the interest rate r_t^M converges to $\hat{r}^M = \delta + \rho_H$, which is independent of θ.

C Appendix – Short-run Dynamics

C.1 Proof of Proposition 5

Observe that in our model, a key transmission channel for shocks to common total factor productivity is the impact on capital allocation $K_t^I = \lambda_t E_t$ and $K_t^M = E_t + \Omega_t - \lambda_t E_t$. Taking partial derivatives using Corollaries 1 to 4, we obtain

$$\begin{aligned}
\frac{\partial K_t^I}{\partial A} &= \frac{\partial \lambda_t}{\partial A} E_t > 0, \\
\frac{\partial K_t^M}{\partial A} &= -\frac{\partial \lambda_t}{\partial A} < 0, \\
\frac{\partial Y_t}{\partial A} &= \frac{Y_t}{A} + (r_t^I - r_t^M)\frac{\partial K_t^I}{\partial A} \\
&= \frac{Y_t}{A} + (r_t^I - r_t^M)\frac{\partial \lambda_t}{\partial A} E_t > 0.
\end{aligned}$$

Note that $\frac{\partial K_t^I}{\partial A}$ and $\frac{\partial Y}{\partial A}$ are increasing in $\frac{\partial \lambda_t}{\partial A} > 0$, i.e. the sensitivity of bank leverage with respect to common total factor productivity destabilizes the economy in *regular recessions*.

[33] Note that we do not claim that the movement from $\hat{E}(\theta)$ to $\hat{E}(\theta')$ is monotonic. However, as $\theta'\lambda_t$ is initially larger than $\delta+\rho_B+1$, a potential overshooting of bank equity above $\hat{E}(\theta')$ later on would not invalidate Proposition (8).

C.2 Proof of Proposition 6

Observe that in our model, a key transmission channel for shocks to bank equity capital is the impact on capital allocation $K_t^I = \lambda_t E_t$ and $K_t^M = E_t + \Omega_t - \lambda_t E_t$. Taking partial derivatives using Corollaries 1 to 4, we obtain

$$\begin{aligned}
\frac{\partial K_t^I}{\partial E_t} &= \frac{\partial \lambda_t}{\partial E_t} E_t + \lambda_t > 0, \\
\frac{\partial K_t^M}{\partial E_t} &= 1 - \frac{\partial \lambda_t}{\partial E_t} E_t - \lambda_t < 0, \\
\frac{\partial Y_t}{\partial E_t} &= r_t^M + (r_t^I - r_t^M) \frac{\partial K_t^I}{\partial E_t} \\
&= r_t^M + (r_t^I - r_t^M) \left(\frac{\partial \lambda_t}{\partial E_t} E_t + \lambda_t \right) > 0.
\end{aligned}$$

Note that $\frac{\partial K_t^I}{\partial E_t}$ and $\frac{\partial Y_t}{\partial E_t}$ are increasing in $\frac{\partial \lambda_t}{\partial E_t} < 0$, i.e. the sensitivity of bank leverage with respect to bank equity capital stabilizes the economy in *banking crises*.

D Appendix – Managing Recoveries: Proof of Proposition 7

Consider a specific shock sequence that leads to a temporary decline in bank equity capital and dies out in finite time, such that long-run bank equity capital converges to $\hat{E}$. Further, define two sequences for bank equity capital: the *laissez faire* sequence $\{E_t^{lf}\}_{t=0}^{\infty}$ and the *balanced budget* sequence $\{E_t^{bb}\}_{t=0}^{\infty}$. In particular, suppose that the planner chooses a feasible allocation characterized by the sequence of bank equity capital $E_t^{bb} = (1-\zeta)E_t^{lf} + \zeta \hat{E}$ where $0 < \zeta \leq 1$. Note that the planner's choice is on equity capital and not the transfer payments that yield the sequence $\{E_t^{bb}\}_{t=0}^{\infty}$.

