

Following Walras Down a Road Which Pareto Did Not Share

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Abstract

Walras and Pareto are considered as founders of the Ecole de Lausanne. However, their points of view on economics as a science were at considerable variance and their appreciation of each other was limited. On the path from Adam Smith's Invisible Hand to Arrow and Debreu's proof of the existence of equilibrium, Walras is thought of as having formulated the General Equilibrium Model and Pareto having improved upon and advanced theory down the path that Walras had envisaged. The purpose of this note is to suggest that this is too simple a picture and that the road to an adequate explanation of the Invisible Hand petered out and Pareto's contribution was instrumental in its demise.

Introduction

A simplistic view of the evolution of recent economic theory, and one which is often recounted to economics students is that of the emergence of liberalism and the idea that individuals left to their own devices would self organize into a satisfactory state, and that this was formalized, through the marginal revolution, improved upon by the Ecole de Lausanne and finally terminated in the Arrow Debreu general equilibrium model. Walras and Pareto are frequently described as belonging to or, indeed as founders of the Ecole de Lausanne in question. This view, conveys the idea that the "invisible hand" of Adam Smith was the starting point for this intellectual route, and that what followed provided the basis for explaining what laid behind this implicit mechanism. The quote usually cited for this is the following.

"Every individual [...] neither intends to promote the public interest, nor knows how much he is promoting it [...] he intends only his own security; and by directing that industry in such a manner as its produce may be of the greatest value, he intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention."

(Smith 1976 [1776], wn, iv.ii.9)

In reality the arguments made by Smith which might be, and have been, interpreted as in favour of laissez faire, stem from a longer tradition going back, at least as far as Hobbes and Locke. Nevertheless, the idea that an economic system would self organize satisfactorily which was promoted by Mill and others, persisted and evolved into a discussion as to the mechanism which would lead to such a state. During this time it became clear that, if markets were to be important, the state in question should be, in some sense, an equilibrium. While this notion was easy to capture in the case of the market for a single good, it was far less evident that a state in which all markets cleared simultaneously existed and even more difficult that such a state would be attained.

The suggestion is usually that Walras was interested in and developed a theory of such an equilibrium, recognized the importance of the interdependence of markets and wished to avoid what was later termed, “partial equilibrium”. He insisted on the idea that markets were interlinked and that what happened on the market for one good had an impact on the other markets. He thus laid the foundations for General Equilibrium theory and Pareto improved and expanded on these. Pareto is therefore considered as having made progress on the path that Walras followed and that culminated in the work of Arrow and Debreu. There is also the frequently repeated assertion that Walras and Pareto collaborated and that each had great respect for the other. My purpose in this short note is to examine the evolution of the thought on this subject and to suggest that actually Walras and Pareto had little in common, that their relationship was far from simple and that the intellectual road which Walras thought economics should and would take, did not correspond to Pareto’s vision of how the economics discipline would evolve.

First, then, I will consider the path that Walras set us on and then explain why this does not coincide with Pareto’s views. One cannot deny that the direction economics, and particularly theoretical economics, took in the 20th century was to a great extent due to Walras’ influence. This was not so much due to his own results but rather a reflection of his vision. He was convinced that economics should have “sound mathematical foundations” and his concern for this is reflected in his correspondence with his contemporaries such as Poincaré. Indeed, his considerable correspondence with Poincaré and some of the leading mathematicians demonstrated his anxiety to show that economics was on the road to becoming a science. He firmly believed that there were fundamental laws which governed the system and which had the same status as the laws of physics. For example he said,

“The law of supply and demand steers the exchange of goods in the same way that the universal law of gravity steers all the movements of the planets. Here already economic law appears in its universal validity and its complexity and nothing can appear so beautiful, in other words great and at the same time so simple as the system of the astronomic world.”

Léon Walras, (1874/1954), Leçon 34

Again, he insisted,

“It took from a hundred to a hundred and fifty or two hundred years for the astronomy of Kepler to become the astronomy of [Newton](#) and [\[\[Laplace\]](#), and for the mechanics of [Galileo](#) to become the mechanics of [d’Alembert](#) and [Lagrange](#). On the other hand, less than a century has elapsed between the publication of [Adam Smith](#)’s work and the contributions of [Cournot](#), [Gossen](#), [Jevons](#), and myself”. Cited in *John Cunningham Wood (1993), p. 32-33.*

He actually went on to say that in the 20th century “justice would be done” and that his achievements along with those of Cournot, Gossen and Jevons would be recognized as invaluable contributions to the advancement of economic science.

Justice was already done, in a certain sense, when Schumpeter said,

« So far as pure theory is concerned, Walras is in my opinion the greatest of all economists. This system of economic equilibrium, uniting, as it does, the quality of « revolutionary » creativeness with the quality of classic synthesis, is the only work by an economist that will stand comparison with the achievements of theoretical physics. Compared with it, most of the theoretical writings of that period – and beyond – however valuable in themselves and however original subjectively, look like boats beside a liner, like inadequate attempts to catch some particular aspect of Walrasian truth. It is the outstanding landmark on the road that economics travels towards the status of a rigorous or exact science and, though outmoded by now, still stands at the back of much of the best theoretical work of our time »
Schumpeter (1954), p.827

Yet this is strange, for someone whose vision of “creative destruction” and a world in constant upheaval, which is so far from the Walrasian concept of equilibrium. Perhaps the key lies in the fact that Schumpeter explicitly suggest that Walras was the giant of “pure economic theory” and therefore not to be hobbled by problems of empirical validation.

Walrasian General Equilibrium

What cannot be contested is that Walras’ specific vision of the nature of equilibrium became a benchmark for modern economic theory. What are the ingredients that characterise the « Walrasian » General Equilibrium model and what was Walras’ own position with respect to them ? Backhouse and Maks (2006) suggest the following caricature of what they describe as « the common university trained scholar’s » understanding of the Walrasian model, « Walras developed the general economic equilibrium model, but did not care about uniqueness and stability of an equilibrium. It is a model with exchange and production only and it assumes an auctioneer who announces price vectors to establish the equilibrium. » Backhouse and Maks (2006) p.1 To this highly restricted vision should be added the underlying characteristics of the model, perfect competition, the nature of equilibrium as a market clearing set of prices, and some notion of perfect information. In addition, one cannot actually leave to one side the problem of stability or how one actually gets to equilibrium and Walras devoted a great deal of time to the discussion of this issue and the nature of Walras’ ideas on the subject has been the subject of an intense debate, (see Walker (1996, 1999) and De Vroey (1999) for example).

This is important since what is at stake is whether Walras had a vision of the economy as a genuinely dynamic process or one which somehow, almost by assumption, arrived at, what he had in mind as, a static equilibrium. The vision of equilibrium as a state rather than as a manifestation of a dynamic process is certainly that which is characteristic of the Arrow-Debreu model. It is also the one which despite, the misleading name used for modern macroeconomic models, Dynamic Stochastic General Equilibrium, underlies them.

Indeed Arrow and Debreu, themselves, said bluntly

“Walras first formulated the state of the economic system at any point of time as the solution of a system of simultaneous equations representing the demand for goods by consumers, the supply of goods by producers and the equilibrium condition that supply equal demand on every market.”

Arrow, Kenneth J., and Gerard Debreu. (1954) p. 265.

Firstly, it should be observed that the preceding description is a very limited account of what constitutes the Walrasian General Equilibrium approach. Furthermore, to suggest that the Walrasian analysis, led us directly to the Arrow-Debreu model would miss an important step, that of the transformation of the underlying problem from one in which marginal utilities were paramount into one based on individuals with preference orders but not cardinal utility functions. That step is due to Pareto.

However, the Arrow-Debreu framework is characterised by its lack of institutional features and that would not be inconsistent with the abstract view that Walras had of an “ideal” economy. But recall that, at the outset, I spelled out the path which many have argued has been taken from the “invisible hand” of Adam Smith to the Arrow-Debreu formulation. If this is what is at stake then there should be a consistent concern along that path with the stability problem, which is that of showing how the “invisible hand” does its work.