To simplify the argument, suppose for a moment that $\zeta = 1$, i.e. $E_t^{bb} = \hat{E}$ for all $t = 0, 1, 2, \ldots$. At the end of period 0, there is an investor-financed capital injection Tr_0 into banks such that $E_1^{bb} = \hat{E}$ and a dividend payout restriction d_0 such that $C_0^{B,bb} = C_0^{B,lf}$, i.e.

$$\begin{aligned}
&E_1^{bb} = d_0((\theta_0 \lambda_0 - \delta_0^E) E_0 + Tr_0) \\
&(1 - d_0)((\theta_0 \lambda_0 - \delta_0^E) E_0 + Tr_0) = (1 - \beta_B)(\theta_0 \lambda_0 - \delta_0^E) E_0.
\end{aligned}$$

Substituting for d_0 and solving for Tr_0 yields $Tr_0 = \hat{E} - \beta_B(\theta_0 \lambda_0 - \delta_0^E) E_0$, which is the total induced increase in next period bank equity capital under the *balanced budget*. Suppose that the capital injection Tr_0 is financed by a tax on investor end-of-period net-wealth. As the investor's consumption-saving policies are linear homogenous in end-of-period net-wealth (see Appendix A.2), we get $\Omega_1^{bb} = \Omega_1^{lf} - \beta_H Tr_0$ and $C_1^{bb} = C_1^{lf} - \beta_H Tr_0$, respectively. Observe that total capital accumulation under the *balanced bailout* policy satisfies $E_1^{bb} + \Omega_1^{bb} = E_1^{lf} + \Omega_1^{lf} + (1-\beta_H) Tr_0 > E_1^{lf} + \Omega_1^{lf}$. Exploiting the properties of the linear homogenous production technology to eliminate

$C_0^W = (1-\alpha)(Y_0^M + Y_0^I)$ from the aggregate resource constraint yields

$$C_0^B + C_0^{I,bb} + E_1^{bb} + \Omega_1^{bb} = (1-\delta_0^E)E_0 + (1-\delta)\Omega_0 + \alpha(Y_0^M + Y_0^I).$$

Thus, the *balanced bailout* is feasible by construction because E_0, Ω_0, Y_0^M, and Y_0^I are given prior to the depreciation shock and C_t^B is constant by the definition of the dividend payout restriction. In period 1, the increase in equity to $E_1^{bb} > E_1^{lf}$ and the decrease in investor wealth $\Omega_1^{bb} < \Omega_1^{lf}$ lead to a decrease in bank leverage $\lambda_1^{bb} < \lambda_1^{lf}$ (see Section 4.1.4). However, note that

$$K_1^{I,bb} = \lambda_1^{bb} E_1^{bb} > \lambda_1^{lf} E_1^{lf} = K_1^{I,lf}.$$

The latter property follows from the following contradiction. Suppose that $\lambda_1^{bb} E_1^{bb} = K_1^{I,bb} \leq K_1^{I,lf} = \lambda_1^{lf} E_1^{lf}$. Hence, $K_1^{M,bb} \geq K_1^{M,lf}$. As a consequence, $r_1^{I,bb} \geq r_1^{I,lf}$ and $r_1^{M,bb} \leq r_1^{M,lf}$. This leads to $\lambda_1^{bb} \geq \lambda_1^{lf}$ which is a contradiction. Because total capital has increased and, in addition, has been re-allocated towards the more efficient production sector I, total output increases relative to the *laissez faire*: $Y_1^{M,bb} + Y_1^{I,bb} > Y_1^{M,lf} + Y_1^{I,lf}$. Furthermore, as bank equity has increased by Tr_0 and investor wealth has decreased by $\beta_H Tr_0$, aggregate end-of-period wealth $(1-\delta_1^B)E_1^{bb} + (1-\delta)\Omega_1^{bb}$ exceeds the *laissez faire* level if and only if $1-\delta_1^E > \beta_H(1-\delta)$, which is the qualification provided in the proposition applicable to period 1. The aggregate resource constraint for period 1