Yet the fundamental problem with the Walrasian framework, as it evolved towards Arrow-Debreu, is the impossibility of showing the stability of an economy under the sort of adjustment process that Walras had in mind, as was later shown by Sonnenschein (1972), Mantel (1974) and Debreu (1974). This contradicts Walras’ assertion that he had “proved” such stability or convergence. This, of course, had it been true might have provided a justification for the elusive “invisible hand”. But above all there is no place in this framework for the out of equilibrium dynamics which would need to be specified if one were to argue that the economy “tends towards equilibrium”. Thus, whilst Walras is to be lauded for his insistence on the interdependence of markets, we should also then be aware that he set us on a path towards economic models which, while admirably internally consistent, seem to be devoid of empirical content.

The separation of the paths of Walras and Pareto

But, I argued at the outset, as have Bridel and Mornati (2009) that it is difficult to picture Walras and Pareto as belonging to the same school since they differed in so many ways. But if this is so, where do the main differences between Walras and Pareto in economic theory lie? Pareto was clearly influenced by Edgeworth and devoted some time early in his work to reconciling Edgeworth’s individualist or, as it is sometimes called, “hedonist” approach with the general equilibrium vision proposed by Walras. However, Pareto made, in so doing, an important step which was to consider ordinal preferences rather than to focus on a cardinal approach. He is therefore thought of as the precursor of modern microeconomic theory in which individuals are characterised by preference orders rather than cardinal utility functions where marginal utility can, in principle be measured. He then developed what we now call the Pareto optimum, a situation in which no individual can be made better off without some other individual being made worse off. The basic idea was already in Edgeworth’s concept of the « contract curve ». This criterion, as is well known, is very weak.

Paradoxically, once one accepts the idea that interpersonal comparisons of utility are impossible, then one has to accept a classification in which the Pareto efficient states constitute an equivalence class. However, the class of Pareto efficient or optimal states is large and no distinction can be made between two of these states, on the basis of distributional considerations for example. This is ironic when one notes that Pareto devoted a considerable

amount of time and energy to developing a law which he hoped covered a large class of income distributions. So the other contribution for which Pareto is best known, his Pareto Law aimed at making just the comparisons that his approach to General Equilibrium ruled out.

He did, however, indicate that Walrasian equilibria were Pareto efficient but stated that they were just special cases. This statement constitutes what is now called « The first fundamental theorem of welfare economics ». In the *Cours d'économie politique (1896)*, Pareto worked within a Walrasian- Edgeworthien framework but in the *Manuel d'économie politique (1909)* Pareto moved to an analysis which could be considered as within the modern framework of rational choice theory. In particular, Pareto insisted on the fact that it was from the interaction between rational individuals that a social state would arise and amongst these were the Walrasian equilibria. This approach, according to Bridel and Mornati (2009), was consistent with Pareto's overall view which could be described as « essentiellement empiriste, déductive-concrète et « vérificationniste » ». Indeed, Pareto abandoned the Walrasian theory based on a non-observable or measurable marginal utility and decided to develop an axiomatic theory of rational choice as his basis. He was then able to move on to consider the collective result of individual taking rational actions. This can clearly be seen in his *Manuel d'économie politique (1909)*. He then went on to work on sociology and wrote the *Traité de sociologie générale (1917-19)*.

But, it is important to observe that when dealing with economics, Pareto claimed that this « satisfactory » collective outcome would only correspond to that of individuals taking rational rather than « non-rational » actions. He thought that the treatment of the latter case was the basic problem of sociology. So the passage from economics to sociology was, for him, characterized by the abandoning of rationality in the sense which he had himself, developed. Indeed, at one point in his *Traité*, he says that rationality is just the varnish that individuals put, ex post, on their non rational actions to justify them. In saying this he is clearly abandoning the path followed by Walras.

But this separation of the paths followed by Walras and Pareto, is just the reflection of the rather hostile relation between the two. Walras saw himself as developing a logically consistent theory. It is again Bridel a successor to Walras and Pareto at the University of Lausanne, together with Mornati, (Bridel, P. & Mornati, F. (2009). who indicate why Walras' view of economics was so different from Pareto's approach.

Walras said, in particular.

“M. Pareto believes that the goal of science is to get closer and closer to reality by successive approximations. But, I believe that the goal of science is to develop a certain ideal model and then to relate reality to that ideal, and that is why I have specified such an ideal”. (Léon Walras, 2000, p. 567.

A Shift in the aims of Economic Theory

But having argued that Pareto's contribution showing that ordinal utility could be used as a basis for General Equilibrium analysis, one has to accept that gradually the path moved from trying to develop a mechanism corresponding to the "Invisible Hand" to one in which the preoccupation became that of showing that there was, in fact, a solution to the simultaneous equation problem. The existence problem, first solved by Wald, (1932) became the central problem for General Equilibrium theory and the idea that the "stability" question as to whether an economy would actually be led to an equilibrium was put on one side. While Debreu, pronounced himself to be a disciple of Walras, he, nevertheless, clearly separated the problem of the welfare properties of the equilibrium of a perfectly competitive economy. from that of how such an equilibrium might be attained. Walras himself made no such distinction. He said for example,

"I take the almost universal regime of free competition in regard to exchange, that which was described by John Stuart Mill, and which consists in raising the price in the case of the quantity demanded exceeding the quantity supplied and lowering it in the case of the quantity supplied exceeding the quantity demanded, and I demonstrate that the process leads to equilibrium by establishing the equality of the quantities supplied and demanded. Whereupon there is thrown at my head the market for English public debt, the system of English auctions, the system of Dutch auctions etc., etc."

(Walras, Letter no. 999 to Von Bortkiewicz, in Jaffé. 1965, 434-43)

Here Walras seems to equate the very notion of free or perfect competition with the adjustment process that would lead an economy to an equilibrium and not just the state of the economy that would constitute an equilibrium.. But, as I have said, this was not what was taken up by those who followed in his footsteps. Thus, the end of the path that they followed (the Arrow Debreu model), was characterized by a view which was a far cry from Walras' original idea. The notion, perfected by Arrow and Debreu, of perfect competition was defined as a situation in which all the actors in the economy are price takers and consider that they have no impact on the announced and commonly known price vector for all goods. It was the development of that view, which could only be justified as a limit case as Aumann (1964) was to explain, but which defied common sense economic logic and aroused the anger of a number of economists and of Hayek in particular.

Hayek made a vigorous attack on this concept of perfect competition and in so doing echoed the Edgeworth's early criticisms of Walras. He attacked the type of adjustment that Walras suggested defined perfect competition and the General Equilibrium model in which it was enshrined, in his 1948 paper *The Meaning of Competition*. He argued that

“the modern theory of competition deals almost exclusively with a state [...] in which it is assumed that the data for the different individuals are fully adjusted to each other, while the problem which requires explanation is the nature of the process by which the data are thus adjusted.”

(Hayek 1948, 94)

That is, the modern theory of competitive equilibrium assumes the situation to exist which a true explanation ought to account for as the result of the competitive process. In other words, we have reached a situation in which the definition of perfectly competitive equilibrium is diametrically opposed to the view that Walras expressed originally and which I cited previously. Walras still had in mind that equilibrium as a result of a process. However, the process he developed was clearly a centralized one and one which had nothing to do with competition as commonly defined. Hayek, thought, paradoxically, in complete accord with Walras, that competition was a process which should lead to equilibrium but in contradiction to Walras he thought that this process had to be completely decentralized if it was to involve competition in a meaningful sense. Indeed his argument uses the definition of Samuel Johnson (1755), who described competition as “the action of endeavouring to gain what another endeavours to gain at the same time”. Hayek expressed this clearly when he said:

“Now, how many of the devices adopted in ordinary life to that end would still be open to a seller in a market in which so-called “perfect competition” prevails? I believe that the answer is exactly none. Advertising, undercutting, and improving (“differentiation”) the goods or services produced are all excluded by definition –“perfect” competition means indeed the absence of all competitive activities.”

(Hayek 1948, 96)

It is difficult to deny that the notion of perfect competition in modern equilibrium models is thought of as providing an analytical framework within which to draw conclusions as to the welfare properties of equilibria rather than as an idealised description of any real economy or market.

There have, of course, been efforts to reconcile a more realistic idea of competition with the equilibrium notion and thus to provide an explanation as to how an economy might achieve an equilibrium state and a particularly interesting effort in this direction is provided by Makowski and Ostroy (2001).