$$C_1^B + C_1^{I,bb} + E_2^{bb} + \Omega_2^{bb} = (1-\delta_1^E)E_1^{bb} + (1-\delta)\Omega_1^{bb} + \alpha(Y_1^{M,bb} + Y_1^{I,bb})$$

reveals that there are now more resources available under the *balanced budget* policy (right hand side of the above resource constraint) than under *laissez faire*. We now implement a similar investor financed transfer scheme $Tr_1 = \hat{E} - \beta_B(\theta_1\lambda_1^{bb} - \delta_1^E)E_1^{bb}$ that compensates for the difference in the banks capital accumulation decision when *balanced bailout* is abandoned in period 1 and the target level of bank equity capital $\hat{E}$. There are two remarks: First, observe that by a similar argument as before, continuing the *balanced bailout* policy in period 1 yields a higher level of aggregate resources at the end of period 2 than quitting the policy in period 1. Second, because under *balanced bailout* aggregate resources at the end of period 1 exceed their *laissez faire* value, a simple redistribution argument applies to obtain that aggregate end of period 2 resources under *balanced bailout* will exceed their respective *laissez faire* value – i.e. no policy intervention from period 0 – as well.

Applying the logic from period 2 onwards establishes the first part of the proposition, i.e. the acceleration on economic recovery in terms of total output. Noting that the bankers consumption path is kept constant by construction and the workers consumption path is shifted upwards by the recovery in output, $C_t^{W,bb} = (1-\alpha)(Y_t^{M,bb} + Y_t^{I,bb}) \geq (1-\alpha)(Y_t^{M,lf} + Y_t^{I,lf})$, we conclude that *balanced bailout* delivers the same lifetime-utility for bankers and a larger lifetime-utility for workers. Finally, as the consumption path for investors is lower at the beginning of the *balanced bailout* and higher later on due to accelerated growth, the overall effect on lifetime-utility of investors is ambiguous.

There are two final remarks. First, note that condition $1 - \delta_t^E > \beta_H(1-\delta)$ is sufficient but over-restrictive and can be relaxed for numerical simulations that allow for a straightforward *ex-post* check when comparing output paths. Second, observe that due to monotonicity in ζ, our results also carry over to *balanced bailout* with $\zeta \leq 1$.

E Appendix – Quantitative Analysis

E.1 Data Analysis and Calibration

The model is calibrated to quarterly frequency based on macroeconomic and bank level data from 1991Q1 to 2017Q4. Data on real activity are taken from the Federal Reserve Economic Data (FRED), the Penn World Table (PWT), and Fernald (2012). Data on the financial sector are compiled from the quarterly bank level Reports of Condition and Income (Call) and from De Fiore and Uhlig (2011). All data are seasonally adjusted using the Census X-13ARIMA and deflated using the GNP deflator provided by FRED.

The saving rate $\overline{s} \doteq \frac{gross\ saving}{gnp} = 0.1801$ and the capital-to-output ratio $\overline{K/Y} = \frac{total\ capital}{gnp} = 12.3763$ are computed from the FRED National Income and Product Accounts and the PWT, respectively. The labor-share of income is taken from the online data update of Fernald (2012) and amounts to $\overline{wL/Y} = 0.6516$. The financial calibration targets are computed from the balance sheet data of all commercial banks. The structure of the bank balance sheet and the construction of consistent time series from the specific Call Report series (four-letter mnemonic and four-digit series number) are depicted in Figure 6. For instance, *rcon2170* is domestic total assets and *rcfd2170* is consolidated total assets, i.e. foreign and domestic. Since 1990 banks either report the *rcon2170* series (when there are only domestic offices) or the *rcfd2170* series (when there are domestic and foreign offices), the summation of both series yields the amount of total assets. A similar logic applies to the other time series in the bank balance sheet. The calibration target for bank leverage is computed as $\overline{\lambda} = \frac{assets}{equity} = 10.7808$. The banks' total net income is taken from the income statement, Call Report series *riad4340*, such that the return on equity is $\overline{r}^B = \frac{net\ income}{equity} = 0.0276$. Finally, the average loan-to-bond finance ratio is taken from De Fiore and Uhlig (2011) and amounts to $\overline{K^I/K^M} = 0.6667$.