But now the question arises as to whether the followers abandoned Walras by taking a path that he had not envisaged and focusing on the existence problem rather than explaining the invisible hand mechanism. The evidence seems to be against this and favours the idea that Walras himself had changed his goal. Whilst he struggled for a long time to show that markets would converge to equilibrium through the tatonnement process which could be thought of a reasonable idealization of what happens on markets, he finally abandoned that idea and admitted that his process should just be viewed as an algorithm to solve the demand and supply equations. The idea that his tatonnement process does this was, of course, shown to be false later, but, since proving that equilibrium even exists required mathematical tools which were not at Walras’ disposal, he can hardly be blamed for that.

The persistence of the Invisible Hand,

Having explained that the development of General Equilibrium Theory finally led to an extensive analysis of equilibrium states and not of how they were achieved, one might be tempted to think that the Invisible Hand question was left, at best, unsolved. But, there are two observations to be made here. Firstly, not all economic theorists were convinced that this was the path to take. Consider Hicks' observation,

“ To some people, (including no doubt Walras himself) the system of simultaneous equations determining a whole price-system seems to have vast significance. They derive intense satisfaction from the contemplation of such a system of subtly interrelated prices; and the further the analysis can be carried (in fact it can be carried a good way)...the better they are pleased, and the profounder the insight into the working of a competitive economic system they feel they get.
(Hicks 1939:p 60)

Secondly as Mark Blaug has pointed out, the idea that we failed to find a satisfactory explanation or proof of an invisible hand mechanism, is far from what, even the most distinguished theorists state and tell their students. Just to take a first example, Rodrik an extremely accomplished theorist says,

“The First Fundamental Theorem is a big deal because it actually proves the Invisible Hand Hypothesis”

Rodrik (2015), p.50.

Yet this is what it does not do. The Invisible Hand Hypothesis is about the process by which individual self interested acts are translated into a socially satisfactory outcome, and on this as we have seen, the First Fundamental Theorem of Welfare Economics is silent.

Blaug (2007) provides a remarkable number of citations of leading economists who have made similar claims, and in particular he mentions Ross Starr (1997, p. 209) who states firmly that .the First Fundamental Theorem of Welfare Economics, is a mathematical statement of Adam Smith's notion of the invisible hand leading to an efficient allocation. He also cites Mas-Colell, Whinston and Green (1995), who repeatedly claim that the First Fundamental Theorem is a formalization of Smith's claims about the Invisible Hand of the market, Arrow and Hahn (1971), Hahn (1982), and many others.

Thus, there is a persistent myth that the Invisible Hand problem has been solved within General Equilibrium Theory whereas the path towards that solution has petered out because, precisely, of results from within that theory. So, in some sense, the trail dies out in 1906 with Pareto's proof of the First Fundamental Theorem of Welfare Economics and as Frank Hahn said, when talking of the attempts to establish stability results and referring to the SMD result, .

“The enterprise was doomed not to be capable of reaching general conclusions in the Walrasian setting. A theorem not directly related to connected with dynamics did the damage.”

Frank Hahn (2002), p.224

Conclusion

The path from Adam Smith to Arrow and Debreu, so often described as a natural and almost inevitable path for economic theory was far from a smooth and continuous process. Its passage by the Ecole de Lausanne, was not a simple one. Walras and Pareto had different ideas as to what economic theory should be, one believing that it was possible to develop a science in the same sense as such sciences as physics, thus one had to construct an ideal economy and then to the other arguing that the only reasonable approach was to modify theory in the light of observations and to progressively approximate reality. Pareto joined Walras at one point using similar mathematical techniques to make progress in building an economic framework based on individual rationality. However, it was he who changed the course of economic theory by introducing ordinal preferences as the basis and putting marginal utility to one side. By proving the First Fundamental Theorem of Welfare Economics, he was changing the orientation of economic theory to one in which economists focused more on the characteristics of the states of an economy rather than worrying about how those states were attained. Pareto himself moved on to sociology, though, in many ways his influence was more profound than that of Walras. As Frank Hahn indicated the Walrasian enterprise was doomed, and Pareto helped to bury it.

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