We next show how to sequentially pin down the time-invariant parameters (including the mean of the time-variant parameters) parameters. Because production sectors are competitive, the output elasticity of capital satisfies $\alpha = 1 - \overline{wL/Y} = 0.3484$. The steady state condition on aggregate capital and the assumption on long-run equalized capital depreciation rates yield $\delta = \overline{s}/\overline{K/Y} = 0.0146$ and $\delta^E = 0.0146$, respectively. Furthermore, the definition of the net-return on equity, $1 + \overline{r}^B = \theta\overline{\lambda} - \delta$, and the steady state condition on bank equity capital, $1 = \beta_B(1 + \overline{r}^B)$, yield $\theta = 0.0276$ and $\beta_B = 0.9731$. Assuming that in the long-run wage rates are equalized, $\hat{w}^M = \hat{w}^I$, we obtain

$$\frac{z^I}{z^M} = \left(\frac{K^I/L^I}{K^M/L^M}\right)^{-\alpha}. \tag{E1}$$

Figure 6: Bank Balance Sheet

Assets rcon2170+rcfd2170 (1991Q1 to 2017Q4)	Liabilities rcon3300+rcfd3300 (1991Q1 to 2017Q4)
Loans rcon2122 (1991Q1 to 2017Q4)	Deposits rcon2200+rcfd2200 (1991Q1 to 2017Q4)
	Residual: Liabilities − Deposits − Equity
Residual: Asset − Loans	Equity rcon3210+rcfd3210 (1991Q1 to 2017Q4)

Given the firms' first-order conditions with respect to capital, we further obtain

$$\frac{r^I}{r^M} = \left(\frac{z^I}{z^M}\right)^{1/\alpha}. \tag{E2}$$

Note that the leverage constraint (condition (3)) and the capital-to-output ratio, $\overline{K/Y} = \frac{K^I+K^M}{Y}$, can be rewritten as

$$r^M = \frac{1-\theta\overline{\lambda}}{\overline{\lambda}(1-r^I/r^M)-1} \tag{E3}$$

$$r^M = \alpha\frac{1+\overline{K^I/K^M}}{(1+r^I/r^M\overline{K^I/K^M})\overline{K/Y}}, \tag{E4}$$

where the second condition uses $Y = (r^M K^M + r^I K^I)/\alpha$. Equalizing both conditions and solving for r^I/r^M yields

$$\frac{r^I}{r^M} = \frac{\alpha(\overline{\lambda}-1)(1+\overline{K^I/K^M}) - (1-\theta\overline{\lambda})\overline{K/Y}}{\alpha\overline{\lambda}(1+\overline{K^I/K^M}) + (1-\theta\overline{\lambda})\overline{K/Y}\,\overline{K^I/K^M}} = 1.0490,$$

which then gives $z^I = (r^I/r^M)^\alpha z^M = 1.0186$, where z^M is normalized to unity. Computing r^M using condition (E3) and plugging it into the steady state condition form investor wealth, $1 = \beta_H(1+r^M-\delta)$ gives $\beta_H = 0.9871$.

Finally, condition (E1) implies that the labor allocation satisfies

$$\frac{L^I}{L^M} = \left(\frac{z^I}{z^M}\right)^{1/\alpha}\overline{K^I/K^M} = 0.6993$$

such that given L is normalized to unity. Thus, $l = L^M/L = 0.5885$.

The stochastic process for total factor productivity is taken from the online data update of Fernald (2012). For the stochastic process on the depreciation rate, we impute an artificial balance sheet consistent with our model by keeping only loans on the asset side and imposing an deposit-to-equity ratio on the liability side consistent with bank leverage. The structure of the imputed balance sheet is depicted in Figure 7 where we imposed $\frac{assets}{equity} = \frac{loans}{equity'}$.

Figure 7: Bank Balance Sheet – Imputed

Assets'	Liabilities'
Loans rcon2122 (1991Q1 to 2017Q4)	Deposits'
	Equity'

Using the bank income statements, we compute the default on loans ($riad4635$) net of the recovery of previously charged off loans (*riad4605*).[34] Keeping deposits fixed, net loan charge-off is equal to net equity charge-off which we use to compute the period t depreciation rate of bank equity as $\delta_t^E = \frac{loan\ charge-off}{equity'}$. Given the time series for λ_t, δ_t^E, and r_t^B, we finally compute the time series of the financial friction using $1 + r_t^B = \theta_t \lambda_t - \delta_t^E$. After de-trending using an HP-filter with smoothing parameter 1600 and disregarding the first four and last four quarters due to well known end point problems, we use the cyclical components of common total factor productivity, bank equity depreciation, and financial friction to estimate the SURE VAR(1) process, condition (16), using FGLS. The estimates of the stochastic process together with the other calibration targets are summarized in Table 2.

[34] Instead of all loans, we also consider an alternative bank balance sheet with commercial and industrial loans (*rcon1763* + *rcon1764* + *rcon1590*) and commercial real estate loans (*rcon1415* + *rcon1420* + *rcon1460* + *rcon1480* for 1991Q1 to 2007Q4 and *rconf158* + *rconf159* + *rcon1420* + *rcon1460* + *rconf160* + *rconf161* for 2008Q1 to 2017Q4). Using the bank income statements, we compute the default on commercial and industrial loans (*riad4645* + *riad4646*) net of the recovery of previously charged off commercial and industrial loans ($riad4608$) and the default on commercial real estate loans (riad3582 + riad3584 + riad3588 + riad3590 for 1991Q1 to 2007Q4 and riadc891 + riadc893 + riad3584 + riad3588 + riadc895 + riadc897 for 2008Q1 to 2017Q4) net of previously charged off commercial and real estate loans (riad3583 + riad3585 + riad3589 + riad3591 for 1991Q1 to 2007Q4 and riadc892 + riadc894 + riad3585 + riad3589 + riadc896 + riadc898 for 2008Q1 to 2017Q4).

E.2 Algorithm

This appendix provides a rough overview of the solution techniques and algorithms used to solve and simulate the model in the quantitative section.

First, we approximate the stochastic process by applying a multi-dimensional generalization of Tauchen's method proposed by Terry and Knotek II (2011). We choose 7 grid points for each dimension of the stochastic process and apply the algorithm to assign the exact position of the grid points and the associated joint Markov transition matrix. There are further 31 grid points for each, total capital and bank equity capital, that we allocated unevenly around the respective steady state values. Thus, the total state space is characterized by a 5 dimensional grid over K, E, A, δ^E, and θ with $31^2 \times 9^3 = 329,623$ grid points.

Second, the solution of the intraperiod equilibrium requires to solve condition (8) for each grid point. Note that this amounts to solving $329,623$ independent non-linear univariate equations. We use a bisection that operates on all conditions simultaneously. Given policy function $\lambda_t(K_t, E_t, A_t, \delta_t^E, \theta_t)$, the remaining variables of the intraperiod allocation ensue in closed form.

Third, we use a time iteration algorithm on the Euler equation with linear interpolation to solve for the interperiod equilibrium.[35] Because there is a closed form solution for bank equity accumulation $E_{t+1}(K_t, E_t, A_t, \delta_t^E, \theta_t)$, we only have to specify an initial guess for total capital accumulation $K_{t+1}(K_t, E_t, A_t, \delta_t^E, \theta_t)$ to iterate over the Euler equation.

Fourth, given the policy functions for bank leverage and the accumulation of total capital and bank equity capita, respectively, we simulate impulse response functions by forward iteration and linear interpolation.

[35] For a detailed description of the time iteration algorithm, see, e.g., Heer and Maussner (2008).